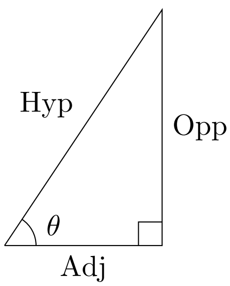
Dr. Joe's Short Trigonometry Guide (rev. 7)

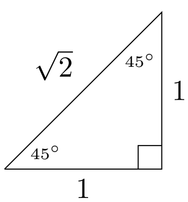
Right Triangles

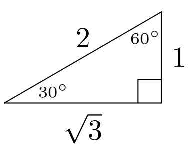


$\sin(\theta) = \frac{\text{Opp}}{\text{Hyp}}$

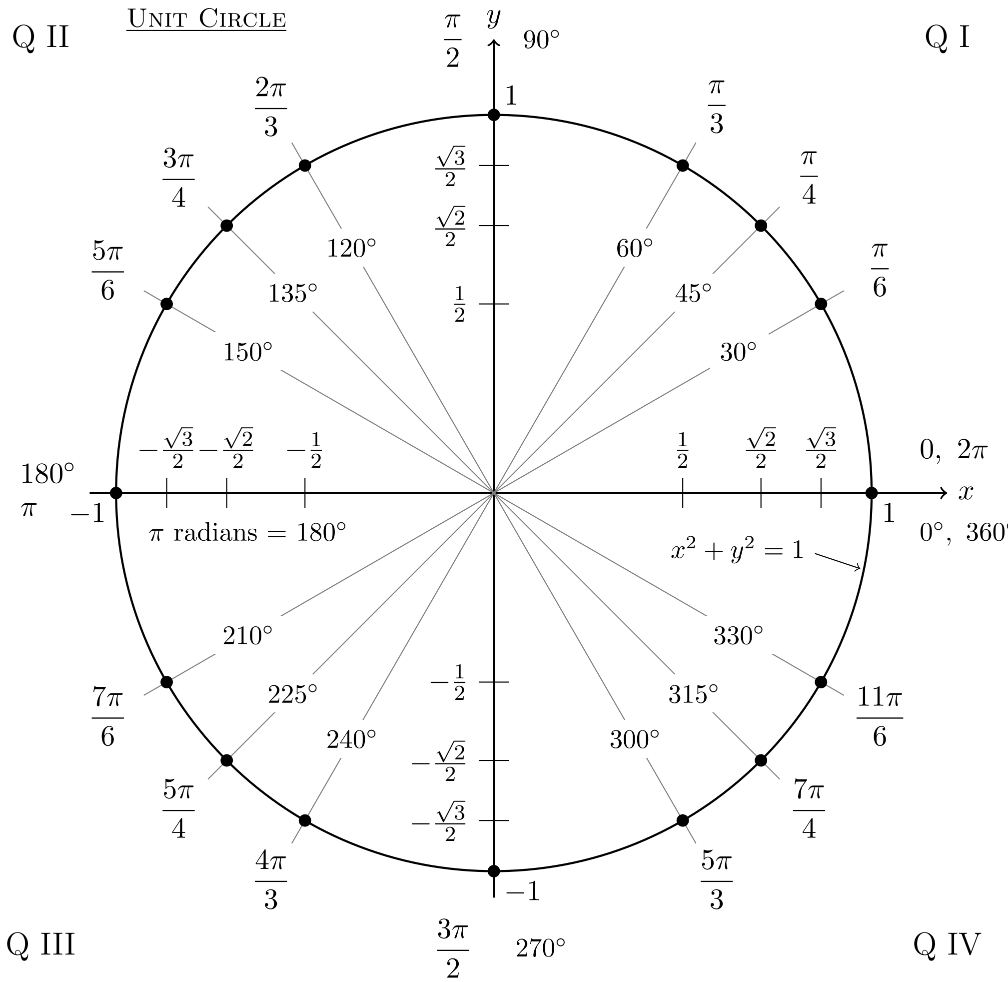
$\cos(\theta) = \frac{\text{Adj}}{\text{Hyp}}$

$\tan(\theta) = \frac{\text{Opp}}{\text{Adj}}$





Unit Circle



Basic Definitions and Identities

If the angle θ is in standard position and (x, y) is the intersection of the terminal side and the **unit circle**:

$\cos(\theta) = x$	$\sec(\theta) = \frac{1}{x} = \frac{1}{\cos(\theta)}$
$\sin(\theta) = y$	$\csc(\theta) = \frac{1}{y} = \frac{1}{\sin(\theta)}$
$\tan(\theta) = \frac{y}{x} = \frac{\sin(\theta)}{\cos(\theta)}$	$\cot(\theta) = \frac{x}{y} = \frac{\cos(\theta)}{\sin(\theta)}$

For the radius-r circle:
$\cos(\theta) = \frac{x}{r}$, $\sin(\theta) = \frac{y}{r}$, $\tan(\theta) = \frac{y}{x}$, etc.

Inverse Trig. Functions

$$\arccos(u) = \theta \iff \begin{cases} \cos(\theta) = u \\ \text{and } 0 \le \theta \le \pi \end{cases}$$

$$\arcsin(u) = \theta \iff \begin{cases} \sin(\theta) = u \\ \text{and } -\frac{\pi}{2} \le \theta \le \frac{\pi}{2} \end{cases}$$

$$\arctan(u) = \theta \iff \begin{cases} \tan(\theta) = u \\ \text{and } -\frac{\pi}{2} < \theta < \frac{\pi}{2} \end{cases}$$

Other Obvious Identities

$\cos^2(\theta) + \sin^2(\theta) = 1$

$\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$

$\sin(\theta) = \cos\left(\frac{\pi}{2} - \theta\right)$

$\cos(-\theta) = \cos(\theta)$

$\sin(-\theta) = -\sin(\theta)$

The Pythagorean Identity and Consequences

$\cos^2(\theta) + \sin^2(\theta) = 1$

$1 + \tan^2(\theta) = \sec^2(\theta)$ [Divide by $\cos^2(\theta)$]

$\cot^2(\theta) + 1 = \csc^2(\theta)$ [Divide by $\sin^2(\theta)$]

Angle Addition and Consequences (when combined with Pythagorean Identity)

$$\left.\begin{aligned} \cos(A+B) &= \cos(A)\cos(B) - \sin(A)\sin(B) \\ \sin(A+B) &= \sin(A)\cos(B) + \cos(A)\sin(B) \end{aligned}\right\} \text{[angle addition]}$$

$$\left.\begin{aligned} \cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ \sin(2\theta) &= 2\sin(\theta)\cos(\theta) \end{aligned}\right\} \text{[double angle]}$$

$$\left.\begin{aligned} \cos^2(\theta) &= \frac{1+\cos(2\theta)}{2} \\ \sin^2(\theta) &= \frac{1-\cos(2\theta)}{2} \end{aligned}\right\} \text{[cosine double angle / power reduction / half-angle]}$$

Law of Sines and Law of Cosines (for all triangles)

$$\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}, \qquad c^2 = a^2 + b^2 - 2ab\cos(C)$$

Other Notes and Hints

* All Students Take Calculus / SOH CAH TOA.
* Memorize the first quadrant and use reference angles (acute angle measured to x-axis).
* Use $\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$ instead of tangent identities to reduce memorization.
* Cosine and Secant are even functions, the other four trig. functions are odd.
* Co-functions: $\text{co}\boxed{\text{sin/sec/tan}}\left(\frac{\pi}{2} - \theta\right) = \boxed{\text{sin/sec/tan}}(\theta)$, and $\boxed{\text{sin/sec/tan}}\left(\frac{\pi}{2} - \theta\right) = \text{co}\boxed{\text{sin/sec/tan}}(\theta)$.

Basic Graphs to Know