Common Issues and Practical Considerations

- We must make sure that the data approximates the SSM, i.e., sufficient truncation has been applied. This can be decided based on the spectrogram. See the `showSpectrogram()` and the `SSM_starttime()` functions.

- When using delay embedding, the number of time delays and the delay time might need to be adjusted. Increasing the number of time delays practically acts as a filter and may lead to more accurate models. These parameters should be chosen to minimize the reconstruction error, or based on the explicit formulas for the delay-embedded tangent space, as in Axås, Haller *Nonlinear Dyn* **111** 22079 (2023).

- If the reduced dynamics exhibits finite-time blowup, the polynomial order might need to be decreased.

- Extrapolating the polynomial vector field of the reduced dynamics to unseen regions of the phase space might produce badly behaved solutions. In these cases, alternative representations such as k-nearest neighbors (kNN) or radial basis functions (RBFs) might improve the extrapolation properties, as in Liu et al. *Chaos* **34** 033140 (2024) and Xu et al. *J. Fluid Mech.* **987** R7 (2024).

- If the observables are directly the phase space coordinates, the leading principal components obtained by SVD might not align with the tangent space of the slow SSM. In these cases, we need to use the subsequent principal components that do align with the slow directions. Alternatively, switching to delay-embedded scalar observables also fixes this issue, as delay embedded slow SSMs tend to be flat. See Cenedese et al. *Nat. Commun.* **13** 872 (2022).

- If the optimization finding the normal form reduced dynamics fails to converge, changing the initial guess might be required. This can be achieved by changing the `'IC_nf'` argument of `IMDynamicsFlow()` from the default value of `'0'` to `'1'` or `'2'`. These set the initial guess based on the modal coefficients or randomly, respectively.