



$$f(t) = \sum_{n=-\infty}^{+\infty} f_0(t) * \delta(t-2n) = \sum_{n=-\infty}^{+\infty} f(t-2n) \quad \text{是周期函数, 周期为 2}$$

2. 推导卷积的微分公式.

$$\text{证明: } \frac{d}{dt} [f_1(t) * f_2(t)] = \frac{d}{dt} \int_{-\infty}^{+\infty} f_1(\tau) f_2(t-\tau) d\tau$$

$$\Leftrightarrow \int_{-\infty}^{+\infty} f_1(\tau) \left( \frac{d}{dt} f_2(t-\tau) \right) d\tau = f_1(t) * \left[ \frac{d}{dt} f_2(t) \right]$$

$$= \frac{d}{dt} \int_{-\infty}^{+\infty} f_1(t-\tau) f_2(\tau) d\tau$$

$$\Leftrightarrow \int_{-\infty}^{+\infty} \frac{d}{dt} f_1(t-\tau) f_2(\tau) d\tau = \left[ \frac{d}{dt} f_1(t) \right] * f_2(t) \quad \square$$

3. 推导卷积的积分公式

$$\text{证明: 令 } m(t) \triangleq \int_{-\infty}^t f_2(\lambda) d\lambda, \text{ 则有 } m(t-\tau) = \int_{-\infty}^{t-\tau} f_2(\lambda) d\lambda = \int_{-\infty}^t f_2(\lambda-\tau) d\lambda$$

$$\text{则 } \int_{-\infty}^t (f_1 * f_2)(\lambda) d\lambda = \int_{-\infty}^t \left[ \int_{-\infty}^{+\infty} f_1(\tau) f_2(\lambda-\tau) d\tau \right] d\lambda$$

$$= \int_{-\infty}^{+\infty} \left[ \int_{-\infty}^t f_1(\tau) f_2(\lambda-\tau) d\lambda \right] d\tau$$

$$= \int_{-\infty}^{+\infty} f_1(\tau) m(t-\tau) d\tau$$

$$= f_1(t) * m(t) = f_1(t) * \int_{-\infty}^t f_2(\lambda) d\lambda \quad \square$$







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4. 推导  $f(t) * u(t) = \int_{-\infty}^t f(\tau) d\tau$

证明:  $\because u(t) = \int_{-\infty}^t \delta(\tau) d\tau$

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$$\therefore f(t) * u(t) = f(t) * \int_{-\infty}^t \delta(\tau) d\tau = \int_{-\infty}^t f(\tau) * \delta(\tau) d\tau = \int_{-\infty}^t f(\tau) d\tau. \quad \square$$

