Homework 1, Total (20 points), Score __________

Name __________, Due Time, January 26, Wednesday

1. (2 points) The first stage in the life of the Hawaiian green sea turtle (eggs and hatchlings) occurs during the first year.

Stage 2, juveniles, extends from year 1 to 16.

Suppose 23% of the hatchlings survive and move to stage 2, while 67.9% of those in Stage 2 remain in that stage each year.

In one year, Stage 1 has 808,988 turtles, and Stage 2 has 715,774 turtles.

a. Give a vector, **p**, with real number elements representing the percentages.

b. Give a vector, **s**, storing the individuals in Stages 1 and 2.

c. Using variables **p** and **s**, not the data, give the vector operation to determine the number of individuals that will be in Stage 2 the following year.

d. Calculate this value

2. (1 point) For the vector **v** = (5, 0, -1), written as a column vector, and the matrix $A = \begin{bmatrix} 1 & 3 & 9 \\ 0 & 5 & 6 \end{bmatrix}$,

a) Calculate $A\mathbf{v}$.

b) For a 5×8 matrix B, give the size of a vector **w** for which we can calculate $B\mathbf{w}$.

c) Give the resulting size of $B\mathbf{w}$.

3. (2 points) For the following matrices:

$$A = \begin{bmatrix} 8 & 5 & 3 & -4 \\ -5 & 1 & 0 & 1 \end{bmatrix},\ B = \begin{bmatrix} 2 & -6 \\ 7 & 1 \\ 4 & 3 \\ -9 & -2 \end{bmatrix},\ C = \begin{bmatrix} 6 & 5 \\ 1 & -3 \\ 2 & -8 \end{bmatrix},\ I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

and $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Evaluate each of the following:

AB, **b.** BA, **c.** CI_2, **d.** I_3C

4. (1 points) Express the following system of equations using a matrix-vector notation:

$$\begin{cases} 2x - y = 7 \\ 6x \quad\ = 5 \end{cases}$$

$$\begin{cases} 2x = 3y - z = 5 \\ x - z = 10 \\ 3x = 2y - 6x = 7 \end{cases}$$

5. (2 points) Suppose an insect has maximum life expectancy of two months. On the average, this animal has 10 offspring in the first month and 300 in the second. The survival rate from the first to the second month of life is only 1%. Assume half the offspring are female. Suppose initially a region has 2 females in their first month of life and 1 in her second.

- **a.** Define the variables of the model.
- **b.** Construct a system of equations for the model.
- **c.** Give the matrix representation for the model.
- **d.** Using matrix multiplication, determine the number of females for each age at time $t = 1$ month expressed to two decimal places.
- **e.** Determine the number of females for each age at time $t = 2$ months.

6. (1 points) Give the Leslie matrix for a system with four classes, where the (female) reproduction rates are 0.2, 1.2, 1.4, and 0.7 for classes 1 to 4, respectively, and the survival rates are 0.3, 0.8, and 0.5 for classes 1 to 3, respectively.

7. (2 points) Consider the Leslie matrix $L = \begin{bmatrix} 5 & 150 \\ 0.01 & 0 \end{bmatrix}$ from a prior exercise with the initial population distribution vector $\boldsymbol{x(0)} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

- **a.** Using a computational tool, for each age class, give the value to which its percentage of the total population converges as time progresses. Express your answer to six significant figures.
- **b.** Using a computational tool, give the number, λ, to which the quotient of each class population at time t over the class population at time t - 1 converges as time progresses. Express your answer to six significant figures.
- **c.** Using the values from Parts a and b, give a vector $\boldsymbol{v}$ that satisfies $L\boldsymbol{v} = \lambda\boldsymbol{v}$.
- **d.** Give another vector $\boldsymbol{v}$ that satisfies the equation from Part c, where a million times more insects are in their first month of life.
- **e.** Give the continuous population growth rate.

8. (2 points) Australian cane toad data from (Lampo and De Leo, 1998)

From	**To**	**Mean Probability**
Egg	Tadpole	0.718
Tadpole	Juvenile	0.05
Juvenile	Adult	0.05
Adult	Adult	0.50

a. Draw a state diagram for the model.
b. Develop a Lefkovitch matrix model, L, using the mean probabilities from Table above with a fecundity of 7,500 female eggs and determine the finite rate of population change, λ. If the animal could maintain such annual population growth, would you anticipate the cane toad population to increase or decrease over time?
c. Use Matlab or NumPy to Calculate the population of the first 20 years, X(0)…X(19) based on the mean probability
d. Based on your answers to c, what is the intrinsic rate of growth of the population?

9. (3 points) Consider the following Leslie matrix representing a population, where the basic unit of time is one year:

$$\begin{bmatrix} 0 & 0.2 & 1.3 & 3.5 \\ 0.1 & 0 & 0 & 0 \\ 0 & 0.2 & 0 & 0 \\ 0 & 0 & 0.4 & 0 \end{bmatrix}$$

a. Give the animal's maximum life span, and describe the meaning of each positive number in the matrix.
b. Draw a state diagram for the animal.
c. Use Matlab to Calculate the population of the first 20 years, X(0)…X(19) assume the initial population is X(0) =[6000, 900, 160, 60].
d. Based on your answers to C, what is the intrinsic rate of growth of the population?
e. What is your conclusion about the species, population is growing, extinct, or stay stable in long term? If stable, what is the proportion of each age group?
f. Determine the sensitivity of λ to the second row, first column parameter (0.1).
g. Determine the sensitivity of λ to the second row, second column parameter.
h. Determine the sensitivity of λ to the third row, second column parameter.
i. Determine the sensitivity of λ to the fourth row, third column parameter.
j. Determine the sensitivity of λ to the fourth row, fourth column parameter.
k. Based on your answers to Parts f-j, where should conservation efforts focus?

10. (4 points) Australian cane toad data from (Lampo and De Leo, 1998)

From	**To**	**Mean Probability**	**Probability Range**
Egg	**Tadpole**	**0.718**	**0.688-0.738**
Tadpole	Juvenile	0.05	0.012-0.176
Juvenile	Adult	0.05	0.03-0.07
Adult	Adult	0.50	0.3-0.7

a. Develop a Lefkovitch matrix model, *L*, using the mean probabilities from Table 14 with a fecundity of 7,500 female eggs and determine the finite rate of population change, λ. If the animal could maintain such annual population growth, would you anticipate the cane toad population to increase or decrease over time?
b. Develop a Lefkovitch matrix model, *L*, Using the lower and upper extremes of the probability and fecundity ranges.
c. Use any computational tool such as MatLab or Python NymPy to Calculate the population of the first 20 years, X(0)…X(19) the lower and upper extremes of the probability and fecundity ranges
d. Based on your answers to b on both the lower and upper extremes of the probability, what are the intrinsic rates of growth of the population λ in both cases?
e. Determine the sensitivity of λ to the second row, first column parameter (0.1).
f. Determine the sensitivity of λ to the second row, second column parameter.
g. Determine the sensitivity of λ to the third row, second column parameter.
h. Determine the sensitivity of λ to the fourth row, third column parameter.
i. Determine the sensitivity of λ to the fourth row, fourth column parameter.

Based on your answers to Parts d-h, where should conservation efforts focus?