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Intro To Algorithms

Drill #2

Ordering Running Times

- Doubling size of input ($n = old\ input\ size,$
  $2n = doubled\ input\ size$)
    - $n^2 \rightarrow slower\ by\ a\ factor\ of\ 4$
        - $(2n)^2 = 4 \times n^2$
    - $n^3 \rightarrow slower\ by\ a\ factor\ of\ 8$
        - $(2n)^3 = 8 \times n^3$
    - $100n^2 \rightarrow\ slower\ by\ a\ factor\ of\ 4$
        - $100(2n)^2 = 4 \times 100n^2$
    - $n\log(n) \rightarrow slower\ by\ a\ factor\ of\ 2$
        - $(2n)\log(2n) = (2n)[\log(n) + \log(2)]$
          $= 2 \times n\log(n) + 2n \qquad (assuming\ \log_2\quad)$
    - $2^n \rightarrow slower\ by\ a\ factor\ of\ 2^n$
        - $2^{2n} = 2^n \times 2^n$
- Increase input by one ($n = old\ input\ size,$
  $n + 1 = new\ input\ size$)
    - $n^2 \rightarrow slower\ by\ a\ factor\ of\ 1$
        - $(n+1)^2 = n^2 + (2n + 1)$
    - $n^3 \rightarrow slower\ by\ a\ factor\ of\ 1$
        - $(n+1)^3 = n^3 + (3n^2 + 3n + 1)$
    - $100n^2 \rightarrow\ slower\ by\ a\ factor\ of\ 1$
        - $100(n+1)^2 = 100n^2 + (200n + 100)$

- $n\log(n) \rightarrow slower\ by\ a\ factor\ of\ 1$
    - $(n+1)\log(n+1) \cong n\log(n) + \log(n)$
- $2^n \rightarrow slower\ by\ a\ factor\ of\ 2$
    - $2^{n+1} = 2 \times 2^n$

Really Understanding Order-Of-Growth

- General Approach
    - Set running time equal to 3600×10^{10} (# of seconds in an hour times # of operations per second), solve for n. Solution is $\lfloor n \rfloor$.
- $n^2 \rightarrow\ n = 6000000$
    - $n^2 = 3.6 \times 10^{13},\ n = \sqrt{3.6 \times 10^{13}} = 6000000$
- $n^3 \rightarrow\ n = 33019$
    - $n^3 = 3.6 \times 10^{13},$
      $n = \sqrt[3]{3.6 \times 10^{13}} = 33019.2724889$
- $100n^2 \rightarrow\ n = 600000$
    - $100n^2 = 3.6 \times 10^{13},\ n^2 = 3.6 \times 10^{11},$
      $n = \sqrt{3.6 \times 10^{11}} = 600000$
- $n\log(n) \rightarrow\ n = 2.8891 \times 10^{12}$
    - $n\log(n) = 3.6 \times 10^{13}, n \cong 2.8891 \times 10^{12}$ ($\log_{10}$)
- $2^n \rightarrow n = 45$
    - $2^n = 3.6 \times 10^{13}, n = \log_2 3.6 \times 10^{13} \cong 45.033$
- $2^{2^n} \rightarrow n = 5$
    - $2^{2^n} = 3.6 \times 10^{13},\ n = \log_2(\log_2 3.6 \times 10^{13}) \cong 5.5$