Program 'balaam' Wednesday 30th October, 2024

Title Fit, simulate, and diagnose auto-logistic actor attribute

 $\qquad \qquad \mathrm{models}.$

sna, network, coda, mvtnorm, MASS ${\bf Imports}$

https://github.com/johankoskinen/ALAAM \mathbf{URL}

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 $\verb"estimate.alaam" \{ \verb"balaam" . R \}$

Estimate Auto-Logistic Actor Attribute Model (ALAAM)

Description

ALAA

estimate.alaam is used for Bayesian estimation of the Auto-Logistic Actor Attribute (ALAAM). The outcome variable is a binary vector \boldsymbol{y} for vertices $V = \{1, \dots, n\}$, a matrix for exogenous covariates \boldsymbol{W} , and a binary adjacency matrix \boldsymbol{X} , defined on the network G(V, E), where $E \subset V^{(2)}$ or $E \subset V^{(2)}$, depending on whether the ties are directed or not, respectively. estimate.alaam is a wrapper for the function BayesALAAM.formula.

Arguments

formula An object of the class formula (see alaam.terms for details

of admissable arguments)

data An object of the class data.frame with dimensions $n \times q$. An object of the class matrix. This matrix needs to be

binary, of dimensions $n \times n$, and have a zero diagonal

Iterations The total number of iterations in the MCMC estimation prevBayes The estimation object from a previous call to

estimate.alaam. With other arguments as specified by default, the estimation will continue the previous

estimation

recalibrate If prevBayes is provided, recalibrate set to TRUE will use

the parameter draws in prevBayes\$Thetas to estimate $\Sigma = cov(\theta \mid Data)$, and set the proposal variance covariance to

 $\Sigma \times (\text{scaling}/\sqrt{p})$, if do.scaling is TRUE

silent If TRUE, some printing to console will be supressed

PropSigma If supplied, the proposal variance covariance matrix $\tilde{\Sigma}$ will be $\Sigma \times (\text{scaling}/\sqrt{p})$, if do.scaling is TRUE, and Σ if do.scaling is FALSE. In iteration t, given the current draw θ_{t-1} , a move to θ^* is proposed from $\theta^* \sim \mathcal{N}_p(\theta_{t-1}, \hat{\Sigma})$ scaling What scaling constant to use to inflate/deflate the variancecovariance matrix in the proposal of parameters in the MCMC do.scaling Scaling PropSigma or not Initial value of any contagion effects to accelerate converinitcontagion gence (will be depreciated) burnin To update θ_{t-1} to θ_t , a draw y^* is made from the likelihood defined by the proposed θ^* . The draw y^* is made using the Metropolis-Hastings (see 4.1 Simulating from the model, Koskinen and Daraganova, 2022) and the number of iterations discarded as burnin is given by burnin. missingCovs DepreciatedmissingPhi User-defined parameter to determine the amount of missing not at random (MNAR) bias for missing values of y (see 4.3) Missing data, Koskinen and Daraganova, 2022) For use of a prior distribution $\pi(\theta)$. Prior distributions are priorSigma limited to the form $\mathcal{N}_p(\boldsymbol{\mu}, \mathbf{S})$, where priorSigma is the $p \times p$ matrix S.priorMu For use of a prior distribution $\pi(\theta)$. Prior distributions are limited to the form $\mathcal{N}_p(\boldsymbol{\mu}, \mathbf{S})$, where priorMu is the $p \times 1$ vector μ . If neither priorSigma nor priorMu provided, an improper prior will be used. scalePrior A special $\mathcal{N}_p(\boldsymbol{\mu}, \mathbf{S})$ prior, where $\mathbf{S} = \text{scalePrior} \mathbf{J}$, where **J** is an information matrix under μ set to the MLE assuming independent observations (see 4.5.1 Prior distributions, Koskinen and Daraganova, 2022). A Boolean vector of the same length as y, indicating canchange whether y_i is fixed, canchange [i] set to FALSE, or variable canchange[i] set to TRUE **MPLE** Logical, indicating whether MCMC will be forced to evaluate the pseudo likelihood, $\tilde{p}(y \mid \theta) = \prod p(y_i \mid y_{-i}\theta)$. If no contagion effect is specified, MPLE will be set to TRUE automatically saveFreq Every saveFreq iteration, the current state of the algo-

rithm will be dumped into BayesALAAMdump.RData and if silent is FALSE, an update will be printed to the console. If not value on saveFreq is provided, it will be set

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to ceiling(Iterations/10)

missFreq How often should imputed missing values be stored initTheta Initial value of $\boldsymbol{\theta}$. This is ignored if prevBayes is provided thinning For iterations $i=1,\ldots,I,$ where I is given by Iterations, only values $\boldsymbol{\theta}_t$ are stored for $t=q+k,q+2k,q+2k,\ldots,q+2M,$ where q is par.burnin and k is thinning. Increasing k will reduce the autocorrelation of the MCMC chains par.burnin See thinning

Details

The algorithm implements the algorithms of Koskinen and Daraganova (2022). A typical model has the form response \sim terms where response is the numeric, binary response vector and terms is a series of admissible terms which specifies a linear predictor for response. A terms specification of the form first + second indicates all the terms in first together with all the terms in second with duplicates removed. The specification first*second indicates the cross (an interaction) of first and second.

The formula has an implied intercept term and will add a vector of constants (1). The intercept cannot be removed.

The function estimate.alaam calls the lower level function BayesALAAM.formula which, itself, supplants the earlier function BayesALAAM.

Formula terms, on the right-hand side of \sim , can be either (column) names of data, admissible network metrics based on adjacency, or contagion terms. For details see alaam.terms.

Value

estimate.alaam returns an 'object' of class estimate.alaam.obj. The functions write.res.table, plotPost, get.gof.distribution, post.deviance.alaam, and more, are defined on 'objects' of class estimate.alaam.obj.

An 'object' of class estimate.alaam.obj returns the following

Thetas A $M \times p$ matrix of posterior draws from the MCMC. Note that even if contagion is specified as 'none', the independent model will have a column of zeros in Thetas for the 'simple' contagion An 'object' of class alaam.obj. This is a version of data ALAAMobj formatted for the specific model specified. ResTab A table summarising the posteriors. priorSigma As defined in call to estimate. alaam, for use in further calls to estimate.alaam with prevBayes priorMu As defined in call to estimate. alaam, for use in further calls to estimate.alaam with prevBayes imputed.obs A matrix with imputed y if the original data had any missing values. silent As defined in call to estimate.alaam, for use in further calls to estimate.alaam with prevBayes As defined in call to estimate. alaam, for use in further calls do.scaling to estimate.alaam with prevBayes PropSigma As defined in call to estimate. alaam, for use in further calls to estimate.alaam with prevBayes MPLE As defined in call to estimate. alaam, for use in further calls to estimate.alaam with prevBayes As defined in call to estimate. alaam, for use in further calls saveFreq to estimate.alaam with prevBayes As defined in call to estimate.alaam, for use in further calls missFreq to estimate.alaam with prevBayes initTheta The last update in the MCMC to be used as a defined in call to estimate.alaam, for use in further calls to estimate.alaam with prevBayes The proportion of proposals θ^* that were accepted. acc.ratio

Examples

```
# Posterior summary:
plotPost(res.ind,showplot=TRUE)
# Simple contagion:
res.cont <- estimate.alaam(agree ~odegree+mood+sex+mood*sex+simple,
                                  my.dat,
                                   adjacency=adj,
                                  Iterations=1000)
# Posterior summary:
plotPost(res.cont,showplot=TRUE)
# Restart estimation with previous results
# and some thinnning and burnin (takes a little longer)
res.cont.2 <- estimate.alaam(agree ~odegree+mood+sex+mood*sex+simple,
                                  my.dat,
                                  adjacency=adj,
                                  Iterations=5000,
                                  prevBayes = res.cont,
                                  recalibrate = TRUE,
                                  par.burnin=50,
                                   thinning = 5)
```

alaam.terms{balaam.R}

Effects for ALAAM

Description

This entry describes and lists the formula objects that are permissible in a call to estimate.alaam.

Specifying models

The social influence model developed by Robins et al. (2001) and later elaborated by Daraganova (2009) and Daraganova and Robins (2013) and now referred to as the autologistic actor-attribute model (ALAAM), is a model for binary nodal attributes $\mathbf{y} = \{Y_i : 1 \le i \le n\}$, conditional on a network adjacency matrix $\mathbf{X} = \{X_{ij} : (i,j) \in V \times V\}$, and a matrix of covariates \mathbf{W}

$$p_{\boldsymbol{\theta}}(\boldsymbol{y}|\mathbf{X}, \mathbf{W}) = \exp\left\{\boldsymbol{\theta}^{\top} z(\boldsymbol{y}, \mathbf{X}, \mathbf{W}) - \psi(\boldsymbol{\theta}; \mathbf{X}, \mathbf{W})\right\},$$

for $\mathbf{y} \in \mathcal{Y} = \{0, 1\}^V$. Here $z(\mathbf{y}, \mathbf{X}, \mathbf{W})$ is a $p \times 1$ vector of statistics calculated for the dependent variable \mathbf{y} , the network \mathbf{X} , and covariates \mathbf{W} . The vector of statistics is defined in three main ways

Covariate effects

Covariate effects are specified in standard 1m syntax. For example, the term

$$\sum_{i=1}^{n} y_i w_{ij}$$

where the name of column j, for example mood, is used in standard fashion +mood on the right-hand side. For a model with variables agree, mood, and sex, where agree is the dependent variable,

agree ~ mood+sex

would specify the model defined by

$$z(\boldsymbol{y}, \mathbf{X}, \mathbf{W}) = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} y_i w_{i,mood} \\ \sum_{i=1}^{n} y_i w_{i,sex} \end{bmatrix}$$

To define and add an interaction effect between mood and sex,

agree ~ mood+sex+mood*sex

which would specify the model defined by

$$z(\boldsymbol{y}, \mathbf{X}, \mathbf{W}) = \begin{bmatrix} \sum_{i=1}^{n} y_i \\ \sum_{i=1}^{n} y_i w_{i,mood} \\ \sum_{i=1}^{n} y_i w_{i,sex} \\ \sum_{i=1}^{n} y_i w_{i,mood} w_{i,sex} \end{bmatrix}$$

Markov effects

If we allow outcome y_i to depend on tie-variables x_{uv} , for which $\{i\} \cap \{u,v\} \neq \emptyset$, the vector of statistics admit as statistics a number of network metrics. In the sequel, we refer to outcomes $y_i = 1$ as a success, and $y_i = 0$ as a failure. These effects are all of the dyadic form

$$z_u(\mathbf{y}, \mathbf{X}, \mathbf{W}) = \sum_{i=1}^n y_i u_i$$

In previous versions of the program, these statistics u_i were pre-calculated and added to **W**. For estimate.alaam, these are calculated internally if requested.

D.C.	T. 1	T
Effect name	Formula u_i	Interpretation
degree	$y_i x_{i\cdot} = y_i \sum_j x_{ij}$	For undirected networks, this measures the
		association of degree centrality and the
	_	probability of success
idegree	$y_i x_{\cdot i} = y_i \sum_j x_{ji}$	For directed networks, this measures the
		association of in-degree centrality and the
	-	probability of success
odegree	$y_i x_{i\cdot} = y_i \sum_j x_{ij}$	For directed networks, this measures the as-
		sociation of out-degree centrality and the
		probability of success
recipties	$y_i \sum_j x_{ij} x_{ji}$	For directed networks, this measures the as-
		sociation of out-degree centrality and the
	7 m N	probability of success
twostar	$y_iinom{x_{i\cdot}}{2}$	For undirected networks, the effect of cen-
		trality over and above degree
intwostar	$y_i {x \cdot i \choose 2}$	For directed networks, the effect of indegree
		centrality over and above indegree
outtwostar	$y_iinom{x_i\cdot}{2}$	For directed networks, the effect of outde-
		gree centrality over and above outdegree
threestars	$y_i {x_i \choose 3}$	For undirected networks, the effect of degree
		centrality over and above two stars
twopath	$y_i(x_{\cdot i}x_{i\cdot} - \sum_j x_{ij}x_{ji})$	For directed networks, the association of
	-	brokerage on the probability of success.
inthreestar	$y_i\binom{x_{\cdot i}}{3}$	For directed networks, the effect of indegree
	, ,	centrality over and above intwostars
outthreestar	$y_i\binom{x_i}{3}$	For directed networks, the effect of outde-
	(0)	gree centrality over and above outtwostars
transties	$y_i \sum_{j,k \neq i} x_{ij} x_{ik} x_{jk}$	For (directed) undirected networks, the ef-
		fect on probability of success of being em-
		bedded in (transitive) triads
indirties	$y_i \sum_j x_{ij} \sum_k (1 - x_{ik}) x_{jk}$	For (directed) undirected networks, the ef-
		fect on probability of success of having ties
		to people that have ties to many people you
		are not directly tied to (see 3.1.3 Indirect
		network and contagion dependencies, Kosk-
		inen and Daraganova, for details)
		, , ,

Contagion effects

Daraganova (2009) proposed a number of dependence assumptions for $\mathcal{Y} \times \mathcal{X}$ and derived the associated statistics (see Koskinen and Daraganova, 2022, for details and proofs). If $z(\boldsymbol{y}, \mathbf{X}, \mathbf{W})$ contain these terms, the responses of y_i and y_j may be conditionally dependent, conditional on \mathbf{X} and $\boldsymbol{y}_{-ij} = \{y_k : k \neq i, j\}$,

and consequently

$$p_{\theta}(y|\mathbf{X}, \mathbf{W}) \neq \prod_{i=1}^{n} \Pr(Y_i = y_i \mid \theta, \mathbf{X}, \mathbf{W}).$$

The most basic form of dependence, direct contagion (see 3.1.2 Network contagion dependence, Koskinen and Daraganova, 2022), yields a statistic z_{DC} , such that

$$\Pr(Y_i = \mid \boldsymbol{y}_{-i}, \mathbf{X}, \boldsymbol{\theta}) = \frac{\exp\{\theta_{DC} z_{DC}(\Delta^+ \boldsymbol{y}, \mathbf{X})\}}{e^{\theta_{DC} z_{DC}(\Delta^- \boldsymbol{y}, \mathbf{X})} + e^{\theta_{DC} z_{DC}(\Delta^+ \boldsymbol{y}, \mathbf{X})}} e^{\boldsymbol{\theta}_{-DC}^\top (z_{-DC}(\Delta^- \boldsymbol{y}, \mathbf{X}, \mathbf{W}) - z_{-DC}(\Delta^+ \boldsymbol{y}, \mathbf{X}, \mathbf{W}))}$$

where $\Delta^+ y$ is the vector y with element i set to 1, and $\Delta^- y$ is the vector y with element i set to 0, and

$$z_{DC}(\boldsymbol{y}, \mathbf{X}) = \sum_{i,j} y_i y_j x_{ij}.$$

Currently, the	contagion effects are	
Effect name	Formula u_i	Interpretaion
simple	$\sum_{i,j} y_i y_j x_{ij}$	is the probability of success increased
recip	$\sum_{i,j} y_i y_j x_{ij} x_{ji}$	by being connected to actors whose outcome is a success is the probability of success increased by being <i>mutually</i> tied to actors whose outcome is a success (directed networks
		only)
indirect	$\sum_{i} y_i \sum_{j} x_{ij} \sum_{k \neq i,j} y_k x_{jk}$	is the probability of success increased by being indirectly connected to actors whose outcome is a success (see 3.1.3
		*
		Indirect network and contagion depen-
		dencies, Koskinen and Daraganova, for details))
closedind	$\sum_{i} y_i \sum_{j} x_{ij} \sum_{k \neq i,j} y_k x_{ik} x_{jk}$	is the probability of success increased
		by being both indirectly and directly
		connected to actors whose outcome is
		a success (see 3.1.3 Indirect network
		and contagion dependencies as well as supplementary material, Koskinen and
		Daraganova, for details))
transitive	$\sum_{i} y_i \sum_{j} x_{ij} y_j \sum_{k \neq i,j} y_k x_{ik} x_{jk}$	is the probability of success increased
		by being embedded in triads where the
		two other members have success on
		the outcome(see 3.1.3 Indirect network and contagion dependencies as well as
		supplementary material, Koskinen and
		Daraganova, for details))
		Darasanova, for details))

Interaction of contagion effects and attributes

If you assume that the dependence, represented by a contagion statistic and parameter, is stronger (weaker), depending on a particular monadic covariate w_{ik} for the focal node i, the contagion effect can be interacted with w_{ik} . Interacting, for example, the (binary) variable **sex** with the direct contagion, the contributions will be

$$\theta_{DC} \sum_{i,j} y_i y_j x_{ij} + \theta_{DC,sex} \sum_{i,j} y_i w_{i,sex} y_j x_{ij}.$$

For $w_{i,sex} = 0$, the contagion parameter is thus

$$\theta_{DC}$$
,

and for $w_{i,sex} = 1$, the contagion parameter is

$$\theta_{DC} + \theta_{DC.sex}$$
.

This can be used to test, for example, if males are more susceptible to social influence than females. An interaction of a covariate with a contagion effect is done using the standard interaction syntax, e.g.

agree ~ mood+sex+simple+sex*simple

Restrictions

- There is no restriction on the number of covariates that can be interacted with a contagion statistic
- Covariate interactions can only be defined for one contagion statistic at a time
- If a covariate interaction with a contagion statistic is defined, the two main effects must be included

 ${\tt post.deviance.alaam}\{{\tt balaam.R}\}$

Calculate posterior deviance from Bayesian ALAAM

Description

post.deviance.alaam calculates the posterior distribution of the deviance

$$D(\boldsymbol{\theta}) = -2\ell(\boldsymbol{\theta}; Data)$$

for posterior draws $\theta_1, \theta_2, \dots, \theta_M$, obtained from estimate.alaam. This function calls the lower order function eval.like.path.alaam that estimates log ratio $\psi(\theta^*) - \psi(\theta)$ of normalising constants, for each value θ in the posterior ALAAMresult\$Thetas, thinned so that that the number of draws is num.outs.

estimation object from a previous call to

The

Arguments

ALAAMresult

пыштесьце	The estimation object from a previous can to								
	estimate.alaam								
numBridges	The number K of bridges used in the call to								
	eval.like.path.alaam, which uses the 'path sampler' to								
	calculate the log ratio $\psi(\boldsymbol{\theta}^*) - \psi(\boldsymbol{\theta})$ of normalising constants								
thinning.like	Passed to eval.like.path.alaam, where a sample								
	$\mathbf{y}_1, \dots, \mathbf{y}_M$ is drawn from $\mathbf{y} \sim p(\mathbf{y} \mid \boldsymbol{\theta}^{(j)})$, for $j = 0, \dots, K$,								
	and the thinning k determines how many iterations are dis-								
	carded between each sample point. Too small values mean								
	that the sample points will be highly correlated.								
sample.size	Passed to eval.like.path.alaam, where a sample								
	y_1, \ldots, y_M is drawn from $y \sim p(y \mid \boldsymbol{\theta}^{(j)})$, for $j = 0, \ldots, K$,								
	and the thinning k determines how many iterations are dis-								
	carded between each sample point. Too small values mean								
	that the sample points will be highly correlated.								
cov.sample.burnin	Passed to eval.like.path.alaam. The number of sample								
•	points that are discarded as burn in when drawing from the								
	likelihood y								
printFreqn	The print frequency								
mult.fact	A constant to set the thinning according to the formula of								
	Snijders (2002)(depreciated)								
num.outs	The number of posterior draws to evaluate $D(\boldsymbol{\theta})$ in								
	-								

Details

Value

The function post.deviance.alaam returns the approximate value of the deviance $D(\boldsymbol{\theta})$, for each $\boldsymbol{\theta}^{(b)}, \boldsymbol{\theta}^{(b+k)}, \boldsymbol{\theta}^{(b+2k)}, \dots, \boldsymbol{\theta}^{(M)}$, in the thinned sample with burnin iterations discarded ALAAMresult\$Thetas. The output, post.devs, can

be used to compare models graphically using plot.deviance.alaam (Aitkin et al., 2017), or by calculating the DIC with alaam.dic.

post.devs An $M \times 1$, where M is given by num.outs, matrix of posterior draws from the distribution $D(\theta)$.

alaam.dic{balaam.R}

Calculate Deviance Information Criterion from Bayesian ALAAM

Description

alaam.dic calculates the posterior distribution of the deviance

$$DIC = E[D(\boldsymbol{\theta})] + V(D(\boldsymbol{\theta}))/2$$

for posterior draws $D(\boldsymbol{\theta}_1), D(\boldsymbol{\theta}_2), \dots, D(\boldsymbol{\theta}_M)$ obtained from post.deviance.alaam. Models with *smaller DIC* should be preferred to models with *larger DIC*.

alaam.dic(Post.dev)

Arguments

Post.dev The object returned by a call to post.deviance.alaam

plot.deviance.alaam{balaam.R}

Plot posterior Deviance from Bayesian ALAAM

Description

plot.deviance.alaam plots the posterior deviance obtained from post.deviance.alaam for up to 4 different models. Models with *smaller* deviance should be preferred to models with *larger* deviance. For rules of thumb, see Aitkin et al. (2017).

Arguments

dev.1	The object returned by a call to post.deviance.alaam
dev.X	Up to 4 objects returned by a calls to post.deviance.alaam
	for different models. These will have to be entered hierar-
	chially. For example, you can use as arguments dev.1 and
	dev.2, but not dev.1 and dev.3 without dev.2.
colPal	The colours of the lines corresponding to the models entered
line.tps	Sets the lty of the lines corresponding to the models entered
mod.names	Sets the names corresponding to the models entered dis-
	played in the legend

Details

The function simply calculates the empirical cumulative distribution function using ecdf for the posterior deviances and returns the plot.

Value

The function plot.deviance.alaam returns a plot for the approximate cumulative distribution function for the posterior deviance returned by post.deviance.alaam. Since models with *smaller* deviance should be preferred to models with *larger* deviance, the relative fit of each model is reflected by their location horizontally. If there is no overlap between model 1 and model 2, and model 1 is to the left of model 2, then this is evidence that model 1 is preferred. See Aitkin et al. (2017) for a guide to interpretation.

References

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