HUST

ĐẠI HỌC BÁCH KHOA HÀ NỘI

HANOI UNIVERSITY OF SCIENCE AND TECHNOLOGY

ONE LOVE. ONE FUTURE.

Applied Algorithm Lab

Range minimum query

- Calculate the sum of the minimal numbers in some intervals of a given sequence
- Given a sequence of n integers $a_0, \ldots, a_{n-1}$.

  Denote rmq(i, j): the minimum of $a_i, a_{i+1}, \ldots, a_j$.

  Given m pairs $(i_1, j_1), \ldots, (i_m, j_m)$

  Compute the sum: $Q = rmq(i_1, j_1) + \ldots + rmq(i_m, j_m)$
- Input: the sequence and the intervals
- Output: the sum

- Example

stdin	stdout
16 2 4 6 1 6 8 7 3 3 5 8 9 1 2 6 4 4 1 5 0 9 1 15 6 10	6 Explain: 1+1+1+3 = 6

- Idea to solve #1:
  - for each pair of (i, j), find the minimum number in the interval
  - sum over all pair

  -> easy, but too many comparisons, high complexity as the number of pairs goes higher

- Idea to solve #2: pre-processing and store some minimum for less-complicated comparisons for answers (faster lookups)
  - Build a table $M[N][\log_2 N]$ that:

$$M[i][t] \text{ is the index of } \min\{a_i, a_{i+1}, \dots, a_{i+2^t-1}\}$$

Then we can answer faster:

$$rmq(i,j) = \min\left(\mathrm{a}_{M[i][k]}, \mathrm{a}_{M[j-2^k+1][k]}\right)$$

with $k = \lfloor \log_2(j - i + 1) \rfloor$

$i + 2^k - 1 \geq j - 2^k$

$\min\{a_i, a_{i+1}, \dots, a_{i+2^k-1}\}$

$\min\{a_{j-2^k+1}, \dots, a_j\}$

- Idea to solve #2: after determine M, it only takes O(1) for each rmq query, compare to O(n) as in idea #1
- To build M: use recursion
  - Base case: $M[\mathrm{i}][0] = i$, storing the index of each element.
  - Recursive case: Uses previous results to compute larger ranges:

$$M[i][j]\text{: the index of } \min(a_{M[i][j-1]}, a_{M[i+2^{j-1}][j-1]})$$

$\min\{a_i, a_{i+1}, \dots, a_{i+2^{j-1}-1}\}$

$\min\{a_{i+2^{j-1}}, \dots, a_{i+2^j-1}\}$

- Complexity: O(n log n)

HUST

hust.edu.vn fb.com/dhbkhn

THANK YOU !