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DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 1

Systems of linear equations

Singular and non-singular matrices

Determinants

Rank of a matrix

Row reduction

Null space

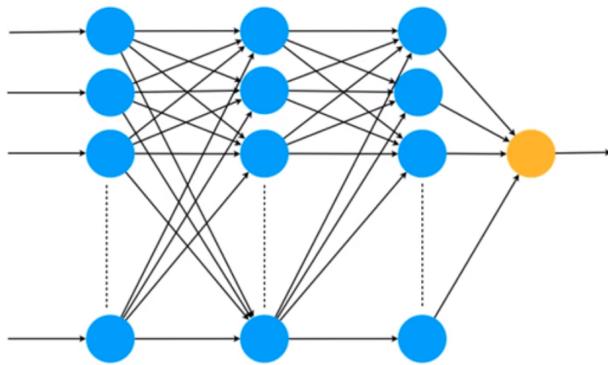


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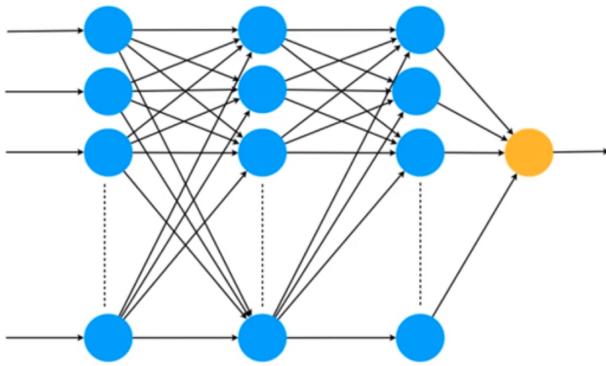
System of Linear Equations

Machine learning motivation

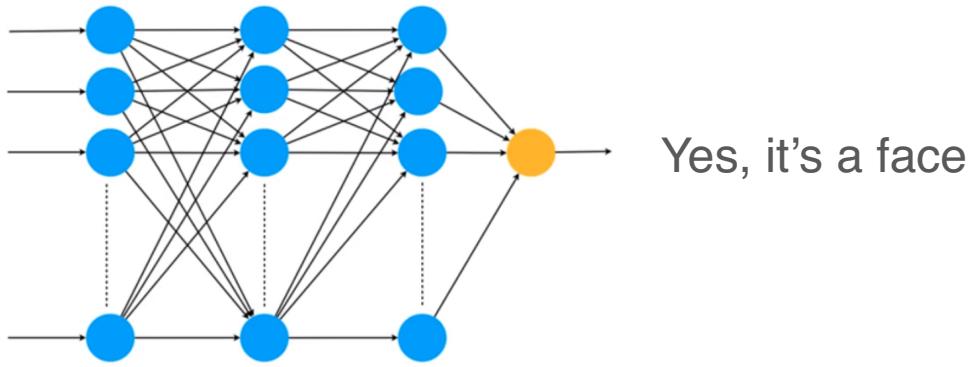
Neural networks - Matrix operations



Neural networks - Matrix operations

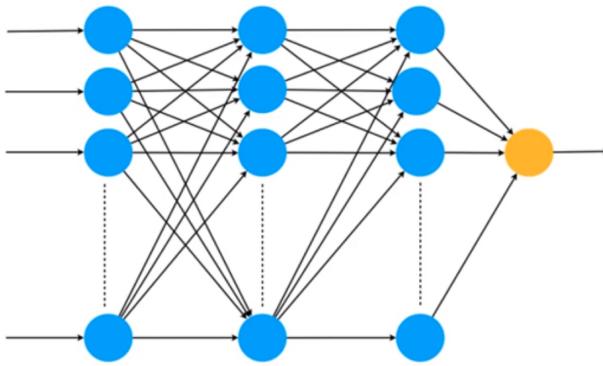


Neural networks - Matrix operations



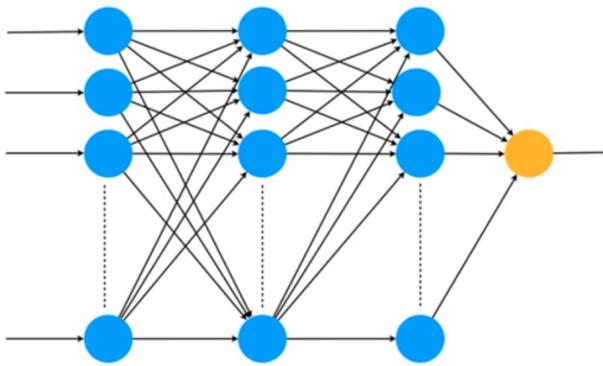
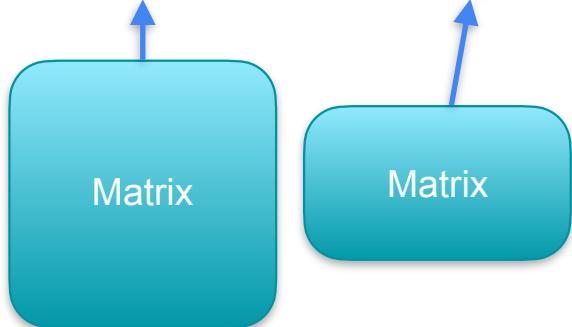
Yes, it's a face

Neural networks - Matrix operations



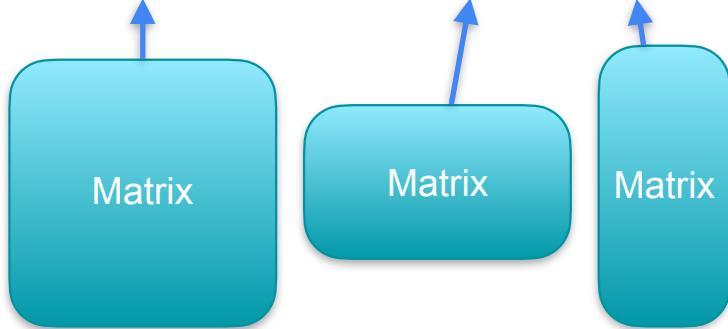
Yes, it's a face

Neural networks - Matrix operations



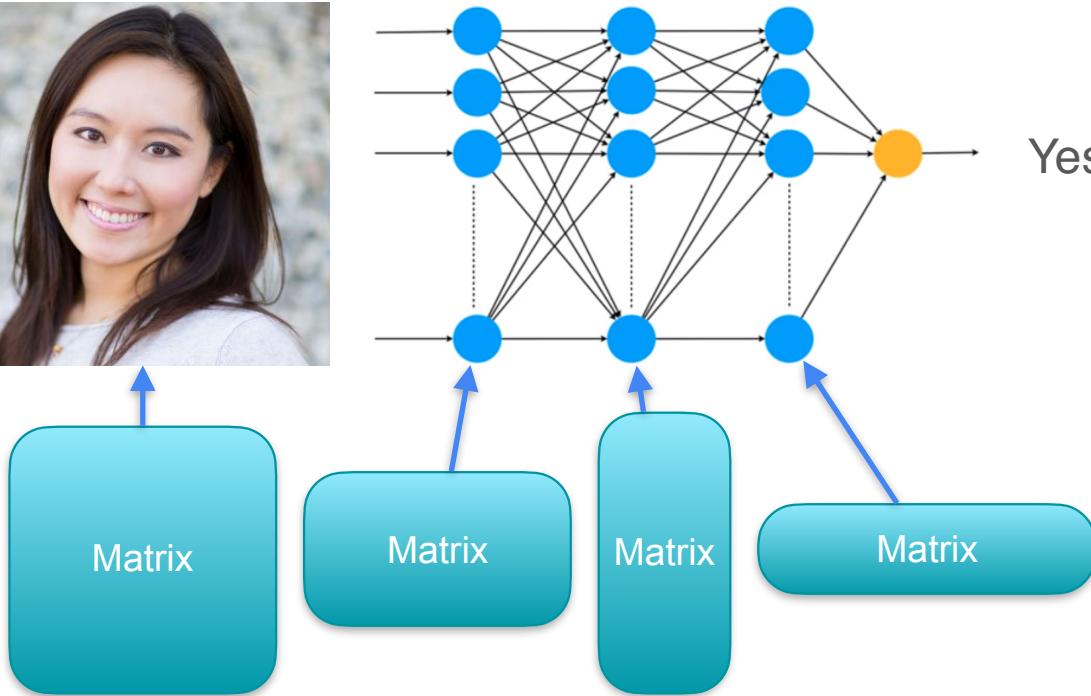
Yes, it's a face

Neural networks - Matrix operations



Yes, it's a face

Neural networks - Matrix operations



Yes, it's a face

Neural networks - image recognition

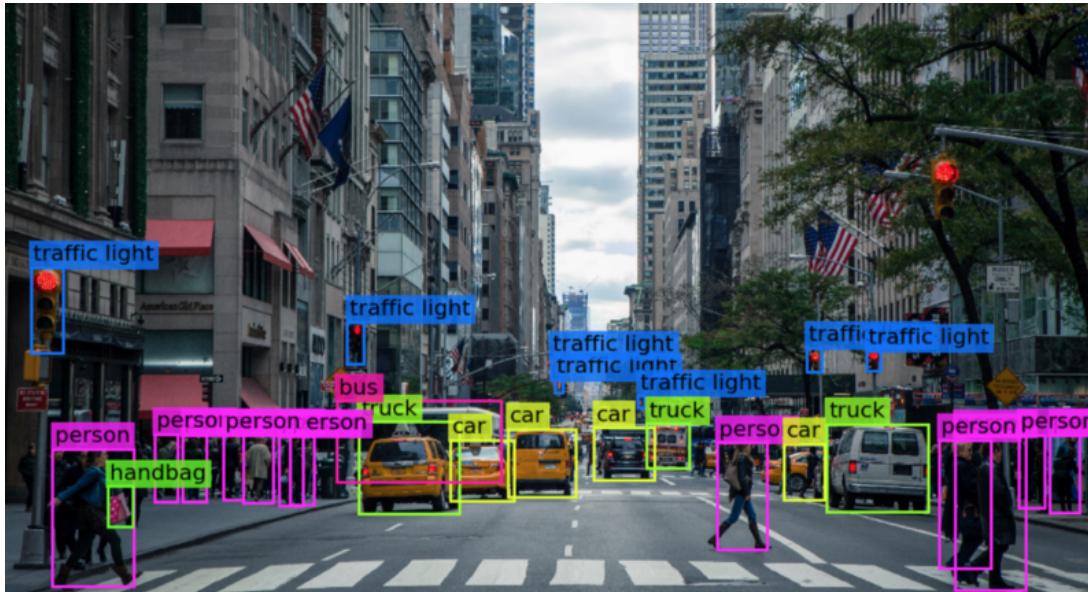


Image recognition in a busy street in New York.

- Image recognition: Getting the computer to see images and recognize what is on them.



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System of Linear Equations

System of sentences

Systems of sentences

Systems of sentences

System 1



The dog is **black**



The cat is **orange**

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**

System 3

 The dog is **black**
 The dog is **white**

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**

System 2

 The dog is **black**
 The dog is **black**

System 3

 The dog is **black**
 The dog is **white**

Complete

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**

Complete

System 2

 The dog is **black**
 The dog is **black**

Redundant

System 3

 The dog is **black**
 The dog is **white**

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**

Complete

System 2

 The dog is **black**
 The dog is **black**

Redundant

System 3

 The dog is **black**
 The dog is **white**

Contradictory

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**

Complete

System 2

 The dog is **black**
 The dog is **black**

Redundant

Singular

System 3

 The dog is **black**
 The dog is **white**

Contradictory

Singular

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**

Complete

Non-singular

more informative

System 2

 The dog is **black**
 The dog is **black**

Redundant

Singular

less informative

System 3

 The dog is **black**
 The dog is **white**

Contradictory

Singular

less informative

Systems of sentences

1 dog
1 cat
1 Bird

System 1

- The dog is **black**
- The cat is **orange**
- The bird is **red**

*complete
(non-singular)*

System 2

- The dog is **black**
- The dog is **black**
- The bird is **red**

*Redundant
(singular)*

System 3

- The dog is **black**
- The dog is **black**
- The dog is **black**

*Redundant
(singular)*

System 4

- The dog is **black**
- The dog is **white**
- The bird is **red**

*contradictory
(singular)*

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**
 The bird is **red**

Complete

Non-singular

System 2

 The dog is **black**
 The dog is **black**
 The bird is **red**

System 3

 The dog is **black**
 The dog is **black**
 The dog is **black**

System 4

 The dog is **black**
 The dog is **white**
 The bird is **red**

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**
 The bird is **red**

Complete

Non-singular

System 2

 The dog is **black**
 The dog is **black**
 The bird is **red**

Redundant

Singular

System 3

 The dog is **black**
 The dog is **black**
 The dog is **black**

System 4

 The dog is **black**
 The dog is **white**
 The bird is **red**

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**
 The bird is **red**

Complete

Non-singular

System 2

 The dog is **black**
 The dog is **black**
 The bird is **red**

Redundant

Singular

System 3

 The dog is **black**
 The dog is **black**
 The dog is **black**

Redundant

Singular

System 4

 The dog is **black**
 The dog is **white**
 The bird is **red**

Systems of sentences

System 1

 The dog is **black**
 The cat is **orange**
 The bird is **red**

Complete

Non-singular

System 2

 The dog is **black**
 The dog is **black**
 The bird is **red**

Redundant

Singular

System 3

 The dog is **black**
 The dog is **black**
 The dog is **black**

Redundant

Singular

System 4

 The dog is **black**
 The dog is **white**
 The bird is **red**

Contradictory

Singular

Quiz: Systems of sentences

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

dog \Rightarrow Black
cat \Rightarrow Orange
bird \Rightarrow Red

Problem 1:

What color is the bird? \rightarrow Red

\circ informative
 \circ (complete system)

Problem 2:

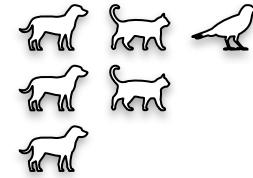
Is this system singular or non-singular?

non singular

Solution: Systems of information

Given this system:

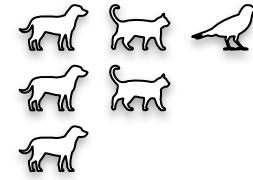
- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.

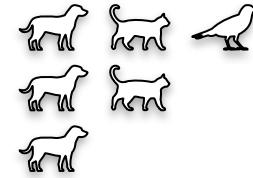


Solution 1:

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



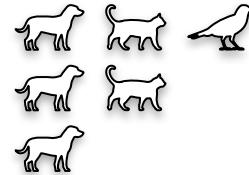
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



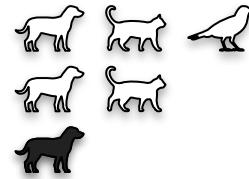
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



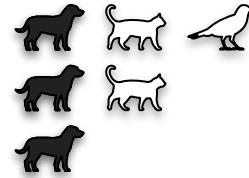
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



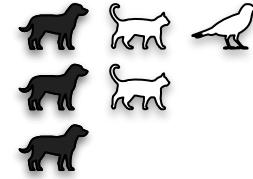
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



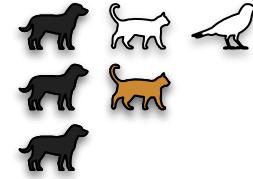
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



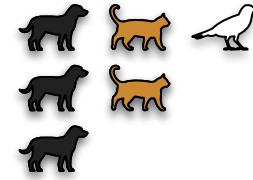
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

- Between the dog, the cat, and the bird, one is red.
- Between the dog and the cat, one is orange.
- The dog is black.



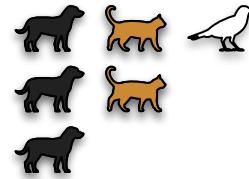
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

-
- Between the dog, the cat, and the bird, one is red.
 - Between the dog and the cat, one is orange.
 - The dog is black.



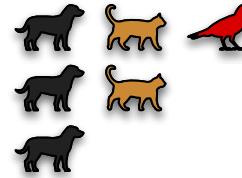
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

-
- Between the dog, the cat, and the bird, one is red.
 - Between the dog and the cat, one is orange.
 - The dog is black.



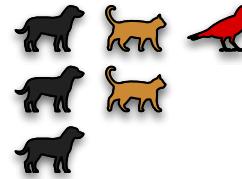
Solution 1:

The bird is red. 

Solution: Systems of information

Given this system:

-
- Between the dog, the cat, and the bird, one is red.
 - Between the dog and the cat, one is orange.
 - The dog is black.



Solution 1:

The bird is red. 

Solution 2:

It is non-singular.   



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System of Linear Equations

System of equations

Sentences → Equations

.

Sentences

Between the dog and
the cat, one is black.



Sentences → Equations

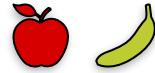
Sentences

Between the dog and the cat, one is black.



Sentences with numbers

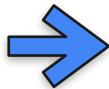
The price of an apple and a banana is \$10.



Sentences → Equations

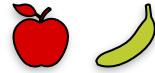
Sentences

Between the dog and the cat, one is black.



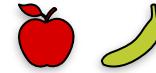
Sentences with numbers

The price of an apple and a banana is \$10.



Equations

$$a + b = 10$$



Quiz: Systems of equations 1

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.

Question: How much does each fruit cost?

$$\begin{array}{rcl} x + y & = & 10 \\ x + 2y & = & 12 \\ \hline y & = & 2 \Rightarrow \text{Banana} \\ x & = & 8 \Rightarrow \text{Apple} \end{array}$$

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought an apple and two bananas and they cost \$12.
- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.
- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍎} + \text{🍌} + \text{🍌} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍎} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍎} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{🍎} + \text{🍌} + \boxed{\text{🍌}} = \$12$$

\$2

- **Solution:** An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

A diagram showing an apple and a banana. A blue arrow points from the banana to the number \$2, indicating that the banana costs \$2.

- Day 2: You bought an apple and two bananas and they cost \$12.

$$\text{apple} + \text{banana} + \boxed{\text{banana}} = \$12$$

A diagram showing an apple and two bananas. The second banana is highlighted with a blue box. A blue arrow points from this boxed banana to the number \$2, indicating that each banana costs \$2.

- Solution: An apple costs \$8, a banana costs \$2.

Solution: Systems of equations 1

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

\$8 + \$2 = \$10

A diagram showing an apple and a banana. An arrow points from the apple to the number \$8, and another arrow points from the banana to the number \$2. A plus sign is placed between the fruit icons, and an equals sign is followed by the dollar amount \$10.

- **Day 2:** You bought an apple and two bananas and they cost \$12.

$$\text{apple} + \text{banana} + \text{banana} = \$12$$

\$8 + \$2 + \$2 = \$12

A diagram showing an apple and two bananas. An arrow points from the apple to the number \$8. Two arrows point from the two bananas to the number \$2 each. A plus sign is placed between the first fruit icon and the first banana icon, and another plus sign is placed between the first banana icon and the second banana icon. An equals sign is followed by the dollar amount \$12.

- **Solution:** An apple costs \$8, a banana costs \$2.

Quiz: Systems of equations 2

You go two days in a row and collect this information:

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

Question: How much does each fruit cost?

Not enough info!

$$x + y = 10 \quad | \quad x = 10 - y$$

$$2x + 2y = 20$$

$$2(10 - y) + 2y = 20 \Rightarrow 20 - 2y + 2y = 20$$

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$20.

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

Solution: Systems of equations 2

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$20.

$$\text{🍎🍎} + \text{🍌🍌} = \$20$$

Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$\text{🍎🍎} + \text{🍌🍌} = \$20$$

Same thing!!!

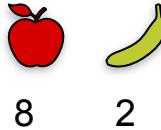
Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2\text{apple} + 2\text{banana} = \$20$$



8 2

Same thing!!!

Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2\text{apple} + 2\text{banana} = \$20$$



8 2

5 5

Same thing!!!

Solution: Systems of equations 2

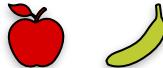
- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2\text{apple} + 2\text{banana} = \$20$$

Same thing!!!



8 2

5 5

8.3 1.7

Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2\text{apple} + 2\text{banana} = \$20$$

Same thing!!!



8 2

5 5

8.3 1.7

0 10

Solution: Systems of equations 2

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10$$

- Day 2: You bought two apples and two bananas and they cost \$20.

$$2\text{apple} + 2\text{banana} = \$20$$

Same thing!!!



8 2

5 5

8.3 1.7

0 10

Infinitely many solutions!

Quiz: Systems of equations 3

You go two days in a row and collect this information:

- Day 1: You bought an apple and a banana and they cost \$10.
- Day 2: You bought two apples and two bananas and they cost \$24.

3 contradictory

Question: How much does each fruit cost?

$$x + y = 10$$

$$2x + 2y = 24$$

$$2(x + y) = 24 \Rightarrow x + y = 12$$

mistake in solution!

$\begin{array}{c} \cancel{x} \\ - \\ \cancel{y} \end{array}$ ye kya hai?

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.
- **Day 2:** You bought two apples and two bananas and they cost \$24.

Solution: Systems of equations 3

- **Day 1:** You bought an apple and a banana and they cost \$10.

$$\text{🍎} + \text{🍌} = \$10$$

- **Day 2:** You bought two apples and two bananas and they cost \$24.

Solution: Systems of equations 3

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10 \quad \rightarrow \quad \text{apple} + \text{apple} + \text{banana} + \text{banana} = \$20$$

- Day 2: You bought two apples and two bananas and they cost \$24.

Solution: Systems of equations 3

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10 \quad \rightarrow \quad \text{apple} + \text{apple} + \text{banana} + \text{banana} = \$20$$

- Day 2: You bought two apples and two bananas and they cost \$24.

$$\text{apple} + \text{apple} + \text{banana} + \text{banana} = \$24$$

Solution: Systems of equations 3

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10 \quad \rightarrow \quad \text{apple} + \text{apple} + \text{banana} + \text{banana} = \$20$$

- Day 2: You bought two apples and two bananas and they cost \$24.

$$\text{apple} + \text{apple} + \text{banana} + \text{banana} = \$24$$

Contradiction!

Solution: Systems of equations 3

- Day 1: You bought an apple and a banana and they cost \$10.

$$\text{apple} + \text{banana} = \$10 \quad \rightarrow \quad \text{apple} + \text{apple} + \text{banana} + \text{banana} = \$20$$

- Day 2: You bought two apples and two bananas and they cost \$24.

$$\text{apple} + \text{apple} + \text{banana} + \text{banana} = \$24$$

Contradiction!

No solutions!

Systems of equations

Systems of equations

System 1

-   $a + b = 10$
-   $a + 2b = 12$

Systems of equations

System 1

-   $a + b = 10$
-   $a + 2b = 12$

System 2

-   $a + b = 10$
-   $2a + 2b = 20$

Systems of equations

System 1

-  +  = 10
-  +  = 12

System 2

-  +  = 10
-  +  = 20

System 3

-  +  = 10
-  +  = 24

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

System 2

- $a + \text{banana icon} = 10$
- $2a + \text{banana icon} = 20$

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Unique solution:

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

System 2

- $a + \text{banana icon} = 10$
- $2a + \text{banana icon} = 20$

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

System 2

- $a + \text{banana icon} = 10$
- $2a + \text{banana icon} = 20$

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Complete

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

System 2

- $a + \text{banana icon} = 10$
- $2a + \text{banana icon} = 20$

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Complete

Non-singular

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + \text{banana icon} = 20$

Infinite solutions

System 3

- $a + \text{banana icon} = 10$
- $2a + \text{banana icon} = 24$

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + \text{banana icon} = 20$

Infinite solutions

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

System 3

- $a + \text{banana icon} = 10$
- $2a + \text{banana icon} = 24$

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\text{apple icon } a = 8, 7$$

$$\text{banana icon } b = 2, 3$$

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\text{apple icon } a = 8, 7, 6$$

$$\text{banana icon } b = 2, 3, 4$$

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\begin{aligned}\text{apple icon } a &= 8 \\ \text{banana icon } b &= 2\end{aligned}$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\begin{aligned}\text{apple icon } a &= 8, 7, 6, \dots \\ \text{banana icon } b &= 2, 3, 4\end{aligned}$$

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\begin{aligned}\text{apple icon } a &= 8, 7, 6, \dots \\ \text{banana icon } b &= 2, 3, 4\end{aligned}$$

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\begin{aligned}\text{apple icon } a &= 8 \\ \text{banana icon } b &= 2\end{aligned}$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\begin{aligned}\text{apple icon } a &= 8, 7, 6, \dots \\ \text{banana icon } b &= 2, 3, 4\end{aligned}$$

Redundant

Singular

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\text{apple icon } a = 8$$

$$\text{banana icon } b = 2$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\begin{aligned}\text{apple icon } a &= 8, 7, 6, \dots \\ \text{banana icon } b &= 2, 3, 4\end{aligned}$$

Redundant

Singular

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

No solution

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\begin{aligned}\text{apple icon } a &= 8 \\ \text{banana icon } b &= 2\end{aligned}$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\begin{aligned}\text{apple icon } a &= 8, 7, 6, \dots \\ \text{banana icon } b &= 2, 3, 4\end{aligned}$$

Redundant

Singular

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

No solution

Contradictory

Systems of equations

System 1

- $a + \text{banana icon} = 10$
- $a + 2\text{banana icon} = 12$

Unique solution:

$$\begin{aligned}\text{apple icon } a &= 8 \\ \text{banana icon } b &= 2\end{aligned}$$

Complete

Non-singular

System 2

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 20$

Infinite solutions

$$\begin{aligned}\text{apple icon } a &= 8, 7, 6, \dots \\ \text{banana icon } b &= 2, 3, 4\end{aligned}$$

Redundant

Singular

System 3

- $a + \text{banana icon} = 10$
- $2a + 2\text{banana icon} = 24$

No solution

Contradictory

Singular

What is a linear equation?

Linear

Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Non-linear

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

Non-linear

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

Non-linear

$$a^2 + b^2 = 10$$

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

What is a linear equation?

Linear

$$a + b = 10$$

$$2a + 3b = 15$$

$$3.4a - 48.99b + 2c = 122.5$$

Numbers

Non-linear

$$a^2 + b^2 = 10$$

$$\sin(a) + b^5 = 15$$

$$2^a - 3^b = 0$$

$$ab^2 + \frac{b}{a} - \frac{3}{b} - \log(c) = 4^a$$



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System of Linear Equations

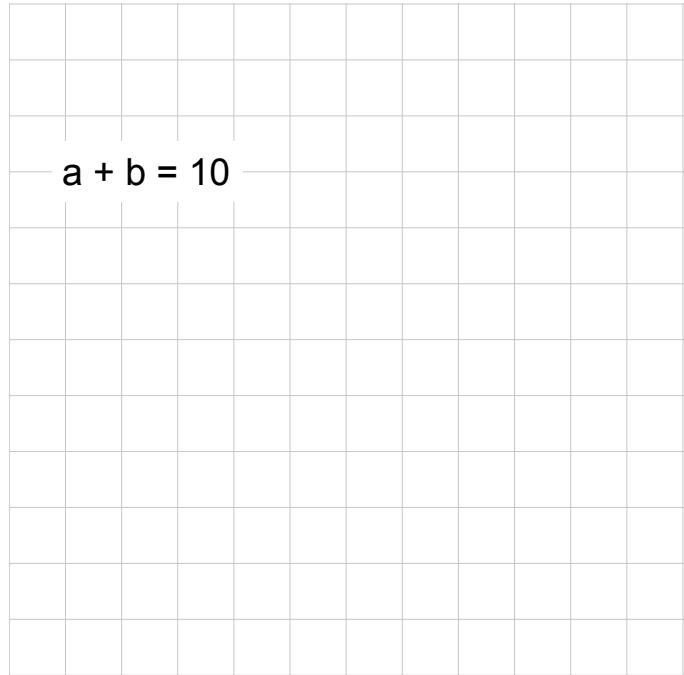
System of equations as lines

Linear equation → line

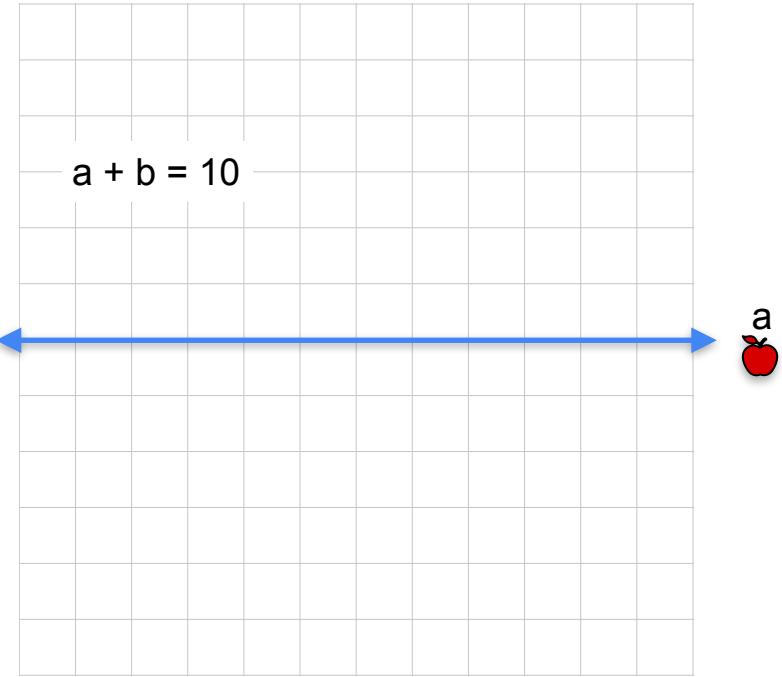
Linear equation → line

$$a + b = 10$$

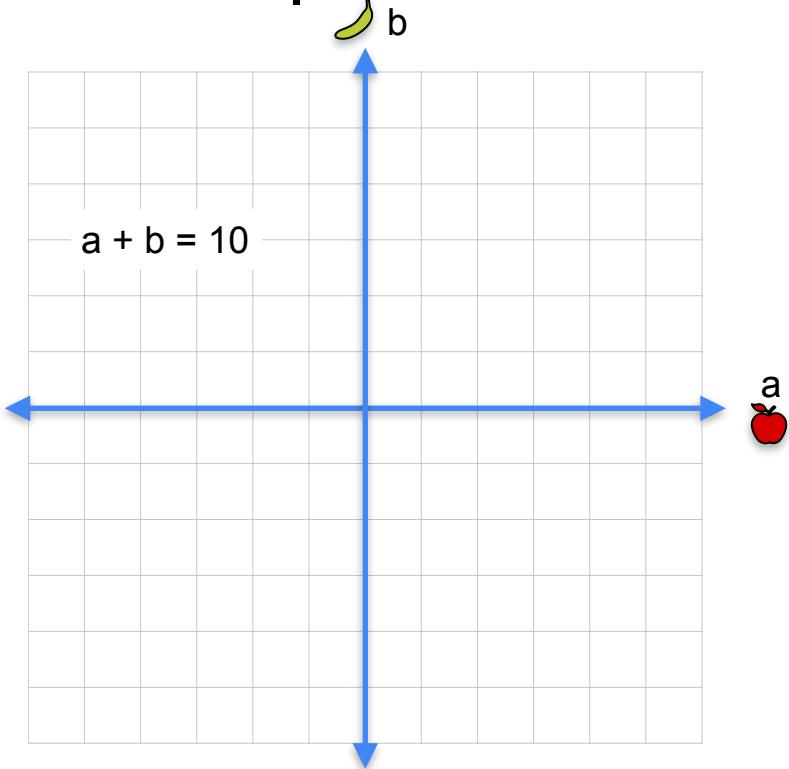
Linear equation → line



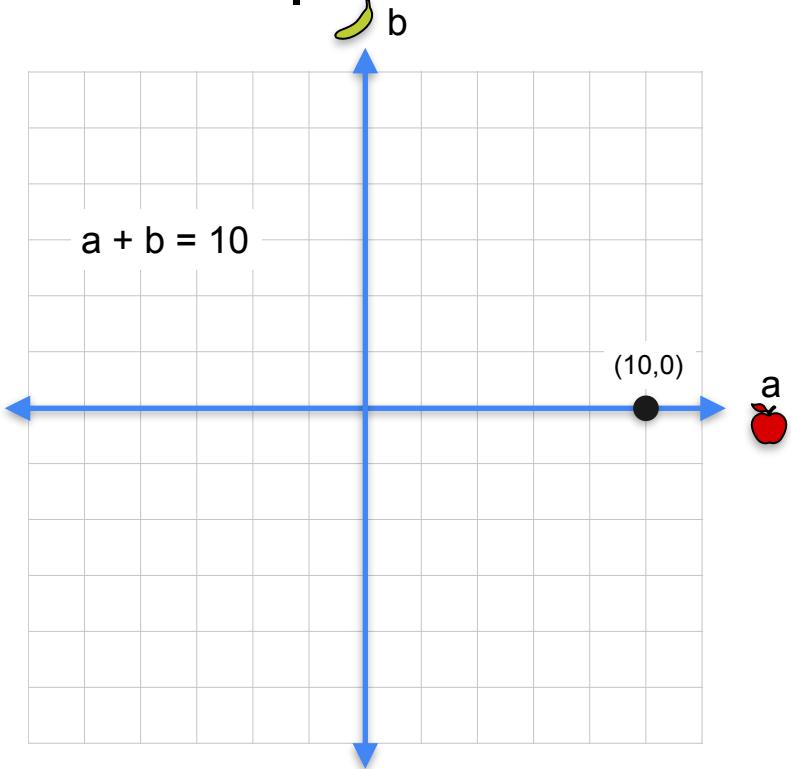
Linear equation → line



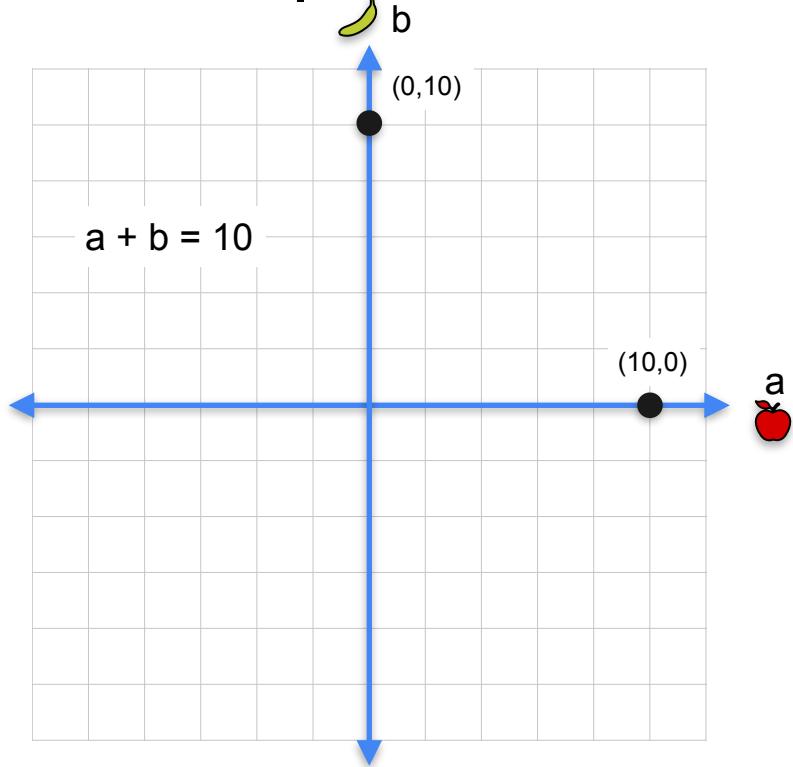
Linear equation → line



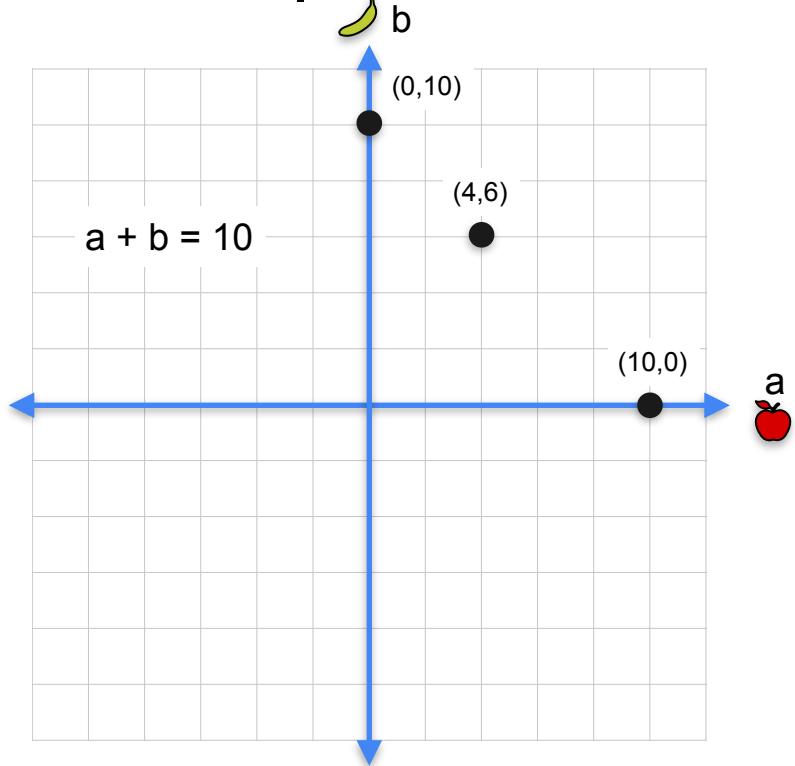
Linear equation → line



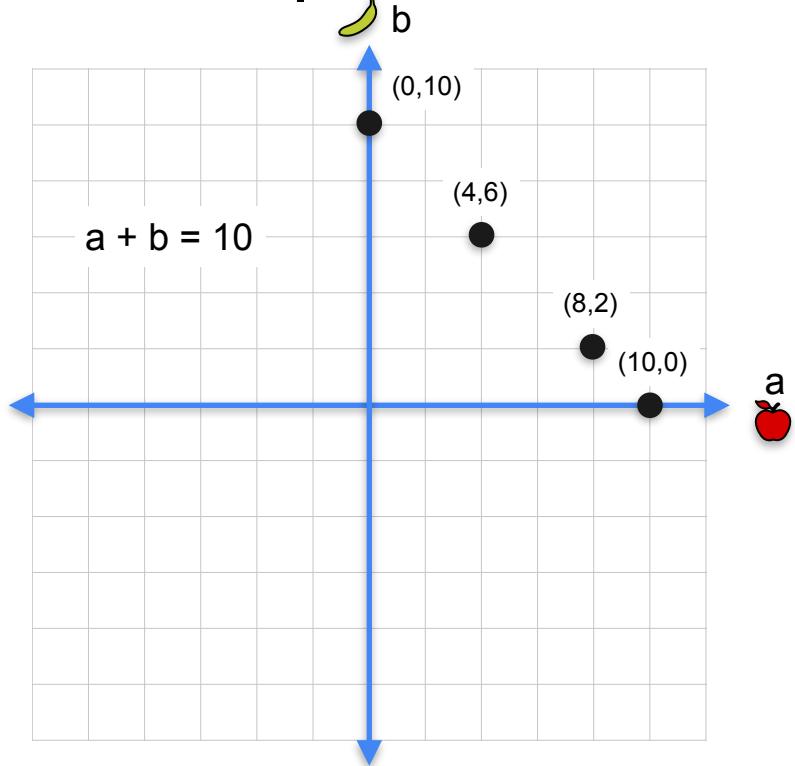
Linear equation → line



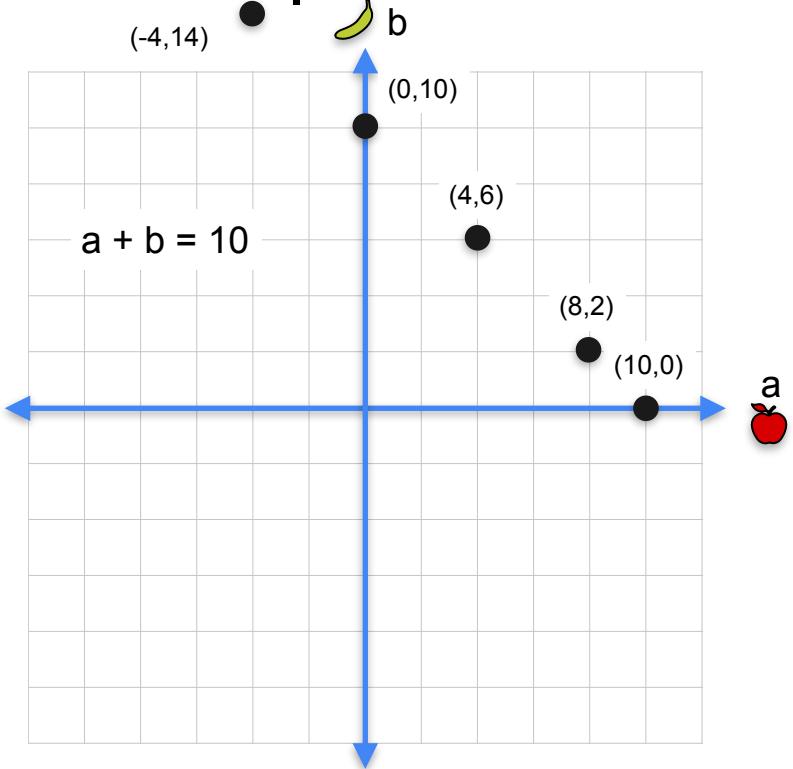
Linear equation → line



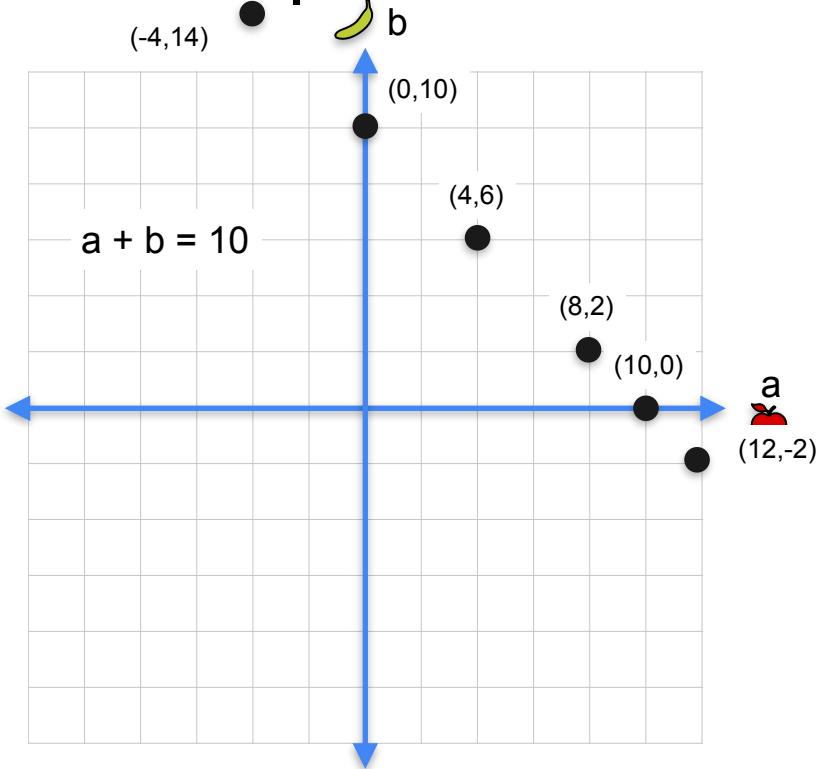
Linear equation → line



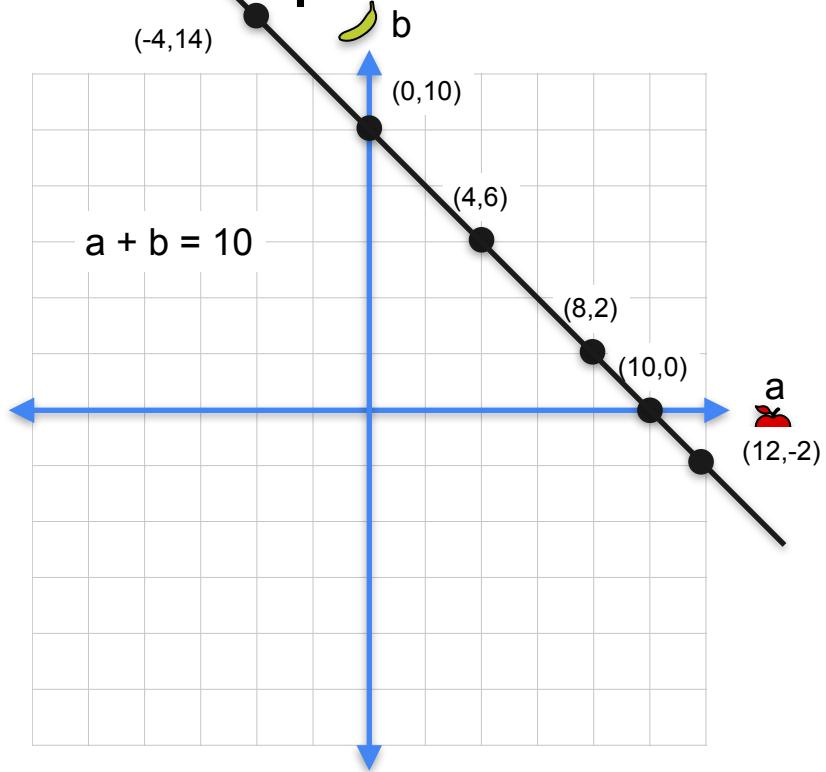
Linear equation → line



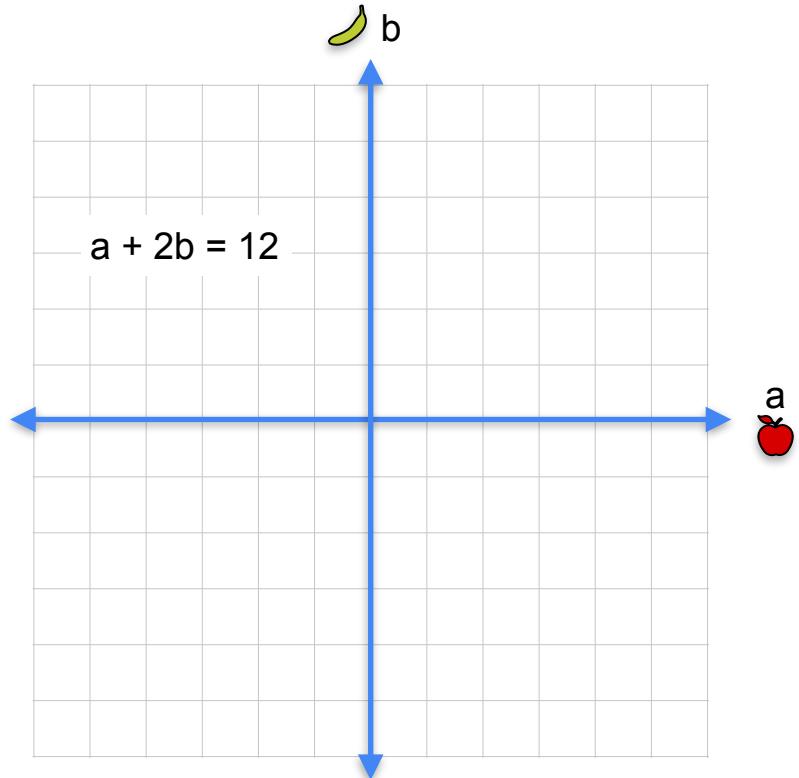
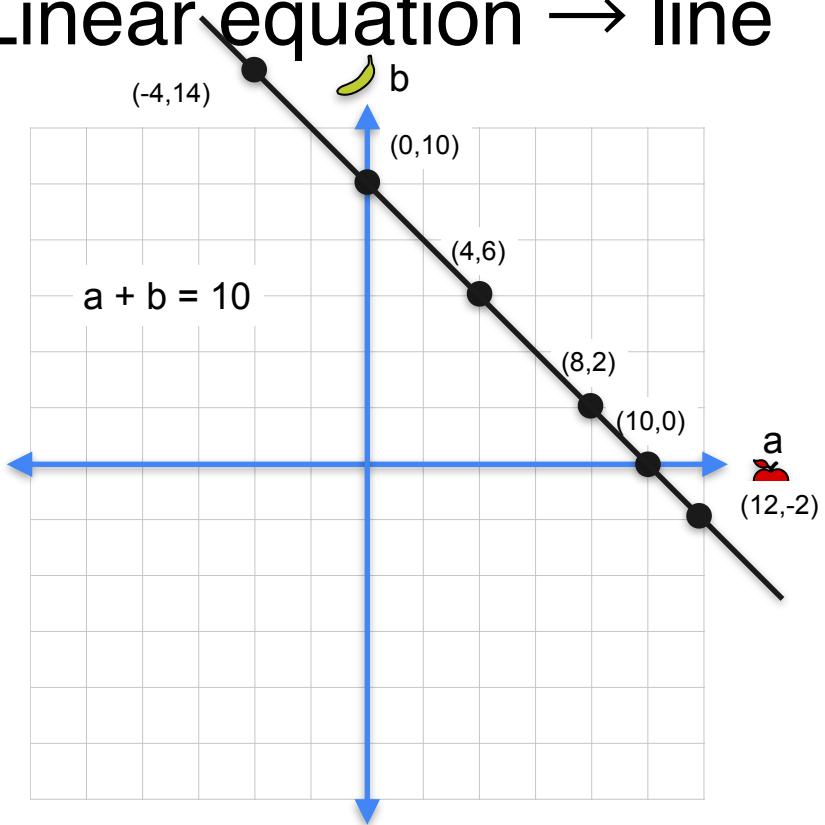
Linear equation → line



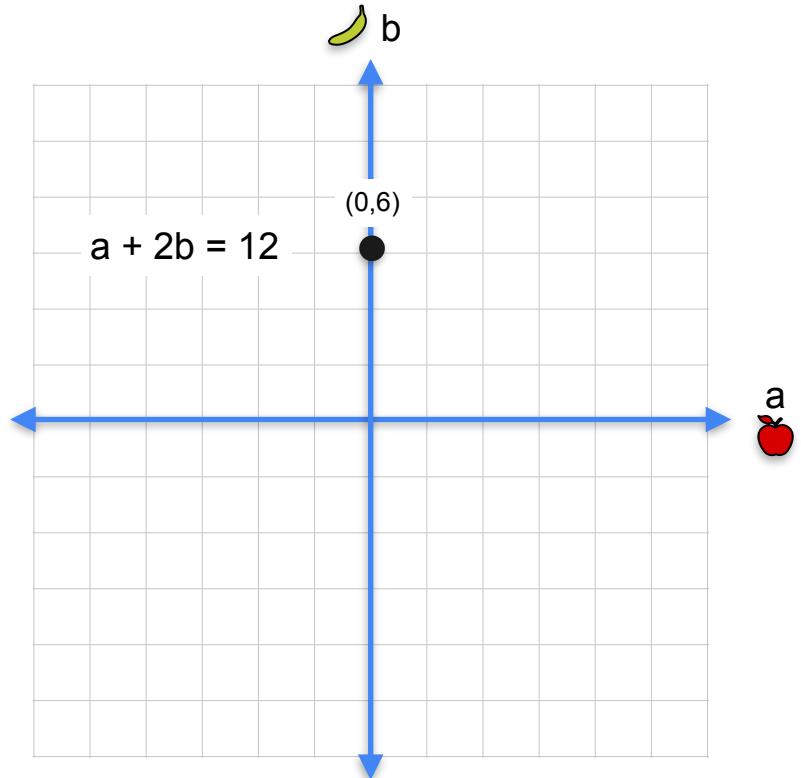
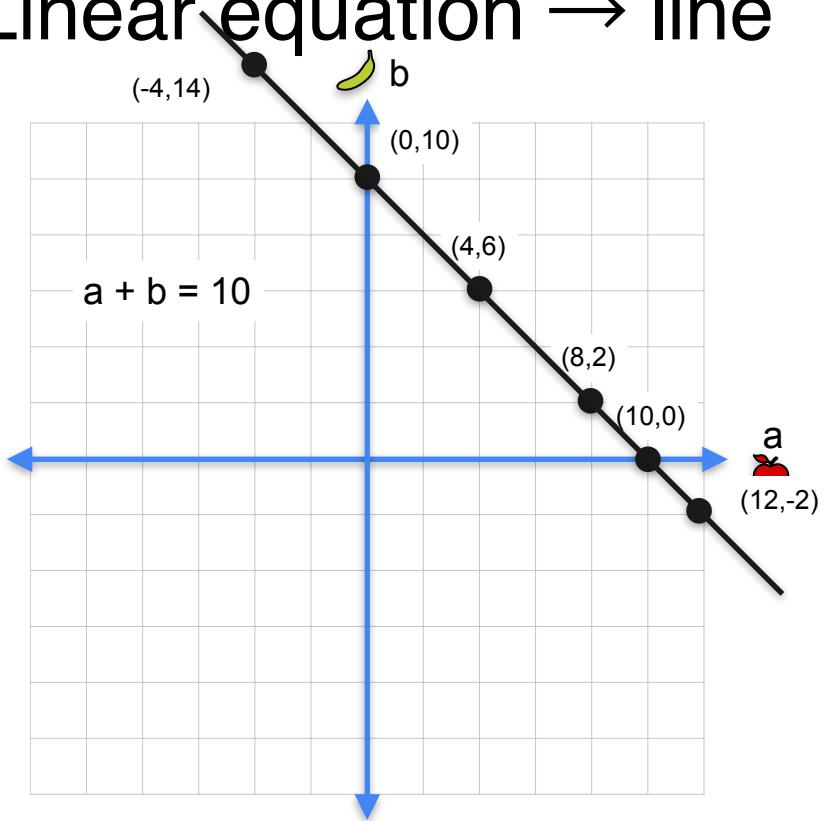
Linear equation → line



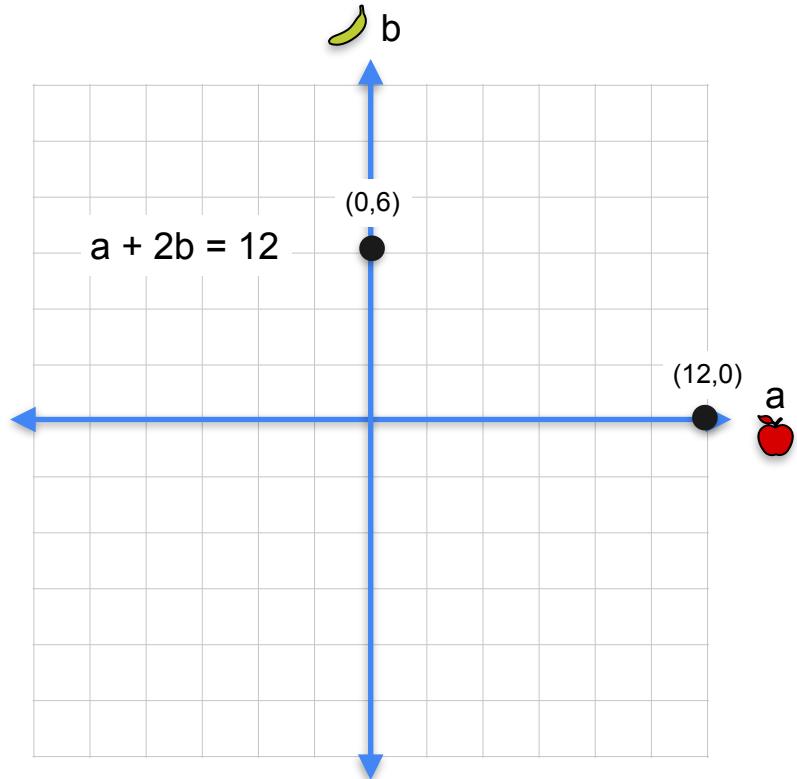
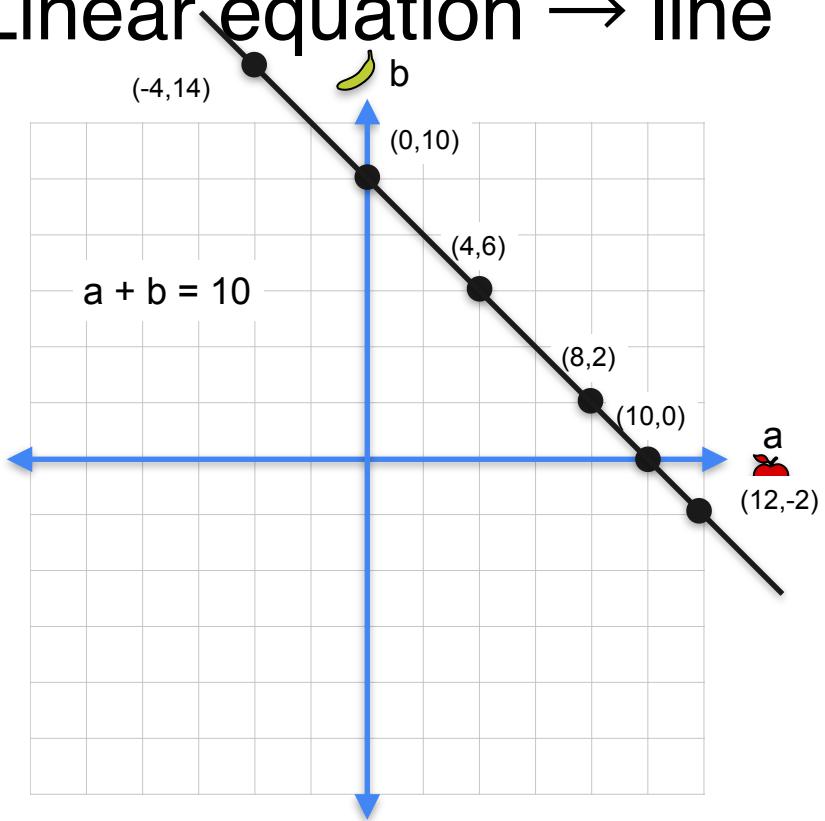
Linear equation → line



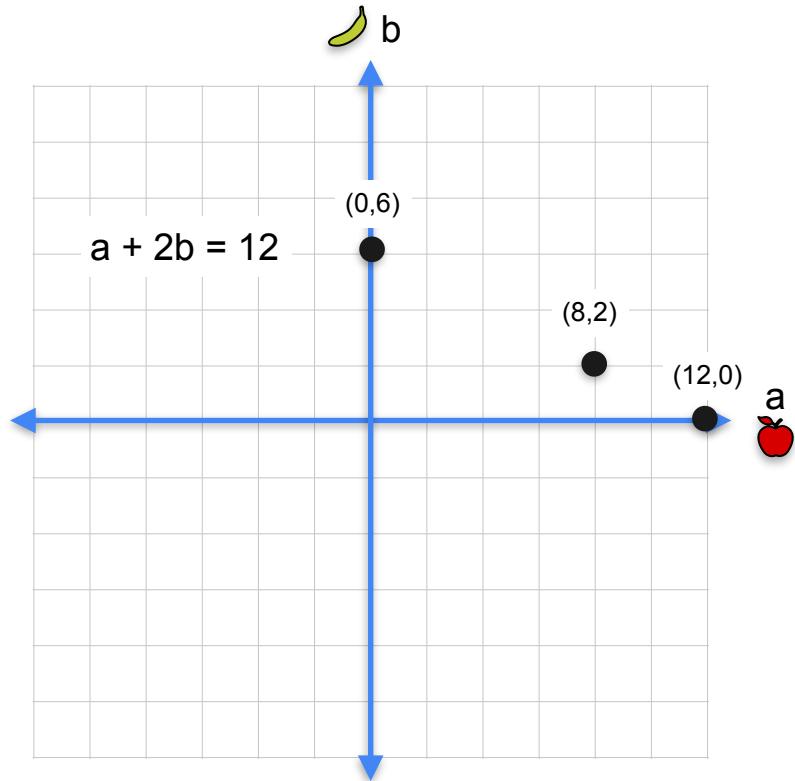
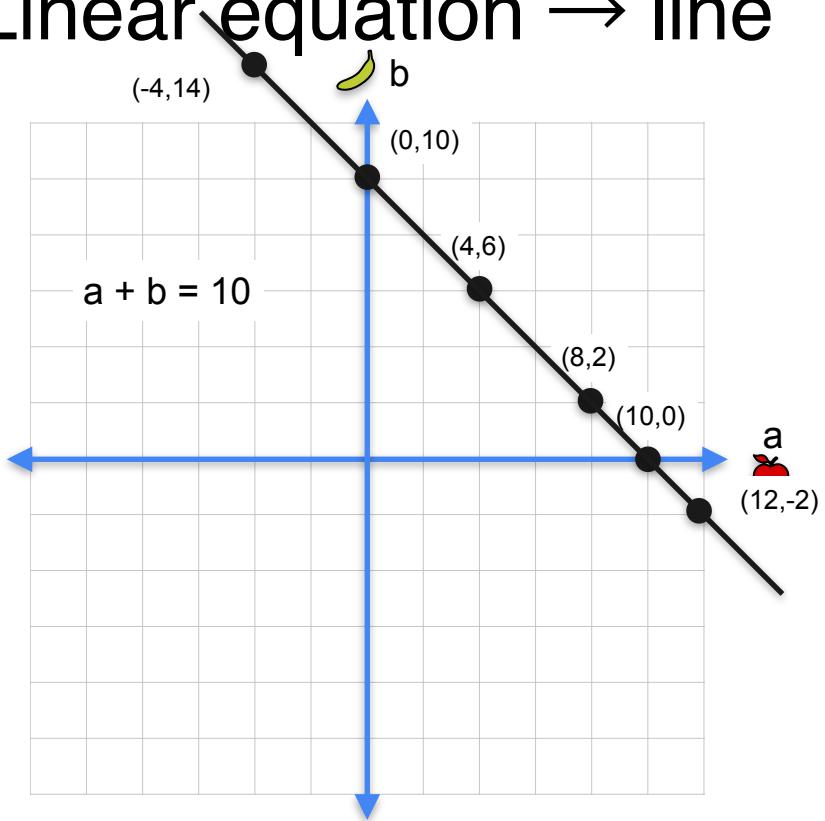
Linear equation → line



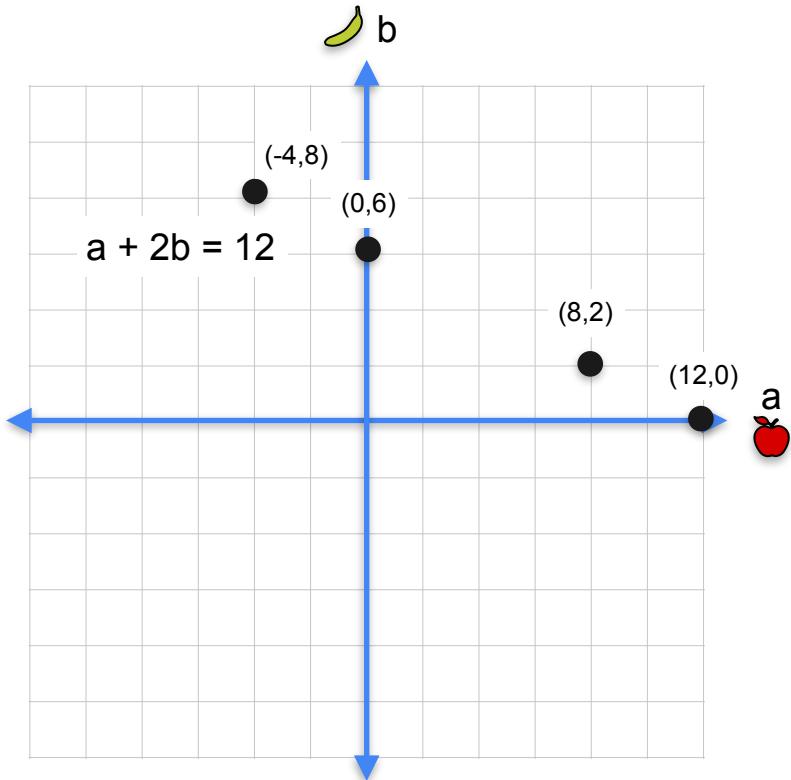
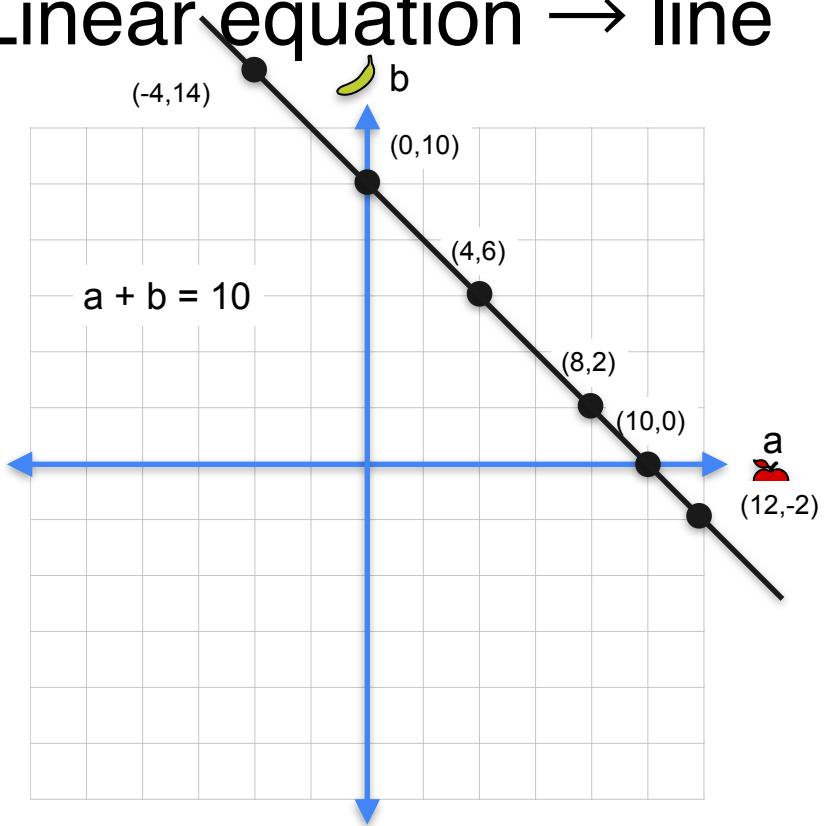
Linear equation → line



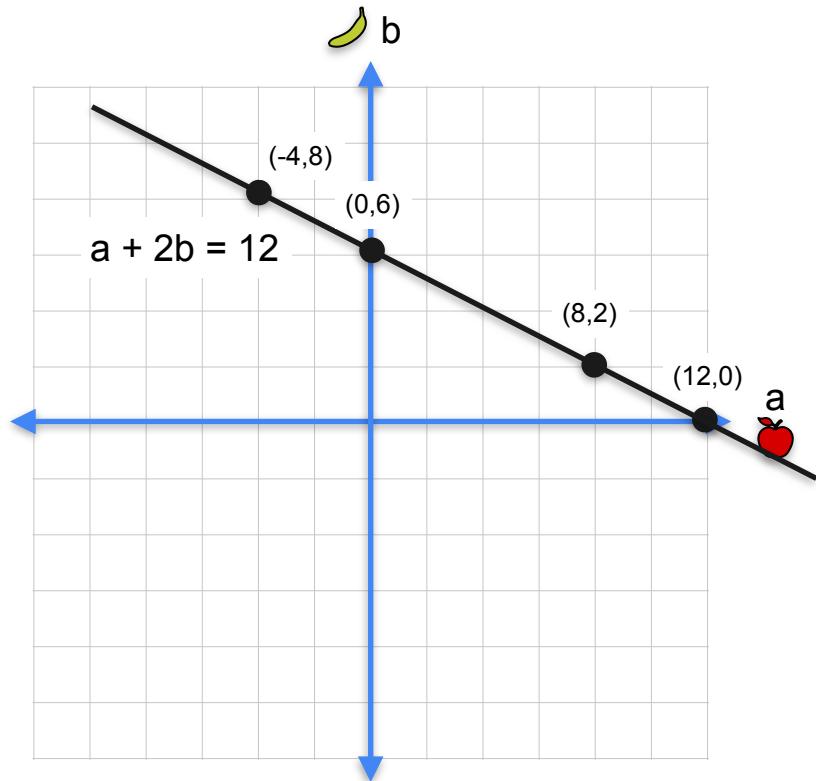
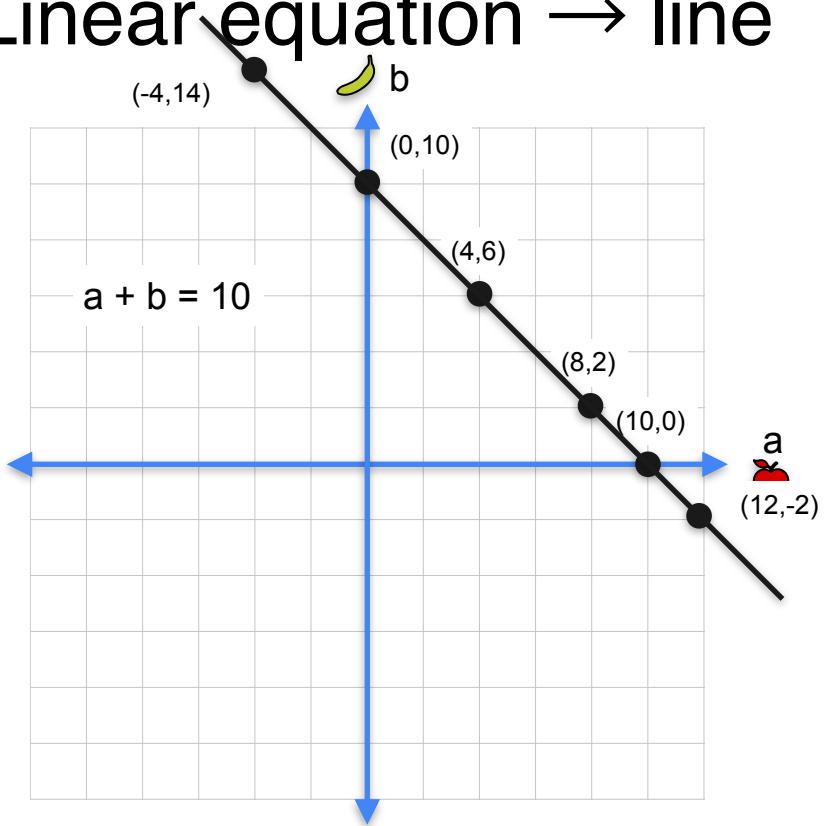
Linear equation → line



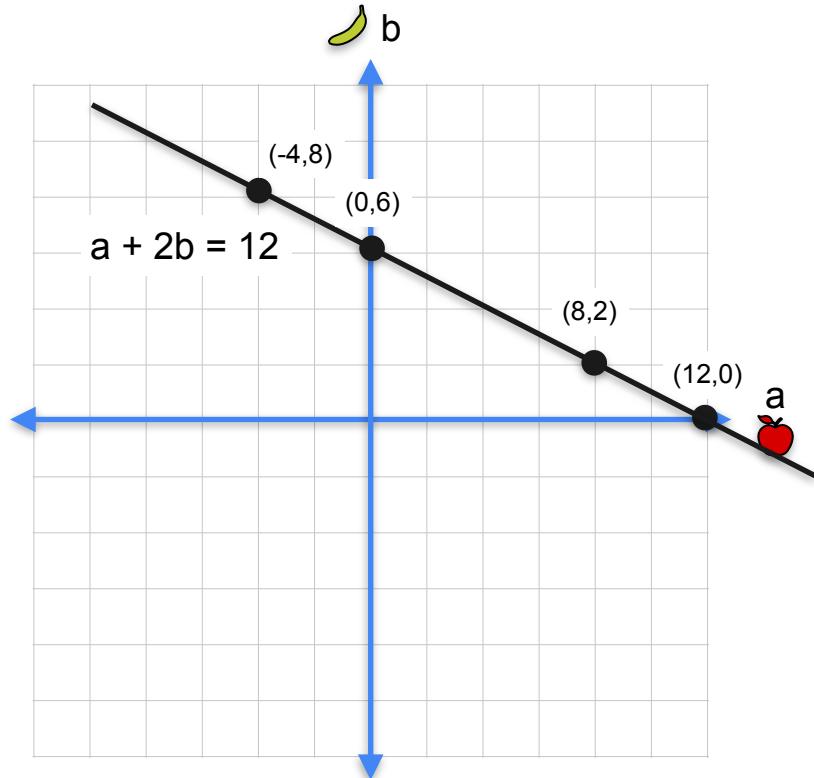
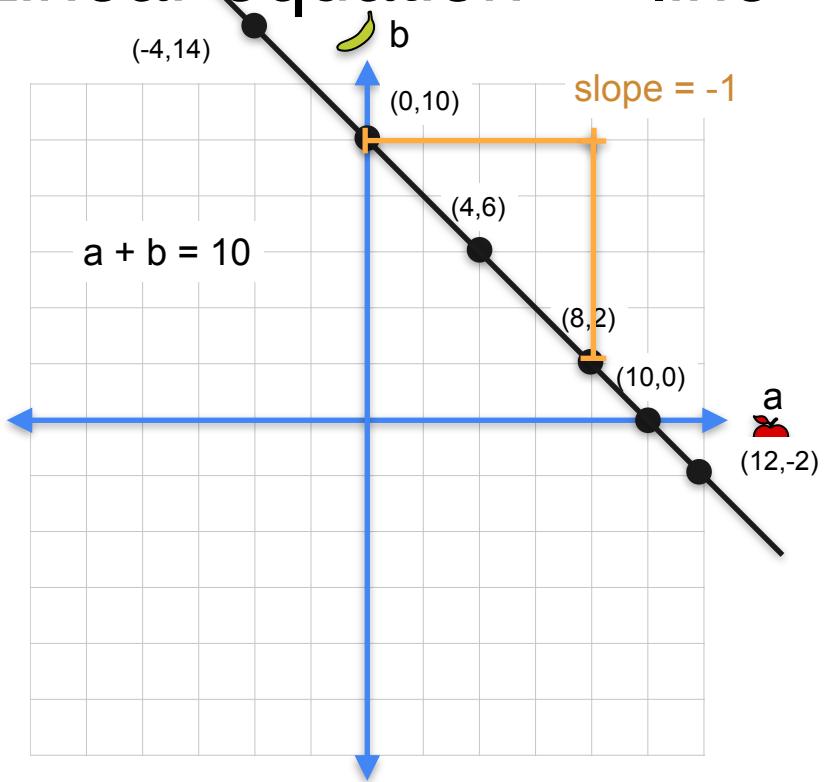
Linear equation → line



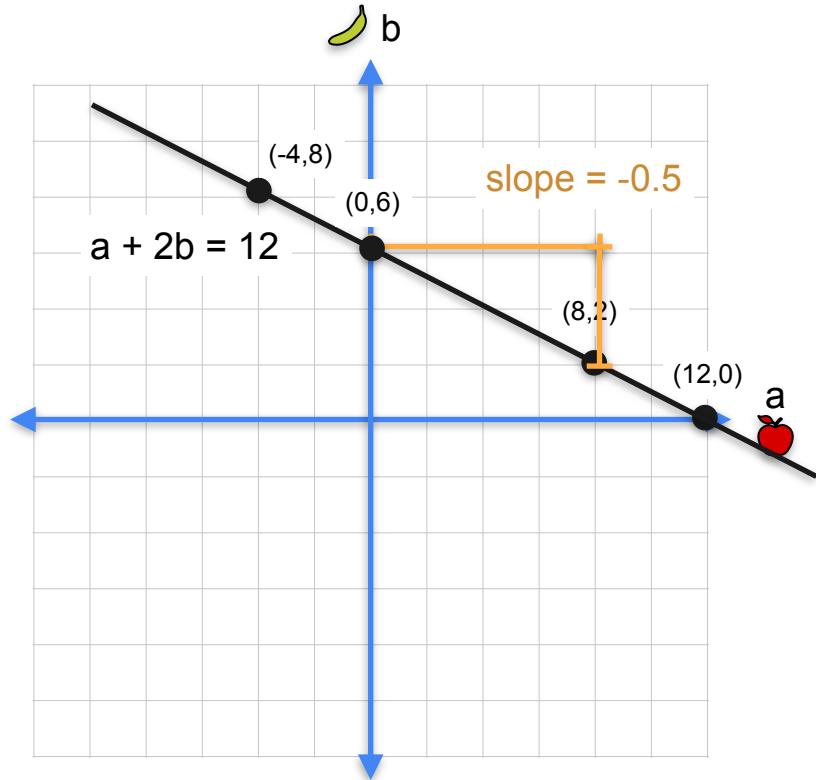
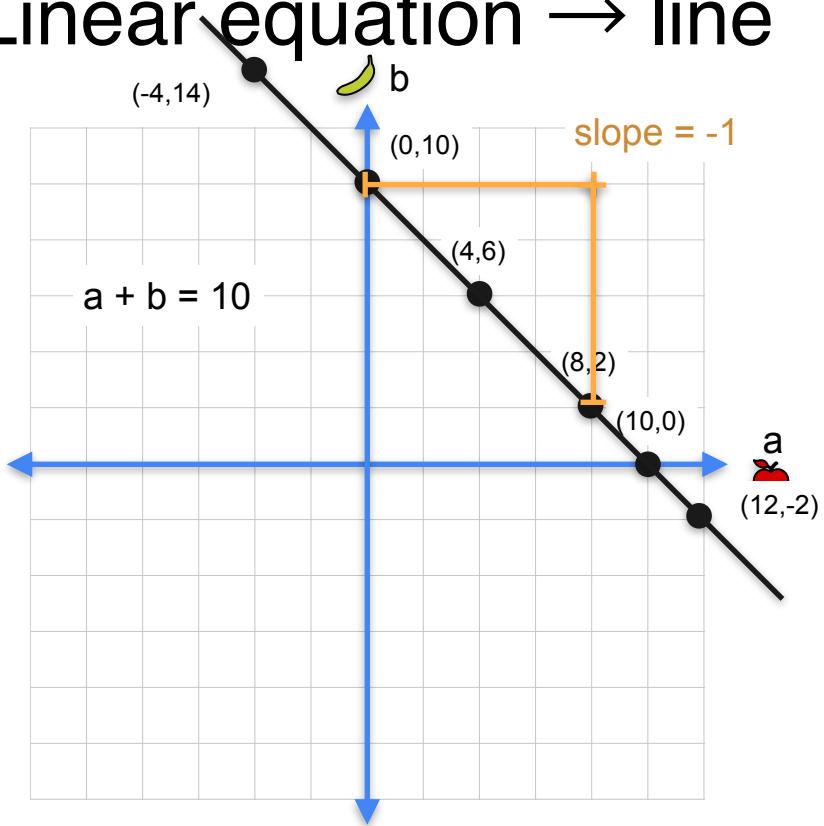
Linear equation → line



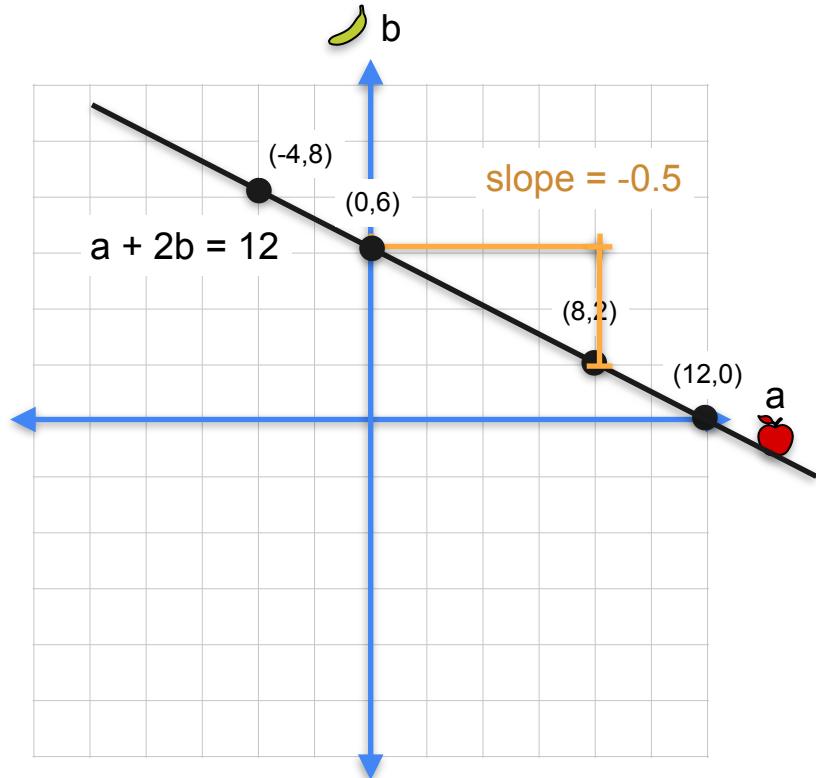
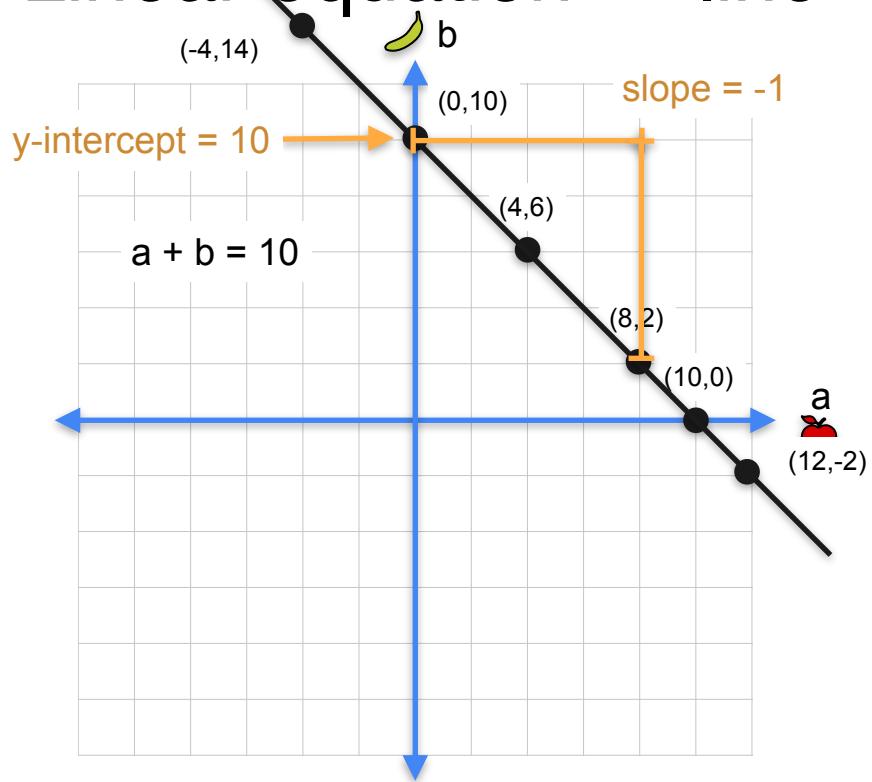
Linear equation → line



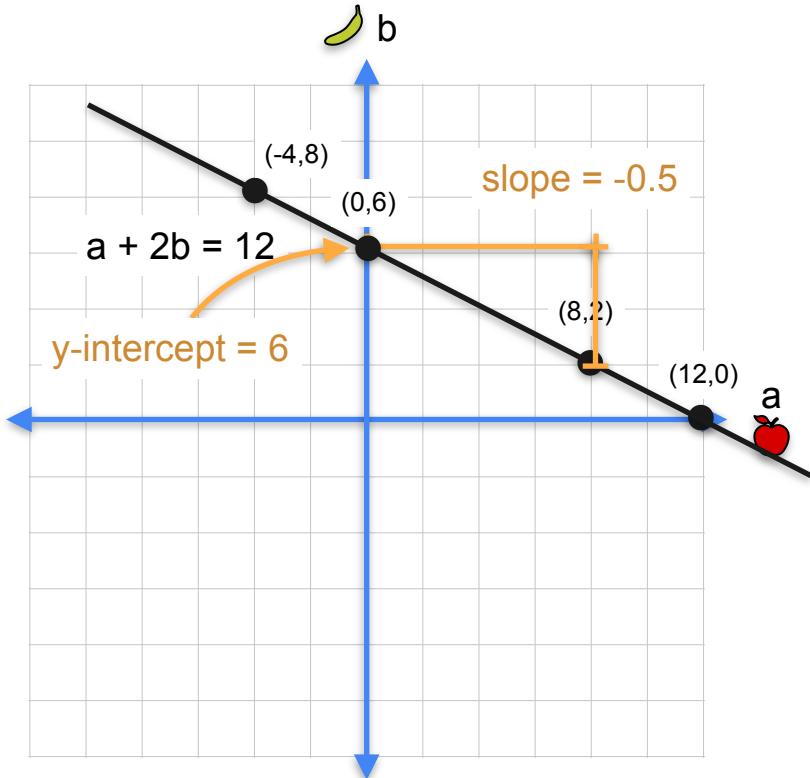
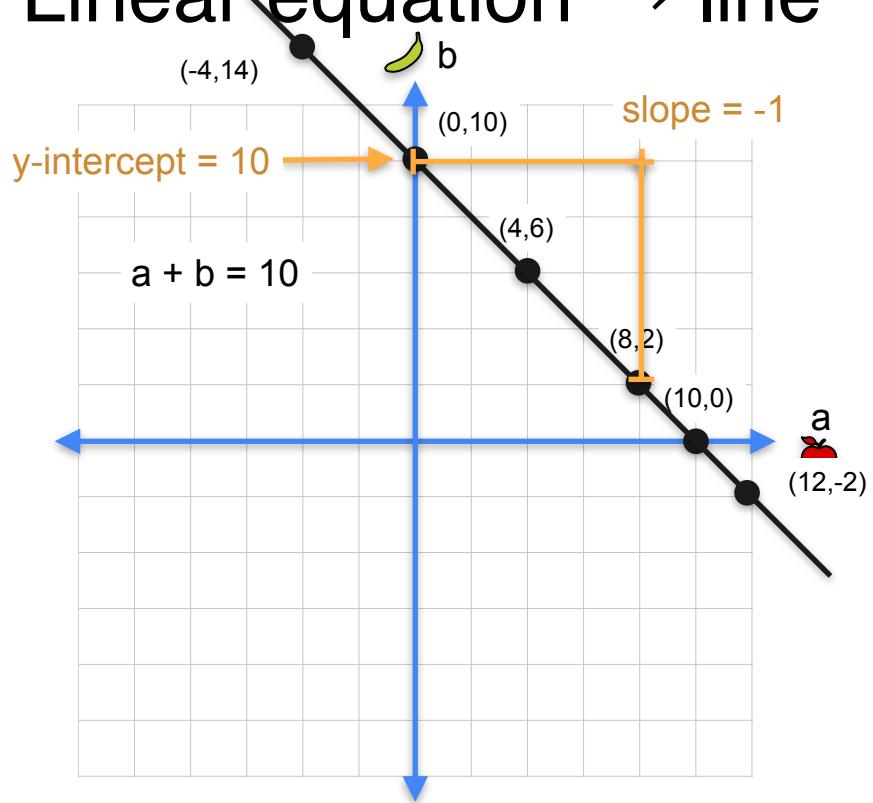
Linear equation → line



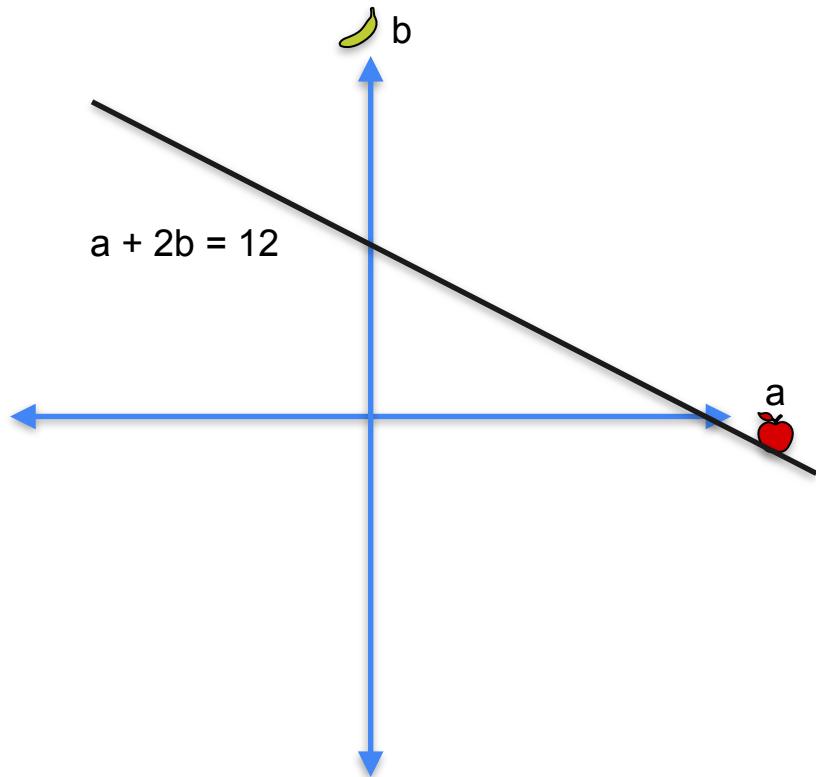
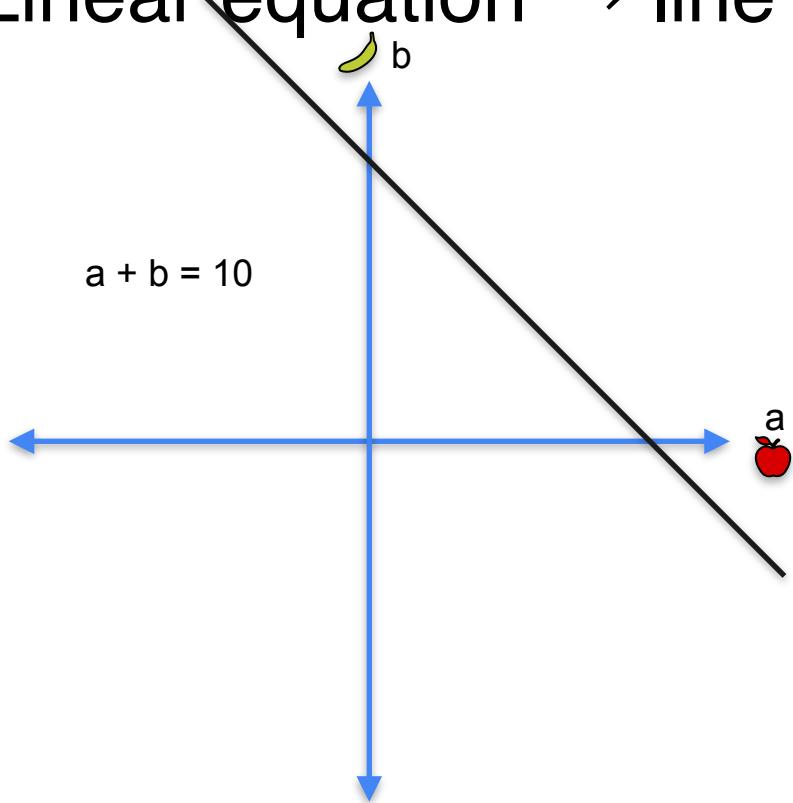
Linear equation → line



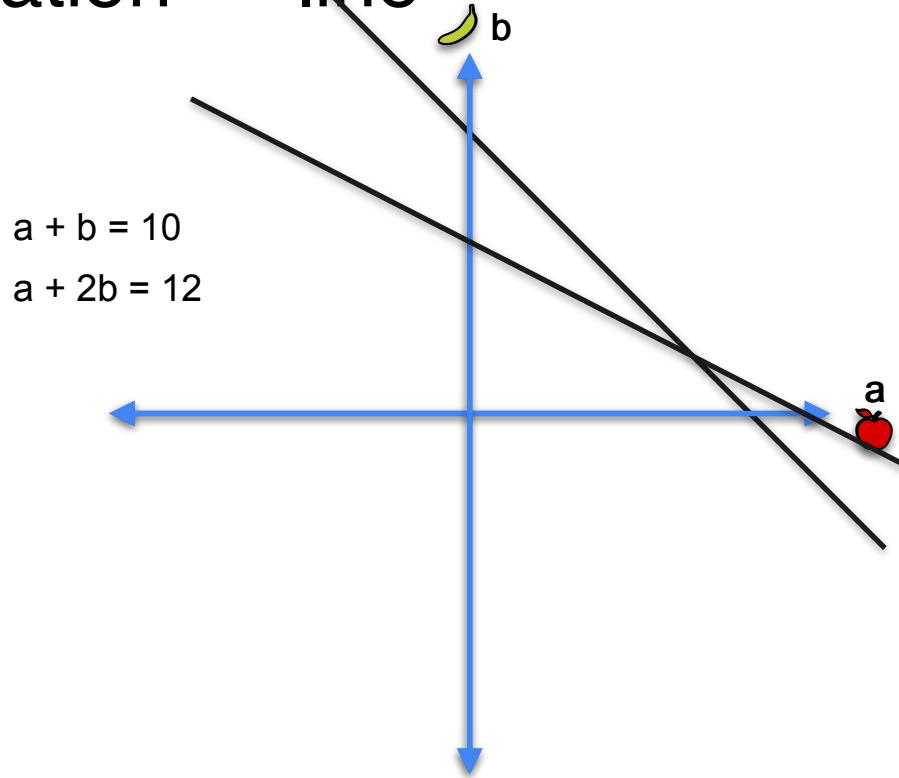
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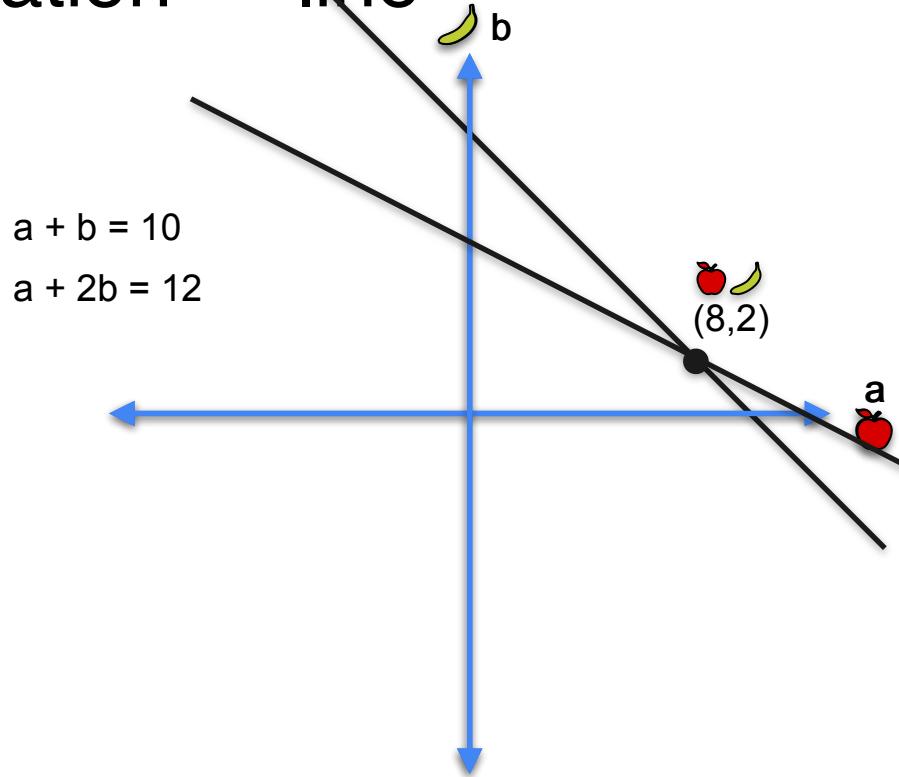
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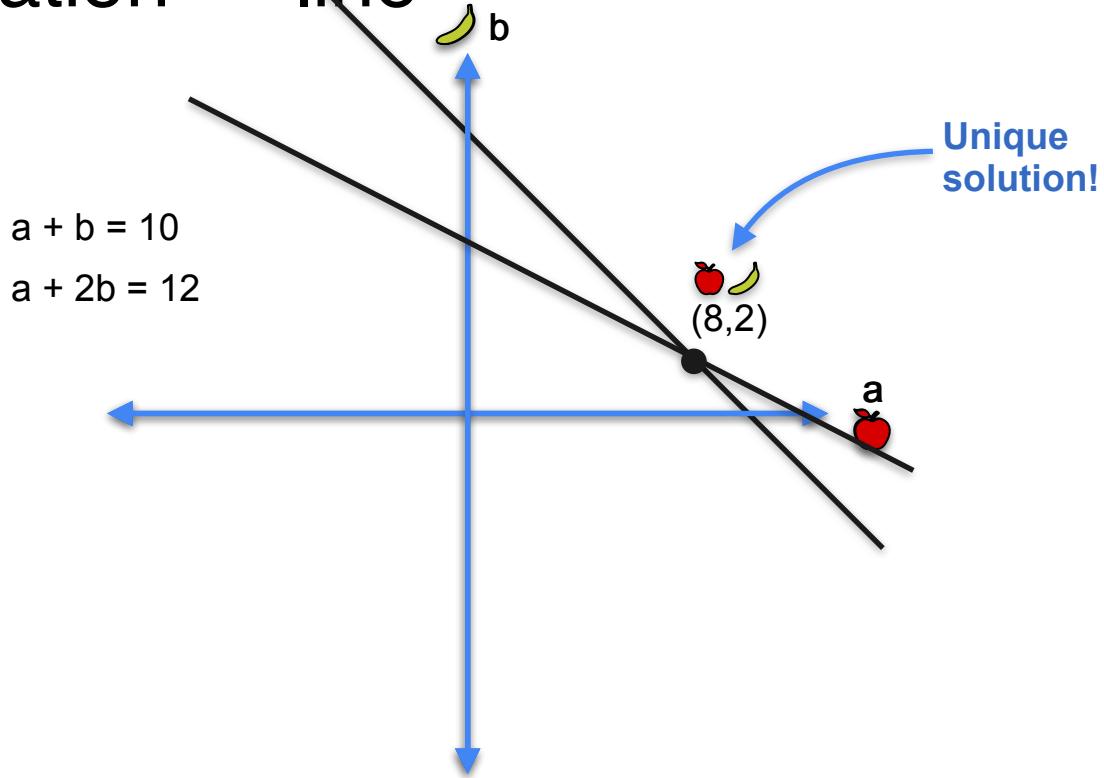
Linear equation → line



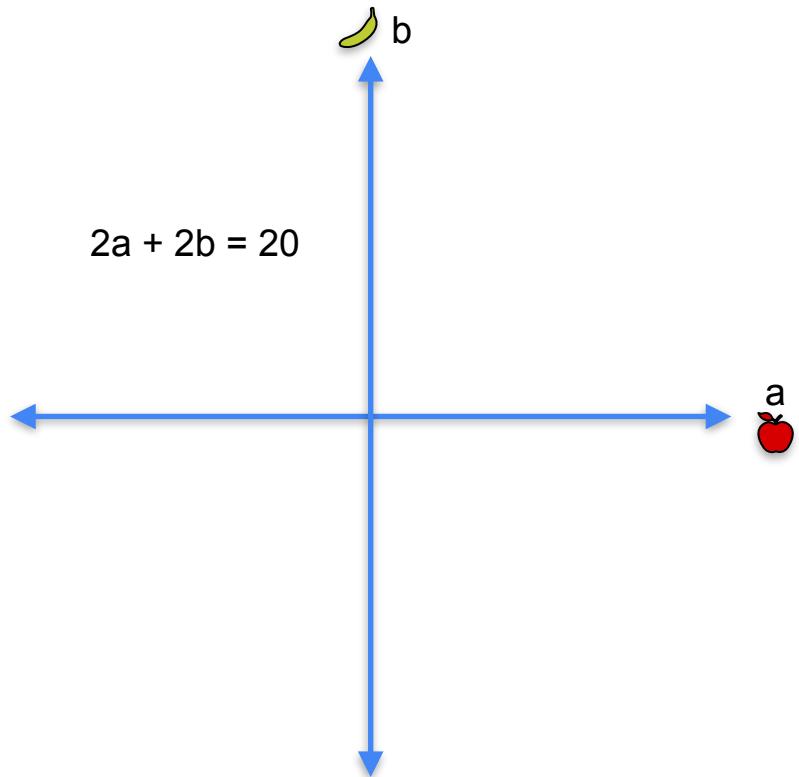
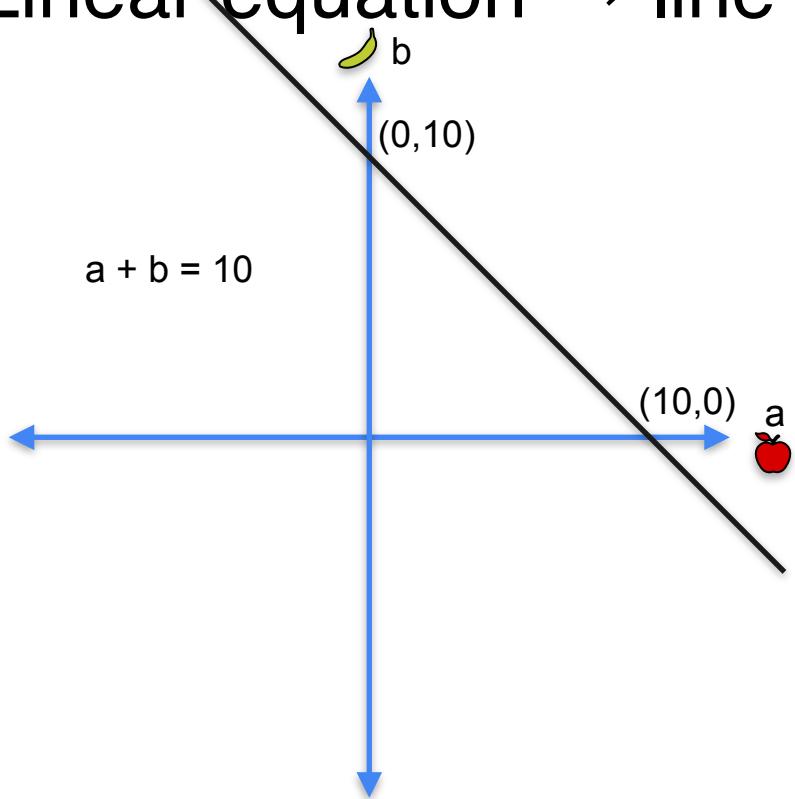
Linear equation → line



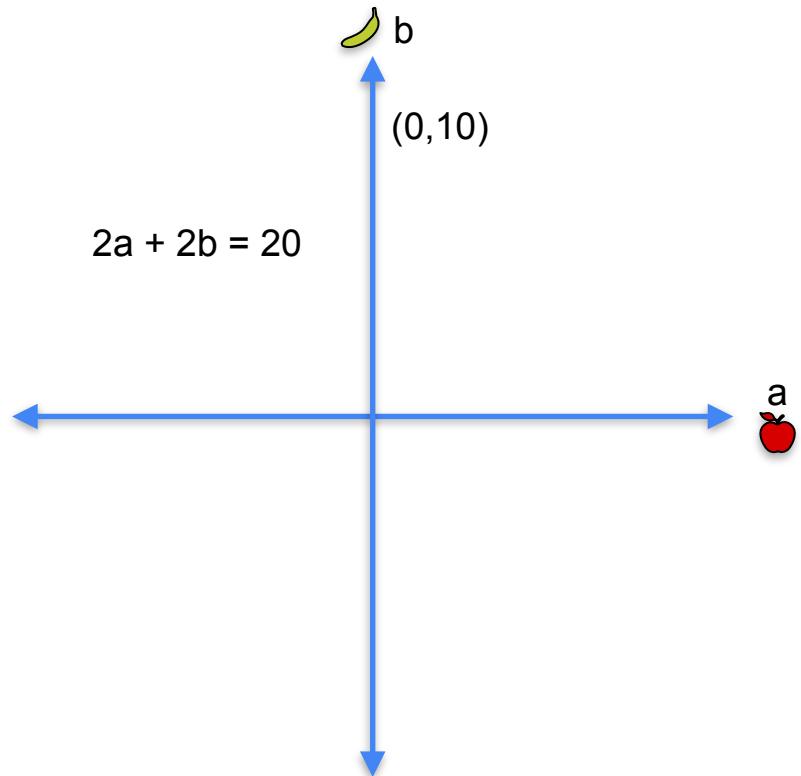
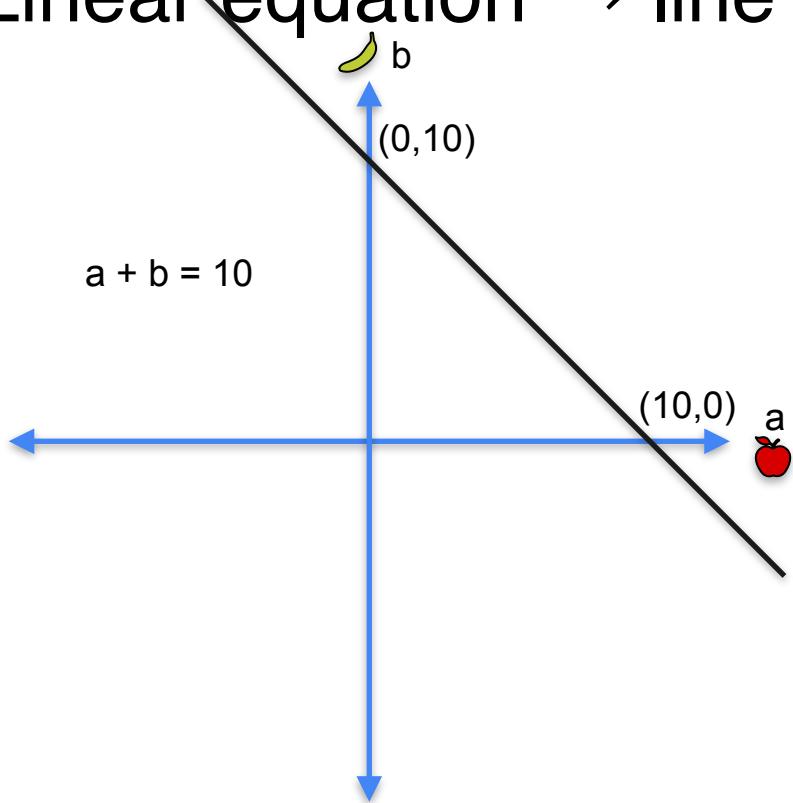
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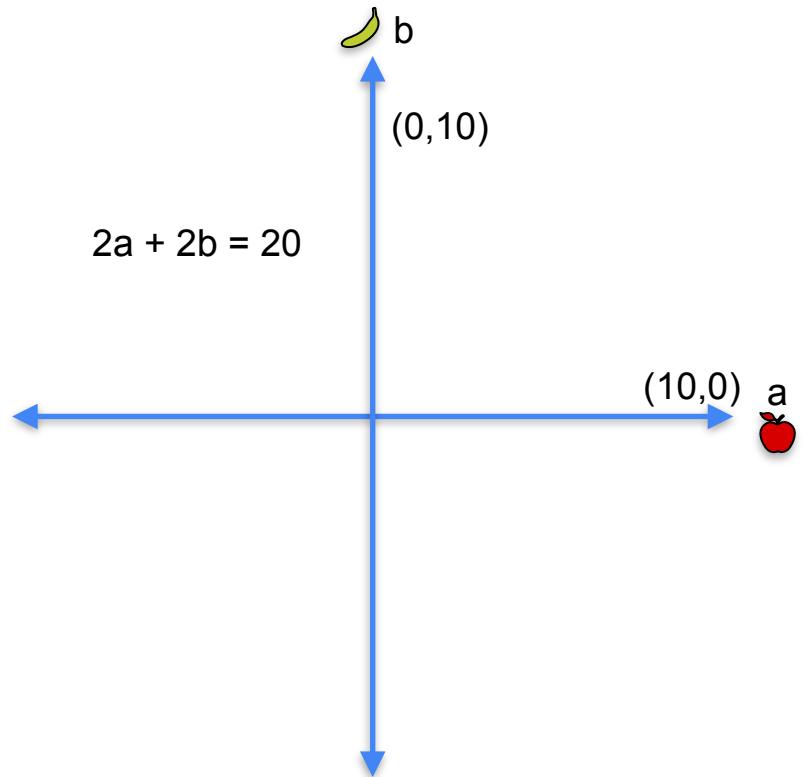
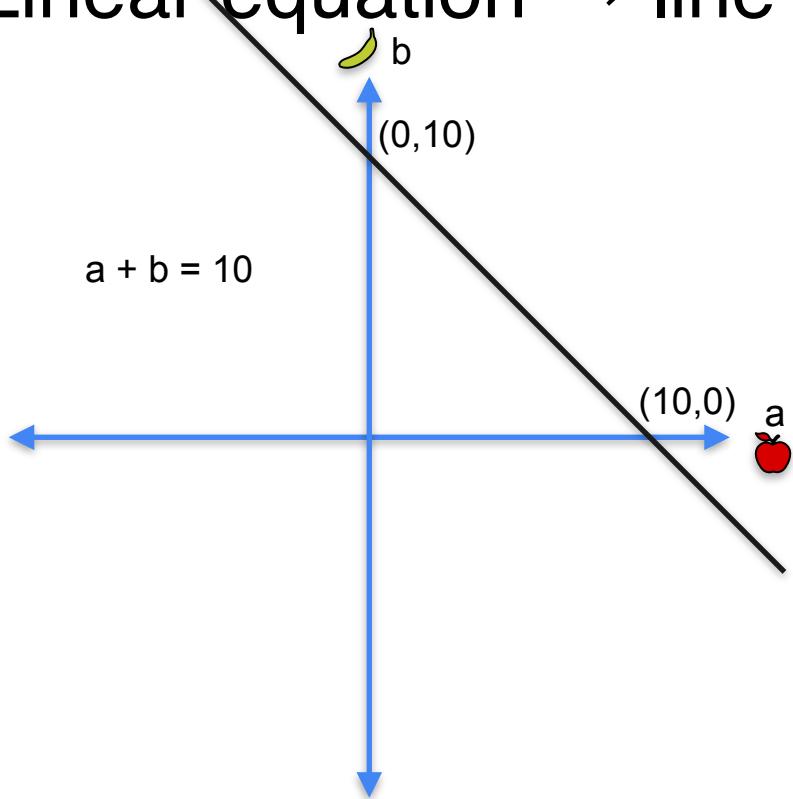
Linear equation → line



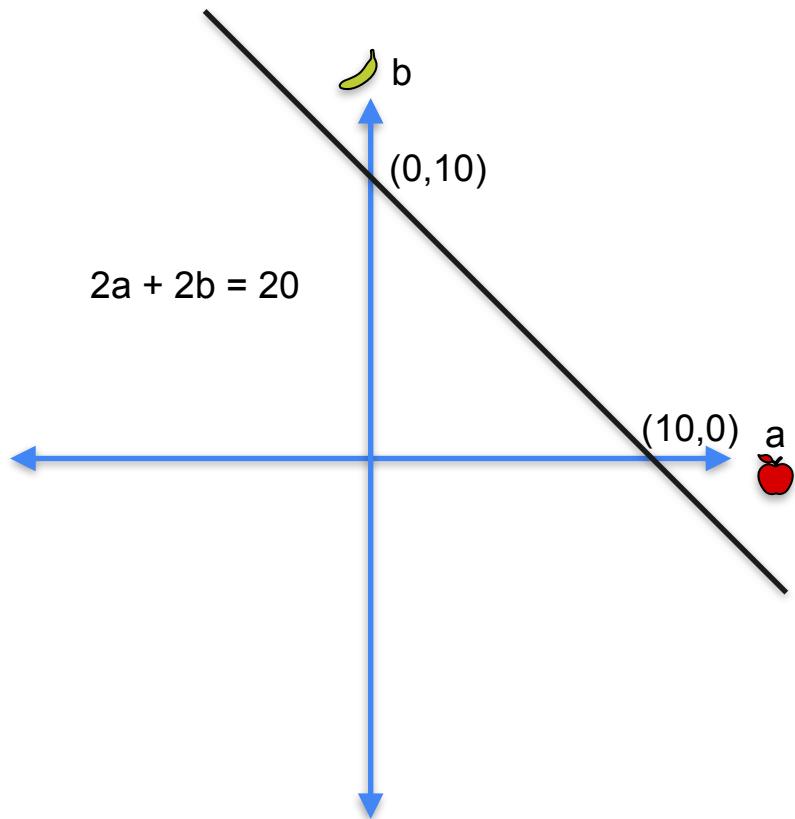
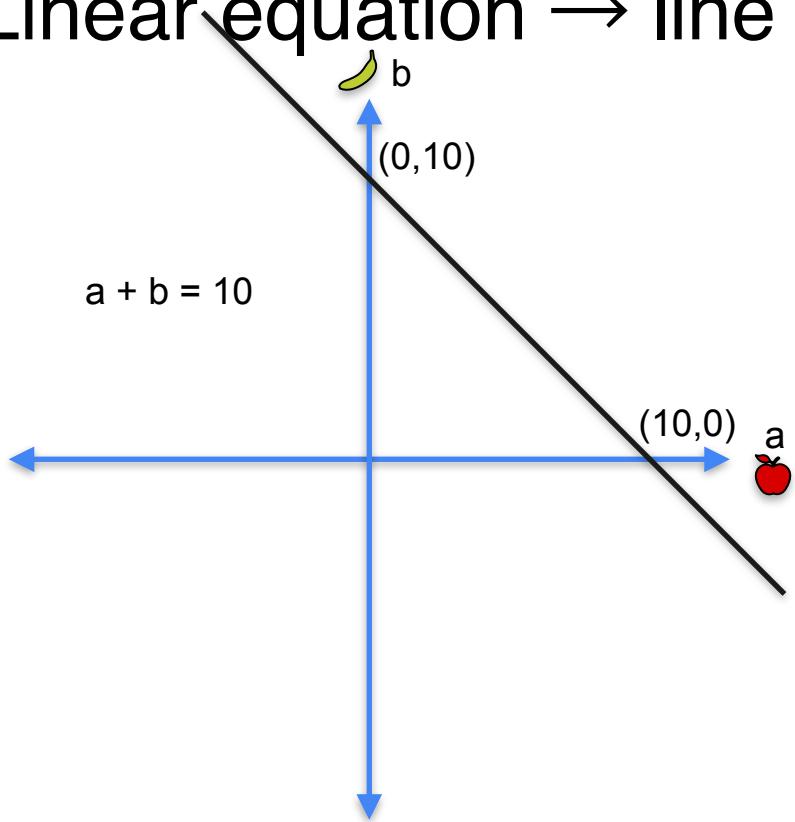
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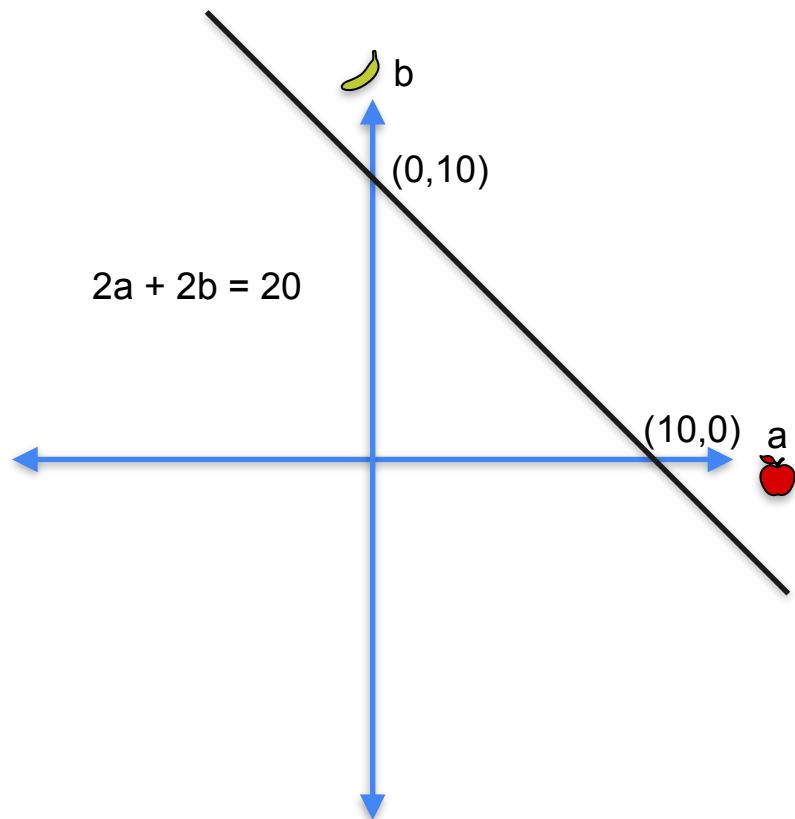
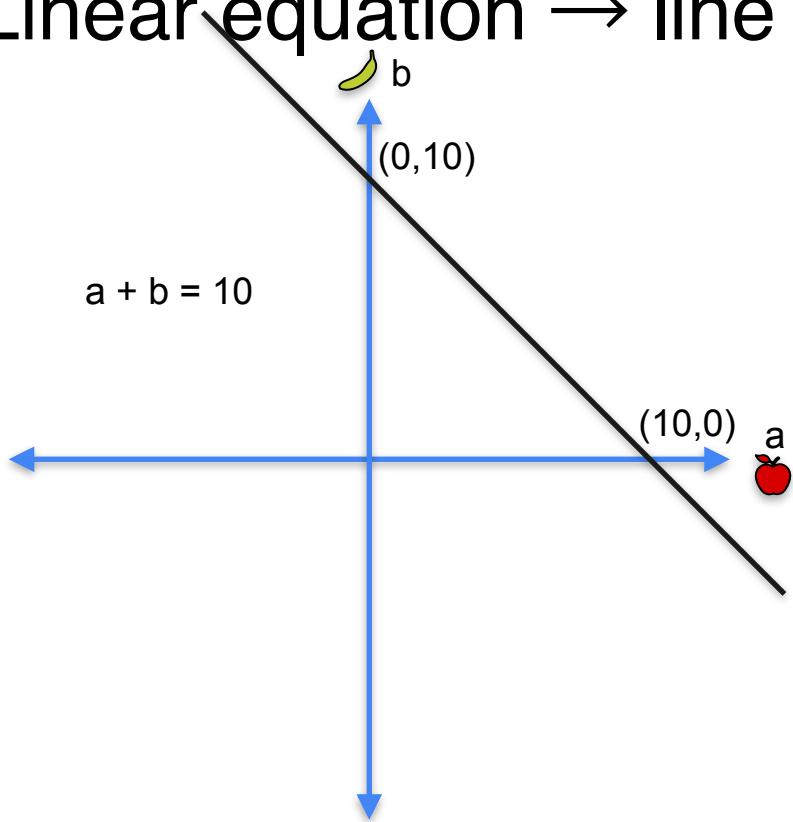
Linear equation → line



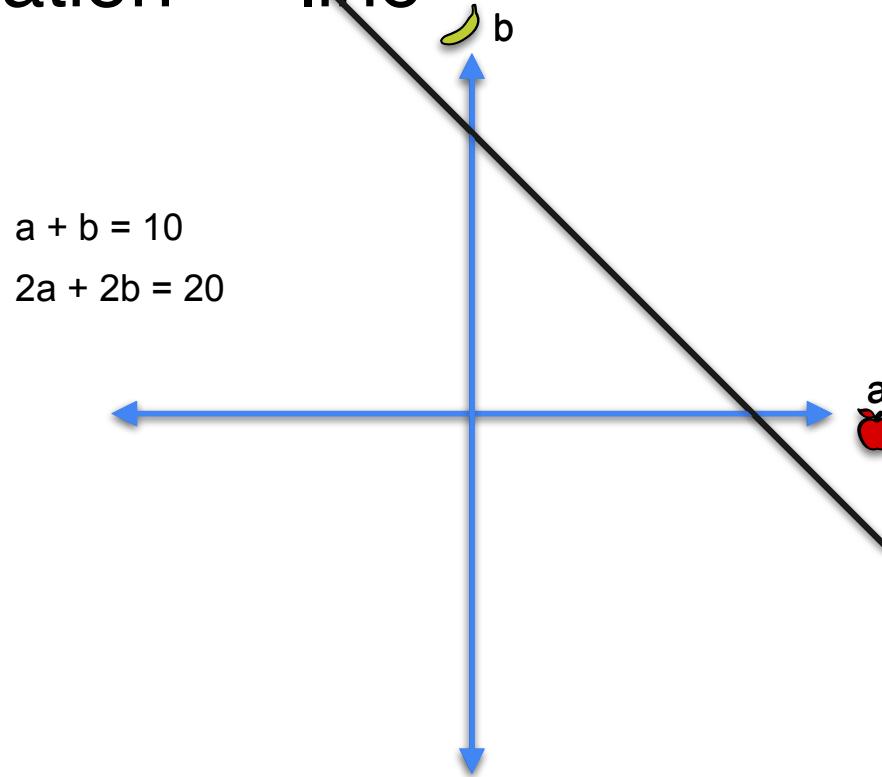
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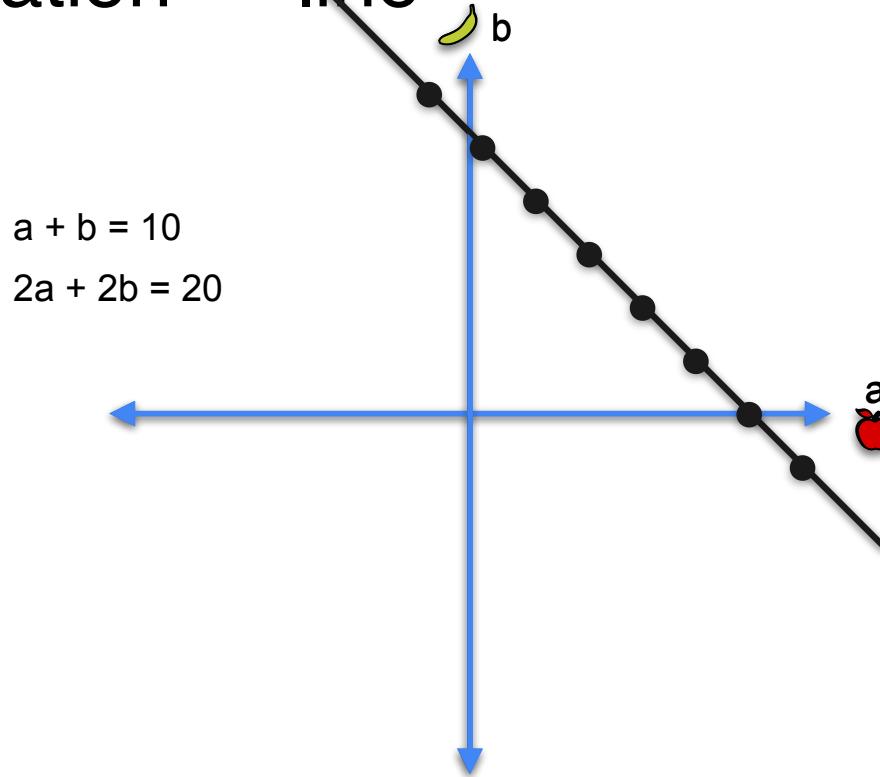
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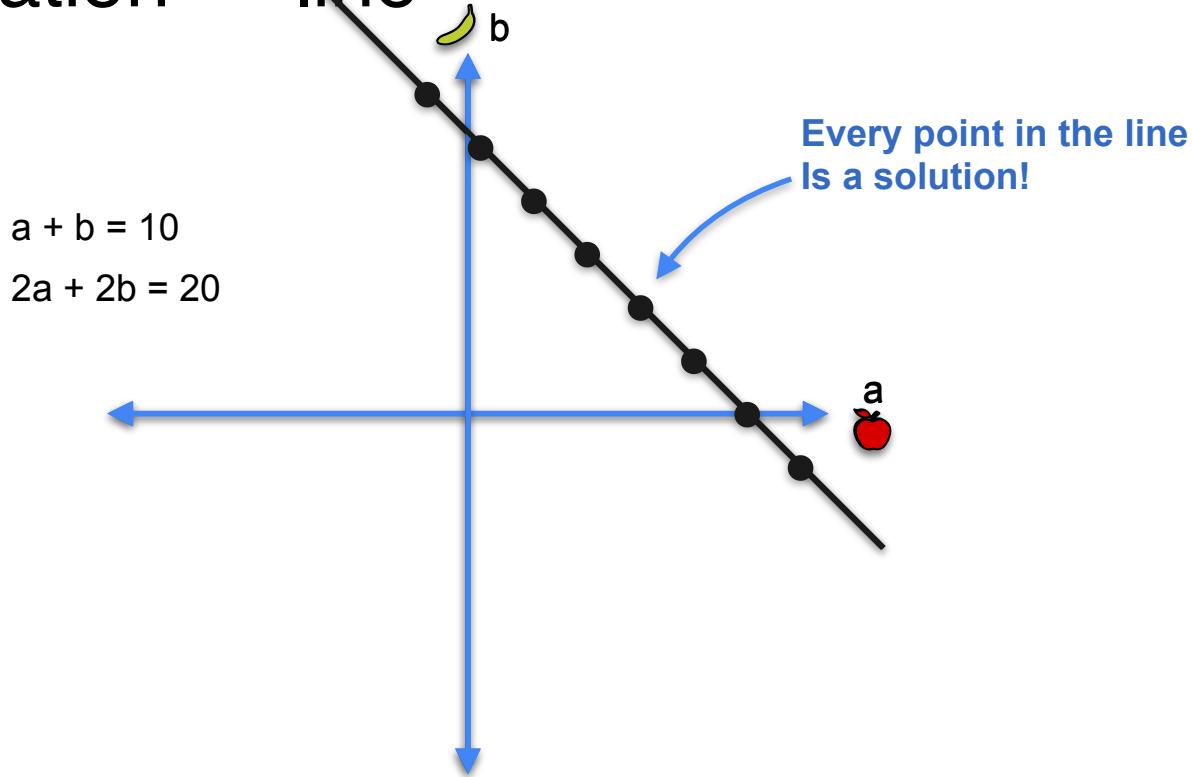
Linear equation → line



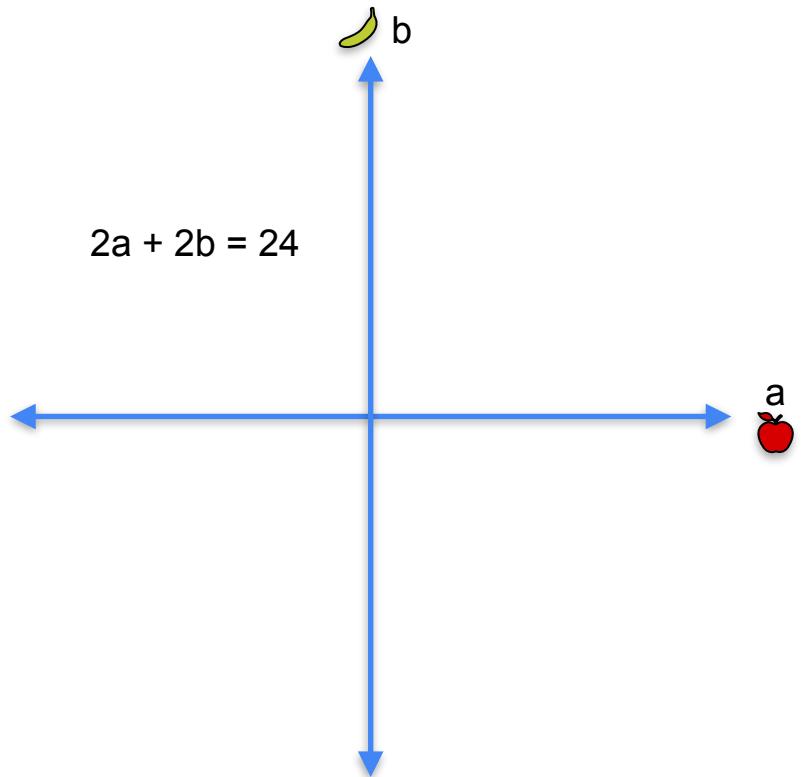
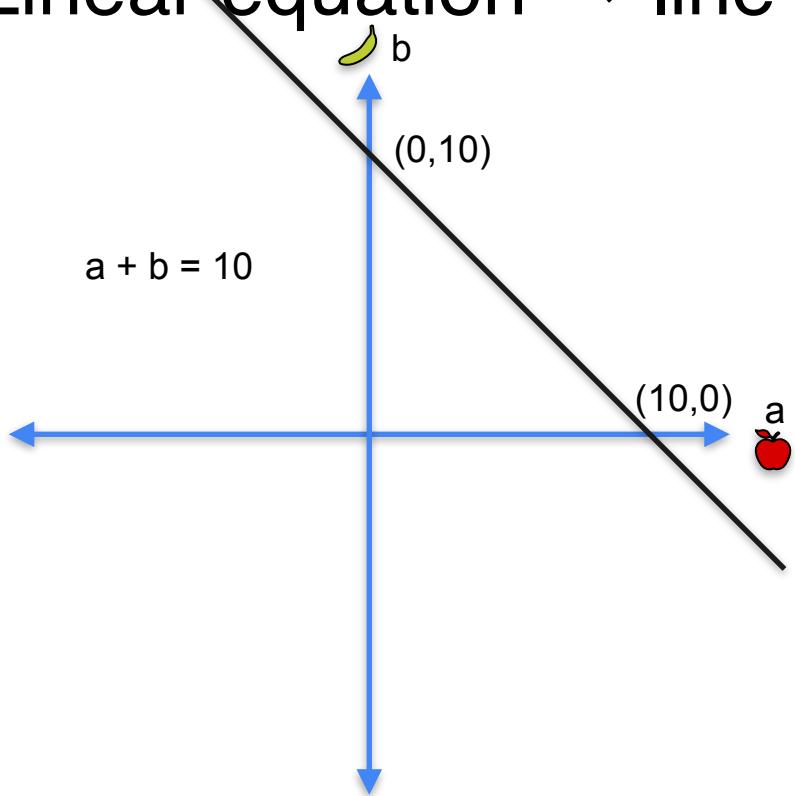
Linear equation → line



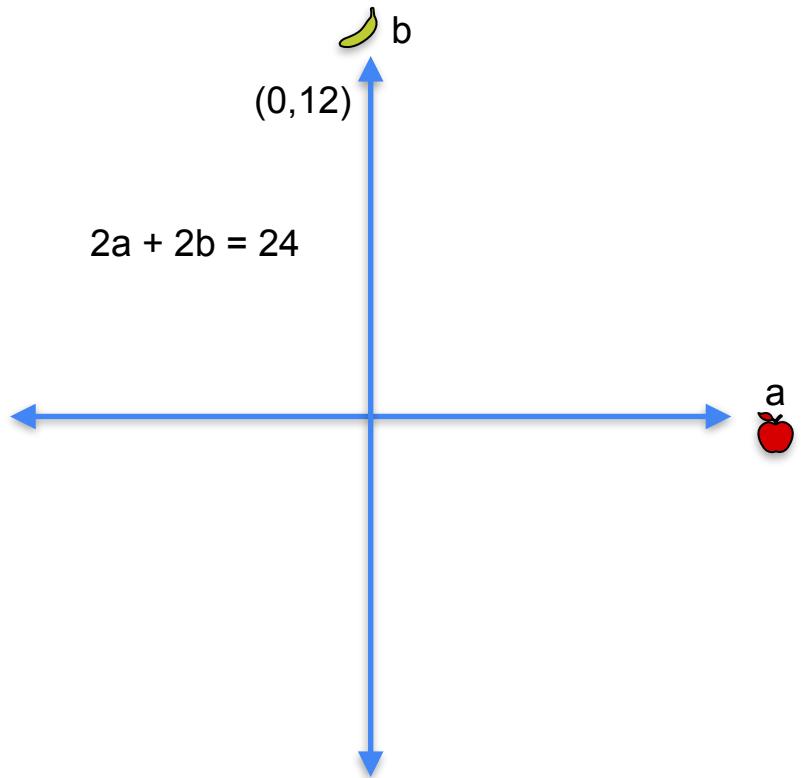
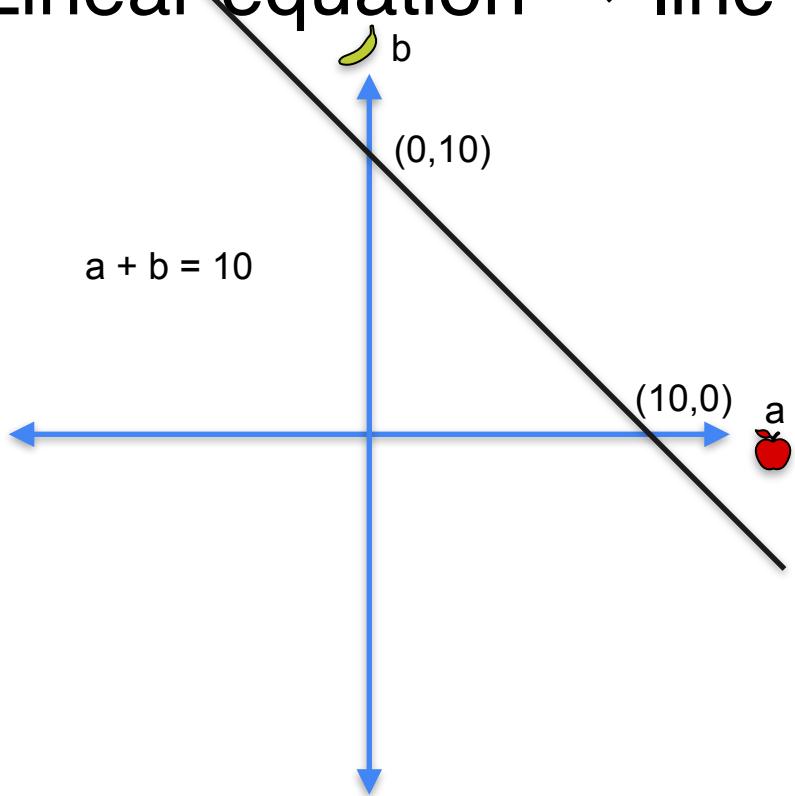
Linear equation → line



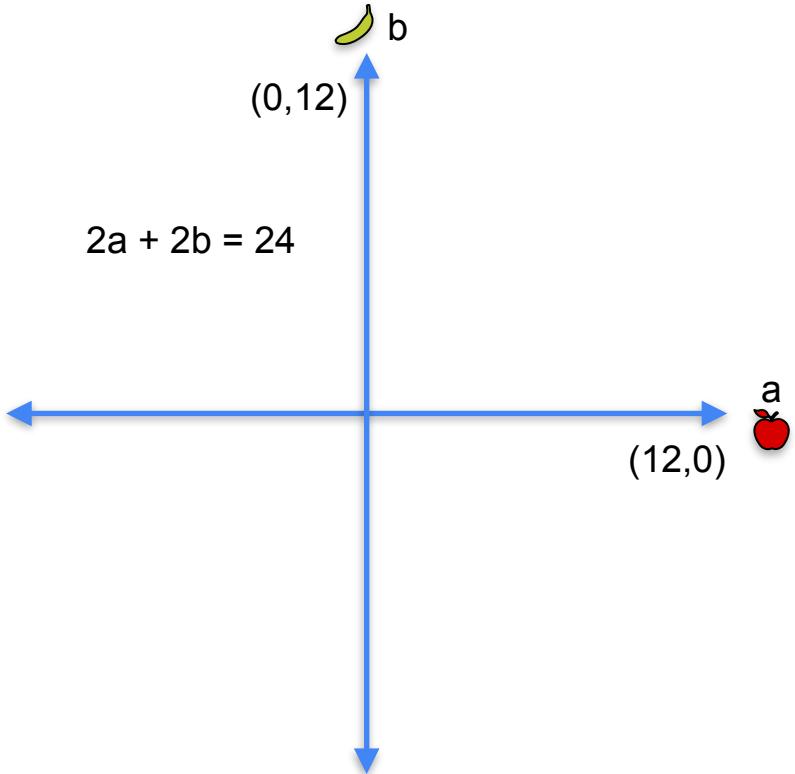
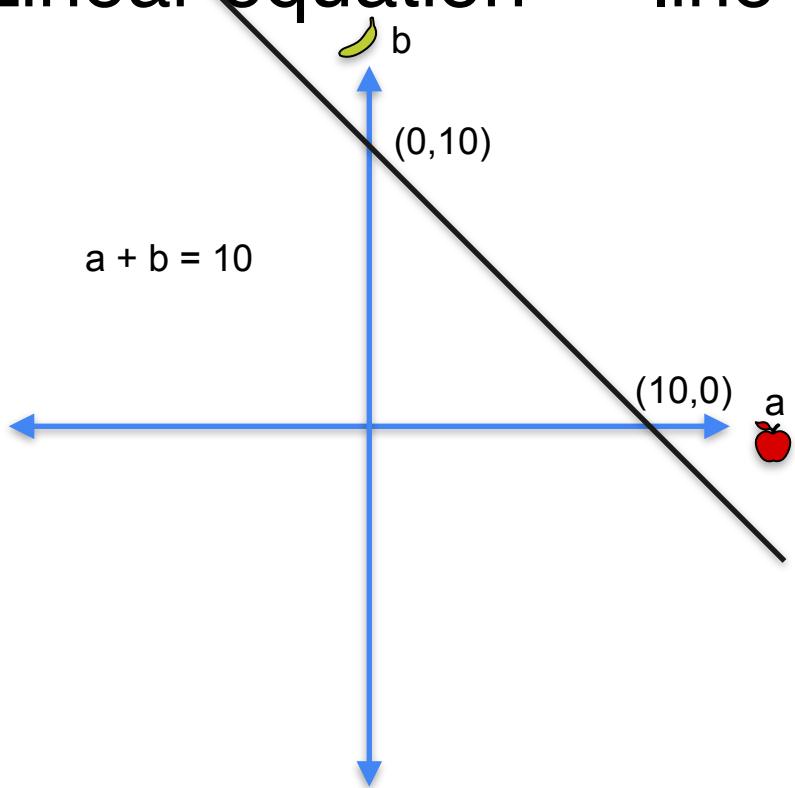
Linear equation → line



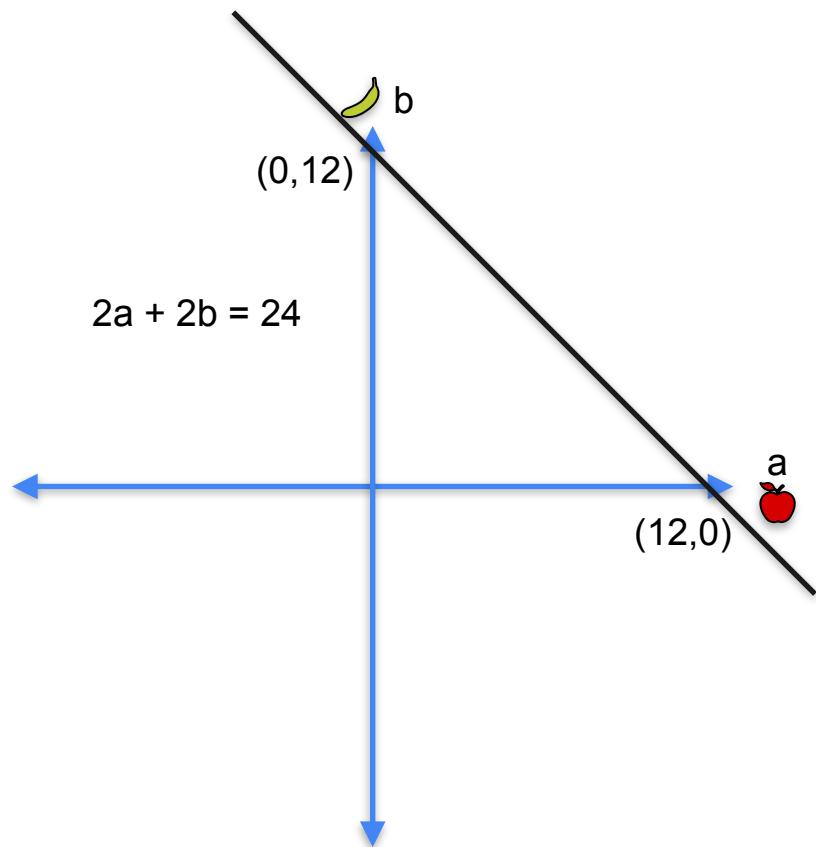
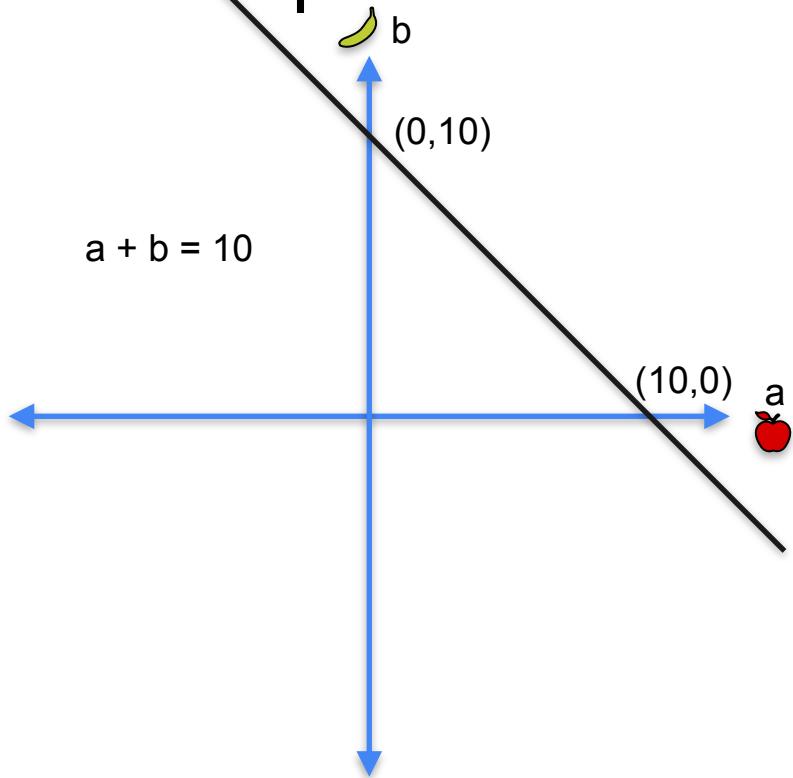
Linear equation → line



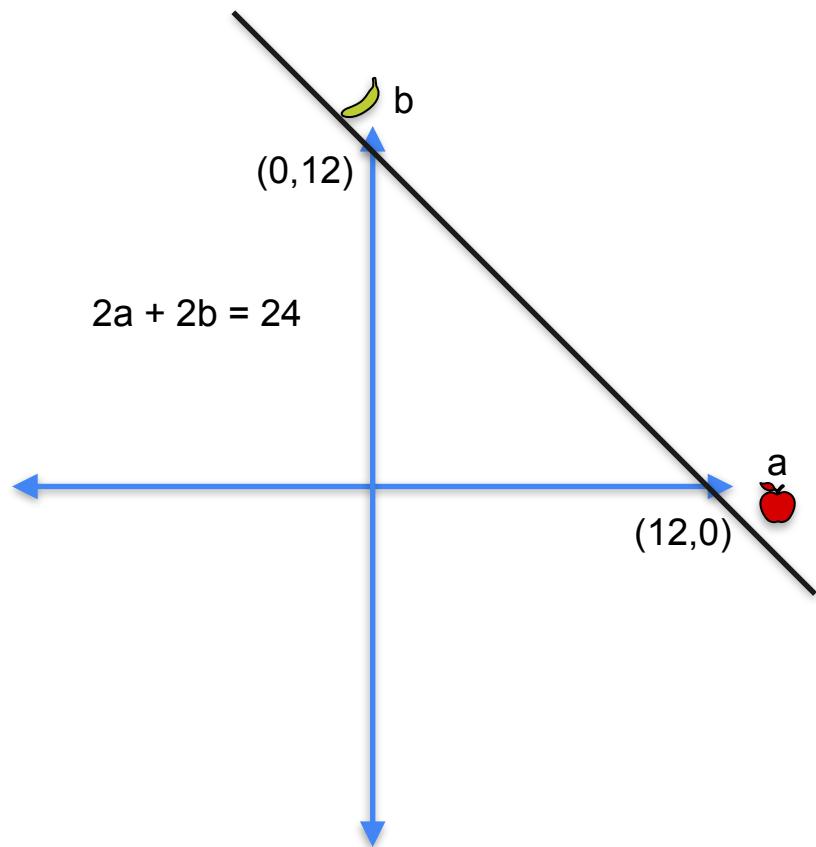
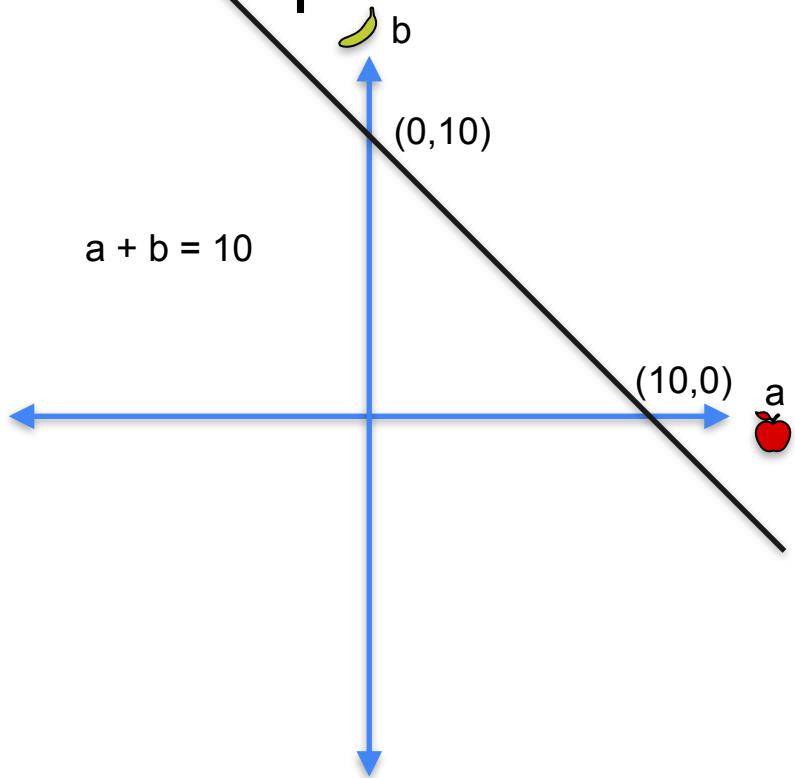
Linear equation → line



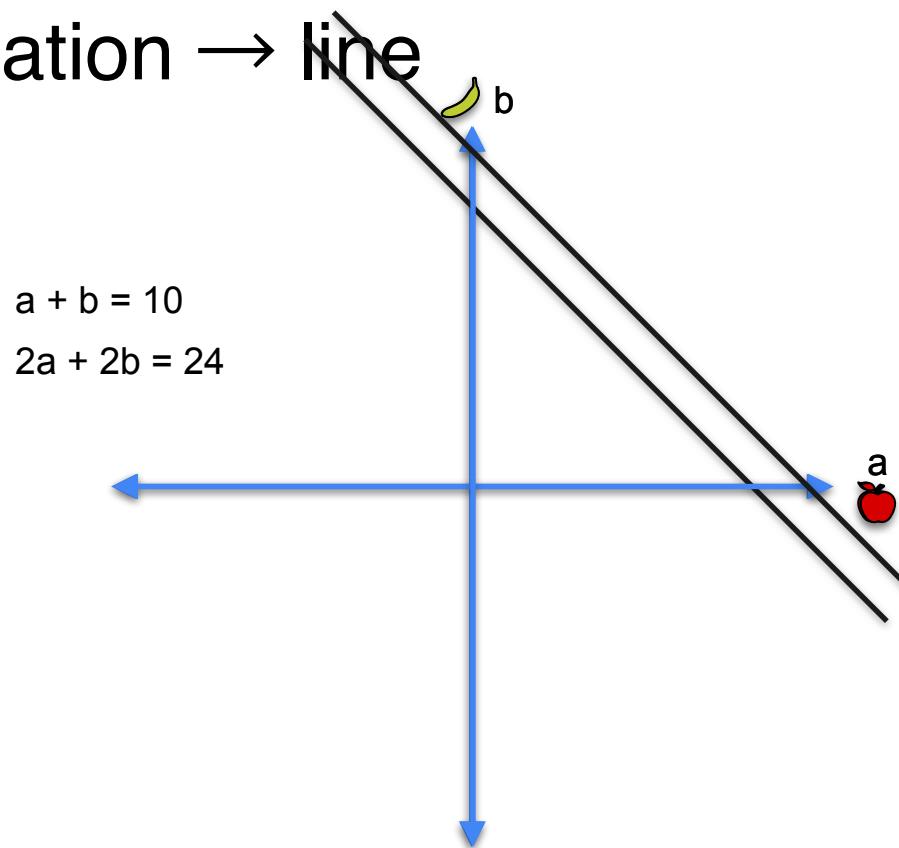
Linear equation → line



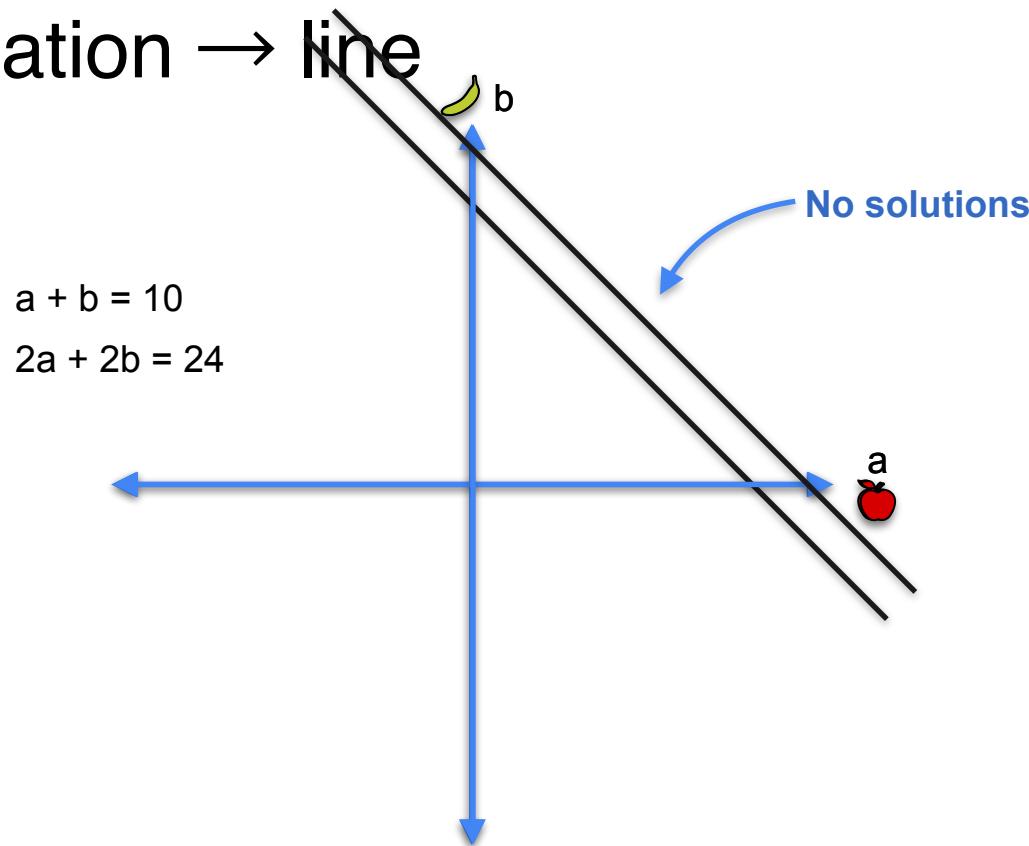
Linear equation → line



Linear equation → line



Linear equation → line



Systems of equations as lines

Systems of equations as lines

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Systems of equations as lines

System 1

- $a + b = 10$

- $a + 2b = 12$


System 2

- $a + b = 10$

- $2a + 2b = 20$


Systems of equations as lines

System 1

- $a + b = 10$

- $a + 2b = 12$


System 2

- $a + b = 10$

- $2a + 2b = 20$


System 3

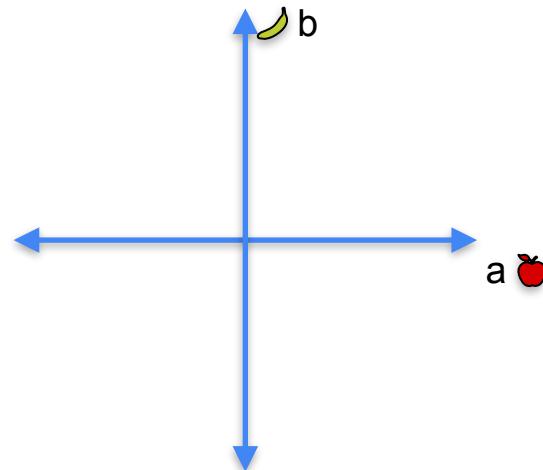
- $a + b = 10$

- $2a + 2b = 24$


Systems of equations as lines

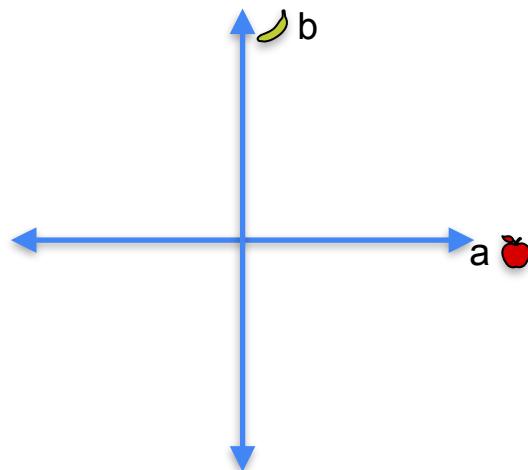
System 1

- $a + b = 10$
 
- $a + 2b = 12$
 



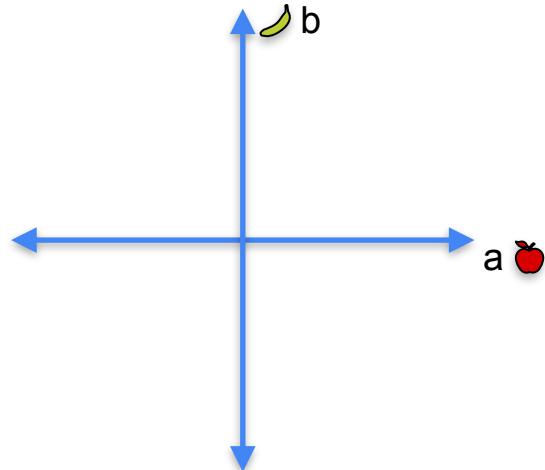
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 



System 3

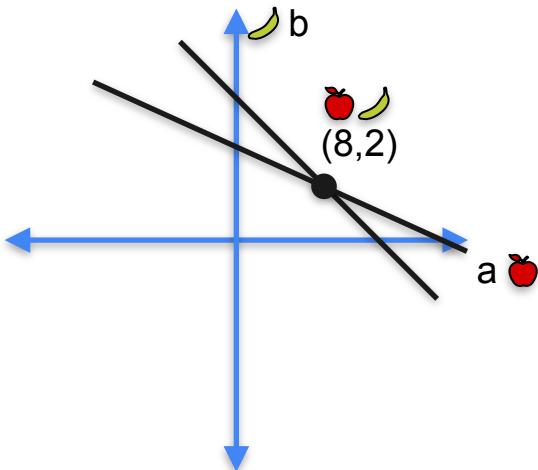
- $a + b = 10$
 
- $2a + 2b = 24$
 



Systems of equations as lines

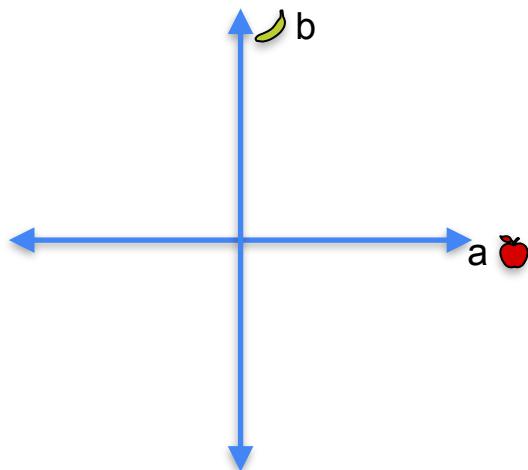
System 1

- $a + b = 10$
 
- $a + 2b = 12$
 



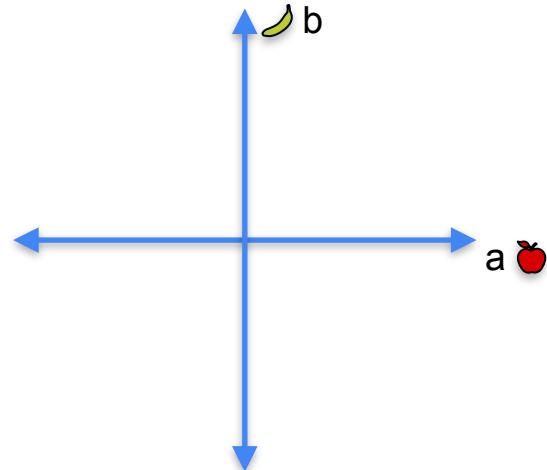
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 



System 3

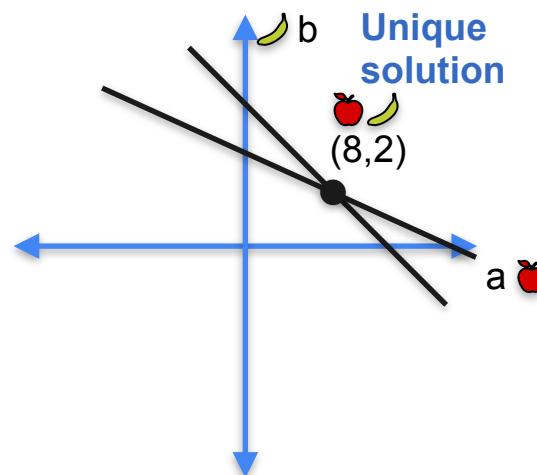
- $a + b = 10$
 
- $2a + 2b = 24$
 



Systems of equations as lines

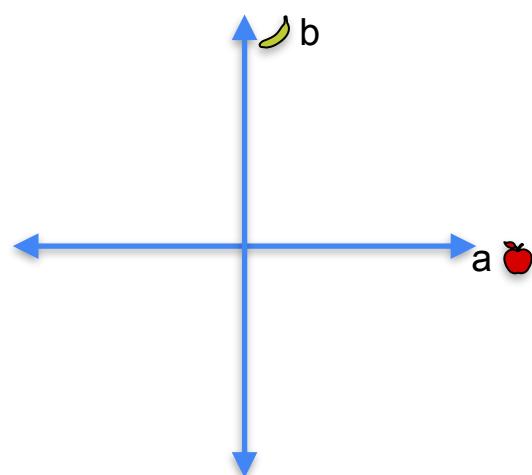
System 1

- $a + b = 10$
 
- $a + 2b = 12$
 



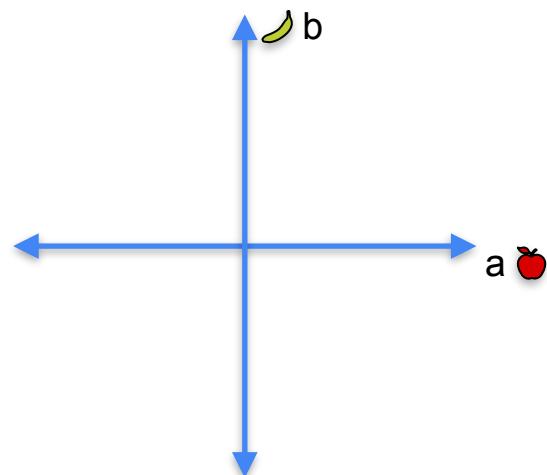
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 



System 3

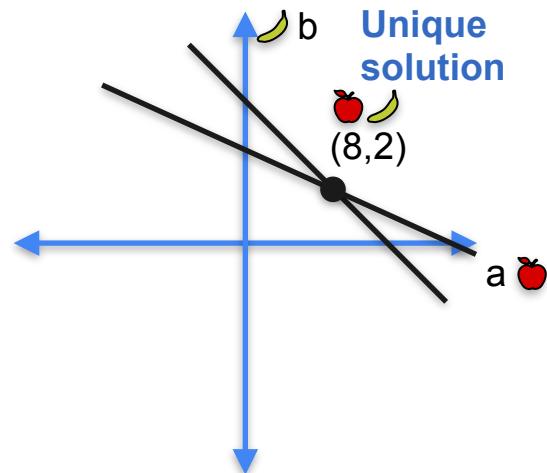
- $a + b = 10$
 
- $2a + 2b = 24$
 



Systems of equations as lines

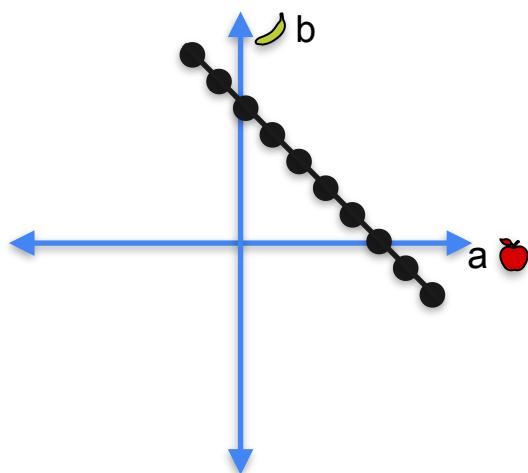
System 1

- $a + b = 10$
 
- $a + 2b = 12$
 



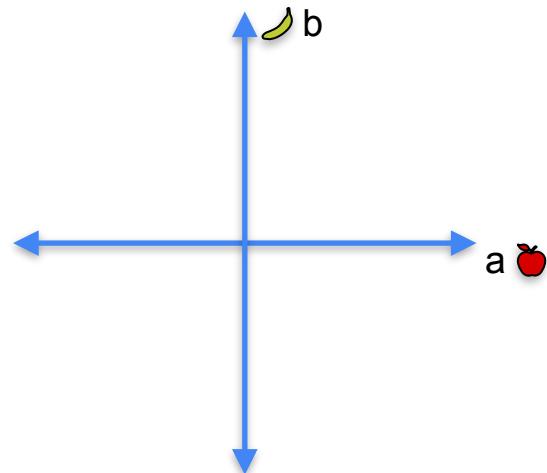
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 



System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 



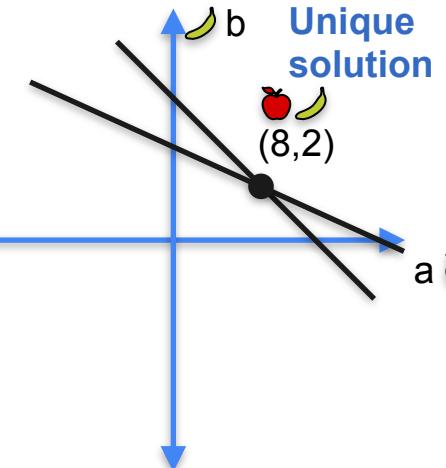
Systems of equations as lines

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

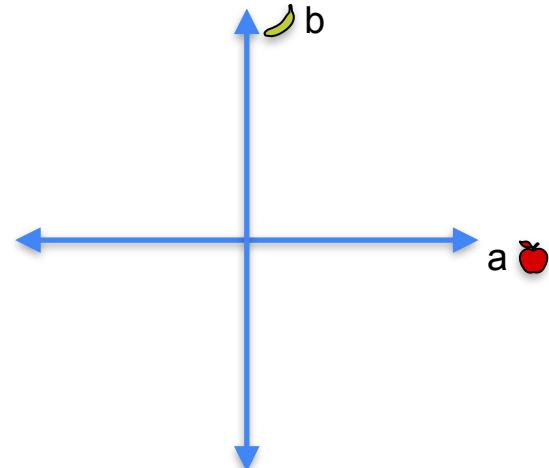
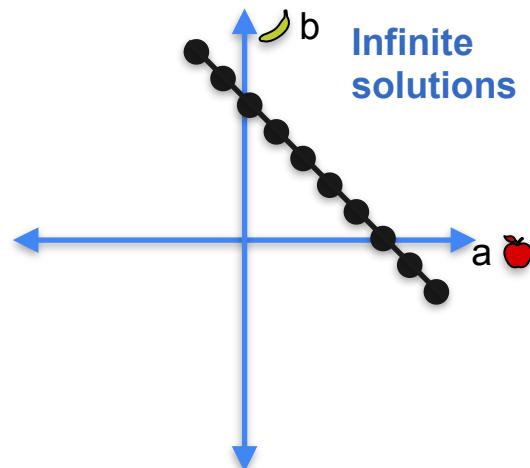
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 



System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 



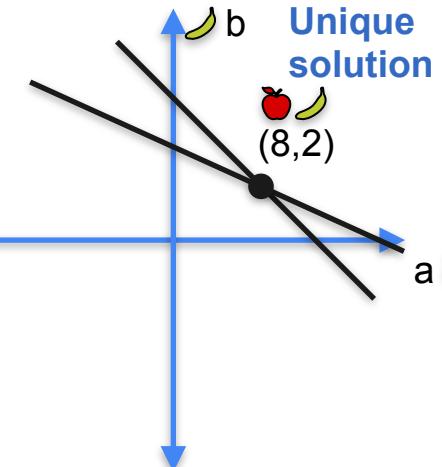
Systems of equations as lines

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

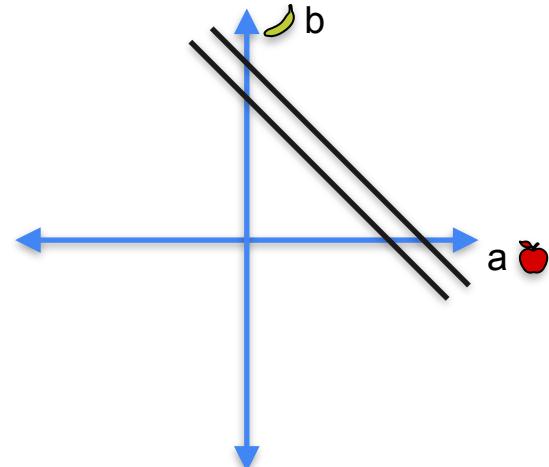
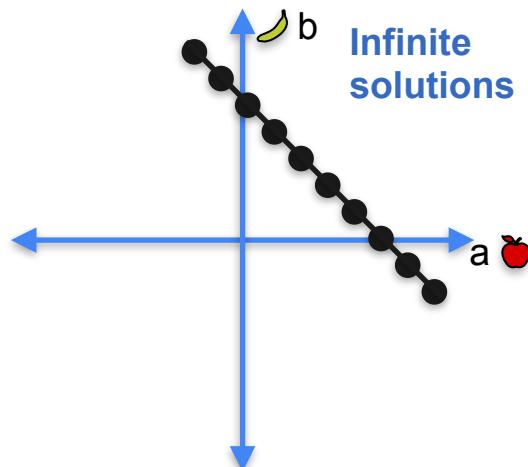
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 



System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 



Systems of equations as lines

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

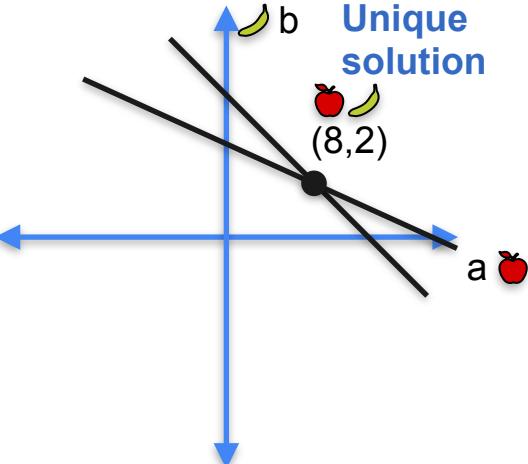
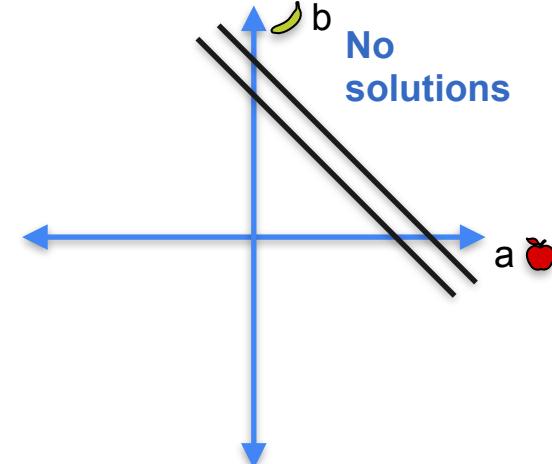
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 



System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 



Systems of equations as lines

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

 **b**
 **a**

Unique solution

(8,2)

**Complete
Non-singular**

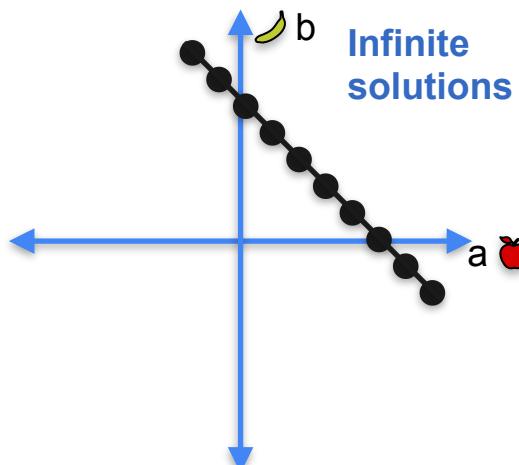
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 

 **b**

 **a**

Infinite solutions



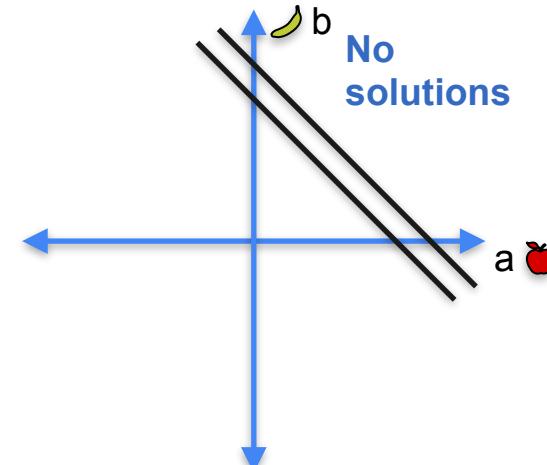
System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 

 **b**

 **a**

No solutions



Systems of equations as lines

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

 **b**
 **a**

 
(8,2)

Unique solution
Complete Non-singular

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 

 **b**
 **a**

Infinite solutions
Redundant Singular

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 

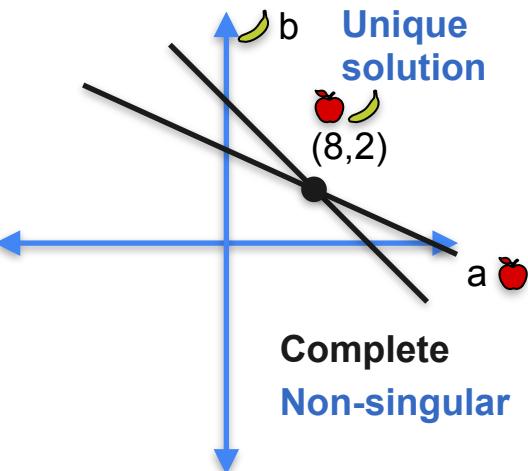
 **b**
 **a**

No solutions

Systems of equations as lines

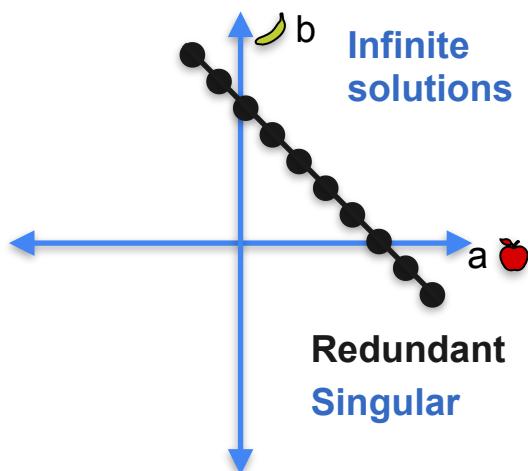
System 1

- $a + b = 10$
 
- $a + 2b = 12$
 



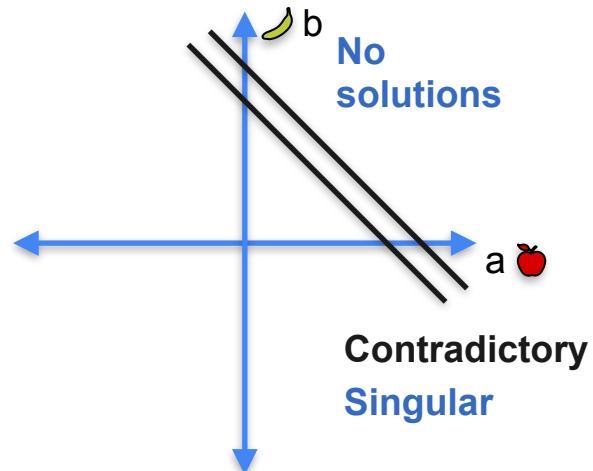
System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 



System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 

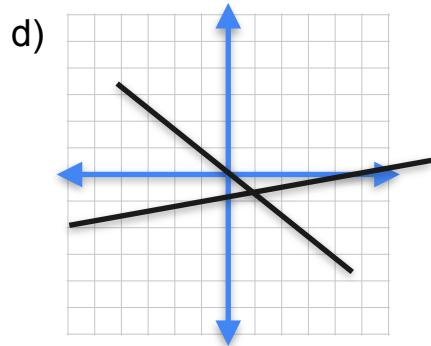
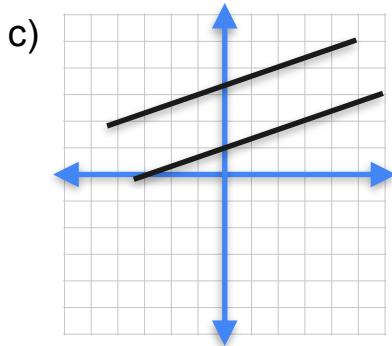
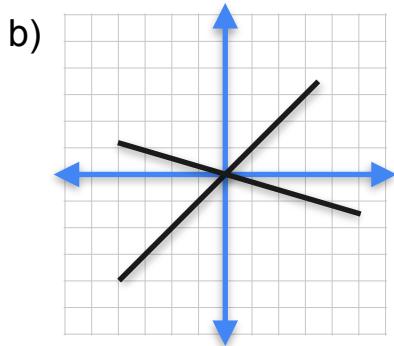
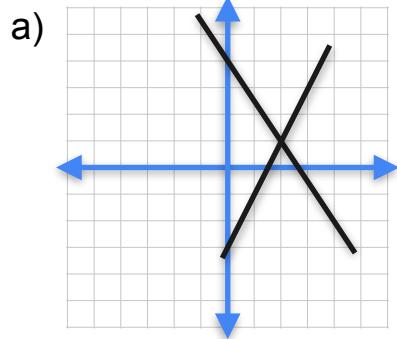


Quiz

Problem 1

Which of the following plots corresponds to the system of equations:

- $3a + 2b = 8$
- $2a - b = 3$



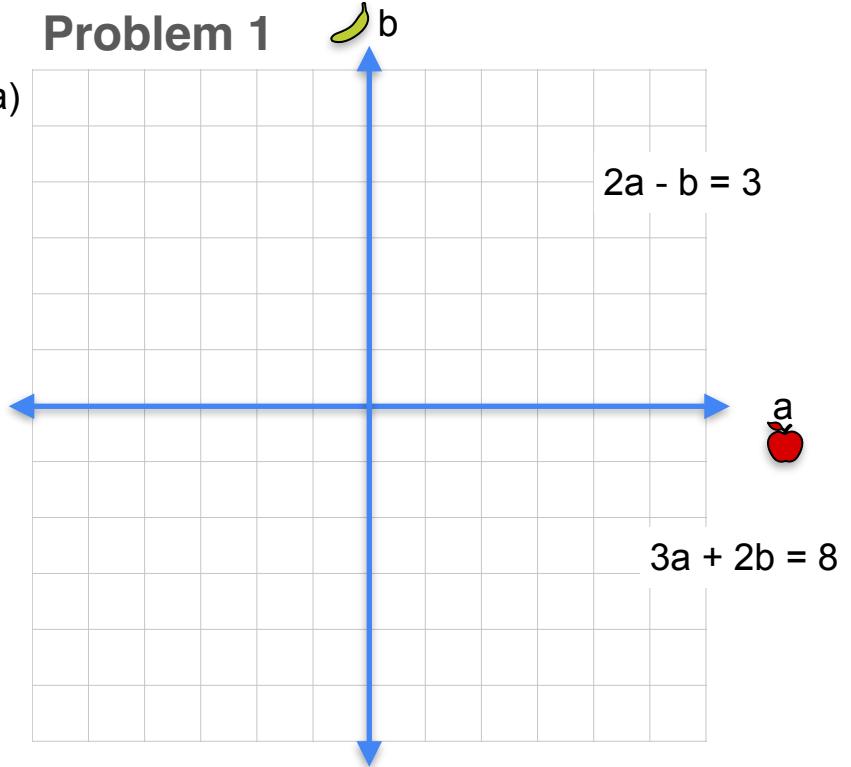
Problem 2

Is this system singular or non-singular?

Solution

Problem 1

a)



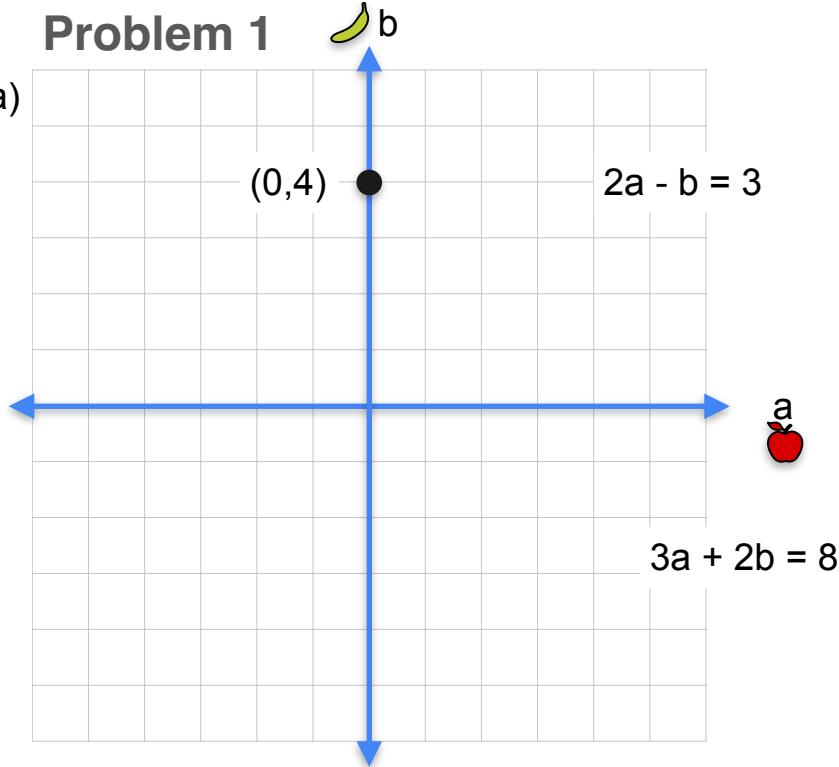
Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

a)



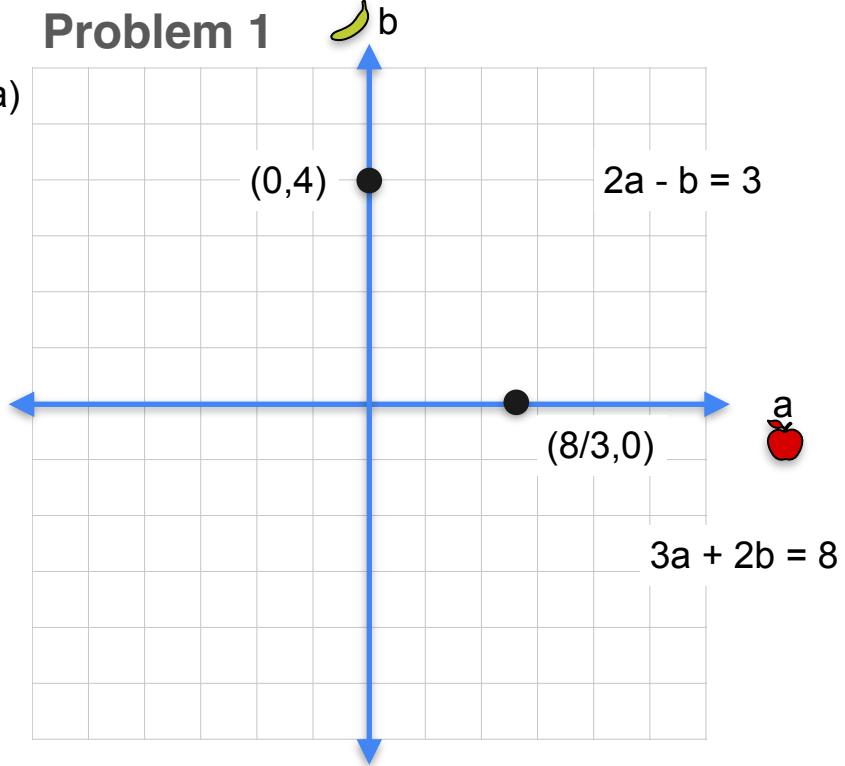
Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

a)



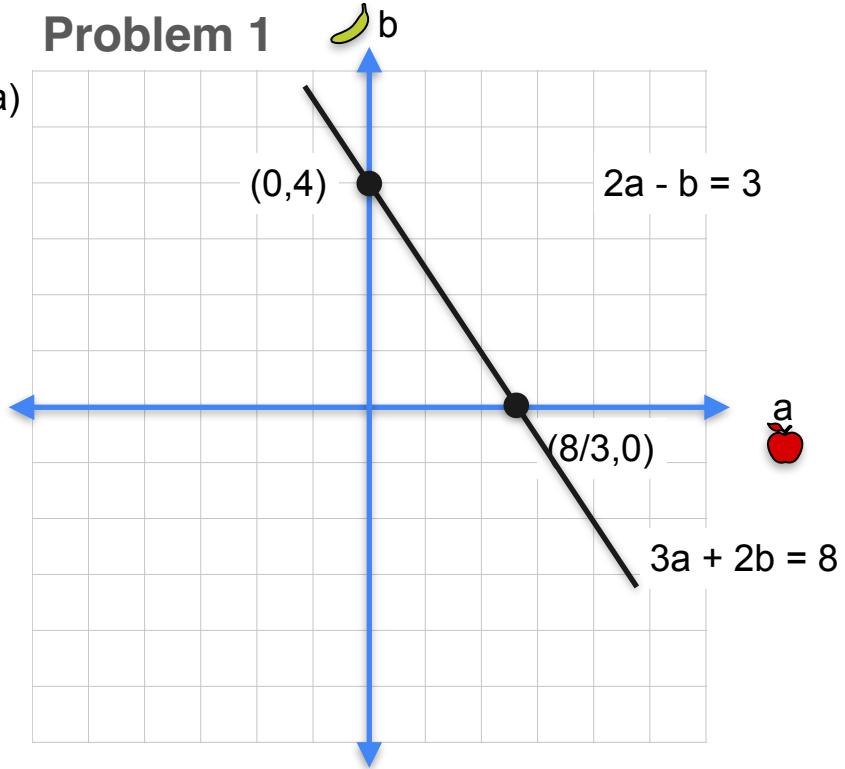
Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

a)



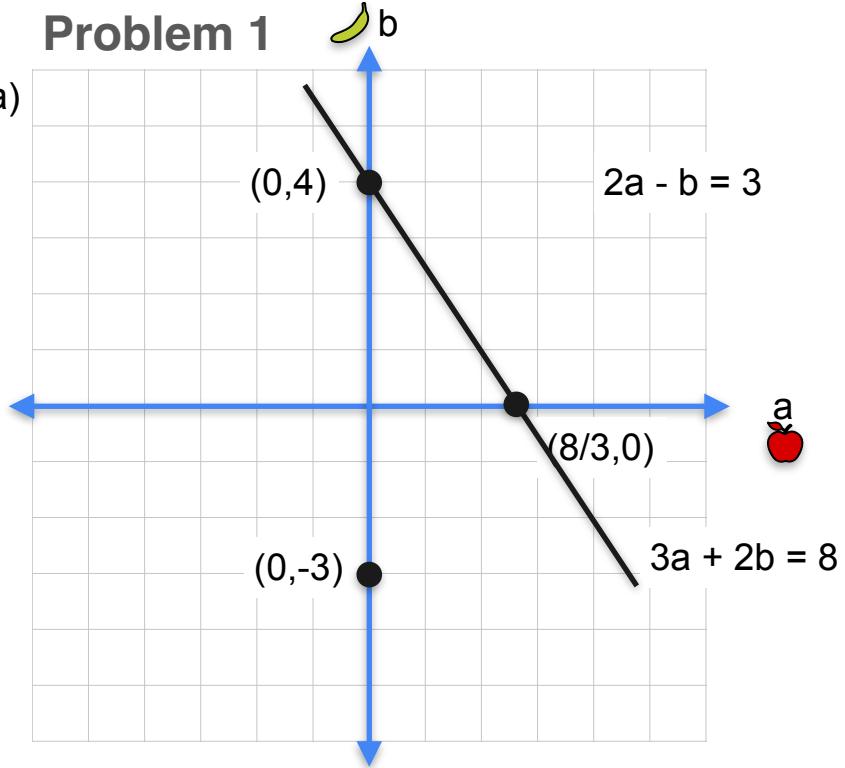
Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

a)



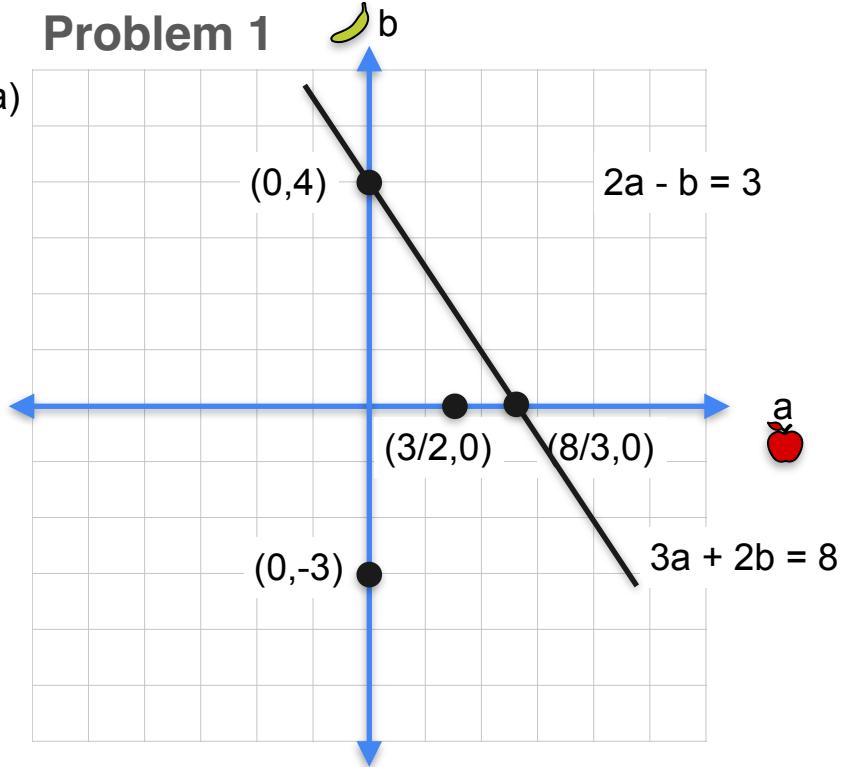
Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

a)



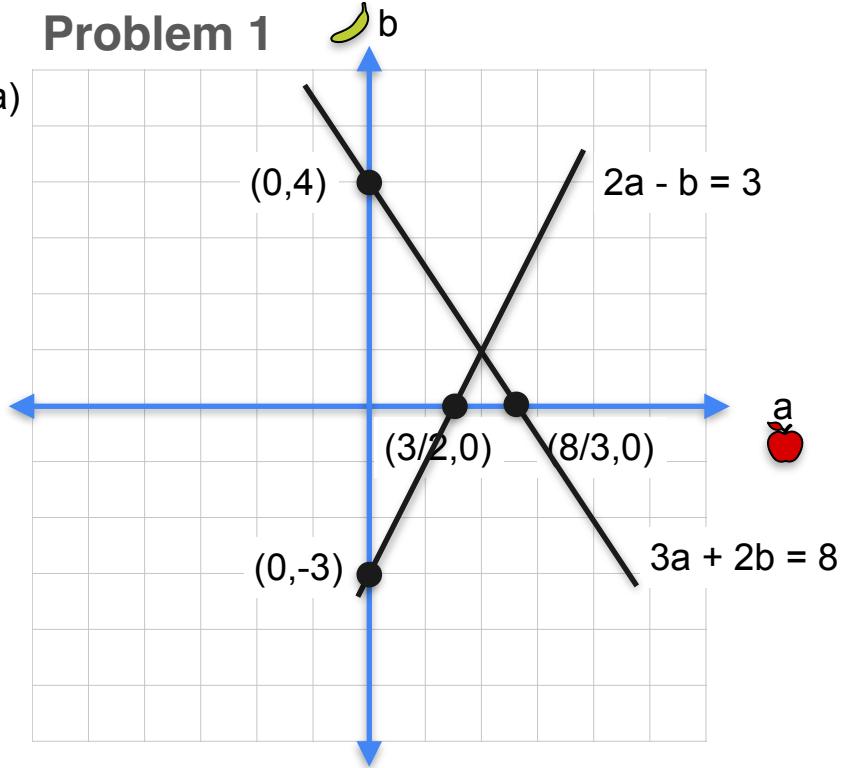
Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

a)



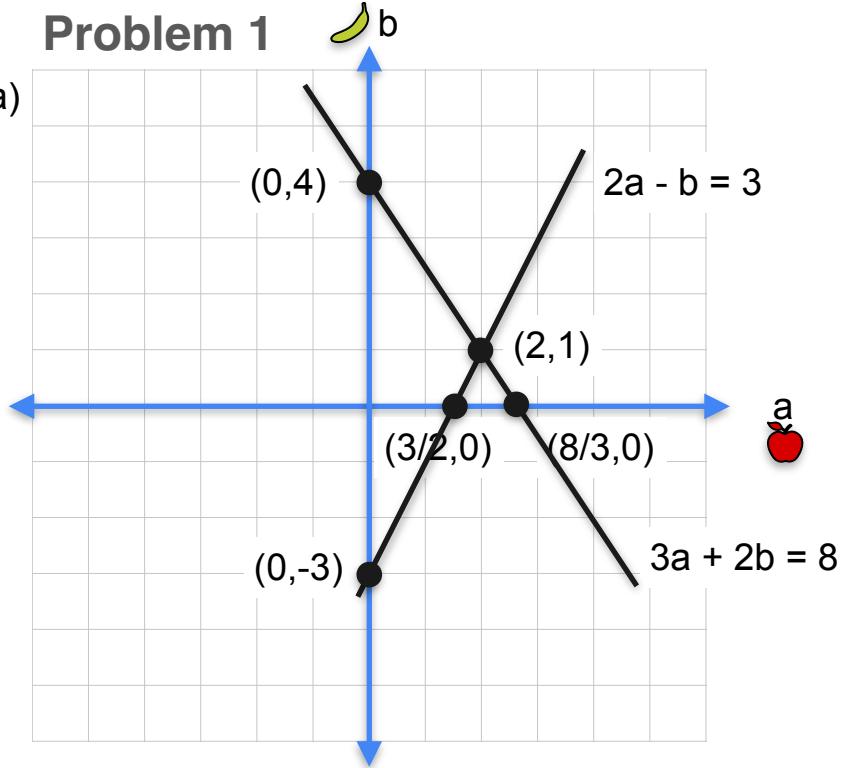
Problem 2

Since the lines cross at a unique point, the system is non-singular.

Solution

Problem 1

a)



Problem 2

Since the lines cross at a unique point, the system is non-singular.



DeepLearning.AI

System of Linear Equations

**A geometric notion of
singularity**

Systems of equations as lines

System 1

- $a + b = 10$
 
- $a + 2b = 12$
 

Unique
solution

(8,2)

Complete
Non-singular

System 2

- $a + b = 10$
 
- $2a + 2b = 20$
 

Infinite
solutions

Redundant
Singular

System 3

- $a + b = 10$
 
- $2a + 2b = 24$
 

No
solutions

Contradictory
Singular

Systems of equations as lines

System 1

- $a + b = 10$
- $a + 2b = 12$

Unique
solution

(8,2)

Complete
Non-singular

System 2

- $a + b = 10$
- $2a + 2b = 20$

Infinite
solutions

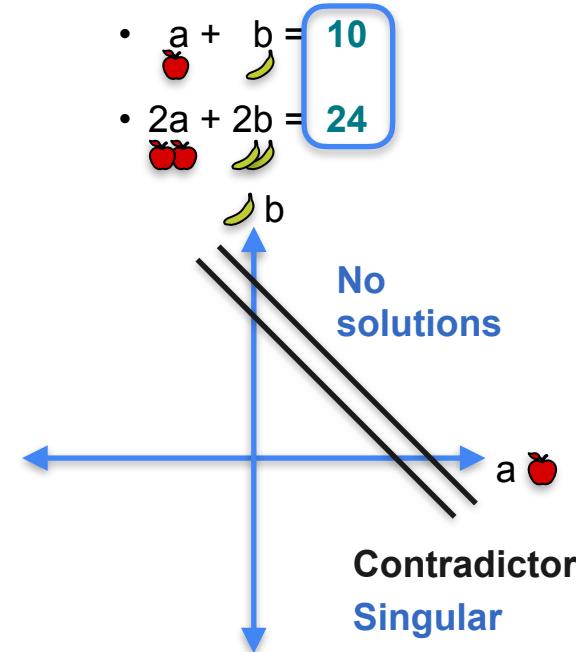
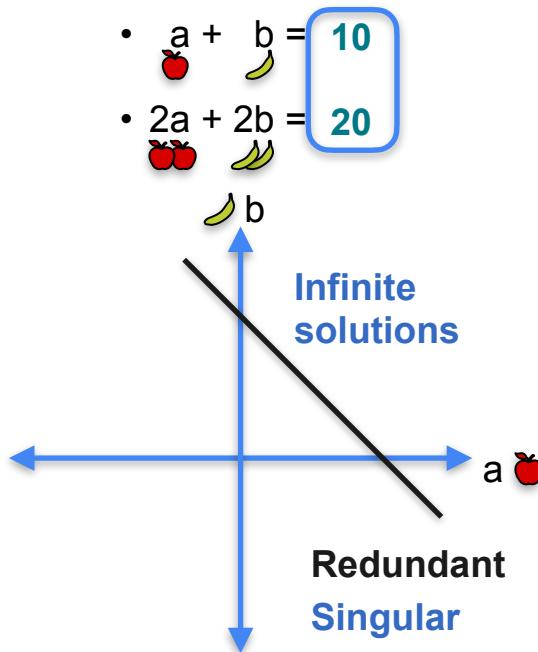
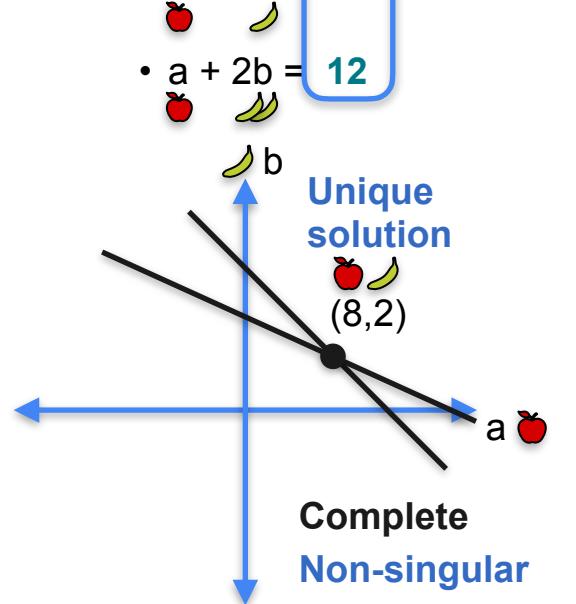
Redundant
Singular

System 3

- $a + b = 10$
- $2a + 2b = 24$

No
solutions

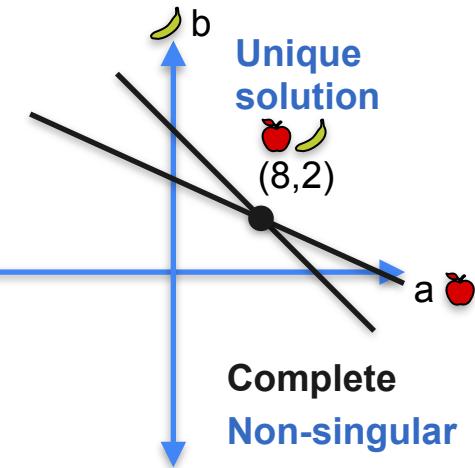
Contradictory
Singular



Systems of equations as lines

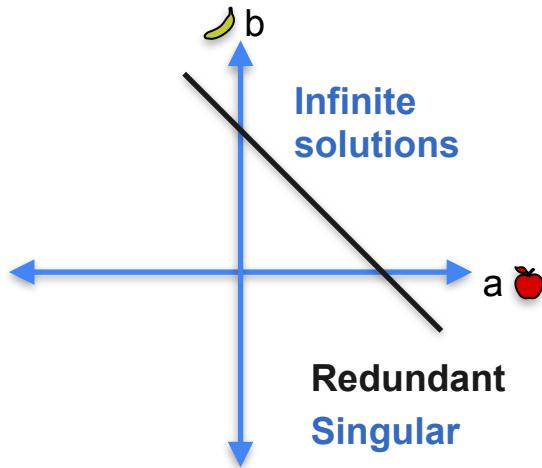
System 1

- $a + b = 0$
- $a + 2b = 0$



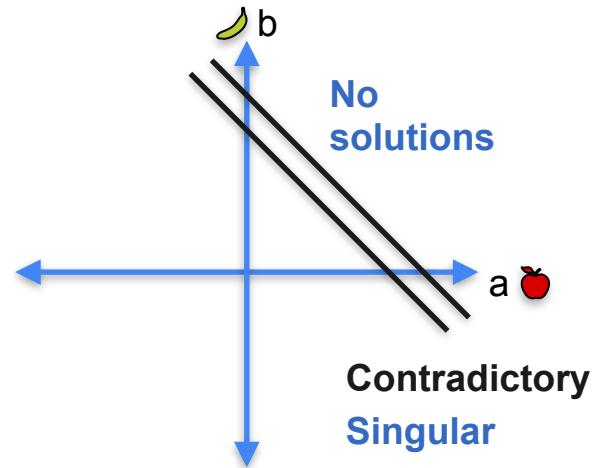
System 2

- $a + b = 10$
- $2a + 2b = 20$



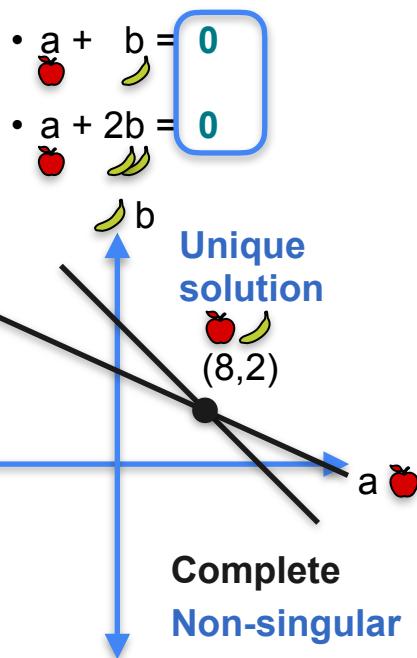
System 3

- $a + b = 10$
- $2a + 2b = 24$

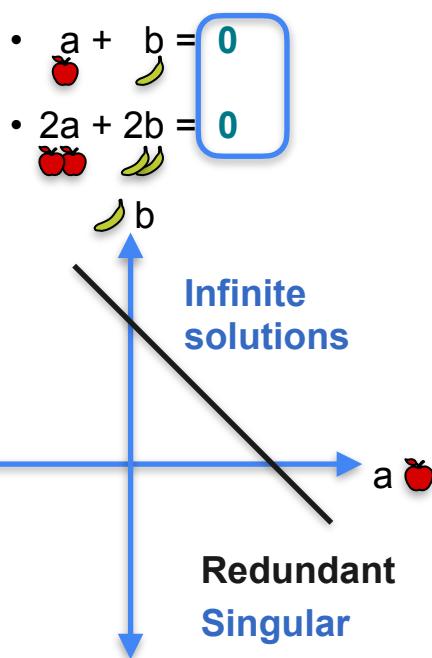


Systems of equations as lines

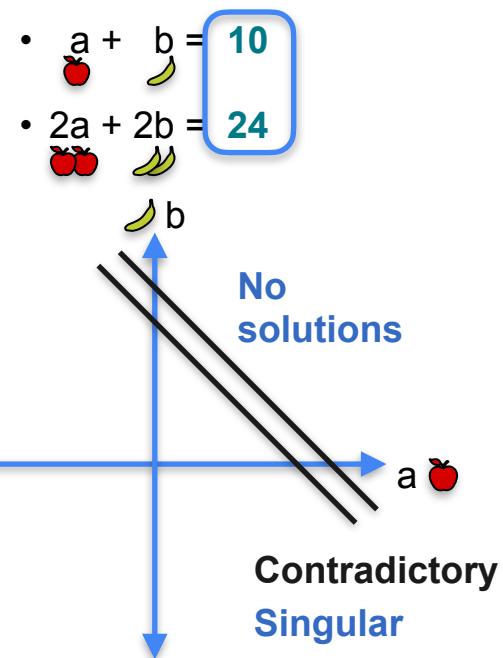
System 1



System 2

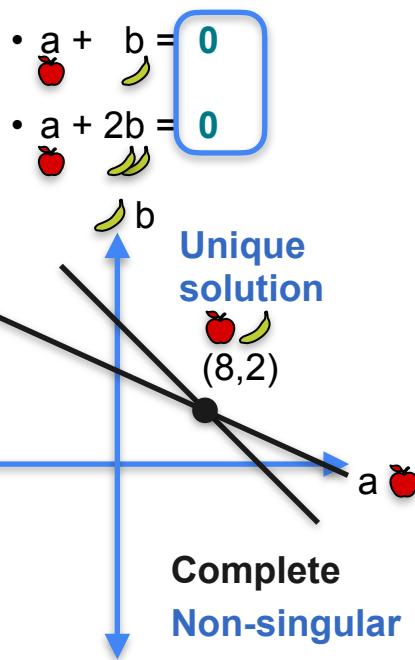


System 3

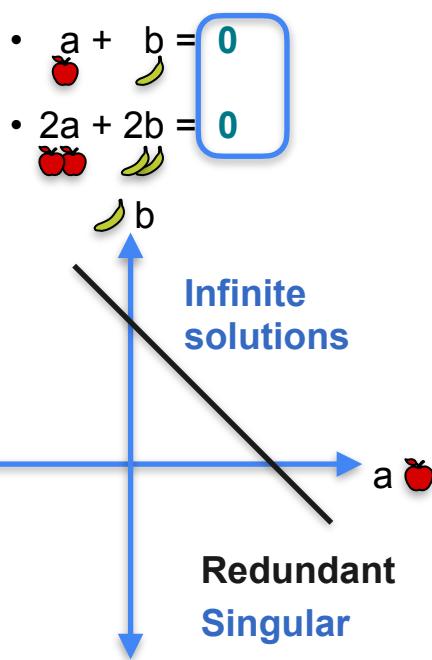


Systems of equations as lines

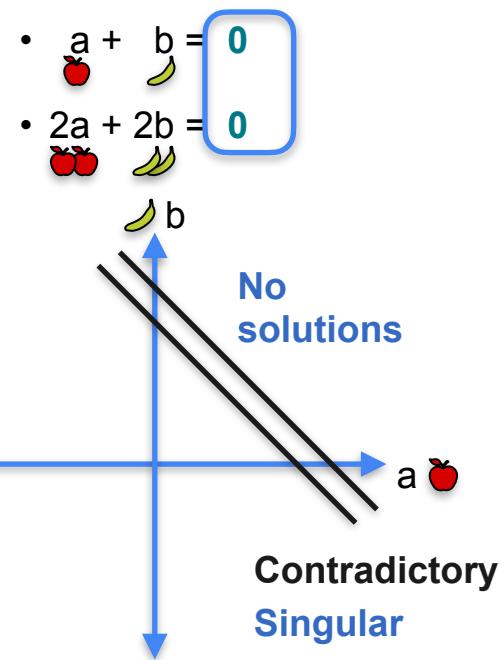
System 1



System 2

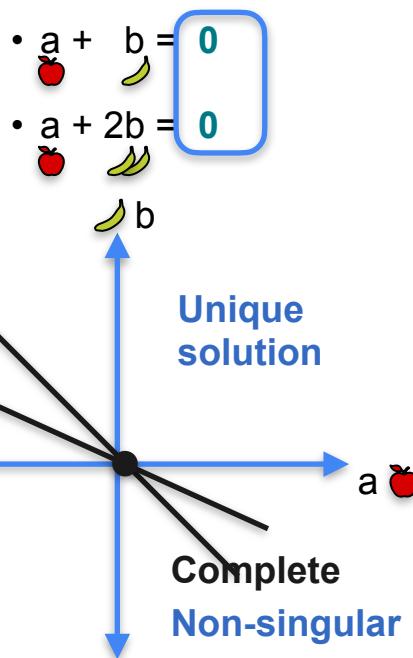


System 3

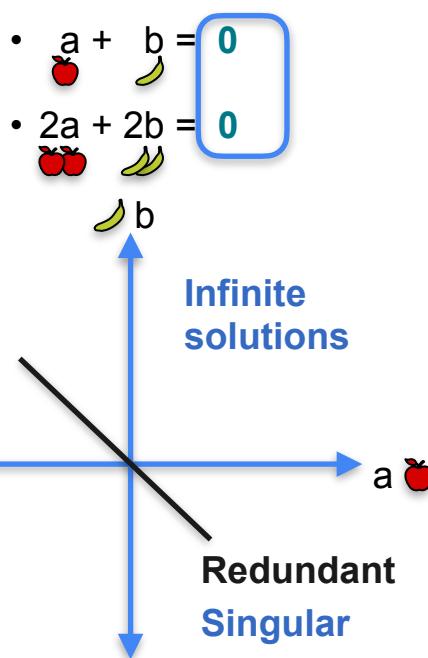


Systems of equations as lines

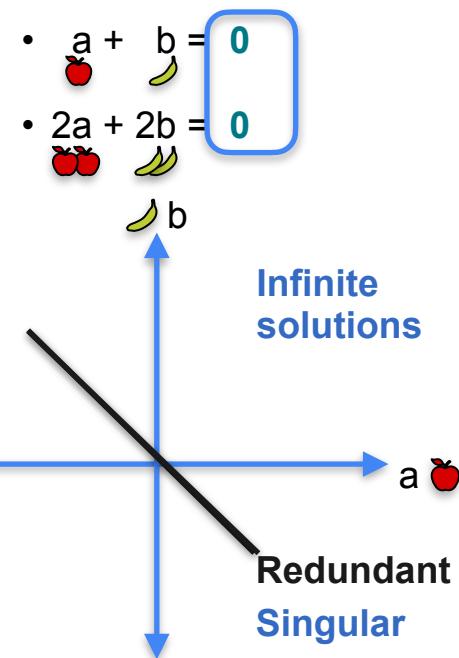
System 1



System 2



System 3





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System of Linear Equations

**Singular vs nonsingular
matrices**

Systems of equations as matrices

System 1

- $a + b = 0$
 
- $a + 2b = 0$
 

System 2

- $a + b = 0$
 
- $2a + 2b = 0$
 

Systems of equations as matrices

System 1

-  +  = **0**
-  + 2 = **0**

	
1	1
1	2

System 2

-  +  = **0**
-  + 2 = **0**

Systems of equations as matrices

System 1

-  +  = **0**
-  + 2 = **0**

	
1	1
1	2

System 2

-  +  = **0**
- 2 + 2 = **0**

	
1	1
2	2

Systems of equations as matrices

System 1

-  +  = **0**
-  + 2 = **0**

	
1	1
1	2

Non-singular
system

(Unique solution)

System 2

-  +  = **0**
- 2 + 2 = **0**

	
1	1
2	2

Systems of equations as matrices

System 1

-  +  = **0**
-  + 2 = **0**

	
1	1
1	2

Non-singular
system

Non-singular
matrix

(Unique solution)

System 2

-  +  = **0**
- 2 + 2 = **0**

	
1	1
2	2

Systems of equations as matrices

System 1

-  +  = **0**
-  + 2 = **0**

	
1	1
1	2

Non-singular
system

Non-singular
matrix

(Unique solution)

System 2

-  +  = **0**
-  + 2 = **0**

	
1	1
2	2

Singular
system

(Infinitely many solutions)

Systems of equations as matrices

System 1

-  +  = **0**
-  + 2 = **0**

	
1	1
1	2

Non-singular
system

Non-singular
matrix

(Unique solution)

System 2

-  +  = **0**
- 2 + 2 = **0**

	
1	1
2	2

Singular
system

Singular
matrix

(Infinitely many solutions)



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System of Linear Equations

**Linear dependence and
independence**

Linear dependence between rows

Non-singular

- $a + b = 0$
 
- $a + 2b = 0$
 

	
1	1
1	2

Singular system

- $a + b = 0$
 
- $2a + 2b = 0$
 

	
1	1
2	2

Linear dependence between rows

Non-singular

- $a + b = 0$
 
- $a + 2b = 0$
 

	
1	1
1	2

Singular system

- $a + b = 0$
 
- $2a + 2b = 0$
 

	
1	1
2	2

Second equation is
a multiple of the
first one

Linear dependence between rows

Non-singular

- $a + b = 0$
 
- $a + 2b = 0$
 

	
1	1
1	2

Singular system

- $a + b = 0$
 
- $2a + 2b = 0$
 

	
1	1
2	2

Second equation is
a multiple of the
first one

Second row is a
multiple of the first
row

Linear dependence between rows

Non-singular

- $a + b = 0$
 
- $a + 2b = 0$
 

	
1	1
1	2

Singular system

- $a + b = 0$
 
- $2a + 2b = 0$
 

	
1	1
2	2

Second equation is
a multiple of the
first one

Second row is a
multiple of the first
row

Rows are
linearly dependent

Linear dependence between rows

Non-singular

- $a + b = 0$
 
- $a + 2b = 0$
 

	
1	1
1	2

No equation is a multiple of the other one

Singular system

- $a + b = 0$
 
- $2a + 2b = 0$
 

	
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are
linearly dependent

Linear dependence between rows

Non-singular

- $a + b = 0$
 
- $a + 2b = 0$
 

	
1	1
1	2

No equation is a multiple of the other one

No row is a multiple of the other one

Singular system

- $a + b = 0$
 
- $2a + 2b = 0$
 

	
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are *linearly dependent*

Linear dependence between rows

Non-singular

- $a + b = 0$
 
- $a + 2b = 0$
 

	
1	1
1	2

No equation is a multiple of the other one

No row is a multiple of the other one

Rows are
linearly independent

Singular system

- $a + b = 0$
 
- $2a + 2b = 0$
 

	
1	1
2	2

Second equation is a multiple of the first one

Second row is a multiple of the first row

Rows are
linearly dependent



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System of Linear Equations

The determinant

Linear dependence between rows

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

Linear dependence between rows

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

Linear dependence between rows

Non-singular matrix



1	1
1	2

Singular matrix



1	1
2	2



1	1
---	---

Linear dependence between rows

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \times 2 =$$

Linear dependence between rows

Non-singular matrix

1	1
1	2

Singular matrix

1	1
2	2



$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Linear dependence between rows

Non-singular matrix

1	1
1	2

Singular matrix

1	1
2	2

$$\begin{matrix} 1 & 1 \end{matrix} \times 2 = \begin{matrix} 2 & 2 \end{matrix}$$

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

A 2x2 matrix with two distinct rows. The first row contains a red apple icon and a yellow banana icon. The second row contains a red apple icon and a blueberry icon. A blue curved arrow points from the right side of the matrix towards the right edge of the slide.

1	1
1	2

Singular matrix

A 2x2 matrix where the second row is a scalar multiple of the first. Both rows contain a red apple icon and a yellow banana icon. A blue curved arrow points from the right side of the matrix towards the right edge of the slide.

1	1
2	2

$$\begin{bmatrix} 1 & 1 \end{bmatrix} \times 2 = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

A 2x2 matrix with blue cells. The first row contains a red apple icon at [250, 110] and a yellow banana icon at [250, 170]. The second row contains the number 1 at [360, 110] and 2 at [360, 170]. A blue curved arrow starts from the bottom right corner of the matrix and points downwards to the right.

1	1
1	2

$$\begin{matrix} 1 & 1 \end{matrix}$$

Singular matrix

A 2x2 matrix with blue cells. The first row contains a red apple icon at [250, 560] and a yellow banana icon at [250, 620]. The second row contains the number 2 at [360, 560] and 2 at [360, 620]. A blue curved arrow starts from the bottom right corner of the matrix and points downwards to the right.

1	1
2	2

$$\begin{matrix} 1 & 1 \end{matrix} \times 2 = \begin{matrix} 2 & 2 \end{matrix}$$

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

1	1
1	2



$$\begin{matrix} 1 & 1 \end{matrix} \times ? =$$

Singular matrix

1	1
2	2



$$\begin{matrix} 1 & 1 \end{matrix} \times 2 = \begin{matrix} 2 & 2 \end{matrix}$$

Rows linearly dependent

Linear dependence between rows

Non-singular matrix

1	1
1	2



$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \times ? = \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array}$$

Singular matrix

1	1
2	2



$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \times 2 = \begin{array}{|c|c|} \hline 2 & 2 \\ \hline \end{array}$$

Rows linearly dependent

Linear dependence between rows

Non-singular matrix



1	1
1	2



$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \times ? = \begin{array}{|c|c|} \hline 1 & 2 \\ \hline \end{array}$$

Rows linearly independent

Singular matrix



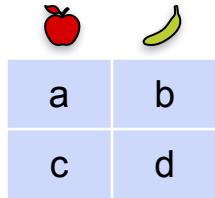
1	1
2	2



$$\begin{array}{|c|c|} \hline 1 & 1 \\ \hline \end{array} \times 2 = \begin{array}{|c|c|} \hline 2 & 2 \\ \hline \end{array}$$

Rows linearly dependent

Determinant



Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant

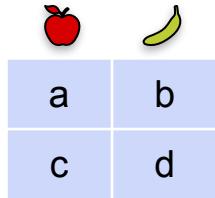
$$ak = c$$

	
a	b
c	d

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant



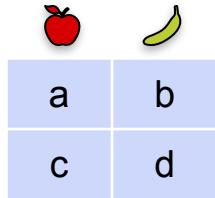
$$ak = c$$

$$bk = d$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant



$$ak = c$$

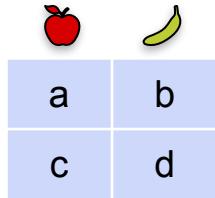
$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant



$$ak = c$$

$$bk = d$$

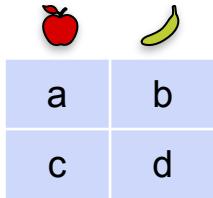
$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

$$ad = bc$$

Determinant



$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

$$ad = bc$$

$$ad - bc = 0$$

Determinant

a	b
c	d

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

Determinant

	
a	b
c	d

$$\text{Determinant} = ad - bc$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{array}{|c|c|} \hline a & b \\ \hline \end{array} * k = \begin{array}{|c|c|} \hline c & d \\ \hline \end{array}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

Determinant

a	b
c	d

$$\text{Determinant} = ad - bc$$

$$\begin{matrix} a \\ & d \end{matrix}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

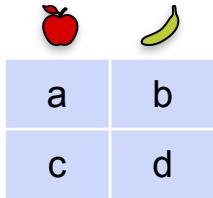
$$\begin{matrix} a & b \end{matrix} * k = \begin{matrix} c & d \end{matrix}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

Determinant



$$\text{Determinant} = ad - bc$$

$$\begin{matrix} a & \\ & d \end{matrix} -$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{matrix} a & b \end{matrix} * k = \begin{matrix} c & d \end{matrix}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

Determinant

	
a	b
c	d

$$\text{Determinant} = ad - bc$$

$$\begin{matrix} a & \\ & d \end{matrix} - \begin{matrix} & b \\ c & \end{matrix}$$

$$ak = c$$

$$bk = d$$

$$\frac{c}{a} = \frac{d}{b} = k$$

Matrix is singular if

$$\begin{matrix} a & b \end{matrix} * k = \begin{matrix} c & d \end{matrix}$$

Determinant

$$ad = bc$$

$$ad - bc = 0$$

Determinant

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

Determinant

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

Determinant

$$1 \begin{matrix} \\ 2 \end{matrix} - \begin{matrix} \\ 1 \end{matrix} 1$$

Determinant

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

Determinant

$$\begin{matrix} 1 & \\ 2 & \end{matrix} - \begin{matrix} & 1 \\ 1 & \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Determinant

Non-singular matrix

	
1	1
1	2

Singular matrix

	
1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 1 \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 2 \end{matrix}$$

Determinant

Non-singular matrix

	
1	1
1	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 1 \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

	
1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 2 \end{matrix}$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

Determinant

Non-singular matrix

1	1
1	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 1 \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 2 \end{matrix}$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

Determinant

Non-singular matrix

1	1
1	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 1 \end{matrix}$$

$$1 \cdot 2 - 1 \cdot 1 = 1$$

Singular matrix

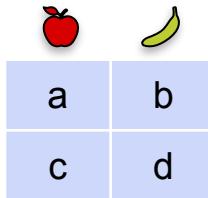
1	1
2	2

Determinant

$$\begin{matrix} 1 & & 1 \\ & - & \\ 2 & & 2 \end{matrix}$$

$$1 \cdot 2 - 2 \cdot 1 = 0$$

Determinant and singularity



$$ad - bc$$

Determinant and singularity

	
a	b
c	d

$$ad - bc$$

Matrix is singular



Determinant is zero

Quiz: Determinant

Problem 1: Find the determinant of the following matrices

Matrix 1

5	1
-1	3

Matrix 2

2	-1
-6	3

Problem 2: Are these matrices singular or non-singular?

Solutions: Determinant

Matrix 1: $\det = 5 \cdot 3 - 1 \cdot (-1) = 15 + 1 = 16$

5	1
-1	3

Non-singular

Matrix 2: $\det = 2 \cdot 3 - (-1) \cdot (-6) = 6 - 6 = 0$

2	-1
-6	3

Singular



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System of Linear Equations

System of equations (3x3)

Quiz: Systems of equations

Problem 1: You're trying to figure out the price of apples, bananas, and cherries at the store. You go three days in a row, and bring this information.

- **Day 1:** You bought an apple, a banana, and a cherry, and paid \$10.
- **Day 2:** You bought an apple, two bananas, and a cherry, and paid \$15.
- **Day 3:** You bought an apple, a banana, and two cherries, and paid \$12.

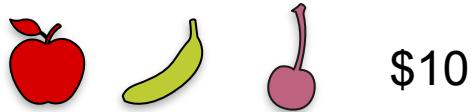
How much does each fruit cost?

Solution: Systems of equations

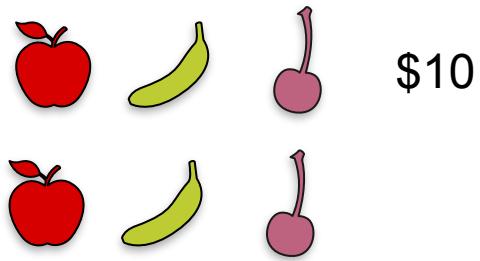
Solution: Systems of equations



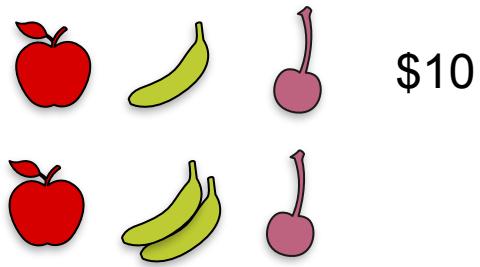
Solution: Systems of equations



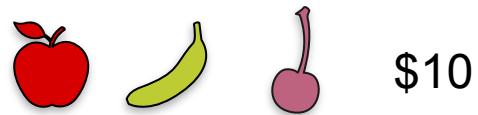
Solution: Systems of equations

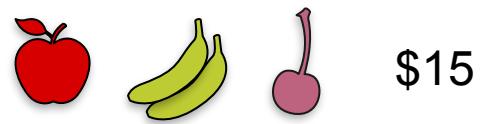


Solution: Systems of equations



Solution: Systems of equations


$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \$10 \end{array}$$


$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \$15 \end{array}$$

Solution: Systems of equations



Solution: Systems of equations

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \\ \$10 \end{array}$$

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \\ \$15 \end{array} \quad \rightarrow \quad \text{banana} \quad \$5$$

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \end{array}$$

Solution: Systems of equations



Solution: Systems of equations

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \\ \$10 \end{array}$$

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \\ \$15 \end{array} \quad \rightarrow \quad \text{banana} \quad \$5$$

$$\begin{array}{c} \text{apple} \quad \text{banana} \quad \text{cherry} \\ \$12 \end{array}$$

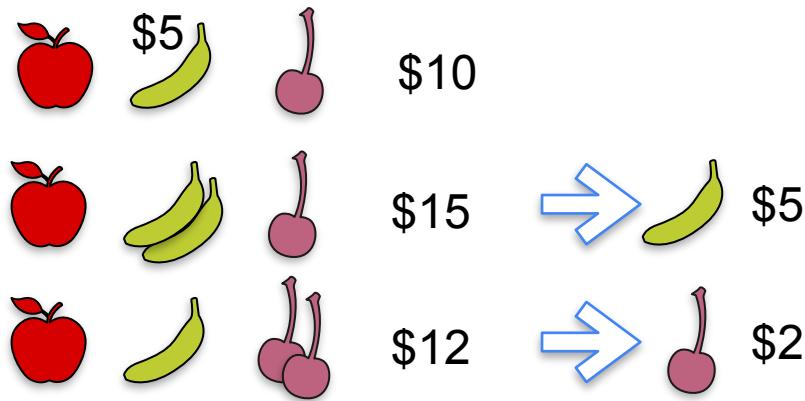
Solution: Systems of equations

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{1} & \text{1} \end{array} \quad \$10$$

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{2} & \text{1} \end{array} \quad \$15 \quad \xrightarrow{\text{banana}} \quad \text{banana} \quad \$5$$

$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ \text{1} & \text{1} & \text{2} \end{array} \quad \$12 \quad \xrightarrow{\text{cherry}} \quad \text{cherry} \quad \$2$$

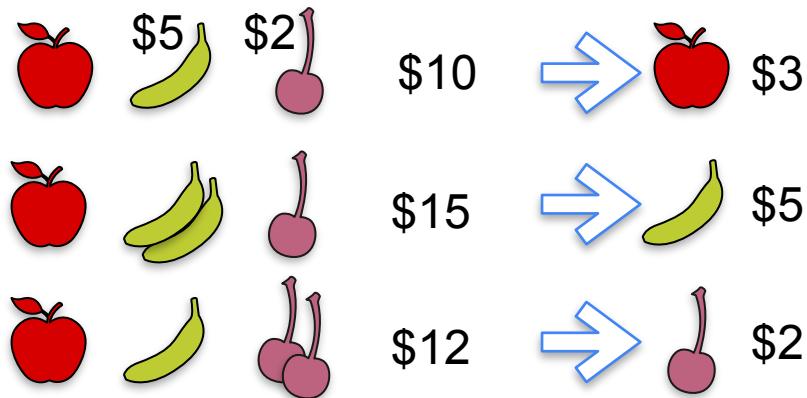
Solution: Systems of equations



Solution: Systems of equations

$$\begin{array}{ccc} \text{apple} & \$5 & \text{banana} \\ & & \$2 \\ & & \text{cherry} \end{array} \quad \$10 \quad \rightarrow \quad \begin{array}{cc} \text{apple} & \$3 \end{array}$$
$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{banana} \\ & 2 & 1 \end{array} \quad \$15 \quad \rightarrow \quad \begin{array}{cc} \text{banana} & \$5 \end{array}$$
$$\begin{array}{ccc} \text{apple} & \text{banana} & \text{cherry} \\ & 1 & 2 \end{array} \quad \$12 \quad \rightarrow \quad \begin{array}{cc} \text{cherry} & \$2 \end{array}$$

Solution: Systems of equations



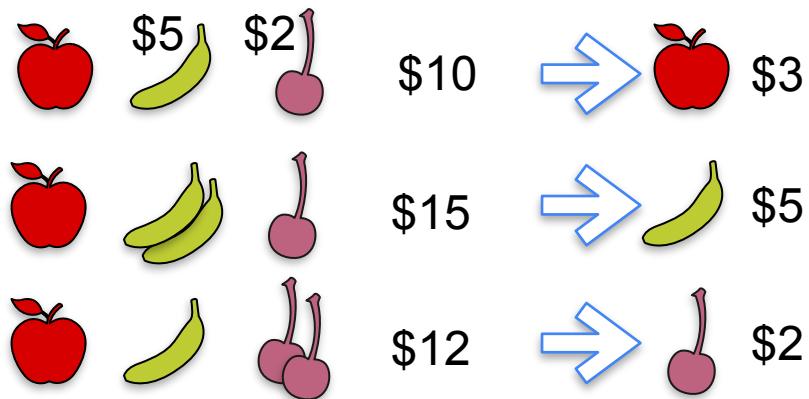
System of equations 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Solution: Systems of equations



System of equations 1

$$\begin{aligned} a + b + c &= 10 \\ a + 2b + c &= 15 \\ a + b + 2c &= 12 \end{aligned}$$

Solution

$$\begin{aligned} a &= 3 \\ b &= 5 \\ c &= 2 \end{aligned}$$

Quiz: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Infinitely many sols.

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Infinitely many sols.

$$c = 5$$

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

$$(0,5,5), (1,4,5), (2,3,5), \dots$$

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

Infinitely many sols.

$$c = 5$$

$$a + b = 5$$

$$(0,5,5), (1,4,5), (2,3,5), \dots$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

No solutions

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

Infinitely many sols.

$$\begin{aligned}c &= 5 \\a + b &= 5 \\(0,5,5), (1,4,5), (2,3,5), \dots\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

No solutions

$$\begin{aligned}\text{From 1st and 2nd:} \\c &= 5 \\\text{From 2nd and 3rd:} \\c &= 3\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

Infinitely many sols.

$$\begin{aligned}c &= 5 \\a + b &= 5 \\(0,5,5), (1,4,5), (2,3,5), \dots\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

No solutions

$$\begin{aligned}\text{From 1st and 2nd:} \\c &= 5 \\\text{From 2nd and 3rd:} \\c &= 3\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Infinitely many solutions

Solutions: More systems of equations

System 2

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 20\end{aligned}$$

Infinitely many sols.

$$\begin{aligned}c &= 5 \\a + b &= 5 \\(0,5,5), (1,4,5), (2,3,5), \dots\end{aligned}$$

System 3

$$\begin{aligned}a + b + c &= 10 \\a + b + 2c &= 15 \\a + b + 3c &= 18\end{aligned}$$

No solutions

$$\begin{aligned}\text{From 1st and 2nd:} \\c &= 5 \\\text{From 2nd and 3rd:} \\c &= 3\end{aligned}$$

System 4

$$\begin{aligned}a + b + c &= 10 \\2a + 2b + 2c &= 20 \\3a + 3b + 3c &= 30\end{aligned}$$

Infinitely many solutions

$$\begin{aligned}\text{Any 3 numbers that add} \\&\text{to 10 work.} \\(0,0,10), (2,7,1), \dots\end{aligned}$$



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System of Linear Equations

**Singular vs non-singular
matrices**

Constants don't matter for singularity

System 1

$$a + b + c = \textcolor{teal}{10}$$

$$a + 2b + c = \textcolor{teal}{15}$$

$$a + b + 2c = \textcolor{teal}{12}$$

System 2

$$a + b + c = \textcolor{teal}{10}$$

$$a + b + 2c = \textcolor{teal}{15}$$

$$a + b + 3c = \textcolor{teal}{20}$$

System 3

$$a + b + c = \textcolor{teal}{10}$$

$$a + b + 2c = \textcolor{teal}{15}$$

$$a + b + 3c = \textcolor{teal}{18}$$

System 4

$$a + b + c = \textcolor{teal}{10}$$

$$2a + 2b + 2c = \textcolor{teal}{15}$$

$$3a + 3b + 3c = \textcolor{teal}{20}$$

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Infinite solutions

No solutions

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = 10$$

$$a + 2b + c = 15$$

$$a + b + 2c = 12$$

Unique solution

Complete

System 2

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 20$$

Infinite solutions

System 3

$$a + b + c = 10$$

$$a + b + 2c = 15$$

$$a + b + 3c = 18$$

No solutions

System 4

$$a + b + c = 10$$

$$2a + 2b + 2c = 15$$

$$3a + 3b + 3c = 20$$

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

Unique solution

Complete

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

Infinite solutions

Redundant

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

No solutions

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Complete

Infinite solutions

Redundant

No solutions

Contradictory

Infinite solutions

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Complete

Infinite solutions

Redundant

No solutions

Contradictory

Infinite solutions

Redundant

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Complete

Non-singular

Infinite solutions

Redundant

No solutions

Contradictory

Infinite solutions

Redundant

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Complete

Non-singular

Infinite solutions

Redundant

Singular

No solutions

Contradictory

Infinite solutions

Redundant

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Complete

Non-singular

Infinite solutions

Redundant

Singular

No solutions

Contradictory

Singular

Infinite solutions

Redundant

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{15}$$

$$3a + 3b + 3c = \mathbf{20}$$

Unique solution

Complete

Non-singular

Infinite solutions

Redundant

Singular

No solutions

Contradictory

Singular

Infinite solutions

Redundant

Singular

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{10}$$

$$a + 2b + c = \mathbf{15}$$

$$a + b + 2c = \mathbf{12}$$



$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

System 2

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{20}$$



$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 3

$$a + b + c = \mathbf{10}$$

$$a + b + 2c = \mathbf{15}$$

$$a + b + 3c = \mathbf{18}$$



$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 4

$$a + b + c = \mathbf{10}$$

$$2a + 2b + 2c = \mathbf{20}$$

$$3a + 3b + 3c = \mathbf{30}$$



$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

System 2

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 3

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 4

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Constants don't matter for singularity

System 1

$$a + b + c = 0$$

$$a + 2b + c = 0$$

$$a + b + 2c = 0$$

System 2

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 3

$$a + b + c = 0$$

$$a + b + 2c = 0$$

$$a + b + 3c = 0$$

System 4

$$a + b + c = 0$$

$$2a + 2b + 2c = 0$$

$$3a + 3b + 3c = 0$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

System 2

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 3

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 4

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e., $a = -b$)

System 3

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 4

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e., $a = -b$)

Redundant

Singular

System 3

System 4

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

$$(i.e., a = -b)$$

Redundant

Singular

System 3

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 4

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Infinite solutions:

$$a + b + c = 0$$

(i.e., $c = -a - b$)

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

Unique solution:

$$a = 0$$

$$b = 0$$

$$c = 0$$

Complete

Non-singular

System 2

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

Infinite solutions:

$$c = 0$$

$$a + b = 0$$

(i.e., $a = -b$)

Redundant

Singular

System 3

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 4

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Infinite solutions:

$$a + b + c = 0$$

(i.e., $c = -a - b$)

Redundant

Singular

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

System 2

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 3

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 4

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Constants don't matter for singularity

System 1

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

1	1	1
1	2	1
1	1	2

System 2

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 3

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

System 4

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

Constants don't matter for singularity

System 1

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + 2b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \end{aligned}$$

1	1	1
1	2	1
1	1	2

System 2

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
1	1	2
1	1	3

System 3

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

System 4

$$\begin{aligned} a + b + c &= \mathbf{0} \\ 2a + 2b + 2c &= \mathbf{0} \\ 3a + 3b + 3c &= \mathbf{0} \end{aligned}$$

Constants don't matter for singularity

System 1

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + 2b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \end{aligned}$$

1	1	1
1	2	1
1	1	2

System 2

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
1	1	2
1	1	3

System 3

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

System 4

$$\begin{aligned} a + b + c &= \mathbf{0} \\ 2a + 2b + 2c &= \mathbf{0} \\ 3a + 3b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
2	2	2
3	3	3

Constants don't matter for singularity

System 1

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + 2b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \end{aligned}$$

1	1	1
1	2	1
1	1	2

Non-singular

System 2

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
1	1	2
1	1	3

System 3

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

System 4

$$\begin{aligned} a + b + c &= \mathbf{0} \\ 2a + 2b + 2c &= \mathbf{0} \\ 3a + 3b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
2	2	2
3	3	3

Constants don't matter for singularity

System 1

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + 2b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \end{aligned}$$

1	1	1
1	2	1
1	1	2

Non-singular

System 2

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
1	1	2
1	1	3

Singular

System 3

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

System 4

$$\begin{aligned} a + b + c &= \mathbf{0} \\ 2a + 2b + 2c &= \mathbf{0} \\ 3a + 3b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
2	2	2
3	3	3

Constants don't matter for singularity

System 1

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + 2b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \end{aligned}$$

1	1	1
1	2	1
1	1	2

Non-singular

System 2

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
1	1	2
1	1	3

Singular

System 3

$$\begin{aligned} a + b + c &= \mathbf{0} \\ a + b + 2c &= \mathbf{0} \\ a + b + 3c &= \mathbf{0} \end{aligned}$$

1	1	1
2	2	2
3	3	3

Singular

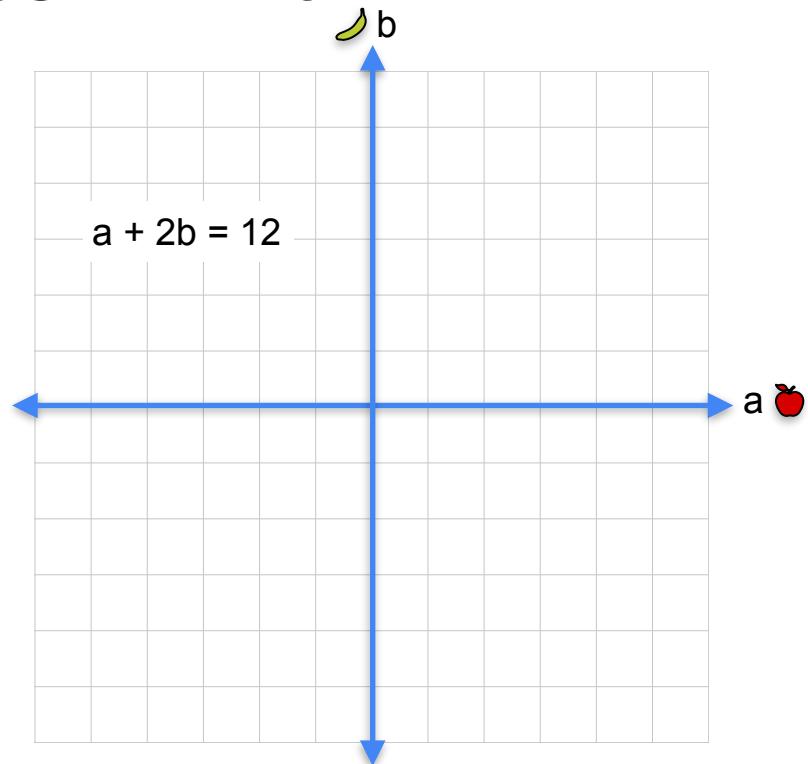
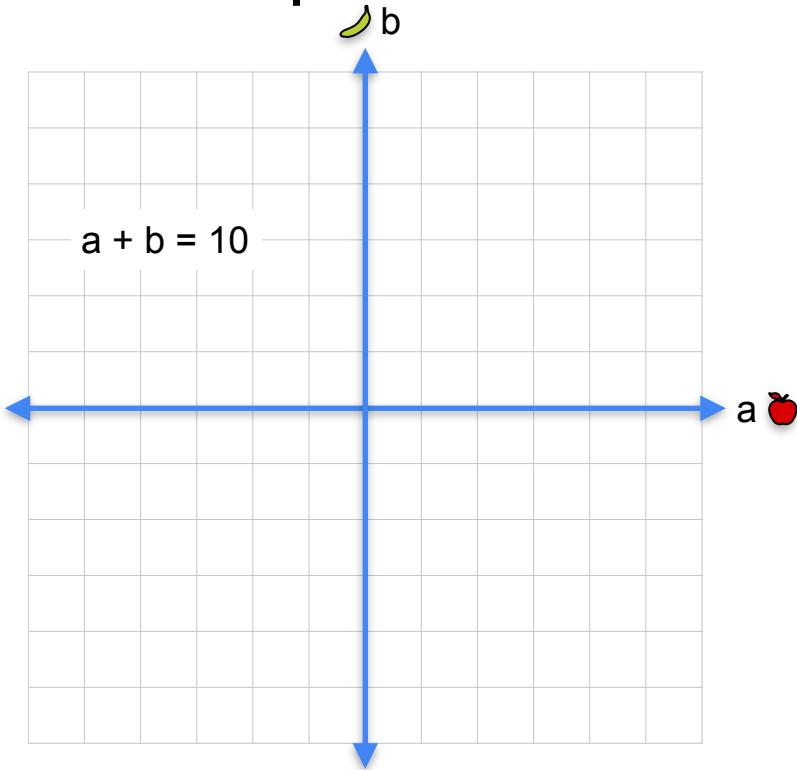


DeepLearning.AI

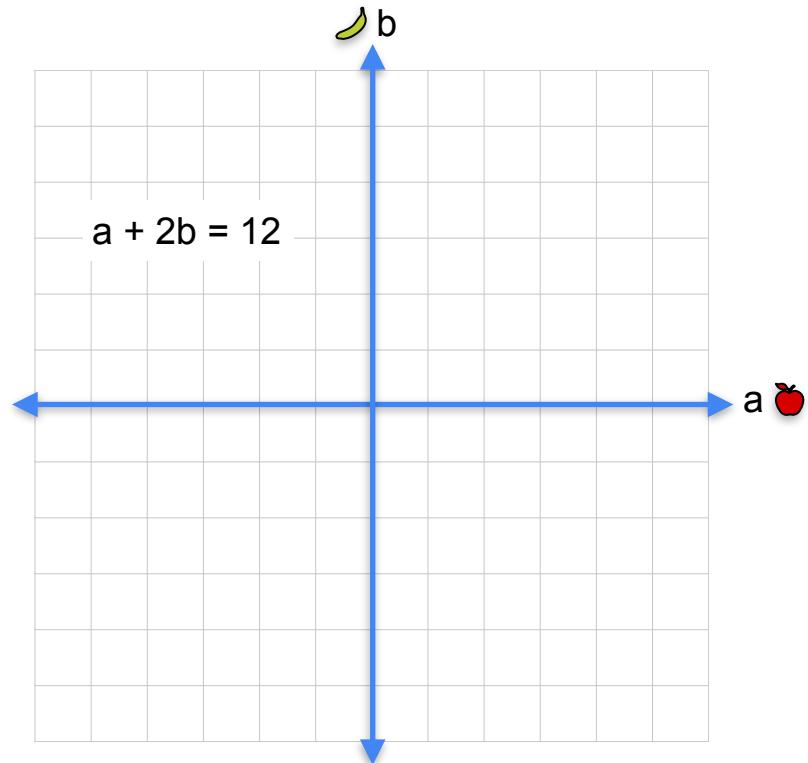
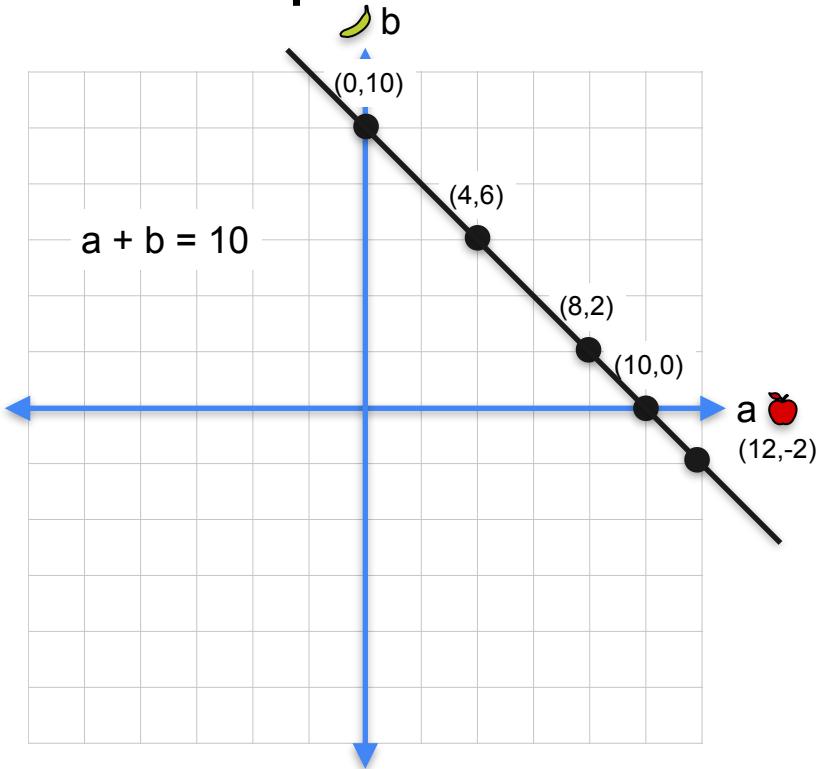
System of Linear Equations

**System of equations as
planes (3x3)**

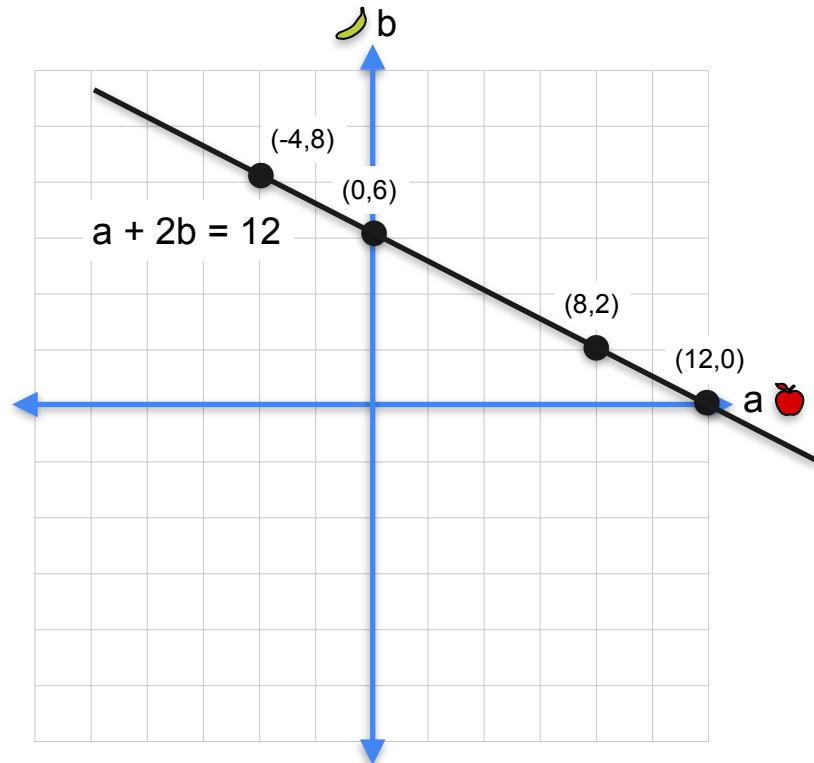
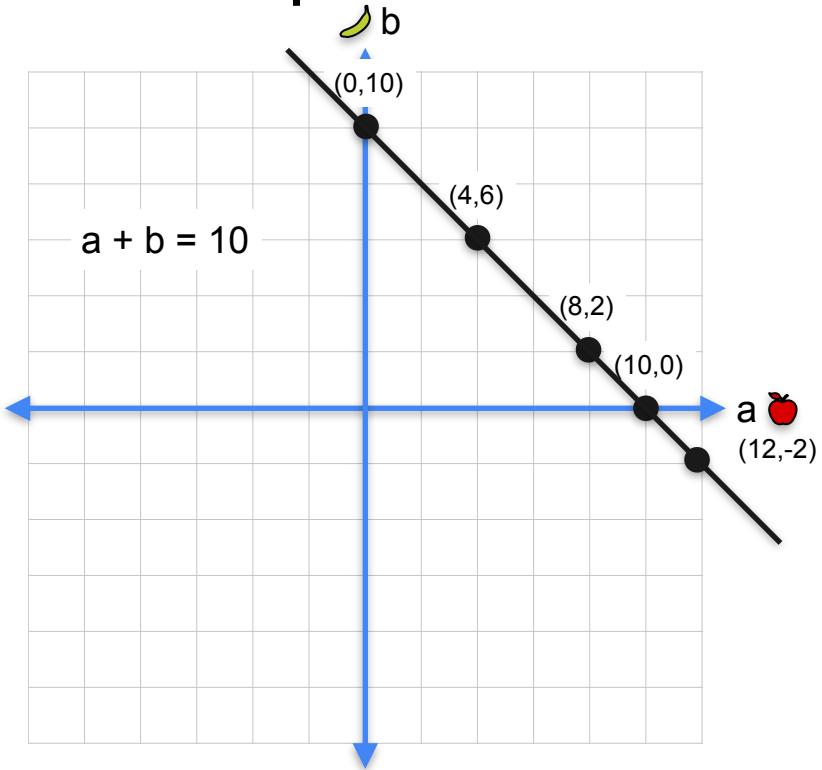
Linear equation in 2 variables -> Line



Linear equation in 2 variables -> Line

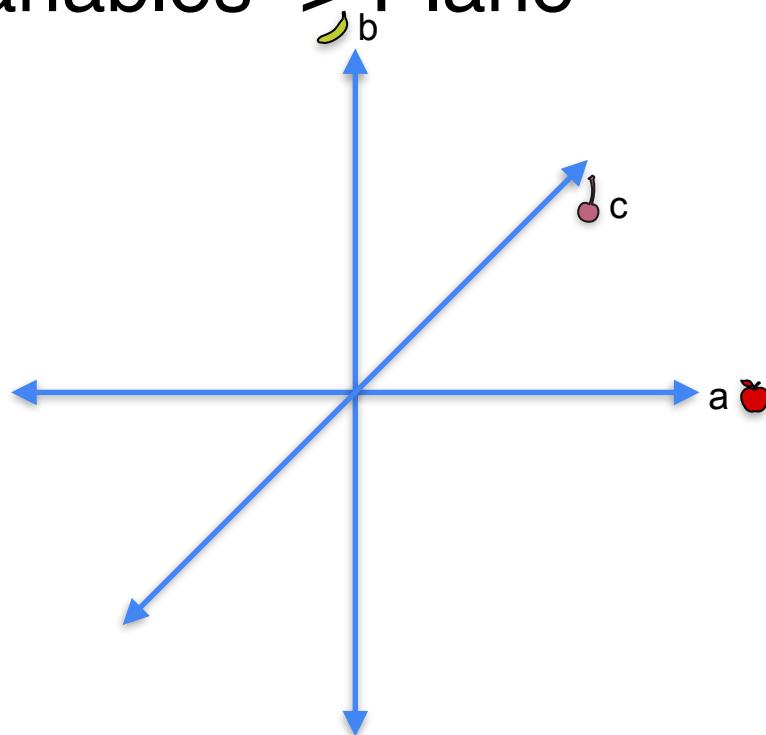


Linear equation in 2 variables -> Line



Linear equation in 3 variables -> Plane

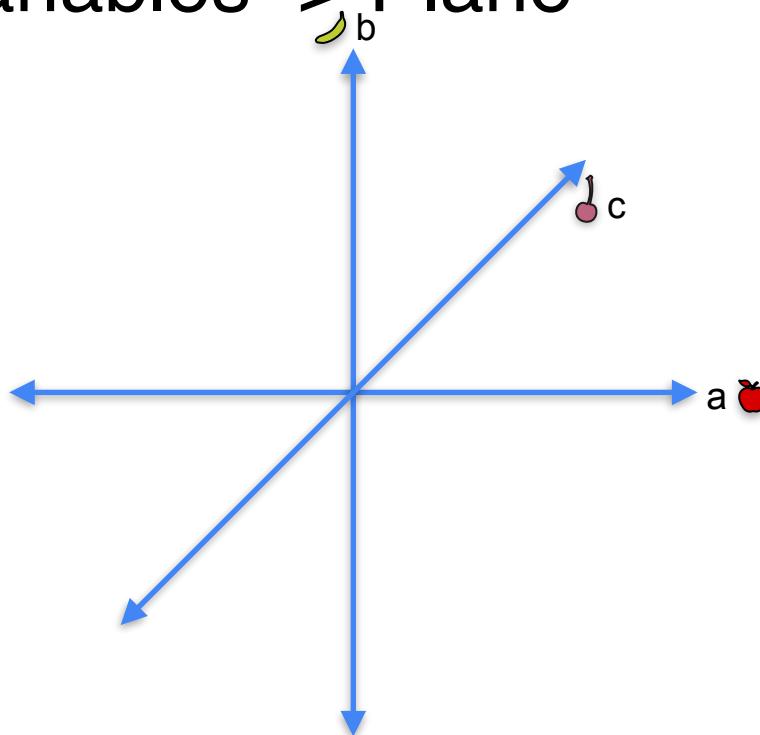
$$a + b + c = 1$$



Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

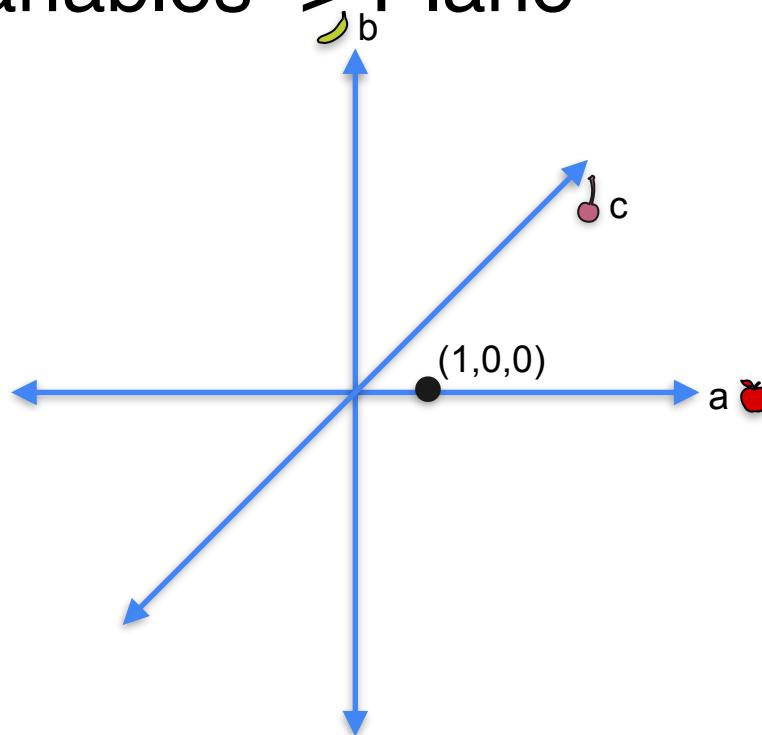
$$1 + 0 + 0 = 1$$



Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

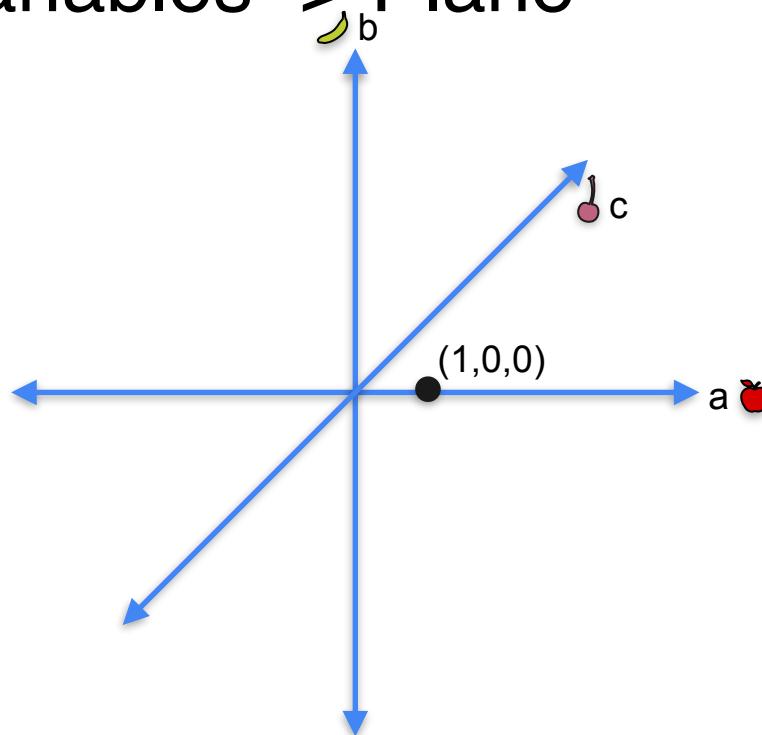


Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

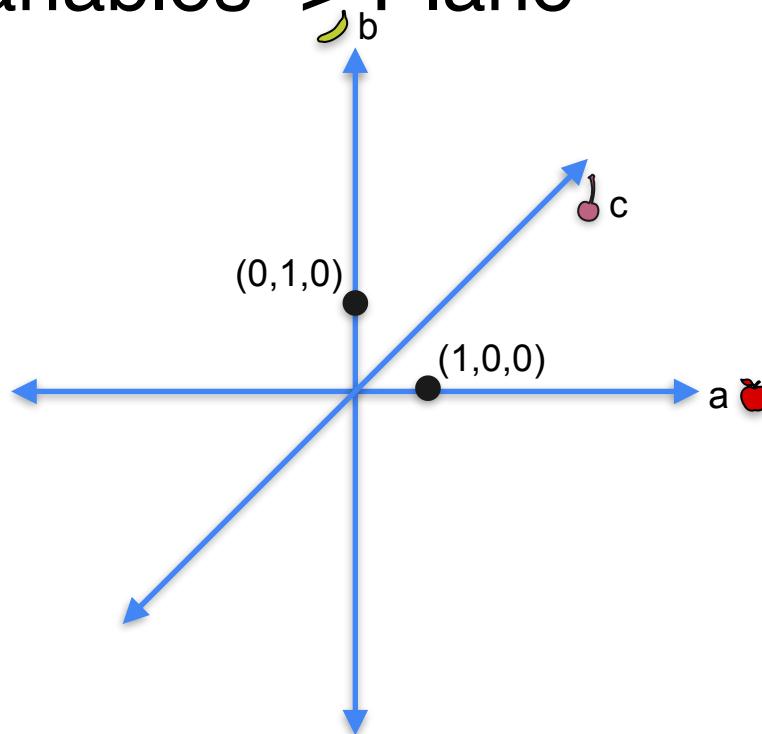


Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$



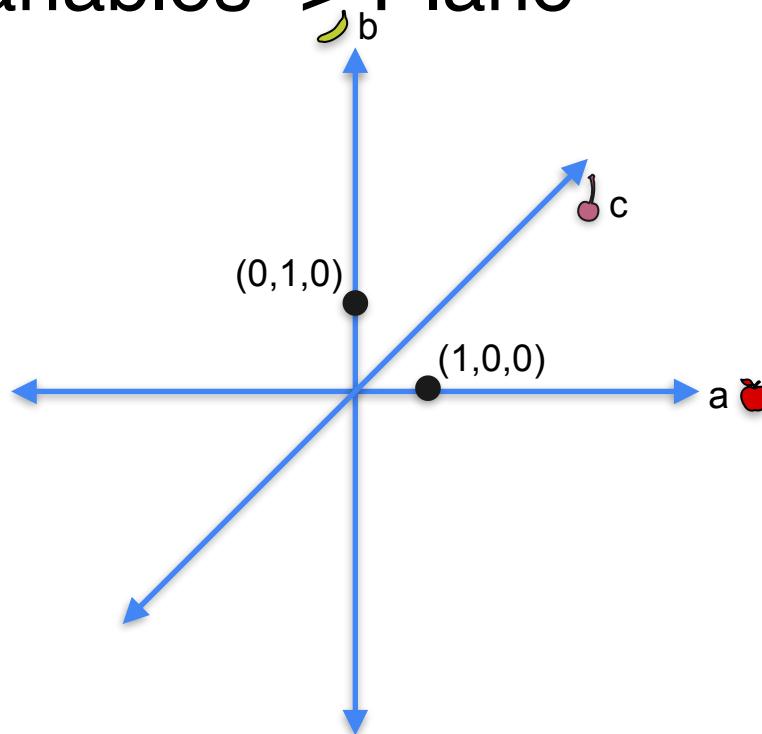
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



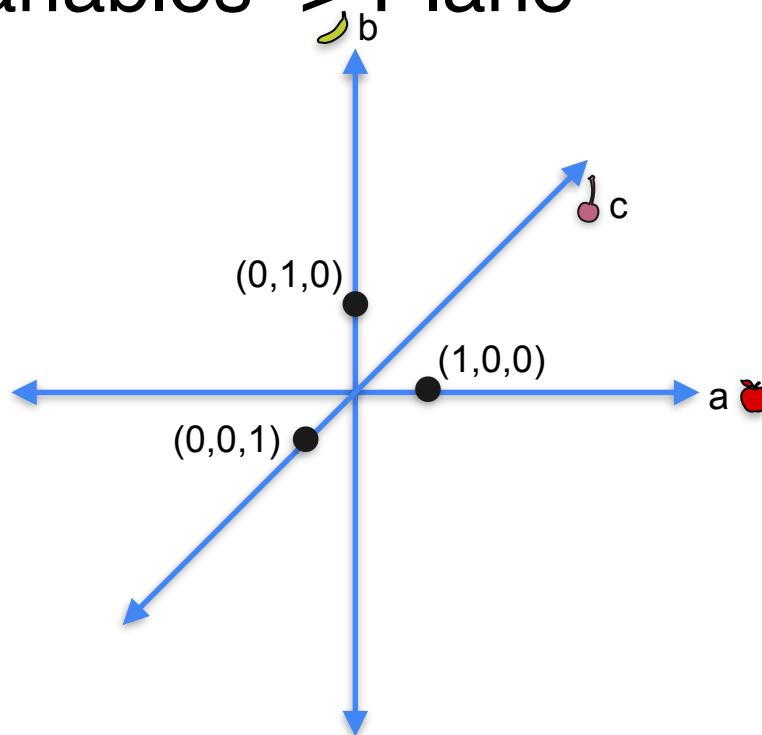
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



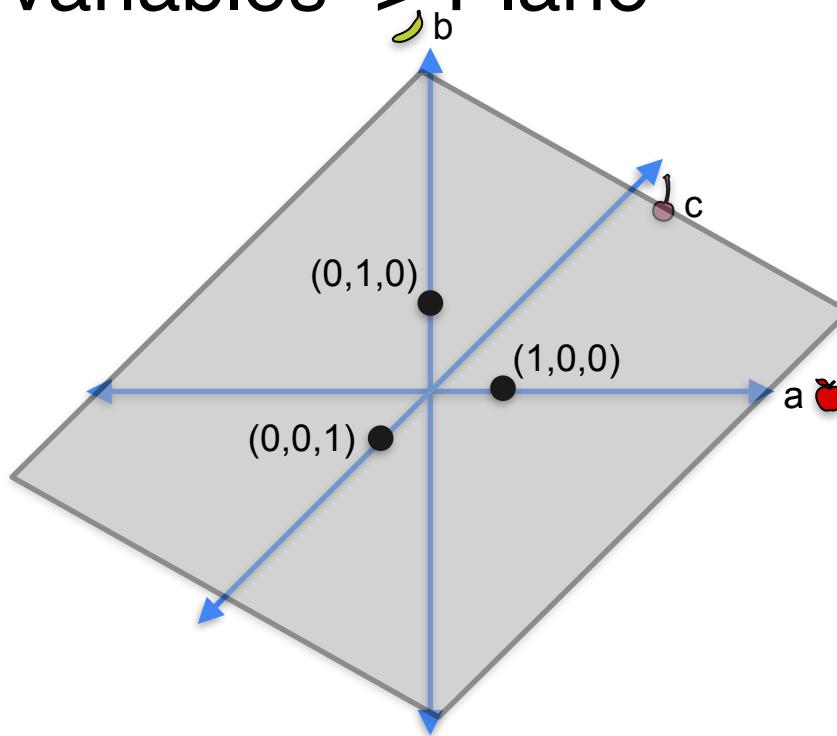
Linear equation in 3 variables -> Plane

$$a + b + c = 1$$

$$1 + 0 + 0 = 1$$

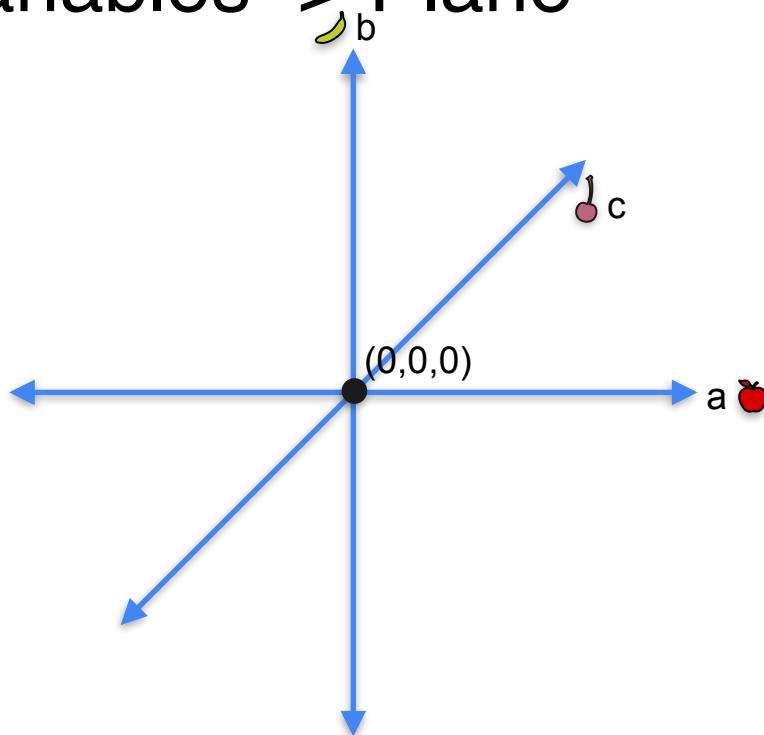
$$0 + 1 + 0 = 1$$

$$0 + 0 + 1 = 1$$



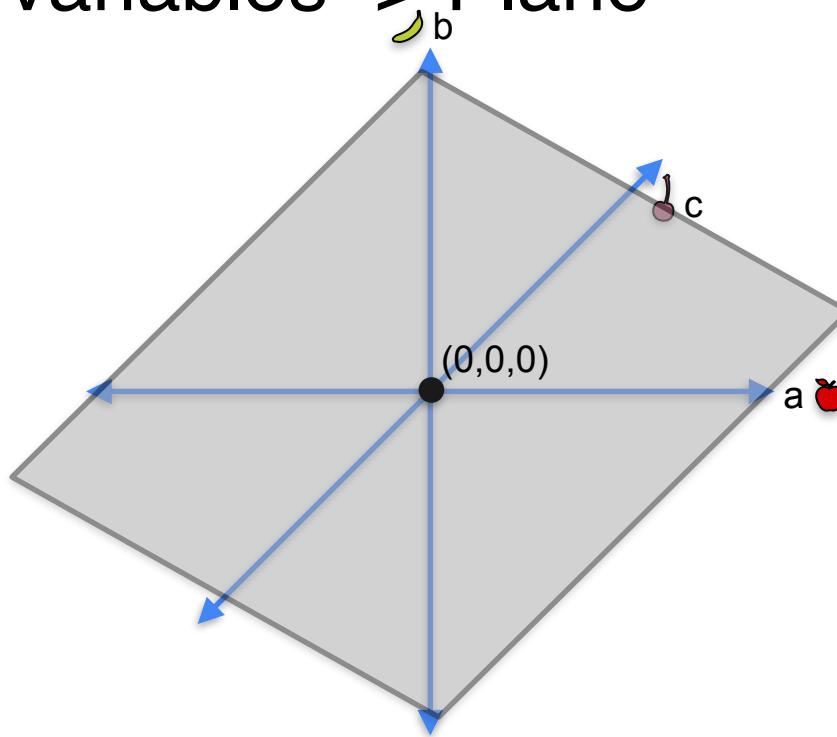
Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = \mathbf{0}$$



Linear equation in 3 variables -> Plane

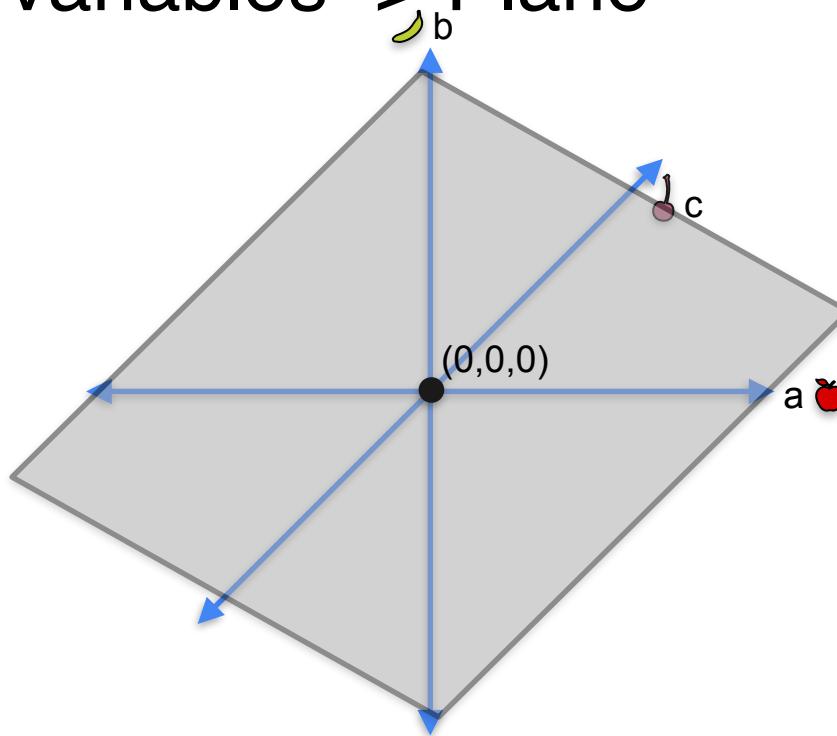
$$3a - 5b + 2c = \mathbf{0}$$



Linear equation in 3 variables -> Plane

$$3a - 5b + 2c = \mathbf{0}$$

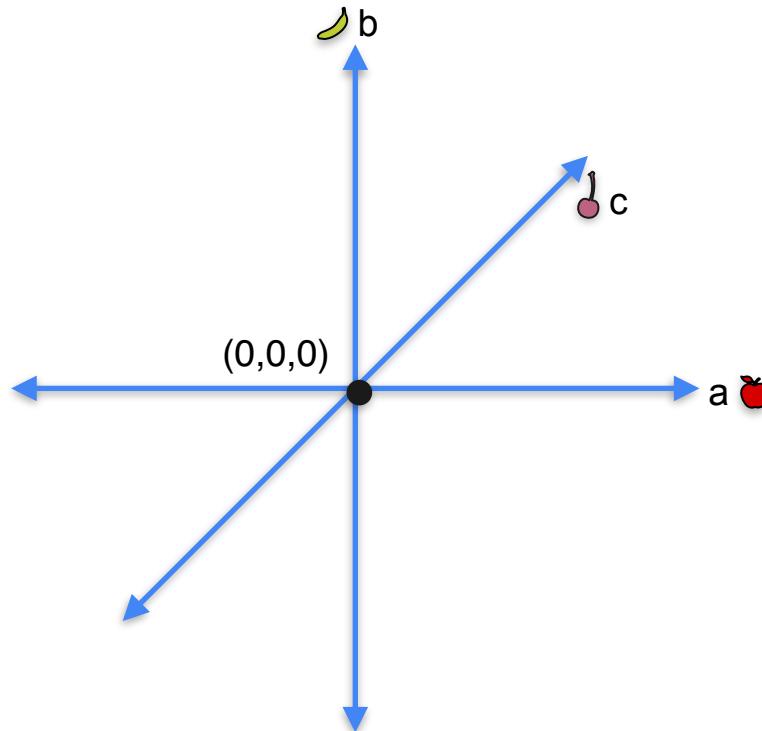
$$3(0) + 5(0) + 2(0) = \mathbf{0}$$



System 1

System 1

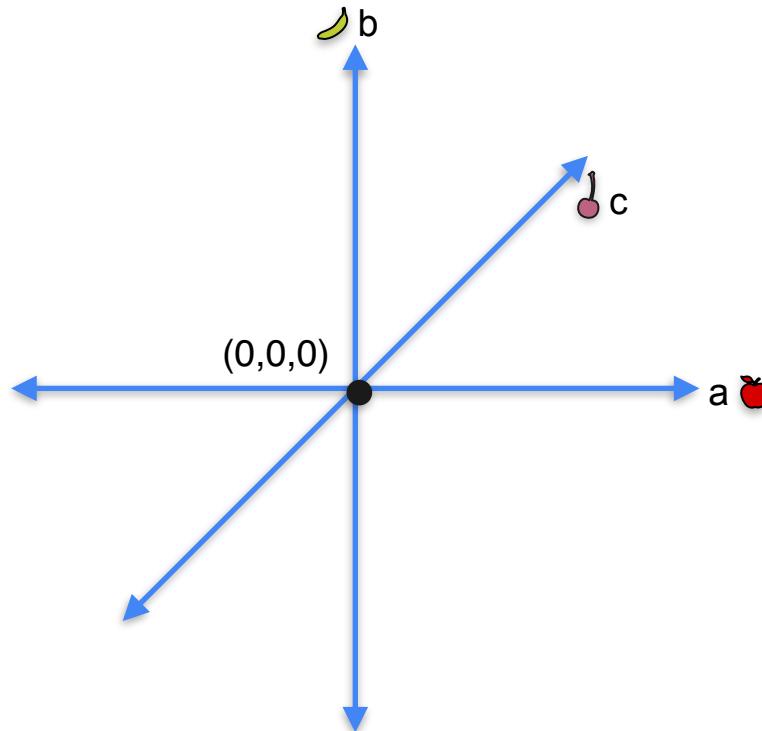
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

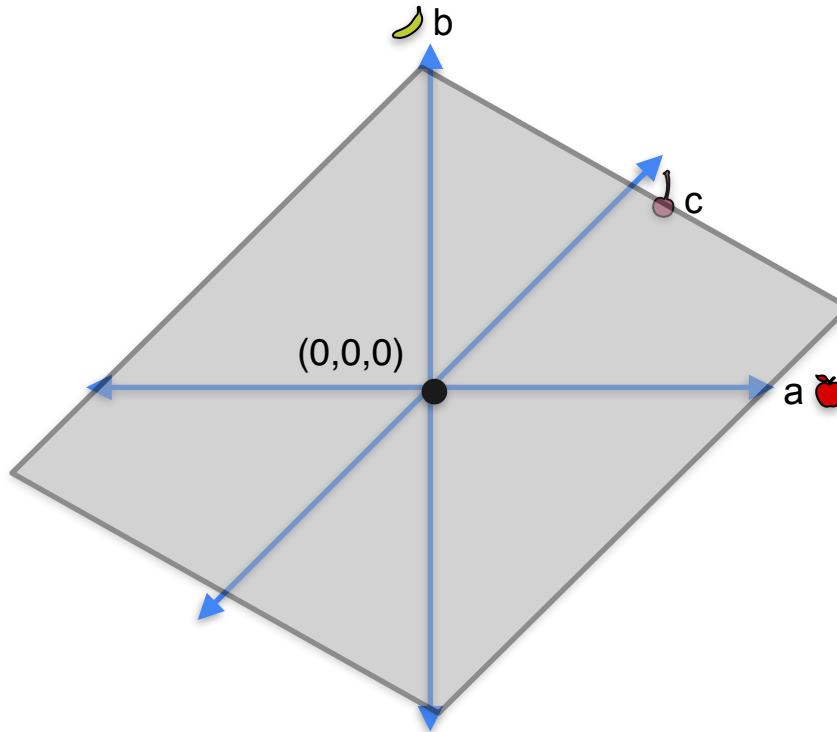
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

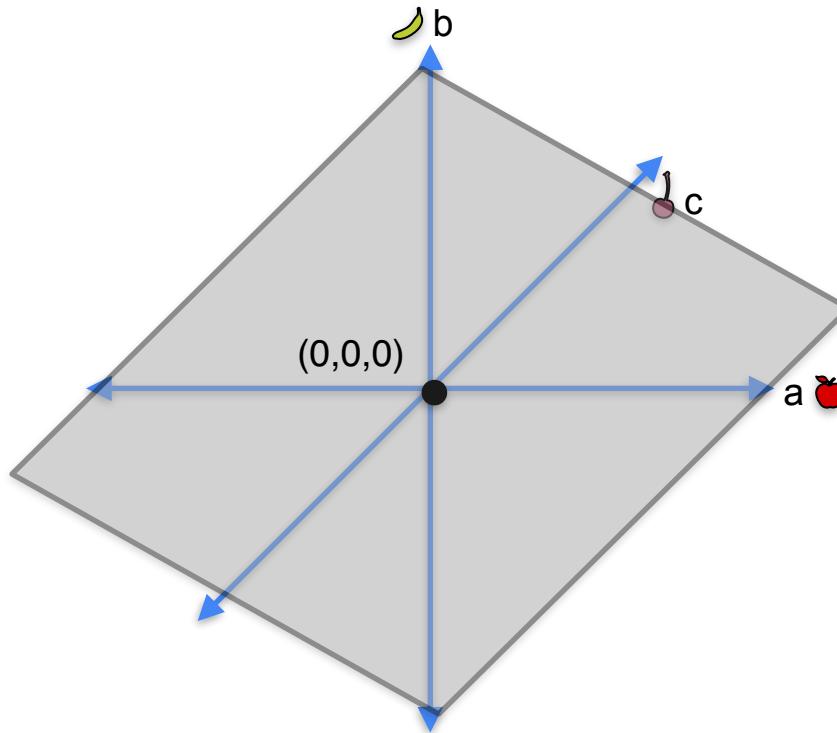
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

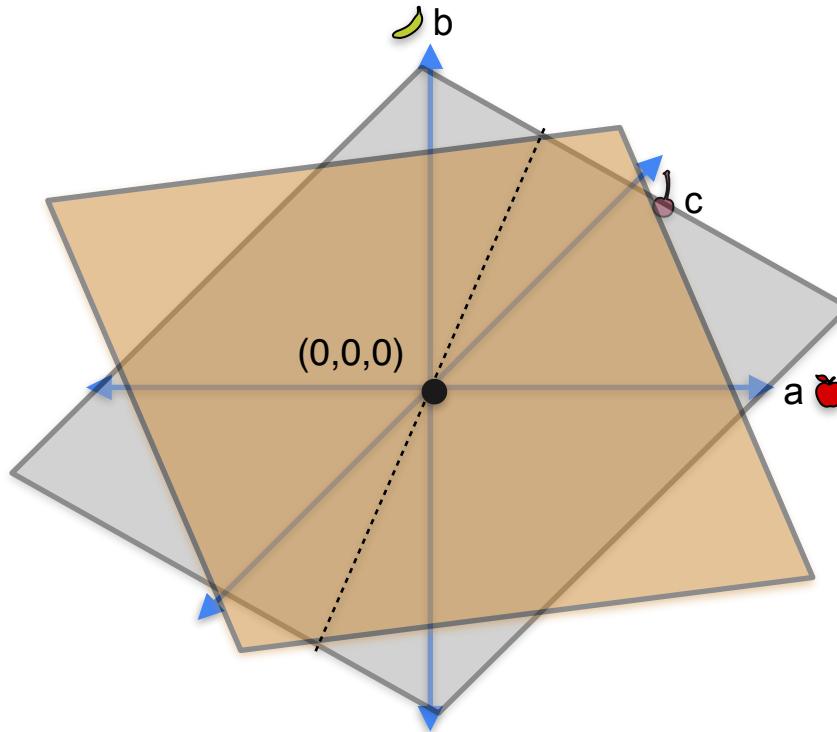
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

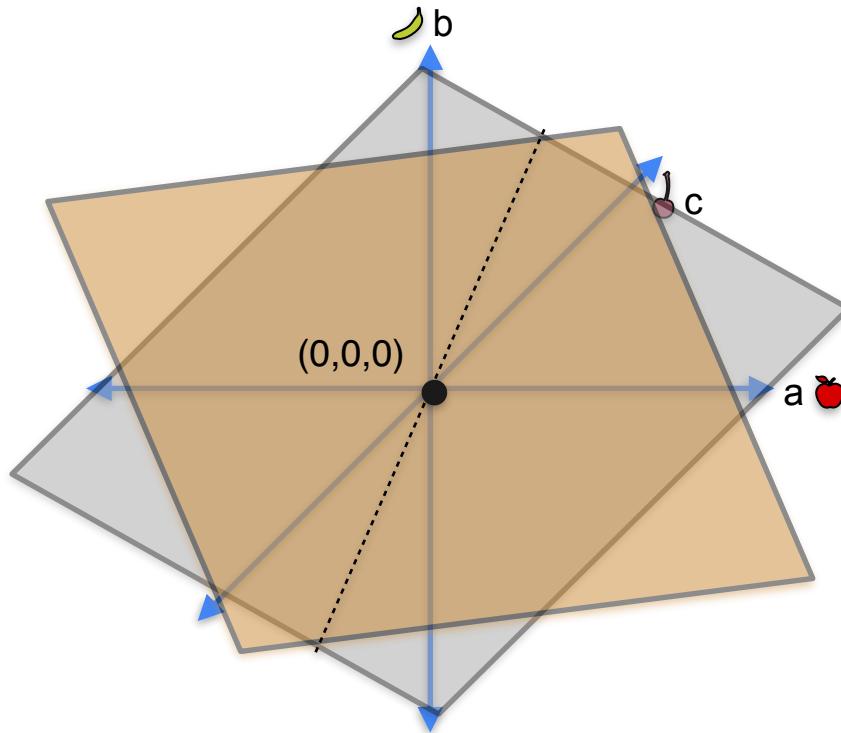
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

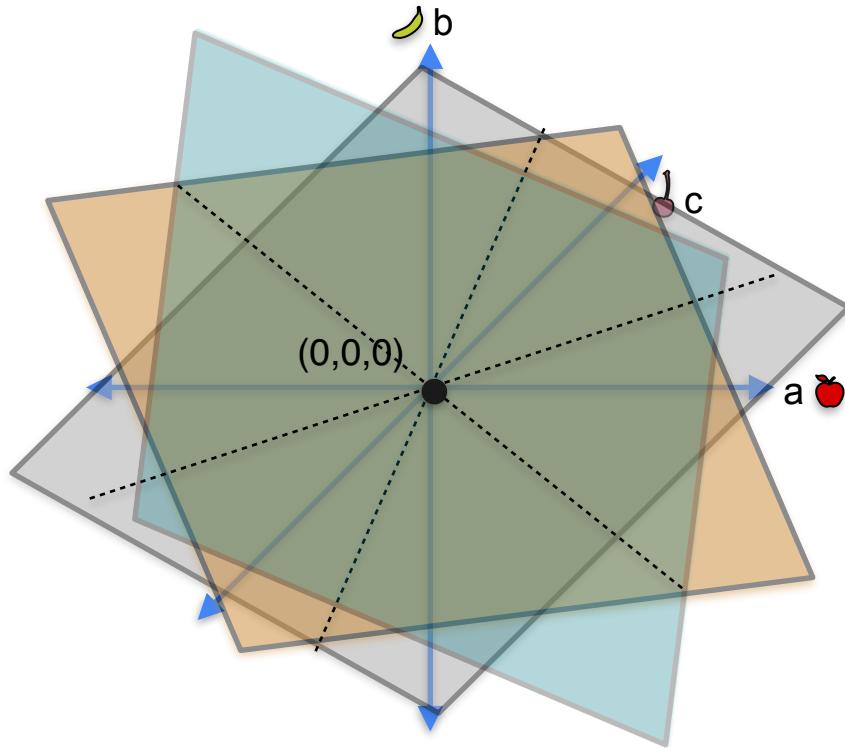
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 1

System 1

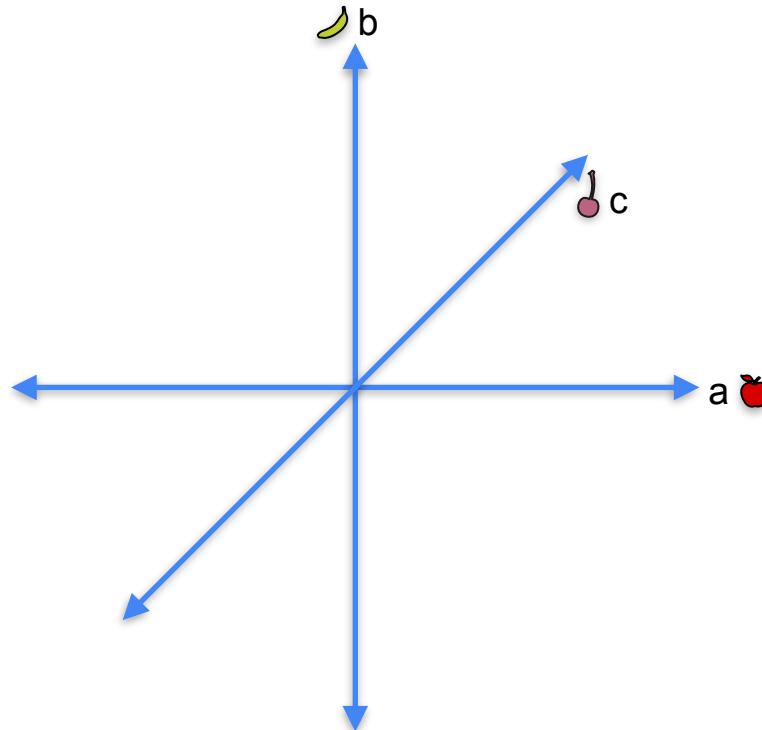
- $a + b + c = \mathbf{0}$
- $a + 2b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$



System 2

System 2

- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

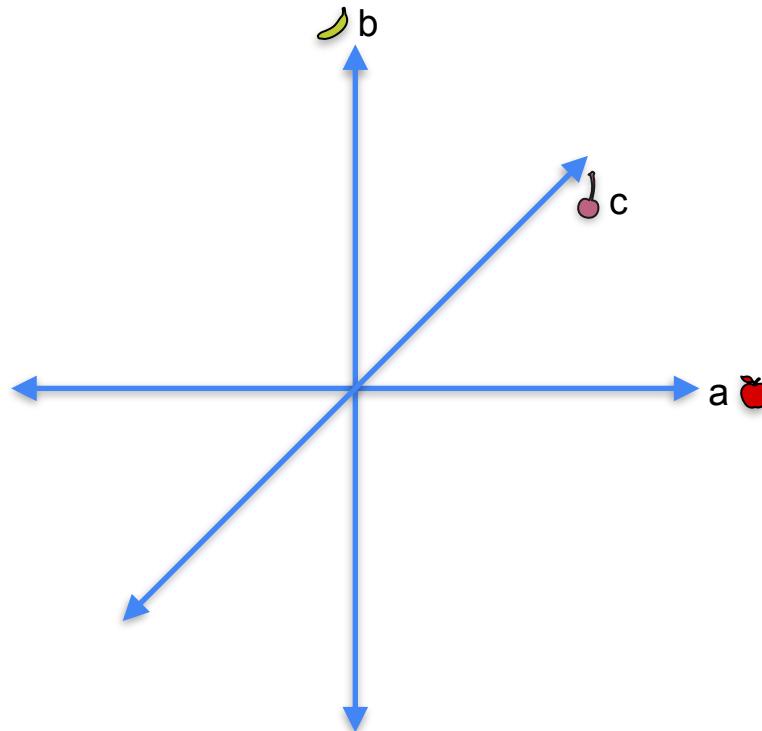
System 2

- $a + b + c = \mathbf{0}$



- $a + b + 2c = \mathbf{0}$

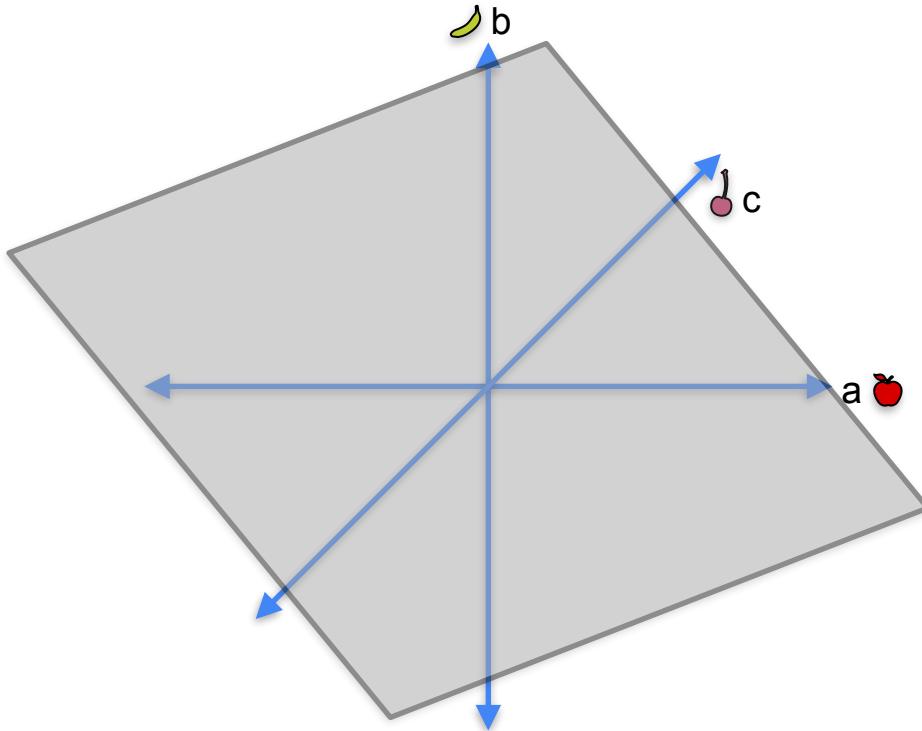
- $a + b + 3c = \mathbf{0}$



System 2

System 2

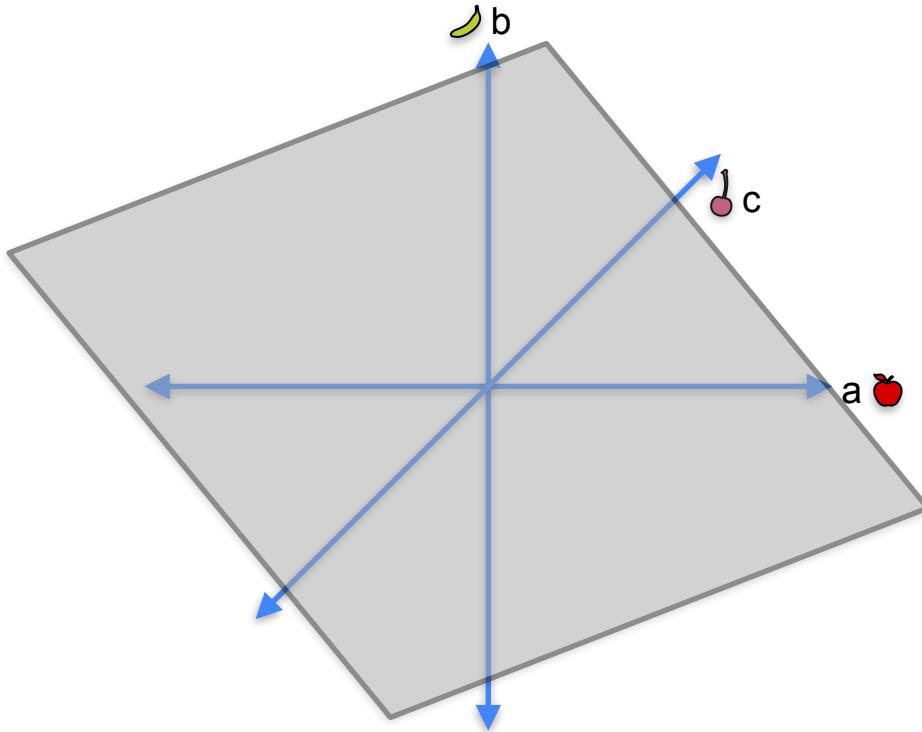
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

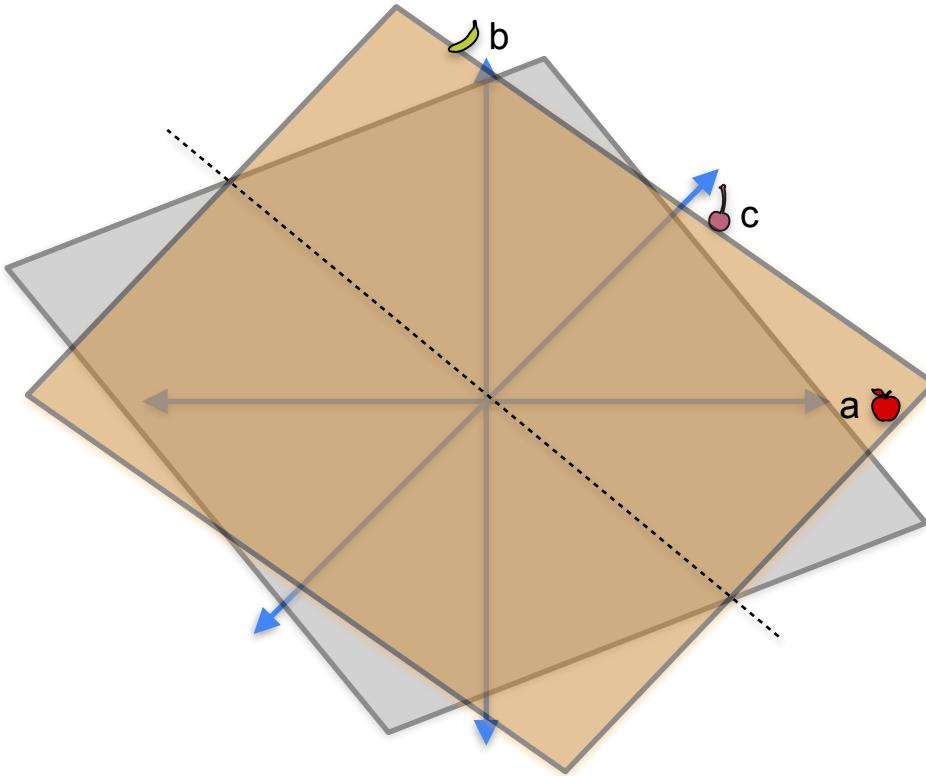
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

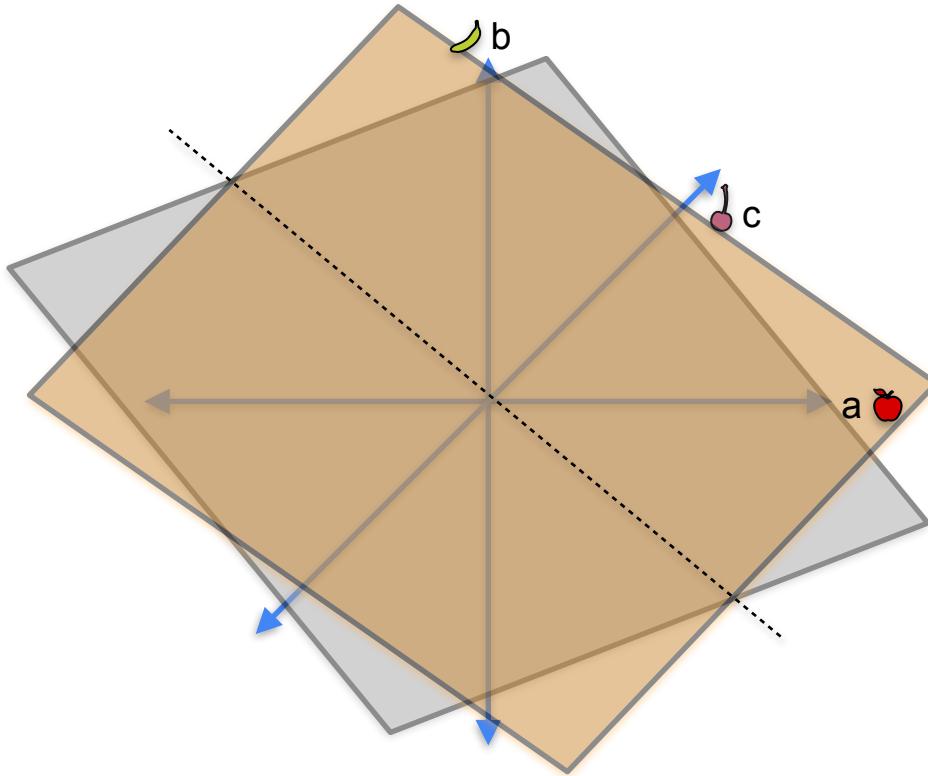
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

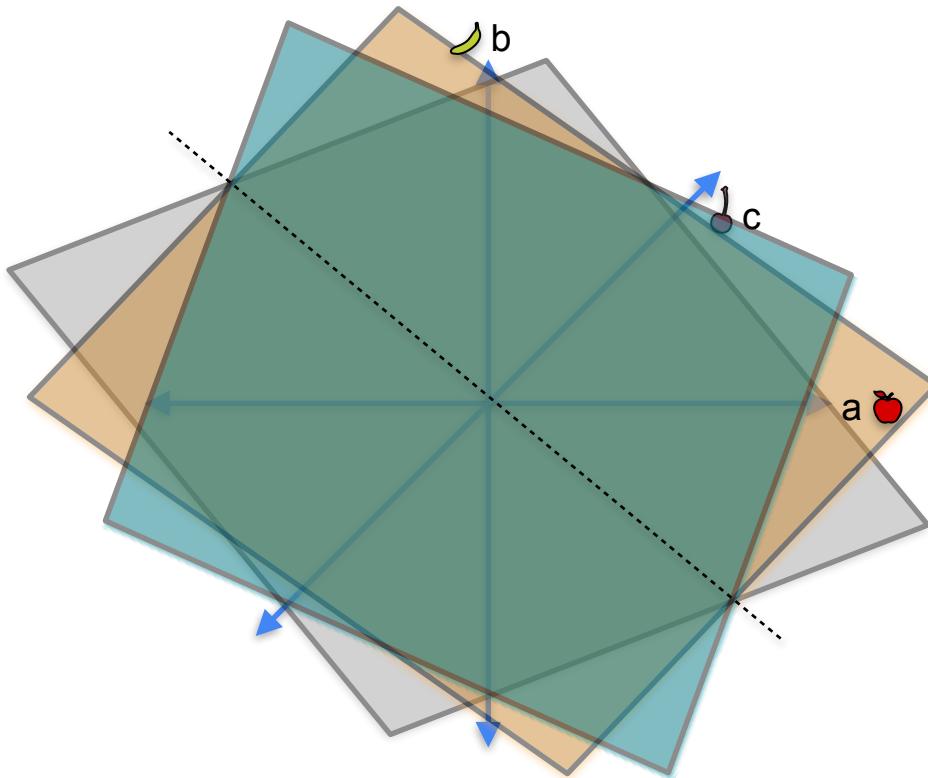
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

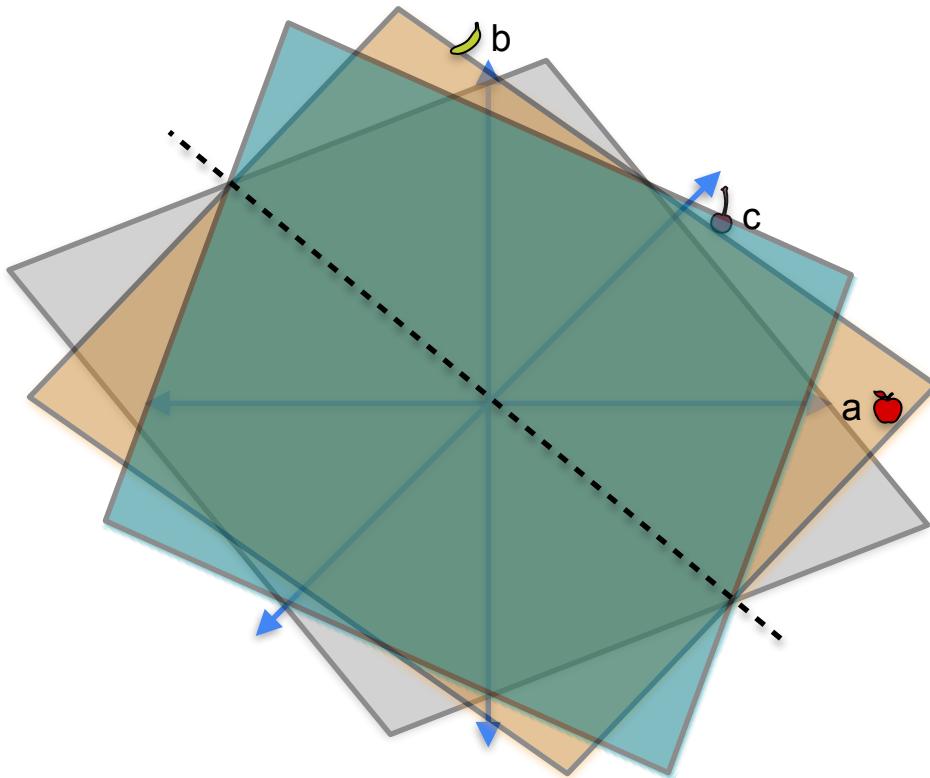
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 2

System 2

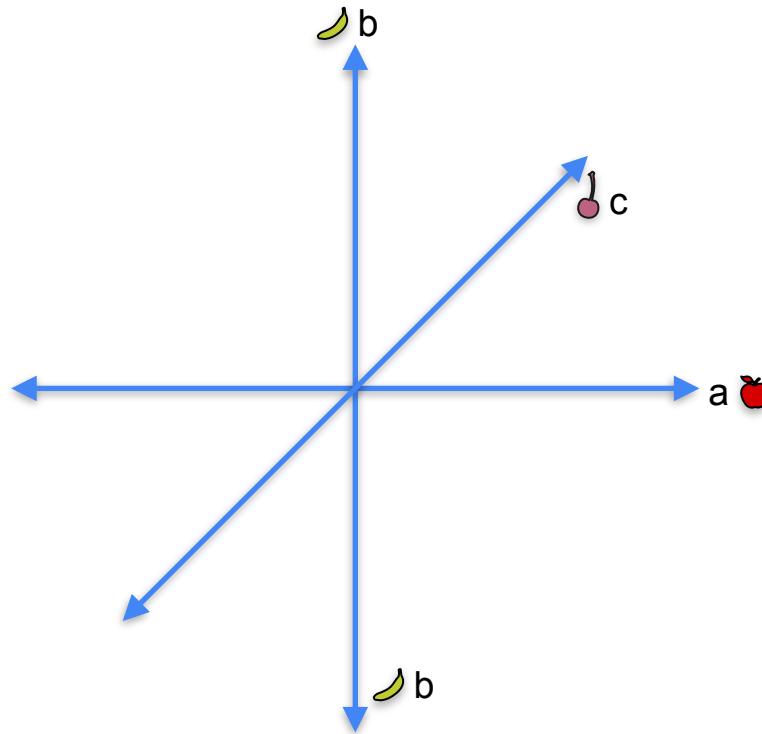
- $a + b + c = \mathbf{0}$
- $a + b + 2c = \mathbf{0}$
- $a + b + 3c = \mathbf{0}$



System 3

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$



System 3

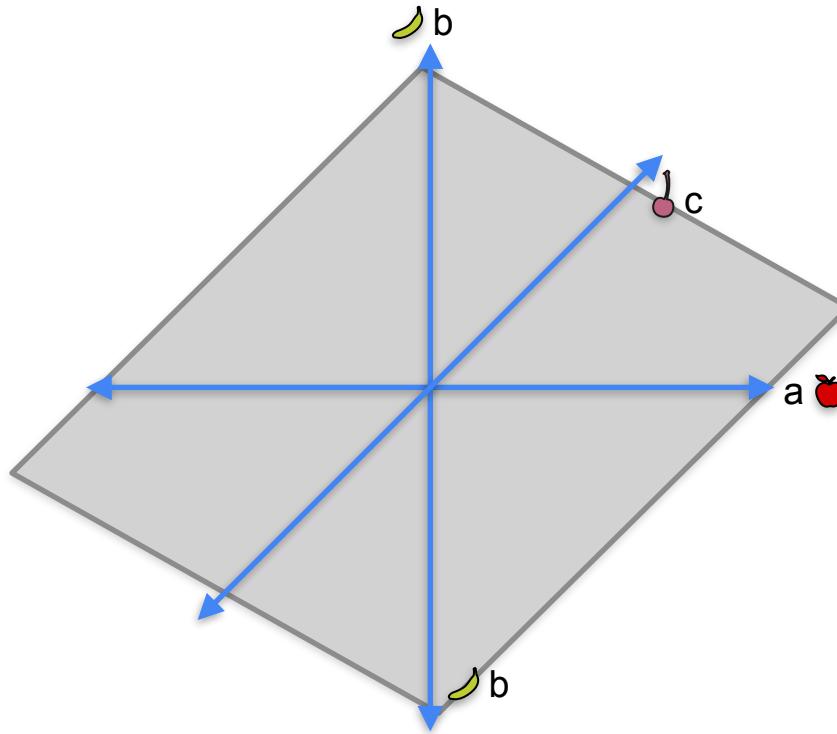
System 3

- $a + b + c = \mathbf{0}$



- $2a + 2b + 2c = \mathbf{0}$

- $3a + 3b + 3c = \mathbf{0}$



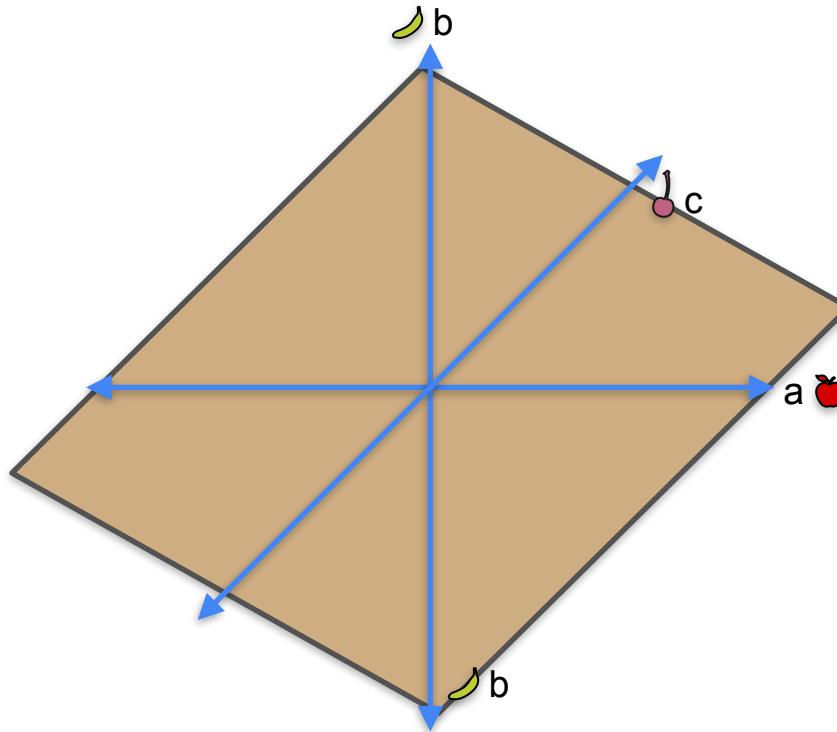
System 3

System 3

- $a + b + c = 0$

- $2a + 2b + 2c = 0$ 

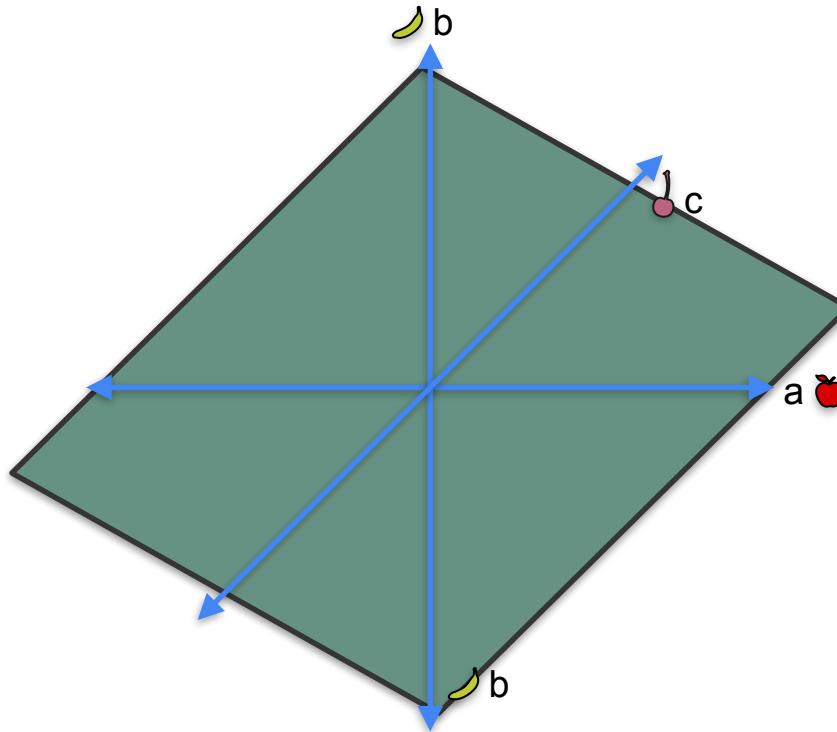
- $3a + 3b + 3c = 0$



System 3

System 3

- $a + b + c = \mathbf{0}$
- $2a + 2b + 2c = \mathbf{0}$
- $3a + 3b + 3c = \mathbf{0}$





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System of Linear Equations

**Linear dependence and
independence (3x3)**

Linear dependence and independence

$$a = \mathbf{1}$$

$$b = \mathbf{2}$$

$$a + b = \mathbf{3}$$

Linear dependence and independence

$$a = 1$$

$$b = 2$$

$$a + b = 3$$

$$a + 0b + 0c = 1$$

Linear dependence and independence

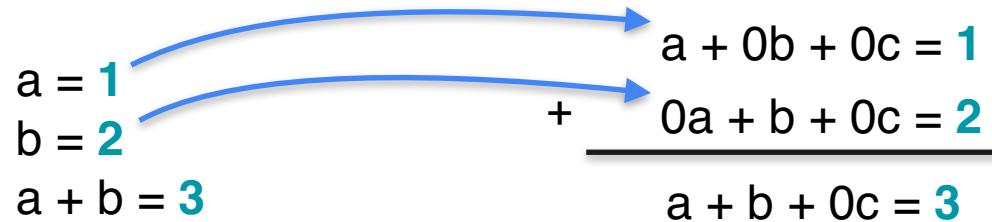
$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{l} \xrightarrow{\text{blue curve}} a + 0b + 0c = 1 \\ \xrightarrow{\text{blue curve}} 0a + b + 0c = 2 \end{array}$$

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} + \\ \hline \end{array} \quad \begin{array}{l} a + 0b + 0c = 1 \\ 0a + b + 0c = 2 \end{array}$$

The diagram illustrates linear dependence. On the left, three equations are listed: $a = 1$, $b = 2$, and $a + b = 3$. On the right, two more equations are shown: $a + 0b + 0c = 1$ and $0a + b + 0c = 2$. A plus sign (+) is placed between the first two equations on the left, and a horizontal line (\hline) is placed between the last two equations on the right. Two blue curved arrows originate from the first two equations and point to the third equation, indicating that the third equation is a linear combination of the first two.

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} + \\ \hline \end{array} \quad \begin{array}{l} a + 0b + 0c = 1 \\ 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$


Linear dependence and independence

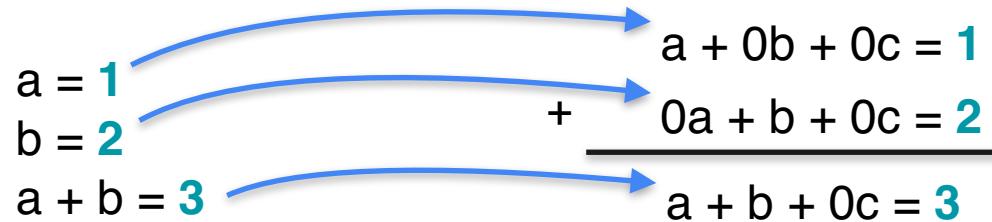
$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} \xrightarrow{\hspace{1cm}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{1cm}} + \quad \quad \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{1cm}} \quad \quad \quad a + b + 0c = 3 \end{array}$$

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} \xrightarrow{\hspace{1cm}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{1cm}} + \quad \quad \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{1cm}} \quad \quad \quad a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} a + 0b + 0c = 1 \\ + 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$


1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} a + 0b + 0c = 1 \\ + 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} \xrightarrow{\hspace{1cm}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{1cm}} + \quad 0a + b + 0c = 2 \\ \hline \xrightarrow{\hspace{1cm}} a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} \xrightarrow{\hspace{1cm}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{1cm}} 0a + b + 0c = 2 \\ + \hline \xrightarrow{\hspace{1cm}} a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} a + 0b + 0c = 1 \\ + 0a + b + 0c = 2 \\ \hline a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$\begin{array}{l} a = 1 \\ b = 2 \\ a + b = 3 \end{array} \quad \begin{array}{r} \xrightarrow{\hspace{1cm}} a + 0b + 0c = 1 \\ \xrightarrow{\hspace{1cm}} 0a + b + 0c = 2 \\ + \quad \quad \quad \hline \xrightarrow{\hspace{1cm}} a + b + 0c = 3 \end{array}$$

1	0	0
0	1	0
1	1	0

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$a + b + c = \mathbf{0}$$

$$2a + 2b + 2c = \mathbf{0}$$

$$3a + 3b + 3c = \mathbf{0}$$

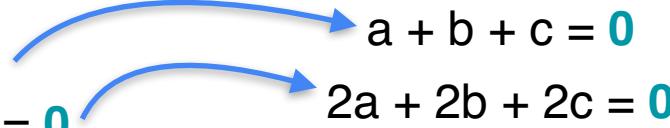
1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$\begin{aligned} a + b + c &= \mathbf{0} \\ 2a + 2b + 2c &= \mathbf{0} \\ 3a + 3b + 3c &= \mathbf{0} \end{aligned}$$


1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$\begin{aligned} a + b + c &= 0 \\ 2a + 2b + 2c &= 0 \\ 3a + 3b + 3c &= 0 \end{aligned}$$


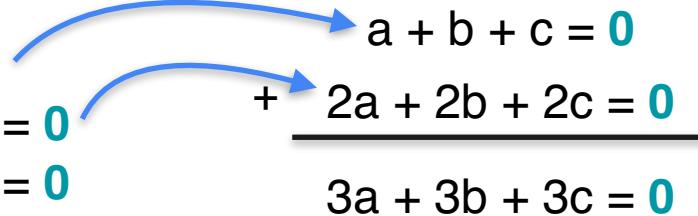
1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + \quad \underline{2a + 2b + 2c = 0} \end{array}$$

1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + \quad 2a + 2b + 2c = 0 \\ \hline 3a + 3b + 3c = 0 \end{array}$$


1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + \quad \underline{2a + 2b + 2c = 0} \\ 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + \quad \quad \quad 2a + 2b + 2c = 0 \\ \hline 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + \quad \quad \quad 2a + 2b + 2c = 0 \\ \hline 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ 2a + 2b + 2c = 0 \\ 3a + 3b + 3c = 0 \end{array} \quad \begin{array}{r} \xrightarrow{\hspace{1cm}} a + b + c = 0 \\ \xrightarrow{\hspace{1cm}} + \quad \xrightarrow{\hspace{1cm}} 2a + 2b + 2c = 0 \\ \hline \xrightarrow{\hspace{1cm}} 3a + 3b + 3c = 0 \end{array}$$

1	1	1
2	2	2
3	3	3

Row 1 + Row 2 = Row 3

Row 3 **depends** on rows 1 and 2

Rows are **linearly dependent**

Linear dependence and independence

$$a + b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

$$a + b + 3c = \mathbf{0}$$

1	1	1
1	1	2
1	1	3

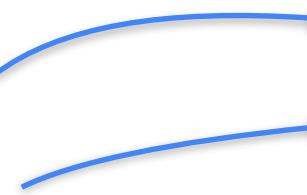
Linear dependence and independence

$$\begin{aligned} a + b + c &= 0 \\ a + b + 2c &= 0 \\ a + b + 3c &= 0 \end{aligned}$$

$$a + b + c = 0$$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{l} a + b + c = 0 \\ a + b + 3c = 0 \end{array}$$


1	1	1
1	1	2
1	1	3

Linear dependence and independence

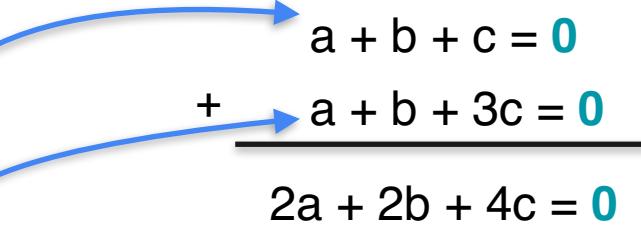
$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array}$$

+

$$a + b + c = 0$$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + \quad a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$


1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

÷ 2

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

÷ 2

$$a + b + 2c = 0$$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \xrightarrow{\quad \quad \quad a + b + c = 0}$$
$$+ \quad \quad \quad \underline{a + b + 3c = 0}$$
$$2a + 2b + 4c = 0$$
$$\downarrow \div 2$$
$$a + b + 2c = 0$$

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

Average of Row 1 and Row 3 is Row 2
Row 2 **depends** on rows 1 and 3

1	1	1
1	1	2
1	1	3

Linear dependence and independence

$$\begin{array}{l} a + b + c = 0 \\ a + b + 2c = 0 \\ a + b + 3c = 0 \end{array} \quad \begin{array}{r} a + b + c = 0 \\ + a + b + 3c = 0 \\ \hline 2a + 2b + 4c = 0 \end{array}$$

A blue curved arrow points from the three equations above to the matrix below. Another blue curved arrow points from the three equations to the sum of the first two. A blue arrow points from the sum to the result $2a + 2b + 4c = 0$. A blue arrow points from the result to the final equation $a + b + 2c = 0$.

1	1	1
1	1	2
1	1	3

Average of Row 1 and Row 3 is Row 2
Row 2 **depends** on rows 1 and 3
Rows are **linearly dependent**

Linear dependence and independence

$$a + b + c = \mathbf{0}$$

$$a + 2b + c = \mathbf{0}$$

$$a + b + 2c = \mathbf{0}$$

1	1	1
1	2	1
1	1	2

Linear dependence and independence

$$a + b + c = 0$$

$a + 2b + c = 0$  No relations between equations

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

Linear dependence and independence

$$a + b + c = 0$$

$a + 2b + c = 0$  No relations between equations

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

No relations between rows

Linear dependence and independence

$$a + b + c = 0$$

$a + 2b + c = 0$  No relations between equations

$$a + b + 2c = 0$$

1	1	1
1	2	1
1	1	2

No relations between rows

Rows are **linearly independent**

Quiz: Linear dependence and independence

Problem: Determine if the following matrices have linearly dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

Dependent (singular)

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

$$\text{Row1} - \text{Row2} = \text{Row3}$$

Dependent (singular)

Dependent (singular)

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

$$\text{Row1} - \text{Row2} = \text{Row3}$$

No relations

Dependent (singular)

Dependent (singular)

Independent
(Non-singular)

Solution: Linear dependence and independence

Problem: Determine if the following matrices have linear dependent or independent rows

1	0	1
0	1	0
3	2	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

$$3\text{Row1} + 2\text{Row2} = \text{Row3}$$

$$\text{Row1} - \text{Row2} = \text{Row3}$$

No relations

$$2\text{Row1} = \text{Row3}$$

Dependent (singular)

Dependent (singular)

**Independent
(Non-singular)**

Dependent (singular)

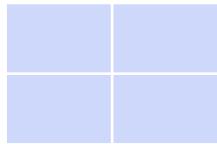


DeepLearning.AI

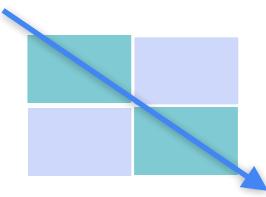
System of Linear Equations

The determinant (3x3)

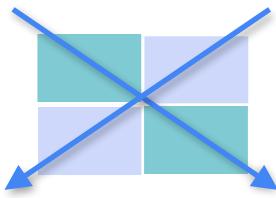
Diagonals in a 3x3 matrix



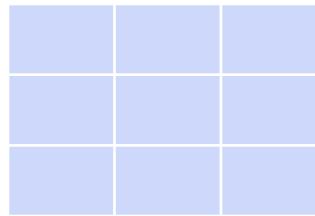
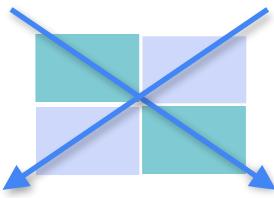
Diagonals in a 3x3 matrix



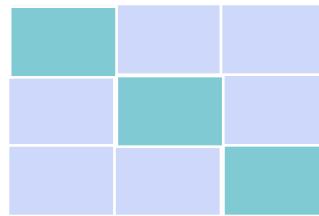
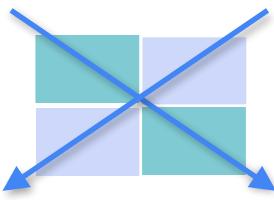
Diagonals in a 3x3 matrix



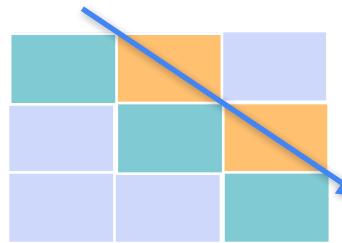
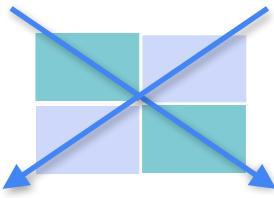
Diagonals in a 3x3 matrix



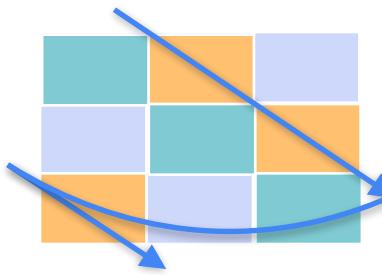
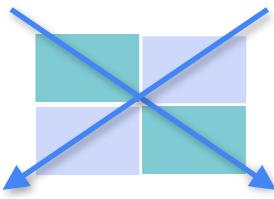
Diagonals in a 3x3 matrix



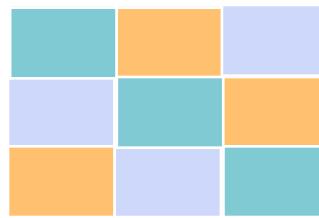
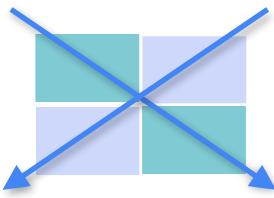
Diagonals in a 3x3 matrix



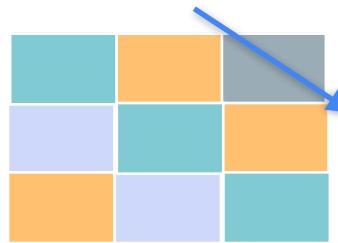
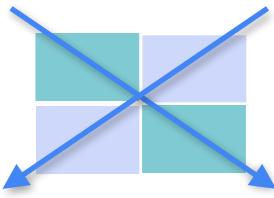
Diagonals in a 3x3 matrix



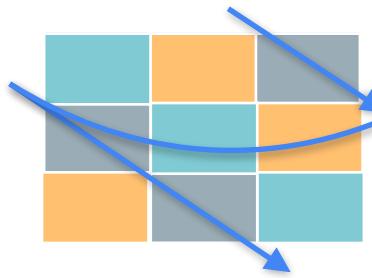
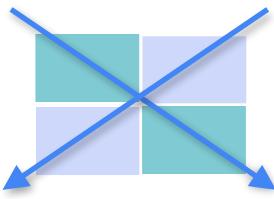
Diagonals in a 3x3 matrix



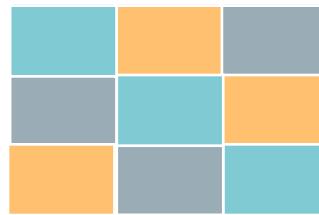
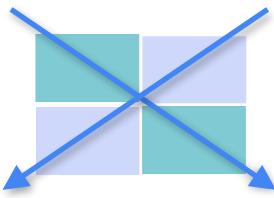
Diagonals in a 3x3 matrix



Diagonals in a 3x3 matrix



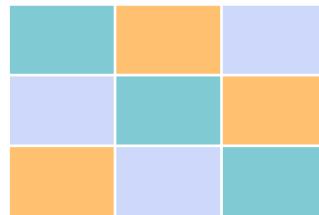
Diagonals in a 3x3 matrix



Determinant

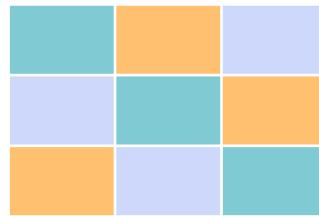
Determinant

Add

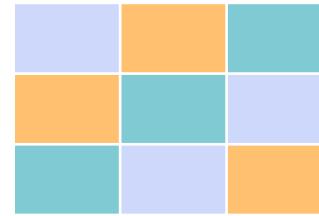


Determinant

Add



Subtract



The determinant

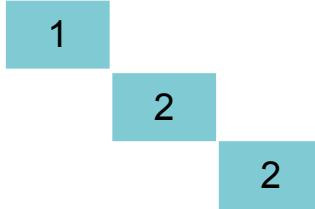
1	1	1
1	2	1
1	1	2

The determinant

1	1	1
1	2	1
1	1	2

The determinant

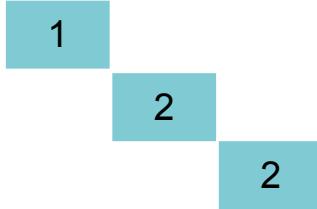
1	1	1
1	2	1
1	1	2



$$+ 1 \cdot 2 \cdot 2$$

The determinant

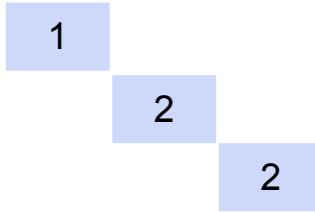
1	1	1
1	2	1
1	1	2



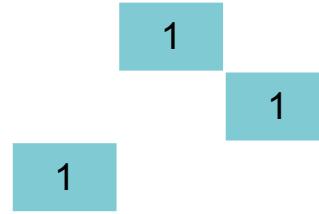
$$+ 1 \cdot 2 \cdot 2$$

The determinant

1	1	1
1	2	1
1	1	2



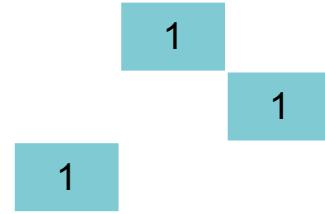
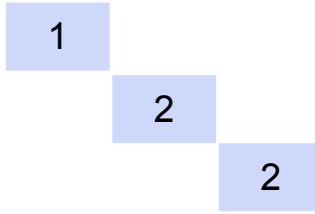
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$

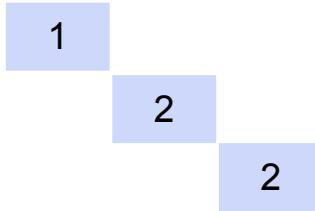
The determinant

1	1	1
1	2	1
1	1	2

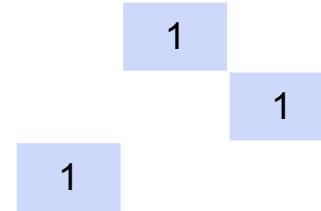


The determinant

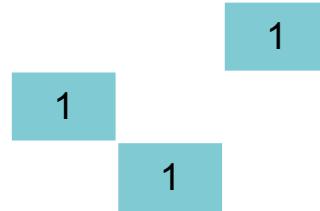
1	1	1
1	2	1
1	1	2



$$+ 1 \cdot 2 \cdot 2$$



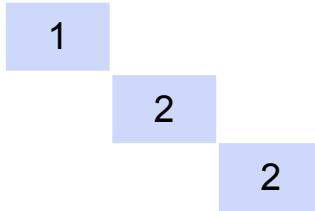
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$

The determinant

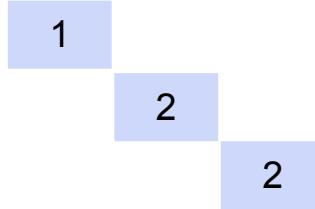
1	1	1
1	2	1
1	1	2



$$+ 1 \cdot 2 \cdot 2 + 1 \cdot 1 \cdot 1 + 1 \cdot 1 \cdot 1$$

The determinant

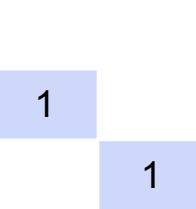
1	1	1
1	2	1
1	1	2



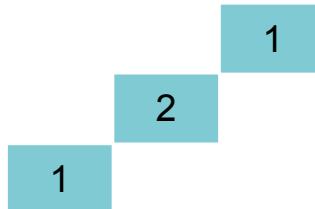
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



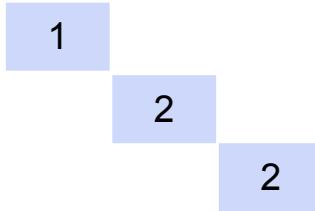
$$+ 1 \cdot 1 \cdot 1$$



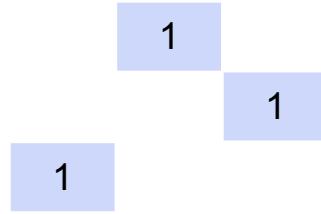
$$- 1 \cdot 2 \cdot 1$$

The determinant

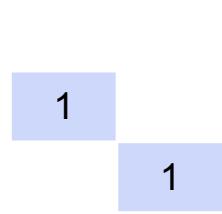
1	1	1
1	2	1
1	1	2



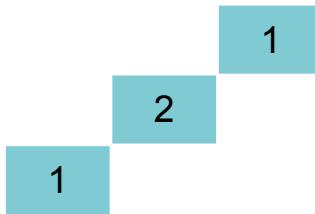
$$+ 1 \cdot 2 \cdot 2$$



$$+ 1 \cdot 1 \cdot 1$$



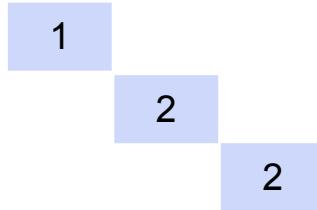
$$+ 1 \cdot 1 \cdot 1$$



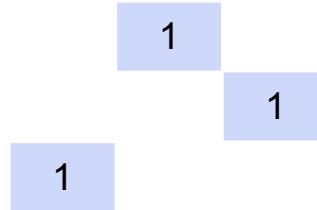
$$- 1 \cdot 2 \cdot 1$$

The determinant

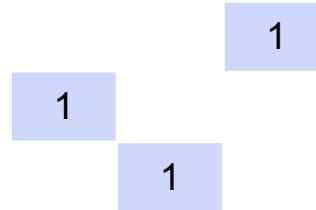
1	1	1
1	2	1
1	1	2



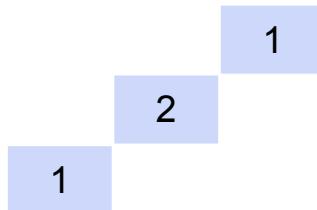
$$+ 1 \cdot 2 \cdot 2$$



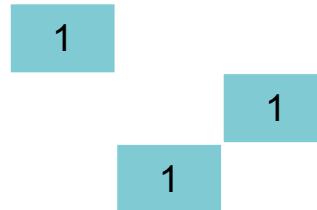
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



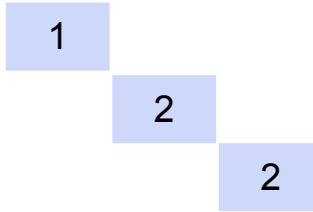
$$- 1 \cdot 2 \cdot 1$$



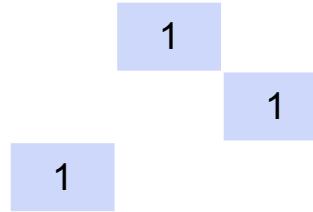
$$- 1 \cdot 1 \cdot 1$$

The determinant

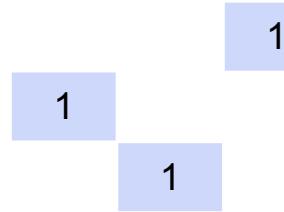
1	1	1
1	2	1
1	1	2



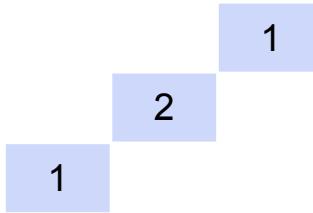
$$+ 1 \cdot 2 \cdot 2$$



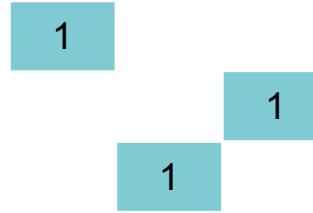
$$+ 1 \cdot 1 \cdot 1$$



$$+ 1 \cdot 1 \cdot 1$$



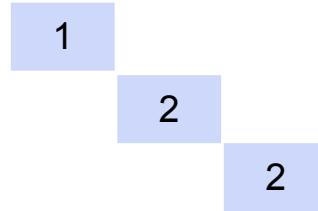
$$- 1 \cdot 2 \cdot 1$$



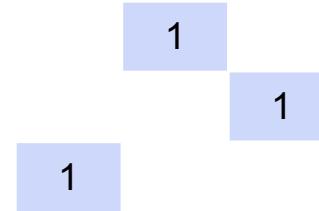
$$- 1 \cdot 1 \cdot 1$$

The determinant

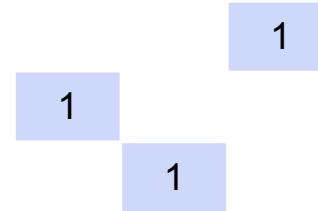
1	1	1
1	2	1
1	1	2



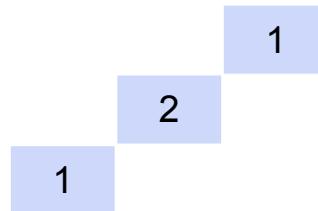
$$+ 1 \cdot 2 \cdot 2$$



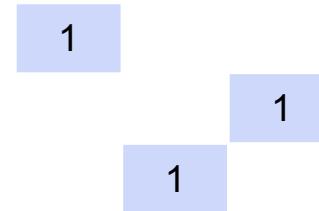
$$+ 1 \cdot 1 \cdot 1$$



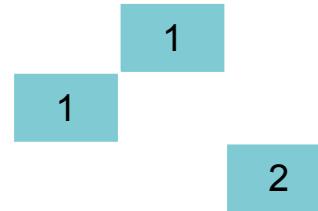
$$+ 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 2 \cdot 1$$



$$- 1 \cdot 1 \cdot 1$$



$$- 1 \cdot 1 \cdot 2$$

The determinant

1	1	1
1	2	1
1	1	2

$$+ 1 \cdot 2 \cdot 2$$

$$+ 1 \cdot 1 \cdot 1$$

$$+ 1 \cdot 1 \cdot 1$$

$$- 1 \cdot 2 \cdot 1$$

$$- 1 \cdot 1 \cdot 1$$

$$- 1 \cdot 1 \cdot 2$$

The determinant

1	1	1
1	2	1
1	1	2

A diagram illustrating the expansion of a 3x3 matrix along the first row. The matrix is shown as a 3x3 grid of light blue squares. The first row contains the values 1, 1, 1. To the right of the grid, the value 1 is enclosed in a light blue square. Below the grid, the value 2 is enclosed in a light blue square. To the right of the 2, the value 4 is enclosed in a large orange circle with a blue border. Further to the right, the value 2 is enclosed in a light blue square. Below the entire row, the expression $+ 1 \cdot 2 \cdot 2$ is written.

A diagram illustrating the expansion of a 3x3 matrix along the second row. The matrix is shown as a 3x3 grid of light blue squares. The second row contains the values 1, 2, 1. To the right of the grid, the value 1 is enclosed in a light blue square. Below the grid, the value 2 is enclosed in a light blue square. To the right of the 2, the value 1 is enclosed in a large orange circle with a blue border. Further to the right, the value 1 is enclosed in a light blue square. Below the entire row, the expression $+ 1 \cdot 1 \cdot 1$ is written.

A diagram illustrating the expansion of a 3x3 matrix along the third row. The matrix is shown as a 3x3 grid of light blue squares. The third row contains the values 1, 1, 2. To the right of the grid, the value 1 is enclosed in a light blue square. Below the grid, the value 1 is enclosed in a light blue square. To the right of the 1, the value 1 is enclosed in a large orange circle with a blue border. Further to the right, the value 1 is enclosed in a light blue square. Below the entire row, the expression $+ 1 \cdot 1 \cdot 1$ is written.

$$\text{Det} = 4 + 1 + 1 \\ - 2 - 1 - 2$$

A diagram illustrating the expansion of a 3x3 matrix along the first column. The matrix is shown as a 3x3 grid of light blue squares. The first column contains the values 1, 2, 1. To the right of the grid, the value 1 is enclosed in a light blue square. Below the grid, the value 2 is enclosed in a light blue square. To the right of the 2, the value 2 is enclosed in a large orange circle with a blue border. Further to the right, the value 1 is enclosed in a light blue square. Below the entire column, the expression $- 1 \cdot 2 \cdot 1$ is written.

A diagram illustrating the expansion of a 3x3 matrix along the second column. The matrix is shown as a 3x3 grid of light blue squares. The second column contains the values 1, 1, 1. To the right of the grid, the value 1 is enclosed in a light blue square. Below the grid, the value 1 is enclosed in a light blue square. To the right of the 1, the value 1 is enclosed in a large orange circle with a blue border. Further to the right, the value 1 is enclosed in a light blue square. Below the entire column, the expression $- 1 \cdot 1 \cdot 1$ is written.

A diagram illustrating the expansion of a 3x3 matrix along the third column. The matrix is shown as a 3x3 grid of light blue squares. The third column contains the values 1, 2, 2. To the right of the grid, the value 1 is enclosed in a light blue square. Below the grid, the value 2 is enclosed in a light blue square. To the right of the 2, the value 2 is enclosed in a large orange circle with a blue border. Further to the right, the value 2 is enclosed in a light blue square. Below the entire column, the expression $- 1 \cdot 1 \cdot 2$ is written.

The determinant

1	1	1
1	2	1
1	1	2

$$+ 1 \cdot 2 \cdot 2$$

A diagram illustrating the calculation of the first term in the determinant expansion. It shows a central orange circle with the number '4' inside, surrounded by three blue squares, each containing the number '2'. Above this group of four squares is another blue square containing the number '1'.

$$+ 1 \cdot 1 \cdot 1$$

A diagram illustrating the calculation of the second term in the determinant expansion. It shows a central orange circle with the number '1' inside, surrounded by three blue squares, each containing the number '1'. Above this group of four squares is another blue square containing the number '1'.

$$+ 1 \cdot 1 \cdot 1$$

A diagram illustrating the calculation of the third term in the determinant expansion. It shows a central orange circle with the number '1' inside, surrounded by three blue squares, each containing the number '1'. Above this group of four squares is another blue square containing the number '1'.

$$\begin{aligned} \text{Det} &= 4 + 1 + 1 \\ &\quad - 2 - 1 - 2 \\ &= 1 \end{aligned}$$

$$- 1 \cdot 2 \cdot 1$$

A diagram illustrating the calculation of the fourth term in the determinant expansion. It shows a central orange circle with the number '2' inside, surrounded by three blue squares, each containing the number '1'. Above this group of four squares is another blue square containing the number '1'.

$$- 1 \cdot 1 \cdot 1$$

A diagram illustrating the calculation of the fifth term in the determinant expansion. It shows a central orange circle with the number '1' inside, surrounded by three blue squares, each containing the number '1'. Above this group of four squares is another blue square containing the number '1'.

$$- 1 \cdot 1 \cdot 2$$

A diagram illustrating the calculation of the sixth term in the determinant expansion. It shows a central orange circle with the number '2' inside, surrounded by three teal squares, each containing the number '1'. Above this group of four squares is another teal square containing the number '1'.

Quiz: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

1	1	1
1	1	2
0	0	-1

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Determinant = 0

Singular

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

Singular

1	1	1
1	1	2
0	0	-1

Determinant = 0

Singular

1	1	1
0	2	2
0	0	3

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

Singular

1	1	1
1	1	2
0	0	-1

Determinant = 0

Singular

1	1	1
0	2	2
0	0	3

Determinant = 6

Non-singular

1	2	5
0	3	-2
2	4	10

Solution: Determinants

Problem: Find the determinant of the following matrices (from the previous quiz). Verify that those with determinant 0 are precisely the singular matrices.

1	0	1
0	1	0
3	3	3

Determinant = 0

Singular

1	1	1
1	1	2
0	0	-1

Determinant = 0

Singular

1	1	1
0	2	2
0	0	3

Determinant = 6

Non-singular

1	2	5
0	3	-2
2	4	10

Determinant = 0

Singular

The determinant

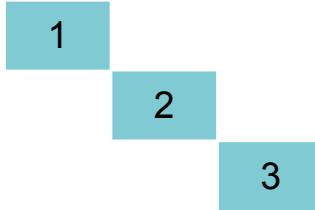
1	1	1
0	2	2
0	0	3

$$\text{Det} = 6+0+0-0-0-0$$

$$= 6$$

The determinant

1	1	1
0	2	2
0	0	3

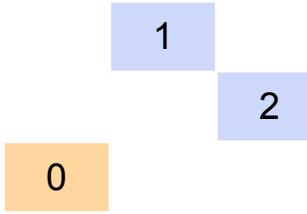
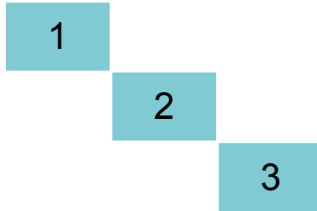


$$\text{Det} = 6+0+0-0-0-0$$

$$= 6$$

The determinant

1	1	1
0	2	2
0	0	3



$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

The determinant

1	1	1
0	2	2
0	0	3

1
2
3

$$+ 1 \cdot 2 \cdot 3$$

1
2

$$+ 1 \cdot 2 \cdot 0$$

0
0

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

The determinant

1	1	1
0	2	2
0	0	3

1
2
3

1
2
0

$$+ 1 \cdot 2 \cdot 3$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

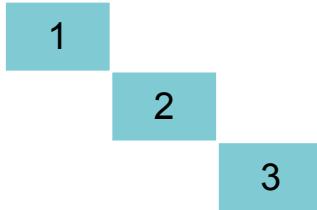
$$= 6$$

1
2
0

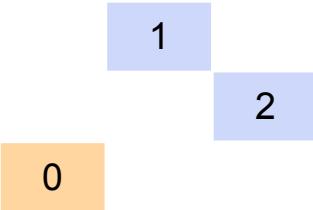
$$- 1 \cdot 2 \cdot 0$$

The determinant

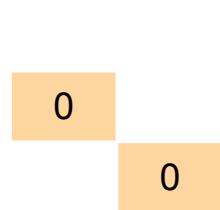
1	1	1
0	2	2
0	0	3



$$+ 1 \cdot 2 \cdot 3$$



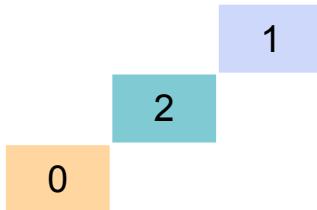
$$+ 1 \cdot 2 \cdot 0$$



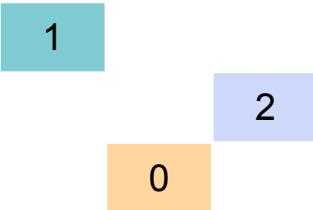
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$



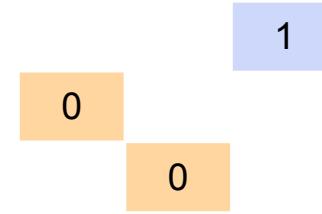
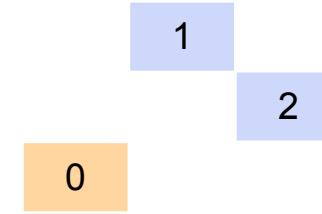
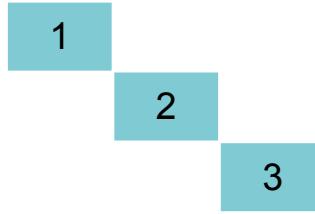
$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$

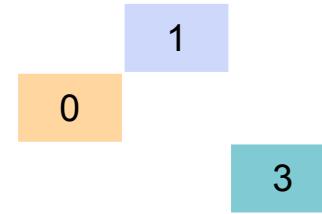
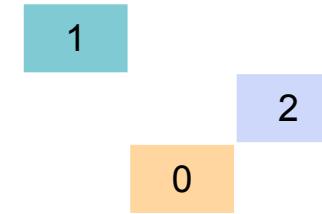
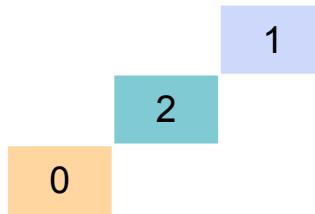
The determinant

1	1	1
0	2	2
0	0	3



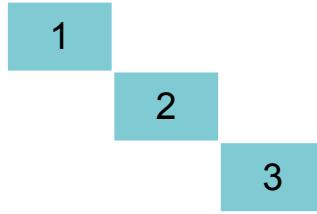
$$\text{Det} = 6+0+0-0-0-0$$

$$= 6$$

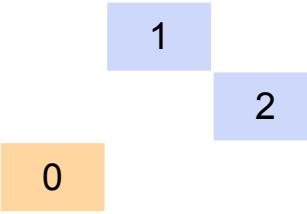


The determinant

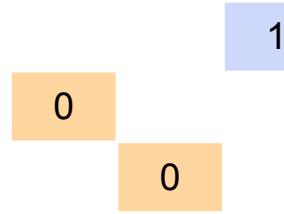
1	1	1
0	2	2
0	0	3



$$+ 1 \cdot 2 \cdot 3$$



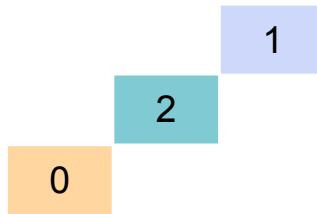
$$+ 1 \cdot 2 \cdot 0$$



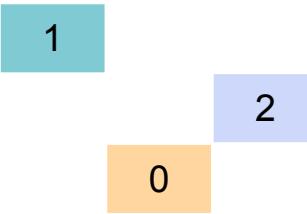
$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

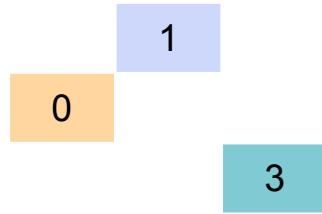
$$= 6$$



$$- 1 \cdot 2 \cdot 0$$



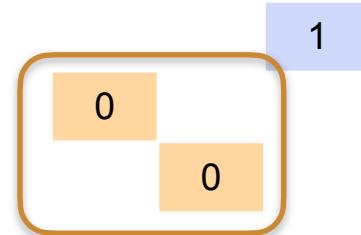
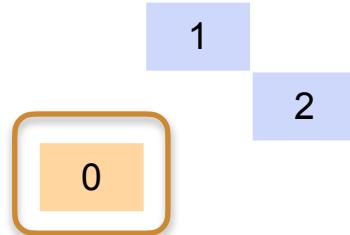
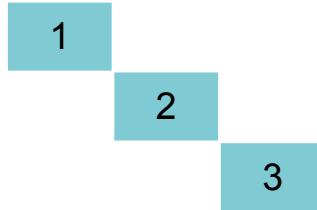
$$- 1 \cdot 2 \cdot 0$$



$$- 1 \cdot 0 \cdot 3$$

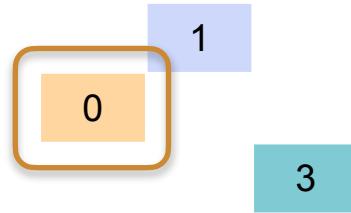
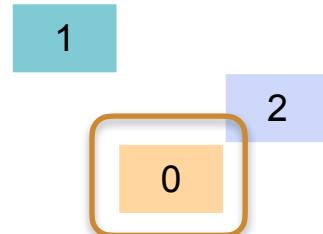
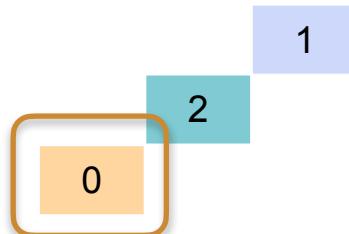
The determinant

1	1	1
0	2	2
0	0	3



$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$



The determinant

1	1	1
0	2	2
0	0	3

$$+ 1 \cdot 2 \cdot 3$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

$$- 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 0 \cdot 3$$

The determinant

$$\begin{matrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{matrix}$$

$$+ 1 \cdot 2 \cdot 3$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

$$\text{Det} = 6 + 0 + 0 - 0 - 0 - 0$$

$$= 6$$

$$- 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 0 \cdot 3$$

The determinant

1	1	1
0	2	2
0	0	0

The determinant

1	1	1
0	2	2
0	0	0

1
2
0

1
2
0

1
0
0

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 2 \cdot 0$$

$$+ 1 \cdot 0 \cdot 0$$

1
2
0

1
2
0

1
0
0

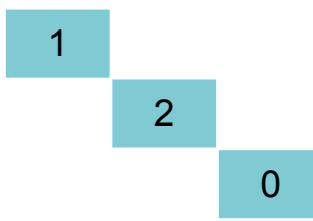
$$- 1 \cdot 2 \cdot 0$$

$$- 1 \cdot 2 \cdot 0$$

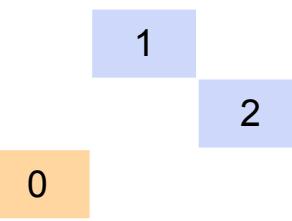
$$- 1 \cdot 0 \cdot 0$$

The determinant

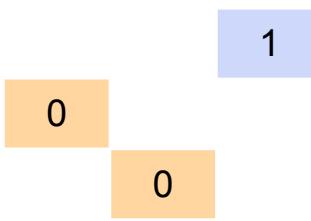
1	1	1
0	2	2
0	0	0



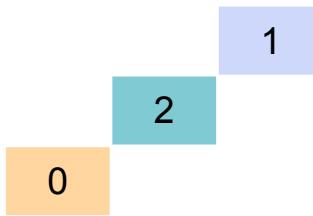
$$+ 1 \cdot 2 \cdot 0$$



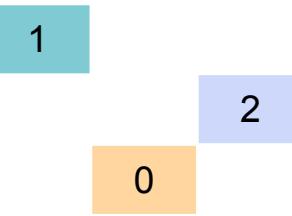
$$+ 1 \cdot 2 \cdot 0$$



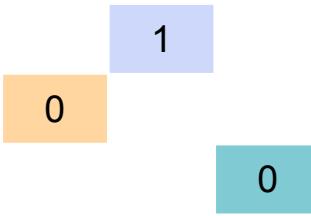
$$+ 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0$$



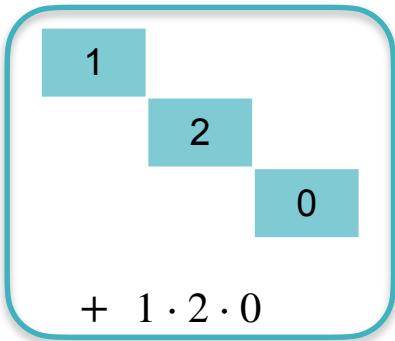
$$- 1 \cdot 2 \cdot 0$$



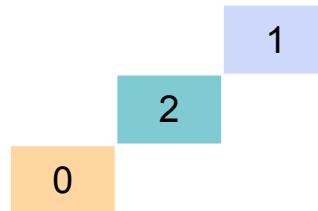
$$- 1 \cdot 0 \cdot 0$$

The determinant

1	1	1
0	2	2
0	0	0



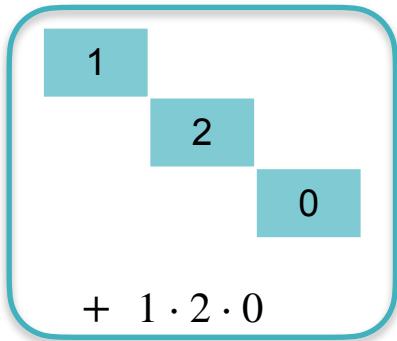
$$+ 1 \cdot 2 \cdot 0 + 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0 - 1 \cdot 0 \cdot 0$$

The determinant

1	1	1
0	2	2
0	0	0



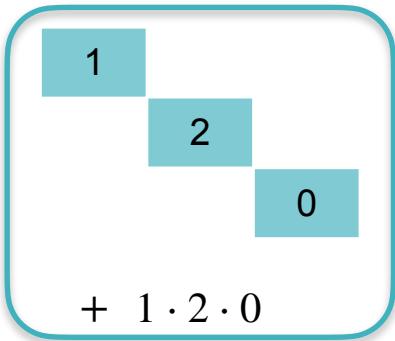
$$\text{Det} = 0+0+0-0-0-0$$

$$+ 1 \cdot 2 \cdot 0 + 1 \cdot 0 \cdot 0$$

$$- 1 \cdot 2 \cdot 0 - 1 \cdot 2 \cdot 0 - 1 \cdot 0 \cdot 0$$

The determinant

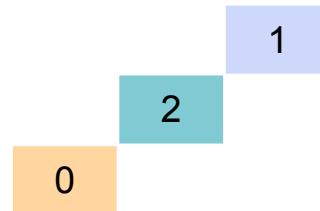
1	1	1
0	2	2
0	0	0



$$\text{Det} = 0+0+0-0-0-0$$

$$= 0$$

$$+ 1 \cdot 2 \cdot 0 + 1 \cdot 0 \cdot 0$$



$$- 1 \cdot 2 \cdot 0 - 1 \cdot 0 \cdot 0$$



DeepLearning.AI

System of Linear Equations

Conclusion