We had reviewed the central limit theorem and what we are going to do now is look at examples of how this central limit theorem applies to hypothesis testing.

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It turns out that some of the calculations that we had done previously when we had looked at the examples of hypothesis testing were not quite correct. If you remember the quality control example, where we were looking at the manufacturing process that was supposed to generate a part with the mean weight of 2.5 lbs and the historical data shows that the mean weight is 2.5 lbs with a standard deviation of 0.12 lbs. When we take the random sample of 45 units, we find that the mean weight in the sample of 45 units is 2.68 lbs and the question was, do we believe that there is a problem with the process.

The way we had done this was like every hypothesis test, set up a null hypothesis first. The null hypothesis is always the hypothesis that whatever variation you see in the sample is simply because of random chance variation. In other words, the null hypothesis says there is no problem with the process. The alternate hypothesis is negation of a null hypothesis. So the alternate hypothesis will say that there is a problem with the process, the weight for unit that this process is producing has increased.

We had decided on a significance level of 5% and we had calculated the P value observing greater than 2.68 lbs when expecting 2.5 lbs. Remember, the P value in any hypothesis test is the random chance probability of observing outcomes more extreme than what is seen in the sample. So more extreme is greater than 2.68 lbs.

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If you want to use excel, that probability can be calculated using the normal distribution function.

1 – Normal Distribution (2.68, 2.5, (0.12), TRUE)

2.68 - The sample outcome
2.5 – Expected population mean
0.12 – Standard deviation

It turns out that however that calculation is not quite correct. Why not?

Let us just think about what does Central Limit theorem said. The Central Limit theorem says that if you take samples of sufficiently large size from an underlying population and the underlying population can have any distribution. The means of the samples will follow a normal distribution with mean equal to the population and a standard deviation equal to population standard deviation by square root of sample size.

The member, the sample size is in the denominator because there is an inverse relationship between size of the sample and the variation expected in the sample averages. The larger the sample, the lesser the variation we expect in the sample averages. The smaller the sample, the greater the variance we will expect in the sample averages. Therefore, the Central Limit theorem says the normal distribution which the sample averages will follow will have these parameters.

Mean = Population mean
Standard deviation = Population Standard deviation / Square root of Sample size

Therefore, our random chance probability formula should now be modified to look like this.

P = 1 – Normal Distribution (2.68, 2.5, (0.12/ (45) ^0.5), TRUE)

It has the same sample outcome (2.68) and the same population mean (2.5), but the Standard deviation is now being divided by the square root of sample size, which is 45. If we calculate this in excel, it turns out to be a number, which is very very close to zero.

Remember, anytime you see 0 as a P value output in excel or any tool, it simply means this is the number that is very very close to 0. P value can never exactly be equal to 0.

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Given this P value, which is a number very very close to 0, what is our conclusion?
If the P value is less than the level of significance, then we reject the null hypothesis. In this

example, we are rejecting the null hypothesis. In other words, we are concluding that there is a problem with the production process.

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Let us look at another example, which is inventory optimization. Suppose you are looking at optimizing inventory cost for your retain chain. As a retailer, you sell many products and you have to keep a certain level of stock for your product, because as people buy the product, you have to replace them on the shelf.
There is usually a lead-time for ordering the product. You can't afford to not have stock for your product.

Now how much inventory do you keep for any product? It depends on a couple of things. It depends on the lead time – how much time does it take to order the product. It depends also on demand – the products that sell very quickly, you probably will want to keep a higher level of inventory. Product that don't sell as often, you may keep lower levels of inventory.

But essentially you do have to make a decision around how much inventory to stock for your product and one of the inputs into that decision is certainly going to be what is the average sales for your product on a daily basis.

If we were looking at category – shelf stable beverages – these are really things like Coke or Pepsi. Lets say that in your store, historical data shows that average daily sales for this category is 310 units with a standard deviation of 85 units. Now in order to optimize inventory cost, you want to take more recent sample. So you take a sample of the last 45 days and you find that in your sample of 45 days, the average daily sales are 338.

The question now that you have to answer is that should you recommend the increase in the inventory levels for in this particular category. Remember, this is not a very easy decision because even though the data in your latest sample says 338, sales are random variable - meaning they are impacted by random chance variation. There is some possibility that the 338 is simply a random chance variation and that your sales have not really gone up.

If you recommend an increase in the inventory levels, remember there is a cost associated with the increase in inventory levels, so you have to be very sure before you make such a recommendation. How do you be very sure? You can calculate what is the likelihood that you are seeing 338 simply because of random chance variation.

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Like any hypothesis test, we will set up a null hypothesis; we will set up an alternate hypothesis; we will decide on a level of significance and then we will calculate the probability of observing outcomes more extreme than what is observed simply because of random chance variation.

Now, in order to calculate the P value, we need a test distribution, but because, the Central Limit theorem allow us to use a normal distribution, in this example, the sample size is greater than 30, we know that the sample averages will be normally distributed. The P value that I will calculate will be based on the normal distribution. In order to calculate P values, I can directly use a tool like excel and put a probability distribution function or I could use a table. But either way, there is really no difference in the outcome. Once I have the P value, I will compare that to my level of significance and then decide whether I want to reject the null hypothesis or not.

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So what will be the null hypothesis here? Remember null hypothesis always, no change, no impact, no difference in the average daily units sold.
Alternative hypothesis is negation of the null hypothesis. There is a change in the daily average units sold.

Decide on the significance level – In this case, because the inventory has a cost, let us say that I want to be 95% confident. Therefore, I will decide on a level of significance of 5%. Now I need to calculate P values of outcomes more extreme that what is observed, so that P value that I will calculate will be

1 – Normal distribution (338, 310, (85/(45)^0.5), TRUE)

Remember, we want to calculate the P values of outcomes more extreme than observed, so that's why we are doing 1 minus less than or equal to 338 and it turns out to 0.013. Of course we can do it using a table.

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If we were you use a distribution table, we would first have to calculate a standardized distance which is nothing but $X - \mu / \sigma / \sqrt{n}$.

Z = (338 – 310) / (85/(45)^0.5),

So the standardized distance of this outcome is 2.209.

Now we need to look up the probability of < Z or > Z in the table. Remember the probability of >Z is simply the 1 – the probability of <Z. So if we look at 2.209, we see the probability of >=Z of 2.209 is 0.9864. Therefore the probability of more extreme outcomes which is >Z is 1 – 0.9864 which is 0.013. Remember whether we use excel or we use the table, we will end up with the same value.

What is our conclusion based on this P value?

If the P value which is less than the level of significance, we will reject the null hypothesis. What is the null hypothesis that we are rejecting? We are rejecting the null hypothesis that the sales are still unchanged. We will accept the null hypothesis that the average daily sales have actually gone up. Therefore, we should recommend an increase in the inventory levels for this particular category.

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