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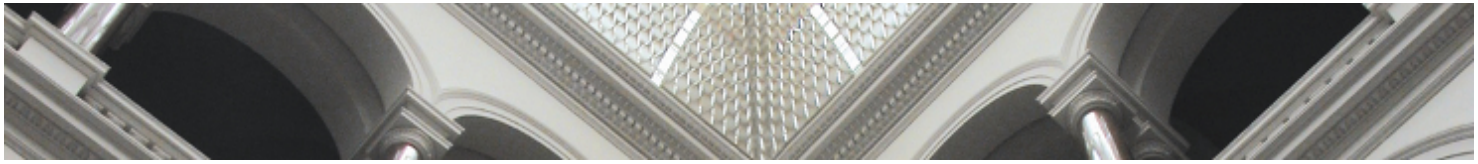
Recurrent Neural Networks

Dr. Grégoire Montavon | Technische Universität Berlin | Machine Learning Group



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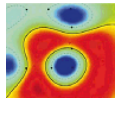
Outline

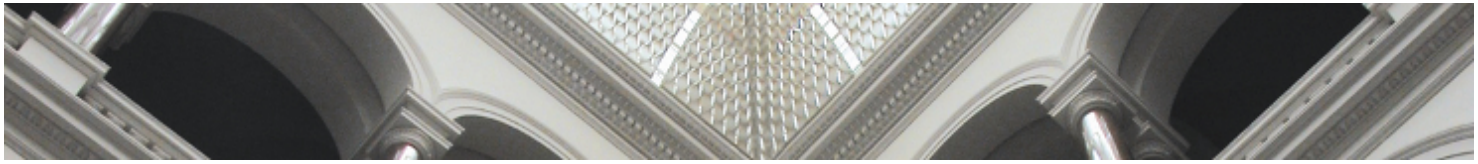
Recurrent Neural Networks

- Overview & Applications
- RNNs from differential equations (Unfolding RNNs)
- Statistics of RNNs

Exploding and Varnishing Gradient Problems

- Echo State Networks (ESNs)
- Gradient Descent with Momentum
- Long Short-Term Memory & Gated RNNs





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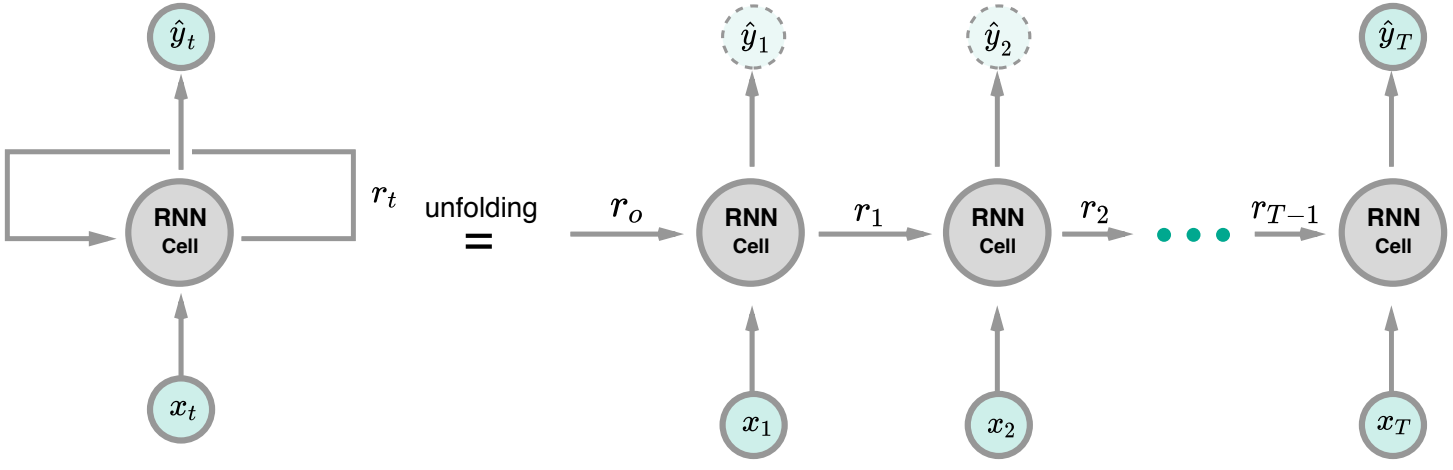
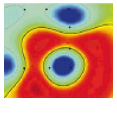
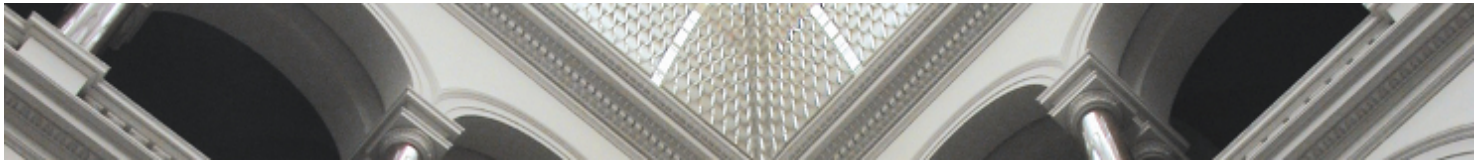
What are Recurrent Neural Networks (RNNs)?

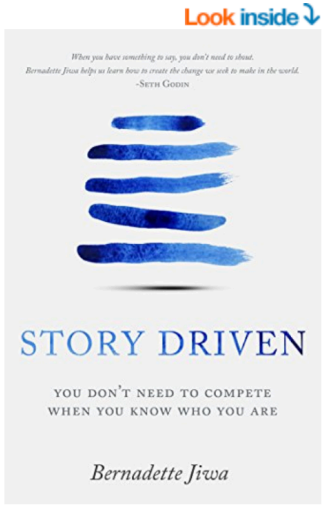


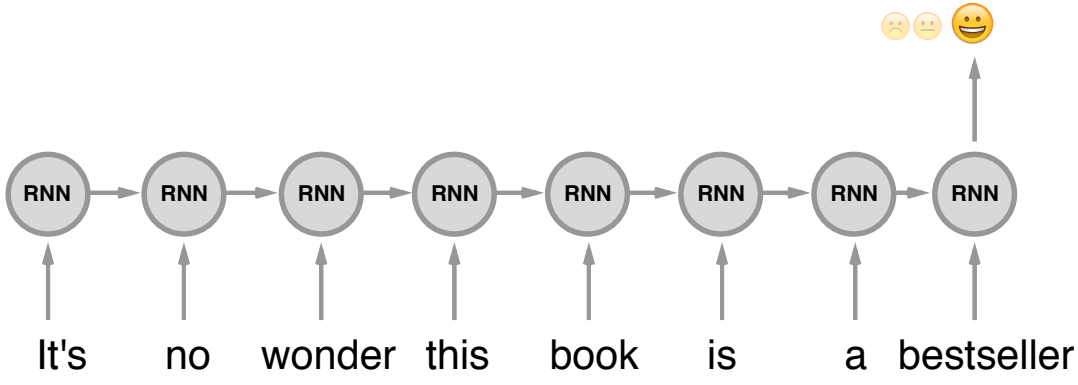




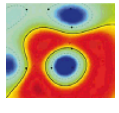
What are Recurrent Neural Networks (RNNs)?

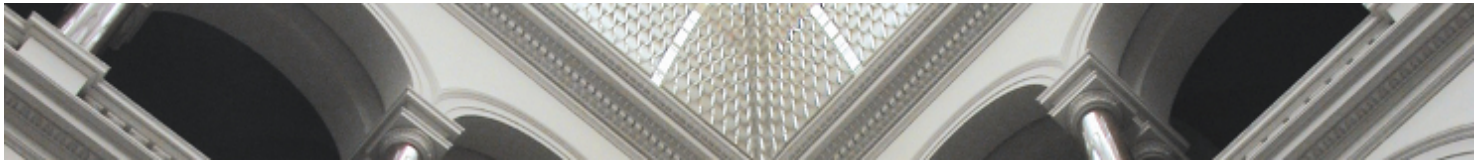
Example Applications





Sentiment Analysis





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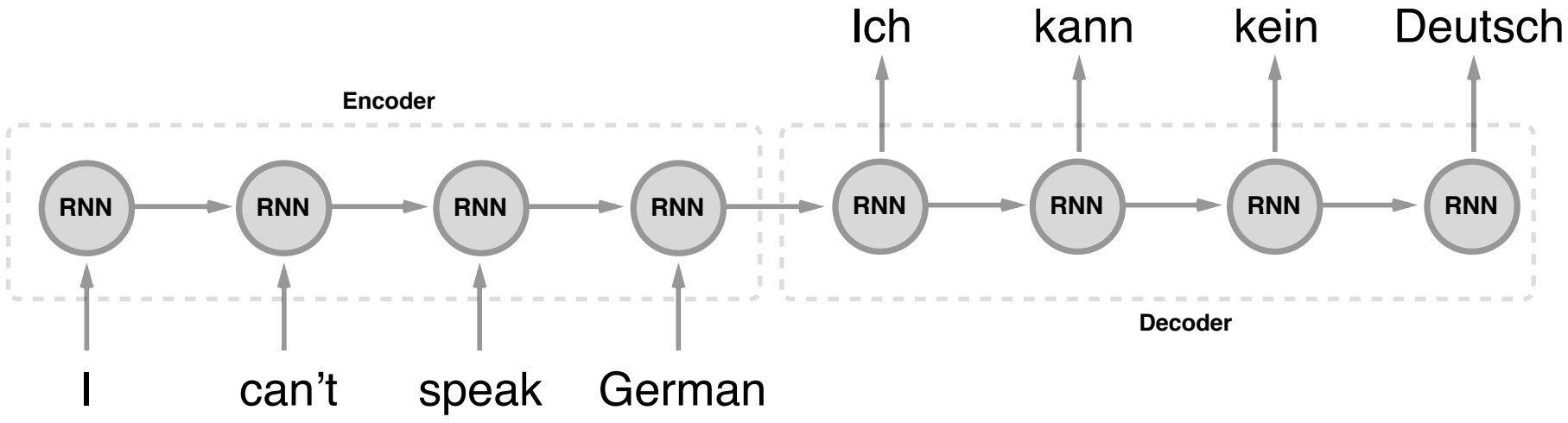
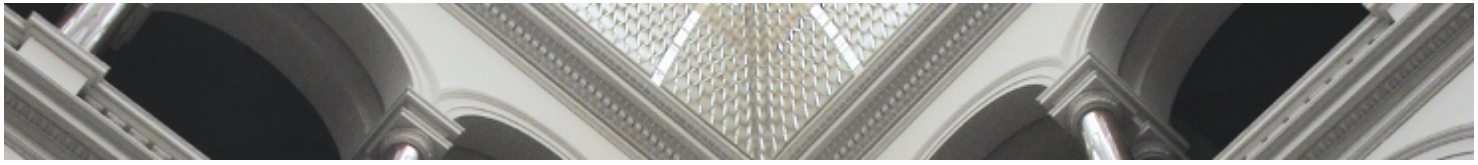
Example Applications



Machine Translation (seq2seq architecture)



What are Recurrent Neural Networks (RNNs)?

Example Applications

Proof. Omitted. □

Lemma 0.1. *Let $\mathcal{C}$ be a set of the construction.*

Let $\mathcal{C}$ be a gerber covering. Let $\mathcal{F}$ be a quasi-coherent sheaves of $\mathcal{O}$-modules. We have to show that

$$\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})$$

.

Proof. This is an algebraic space with the composition of sheaves $\mathcal{F}$ on $X_{\acute{e}tale}$ we have

$$\mathcal{O}_X(\mathcal{F}) = \{morph_1 \times_{\mathcal{O}_X} (\mathcal{G}, \mathcal{F})\}$$

where $\mathcal{G}$ defines an isomorphism $\mathcal{F} \to \mathcal{F}$ of $\mathcal{O}$-modules. □

Lemma 0.2. *This is an integer $\mathcal{Z}$ is injective.*

Proof. See Spaces, Lemma ??. □

Lemma 0.3. *Let S be a scheme. Let X be a scheme and X is an affine open covering. Let $\mathcal{U} \subset \mathcal{X}$ be a canonical and locally of finite type. Let X be a scheme. Let X be a scheme which is equal to the formal complex.*

The following to the construction of the lemma follows.

Let X be a scheme. Let X be a scheme covering. Let

$$b : X \to Y' \to Y \to Y \to Y' \times_X Y \to X.$$

be a morphism of algebraic spaces over S and Y.

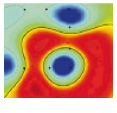
Proof. Let X be a nonzero scheme of X. Let X be an algebraic space. Let $\mathcal{F}$ be a quasi-coherent sheaf of $\mathcal{O}_X$-modules. The following are equivalent

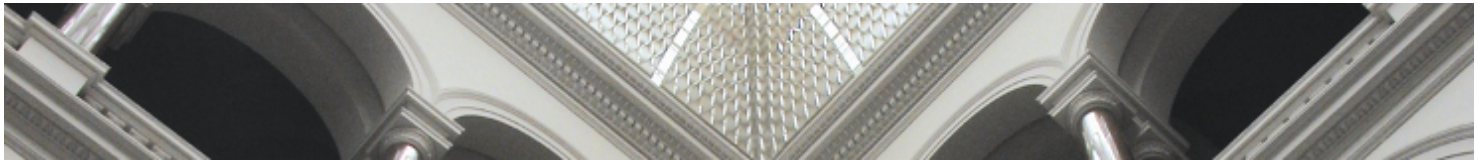
(1) $\mathcal{F}$ is an algebraic space over S.
(2) If X is an affine open covering.

Consider a common structure on X and X the functor $\mathcal{O}_X(U)$ which is locally of finite type. □

An RNN learns to create a LaTeX document.

`http://karpathy.github.io/2015/05/21/rnn-effectiveness/`





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Unfolded computations of an RNN

Assume that $\theta = \{W^{\text{in}}, W^{\text{rec}}\}$, $\boldsymbol{r}_0 = \mathbf{0}$, and the **forward pass** is constructed as follows:

$$\boldsymbol{r}_1 = W^{\text{in}}\boldsymbol{x}_1 + W^{\text{rec}}\sigma(\boldsymbol{r}_0)$$

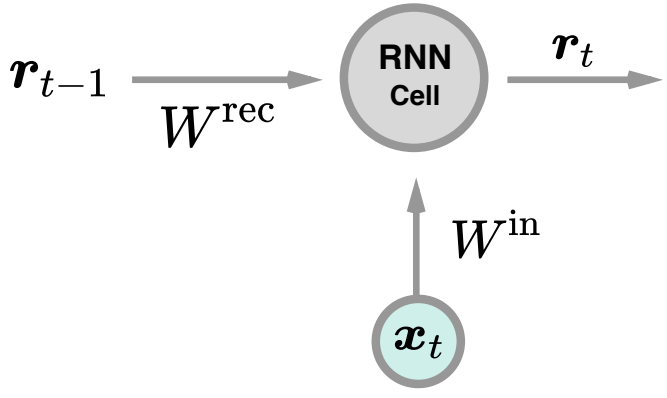
$$\vdots$$

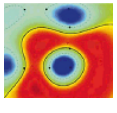
$$\boldsymbol{r}_t = W^{\text{in}}\boldsymbol{x}_t + W^{\text{rec}}\sigma(\boldsymbol{r}_{t-1})$$

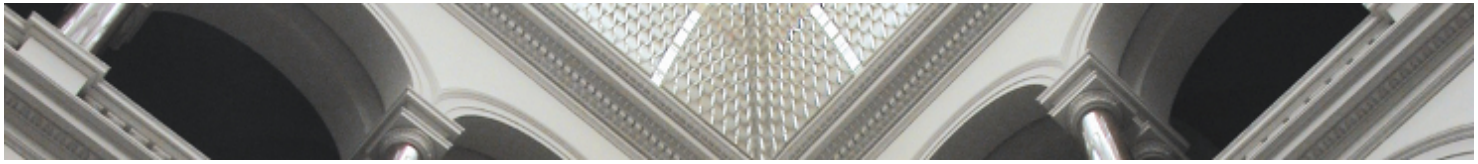
$$\vdots$$

$$\boldsymbol{r}_T = W^{\text{in}}\boldsymbol{x}_T + W^{\text{rec}}\sigma(\boldsymbol{r}_{T-1})$$

$$\hat{\boldsymbol{y}} = \boldsymbol{r}_T$$







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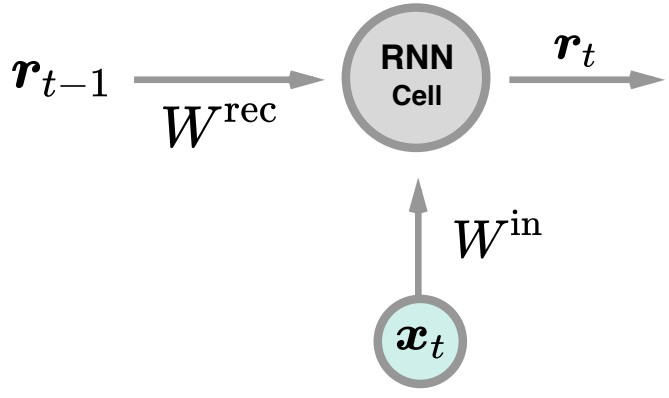
Unfolded computations of an RNN

Backpropagation Through Time can be derivsed by

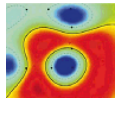
$$\frac{\partial E}{\partial \theta} = \sum_{k=1}^{T} \frac{\partial E}{\partial \boldsymbol{r}_T} \frac{\partial \boldsymbol{r}_T}{\partial \boldsymbol{r}_k} \frac{\partial^+ \boldsymbol{r}_k}{\partial \theta}$$

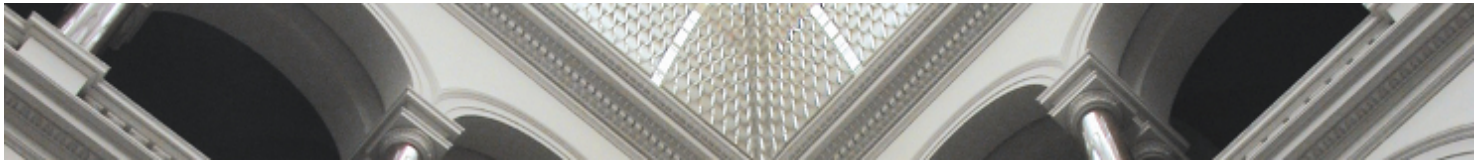
$$\frac{\partial \boldsymbol{r}_T}{\partial \boldsymbol{r}_k} = \prod_{T \geq i > k} \frac{\partial \boldsymbol{r}_i}{\partial \boldsymbol{r}_{i-1}}$$

$$= \prod_{T \geq i > k} \underbrace{(W^{\text{rec}})^T}_{!} \operatorname{diag}(\sigma'(\boldsymbol{r}_{i-1}))$$



$\frac{\partial^+ \boldsymbol{r}_{t+1}}{\partial \theta}$ is the "immediate"partial derivative. For example, any row in the matrix $\frac{\partial^+ \boldsymbol{r}_k}{\partial W^{\text{rec}}}$ is $\sigma(\boldsymbol{r}_t)$ [PMB13].





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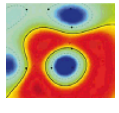
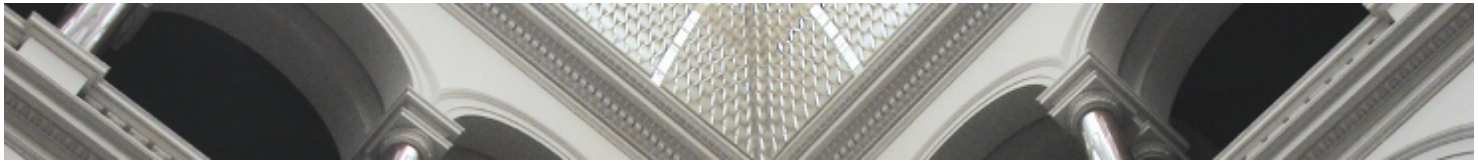
Exploding and Varnishing Gradient Problems

- The exponential term of W^{rec} will dominate the calculation of $\frac{\partial E}{\partial \theta}$.
- Two problems can happen:
  - Exploding gradient ($\left\|\nabla_{\boldsymbol{\theta}} E\right\| \to \infty$)
  - Varnishing gradient ($\left\|\nabla_{\boldsymbol{\theta}} E\right\| \to 0$)
- The situation is determined by **spectral radius** of the matrix $\rho\left(W^{\text{rec}}\right)$.
  - *Spectral radius* of a matrix is its largest absolute eigenvalue.





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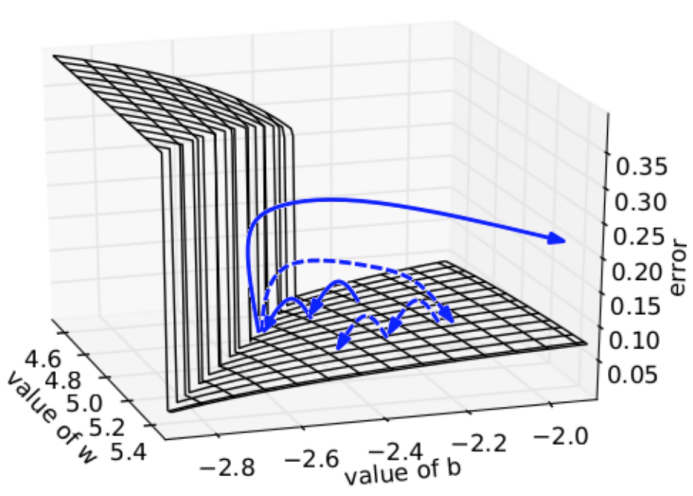
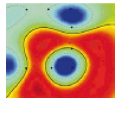
Exploding Gradient

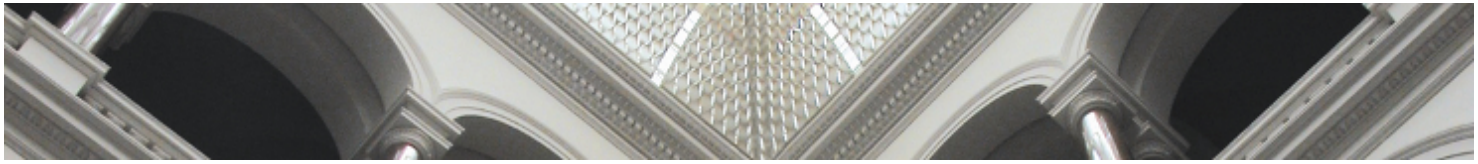
– happens when $\rho(W^{\text{rec}}) > 1$ (for linear cases).



Error Function [PMB13].

– causes training *unstable*.





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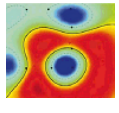
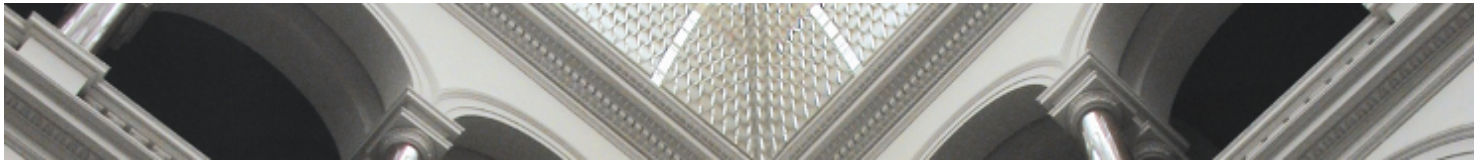
Exploding Gradient

Exploding gradient can be alleviated by **clipping** the norm of the gradient.

$\epsilon \leftarrow$ *threshold* ;
$\hat{\boldsymbol{g}} \leftarrow \nabla_{\theta} J$;
if $\|\hat{\boldsymbol{g}}\| \geq$ *threshold* **then**
  $\hat{\boldsymbol{g}} \leftarrow \frac{\epsilon}{\|\hat{\boldsymbol{g}}\|} \hat{\boldsymbol{g}}$;
end

Algorithm 1: Gradient Clipping [PMB13].





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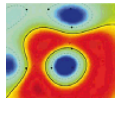
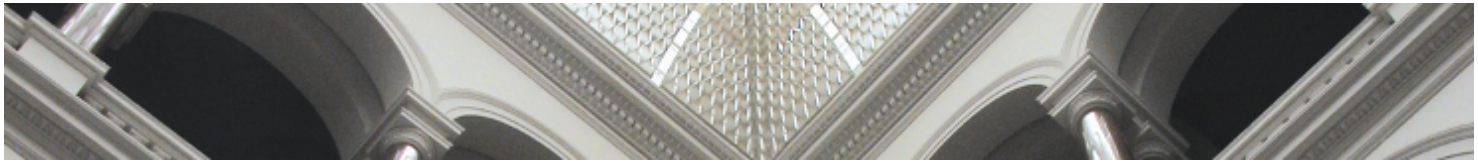
Varnishing Gradient

- happens when $\rho(W^{\text{rec}}) < 1$.
- causes RNNs to learn long-term dependencies with a slow progress.
- **Solutions**
  - Echo State Networks (ESNs)
  - Momentum Technique
  - Long Short-Term Memory Networks





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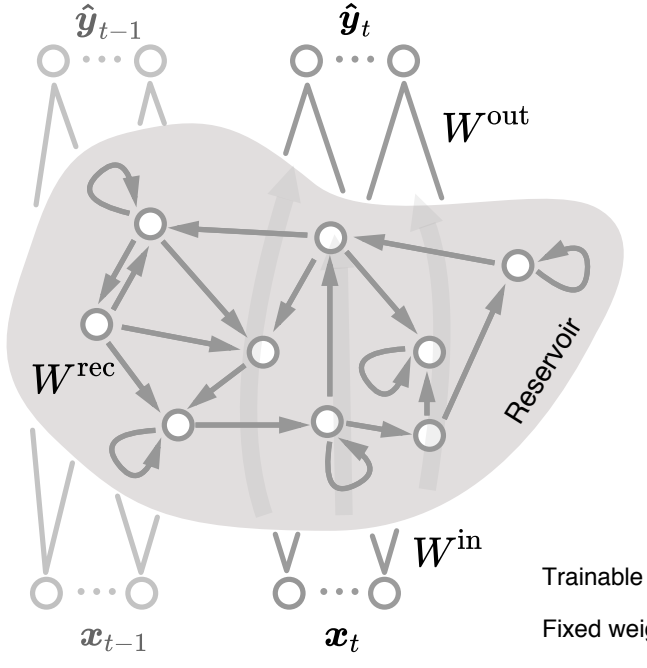
Echo State Networks [Jae01]

The idea of ESNs is to create a large random reservoir RNN (W^{rec}, W^{in}). The matrix W^{rec} is carefully constructed such that reservoir can keep signal **echoed** in the network for long time.

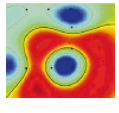
$$\tilde{\boldsymbol{h}}_t = \tanh(W^{\text{in}}\boldsymbol{x}_t + W^{\text{rec}}\boldsymbol{h}_{t-1}),$$
$$\boldsymbol{h}_t = (1-\alpha)\boldsymbol{h}_{t-1} + \alpha\tilde{\boldsymbol{h}}_t,$$
$$\hat{\boldsymbol{y}}_t = W^{\text{out}}\text{concat}(\boldsymbol{x}_t, \boldsymbol{h}_t),$$

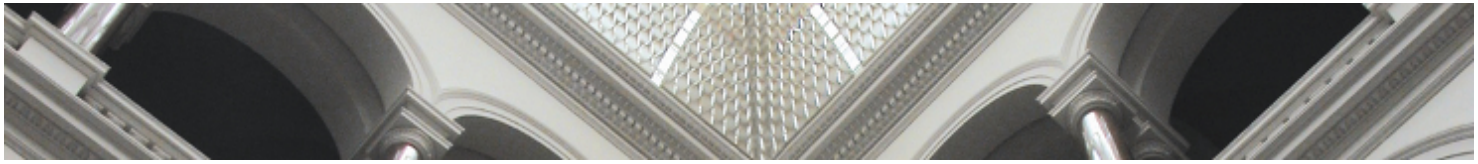
where $W^{\text{rec}} \in \mathbb{R}^{M\times M}$ with M is the number of reservior units, and $\alpha \in (0, 1]$ is the leaking rate.



Trainable weights : W^{out}

Fixed weights : $W^{\text{rec}}, W^{\text{in}}$





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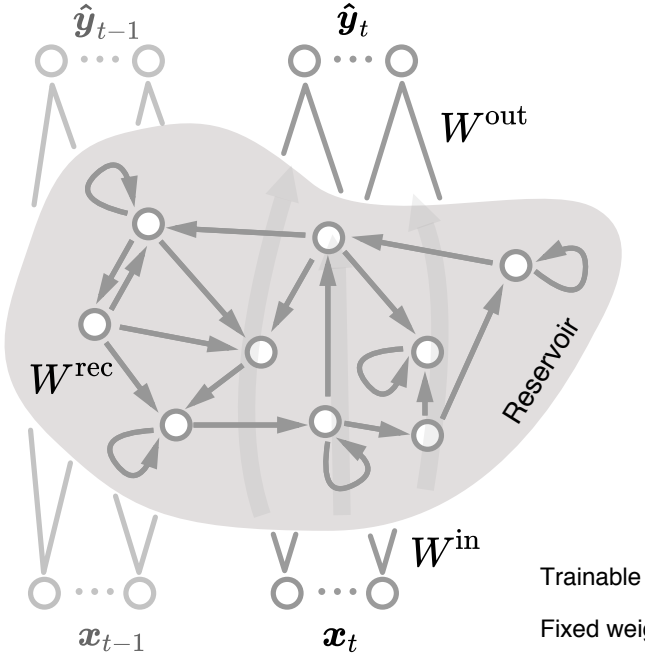
Echo State Networks [Jae01]

To have the echo state property, the W^{rec} is typically initialized as

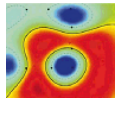
$$W^{\text{rec}} \leftarrow \frac{\xi}{\rho(W^{\text{rec}})} W^{\text{rec}},$$

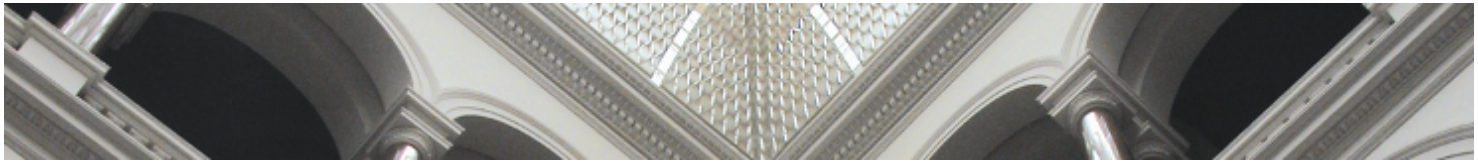
where ξ is a hyperparameter.



Trainable weights : W^{out}

Fixed weights : $W^{\text{rec}}, W^{\text{in}}$





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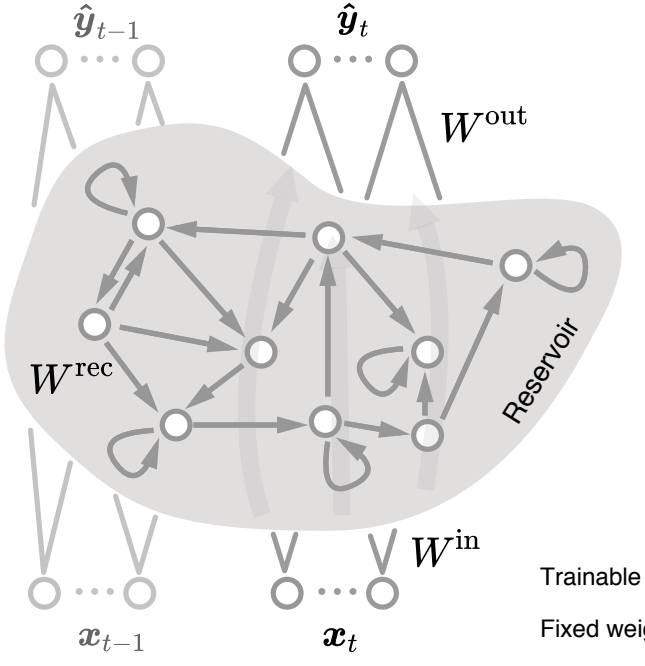
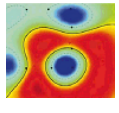
Echo State Networks [Jae01]

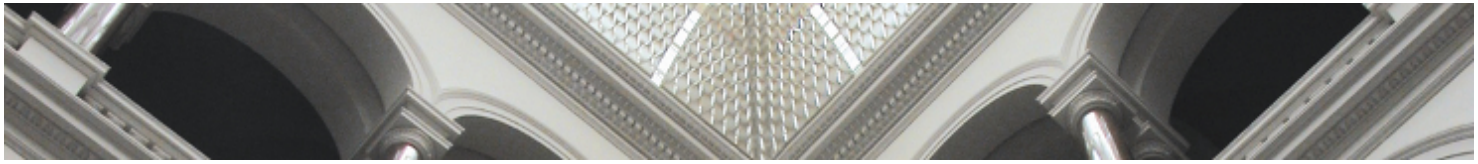
ESNs can be viewed as a kernel trick mapping input to a high-dimensional space and learn a linear classifier (W^{out}) there.



Trainable weights : W^{out}

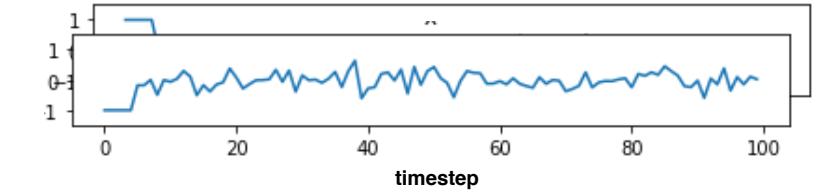
Fixed weights : $W^{\text{rec}}, W^{\text{in}}$

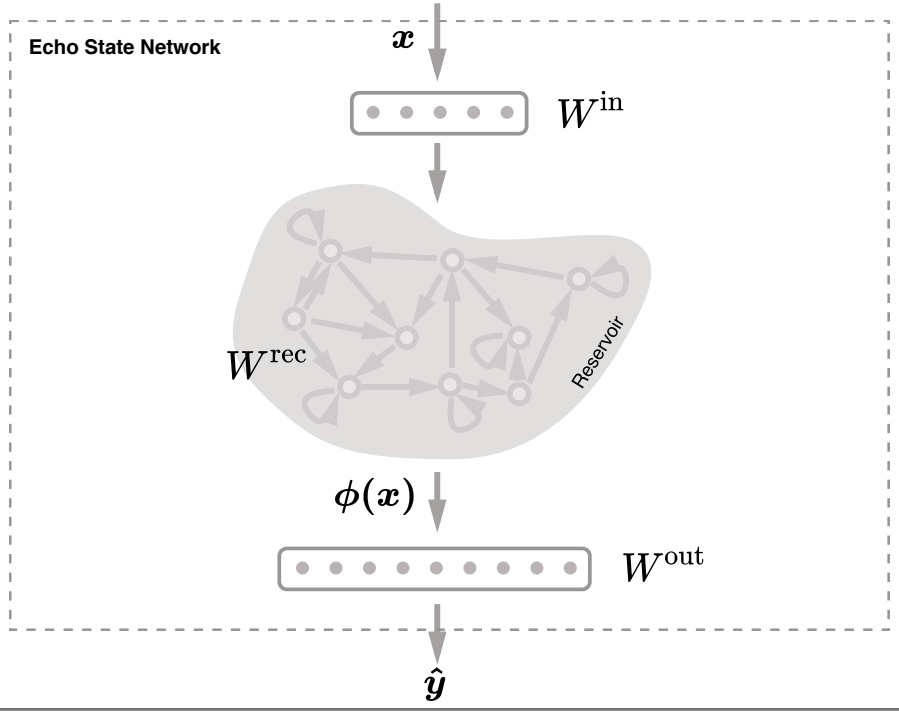


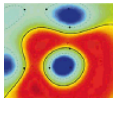


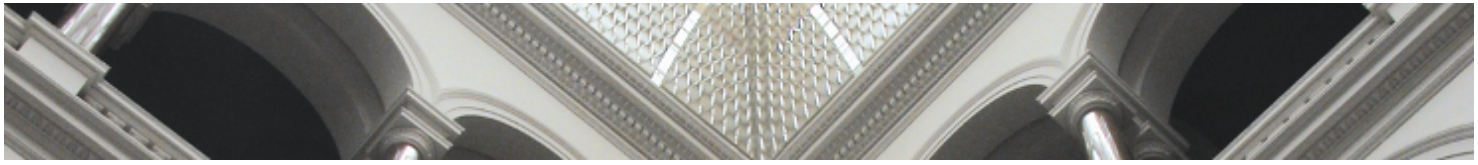
Example : ESNs for signal classification

Two classes of signal $\{-1,1\}$

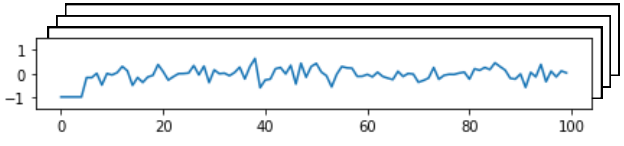








Example : ESNs for signal classification

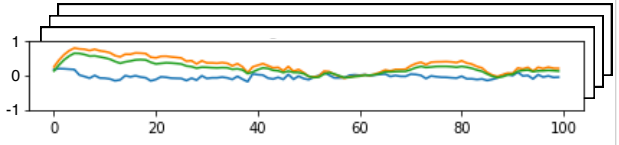


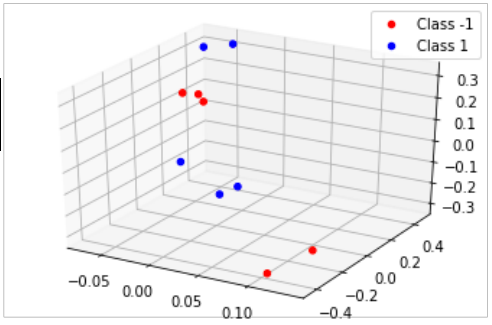
ESNs with 3 reserviors

Hidden States

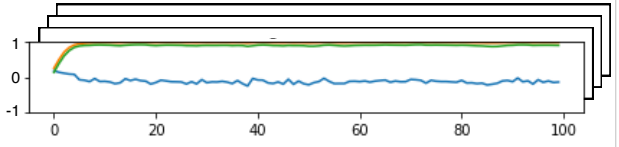
$\phi(\boldsymbol{x})$

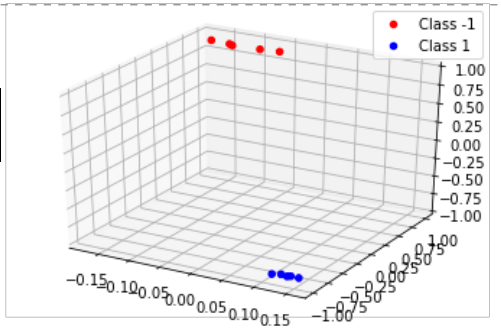
unnormalized W^{rec}

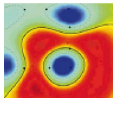


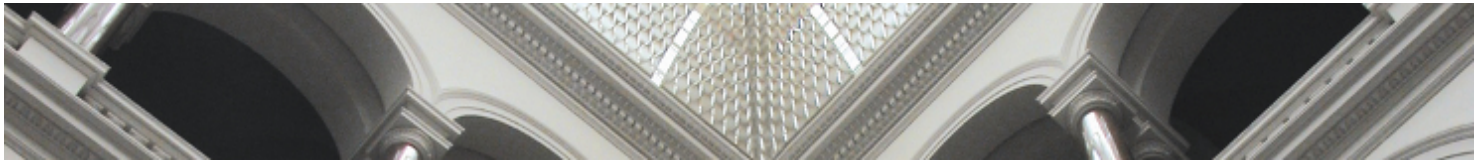


properly normalized W^{rec}









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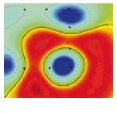
Advantages and Disadvantages of ESNs

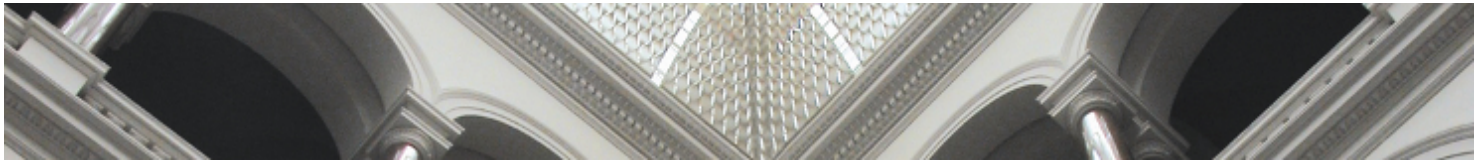
Advantages

- Can be trained very fast.
- Work well for low dimensional input.

Disadvantages

- Require experiences to initialize W^{rec} sensibly.
- Require a big reservoir to solve complex problems.





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Gradient Descent with Momentum

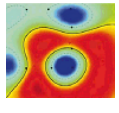
- Standard Gradient Descent

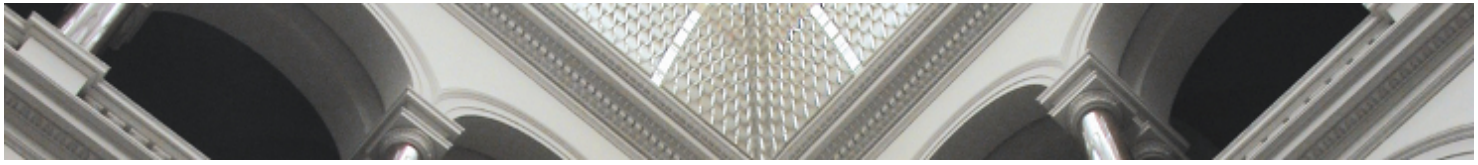
$$w \leftarrow w - \lambda \frac{\partial J}{\partial w}$$

- Adam [KB14]

$$w \leftarrow w - \lambda \frac{\partial J}{\partial w} + \textit{someformular}$$

- RMSProp [TH]





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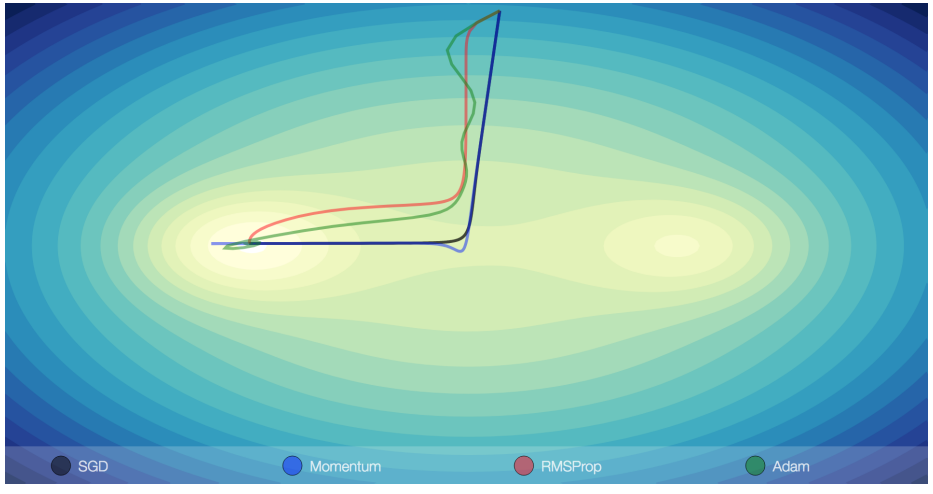
Gradient Descent with Momentum



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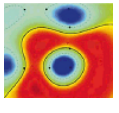

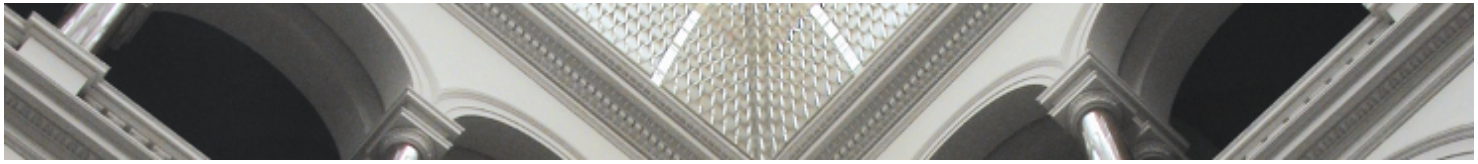

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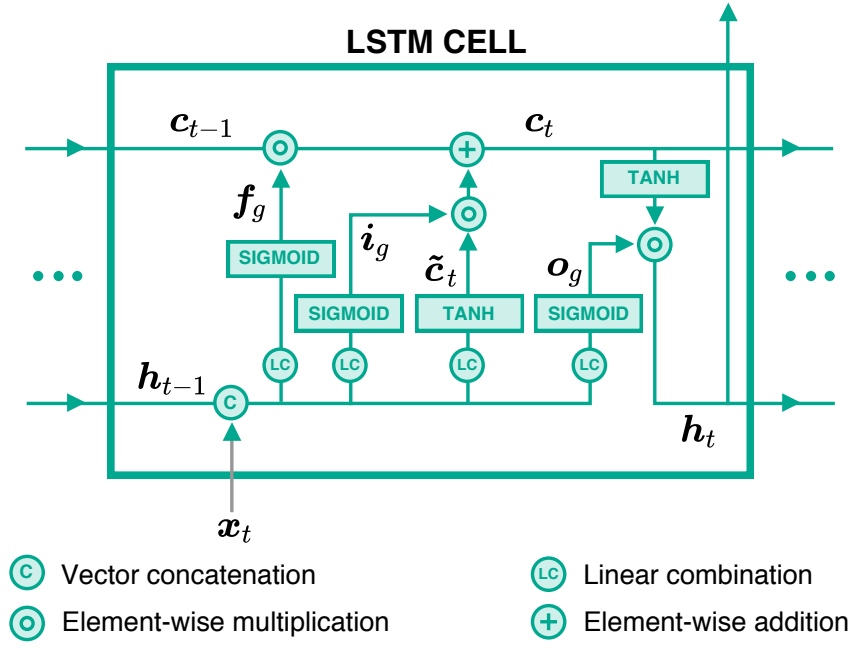
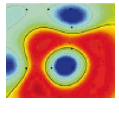
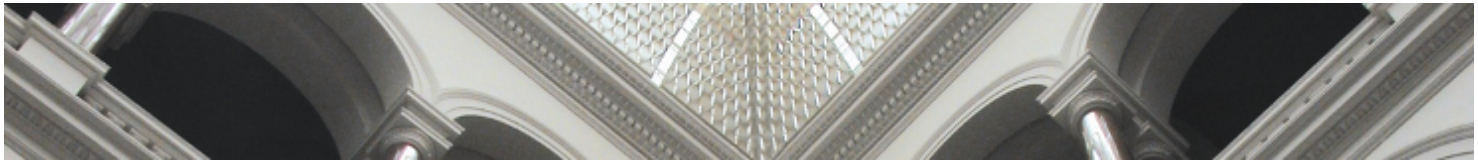
Long Short-Term Memory (LSTM) [HS97]

- Use gating mechanisms and additive updates for $\boldsymbol{c}_t$.
  - Eliminate exponential decay factors in the gradient computation

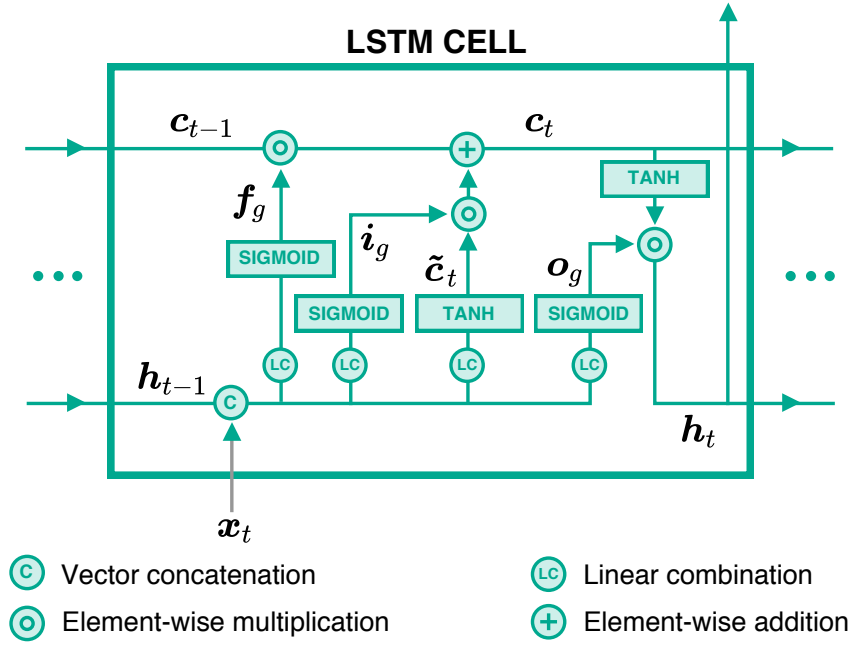


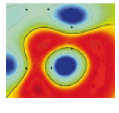


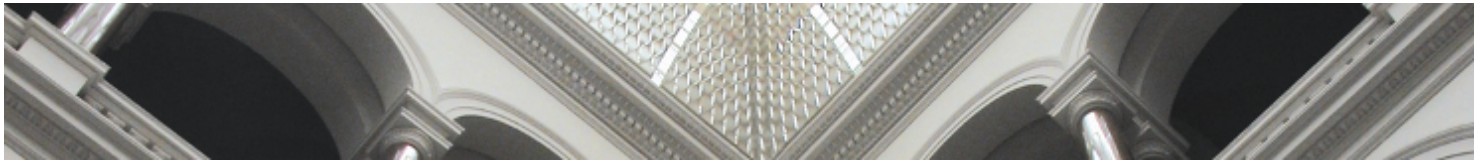


Long Short-Term Memory (LSTM) [HS97]

- Use gating mechanisms and additive updates for $\boldsymbol{c}_t$.
  - Eliminate exponential decay factors in the gradient computation
- Employ 3 gates:
  - Input gate $\boldsymbol{i}_g$
  - Forget gate $\boldsymbol{f}_g$
  - Output gate $\boldsymbol{o}_g$







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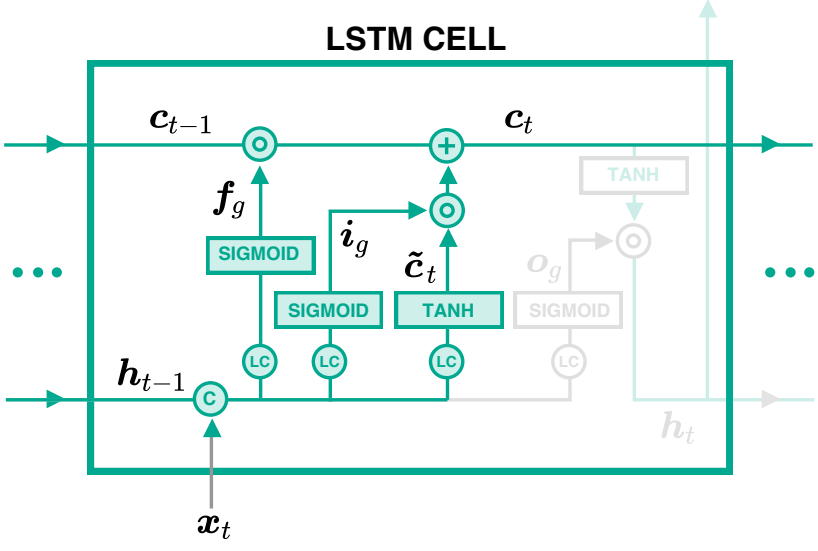
Computations in an LSTM cell

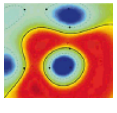


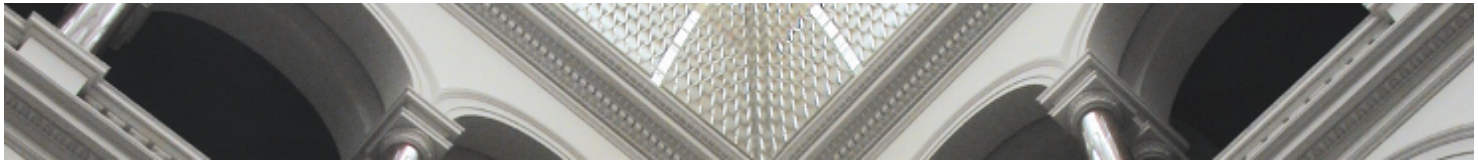
$$f_g = \text{sigm}(U^{(f)} x_t + V^{(f)} h_{t-1})$$

$$i_g = \text{sigm}(U^{(i)} x_t + V^{(i)} h_{t-1})$$

$$\tilde{c}_t = \tanh(U^{(c)} x_t + V^{(c)} h_{t-1})$$

$$c_t = f_g \circ c_{t-1} + i_g \circ \tilde{c}_t$$





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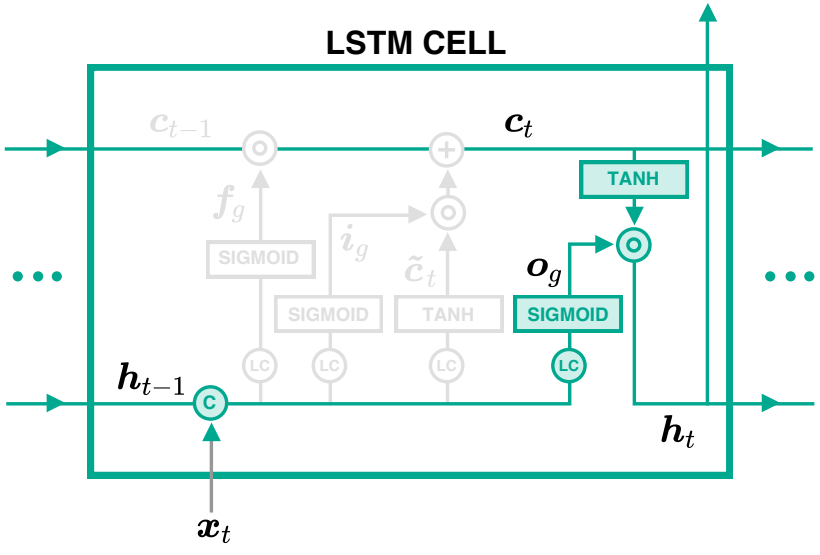
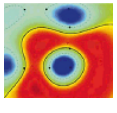
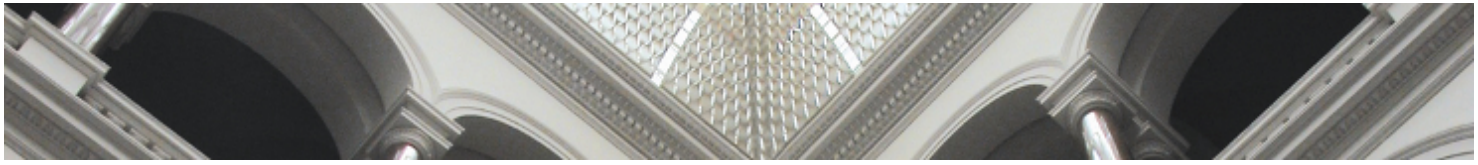
Computations in an LSTM cell



$$o_g = \mathrm{sigm}(U^{(o)} x_t + V^{(o)} h_{t-1})$$

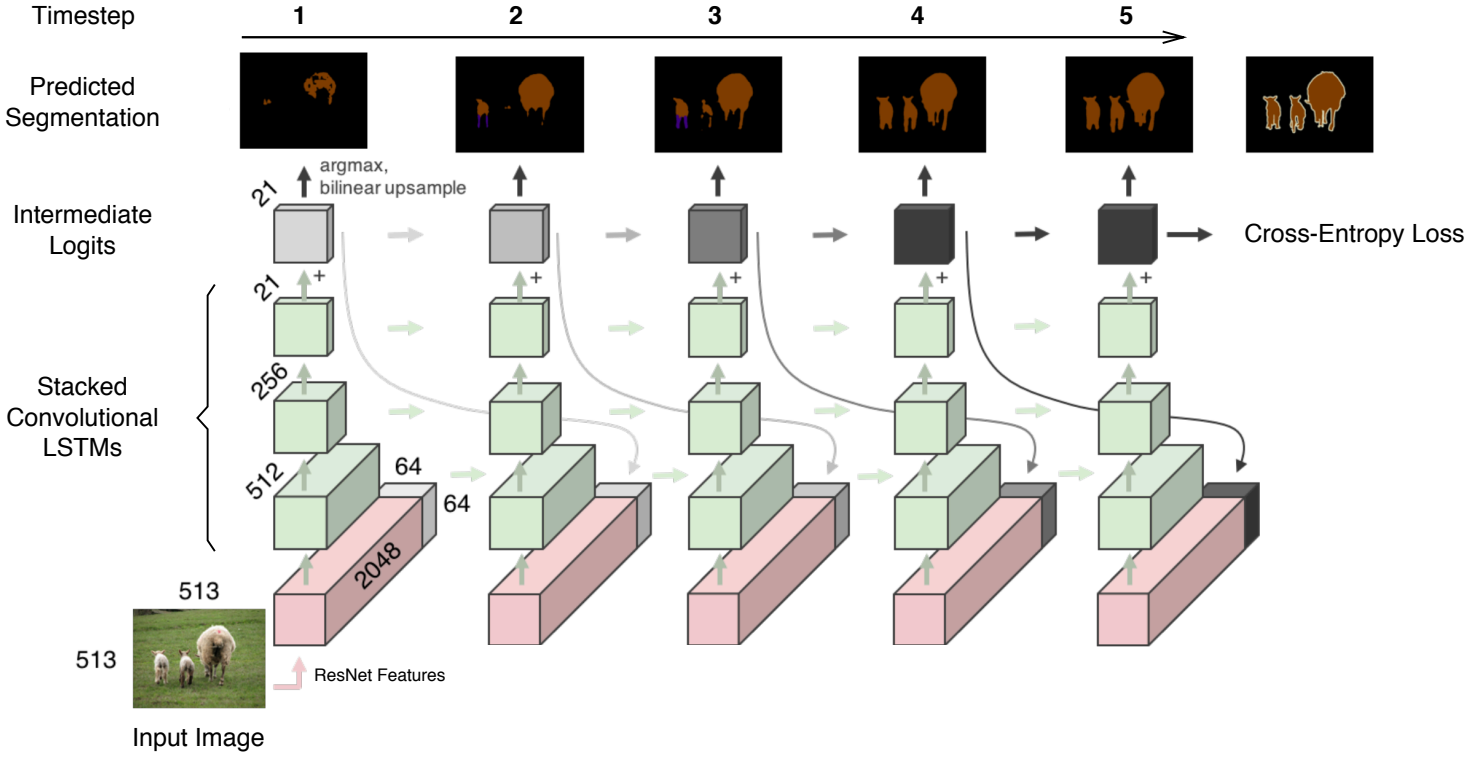
$$h_t = o_g \circ \tanh(c_t)$$

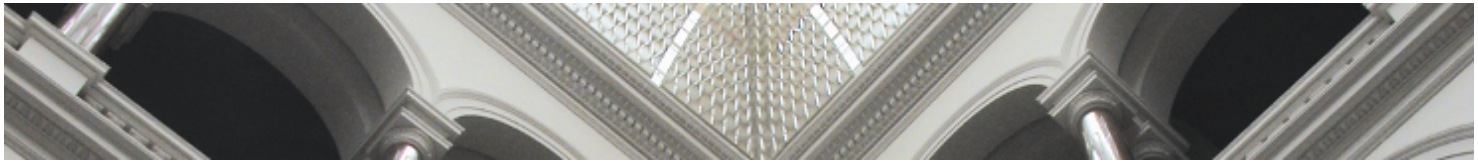




Deep RNNs

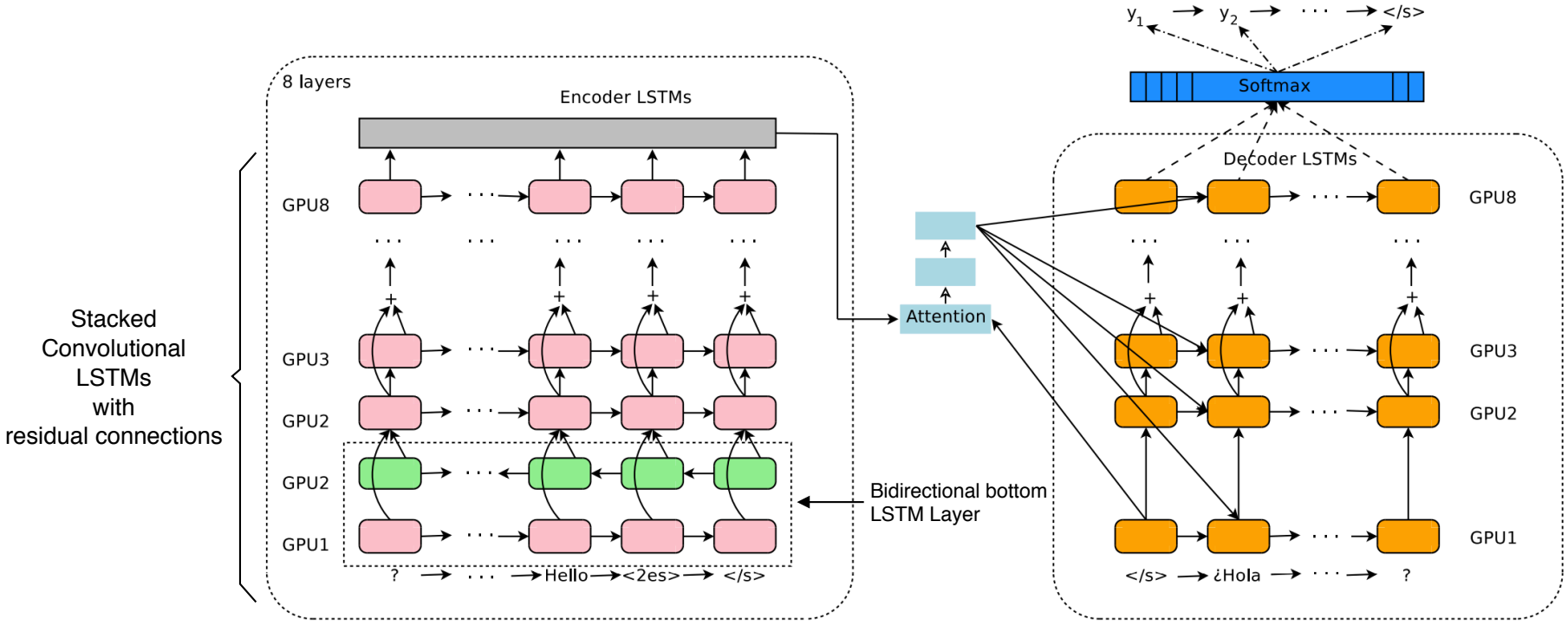
- An RNN for image segmentation [MSMS17].

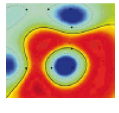


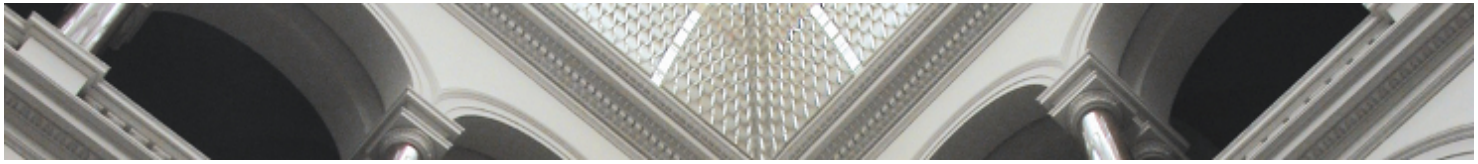


Deep RNNs

- Google's Neural Machine Translation System [WSC+16].







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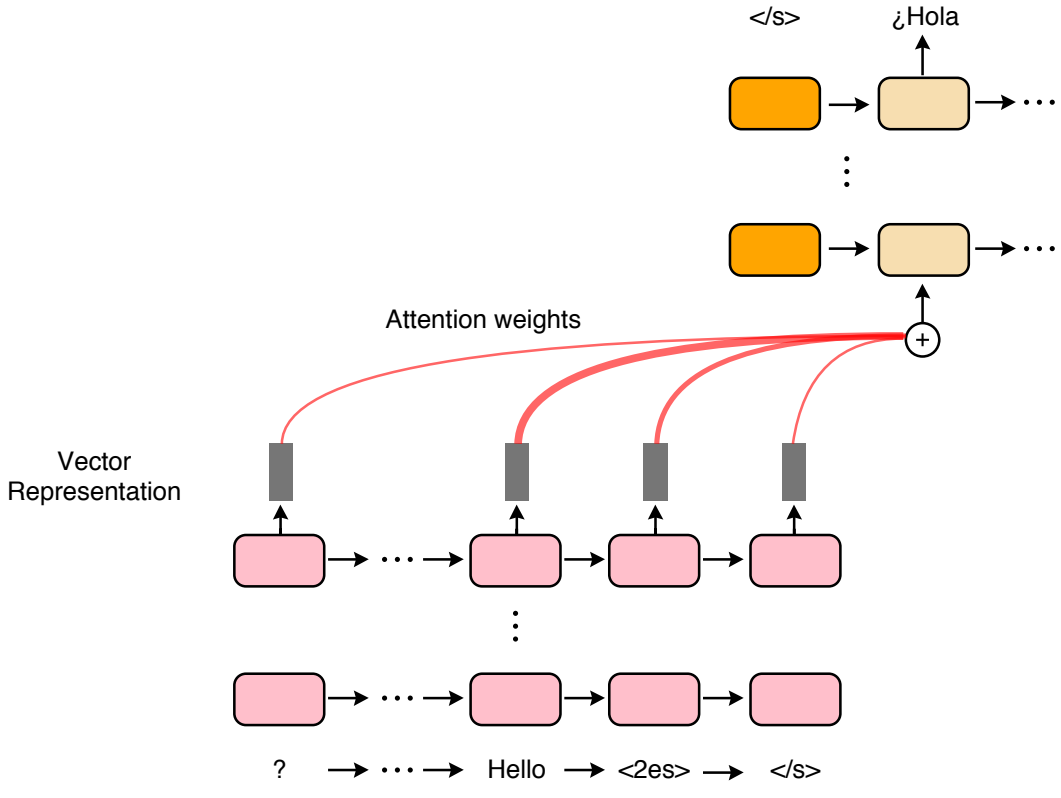

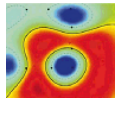

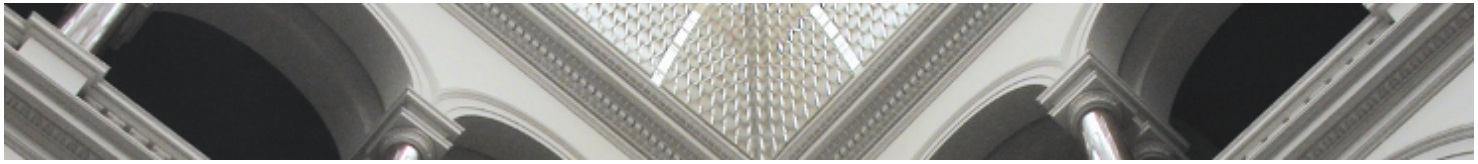

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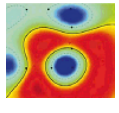

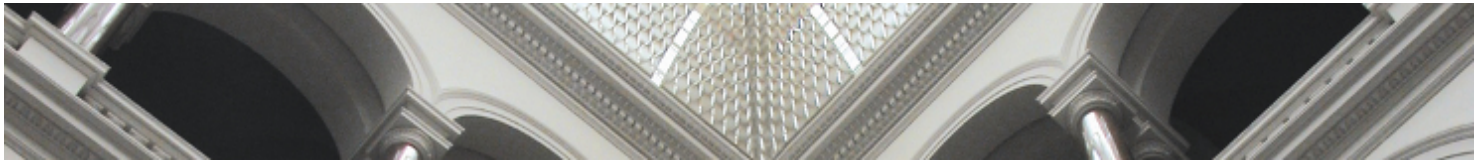

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