Online Appendices for "Credit crunches from occasionally binding bank borrowing constraints"

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Abstract

This contains the online appendices for the paper: "Credit crunches from occasionally binding bank borrowing constraints" by Tom D. Holden, Paul Levine and Jonathan M. Swarbrick.

Appendix A Proofs

Proof 1 (Proof of Proposition 1) *Substituting equation* (14) *into* (13) *gives*

$$\lambda_t^D = 1 - \lambda_t^E - (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\left(1-\kappa_t\right)\mathcal{N}_{1,t+1}\right] - \left(1-\kappa_t\right). \tag{1}$$

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Suppose that $\lambda_t^E > 0$. Then $E_t = 0$ by complementary slackness, so, from the definition of κ_t, the previous equation becomes:

$$\lambda_t^D + \lambda_t^E + (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{1,t+1}\right] = 0, \tag{2}$$

and so $\lambda_t^D = \lambda_t^E = (1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{1,t+1}\right] = 0$ giving the required contradiction. □

Proof 2 (Proof of Proposition 2) *Substituting $\mathcal{H}_t = 1$ into equation (11) leads to:*

$$\mathcal{M}_t = \frac{\left(1-\lambda_t^B\right)(1-\kappa_t)(1-(1-\iota)(1-\theta))}{(1-\kappa_t)(1-(1-\iota)(1-\theta)) - \lambda_t^B} \tag{3}$$

Since $0 \leq (1-\kappa_t)(1-(1-\iota)(1-\theta)) < 1$, it follows that $\mathcal{M}_t = 1$ if and only if $\lambda_t^B = 0$. Given that $\mathcal{M}_t \geq 1$ as a bank can always sell itself to another bank for $\hat{V}_t$, independent of its history of dividend payments, this also implies that $\mathcal{M}_t > 1$ if and only if $\lambda_t^B > 0$. □

Proof 3 (Proof of Proposition 3) *Using equation (8), we have that the steady-state value of λ_t^B is given by:*

$$\lambda^B = \iota(1-\kappa)(1-(1-\iota)(1-\theta)) \in (0,1), \tag{4}$$

where throughout this document, values without time subscripts will refer to steady-states. This implies that the borrowing constraint binds with positive ι but $\lim_{\iota\to 0^+}\lambda^B = 0$. As λ^B is the Lagrange multiplier on the borrowing constraint, the claim follows. □

Proof 4 (Proof of Corollaries 1 and 2) *The results for $\mathcal{M}$ in Corollaries 1 and 2 follow immediately from Proposition 2. Indeed, from equation (3), we find:*

$$\mathcal{M} = \frac{1-\iota(1-\kappa)\left[1-(1-\iota)(1-\theta)\right]}{1-\iota} > 1 \tag{5}$$

and so in the limit as $\iota \to 0^+$, we have $\mathcal{M} \to 1$. The same is true for $\mathcal{N}_i$ as:

$$\mathcal{N}_i = \mathcal{Z}_i \frac{(1-\iota)(1-\theta)}{1-(1-\iota)(1-\theta)} R^i, \quad i = 1, \cdots, \tau - 1, \tag{6}$$

where:

$$\mathcal{Z}_i \equiv \frac{1-(1-\iota)^{\tau-i}}{\iota}\frac{\lambda^B}{1-\lambda^B}\frac{\mathcal{M}}{1-\kappa} > 0, \quad i = 1, \cdots, \tau - 1. \tag{7}$$

So $\mathcal{N}_i > 0$ for $i = 1, \cdots, \tau-1$, but as $\iota \to 0$, $\mathcal{Z}_i \to 0$ and $\mathcal{N}_i \to 0$. □

Proof 5 (Proof of Corollary 3) *The value of the bank is given by:*

$$V = \mathcal{M}\hat{V} + \sum_{i=1}^{\tau-1} \mathcal{N}_i D. \tag{8}$$

Hence, the value of a bank is always greater than its book value for $\iota > 0$, but $\lim_{\iota \to 0^+} V = \hat{V}$.

Now, banks must pay dividends in steady state, at least with $\iota > 0$, for, suppose they did not. Then, their steady-state value would be zero, by the definition of bank value, and so since book value is always weakly below value, their steady-state book value would be non-positive. However, since equity issuance is always strictly positive with $\iota > 0$, steady-state book-value would be infinite without dividend payments, giving the required contradiction. Consequently:

$$\lambda^D = \kappa - (1-\iota)\frac{\beta}{\Pi^*}\mathcal{N}_1 (1-\kappa) = 0, \tag{9}$$

so:

$$\kappa = \frac{(1-\iota)\frac{\beta}{\Pi^*}\mathcal{N}^1}{1 + (1-\iota)\frac{\beta}{\Pi^*}\mathcal{N}_1} > 0. \tag{10}$$

It follows from $\lim_{\iota \to 0^+} \mathcal{N}_1 = 0$, that $\lim_{\iota \to 0^+} \kappa = 0$ and so there is no equity issuance in the limit. □

Proof 6 (Proof of Corollary 4) *Note:*

$$R = (1 - \iota(1-\kappa)[1 - (1-\iota)(1-\theta)]) R^K, \tag{11}$$

so $R^K > R$ but $\lim_{\iota \to 0^+} R^K = R$. □

Proof 7 (Proof of Proposition 4) *First suppose that $\kappa = 0$. In this case, the first order condition with respect to dividend payments becomes:*

$$\lambda_t^D = -(1-\iota)\mathbb{E}_t\left[\Lambda_{t,t+1}\mathcal{N}_{1,t+1}\right]. \tag{12}$$

Now, it follows from the definition of $\mathcal{N}_{1,t}$ in equations (9) to (10), that $\mathcal{N}_{1,t} \geq 0$ for all t, since $\lambda_t^B \in [0,1]$, by equation (12). Hence, since $\lambda_t^D \geq 0$, equation (12) in fact implies that $\lambda_t^D = \mathcal{N}_{1,t} = 0$ for all t. Consequently, again by equations (9) to (10), we must also have that $\lambda_t^B = 0$ for all t, which in turn implies that $\mathcal{M}_t = 1$ for all t, by Proposition 2. Using this in the definitions of the pricing kernels for bank and firm equity, we find *that when $\iota = 0$ as well, $\Lambda_{t,t+1} = \Xi_{t,t+1}$ for all t, so financial intermediation is efficient. The bank is never financially constrained as they can always raise equity finance at no cost.*

Next, suppose that $\theta = 0$. Recall the borrowing constraint is of the form:

$$B_t \leq \mathcal{A}\hat{V}_t + \mathcal{F}_{1,t}D_{t-1} + \cdots + \mathcal{F}_{\tau-1,t}D_{t-\tau+1}. \tag{13}$$

If $\theta = 0$, then as $\iota \to 0$, it follows from the solutions of the coefficients in equations (6) and (7), that $\mathcal{A}_t \to \infty$ and $\mathcal{F}_{i,t} \to \infty$ for $i = 1, ..., \tau - 1$. So in the limit as $\iota \to 0$, borrowing becomes unlimited. As in the previous case, it follows that for all t, $\lambda_t^B = 0$, $\mathcal{M}_t = 1$ and $\Lambda_{t,t+1} = \Xi_{t,t+1}$ if $\iota = 0$, and so financial intermediation is efficient.

Proof 8 (Proof of Proposition 5) *As $\nu \to \infty$, the marginal issuance cost $\partial\kappa_t/\partial E_t \to \infty$, and so $E_t, \kappa_t = 0$. $\tau = 0$ implies that creditors cannot reclaim previous dividend payments and the terms on D_{t-i} in the borrowing constraint and value function, $\mathcal{F}_{i,t}, \mathcal{N}_{i,t}$ are dropped. Providing $1 - \sigma$ is sufficiently high that the constraint is always binding, it can then be shown that retained earnings are always cheaper than debt, so $D_t = 0$. Substituting the balance sheet of the bank, $\hat{V}_t = S_t - B_t$, and the value of the bank defined in equation (5) into the borrowing constraint (4) yields $V_t \geq S_t\mathcal{M}_t/(\mathcal{A} + 1)$. Then, using the definition of $\mathcal{A}_t$ in equation (6) implies:*

$$V_t \geq [1 - \sigma(1 - \theta)] S_t \tag{14}$$

which is equivalent to the borrowing constraint in GK for appropriately chosen parameters.[1]

Appendix B Gertler & Kiyotaki (2010) model

We describe a version of the GK model extension with equity issuance. The household and firm sectors are identical to our model, the difference is on the intermediation of funds between these two sectors. Every period, banks face a constant probability, $1-\sigma_B$,

[1] GK use notation Q_tS_t for the value of bank assets whereas we use S_t. In GK, Θ is in place of $[1 - \sigma(1 - \theta)]$; this difference occurs due to alternative timing assumptions. In our model with the GK settings, the amount that can be reclaimed against excludes the proportion of assets paid as usual when the proportion $1 - \sigma$ of bankers exit.

of exiting and paying the household a dividend. No dividend is paid if the bank continues, the bank decides on debt and outside equity finance and issues loans to non-financial firms. When a bank exits, a new bank takes is place and is transferred a fraction ξ_B of the exiting banks' net worth. Bank activity is subject to financial constraints as the inside shareholders can divert assets. In particular bank j solves

$$\begin{aligned} V_t^j = \max_{S_t, B_t, E_t} & \mathbb{E}_t \{ (1-\sigma_B) N_{t+1}^j + \sigma_B \Lambda_{t,t+1} V_{t+1}^j \} \\ \text{s.t.} \quad & V_t^j \geq \Theta \left(x_t^j \right) S_t^j \\ & N_t^j = R_t^K S_{t-1}^j - R_t^E Q_{t-1}^E E_{t-1}^j - R_{t-1} B_{t-1}^j \\ & S_t^j = B_t^j + Q_t^E E_t^j + N_t^j \end{aligned}$$

where E_t is the stock of outside equity, rather than new issuance of inside equity as in our model, Q_t^E is the price of equity, and R_t^E is the rate of return on outside equity. Where each unit of $E_t Q_t^E$ is a claim on one unit of S_t, itself a claim on a unit of $Q_t K_t$. The proportion of divertable assets, θ is a quadratic function of $x_t \equiv Q_t^E E_t / S_t$:

$$\theta \left(x_t^j \right) = \bar{\theta} \left(1 + \epsilon x_t + \frac{\kappa^{GK}}{2} x_t^2 \right)$$

Dropping bank indices, this leads to demand equations for debt and equity finance

$$\begin{aligned} \nu_t^b &= \phi_t \left(\theta \left(x_t \right) - \left[\mu_t^s + \mu_t^e x_t \right] \right) \\ \mu_t^e &= \left[\mu_t^s + \mu_t^e x_t \right] \frac{\theta' \left(x_t \right)}{\theta \left(x_t \right)} \end{aligned}$$

with $\phi_t \equiv \frac{S_t}{N_t}$ and where

$$\begin{aligned} \Omega &\equiv 1 - \sigma_B + \sigma_B \theta \left(x_t \right) \phi \\ \mu_t^s &\equiv \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_{t+1}^K - R_t \right) \right] \\ \nu_t^b &\equiv \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} R_t \right] \\ \mu_t^e &\equiv \mathbb{E}_t \left[\Lambda_{t,t+1} \Omega_{t+1} \left(R_t - R_{t+1}^E \right) \right] \end{aligned}$$

Finally, the demand for outside equity must satisfy

$$1 = \mathbb{E}_t \left[\Lambda_{t,t+1} R_{t+1}^E \right].$$

Appendix C Additional impulse response functions

In addition to the impulse response function to an adverse capital quality shock in the paper, here we show a positive capital quality shock that highlights the asymmetry. Plots of the responses to a positive 5 percent capital quality shock are shown in figure 1. The same is true for shocks to total factor productivity. As a negative productivity shock decreases the continuation value of the bank, or the value of future profits, the constraint tightens. As there is also a decline in the value of bank assets, which acts in the opposite direction, a large shock is required to cause the borrowing constraint to bind sufficiently to have a large impact. There is a small financial accelerator for adverse shocks, but as shown in figure 2, this effect is not persistent and the model converges quickly to the RBC model. As shown in figure 3, for positive technology shock there is little difference between our model and the RBC model.

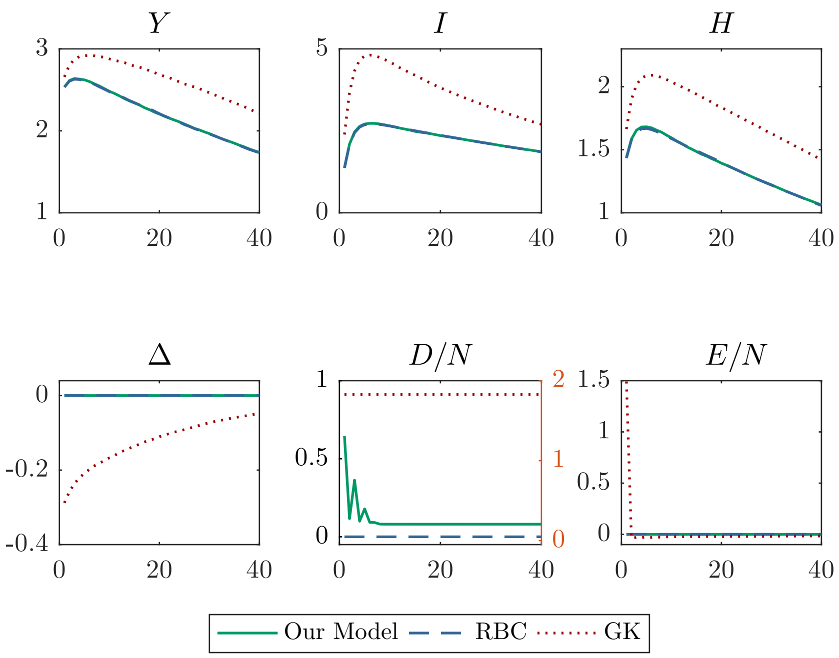

Figure 1: Median impulse response functions to a positive capital quality shock for our model, and the RBC and GK models. The shock is defined as a one time 5 percent increase in the capital stock. Plots show percent deviation from ergodic median for Y, I, and H and level deviation for $\Delta = \mathbb{E}_t\left[R^K_{t+1} - R_t\right]$, D/N and E/N. The left axis of the D/N plot corresponds to our model, the right to GK.

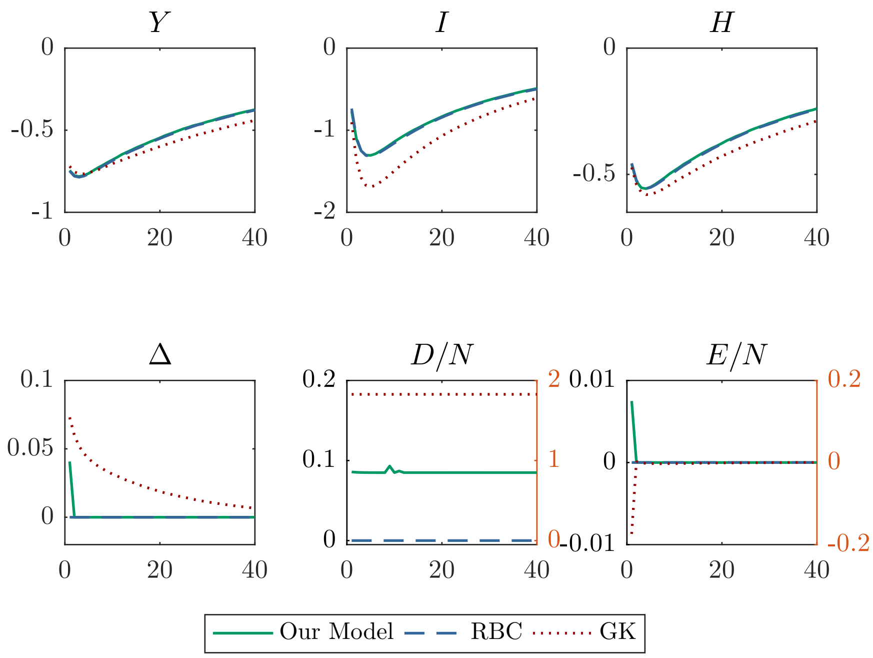

Figure 2: Median impulse response functions to a 1-standard deviation, negative productivity shock in our model, and the RBC and GK models. Plots show percent deviations from ergodic median for Y, I, and H and percentage point deviations for $\Delta = \mathbb{E}_t\left[R^K_{t+1} - R_t\right]$, D/N and E/N. The left axes of the last two plots correspond to our model, the right to GK.

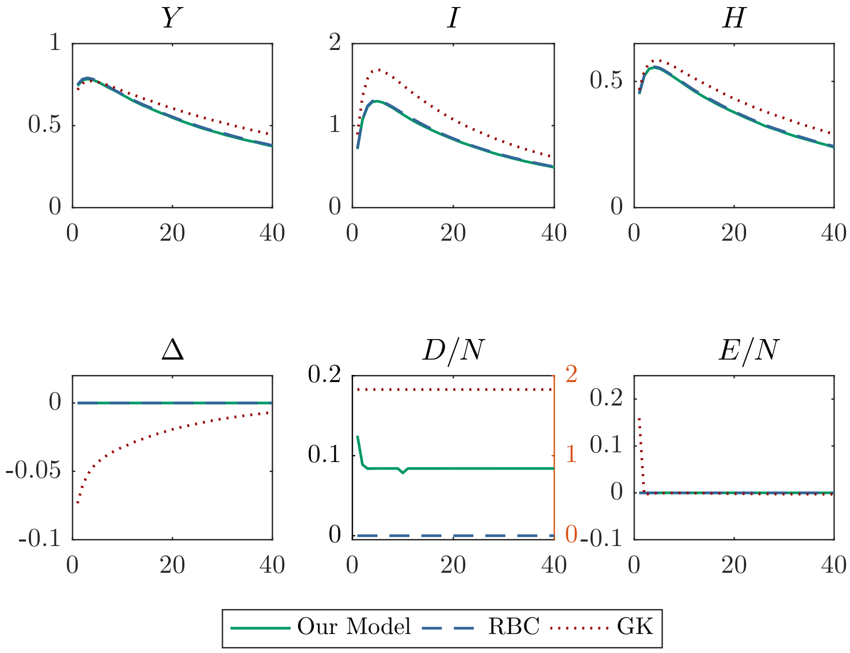

Figure 3: Median impulse response functions to a 1-standard deviation, positive productivity shock in our model, and the RBC and GK models. Plots show percent deviations from ergodic median for Y, I, and H and percentage point deviations for $\Delta = \mathbb{E}_t\left[R^K_{t+1} - R_t\right]$, D/N and E/N. The left axis of the D/N plot corresponds to our model, the right to GK.