 Fall 2022

- CSC545/645 Artificial Intelligence - Assignment 10

Due date: Thursday, December 1, 2022, 2:00 pm. Please create a folder called assignment10 in your local working copy of the repository and place all files and folders necessary for the assignment in this folder. Once done with the assignment, add the files and folders to the repo with svn add *files, folders* and then commit with svn ci -m "SOME USEFUL MESSAGE" *files, folders* .

Exercise 10.1 [20/25 points (CSC545/645)]
Read chapter "Probabilistic Reasoning over Time" of the textbook (chapter 15 in the 3rd edition, chapter 14 in the 4th edition).

1. A professor wants to know if students are getting enough sleep. Each day, the professor observes whether the students sleep in class and whether they have red eyes. The professor has the following domain theory:

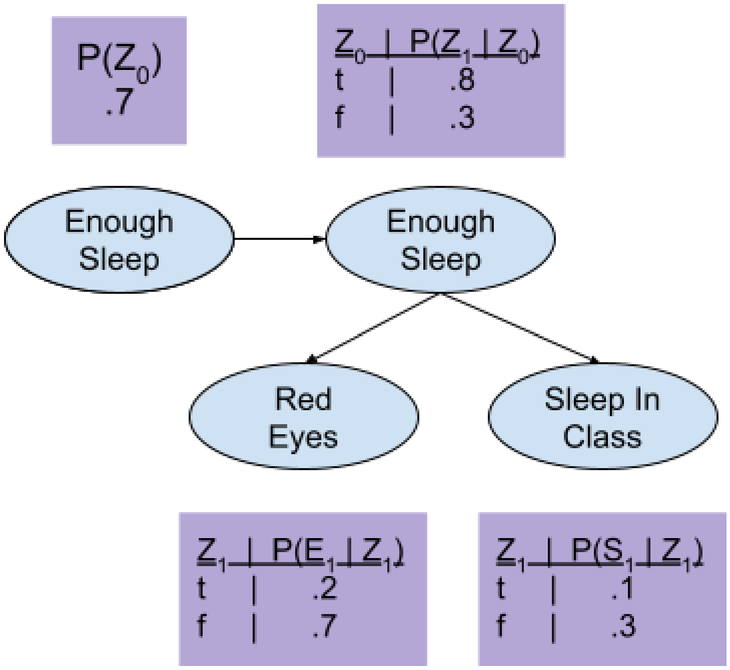
   (a) The prior probability of getting enough sleep, with no observations, is 0.7.

   (b) The probability of getting enough sleep on night *t* is 0.8 given that the student got enough sleep the previous night, and 0.3 if not.

   (c) The probability of having red eyes is 0.2 if the student got enough sleep, and 0.7 if not.

   (d) The probability of sleeping in class is 0.1 if the student got enough sleep, and 0.3 if not.

   Formulate this information as a dynamic Bayesian network (DBN) that the professor could use to filter or predict from a sequence of observations. Then reformulate it as a hidden Markov model that has only a single observation variable. Give the complete probability tables for the model.
   [10 points]



This DBN is already an HMM since it only has one observable variable, ‘Enough Sleep’.

Z_0			Z_{t+1}	Z_t	$P(Z_{t+1} \mid Z_t)$		E_1	Z_1	$P(E_1 \mid Z_1)$		S_1	Z_1	$P(S_1 \mid Z_1)$
			t	t	.8		t	t	.2		t	t	.1
t	.7		t	f	.3		t	f	.7		t	f	.3
f	.3		f	t	.2		f	t	.8		f	t	.9
			f	f	.7		f	f	.3		f	f	.7

2. For the DBN specified in Exercise 10.1 and for the evidence values

e_1 = not red eyes, not sleeping in class
e_2 = red eyes, not sleeping in class
e_3 = red eyes, sleeping in class

perform the following computations:

(a) State estimation: Compute $P(EnoughSleep_t|e_{1:t})$ for each of t = 1,2,3.

First compute Z_1 just based on Z_0 then update based on the e's.

t = 1

$P(Z_1) = \Sigma P(Z_1 \mid Z_0) = P(Z_1 \mid Z_0) * P(Z_0) + P(Z_1 \mid \sim Z_0) * P(\sim Z_0) = .8 * .7 + .3 * .3 = .65$

$P(\sim Z_1) = 1 - .65 = .35$

$P(Z_1 \mid e_1) = a * P(e_1 \mid Z_1) * P(Z_1) = a * .8 * .9 * .65 = a * .468$

$P(\sim Z_1 \mid e_1) = a * P(e_1 \mid \sim Z_1) * P(\sim Z_1) = a * .3 * .7 * .35 = a * .0735$

So a = 1 / (.468 + .0735) = 1/.5415

$\mathbf{P(Z_1 \mid e_1) = .8642}$

$\mathbf{P(\sim Z_1 \mid e_1) = .1357}$

t = 2 (Letting $P(Z_1) = P(Z_1 \mid e_1)$ for all below calculations)

$P(Z_2) = \Sigma P(Z_2 \mid Z_1) = P(Z_2 \mid Z_1) * P(Z_1) + P(Z_2 \mid \sim Z_1) * P(\sim Z_1) = .8 * .8642 + .3 * .1357$
$= .73207$

$P(\sim Z_2) = 1 - .73207 = .2679$

$P(Z_2 \mid e_{1:2}) = a * P(e_{1:2} \mid Z_2) * P(Z_2) = a * .2 * .9 * .73207 = a * .1318$

$P(\sim Z_2 \mid e_{1:2}) = a * P(e_{1:2} \mid \sim Z_2) * P(\sim Z_2) = a * .7 * .7 * .2679 = a * .1313$

So a = 1 / (.1318 + .1313) = 1/.2631

$\mathbf{P(Z_2 \mid e_2) = .501}$

$\mathbf{P(\sim Z_2 \mid e_2) = .499}$

t = 3

$P(Z_3) = \Sigma P(Z_3 \mid Z_2) = P(Z_3 \mid Z_2) * P(Z_2) + P(Z_3 \mid \sim Z_2) * P(\sim Z_2) = .8 * .4348 + .3 * .55 = .5505$

$P(\sim Z_2) = 1 - .5505 = .45$

$P(Z_3 \mid e_{1:3}) = a * P(e_{1:3} \mid Z_3) * P(Z_3) = a * .2 * .1 * .5505 = a * .01101$

$P(\sim Z_3 \mid e_{1:3}) = a * P(e_{1:3} \mid \sim Z_3) * P(\sim Z_3) = a * .7 * .3 * .45 = a * .0945$

So $a = 1 / (.01101 + .0945) = 1/.10551$

$P(Z_3 \mid e_3) = .104$

$P(\sim Z_3 \mid e_3) = .896$

(b) Smoothing: Compute $P(EnoughSleep_t \mid e_{1:3})$ for each of t = 1, 2, 3.

t = 3

$P(Z_3 \mid e_{1:3}) = f_{1:3}$ = answer from part a: **<.104, .896>**

t = 2

$P(Z_2 \mid e_{1:3}) = a * f_{1:2} \times b_{3:3}$

$b_{3:3} = P(e_{3:3} \mid Z_2) = \Sigma_{z3} P(e_3 \mid Z_3) * P(Z_3 \mid Z_2)$

$= (.2 * .1 * <.8, .2>) + (.3 * .7 * <.2, .8>) = <.058, .178>$

$P(Z_2 \mid e_{1:3}) = a * <.501, .499> \times <.058, .178> =$ **<.28, .72>**

t = 1

$P(Z_1 \mid e_{1:3}) = a * f_{1:1} \times b_{2:3}$

$b_{2:3} = P(e_{3:3} \mid Z_1) = \Sigma_{z2} P(e_2 \mid Z_2) * P(e_{3:3} \mid Z_2) * P(Z_2 \mid Z_1)$

$= (.2 * .9 * .2 * .1 * <.8, .2>) + (.7 * .7 * .3 * .7 <.2, .8>) = <.04938, .08952>$

$P(Z_2 \mid e_{1:3}) = a * <.8642, .1357> \times <.04938, .08952> =$ **<.72, .28>**

(c) Compare the filtered and smoothed probabilities for *t* = 1 and *t* = 2.

My answer for t=2 is a descent amount lower in backtracking. Since on night t=3 the odds that we did not get a good nights sleep were very high ($\sim Z_3 = .9$), the odds we did not get a good nights sleep on t=2 should have been pretty high just based on that. Similarly though, obviously backtracking also uses the nights before (t_1) which was a night where we probably did sleep well, so weighing one night of (probably) good sleep and one night of bad sleep may have evened out, but knowing we had red eyes dragged down the odds of having good sleep pretty strong. Similarly, the prediction for t=1 was also only slightly lower. The evidence significantly indicates we got a good night's sleep, but nights 2 and 3 we probably didnt sleep well, so although it went up much higher compared to the increase from t=3 to t=2 it didnt entirely catch up to our initial "forward" predictions.

[10 points]