We have reviewed two kinds of Chi square tests. Association tests and goodness of fit tests. We will now look at a slightly different Chi square test which is a Chi square test of variance.

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Why is it different? The test that we have looked at, the test of association, the goodness of fit test are called non-parameter tests. What does that mean? In order to apply this test, we do not need the underlying population to follow any specific distribution. If you remember the examples we used for association and goodness of fit, that particular data distribution itself did not need to follow any particular kind of distribution.

If you look at t-test or normal distribution tests that we have looked at before, and f-test, we do need the data to follow a certain distribution. Sometimes we say the samples must come from populations that are normally distributed. Those are called parametric tests. Because the parameters of the underlying distributions are specified and are important.

There are also non-parametric tests where it doesn't matter what the data distribution of the underlying population is like. The goodness of fit test, and test association are essentially non-parametric test. Which tests are preferable? Parametric or non-parametric? Logically we would prefer non-parametric test. Because we don't have to worry about the distribution of the underlying data having to follow a specified distribution. We can apply the test irrespective of what distribution the data follows.

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But in real life, people still extensively use parametric tests. Why is that? Because non-parametric tests are somewhat less powerful than equivalent parametric test. For every parametric test, the reason equivalent non-parametric test. But people still end up using parametric tests more often than non-parametric tests. That is because with the same information, non-parametric tests are less powerful. You need more information or more data to arrive at a conclusion with a same level of confidence when you are dealing with the non-parametric test as compared to a parametric test.

In a parametric test, you are able to attain your required level of confidence with less data. Because the parameters of the distribution are providing some data. So non-parametric test required more data or more information to arrive at the same level of confidence. Also it turns out as sample size increases, non-parametric test distributions approximate normal distribution.

If you have large sample sizes, we can use parametric distributions. Non-parametric distributions don't really add any value. If you have smaller samples, then you can use the non-parametric tests, but you will not be able to achieve the same level of confidence that we achieve with the parametric test for sample size that's exactly the same.

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The Chi square test that we are going to review now is a different Chi square test. It is the parametric test of variance. So this is not based on observed frequencies or expected frequencies. This is a Chi square test of variance. What is this? Remember that when we started understanding hypothesis testing, we said we will rely extensively on the central limit theorem for a lot of hypothesis testing.

Because the central limit theorem says that the distribution of sample means will follow the normal distribution, irrespective of the underlying population distribution. Therefore when we want to check for the difference in the sample mean compared to a population mean. We can end up using a normal distribution as an approximate test distribution.

What if I wanted to check the difference in the variance of a sample against a population variance? Will the variance of sample means follow a normal distribution. No, it is only the means of sample that will follow a normal distribution. The variance is the square term and it turns out the variance will follow what is called a Chi square distribution.

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Therefore in order to check for difference in variance between a sample and a population, we will use a test distribution which is a Chi square distribution. The test statistic is computed as

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

Where

N = sample size

S = sample standard deviation

σ = population standard deviation

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To understand let's use a simple example. Let's say that the call centre is experimenting with different approaches to improve customer experience typically with the aim of consistent call resolution time. Remember they are interested in consistency, not necessarily average call resolution time being reduced. Currently average resolution time is 6.5 minutes with a variance of 4.5 minutes.

A new approach has been tested and that results in average resolution time of 6 minutes with a variance of 3 minutes across 30 calls. Is the new approach sufficiently different from the standard to justify investment in it?

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Remember we are interested in variance. So we want to check is the difference in variance of 4.5 vs 3 are significant variance or not? Therefore we run a Chi square test of variance and we calculate a test statistic as

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

So 29*3^2/ (4.5^2) =12.88. Degrees of freedom= n-1 = 29.

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If we look that up in table we can calculate the p value or of course we can always use excel. In excel if you want to use a Chi square distribution, remember this is not a Chi square test. This is a Chi square distribution test. I will say Chi square distribution outcome 12.88, Degrees of freedom 29, true and when I do that I will get a p value.

The p value that I generate is 0.002 and therefore I will reject the null hypothesis and conclude that the variance of call has reduced. So just to see how we do this in excel. We say chisq.dist, because remember this is not a standard Chi square test. 12.8, degrees of freedom is 29, and cumulative is true and that gives me a p value of 0.004 rounded of. Therefore I reject the null hypothesis and conclude that there is difference in the variance post the new approach.

Because now we are rejecting the null hypothesis that there is no difference in variance between the sample and the population. So that is an example of a Chi square test of variance. Remember the Chi square test of variance is a different Chi square test. It is a parametric test as supposed to the non-parametric test of association and goodness of fit test.