Naive Laplacian Deformation

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1 INTRODUCTION

In computer graphics, the mesh is a useful way to represents the geometry of objects. Sometimes people need to deform the mesh and thus requiring to find a new position for each vertex. Naive Laplacian Deformation is one way to solve this problem. In this method, geometric details are encoded by using differential coordinates for each vertex. People can select some points to move and the deformation problem can be formulated as a least square problem, which contains two-part: the regularization part and the matching part. In this work, I implement the Naive Laplacian deformation without considering the local rotations.

2 METHODOLOGY

In this paper, we describe the mesh by a pair $\{V, E\}$ where $V = \{v_1, \cdots, v_n\}$ is the vertices set, and v_i is the position of each vertex. E represents the edge set connects two vertices. The one-ring neighbors of vertex i can be described as $N_i = \{j \,|\, (i,j) \in E\}$.

Considerating that it is difficult to do mesh operations in the absolute coordinates, we instead use the differential δ_i to represent the mesh. Namely, the position of each vertex i can be described by the difference between i and it's one-ring neighbors as follow.

$$\delta_i = v_i - \frac{\sum_{j \in N_i} w_{ij} v_j}{\sum_{j \in N_i} w_{ij}} \tag{1}$$

In the above equation, w_{ij} is the weight of the j-th neighbor of vertex i. It can have different forms, like uniform weights. And in our experiment, we choose the cotangent weights as shown in Fig.1. Since equation 1 is a linear combination of a vertex and its neighbors, by defining the Laplacian Matrix L, we can use the following equation to construct differential coordinates for all vertices.

$$LV = D \tag{2}$$

$$L_{ij} = \begin{cases} 1, \mathrm{i} = \mathrm{j} \\ -\frac{w_{ij}}{\sum_{j \in N_i} w_{ij}}, (\mathrm{i}, \mathrm{j}) \in \mathrm{E} \\ 0, \text{otherwise} \end{cases} \tag{3}$$

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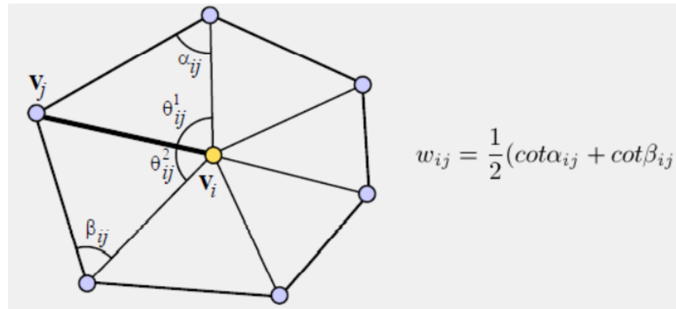

Fig. 1. Cotangent Weights

$$V = \begin{bmatrix} v_1^x & v_1^y & v_1^z \\ \vdots & \vdots & \vdots \\ v_n^x & v_n^y & v_n^x \end{bmatrix} \tag{4}$$

$$D = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_n \end{bmatrix} \tag{5}$$

We then need to recover absolute coordinate from differential coordinate. A simply result to equation is $V = L^{-1}D$. But L may be singular, so we need to add some constraints. And also sometimes we want to select some points and move them to specific location. Thus, we need to expand the equation 2 to the following form and define the selected points set as C.

$$\left(\frac{\mathrm{L}}{wI_{m \times m}|0} \right) V = \left(\begin{array}{c} D \\ \mathrm{w}c_{1:m} \end{array} \right) \tag{6}$$

To find the solution of the above equation, is equally to solve the a least square equation.

$$\tilde{V} = \arg\min_{V} (\|LV - D\|^2 + \sum_{j \in C} w^2 \left|v_j - c_j\right|^2)$$

Here, the first part $\|LV - D\|^2$ is the smoothness constraints, and it keep the shape information of the origin mesh.The second part $\sum_{j \in C} w^2 \left|v_j - c_j\right|^2$ add some position constraints, which can let the mesh move toward the user set position. w is a weight to balance these two parts of constraints.

We know that every least square problem can be formulated into this form.

$$\tilde{X} = \arg\min_{x} (\|Ax - b\|^2) \tag{7}$$

And equation 7 can be solved by doing Cholesky factorization for A.

3 RESULTS

We use the interface and the dinosaur object to test the implement Naive Laplacian Deformation. As we can see in Fig. 3, we lift the tail of the dinosaur and change the legs and hands position of this dinosaur.

Fig. 2. Dinosaur before Naive Laplacian Deformation

Fig. 3. Dinosaur after Naive Laplacian Deformation

4 DISCUSSIONS

When looking into the two parts of the least square problem, w is a weight to balance origin shape constraints and target position constraints. If w is larger, it means that after deformation, select points are more likely to get closer to the target position, while the original shape cannot be preserved very well. If w is set to small, it means the mesh will likely to remain as the original one.

5 CONCLUSION

The Naive Laplacian Deformation using differential coordinates to represent vertices, and the mesh deformation problem can be formulated as a least square problem. The least square problem has two parts. One is the regularization term which helps to preserve the original shape of the mesh. The other is the matching term which moves the user selected vertices closer to their target positions. And experiments show that Naive Laplacian Deformation has good results.