

EDDA - Assignment 2 - Group 77

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Exercise 1

If left alone bread will become moldy, rot or decay otherwise. To investigate the influence of temperature and humidity on this process, the time to decay was measured for 18 slices of white bread, which were placed in 3 different environments and humidified or not. The data are given in the file bread.txt, with the first column time to decay in hours, the second column the environment (cold, warm or intermediate temperature) and the third column the humidity.

a) The 18 slices came from a single loaf, but were randomized to the 6 combinations of conditions. Present an R-code for this randomization process.

```
data_bread <- read.table(file="data/bread.txt",header=TRUE)

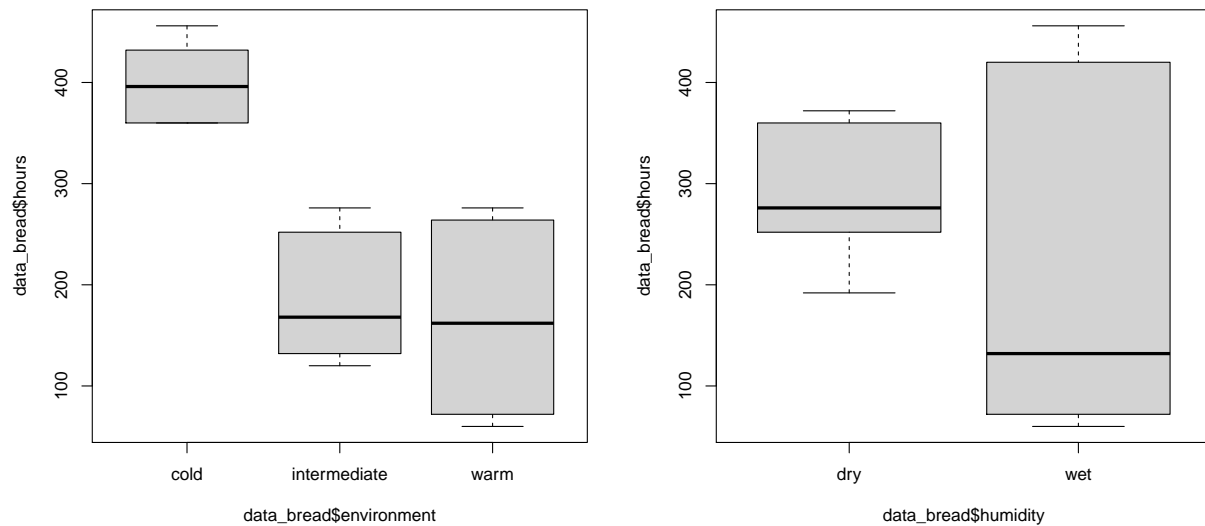
humid <- factor(rep(c("dry","wet"),each = 9))
temp <- factor(rep(c("cold", "intermediate","warm"),times = 6))

data.frame(humid,temp,slices = sample(1:18))
```

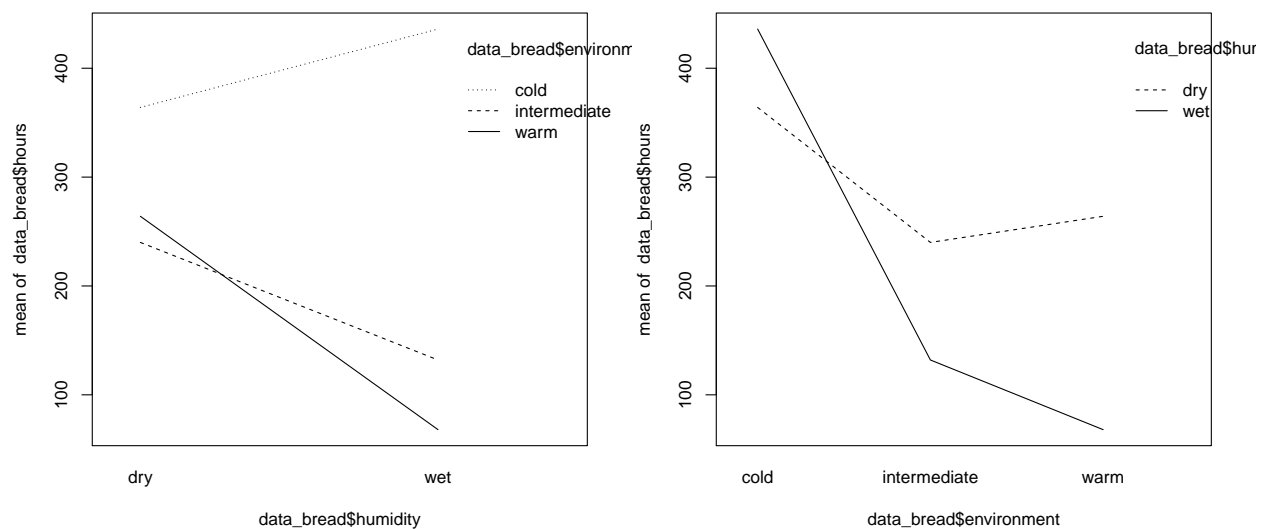
```
##      humid      temp slices
## 1    dry      cold     12
## 2    dry intermediate     3
## 3    dry      warm      2
## 4    dry      cold     10
## 5    dry intermediate     17
## 6    dry      warm     11
## 7    dry      cold      1
## 8    dry intermediate     13
## 9    dry      warm      8
## 10   wet      cold      6
## 11   wet intermediate     4
## 12   wet      warm     15
## 13   wet      cold      7
## 14   wet intermediate     14
## 15   wet      warm     16
## 16   wet      cold      9
## 17   wet intermediate     5
## 18   wet      warm     18
```

b) Make two boxplots of hours versus the two factors and two interaction plots (keeping the two factors fixed in turn).

```
par(mfrow=c(1,2))
boxplot(data_bread$hours~data_bread$environment)
boxplot(data_bread$hours~data_bread$humidity)
```



```
interaction.plot(data_bread$humidity,data_bread$environment,data_bread$hours)
interaction.plot(data_bread$environment,data_bread$humidity,data_bread$hours)
```



c) Perform an analysis of variance to test for effect of the factors temperature, humidity, and the interaction. Describe the interaction effect in words.

```
attach(data_bread)
environment=as.factor(environment)
humidity=as.factor(humidity)
dataaov=lm(hours~humidity*environment,data=data_bread)
anova(dataaov)

## Analysis of Variance Table
##
## Response: hours
##
##           Df Sum Sq Mean Sq F value    Pr(>F)
## humidity      1  26912   26912    62.3 4.3e-06 ***
## environment    2 201904  100952   233.7 2.5e-10 ***
## humidity:environment  2  55984   27992    64.8 3.7e-07 ***
## Residuals     12   5184     432
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(dataaov)
```

```
##
## Call:
## lm(formula = hours ~ humidity * environment, data = data_bread)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
##     -48      -7         0        11        36
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)         364         12   30.33 1.0e-12 ***
## humiditywet          72         17    4.24  0.0011 **
## environmentintermediate -124         17   -7.31 9.4e-06 ***
## environmentwarm       -100         17   -5.89 7.3e-05 ***
## humiditywet:environmentintermediate -180         24   -7.50 7.2e-06 ***
## humiditywet:environmentwarm       -268         24  -11.17 1.1e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 20.8 on 12 degrees of freedom
## Multiple R-squared:  0.982, Adjusted R-squared:  0.975
## F-statistic: 132 on 5 and 12 DF, p-value: 4.68e-10
```

When looking at the two-way anova model we see that it consists of the following terms: $Y_{ijk} = \mu_{ij} + e_{ijk}$
 $= \mu + \alpha_i + \beta_j + \gamma_{ij} + e_{ijk}$ We decompose the formula in this way such that μ is the overall mean,
 α_i and β_j are the main effect of level i and j of the first factor and second factor respectively and γ_{ij} the
interaction effect.

In order to test the effect of the temperature, humidity, and the interaction we set up 3 hypotheses which
are: H_{AB} : $\gamma_{ij} = 0$ for every (i, j) (no interactions between factor A and B)

H_A : $\alpha_i = 0$ for every i (no main effect of factor A)

H_B : $\beta_j = 0$ for every j (no main effect of factor B)

We use the test statistics F_{AB} for H_{AB} , F_A for H_A and F_B for H_B where F is the F-distribution.

To see if the Hypotheses can be rejected we want to look at the probability that $P(F > f_{AB})$, $P(F > f_A)$ and $P(F > f_B)$, the bigger the F value the lower the probability that the Hypothesis lays under a F-distribution and therefore the Hypothesis can be rejected.

We see that the humidity has a p-value of 4.3e-06, environment a p-value of 2.5e-10 and the interaction between the two (humidity:environment) shows a p-value of 3.7e-07. This means that humidity, environment and the interaction effect between humidity and environment have a significant influence on the hours, which means we can reject H_A , H_B and H_{AB} .

The interaction effect looks at the difference of differences, for example: it looks at the difference in hours for environment = cold and environment = warm for humidity = wet. Then it looks the difference between environment = cold and environment = warm for humidity = dry. It then looks at the difference between those differences and when this difference is high it shows that there is indeed interaction.

d) Which of the two factors has the greatest (numerical) influence on the decay? Is this a good question?

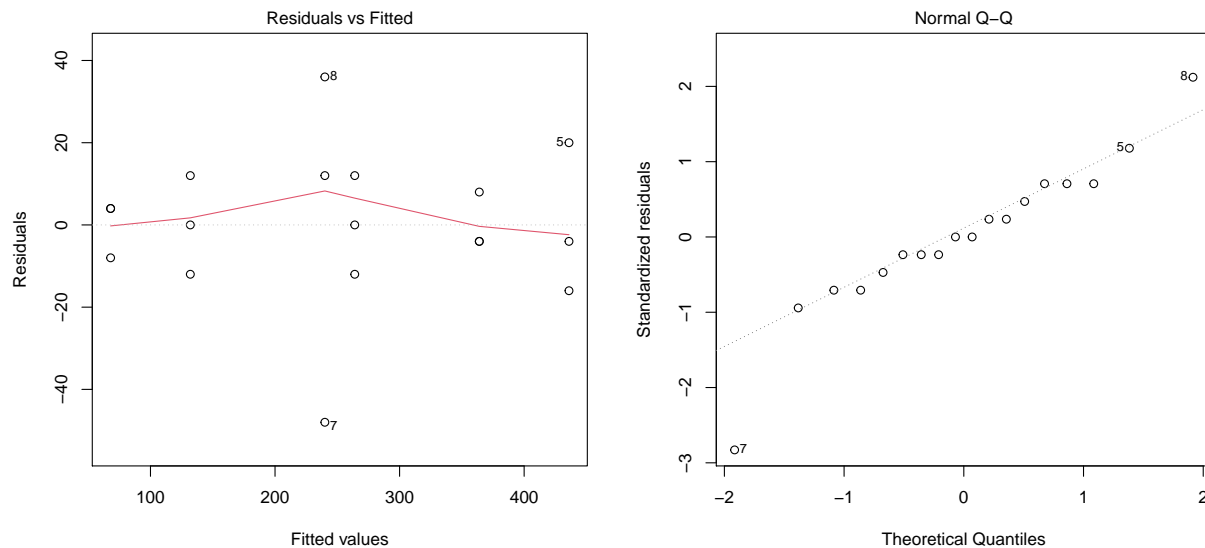
```
# Without interaction
humidity=as.factor(humidity)
environment=as.factor(environment)
dataaov=lm(hours~humidity+environment)
anova(dataaov)

## Analysis of Variance Table
##
## Response: hours
##          Df Sum Sq Mean Sq F value    Pr(>F)
## humidity    1  26912   26912     6.16   0.026 *
## environment  2 201904  100952    23.11 3.7e-05 ***
## Residuals   14   61168    4369
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

When we want to know which factor has the greatest influence we want to use the additive model as used above. This shows a p-value of 0.026 for humidity and a p-value of 3.7e-05 for environment. This means that the environment has the greatest influence.

e) Check the model assumptions by using relevant diagnostic tools. Are there any outliers?

```
par(mfrow=c(1,2))
dataaov2=lm(hours~humidity*environment,data=data_bread);
plot(dataaov2, 1)
plot(dataaov2, 2)
```



The qqplot shows a somewhat linear line which means that based on the qqplot we can state that the data is normally distributed. We also looked at the spread of the residuals, which showed that there are three outliers which are number 5, 7 and 8 which can be observed in both plot.

Exercise 2

A researcher is interested in the time it takes a student to find a certain product on the internet using a search engine. There are three different types of interfaces with the search engine and especially the effect of these interfaces is of importance. There are five different types of students, indicating their level of computer skill (the lower the value of this indicator, the better the computer skill of the corresponding student). Fifteen students are selected; three from each group with a certain level of computer skill. The data is given in the file search.txt. Assume that the experiment was run according to a randomized block design which you make in a). (Beware that the levels of the factors are coded by numbers.)

a) Number the selected students 1 to 15 and show how (by using R) the students could be randomized to the interfaces in a randomized block design.

```
interface <- factor(rep(c(1,2,3),each = 5))
skill <- factor(rep(c(1,2,3,4,5),times = 3))
students <- c(1:15)
block <- data.frame(students,skill,interface); block
```

```
##      students skill interface
## 1           1     1         1
## 2           2     2         1
## 3           3     3         1
## 4           4     4         1
## 5           5     5         1
## 6           6     1         2
## 7           7     2         2
## 8           8     3         2
## 9           9     4         2
## 10          10     5         2
```

```
## 11      11      1      3
## 12      12      2      3
## 13      13      3      3
## 14      14      4      3
## 15      15      5      3
```

b) Test the null hypothesis that the search time is the same for all interfaces. What type of interface does require the longest search time? For which combination of skill level and type of interface is the search time the shortest? Estimate the time it takes a typical user of skill level 3 to find the product on the website if the website uses interface 3.

```
data_search <- read.table(file="data/search.txt",header=TRUE)
data_search$skill <- as.factor(data_search$skill)
data_search$interface <- as.factor(data_search$interface)

aovsearch = lm(time~interface+skill, data= data_search)

anova(aovsearch)
```

```
## Analysis of Variance Table
##
## Response: time
##          Df Sum Sq Mean Sq F value Pr(>F)
## interface  2   50.5   25.23    7.82  0.013 *
## skill      4   80.1   20.01    6.21  0.014 *
## Residuals  8   25.8    3.23
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

```
summary(aovsearch)
```

```
##
## Call:
## lm(formula = time ~ interface + skill, data = data_search)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2.573 -0.697  0.387  1.057  1.787
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)    15.01      1.23    12.24 1.8e-06 ***
## interface2      2.70      1.14     2.38  0.0447 *
## interface3      4.46      1.14     3.93  0.0044 **
## skill12         1.30      1.47     0.89  0.4012
## skill13         3.03      1.47     2.07  0.0724 .
## skill14         5.30      1.47     3.61  0.0068 **
## skill15         6.10      1.47     4.16  0.0032 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1.8 on 8 degrees of freedom
## Multiple R-squared:  0.835, Adjusted R-squared:  0.711
## F-statistic: 6.74 on 6 and 8 DF, p-value: 0.0084
```

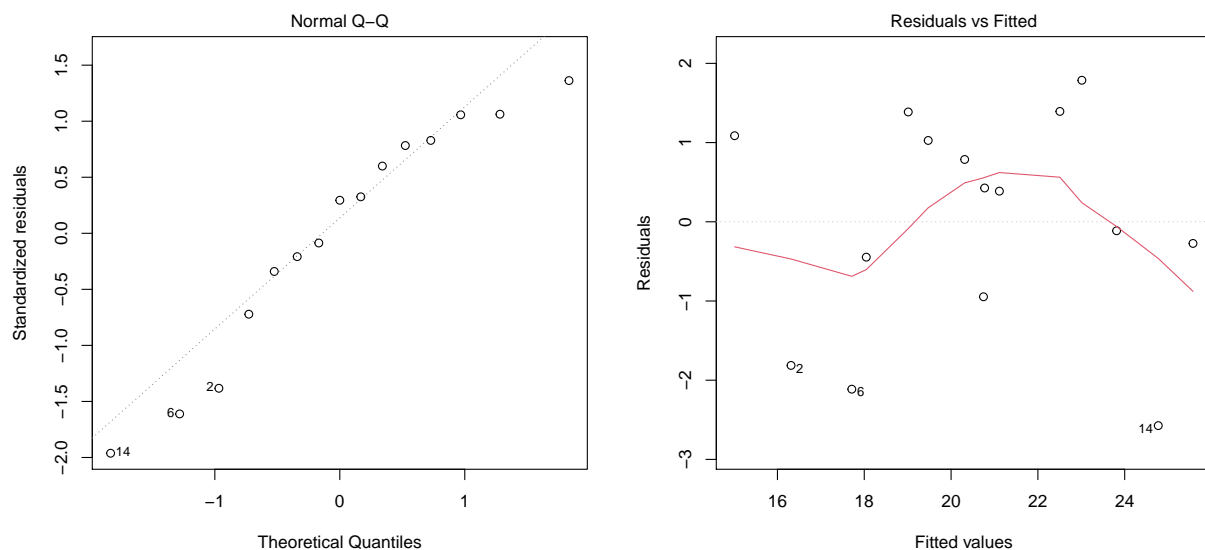
```
# Estimate interface 3 and skill 3:
Y = 15.01+4.46+3.03+1.8
Y
```

```
## [1] 24.3
```

Looking at the additive ANOVA test we can conclude that there is a significant main effect of the interface. Furthermore, the summary shows that interface three gives the highest alpha parameter value, making the time it takes for this interface the longest. For the shortest search time, interface 1 can be combined with skill levels 1,2 or 3 since all three have the lowest alpha parameter values without being significant. For the estimation of time it takes a typical user of skill level 3 using interface 3 we can calculate \hat{Y} by summing the estimates and adding the error, giving a time of 24.3 units.

c) Check the model assumptions by using relevant diagnostic tools.

```
par(mfrow=c(1,2))
plot(aovsearch,2)
plot(aovsearch,1)
```



As shown in the above QQ-plot and the residuals-fitted plot there are some outliers that raises some doubt about the normality of the data.

d) Perform the Friedman test to test whether there is an effect of interface.

```
friedman.test(data_search$time, data_search$interface, data_search$skill)
```

```
##
## Friedman rank sum test
##
## data: data_search$time, data_search$interface and data_search$skill
## Friedman chi-squared = 6, df = 2, p-value = 0.04
```

P-value is significant thus H_0 is not rejected and therefore there is a significant effect of the interface.

e) Test the null hypothesis that the search time is the same for all interfaces by a one-way ANOVA test, ignoring the variable skill. Is it right/wrong or useful/not useful to perform this test on this dataset?

```
aovsearch = lm(data_search$time~data_search$interface)
anova(aovsearch)

## Analysis of Variance Table
##
## Response: data_search$time
##              Df Sum Sq Mean Sq F value Pr(>F)
## data_search$interface  2    50.5    25.23    2.86  0.096 .
## Residuals              12   105.9     8.82
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

is it not useless also to ignore skill since the time is clearly also depended on this variable, you can not simply ignore such a variable right?

Looking at the p-value of the one-way ANOVA test, we see that is not significant. We could therefore conclude that the interfaces does not have a significant effect on the search time. However, since the data originates from a random block design, it is not correct to use this test since it leaves out important interactions.

Exercise 3

In a study on the effect of feedingstuffs on lactation a sample of nine cows were fed with two types of food, and their milk production was measured. All cows were fed both types of food, during two periods, with a neutral period in-between to try and wash out carry-over effects. The order of the types of food was randomized over the cows. The observed data can be found in the file cow.txt, where A and B refer to the types of feedingstuffs.

a) Test whether the type of feedingstuffs influences milk production using an ordinary “fixed effects” model, fitted with lm. Estimate the difference in milk production.

```
# read data
data <- read.table(file="data/cow.txt",header=TRUE)
data$treatment <- as.factor(data$treatment); data$order <- as.factor(data$order)
data$id <- as.factor(data$id); data$per <- as.factor(data$per)

# perform fixed effects model analysis
fixed_aov <- lm(milk ~ id + per + treatment, data = data)
anova(fixed_aov)

## Analysis of Variance Table
##
## Response: milk
##              Df Sum Sq Mean Sq F value  Pr(>F)
## id              8   2467    308.4  124.48 7.5e-07 ***
## per             1     25     24.5    9.89  0.016 *
## treatment       1      1      1.2    0.47  0.517
## Residuals       7      17      2.5
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```


From the results of fixed effects model above we see that the p-value for treatment is > 0.05 , therefore we can not conclude that there is a significant effect of the treatment.

b) Repeat a) and b) by performing a mixed effects analysis, modelling the cow effect as a random effect (use the function `lmer`). Compare your results to the results found by using a mixed effects model.

```
attach(data)
mixed_avo <- lmer(milk ~ treatment + order + per + (1|id),REML=FALSE)
mixed_avo_1 <- lmer(milk ~ order + per + (1|id),REML=FALSE)
anova(mixed_avo_1, mixed_avo)
```

```
## Data: NULL
## Models:
## mixed_avo_1: milk ~ order + per + (1 | id)
## mixed_avo: milk ~ treatment + order + per + (1 | id)
##          npar AIC BIC logLik deviance Chisq Df Pr(>Chisq)
## mixed_avo_1    5 118 122  -53.9      108
## mixed_avo      6 119 125  -53.7      107  0.58  1      0.45
```

The code above performed an ANOVA test between the random effect model with and without treatment in it. From the p-value of >0.05 we can not say that there is significant difference between the two models, therefore there is no significant effect of the treatment.

c Study the commands:

```
t.test(milk[treatment=="A"],milk[treatment=="B"], paired=TRUE)

##
## Paired t-test
##
## data:  milk[treatment == "A"] and milk[treatment == "B"]
## t = 0.2, df = 8, p-value = 0.8
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -2.27  2.76
## sample estimates:
## mean of the differences
##                0.244
```

Exercise 4

Stochastic models for word counts are used in quantitative studies on literary styles. Statistical analysis of the counts can, for example, be used to solve controversies about true author ships. Another example is the analysis of word frequencies in relation to Jane Austen's novel Sanditon. At the time Austen died, this novel was only partly completed. Austen, however, had made a summary for the remaining part. An admirer of Austen's work finished the novel, imitating Austen's style as much as possible. The file `austen.txt` contains counts of different words in some of Austen's novels: chapters 1 and 3 of *Sense and Sensibility* (stored in the `Sense` column), chapters 1, 2 and 3 of *Emma* (column `Emma`), chapters 1 and 6 of *Sanditon* (both written by Austen herself, column `Sand1`) and chapters 12 and 24 of *Sanditon* (both written by the admirer, `Sand2`)

a) Discuss whether a contingency table test for independence or for homogeneity is most appropriate here.

The contingency table test for homogeneity is appropriate because we want to know if the fan writer imitates Austen in a good way. This means that we want to test whether or not the different columns of data in the

table come from the same population (writer) or not, which would be the case if the fan imitated Austen correctly. The H_0 of the contingency table test for homogeneity states that the distribution of the words is the same for the stories.

b) Using the given data set, investigate whether Austen herself was consistent in her different novels. Where are the main inconsistencies?

```
data=read.table(file="data/austen.txt",header=TRUE)
austen = data[,1:3]
z = chisq.test(austen)
z
```

```
##
## Pearson's Chi-squared test
##
## data: austen
## X-squared = 12, df = 10, p-value = 0.3
```

```
residuals(z)
```

```
##      Sense  Emma  Sand1
## a      -1.0300 -0.129  1.594
## an      0.4473 -0.159 -0.375
## this    0.0513  0.294 -0.504
## that    0.7482  0.287 -1.442
## with   -0.0475  0.521 -0.704
## without  1.0654 -1.588  0.893
```

She is not inconsistent as the p-value is above 0.05. This means that we cannot reject the H_0 . She does however have some main inconsistency, which where the words “a”, “that” and “without”. As can be seen in the residual table above.

```
z = chisq.test(data)
z
```

```
##
## Pearson's Chi-squared test
##
## data: data
## X-squared = 46, df = 15, p-value = 6e-05
```

```
residuals(z)
```

```
##      Sense      Emma  Sand1  Sand2
## a      -1.015 -0.112093  1.606 -0.0589
## an      -0.591 -1.219955 -1.067  3.7282
## this    0.139  0.390490 -0.444 -0.3267
## that    1.594  1.179849 -0.910 -3.0493
## with   -0.512  0.000192 -1.025  1.7482
## without  1.392 -1.341196  1.137 -1.0696
```

The fan is inconsistent as the p-value of the test is below 0.05. Therefore we have to reject the H_0 and accept that the distribution of the words in the stories are not the same. Because Austen herself did not have this inconsistency we can say that the inconsistency is caused by the fan writer. The main inconsistencies were for the words “that” and “an”. As can be seen in the residual table above.

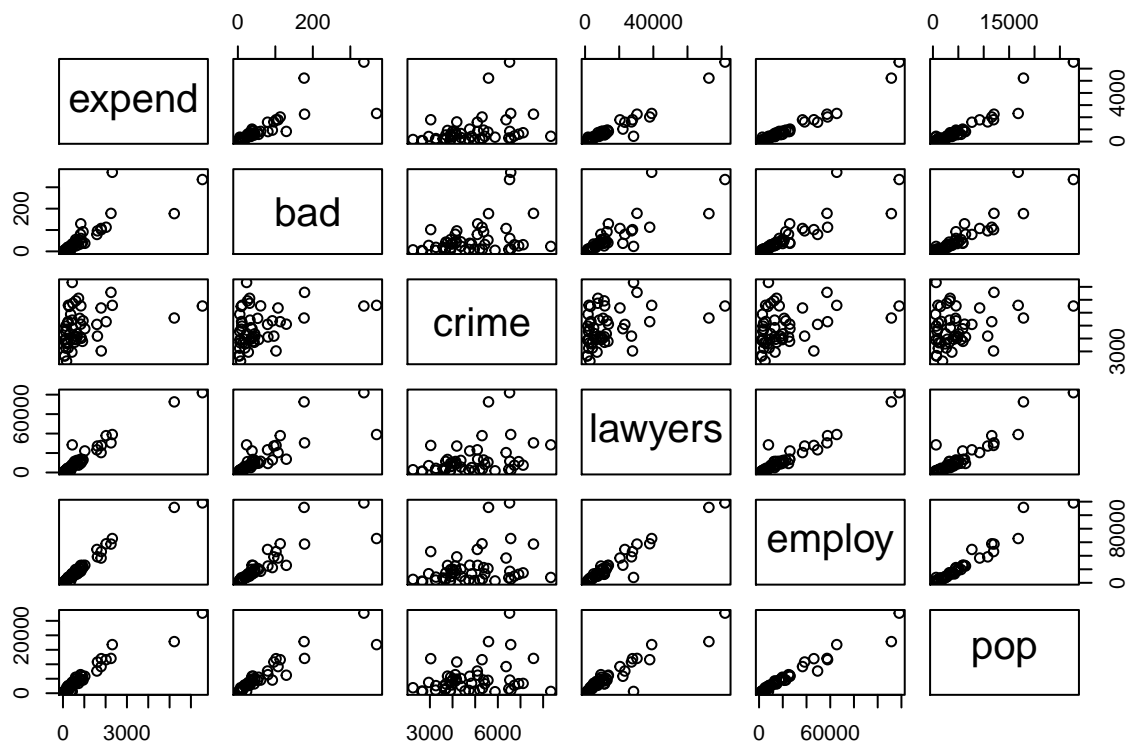
Exercise 5

The data in `expenses crime.txt` were obtained to determine factors related to state expenditures on criminal activities (courts, police, etc.) The variables are: `state` (indicating the state in the USA), `expend` (state expenditures on criminal activities in \$1000), `bad` (crime rate per 100000), `crime` (number of persons under criminal supervision), `lawyers` (number of lawyers in the state), `employ` (number of persons employed in the state) and `pop` (population of the state in 1000). In the regression analysis, take `expend` as response variable and `bad`, `crime`, `lawyers`, `employ` and `pop` as explanatory variables.

a) Make some graphical summaries of the data. Investigate the problem of potential and influence points, and the problem of collinearity.

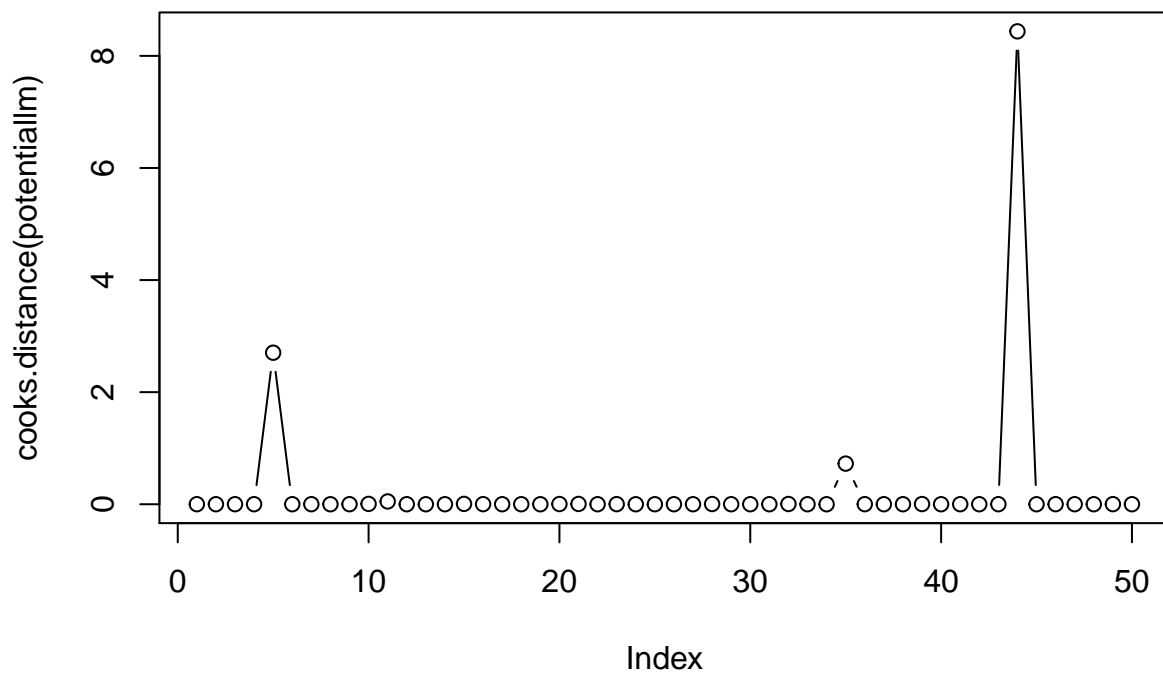
```
data_crime = read.table(file="data/expensescrime.txt",header=TRUE)

plot(data_crime[,c(2,3,4, 5, 6, 7)])
```

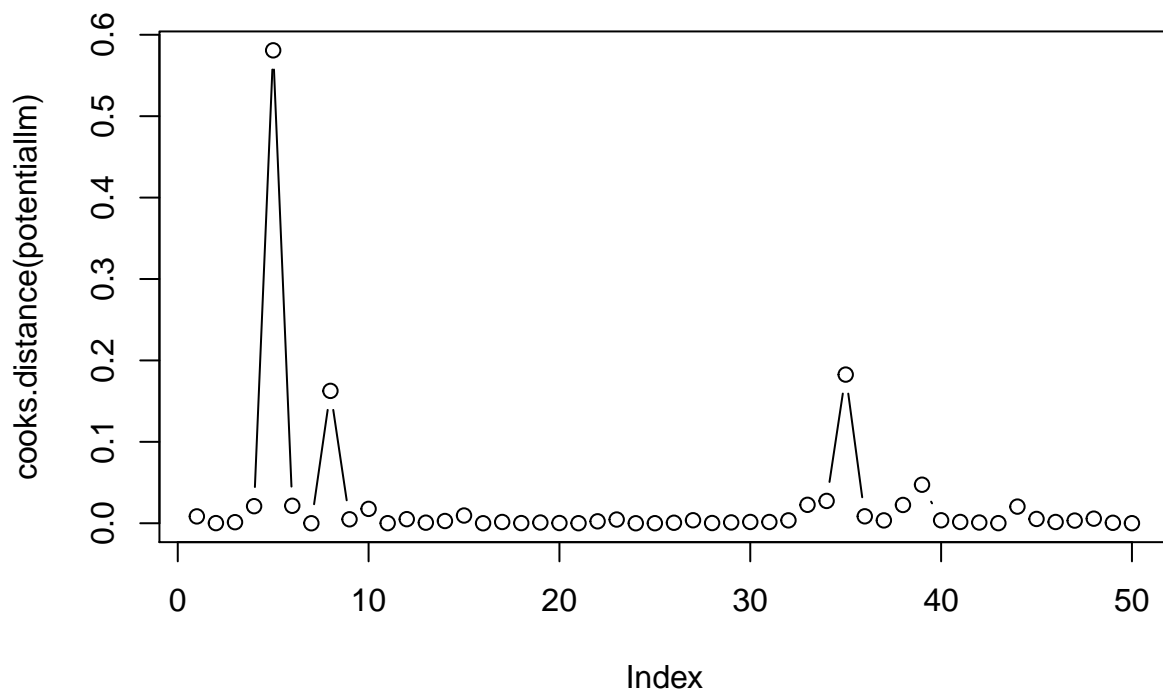


```
regression_data = data_crime[2:7]

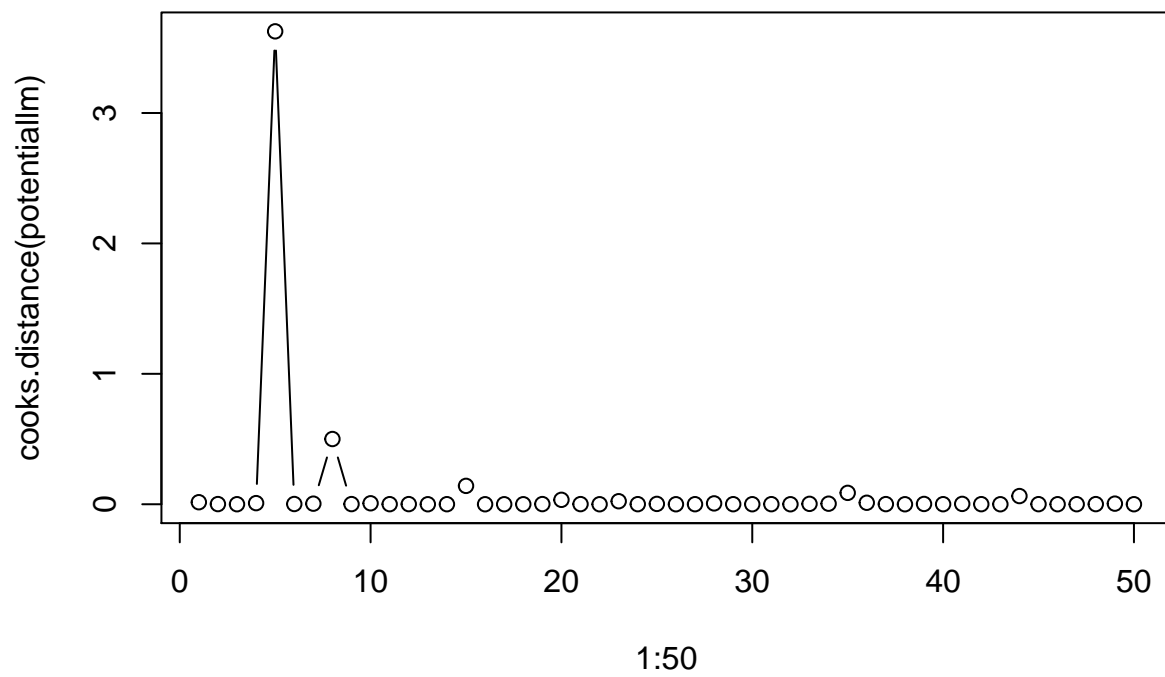
par(mfrow=c(1,1))
potentiallm = lm(expend~bad, data = regression_data)
# potentiallm
# round(cooks.distance(potentiallm),2)
plot(cooks.distance(potentiallm), type="b")
```



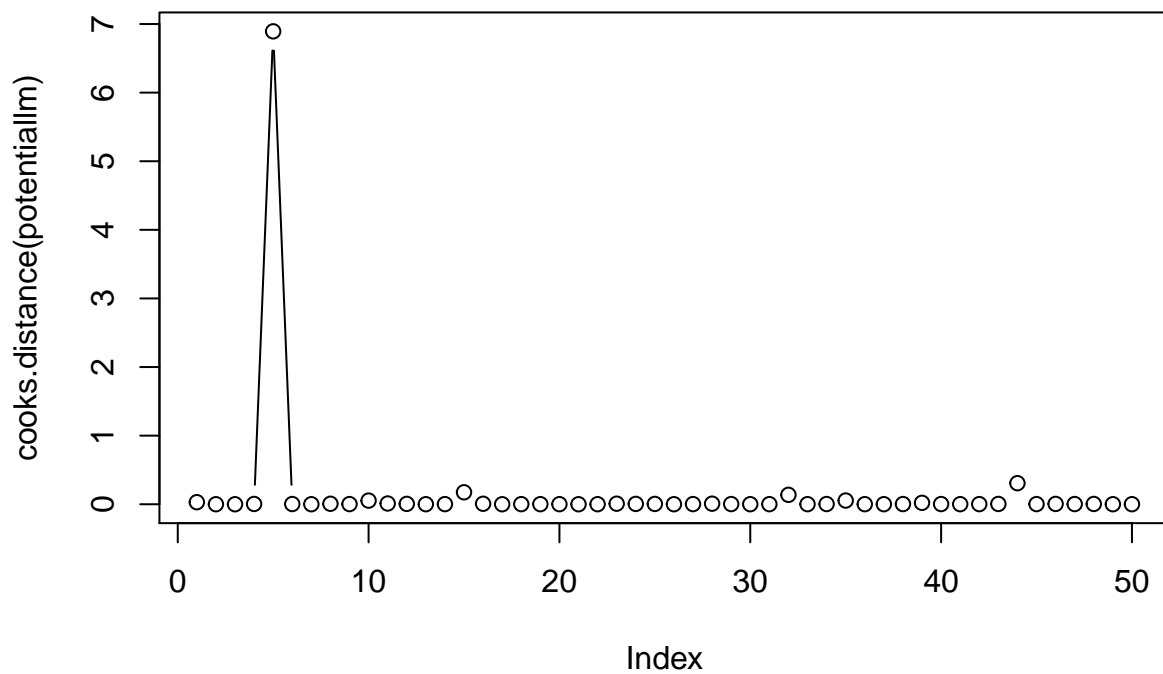
```
potentiallm = lm(expend~crime, data = regression_data)
# potentiallm
# round(cooks.distance(potentiallm),2)
plot(cooks.distance(potentiallm), type="b")
```



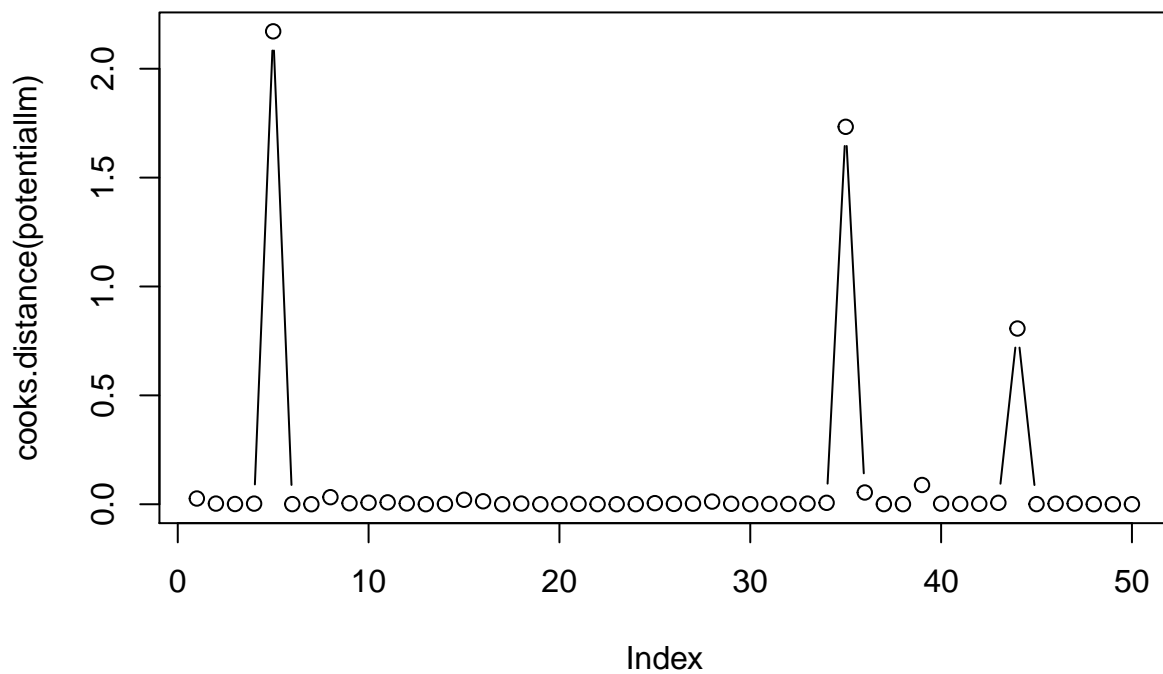
```
potentiallm = lm(expend~lawyers, data = regression_data)
# potentiallm
# round(cooks.distance(potentiallm),2)
plot(1:50,cooks.distance(potentiallm), type="b")
```



```
potentiallm = lm(expend~employ, data = regression_data)
# potentiallm
# round(cooks.distance(potentiallm),2)
plot(cooks.distance(potentiallm), type="b")
```



```
potentiallm = lm(expend~pop, data = regression_data)
# potentiallm
# round(cooks.distance(potentiallm),2)
plot(cooks.distance(potentiallm), type="b")
```

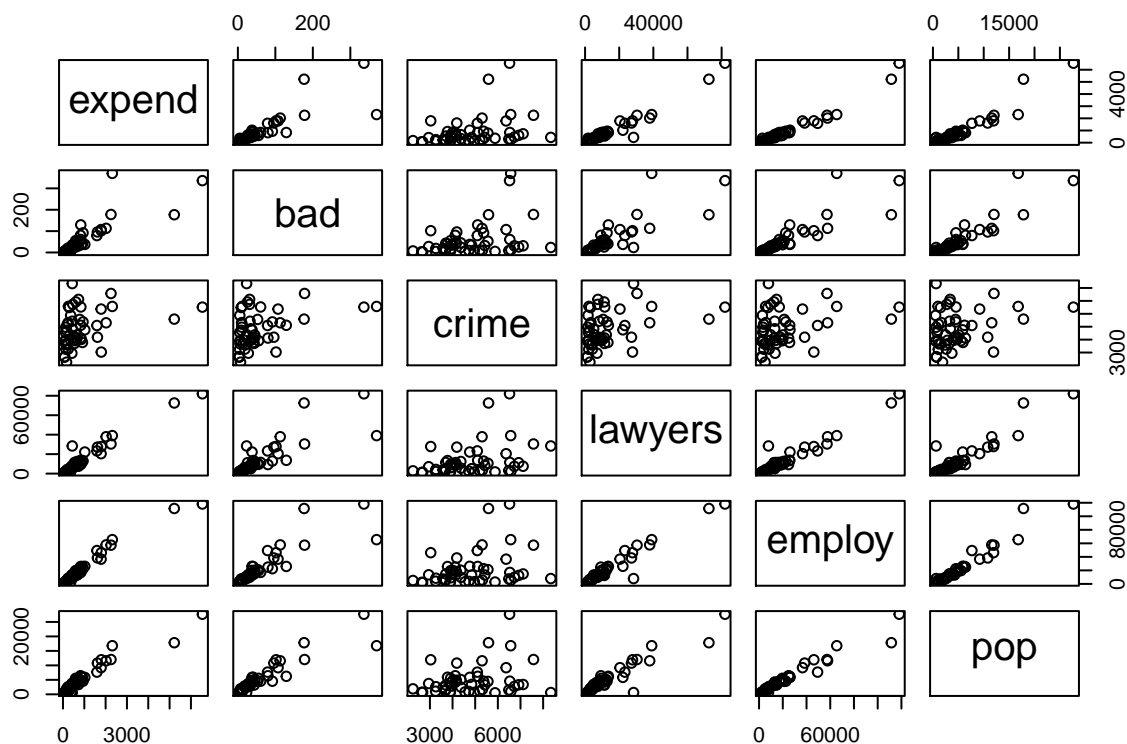


```
# Collinearity
```

```
round(cor(regression_data),2)
```

```
##      expend  bad crime lawyers employ pop
## expend    1.00 0.83 0.33   0.97  0.98 0.95
## bad        0.83 1.00 0.37   0.83  0.87 0.92
## crime      0.33 0.37 1.00   0.37  0.30 0.27
## lawyers    0.97 0.83 0.37   1.00  0.97 0.93
## employ     0.98 0.87 0.30   0.97  1.00 0.97
## pop        0.95 0.92 0.27   0.93  0.97 1.00
```

```
pairs(regression_data)
```

```
# We see that employee and lawyers are strongly correlated(0.97)
# We see that employee and crime rate per 100000 are strongly correlated(0.87)
# We see that lawyers and crime rate per 100000 are strongly correlated(0.83)
# We see a correlation between pop and bad and pop and lawyers and pop and employ
```

```
regressionlm=lm(expend~bad+crime+lawyers+employ, data=regression_data)
car::vif(regressionlm)
```

```
## Registered S3 methods overwritten by 'car':
```

```
## method from
## influence.merMod lme4
## cooks.distance.influence.merMod lme4
## dfbeta.influence.merMod lme4
## dfbetas.influence.merMod lme4
```

```
## bad crime lawyers employ
## 4.42 1.30 16.58 20.87
```

```
# We see a value above 5 for lawyers and employees which means we need to take one out
```

```
regressionlm=lm(expend~bad+crime+lawyers, data=regression_data)
car::vif(regressionlm)
```

```
## bad crime lawyers
## 3.26 1.17 3.27
```

```
# Now it looks good
```

b) Fit a linear regression model to the data. Use both the step-up and the step-down method to find the best model. If step-up and step-down yield two different models, choose one and motivate your choice.

```
# Step-up method
```

```
summary(lm(expend~bad, data=regression_data)) #0.694
```

```
##
## Call:
## lm(formula = expend ~ bad, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2742.7  -133.1   -75.6   110.9  2739.2
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   128.34     117.77    1.09   0.28
## bad           13.31       1.28   10.43 6.2e-14 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 668 on 48 degrees of freedom
## Multiple R-squared:  0.694, Adjusted R-squared:  0.688
## F-statistic: 109 on 1 and 48 DF, p-value: 6.17e-14
```

```
summary(lm(expend~crime, data=regression_data)) #0.1
```

```
##
## Call:
## lm(formula = expend ~ crime, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1423    -583    -181     138     5196
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -500.284     585.908  -0.85   0.397
## crime         0.283       0.117    2.42   0.019 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 1140 on 48 degrees of freedom
## Multiple R-squared:  0.109, Adjusted R-squared:  0.0901
## F-statistic: 5.85 on 1 and 48 DF, p-value: 0.0194
```

```
summary(lm(expend~lawyers, data=regression_data)) #0.9369
```

```
##
## Call:
## lm(formula = expend ~ lawyers, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1503.7   -28.9    36.3    94.5   822.9
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -62.68063   55.13018   -1.14    0.26
## lawyers      0.07047    0.00264   26.70 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 303 on 48 degrees of freedom
## Multiple R-squared:  0.937, Adjusted R-squared:  0.936
## F-statistic: 713 on 1 and 48 DF, p-value: <2e-16
```

```
summary(lm(expend~employ, data=regression_data))#0.954
```

```
##
## Call:
## lm(formula = expend ~ employ, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -636.8   -85.0    50.1   106.1  1120.3
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.20e+02  4.82e+01   -2.5    0.016 *
## employ       4.69e-02  1.49e-03   31.5 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 260 on 48 degrees of freedom
## Multiple R-squared:  0.954, Adjusted R-squared:  0.953
## F-statistic: 991 on 1 and 48 DF, p-value: <2e-16
```

```
summary(lm(expend~pop, data=regression_data)) # 0.907
```

```
##
## Call:
## lm(formula = expend ~ pop, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1148.3  -161.1    26.1   138.1  1533.0
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -195.8440   71.3698   -2.74  0.0085 **
```

```
## pop          0.2178      0.0101   21.66   <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 368 on 48 degrees of freedom
## Multiple R-squared:  0.907, Adjusted R-squared:  0.905
## F-statistic: 469 on 1 and 48 DF, p-value: <2e-16
```

```
summary(lm(expend~employ+bad, data=regression_data))
```

```
##
## Call:
## lm(formula = expend ~ employ + bad, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -655.3 -100.0   39.1  102.3 1149.7
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.20e+02  4.81e+01  -2.49   0.016 *
## employ       4.97e-02  3.01e-03  16.49  <2e-16 ***
## bad         -1.08e+00  1.00e+00  -1.08   0.286
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 259 on 47 degrees of freedom
## Multiple R-squared:  0.955, Adjusted R-squared:  0.953
## F-statistic: 498 on 2 and 47 DF, p-value: <2e-16
```

```
summary(lm(expend~employ+crime, data=regression_data))
```

```
##
## Call:
## lm(formula = expend ~ employ + crime, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -666.9  -84.3   56.7  101.4 1119.0
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -2.56e+02  1.33e+02  -1.92   0.061 .
## employ       4.64e-02  1.56e-03  29.71  <2e-16 ***
## crime        3.03e-02  2.79e-02   1.09   0.282
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 259 on 47 degrees of freedom
## Multiple R-squared:  0.955, Adjusted R-squared:  0.953
## F-statistic: 498 on 2 and 47 DF, p-value: <2e-16
```

```
summary(lm(expend~employ+pop, data=regression_data))
```

```
##
## Call:
## lm(formula = expend ~ employ + pop, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -689.4   -96.3    46.2   113.2  1065.1
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.31e+02   5.15e+01  -2.55   0.014 *
## employ       4.31e-02   6.21e-03   6.94   1e-08 ***
## pop         1.84e-02   2.96e-02   0.62   0.538
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 261 on 47 degrees of freedom
## Multiple R-squared:  0.954, Adjusted R-squared:  0.952
## F-statistic: 489 on 2 and 47 DF, p-value: <2e-16
```

```
summary(lm(expend~employ+lawyers, data=regression_data)) #0.9631 ==> only significant model
```

```
##
## Call:
## lm(formula = expend ~ employ + lawyers, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -599.8   -93.4    38.4    94.8   931.6
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.15e+02   4.36e+01  -2.63   0.0115 *
## employ       2.98e-02   5.15e-03   5.77   5.9e-07 ***
## lawyers      2.69e-02   7.82e-03   3.44   0.0012 **
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 234 on 47 degrees of freedom
## Multiple R-squared:  0.963, Adjusted R-squared:  0.962
## F-statistic: 613 on 2 and 47 DF, p-value: <2e-16
```

```
# expend = -1.146e+02 + 2.690e-02*lawyers + 2.976e-02*employ + error
# Step-down
```

```
summary(lm(expend~bad+crime+lawyers+employ + pop, data=regression_data))
```

```
##
## Call:
## lm(formula = expend ~ bad + crime + lawyers + employ + pop, data = regression_data)
```

```
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -638.7   -92.6    23.1   117.7   792.5
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -3.14e+02   1.42e+02  -2.21   0.0322 *
## bad         -2.90e+00   1.25e+00  -2.32   0.0251 *
## crime        3.42e-02   2.84e-02   1.21   0.2345
## lawyers      2.31e-02   8.08e-03   2.86   0.0064 **
## employ       2.27e-02   7.50e-03   3.03   0.0041 **
## pop          8.06e-02   3.55e-02   2.27   0.0281 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 227 on 44 degrees of freedom
## Multiple R-squared:  0.968, Adjusted R-squared:  0.964
## F-statistic: 264 on 5 and 44 DF, p-value: <2e-16
```

```
summary(lm(expend~lawyers+employ+bad + pop, data=regression_data))
```

```
##
## Call:
## lm(formula = expend ~ lawyers + employ + bad + pop, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -635.8   -79.6    19.7   116.5   799.0
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.52e+02   4.65e+01  -3.27   0.0020 **
## lawyers      2.65e-02   7.62e-03   3.48   0.0011 **
## employ       2.26e-02   7.54e-03   3.00   0.0044 **
## bad         -2.27e+00   1.14e+00  -1.99   0.0529 .
## pop          6.54e-02   3.33e-02   1.96   0.0560 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 228 on 45 degrees of freedom
## Multiple R-squared:  0.967, Adjusted R-squared:  0.964
## F-statistic: 326 on 4 and 45 DF, p-value: <2e-16
```

```
summary(lm(expend~lawyers+employ + bad, data=regression_data))
```

```
##
## Call:
## lm(formula = expend ~ lawyers + employ + bad, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -631.8   -94.9    32.2    92.4   958.6
```

```
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.14e+02  4.36e+01  -2.62  0.0118 *
## lawyers      2.63e-02  7.85e-03   3.36  0.0016 **
## employ       3.23e-02  5.85e-03   5.53  1.5e-06 ***
## bad          -8.55e-01  9.12e-01  -0.94  0.3530
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 235 on 46 degrees of freedom
## Multiple R-squared:  0.964, Adjusted R-squared:  0.961
## F-statistic:  408 on 3 and 46 DF,  p-value: <2e-16
```

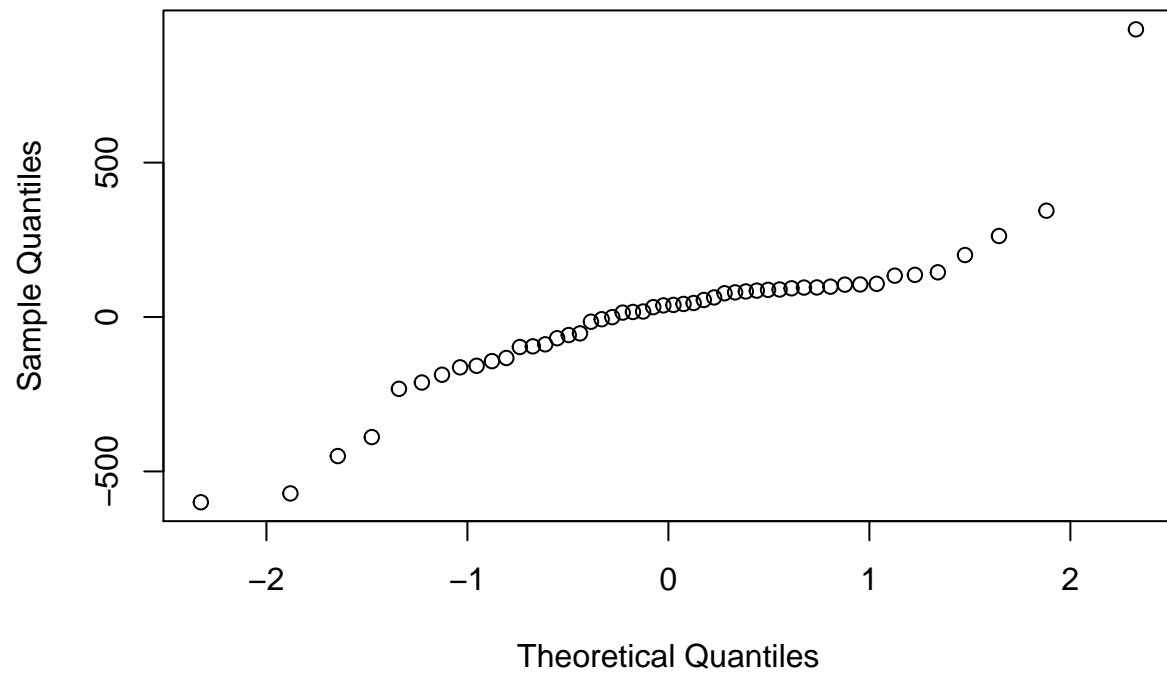
```
summary(lm(expend~lawyers+employ , data=regression_data))
```

```
##
## Call:
## lm(formula = expend ~ lawyers + employ, data = regression_data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -599.8  -93.4   38.4   94.8  931.6
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) -1.15e+02  4.36e+01  -2.63  0.0115 *
## lawyers      2.69e-02  7.82e-03   3.44  0.0012 **
## employ       2.98e-02  5.15e-03   5.77  5.9e-07 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 234 on 47 degrees of freedom
## Multiple R-squared:  0.963, Adjusted R-squared:  0.962
## F-statistic:  613 on 2 and 47 DF,  p-value: <2e-16
```

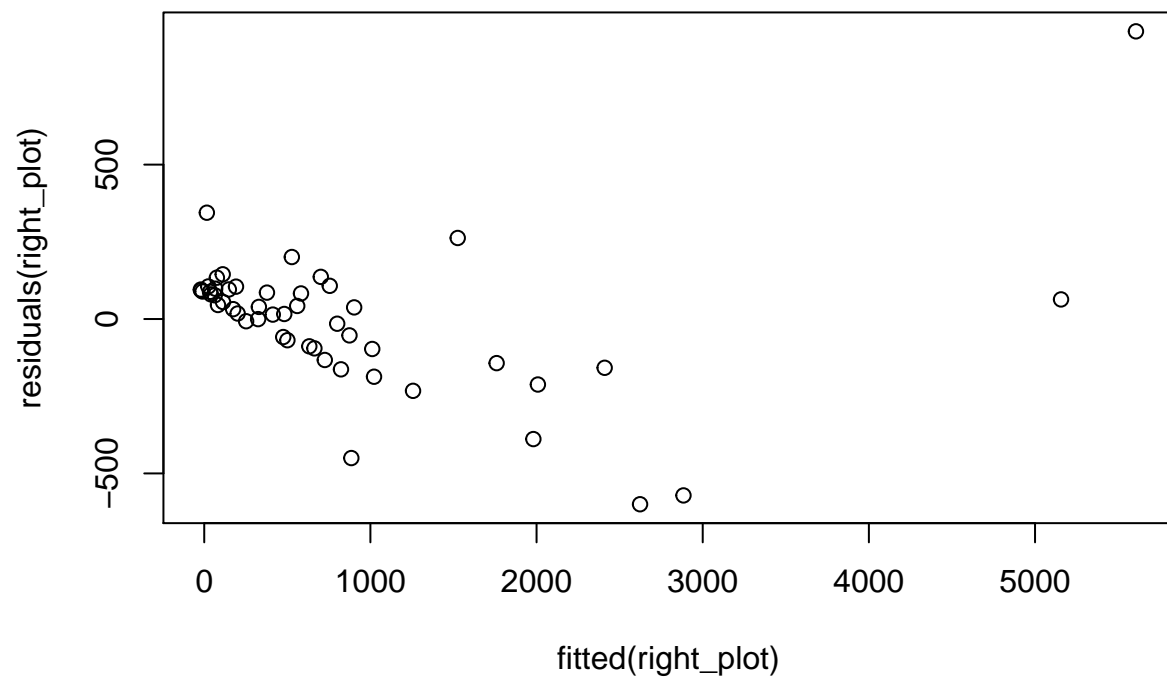
c) Check the model assumptions (of the resulting model from b)) by using relevant diagnostic tools.

```
right_plot = lm(expend~lawyers+employ , data=regression_data)
qqnorm(residuals(right_plot))
```

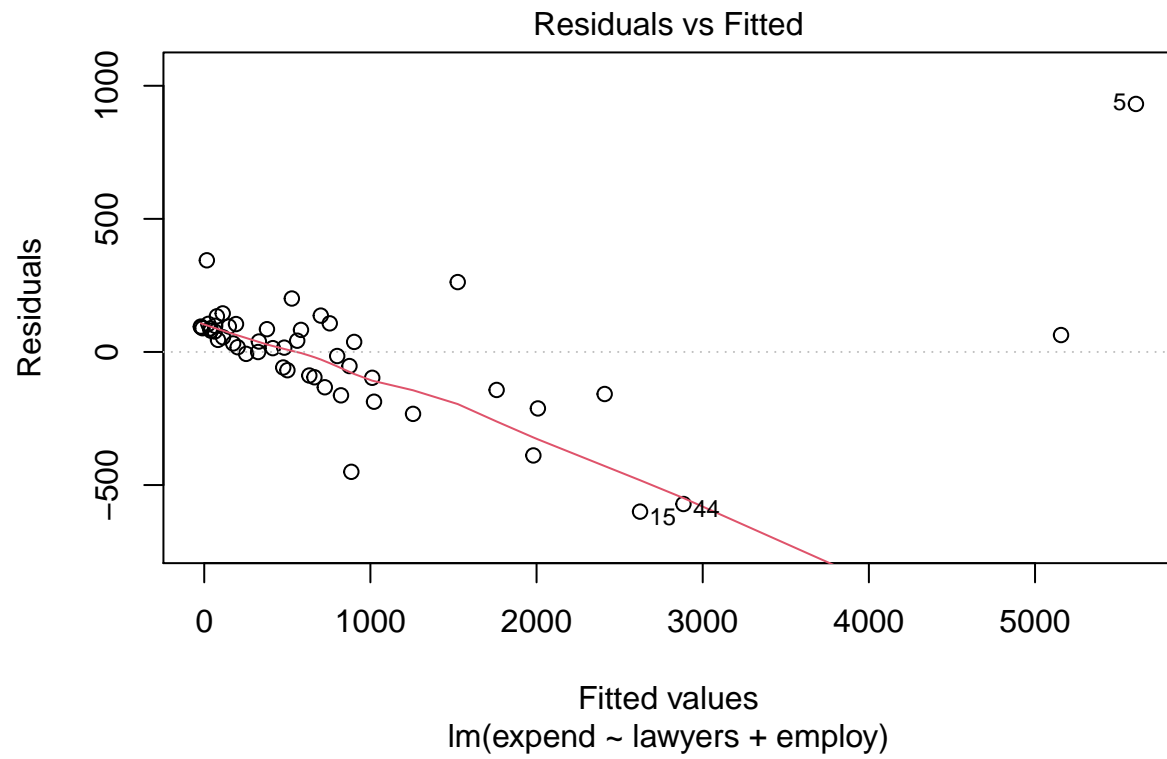
Normal Q-Q Plot



```
plot(fitted(right_plot), residuals(right_plot))
```

```
plot(right_plot, 1)
```



```
plot(right_plot, 2)
```

