

DATA SCIENCE WITH R

REGRESSION ANALYSIS

Overview



Simple Linear Regression

Multiple Linear Regression

Regression Assumptions

Implementation in SAS



Regression

SIMPLE LINEAR REGRESSION

- ✓ Concepts - OLS
- ✓ How to Run
- ✓ **Interpret Results**



OLS Results: Confidence Levels

The 95% confidence interval tell us – for a 1 week increase in gestation period, we expect to see an increase in birthweight of between 156.5 and 176.4 gms 95% of the time.

| | <i>Coefficients</i> | <i>Standard Error</i> | <i>t Stat</i> | <i>P-value</i> | <i>Lower 95%</i> | <i>Upper 95%</i> |
|-----------|---------------------|-----------------------|---------------|----------------|------------------|------------------|
| Intercept | -3245.446394 | 197.0110519 | -16.4734 | 9.95259E-55 | -3632.001323 | -2858.891465 |
| gestate | 166.4462854 | 5.060260218 | 32.89283 | 2.54E-166 | 156.5175606 | 176.3750103 |



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If we run regression models on multiple random samples from the same population many times, then 95% of the time the point estimate of the coefficient on the independent variable of interest will lie within the lower and upper bounds calculated



OLS Results Interpretation

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Or, a better way to put it is, how much of the variance in Y is explained by X ?



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The mathematical calculation is:

$$R^2 \equiv 1 - \frac{SS_{\text{err}}}{SS_{\text{tot}}} \quad \text{Where,} \quad SS_{\text{tot}} = \sum_i (y_i - \bar{y})^2, \quad SS_{\text{err}} = \sum_i (y_i - f_i)^2$$



OLS Results Interpretation

SUMMARY OUTPUT

| <i>Regression Statistics</i> | |
|------------------------------|-------------|
| Multiple R | 0.702085646 |
| R Square | 0.492924254 |
| Adjusted R Square | 0.49246866 |
| Standard Error | 451.3259178 |
| Observations | 1115 |

ANOVA

| | <i>df</i> | <i>SS</i> | <i>MS</i> | <i>F</i> |
|------------|-----------|-------------|-----------|-------------|
| Regression | 1 | 220385522.7 | 2.2E+08 | 1081.938347 |
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- However, R^2 is not the only indicator of model fit
- It is possible to have the same R^2 but different models with different fit
- R^2 also increases with addition of variables, whether relevant or not, it is better to use the adjusted R^2 measure



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In this example, $p \text{ value} < 0.05$, so we conclude that at least one of the beta coefficients is significant (in this case, we have only one beta)



Coming Up

Regression Analysis

Multiple Linear Regression



THANK YOU

