


Recurrent Neural Networks

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Outline

Recurrent Neural Networks

Overview & Applications

RNNs from differential equations (Unfolding RNNs)

Statistics of RNNs

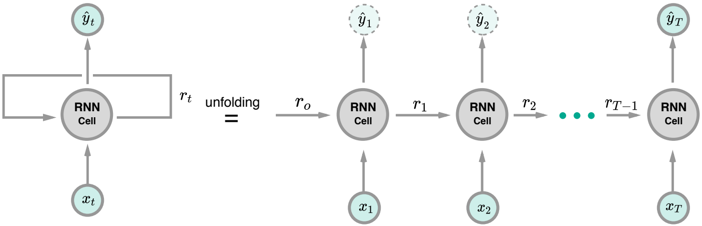
Exploding and Varnishing Gradient Problems

Echo State Networks (ESNs)

Gradient Descent with Momentum

Long Short-Term Memory & Gated RNNs

What are Recurrent Neural Networks (RNNs)?



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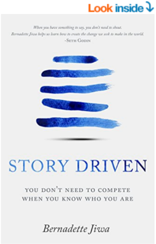
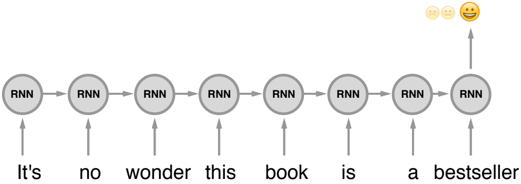
Example Applications

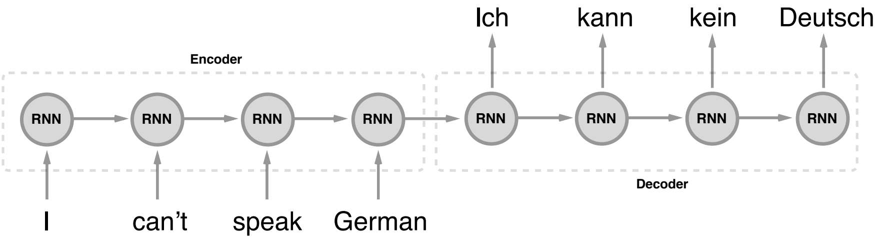




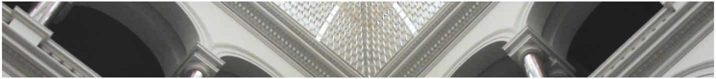
Sentiment Analysis

What are Recurrent Neural Networks (RNNs)?

Example Applications



Machine Translation (seq2seq architecture)



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What are Recurrent Neural Networks (RNNs)?

Example Applications

Proof. Omitted. □

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*
Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{étale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. □

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. □

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

be a morphism of algebraic spaces over S and Y.

Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. □

An RNN learns to create a LaTeX document.

`http://karpathy.github.io/2015/05/21/rnn-effectiveness/`

Unfolded computations of an RNN

Assume that $\theta = \{W^{\text{in}}, W^{\text{rec}}\}$, $\boldsymbol{r}_0 = \mathbf{0}$, and the **forward pass** is constructed as follows:

$$\boldsymbol{r}_1 = W^{\text{in}}\boldsymbol{x}_1 + W^{\text{rec}}\sigma(\boldsymbol{r}_0)$$

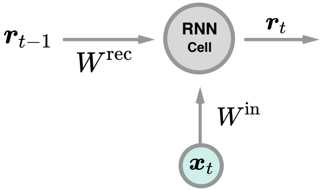
$$\vdots$$

$$\boldsymbol{r}_t = W^{\text{in}}\boldsymbol{x}_t + W^{\text{rec}}\sigma(\boldsymbol{r}_{t-1})$$

$$\vdots$$

$$\boldsymbol{r}_T = W^{\text{in}}\boldsymbol{x}_T + W^{\text{rec}}\sigma(\boldsymbol{r}_{T-1})$$

$$\hat{\boldsymbol{y}} = \boldsymbol{r}_T$$



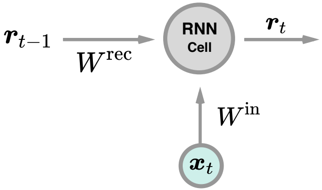
Unfolded computations of an RNN

Backpropagation Through Time can be derivsed by

$$\frac{\partial E}{\partial \theta} = \sum_{k=1}^{T} \frac{\partial E}{\partial \boldsymbol{r}_T} \frac{\partial \boldsymbol{r}_T}{\partial \boldsymbol{r}_k} \frac{\partial^+ \boldsymbol{r}_k}{\partial \theta}$$

$$\frac{\partial \boldsymbol{r}_T}{\partial \boldsymbol{r}_k} = \prod_{T \geq i > k} \frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{r}_{i-1}}$$

$$= \prod_{T \geq i > k} \underbrace{(W^{\text{rec}})^{T}}_{!} \sigma'(\boldsymbol{r}_{i-1})$$



$\frac{\partial^+ \boldsymbol{r}_{t+1}}{\partial \theta}$ is the “immediate”partial derivative. For example, any row in the matrix $\frac{\partial^+ \boldsymbol{r}_k}{\partial W^{\text{rec}}}$ is $\sigma(\boldsymbol{r}_t)$ [PMB13].

Exploding and Varnishing Gradient Problems

– The exponential term of W^{rec} will dominate the calculation of $\frac{\partial E}{\partial \theta}$.

Exploding and Varnishing Gradient Problems

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- Two problems can happen:
  - Exploding gradient ($\|\nabla_{\boldsymbol{\theta}} E\| \to \infty$)
  - Varnishing gradient ($\|\nabla_{\boldsymbol{\theta}} E\| \to 0$)

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- The exponential term of W^{rec} will dominate the calculation of $\frac{\partial E}{\partial \theta}$.
- Two problems can happen:
  - Exploding gradient ($\|\nabla_{\boldsymbol{\theta}} E\| \rightarrow \infty$)
  - Varnishing gradient ($\|\nabla_{\boldsymbol{\theta}} E\| \rightarrow 0$)
- The situation is determined by **spectral radius** of the matrix $\rho(W^{\text{rec}})$.
  - *Spectral radius* of a matrix is its largest absolute eigenvalue.

Exploding Gradient

- happens when $\rho(W^{\text{rec}}) > 1$.

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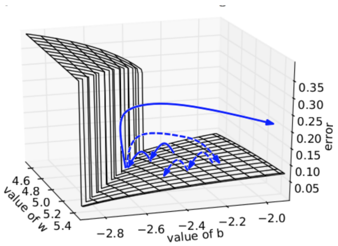
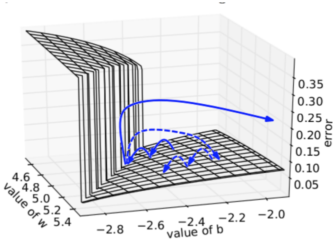


Error Function [PMB13].

Exploding Gradient

– happens when $\rho(W^{\text{rec}}) > 1$.



Error Function [PMB13].

– causes training *unstable*.

Exploding Gradient

Exploding gradient can be alleviated by **clipping** the norm of the gradient.

$\epsilon \leftarrow$ *threshold* ;

$\hat{\boldsymbol{g}} \leftarrow \nabla_{\boldsymbol{\theta}} J$;

if $||\hat{\boldsymbol{g}}|| \geq$ *threshold* **then**

$\quad \hat{\boldsymbol{g}} \leftarrow \frac{\epsilon}{||\hat{\boldsymbol{g}}||} \hat{\boldsymbol{g}}$;

end

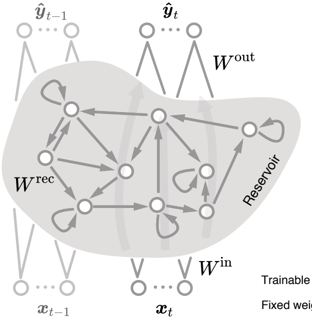
Algorithm 1: Gradient Clipping [PMB13].

Varnishing Gradient

- happens when $\rho(W^{\text{rec}}) < 1$.
- causes RNNs to learn long-term dependencies with a slow progress.
- **Solutions**
  - Echo State Networks
  - Momentum Technique
  - Long Short-Term Memory Networks

Echo State Networks [Jae01]

The idea of ESNs is to create a large random reservoir RNN (W^{rec}, W^{in}). The matrix W^{rec} is carefully constructed such that reservoir can keep signal **echoed** in the network for long time.



Trainable weights : W^{out}

Fixed weights : $W^{\text{rec}}, W^{\text{in}}$

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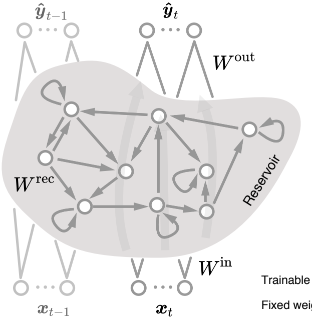
– $W^{\text{rec}} \leftarrow \frac{\xi}{\rho(W^{\text{rec}})} W^{\text{rec}}$



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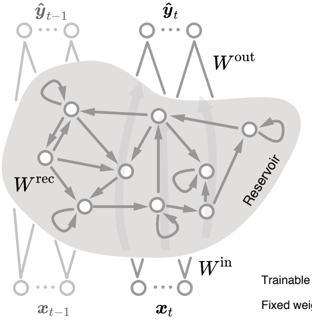
ESNs can be viewed as a kernel trick mapping input to a high-dimensional space and learn a linear classifier (W^{out}) there.



Trainable weights : W^{out}

Fixed weights : W^{rec}, W^{in}

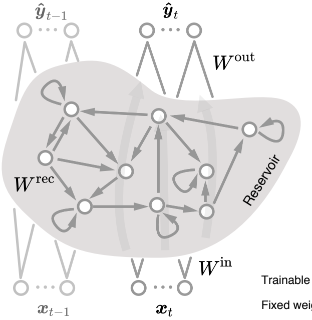
Echo State Networks [Jae01]

Advantages

- Can be trained very fast.
- Work well for low dimensional input.

Disadvantages

- Require experiences to initialize W^{rec} sensibly.
- Require a big reservoir to solve complex problems.



Trainable weights : W^{out}

Fixed weights : $W^{\text{rec}}, W^{\text{in}}$

Gradient Descent with Momentum

– Standard Gradient Descent

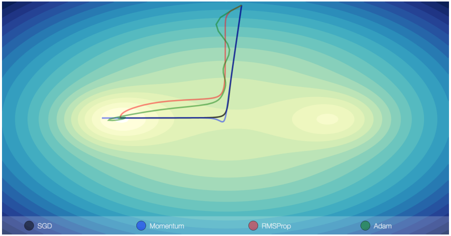
$$w \leftarrow w - \lambda \frac{\partial J}{\partial w}$$

– Adam [KB14]

$$w \leftarrow w - \lambda \frac{\partial J}{\partial w} + someformular$$

– RMSProp [TH]

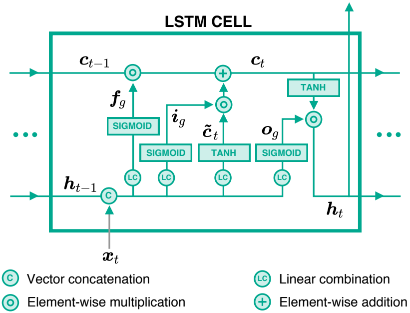
Gradient Descent with Momentum



https://cdn.rawgit.com/EmilienDupont/aaf429be5705b219aaaf8d691e27ca87/raw/d5c6bd2483479c8187f5b49e329d84651b930e09/index.html

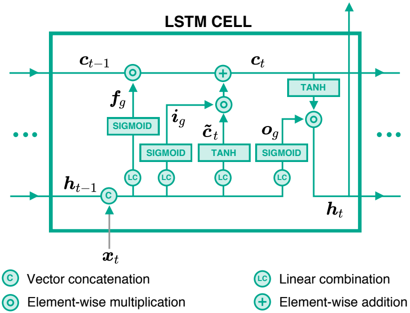
Long Short-Term Memory (LSTM) [HS97]

- Use gating mechanisms and additive updates for $\boldsymbol{c}_t$.
  - Eliminate exponential decay factors in the gradient computation

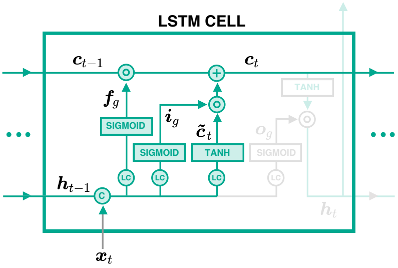


Long Short-Term Memory (LSTM) [HS97]

- Use gating mechanisms and additive updates for $\boldsymbol{c}_t$.
  - Eliminate exponential decay factors in the gradient computation
- Employ 3 gates:
  - Input gate $\boldsymbol{i}_g$
  - Forget gate $\boldsymbol{f}_g$
  - Output gate $\boldsymbol{o}_g$



Computations in an LSTM cell



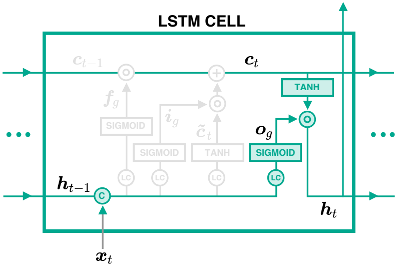
$$\boldsymbol{f}_g = \mathrm{sigm}(U^{(f)}\boldsymbol{x}_t + V^{(f)}\boldsymbol{h}_{t-1})$$

$$\boldsymbol{i}_g = \mathrm{sigm}(U^{(i)}\boldsymbol{x}_t + V^{(i)}\boldsymbol{h}_{t-1})$$

$$\tilde{\boldsymbol{c}}_t = \tanh(U^{(c)}\boldsymbol{x}_t + V^{(c)}\boldsymbol{h}_{t-1})$$

$$\boldsymbol{c}_t = \boldsymbol{f}_g \circ \boldsymbol{c}_{t-1} + \boldsymbol{i}_g \circ \tilde{\boldsymbol{c}}_t$$

Computations in an LSTM cell



$$\boldsymbol{o}_g = \mathrm{sigm}(U^{(o)}\boldsymbol{x}_t + V^{(o)}\boldsymbol{h}_{t-1})$$

$$\boldsymbol{h}_t = \boldsymbol{o}_g \circ \tanh(\boldsymbol{c}_t)$$

References I

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