DATA SCIENCE WITH R

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REGRESSION ANALYSIS

Overview

Simple Linear Regression

Multiple Linear Regression

Regression Assumptions

Implementation in SAS

Regression

SIMPLE LINEAR REGRESSION

- ✓ **Concepts - OLS**
- ✓ How to Run
- ✓ Interpret Results

Simple Linear Regression

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A linear relationship between two variables is essentially a straight line relationship

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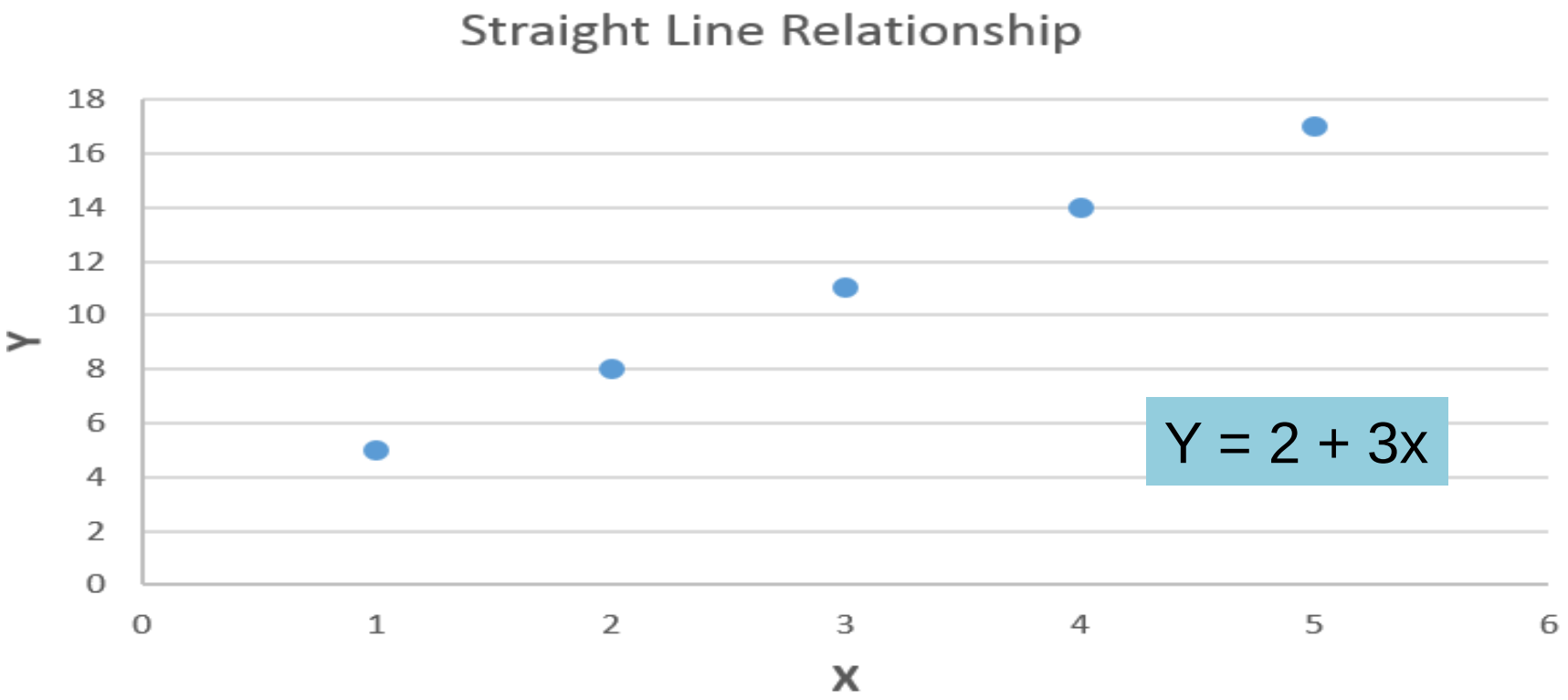
Intercept is the value of Y when X = 0.

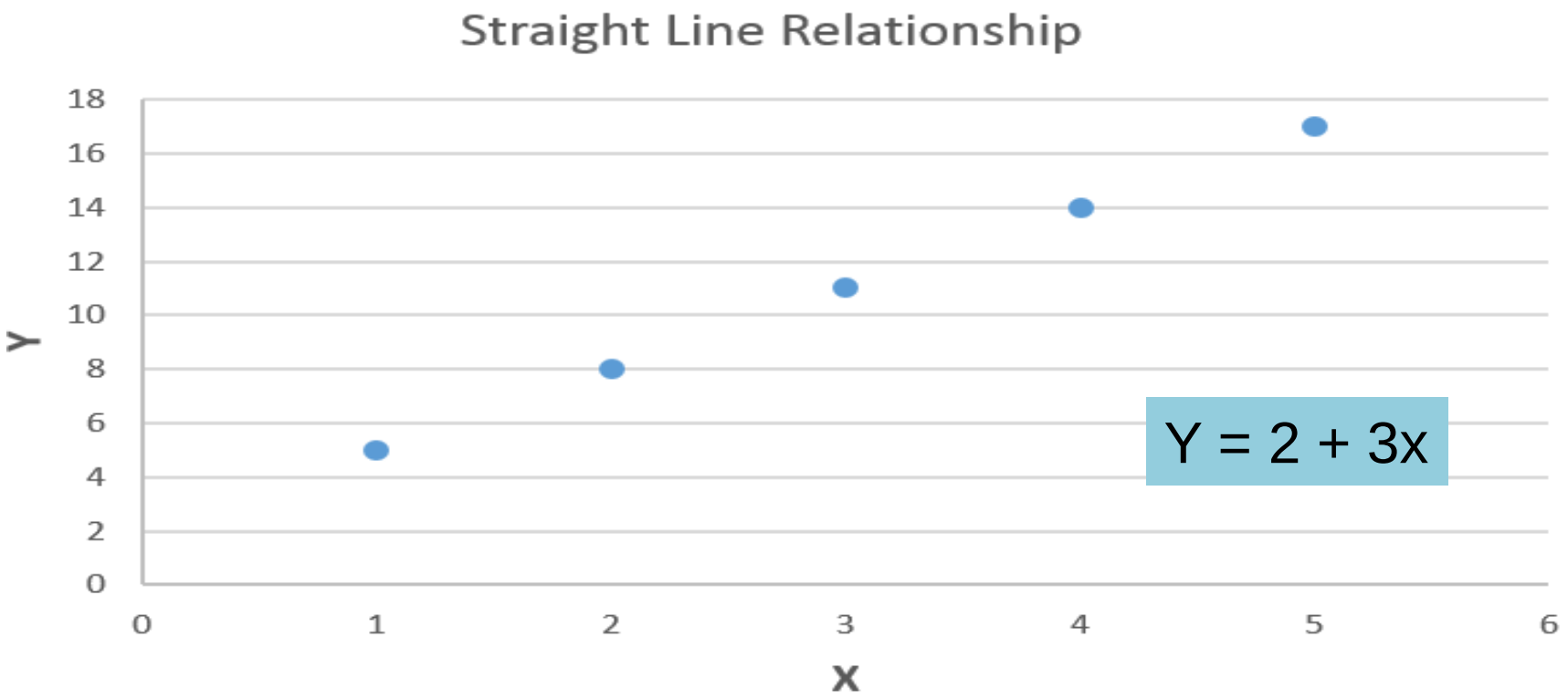
- What if A = 0? Then the line passes through the origin, and Y is directly proportional to X

Simple Linear Regression



Intercept = 2
Slope = 3

Simple Linear Regression



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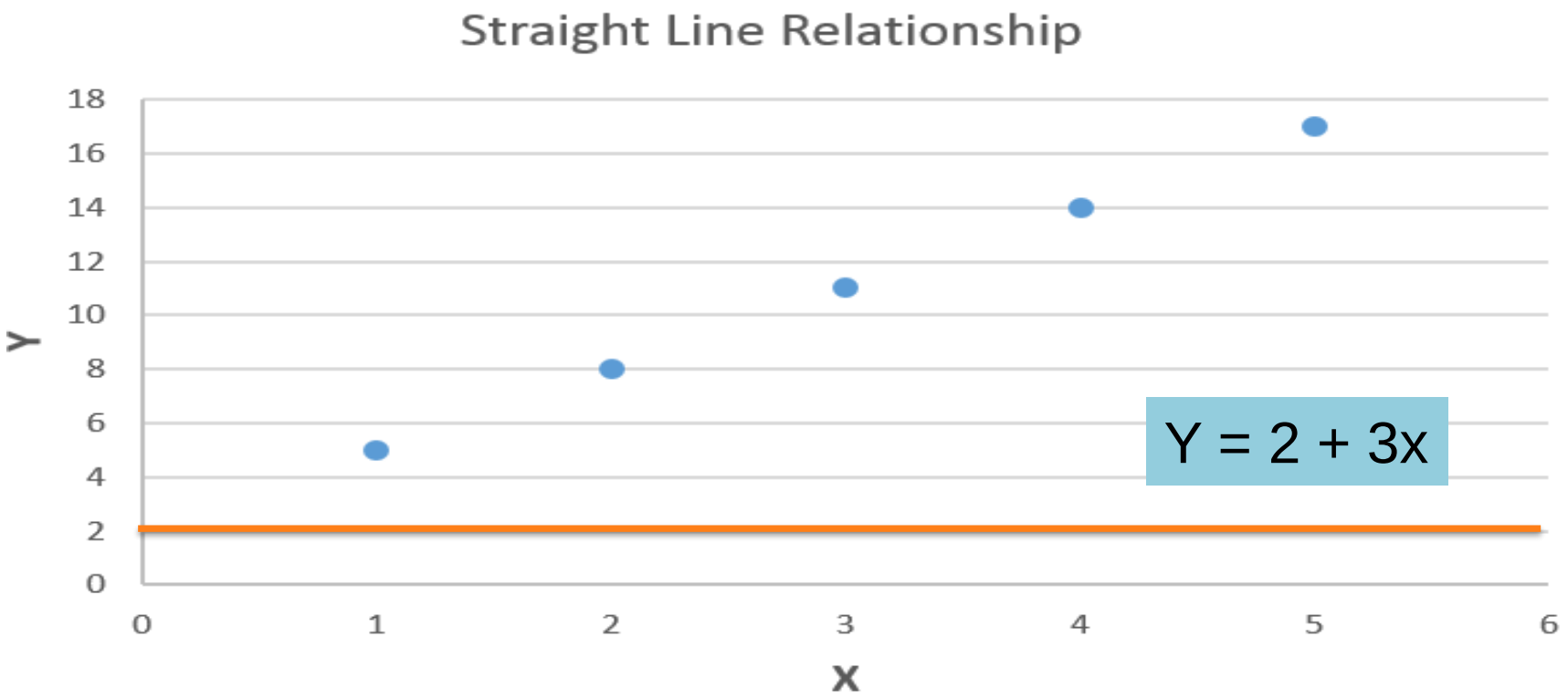
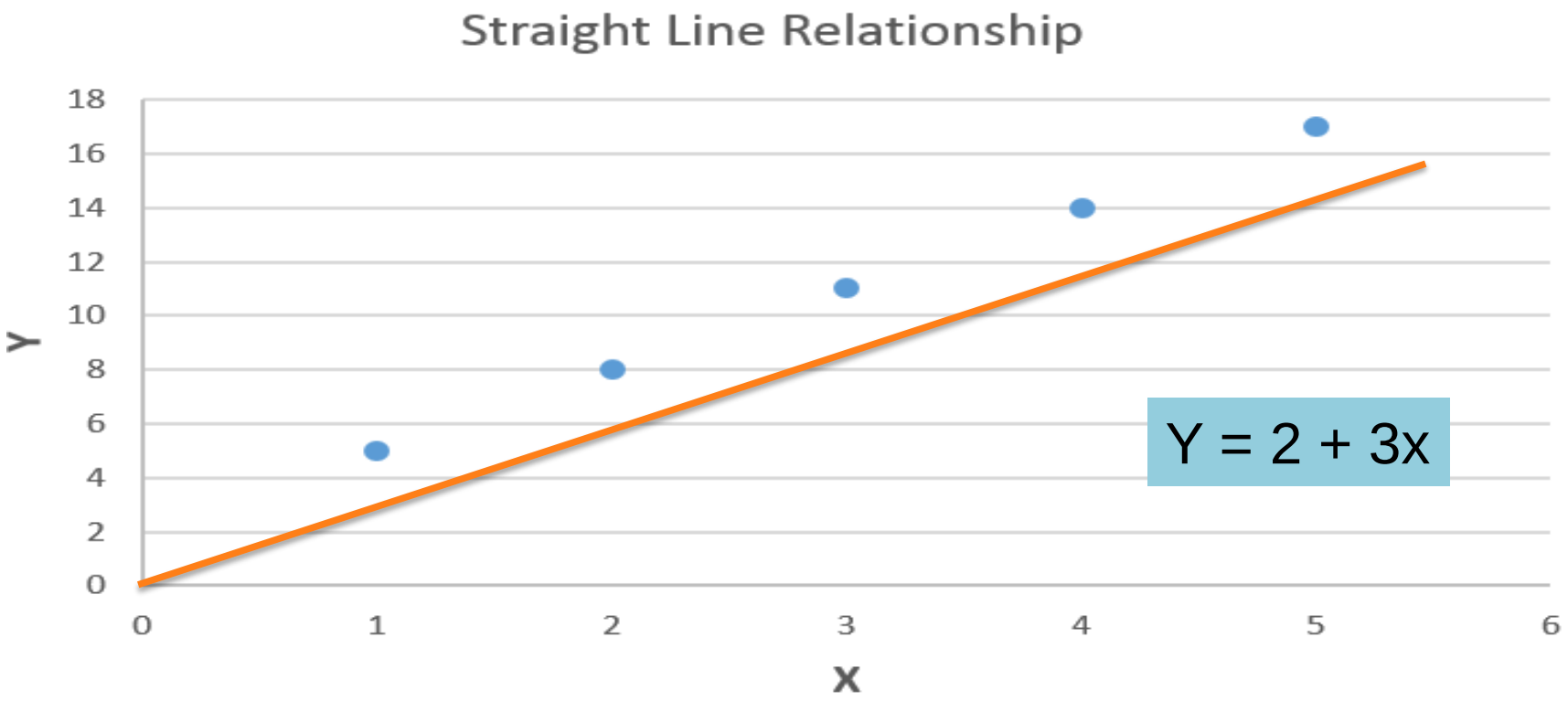
For a unit change in X, Y Changes by a constant amount (3)

Simple Linear Regression



If m = 0?

Simple Linear Regression



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e = Error

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Therefore, in order to understand the relationship between X and Y, we need to figure out what the values of the BETAs are

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TERMINOLOGY

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Error:e:u: *The impact of the unobserved variables on the dependent variable, usually calculated as the difference between the predicted value of Y given the estimated regression function and the actual value of Y*

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$$Birthweight = \beta_0 + \beta_1 * Gestation\ Period + e$$

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We now need to estimate what the beta coefficients values are, from the data available to us, that will best capture the relationship between Birthweight and Gestation Period

Ordinary Least Squares Regression

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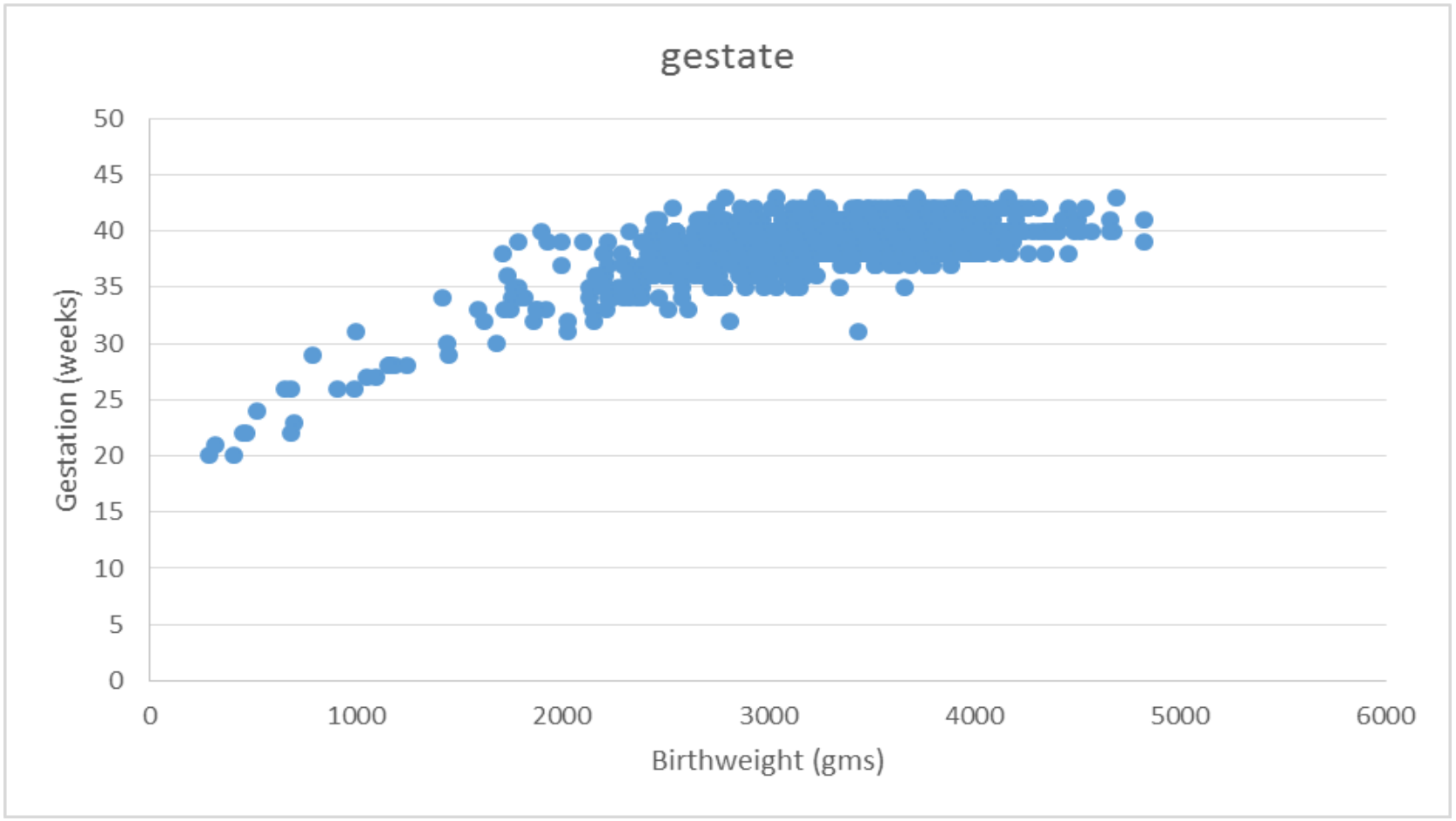
- **Coefficients**: Betas
- **Minimizes**: Least
- **Sum of Squared Differences:** Square of residuals
- **Estimated Line:** Regression Line
- **Actual Values:** Values in data set

OLS Regression

A scatter plot of values of birth weight and gestation period



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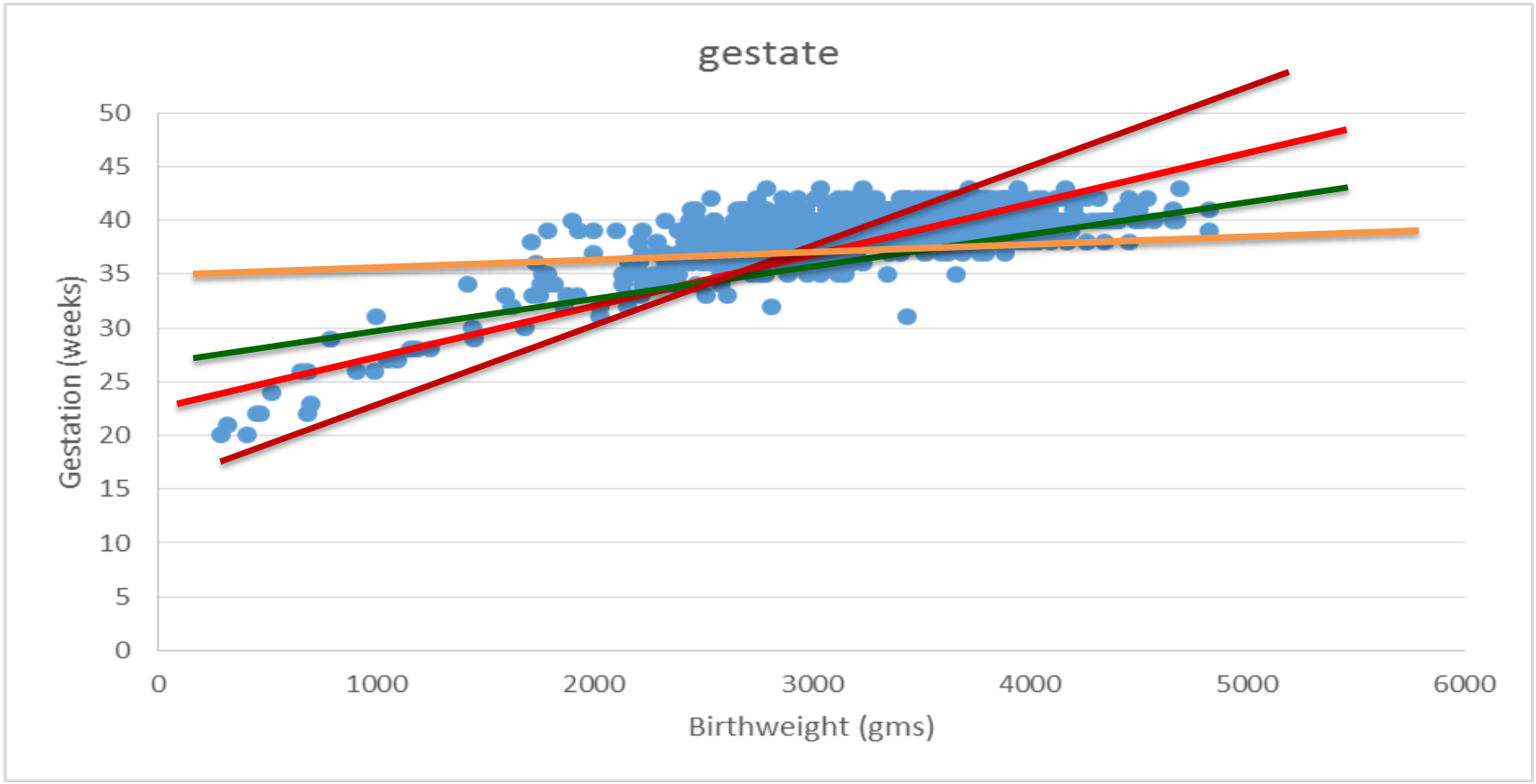

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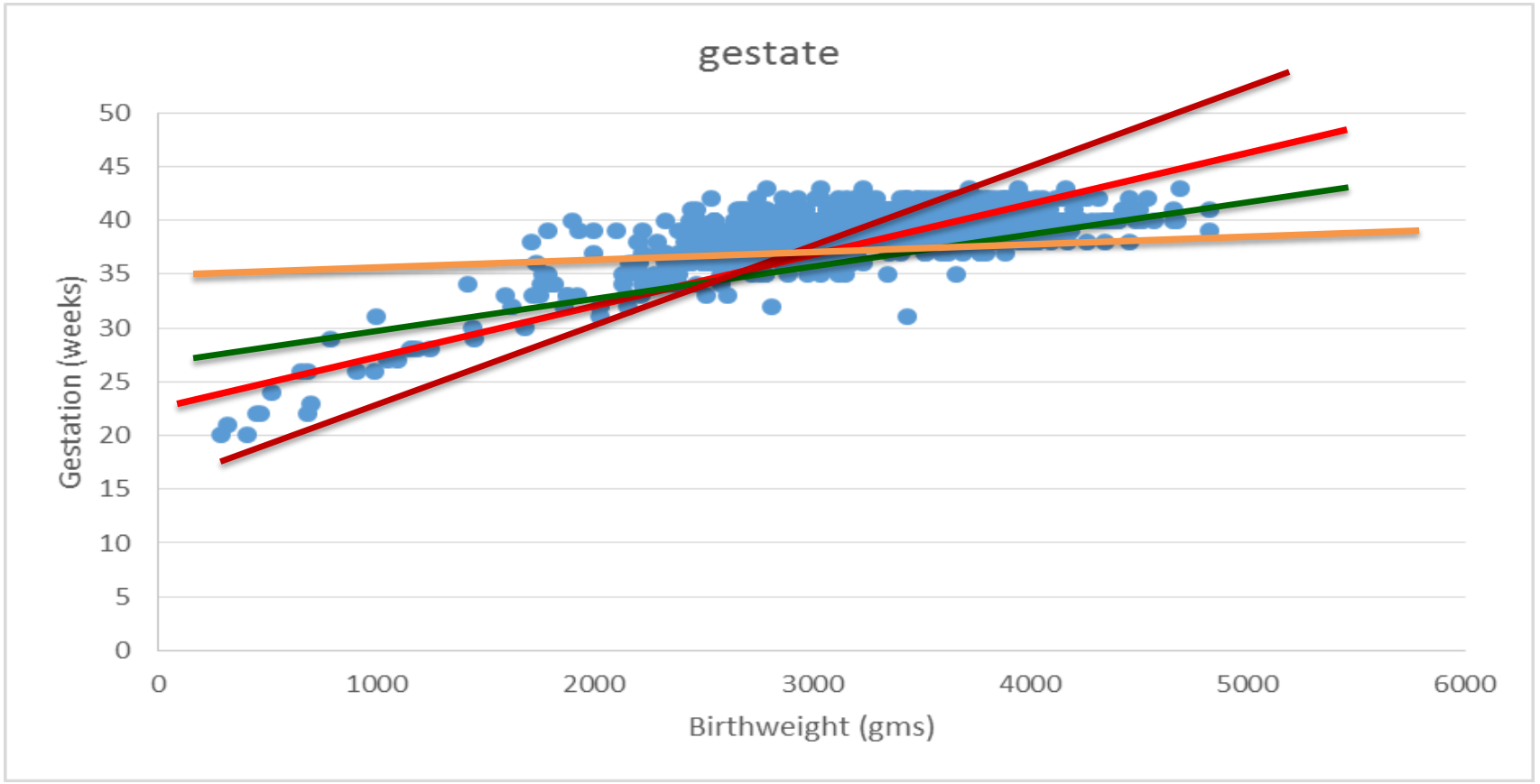


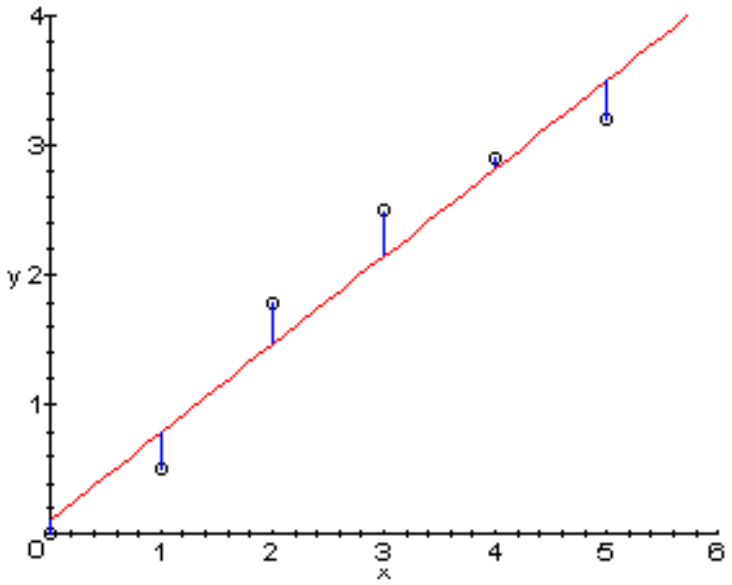
- Is there a linear relationship?
- Would it be possible to fit a straight line through these points?
- How many straight lines?

OLS Regression

Clearly we can fit many straight lines that each will cover some of the points

Is there a straight line that can hit all points?

One way of choosing a line among all possible lines is to identify the line that would explain most variation in Y -
In other words, have least total error



OLS Regression

OLS Estimates

The Ordinary Least Squares regression find that line by looking at the residuals (or the difference between the points on each line and actual Y) and minimizing the sum of their squares

Why sum of squares?

Positive and Negative Differences

Mathematically, minimize $$Q = \sum_{i=1}^{N} (Y_i - b_0 - b_1 X_i)^2$$

OLS Regression

Using differential calculus, we will get

$$b_o = \frac{\Sigma X_i^2 \Sigma Y_i - \Sigma X_i \Sigma X_i Y_i}{n\Sigma X_i^2 - (\Sigma X_i)^2}$$

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We can be sure that given the data, the Ordinary Least Squares estimate line minimizes errors more than any other line that we choose

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Once we estimate the coefficients, we have an equation like this:

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Remember, this is the best fitted line, but this line will not cover every single point on the scatter plot

To Be Continued

Regression Analysis

Simple Linear Regression

THANK YOU