

DATA SCIENCE WITH R

HYPOTHESIS TESTING

Introduction to Hypothesis Testing

Basic Framework of a Hypothesis Test

Distance Measures

Central Limit Theorem



Types of Hypothesis Tests



Population Std Deviation Not Known



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Example: IQ Testing

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2. The distribution of sample means will follow **T (μ , $s/(\sqrt{n})$)**



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Conclusion – Reject Null Hypothesis



Directional Tests



One Tailed v/s Two Tailed Tests

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One Tailed v/s Two Tailed Tests

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OR

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Which is appropriate?



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- You could be liable for fines if your packaged **weight is** $<$ what is printed on the package.



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- You are not interested in testing if **packaged weight is greater**
- Use a one tail test



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If, however, you do not have a strong reason for believing that sample outcome has to be either greater than or less than an expected population mean, use a **two tail test**



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A manufacturing process has to generate auto parts with weights of exactly 0.8 lbs. If you were running a quality check, you will want to check if the process is producing units of exactly 0.8 lbs or different from 0.8 lbs.



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H1: Process producing weights different from 0.8 lbs
- This would therefore be a two tail test



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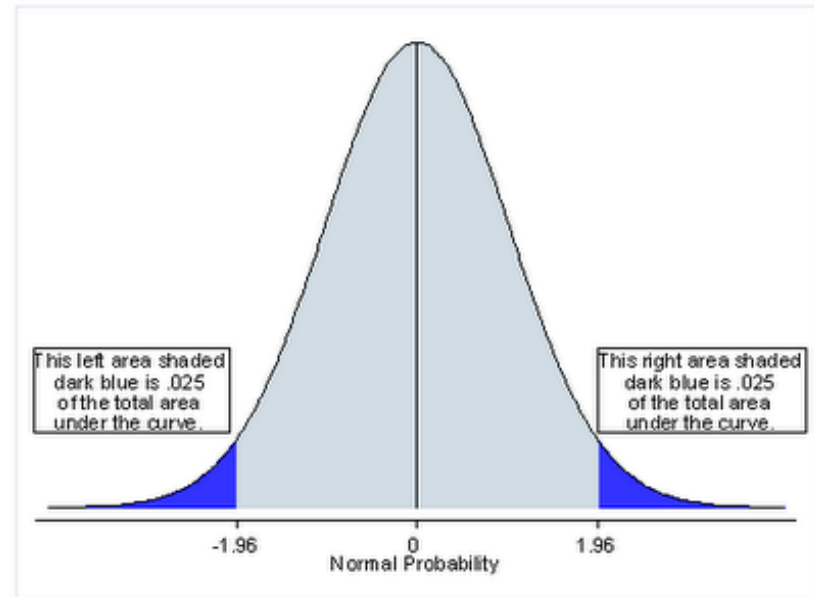
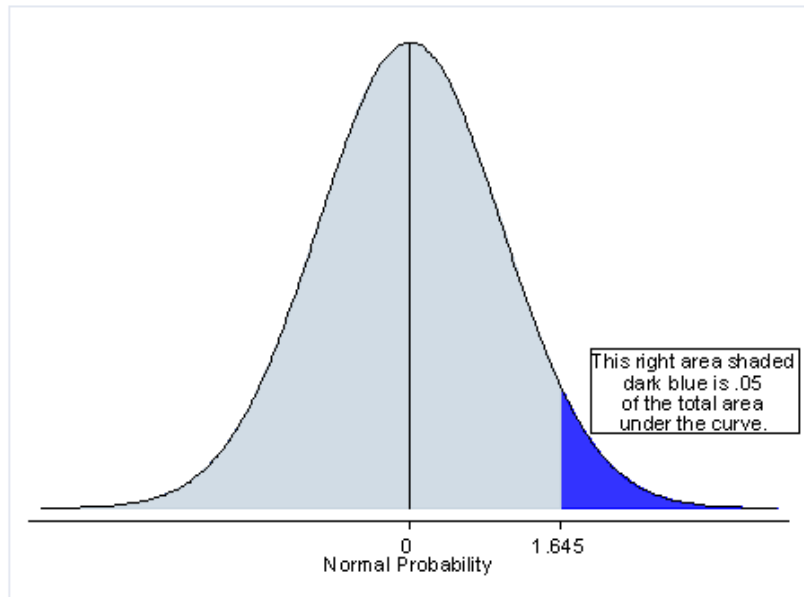
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What are the implications of one tail v/s two tail?

One Tailed v/s Two Tailed Tests

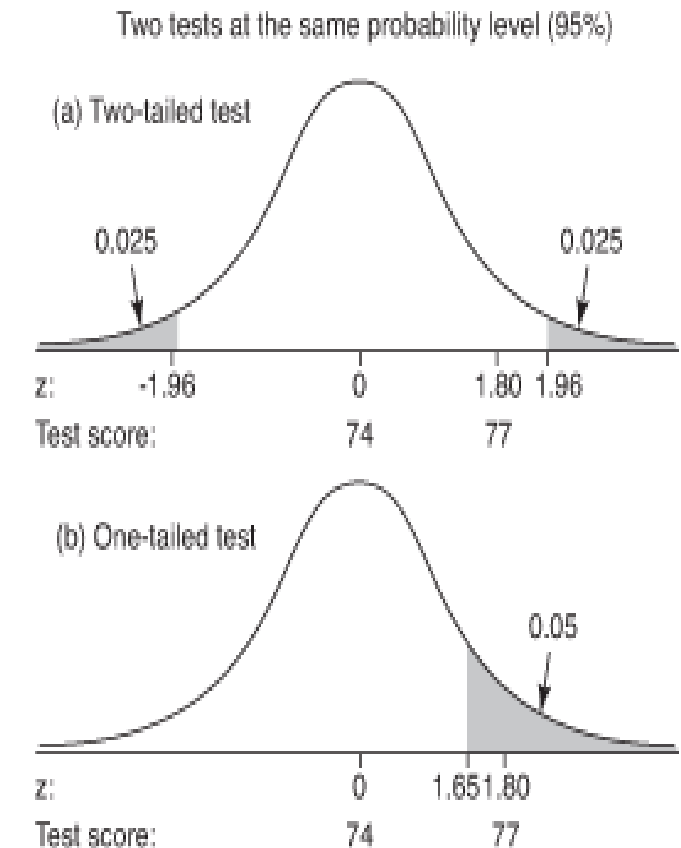
- If you are testing $X > \mu$ or $X < \mu$, rejection region will fall on one side of the distribution curve
- If you are testing $X < > \mu$, then rejection region will fall symmetrically on both sides of the curve



One Tailed v/s Two Tailed Tests

In terms of practical application to hypothesis tests,

If using a two tail test, the rejection criteria for a 5% level of significance:



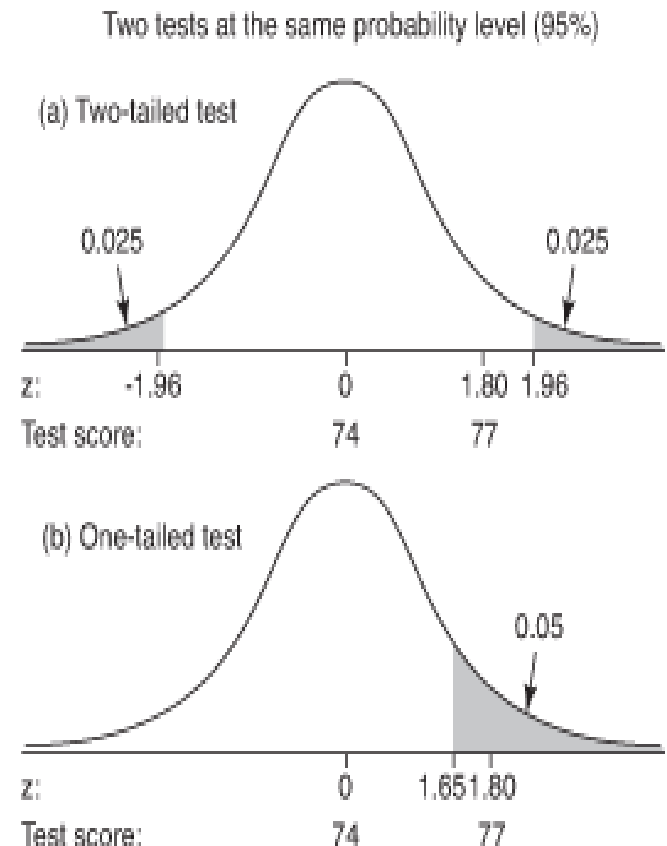
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In terms of practical application to hypothesis tests,

If using a two tail test, the rejection criteria for a 5% level of significance:

**Reject Null if p-value is less than 0.025:
(0.05/2)**

So for any pre-determined significance level, adjust rejection criteria if using a two tail test



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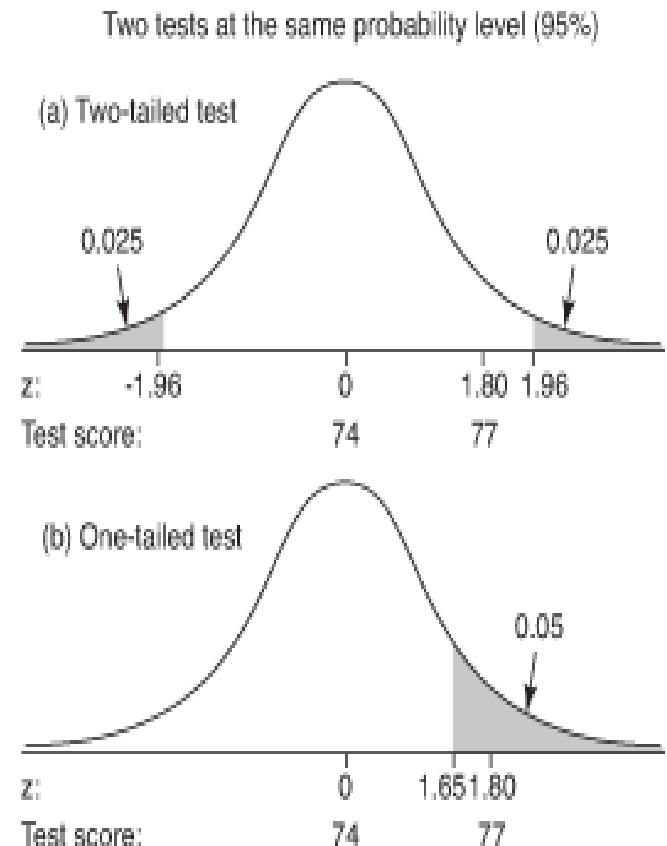
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Which is a stricter test, one-tail or two-tail?



Coming Up

Types of Hypothesis Tests:

Two Sample Tests



THANK YOU

