We use A Task structure $\{\text{stat, result}\}\$ to keep the status $(\text{running or done})\$ and the result $(\perp \text{ or val})\$ of a task and associate each task with a task identifier. The tasks are global variables can be created when a procedure is posted or asynchronously called. We mark the execution frames of tasks with the corresponding task identifiers. These task identifiers are used while creating, waiting for and returning a task.

CALL

$$\frac{l' \in e(g, l)}{\langle g, \langle l, S[call \ x \coloneqq p(e)] \rangle w, m \rangle \rightarrow \langle g, \langle l', s_n \rangle \langle l, S[x \coloneqq \star] \rangle w, m \rangle}$$

RETURN-CALL

$$\frac{v \in e(g,l)}{\langle g, \langle l, return \; e \rangle \langle l', S[x \coloneqq \star] \rangle w, m \rangle \; \rightarrow \langle g, \langle l', S[x \coloneqq v] \rangle w, m \rangle}$$

POST

$$\frac{l' \in e(g, l) \quad g' = g \cup \{t. stat = running, \ t. result = \bot\} \quad w' = \langle l', s_p \rangle^t}{\langle g, \langle l, S[post \ t \coloneqq p(e)] \rangle w, m \rangle \rightarrow \langle g', \langle l, S[skip] \rangle w, give(m, w') \rangle}$$

RETURN-POST

$$\frac{g' = g[t \to \{t. stat = done, t. result = e\}]}{\langle g, \langle l, return e \rangle^t, m \rangle \to \langle g', \varepsilon, m \rangle}$$

ASYNC

$$\frac{l' \in e(g, l) \quad g' = g \cup \{t.stat = running, \ t.result = \bot\}}{\langle g, \langle l, S[async \ t := p(e)]w \rangle, m \rangle \rightarrow \langle g', \langle l', s_p \rangle^t \langle l, S[skip]w \rangle, m \rangle}$$

RETURN-ASYNC

$$\frac{l' \in e(g,l) \quad g' = g[t \to \{t.stat = done, t.result = e\}]}{\langle g, \langle l, return e \rangle^t \langle l', w \rangle, m \rangle \to \langle g', \langle l', w \rangle, m \rangle}$$

POST-WAIT-CONTINUE

$$\frac{l' \in e(g,l) \quad \{t.stat = done, t.result = v\} \in g}{\langle g, \langle l, S[x \coloneqq wait \ t]w \rangle^{t'}, m \rangle \rightarrow \langle g, \langle l, S[x \coloneqq v]w \rangle^{t'}, m \rangle}$$

ASYNC-WAIT-CONTINUE

$$\frac{l' \in e(g,l)}{\langle g, \langle l, S[x \coloneqq wait \ t]w \rangle^{t\prime} \langle l', w' \rangle^{t\prime\prime}, m \rangle \rightarrow \langle g, \langle l, S[x \coloneqq v]w \rangle^{t\prime} \langle l', w' \rangle^{t\prime\prime}, m \rangle}{\langle g, \langle l, S[x \coloneqq v]w \rangle^{t\prime} \langle l', w' \rangle^{t\prime\prime}, m \rangle}$$

ASYNC-WAIT (async call returns to its caller)

$$\frac{l' \in e(g, l) \quad \{t. stat = running, \ t. result = \bot\} \in g \quad w'' = \langle l, S[x \coloneqq wait \ t]w \rangle^{t'}}{\langle g, \langle l, S[x \coloneqq wait \ t]w \rangle^{t'} \langle l', w' \rangle^{t''}, m \rangle \rightarrow \langle g', \langle l', w' \rangle^{t''}, give(m, w'')}}$$

POST-WAIT (no caller, dispatch a new task)

$$\frac{\{t. stat = running, \ t. result = \bot\} \in g \qquad w' = \langle l, S[x \coloneqq wait \ t]w\rangle^{t'}}{\langle g, \langle l, S[x \coloneqq wait \ t]w\rangle^{t'}, m\rangle \rightarrow \langle g, \varepsilon, give(m, w')\rangle}$$

How to introduce delays?

An idea:

I assume that a task waiting for a delayed task also delays. Otherwise, I will wait in the same statement even if it is scheduled (ASYNC-WAIT or POST-WAIT). It should execute the transition in DELAY in order to be able to continue its execution.
(A scheduler executes the non-delaying tasks first. When all tasks are completed/delayed, it takes and executes the tasks in delayed buffer)

DELAY $\frac{\langle m', w \rangle \in take(m, g)}{\langle g, \varepsilon, m \rangle \rightarrow \langle g, \varepsilon, delay(m', w) \rangle}$