DR. JOE'S SHORT TRIGONOMETRY GUIDE (rev. 6)
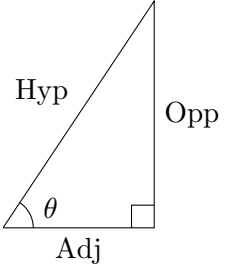
RIGHT TRIANGLES
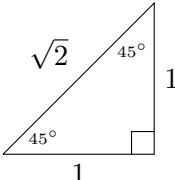

$$\sin(\theta) = \frac{\text{Opp}}{\text{Hyp}}$$

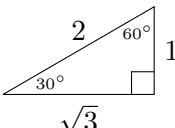$$\cos(\theta) = \frac{\text{Adj}}{\text{Hyp}}$$
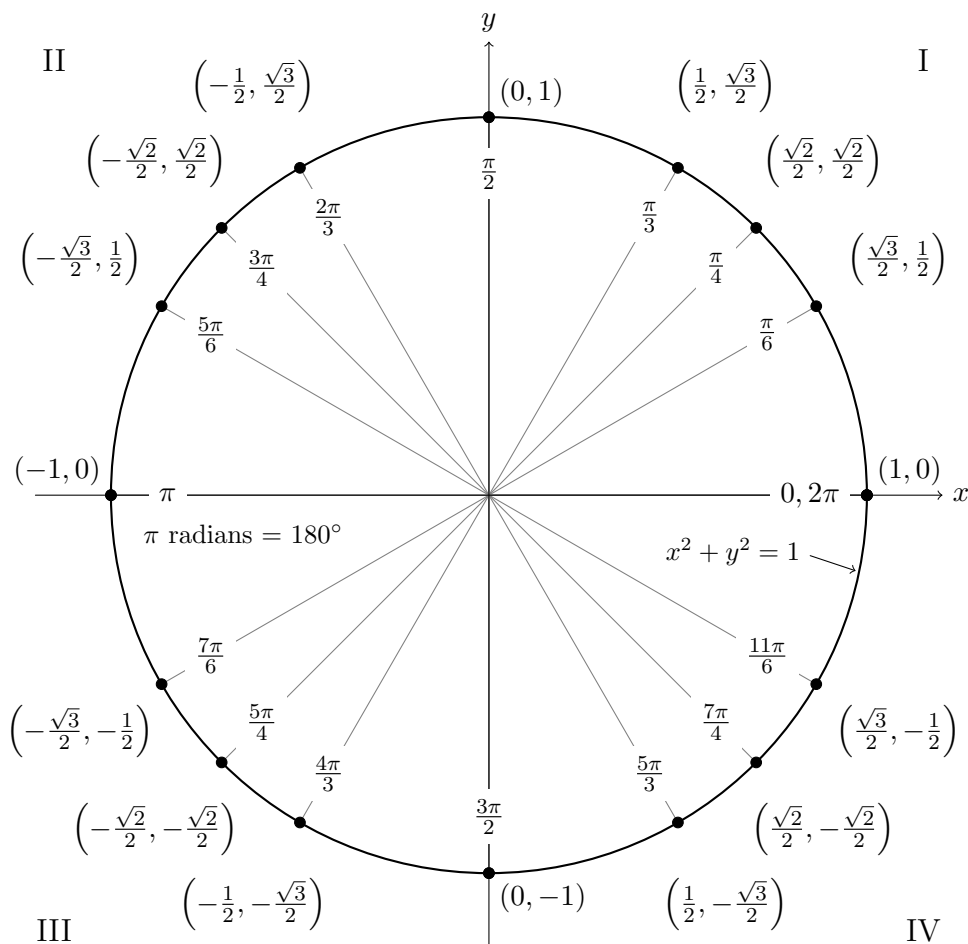
$$\tan(\theta) = \frac{\text{Opp}}{\text{Adj}}$$

Unit circle labels:

II   I

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ $(0,1)$ $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\left(-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$ $\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\frac{2\pi}{3}$ $\frac{\pi}{2}$ $\frac{\pi}{3}$

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ $\frac{3\pi}{4}$ $\frac{\pi}{4}$ $\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

$\frac{5\pi}{6}$ $\frac{\pi}{6}$

$(-1,0)$ π $0, 2\pi$ $(1,0)$

$\pi \text{ radians} = 180°$

$x^2 + y^2 = 1$

$\frac{7\pi}{6}$ $\frac{11\pi}{6}$

$\left(-\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$ $\frac{5\pi}{4}$ $\frac{7\pi}{4}$ $\left(\frac{\sqrt{3}}{2}, -\frac{1}{2}\right)$

$\frac{4\pi}{3}$ $\frac{3\pi}{2}$ $\frac{5\pi}{3}$

$\left(-\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$ $\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$ $(0,-1)$ $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

III   IV

BASIC DEFINITIONS AND IDENTITIES

If the angle θ is in standard position and (x, y) is the intersection of the terminal side and the **unit circle**:

$$\cos(\theta) = x \qquad \sec(\theta) = \frac{1}{x} = \frac{1}{\cos(\theta)}$$

$$\sin(\theta) = y \qquad \csc(\theta) = \frac{1}{y} = \frac{1}{\sin(\theta)}$$

$$\tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta) = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$$

For the radius-r circle:
$$\cos(\theta) = \frac{x}{r}, \ \sin(\theta) = \frac{y}{r}, \ \tan(\theta) = \frac{y}{x}, \text{ etc.}$$

INVERSE TRIG. FUNCTIONS

$$\arccos(u) = \theta \iff \begin{cases} \cos(\theta) = u \\ \text{and } 0 \le \theta \le \pi \end{cases}$$

$$\arcsin(u) = \theta \iff \begin{cases} \sin(\theta) = u \\ \text{and } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \end{cases}$$

$$\arctan(u) = \theta \iff \begin{cases} \tan(\theta) = u \\ \text{and } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \end{cases}$$

OTHER OBVIOUS IDENTITIES

$$\cos^2(\theta) + \sin^2(\theta) = 1$$
$$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$$
$$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$$
$$\cos(-\theta) = \cos(\theta)$$
$$\sin(-\theta) = -\sin(\theta)$$

The Pythagorean Identity and Consequences

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\quad 1 + \tan^2(\theta) = \sec^2(\theta) \qquad$ [Divide by $\cos^2(\theta)$]

$\quad \cot^2(\theta) + 1 = \csc^2(\theta) \qquad$ [Divide by $\sin^2(\theta)$]

Angle Addition and Consequences (when combined with Pythagorean Identity)

$\left. \begin{array}{l} \cos(A + B) = \cos(A)\cos(B) - \sin(A)\sin(B) \\ \sin(A + B) = \sin(A)\cos(B) + \cos(A)\sin(B) \end{array} \right\}$ [angle addition]

$\sin(2\theta) = 2\sin(\theta)\cos(\theta) \quad$ [sine double-angle]

$\left. \begin{array}{l} \cos^2(\theta) = \dfrac{1 + \cos(2\theta)}{2} \\[4mm] \sin^2(\theta) = \dfrac{1 - \cos(2\theta)}{2} \end{array} \right\}$ [cosine double angle / power reduction / half-angle]

Law of Sines and Law of Cosines (for all triangles)

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}, \quad c^2 = a^2 + b^2 - 2ab\cos(C)$$

Other Notes and Hints

* All Students Take Calculus / SOH CAH TOA.
* Memorize the first quadrant and use reference angles (acute angle measured to x-axis).
* Use $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ instead of tangent identities to reduce memorization.
* Cosine and Secant are even functions, the other four trig. functions are odd.
* Co-functions: $\text{co}\boxed{\text{sin/sec/tan}}\left(\frac{\pi}{2} - \theta\right) = \boxed{\text{sin/sec/tan}}(\theta)$, and $\boxed{\text{sin/sec/tan}}\left(\frac{\pi}{2} - \theta\right) = \text{co}\boxed{\text{sin/sec/tan}}(\theta)$.

Basic Graphs to Know

$y = \sin(x)$

$y = \cos(x)$

$y = \tan(x)$

period π unlike sine/cosine