DeepLearning.AI

Math for Machine Learning

Linear algebra - Week 3

Vectors

Matrices

Dot product

Matrix multiplication

Linear transformations

week 03

Generative Adversarial Neural Networks

→ image generation { deepfakes }

→ text to image

Vector Algebra

$$2a + 4b + c = 28$$

$$\begin{bmatrix} 2 & 4 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = 28$$

Vector Algebra

System of equations

$$a+b+c=10$$
$$a+2b+c=15$$
$$a+b+2c=12$$

Matrix product

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 10 \\ 15 \\ 12 \end{bmatrix}$$

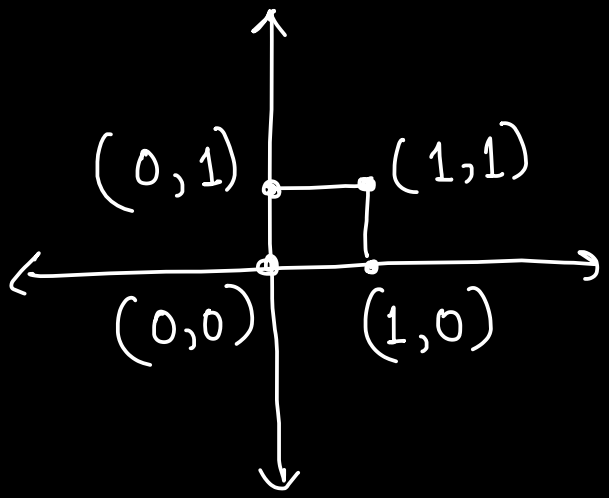
matrix vector

Vector Algebra

$$\begin{bmatrix} -1 & 5 & 2 \end{bmatrix} \begin{bmatrix} -3 \\ 6 \\ -4 \end{bmatrix} = 3 + 30 - 8$$
$$= 25$$

Linear Transformations

Matrices as linear transformation → change of coordinates



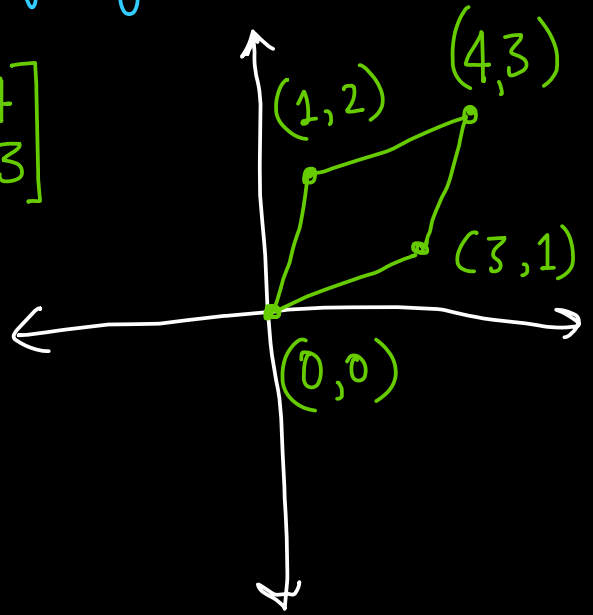
$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$$

$(0,0) \to (0,0)$

$(1,0) \to (3,1)$

$(0,1) \to (1,2)$

$(1,1) \to (4,3)$



Linear Transformations

Inverse

$$\begin{bmatrix} 3 & 1 \\ 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} a \\ c \end{bmatrix} = 1$$

$$\begin{bmatrix} 3 & 1 \end{bmatrix} \cdot \begin{bmatrix} b \\ d \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} a \\ c \end{bmatrix} = 0$$

$$\begin{bmatrix} 1 & 2 \end{bmatrix} \cdot \begin{bmatrix} b \\ d \end{bmatrix} = 1$$

$$\begin{aligned} 3a + c &= 1 \\ 3b + d &= 0 \\ a + 2c &= 0 \\ b + 2d &= 1 \end{aligned} \quad \Rightarrow \quad \begin{aligned} a &= 2/5 \\ b &= -1/5 \\ c &= -1/5 \\ d &= 3/5 \end{aligned}$$

Linear Transformations

$$\begin{bmatrix} 5 & 2 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} = 8 \begin{bmatrix} 1/4 & -1/4 \\ -1/8 & 5/8 \end{bmatrix}$$

$$\begin{aligned} 5a + 2c &= 1 \\ -\ a + 2c &= 0 \\ \hline 4a &= 1, \quad a = 1/4 \end{aligned}$$

$$\begin{aligned} 5b + 2d &= 0 \\ -\ b + 2d &= 1 \\ \hline 4b &= -1 \end{aligned}$$

$$b = -1/4$$

$$2c = -1/4$$

$$c = -1/8$$

$$2d = 1 + 1/4 = 5/4$$

$$d = 5/8$$

$$\Rightarrow \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$$

Linear Transformations

$$\begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\begin{array}{l|l} a+c=1 & b+d=0 \\ 2a+2c=0 & 2b+2d=1 \end{array}$$

Linear Transformations

Non singular Matrices always have an inverse

(invertible matrices)

$|M| \neq 0$

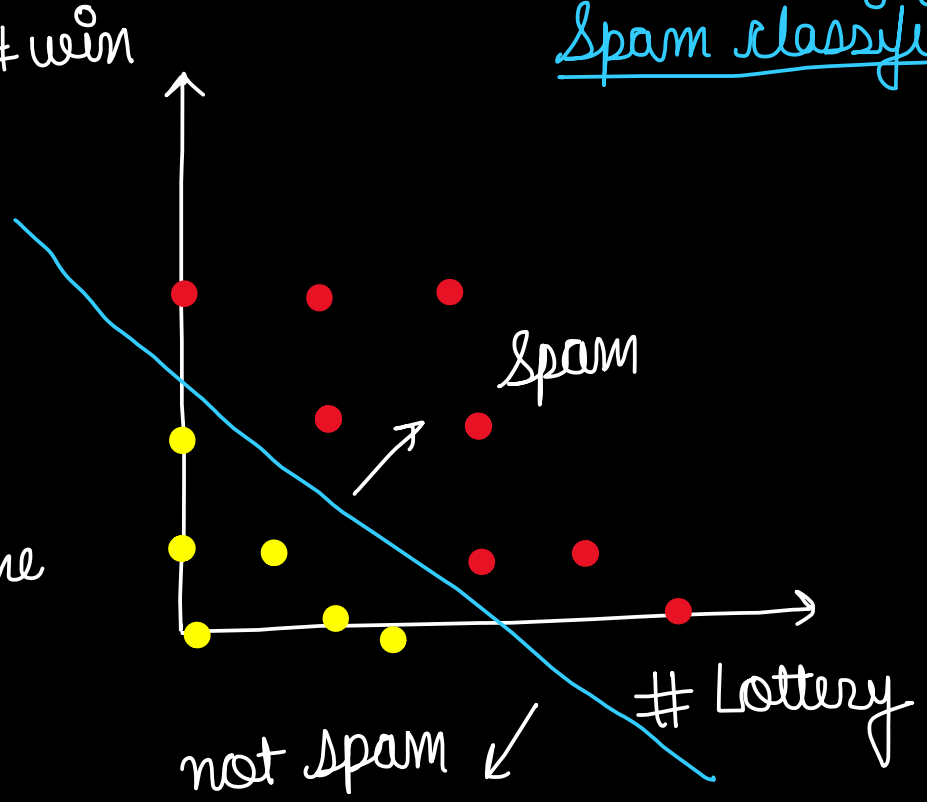
Singular Matrices do not have an inverse

(non-invertible matrices)

$|M| = 0$

Neural Networks and Matrices

Spam classifier



$(1 \cdot \text{win} + 1 \cdot \text{Lottery} = 1.5)$

equation of the line

Neural Networks and Matrices

Spam	Lottery	Win
Yes	1	1
Yes	2	1
No	0	0
Yes	0	2
No	0	1
No	1	0
Yes	2	2
Yes	2	0
Yes	1	2

Model
1
1

=

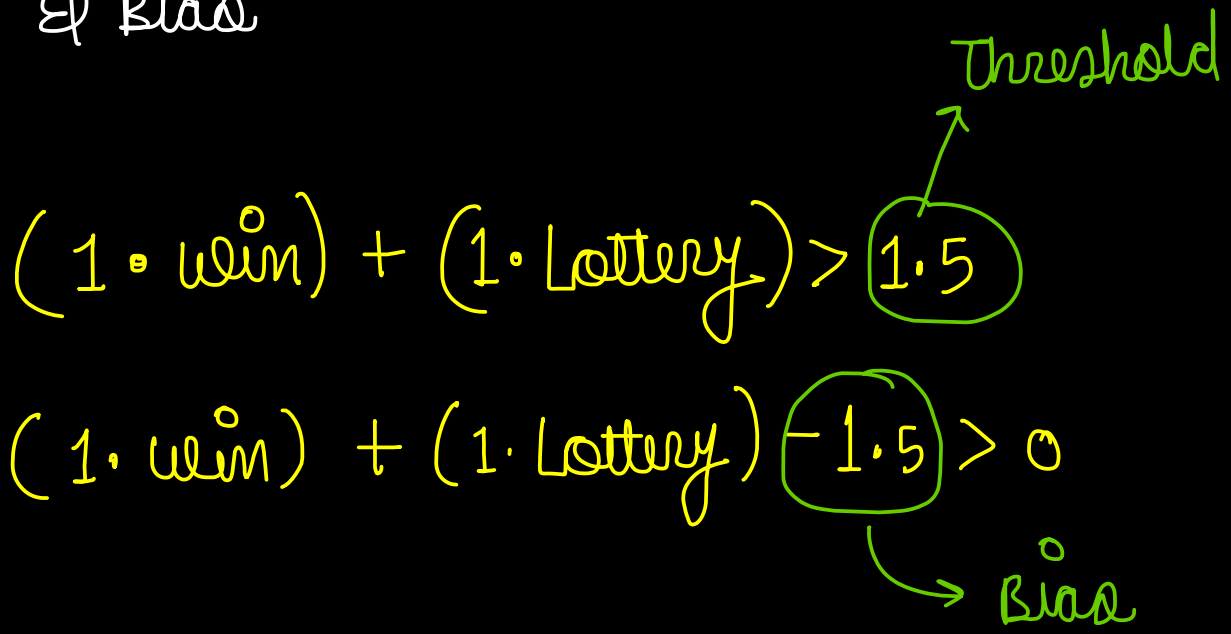
Prod
2
3
0
2
1
1
4
2
3

Check: >1.5?

Check
Yes
Yes
No
Yes
No
No
Yes
Yes
Yes

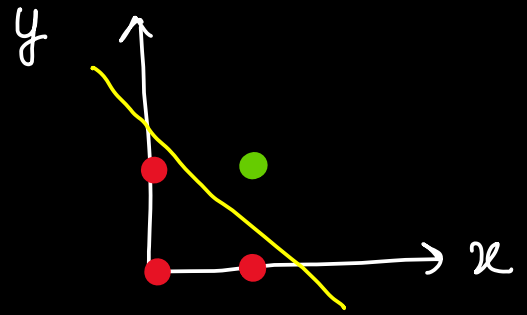
Neural Networks and Matrices

Threshold & Bias



Neural Networks and Matrices

AND operator perceptron



AND	x	y
NO	0	0
NO	0	1
NO	1	0
YES	1	1

Model

$$\cdot \begin{bmatrix} 1 \\ 1 \end{bmatrix} =$$

Dot product

$$\begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

check > 1.5

Threshold

$$\begin{bmatrix} \text{NO} \\ \text{NO} \\ \text{NO} \\ \text{YES} \end{bmatrix}$$

Neural Networks and Matrices