Fall 2022 - CSC545/645 Artificial Intelligence - Assignment 7

Due date: Thursday, October 27, 2022, 2:00pm. Please create a folder called assignment7 in your local working copy of the repository and place all files and folders necessary for the assignment in this folder. Once done with the assignment, add the files and folders to the repo with svn add *files,folders* and then commmit with svn ci -m "SOME USEFUL MESSAGE" *files,folders* .

Exercise 7.1 [20 points]

Read the chapter *Quantifying Uncertainty* of the textbook (chapter 13 in the third edition, chapter 12 in the 4th edition).

Agent 1		Agent 2		Outcomes and payoffs to Agent 1			
Proposition	Belief	Bet	Stakes	a, b	$a, \neg b$	$\neg a, b$	$\neg a, \neg b$
a	0.4	a	4 to 6	−6	−6	4	4
b	0.3	b	3 to 7	−7	3	−7	3
$a \vee b$	0.8	$\neg(a \vee b)$	2 to 8	2	2	2	−8
				−11	−1	−1	−1

Figure 13.2 Because Agent 1 has inconsistent beliefs, Agent 2 is able to devise a set of bets that guarantees a loss for Agent 1, no matter what the outcome of a and b.

1. Would it be rational for an agent to hold the three beliefs:

$$P(A) = 0.4,\ P(B) = 0.3, \text{ and } P(A \vee B) = 0.5?$$

If so, what range of probabilities would be rational for the agent to hold for $A \wedge B$? Make up a table like the one in Figure 13.2 on the right and show how it supports your argument about rationality.

Prop 1	Belief 1	Bet 2	Stakes 2	a, b	a, ~b	~a, b	~a, ~b
a	.4	a	4 to 6	-6	-6	4	4
b	.3	b	3 to 7	-7	3	-7	3
a v b	.5	~(a v b)	5 to 5	5	5	5	-5
				-8	2	2	2

Even if you switched around any of the bets for a, b, or (a v b) there is not a case in which agent 1 always loses. Therefore, this is a rational belief system. Then, since ~(a v b) == ~ a n ~b, but the odds are 50/50 anyway, P(a n b) could be anything up to .5

Then draw another version of the table where $P(A \vee B) = 0.7$. Explain why it is rational to have this probability, even though the table shows one case that is a loss and three that just break even. (Hint: what is Agent 1 committed to about the probability of each of the four cases, especially the case that is a loss?) [8 points]

Prop 1	Belief 1	Bet 2	Stakes 2	a, b	a, ~b	~a, b	~a, ~b
a	.4	a	4 to 6	-6	-6	4	4
b	.3	b	3 to 7	-7	3	-7	3
a v b	.7	~(a v b)	3 to 7	3	3	3	-7
				-10	0	0	0

Since Agent 2 isnt making Agent 1 LOSE every time, this still means Agent 1 has rational beliefs. The only thing this shows is that Agent 1 is a little too comitted to the idea that both a and b will be true at the same time.

2. Given the full joint distribution shown in the figure below, calculate the following. Please elaborate your answer, we would like to see your calculations and the results rather than the results alone. Also note the bold (**P**) and regular (*P*) as well as upper/lower case writing). [4 points]

	toothache		¬toothache	
	catch	¬catch	catch	¬catch
cavity	0.108	0.012	0.072	0.008
¬cavity	0.016	0.064	0.144	0.576

(a) *P(toothache)*
The whole table under toothache = .108 + .012 + .016 + .064 **= .2**

(b) **P***(Cavity)*

The <true, false> probabilities of cavity given all known information =

P(Cavity, toothache, catch) + P(Cavity, ~toothache, catch)

+ P(Cavity, toothache, ~catch) + P(Cavity, ~toothache ~catch)

= <.108, .016> + <.012, .064> + <.072, .144> + <.008, .576>

= <.2, .8>

(c) **P**(*Toothache* | *cavity)*

The <true, false> odds of having a toothache given cavity is true

= P(Toothache, cavity, catch) + P(Toothache, cavity, ~catch)

= <.108, .072> + <.012, .008> = **<.12, .02>**

(d) **P**(*Cavity* | *toothache* ∨ *catch)*

The <true, false> probability of having a cavity given both a toothache and catching

P(Cavity, toothache, catch) = **<.108, .016>**

3. After your yearly checkup, the doctor has bad news and good news. The bad news is that you tested positive for a serious disease, and that the test is 99% accurate (i.e., the probability of testing positive given that you have the disease is 0.99, as is the probability of testing negative given that you don't have the disease). The good news is that this is a rare disease, striking only one in 10,000 people of your age. Why is it good news that the disease is rare? What are the chances that you actually have the disease?
[2 points]

Given:
P(T | D) = .99   P(D) = .0001   therefore   P(~D) = .9999   P(T | ~D) = .01
so
P(T) = P(T | ~D) P(~D) + P(T | D) P(D) = .01 * .9999 + .99 * .0001 = .010098

P(D | T) = P(T | D) * P(D) / P(T) = [.99 * .0001] / .010098 = .00980

This means there is only a .98% chance you actually have the disease.

4. Deciding to put probability theory to good use, we encounter a slot machine with three independent wheels, each producing one of the four symbols BAR, BELL, LEMON, or CHERRY with equal probability. The slot machine has the following payout scheme for a bet of 1 coin (where “?” denotes that we don’t care what comes up for that wheel):

BAR/BAR/BAR pays 20 coins
BELL/BELL/BELL pays 15 coins
LEMON/LEMON/LEMON pays 5coins
CHERRY/CHERRY/CHERRY pays 3 coins
CHERRY/CHERRY/? pays 2 coins
CHERRY/?/? pays 1 coin

(a) Compute the expected “payback” percentage of the machine. In other words, for each coin played, what is the expected coin return?

$(\frac{1}{4})^3 * 20 + (\frac{1}{4})^3 * 15 + (\frac{1}{4})^3 * 5 + (\frac{1}{4})^3 * 3 + (\frac{1}{4})^2 * 2 + (\frac{1}{4}) =$ **1.046875**

Is the return based on a coin. So, after paying one coin, net profit of **.046875**.

(b) Compute the probability that playing the slot machine once will result in a win.

Total number of way to win / Total different slot states = 19 / 64 = **.296875**

(c) Estimate the mean and median number of plays you can expect to make until you go broke, if you start with 10 coins. You can run a simulation (a small Python program for example) to estimate this, rather than trying to compute an exact answer. [6 points]

Running my python code for 10000 simulations starting with 10 coins, I had an average numner of pulls to be ~218 and a median number of 21. This is because sometimes the system can just go on huge lucky streaks!