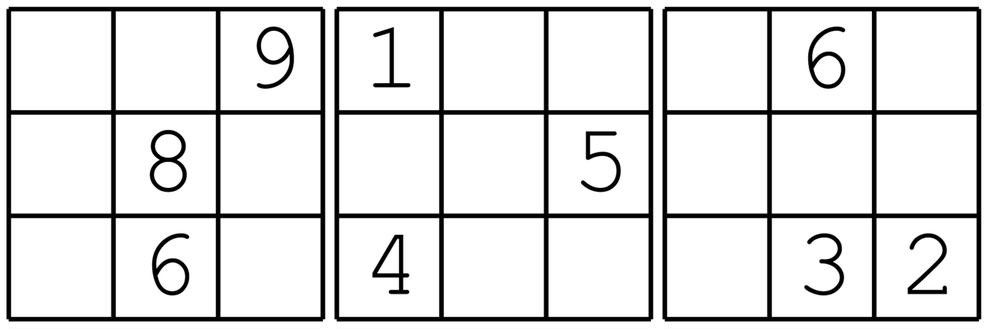
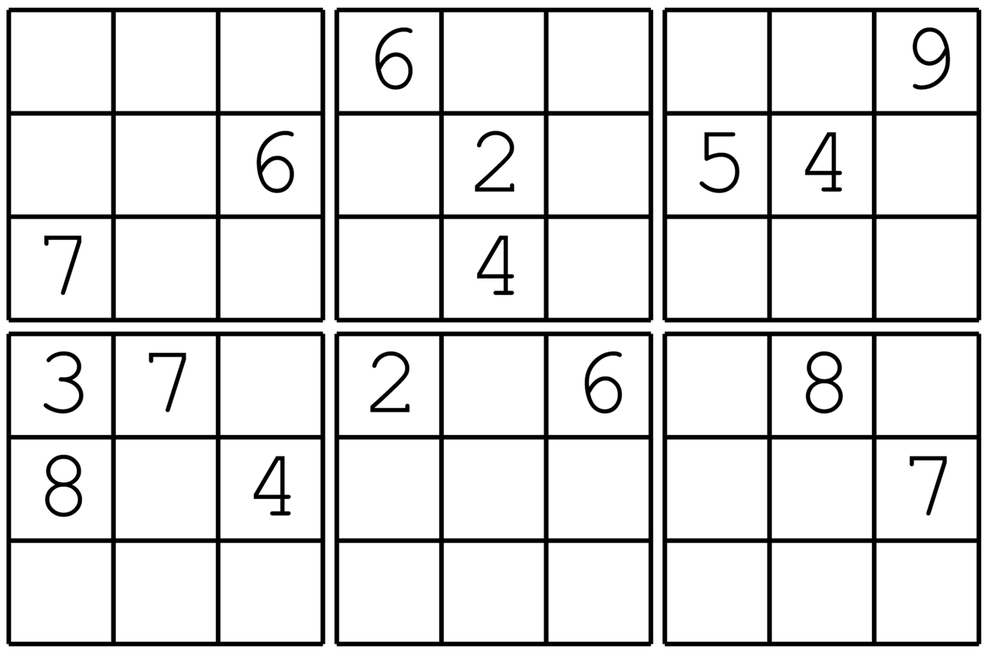
# Sudoku solving with constraint propagation

## Introduction

Sudoku is a constraint satisfaction problem (a problem whose solution requires a set of variables and which imposes constraints on the values they can take). In Sudoku the variables are the cells and the constraints are that rows, columns and boxes must contain each digit exactly once.





## Definitions

- Square: Cell.
- Peers: All the square's neighbors; neighbors are squares that are in the same unit of the squares.
- Unit: A collection of squares, for each row, column, and region.
- Region: Boxes containing 3x3 squares.

## Algorithm

- There is a partly filled Sudoku field.
- Each square holds possible values, which can be added there.
- Algorithm uses two main strategies:
  - If a square has only one possible value, then eliminate that value from the square's peers.
  - If a unit has only one possible place for a value, then put the value there.
- If any more values can't be eliminated:
  - Adding possible value for empty square using depth-first search.
  - Empty square will be chosen with Minimum remaining values heuristic.
  - After adding a value to the square, it uses the main strategies again.
- If contradiction arises then algorithm uses recursive depth-first search and backtracking to guess another value for a square or move to the next square.

## Results

- Solved 2365 of 2365 hard puzzles.
  Average: 4.8998 ms, max 734.0 ms.
  Hardest to solve sudoku:

```
. . . |. 6 9 |. . .
8 . . |. . 4 |. . .
. 7 . |. . . |6 . 1
------+------+------
. . . |. . . |. . 2
3 . . |. . 1 |7 . .
. . . |8 . 6 |1 3 .
------+------+------
. . . |. . . |4 . .
1 . 4 |. 9 . |. 6 .
. 2 3 |. 5 . |. . .
```

Solved:

```
5 3 1 |7 6 9 |2 8 4
8 6 9 |1 2 4 |3 7 5
4 7 2 |5 8 3 |6 9 1
------+------+------
7 1 6 |9 3 5 |8 4 2
3 9 8 |2 4 1 |7 5 6
2 4 5 |8 7 6 |1 3 9
------+------+------
9 5 7 |6 1 8 |4 2 3
1 8 4 |3 9 2 |5 6 7
6 2 3 |4 5 7 |9 1 8
```

References:
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http://en.wikipedia.org/wiki/Constraint_propagation
http://cordobaforever.files.wordpress.com/2011/06/sudoku-13-de-junio.jpg
http://norvig.com/sudoku.html
http://www2.warwick.ac.uk/fac/sci/moac/currentstudents/peter_cock/python/sudoku/

Code:
https://github.com/joosep/sudoku

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