DATA SCIENCE WITH R

HYPOTHESIS TESTING

Introduction to Hypothesis Testing

Basic Framework of a Hypothesis Test

Distance Measures

Central Limit Theorem

Types of Hypothesis Tests

Multiple Sample Tests

Agenda

Anova

- One Way
- Two Way
- Post Hoc Tests

Chi Square

- Association Tests
- Goodness-of-fit Tests

Chi Square Parametric

- Tests of Variance

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Anova

We have reviewed hypothesis tests of two types:

1. Single Sample: Testing a sample outcome against an expected population outcome

2. Two Sample: Testing the difference between two sample means

In situations where we want to compare means across multiple samples -

Can we use multiple sets of t-tests? For example, to test for difference between three samples:

Mean 1 = Mean 2,
Mean 2 = Mean 3,
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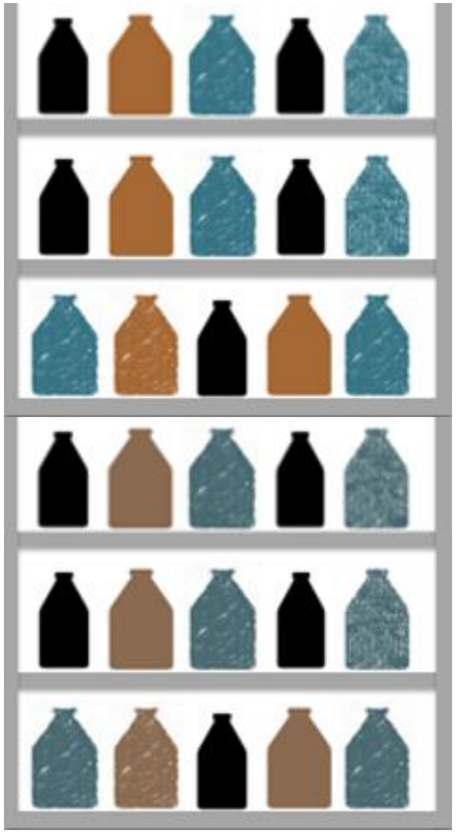
Example:

- A retailer wants to understand shelving height impacts on sales. That is, do sales of a particular brand change significantly if they are placed at eye level, or at lower levels or higher levels?



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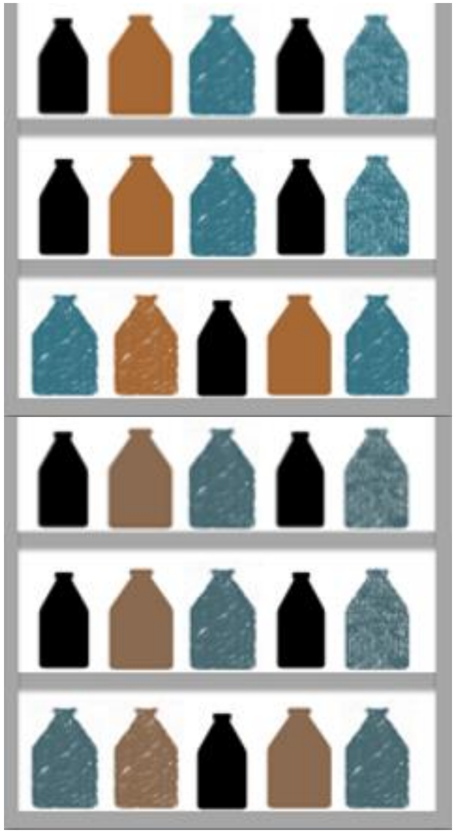
- One way to test this "hypothesis" – store the same product at different shelves and record sales for a fixed number of days at each height



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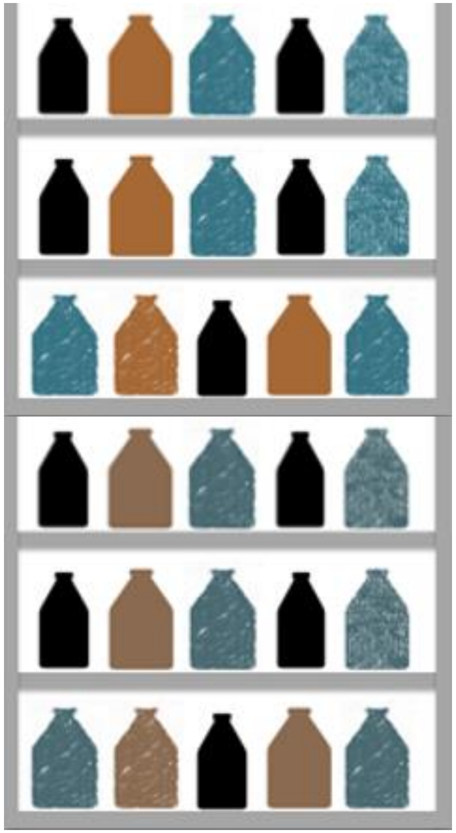
- One way to test this "hypothesis" – store the same product at different shelves and record sales for a fixed number of days at each height
- Look at sales averages for each height, and then run a test to see if any observed differences are **statistically significant**



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Below table lists total sales for 10 days, when the brand was stocked in shelves at different heights

We need to determine if height has an impact on total sales, i.e., are the differences observed in the sample means statistically significant?

	Shelf 1	Shelf 2	Shelf 3	Shelf 4	Shelf 5
	210.5	198.1	170.5	167.1	188.5
	198.1	189	225.5	167.9	167.9
	145.3	210.3	158	175.5	176.5
	185.5	254.4	139.4	175	152
	189.1	210.3	156.4	149.1	164.5
	135.9	160.9	217.1	189.3	171.7
	180	120.8	189.1	198.2	158.9
	149.4	167.8	158.2	205	177.9
	176.4	148.9	218.1	233.5	189.1
	229	190.4	178.9	167.9	187.1
	179.92	185.09	181.12	182.85	173.41
Avg	179.92	185.09	181.12	182.85	173.41

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Between Groups Variance	Sum of squared differences between each group mean and the overall mean	Sum of Squares Between SSB

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ANOVA looks at a ratio of the two methods of estimating variance – if the ratio is similar, then the null hypothesis is unlikely to be rejected

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If the independent variable has no impact, then within group variation and between group variation should be similar with any small differences attributable to random sampling error

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DFW: n-k, -n # of observations

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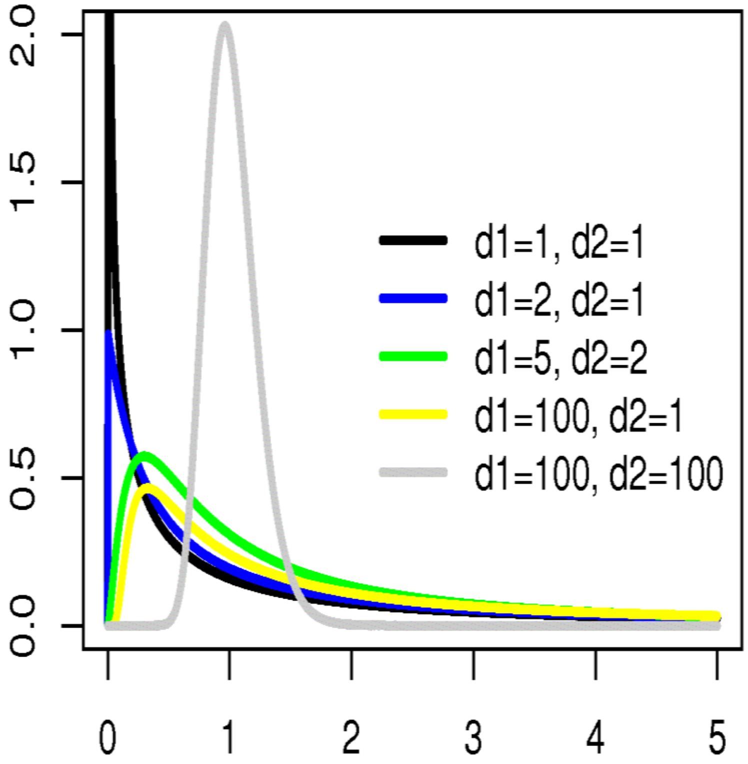
The Test Stat follows an **F-Distribution**



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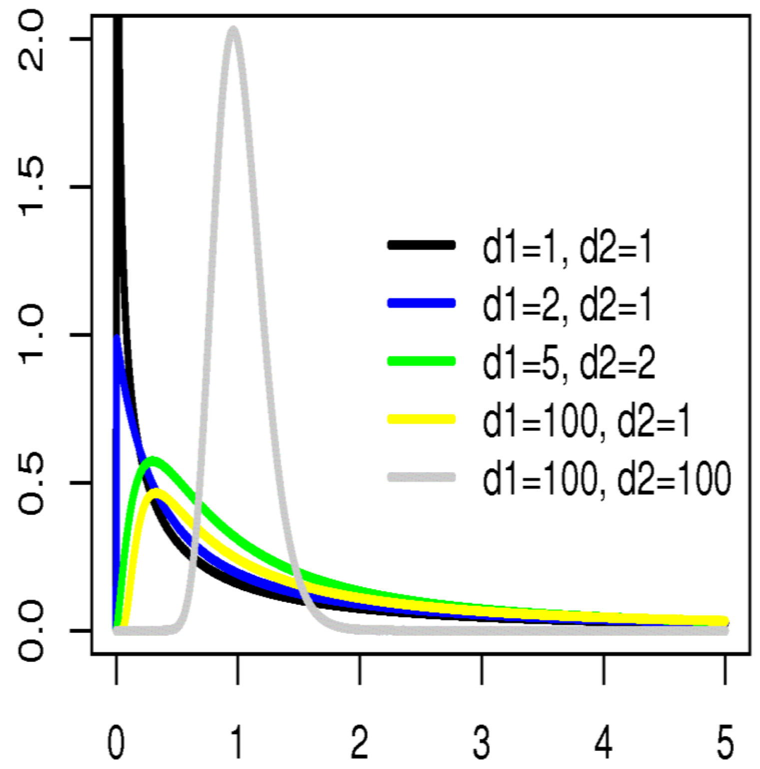
The Test Stat follows an **F-Distribution**

Any random variate of F-distribution can be characterized as the ratio of two Chi Square Distributions

$$\frac{U_1/d_1}{U_2/d_2}$$

where U_1 and U_2 are Chi Square Dist with d_1 and d_2 df



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- What would be constructed as a test-statistic?
- Ratio of Within Group Variation to Between Group Variation

Coming Up

Anova:

Tests Statistic Calculations

THANK YOU