DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 4

Bases

Span

Orthogonal and orthonormal bases

Orthogonal and orthonormal matrices

Determinants In-Depth

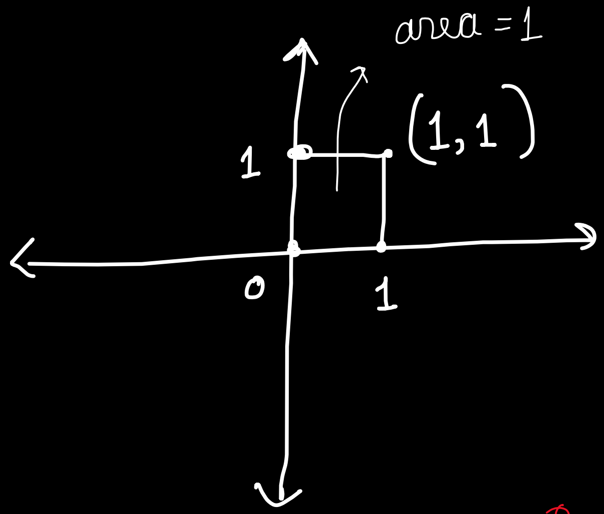
Dimensionality Reduction Algorithm ⇒ (PCA)

(Principal component Analysis)

Rank ⇒ Dimension of image of an linear transformation

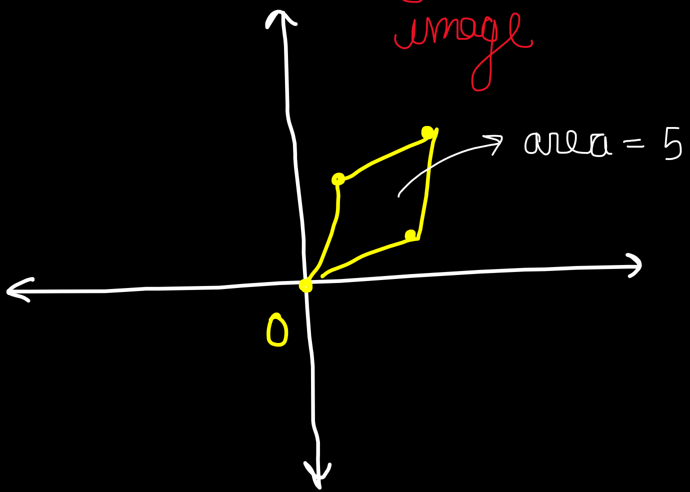
Determinants In-Depth

(Non singular) <u>Determinant as an area</u>



$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}$$

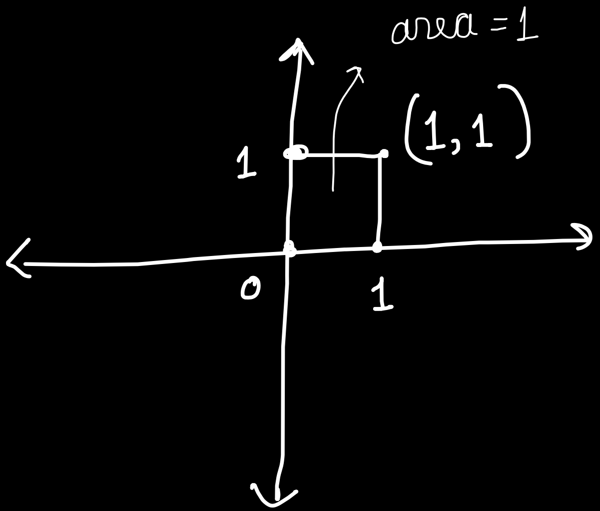
$$\Delta = 3(2) - 1$$
$$= 6 - 1$$
$$= 5$$



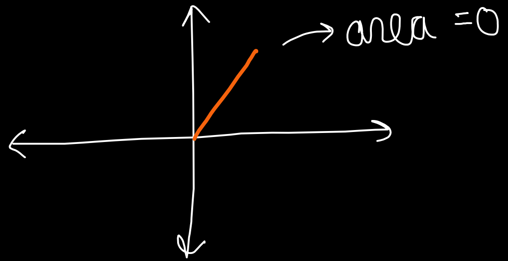
determinant of a matrix is area of the linear transformation of a unit square

Determinants In-Depth

(singular) Determinant as an area

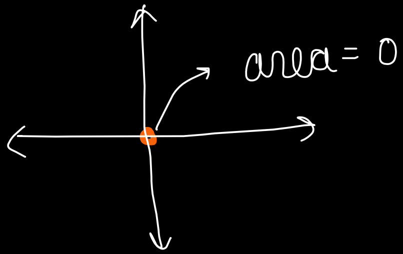


$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \quad \Delta = 2 - 2 = 0$$



$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Delta = 0$



Determinants In-Depth

$$\det(A, B) = \det(A)\det(B)$$

if one matrix is singular, $\det(\ ) = 0$

(non-singular matrix) · (singular matrix) = (singular matrix)

Determinants In-Depth

$$\begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix} \Rightarrow \quad 0.24 - 0.04 = 0.2$$

$$\begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix} \Rightarrow \quad 0.25\{0.625 - 0.125) \quad \Rightarrow \quad \begin{array}{r} 625 \\ 125 \\ \hline 500 \end{array}$$

$$0.125$$

Determinants In-Depth

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.4 & -0.2 \\ -0.2 & 0.6 \end{bmatrix}$$

det = 5 det = 0.2

$$5^{-1} = 0.2$$

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix}^{-1} = \begin{bmatrix} 0.25 & -0.25 \\ -0.125 & 0.625 \end{bmatrix}$$

det = 8 det = 0.125

$$8^{-1} = 0.125$$

(Non Invertible Matrices) determinant = 0, 0^{-1} = Not Def.

So no inverse

Determinants In-Depth

$$\det(A^{-1}) = 1/\det(A)$$

$$\Rightarrow \det(AB) = \det(A)\det(B)$$

$$\Rightarrow \det(AA^{-1}) = \det(A)\det(A^{-1})$$

$$\Rightarrow \det(I) = \det(A)\det(A^{-1})$$

$$\Rightarrow 1 = \det(A)\det(A^{-1})$$

Determinants In-Depth

Determinants In-Depth

Eigenvalues & Eigenvectors

Bases

any 2 vectors on a plane
but not in the same direction
so that we can reach any
point on plane using those
two vectors

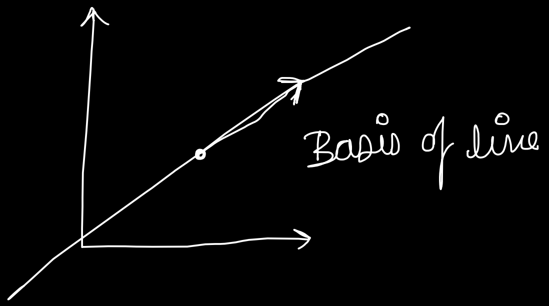
Span

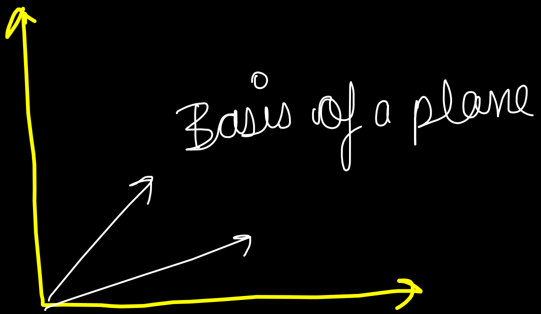
all the set of points
which can be covered
by travelling in direction
of vectors.

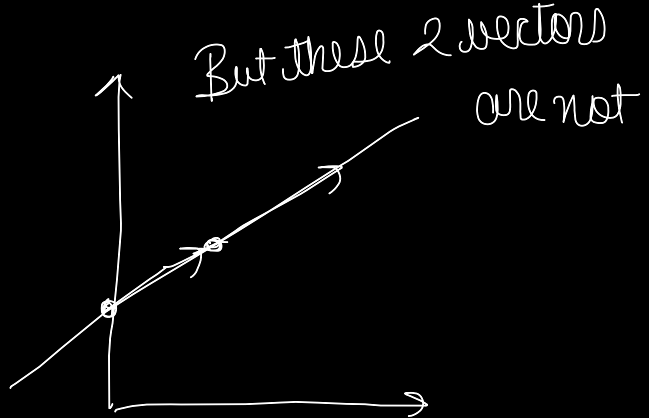
Span of Bases = "plane"

Eigenvalues & Eigenvectors

Basis → minimal spanning set



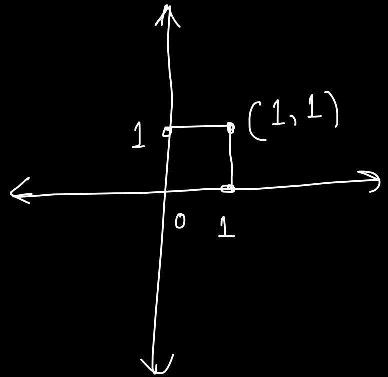


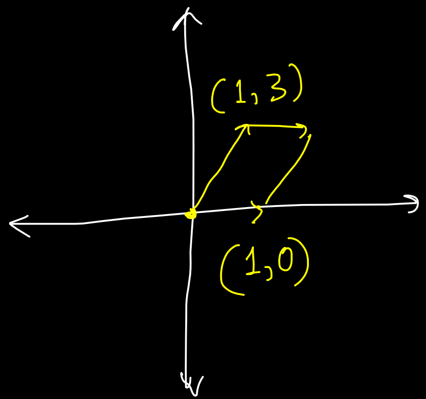


Eigenvalues & Eigenvectors

Basis

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$$





Eigenvalues & Eigenvectors

Finding Eigen Values

If λ is an eigen value:

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$$

For ∞ (x, y)

$$\boxed{\begin{bmatrix} \lambda x \\ \lambda y \end{bmatrix} = \lambda \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow}$$

$$\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

∞ solutions

$$\det\begin{bmatrix} 2-\lambda & 1 \\ 0 & 3-\lambda \end{bmatrix} = 0$$

Characteristic Polynomial: $(2-\lambda)(3-\lambda) - 1\cdot 0 = 0$

Eigenvalues & Eigenvectors

Finding Eigen vectors

Eigen Values : $\lambda = 2$

$\lambda = 3$

Solve equations

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 2 \begin{bmatrix} x \\ y \end{bmatrix}$$

$2x + y = 2x$ $\quad x = 1$

$0x + 3y = 2y$ $\quad y = 0$

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 3 \begin{bmatrix} x \\ y \end{bmatrix}$$

$2x + y = 3x$ $\quad x = 1$

$0x + 3y = 3y$ $\quad y = 1$

$$\begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Eigenvalues & Eigenvectors

$$\begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix} \ni \begin{bmatrix} 9-\lambda & 4 \\ 4 & 3-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0 \qquad \begin{bmatrix} 9 & 4 \\ 4 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$(9-\lambda)(3-\lambda) - 16 = 0$$

$$27 - 9\lambda - 3\lambda + \lambda^2 - 16 = 0$$

$$\lambda^2 - 12\lambda + 11 = 0$$

$$\lambda^2 - \lambda - 11\lambda + 11 = 0$$

$$\lambda(\lambda-1) - 11(\lambda-1) = 0$$

$$(\lambda-1)(\lambda-11) = 0$$

$$\lambda = 1, 11$$

$$9x + 4y = x$$

$$4x + 3y = y$$

$$y = -2x$$

$$9x - 8x = x$$

$$9x = 9x$$

$$\begin{matrix} x = \\ y = \end{matrix} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$$

$$9x + 4y = 11x$$

$$4x + 3y = 11y$$

$$4x = 8y$$

$$x = 2y$$

$$18y + 4y = 22y$$

$$\begin{matrix} x = \\ y = \end{matrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

Eigenvalues & Eigenvectors

$$\begin{bmatrix} 2 & 1 \\ -3 & 6 \end{bmatrix} \Rightarrow \begin{bmatrix} 2-\lambda & 1 \\ -3 & 6-\lambda \end{bmatrix}$$

$$(2-\lambda)(6-\lambda) + 3 = 0$$

$$12 - 2\lambda - 6\lambda + \lambda^2 + 3 = 0$$

$$\lambda^2 - 3\lambda - 5\lambda + 15 = 0$$

$$\lambda(\lambda - 3) - 5(\lambda - 3) = 0$$

$$\lambda = 3, 5 \longrightarrow \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\hookrightarrow \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$2x + 1y = 3x \Rightarrow y = x$$

$$-3x + 6y = 3y$$

$$2x + y = 5x \Rightarrow y = 3x$$

$$-3x + 6y = 5y$$

$$-3x + 18x = 15x$$

$$x = 1$$

Eigenvalues & Eigenvectors

$$(1-\lambda)^3$$

$$\begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 4 \end{bmatrix} \quad \begin{bmatrix} 1-\lambda & 2 \\ 0 & 4-\lambda \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = 0$$

$$x + 2y = x$$
$$4y = y$$

$$x + 2y = 4x$$
$$4y = 4y$$

$$(1-\lambda)(4-\lambda) = 0$$
$$4 - \lambda - 4\lambda + \lambda^2$$
$$\lambda^2 - \lambda - 4\lambda + 4 = 0$$
$$\lambda(\lambda - 1) - 4(\lambda - 1) = 0$$
$$\lambda = 1, 4$$

Eigenvalues & Eigenvectors

$$(2-\lambda)(1-\lambda) - a = 0$$

$$2 + \lambda^2 - 3\lambda - a = 0$$

$$9 - 4(2-a) > 0$$

$$9 - 8 + 4a > 0$$

$$a > -1/4$$

Eigenvalues & Eigenvectors

Eigenvalues & Eigenvectors