SHRI MATA VAISHNO DEVI UNIVERSITY, KATRA
School of Mathematics
M.Sc. Mathematics Minor Examination (Odd Sem.) 2018-19

Entry No: 18BEC066 | Total Number of Pages:[01]
Date: | Total Number of Questions:[04]

Course Title: Engineering Mathematics- I
Course Code: MTL1025

Time Allowed: 1.5 Hours | Max Marks:[10]

NOTE: Attempt All Questions.

Q1.	State Leibnitz Theorem and use it to prove that $x^{n-1}y_n = (-1)^n(n-2)!$, $n \geq 2$ for the function $y = x \log x$.	[02]
Q2. ✓	(a)State Euler theorem and hence show that if $u = \cot^{-1}\left(\frac{x-y}{x^3+y^3}\right)$, then $$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u.$$	[01]
	(b) Evaluate $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the implicit relation given by the equation $x^5 + y^5 - 5a^3xy = 0$.	[02]
Q3. ✓	(a) Find all the asymptotes of the curve $$x^3 - 2x^2y + xy^2 + x^2 - xy + 2 = 0.$$	[02]
	(b) Define Jacobian and if $x = u(1-v)$, $y = uv$ then compute J and J' and prove $JJ' = 1$.	[01]
Q4.	Using Taylor's series expansion show that $$x^2 + 4xy + 2y^2 + 6x + 4y + 1 = (x-1)^2 + 4(x-1)(y+2) + 2(y+2)^2.$$	[02]
	OR	
	Find Taylor's series expansion for the function $x^2+4xy+2y^2+6x+4y+1$ about the point $(1, -2)$.	[02]

Course Outcomes

After successful completion of this Course, students shall be able to;
(1) apply Leibnitzs theorem to find n^{th} derivative.
(2) compute total derivative, partial derivatives and asymptotes of an algebraic curve.
(3) expand Taylors series, Macluarins series.
(4) find the derivatives of composite, implicit functions and Maxima Minima of functions of two variables.

SHRI MATA VAISHNO DEVI UNIVERSITY, KATRA

School of Mathematics

B.Tech (CSE/ME/CE/ECE/EE) (Odd Sem.)Major 2018-19

Entry No: 18BME012 | Total Number of Pages:[02]

Date: | Total Number of Questions:[05]

Course Title: Engineering Mathematics- I

Course Code: MTL1025

Time Allowed: 3 Hours | Max Marks:[50]

NOTE: Attempt All Questions.

Q1. Reduce the matrix

$$A = \begin{pmatrix} 1 & 0 & 2 & 1 \\ 0 & 1 & -2 & 1 \\ 1 & -1 & 4 & 0 \\ -2 & 2 & 8 & 0 \end{pmatrix}$$

into their normal form and find its rank. [05]

(ii) What values of λ and μ do the system of equations [05]

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = \mu.$$

have (i) no solution (ii) unique solution (iii) more than one solution.

Q2. (i)Define eigen values and eigen vectors of a square matrix and hence find the eigen values and eigen vector for the matrix [05]

$$\begin{pmatrix} 6 & -6 & 5 \\ 14 & -13 & 10 \\ 7 & -6 & 4 \end{pmatrix}$$

(ii)State Cayley-Hamilton Theorem and verify it for the matrix [05]

$$A = \begin{pmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}.$$

Further find A^{-1} and A^4.

Q3. (i) Diagonalize the matrix [05]

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & 2 \\ 0 & 1 & 1 \end{pmatrix}.$$

(ii) Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid [05]

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$$

Q4.	(i) If $u = f(\theta, \phi)$, $x + y = 2e^{\theta}\cos\phi$ and $x - y = 2ie^{\theta}\sin\phi$. Then show that $$\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 u}{\partial \phi^2} = 4xy\frac{\partial^2 u}{\partial x \partial y}$$	[05]
	(ii)Prove that $$\int_{-a}^{a} f(x)dx = \int_{0}^{a} [f(x) + f(-x)]dx.$$ Hence evaluate $$\int_{\pi/2}^{-\pi/2} \theta \sin\theta \cos\theta d\theta.$$	[05]
	OR	
	Evaluate $\int_{0}^{\pi/2} \log(\sin x)dx$.	[05]
Q5.	(i) Obtain the reduction formula for $$I_n = \int_{0}^{\infty} e^{-x} \sin^n x dx$$ and show that $(1 + n^2)I_n = n(n-1)I_{n-2}$	[05]
	(ii) Evaluate the following using Beta and gamma functions: $$\int_{0}^{\pi/2} \sin^3 x \cos^{5/2} x dx.$$	[05]
	OR	
	Prove that the area common to the two parabolas $x^2 = 4ay$ and $y^2 = 4ax$ is $\frac{16a^2}{3}$.	5

Course Outcomes

After successful completion of this Course, students shall be able to;

(1) identify the extrema of a function on an interval and classify them as minima , maxima or saddle points.

(2) interpret the definite integral geometrically as the area under a curve.

(3) evaluate a definite integral using an anti-derivative (Fundamental Theorem of Calculus).

(4) understand the fundamentals of Integral calculus to understand their applications to length, area, volume, surface of revolution, moments and centre of gravity.

(5)understand the improper integrals and Beta and Gamma functions and their applications.

$-1+\lambda+\lambda-\lambda^2$

$\lambda^2+2\lambda-1=0$

$x^{n-1}(1-x)^m$

SHRI MATA VAISHNO DEVI UNIVERSITY, KATRA

School of Mathematics

B.Tech (CSE/ME/CE/ECE/EE) (Odd Sem.)Minor(I&II) Examination

2019-20

Entry No: 19BCS051 | Total Number of Pages:[01]

Date: | Total Number of Questions:[07]

Course Title: Engineering Mathematics-I

Course Code: MTL 1025

Time Allowed: 90 minutes | Max Marks:[30]

NOTE: Attempt any six Questions.

Q1.	State Euler's theorem and use it to find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$, where $u = \sin^{-1}\left(\frac{x^{\frac{1}{2}}+y^{\frac{1}{2}}}{x^{\frac{1}{4}}+y^{\frac{1}{4}}}\right)$.	[05]
Q2.	Define a double point and find the position and nature of the double points on the curve $x^4 - 2ay^3 - 3a^2y^2 - 2a^2x^2 + a^4 = 0$. $a^4 - 2a^4 + a^4 = 0$ $a^4 + 2a^4 - 3a^4 - 2a^4 + a^4$	[05]
Q3.	A rectangular box open at the top is to have a volume of 32cc. Find the dimension of the box which requires least amount of material for its construction.	[05]
Q4.	Define maxima and minima of the function of two variables and examine the function $f(x,y) = 2(x-y)^2 - x^4 - y^4$ for the extreme values.	[05]
Q5.	Find the asymptotes of the curve $x^3+2x^2y-xy^2-2y^3+4y^2+2xy+y-1=0$.	[05]
Q6.	If $x = u(1-v)$, $y = uv$, then compute J and J' and prove $JJ' = 1$.	[05]
Q7.	Find the reduction formula for $\int \tan^n x$. Further show that $I_{n+2} + I_n = \frac{t^{n+1}}{n+1}$, where $I_n = \int \frac{t^n}{1+t^2}dt$. Hence evaluate I_6. *OR* Define point of inflexion of a curve and determine the intervals in which the curve $y = 3x^5 - 40x^3 + 3x - 20$ is concave upwards or downwards.	[05]

Course Outcomes

After successful completion of this Course, students shall be able to;

(1) identify the extreme of a function of two variables and classify them as minima, maxima or saddle points.

(2) find the derivatives of composite and implicit functions.

(3) compute total derivative, partial derivative and asymptotes of an algebraic curve.

(4) expand the reduction formulae and their applications.

$(0,-a)$ ✓ $0 - 2a(-a^3) - 3a^2a^2 + a^4$

$2a^4$

$(0,0)$ X , $(a,-a) \Rightarrow a^4 + 2a^4 - 3a^4 - 2a^4 + a^4$

$(a,0)$ ✓

X

$f(a,-a): a^4 + 2a^4 - 3a^4 - 2a^4 + a^4$ X

SHRI MATA VAISHNO DEVI UNIVERSITY, KATRA
School of Mathematics
B.Tech (CSE/ME/CE/ECE/EE) (Odd Sem.)Major 2019-20

Entry No: 19BCE021
Date:

Total Number of Pages:[02]
Total Number of Questions:[05]

Course Title: Engineering Mathematics- I
Course Code: MTL1025

Time Allowed: 3 Hours
NOTE: Attempt All Questions.
Max Marks:[50]

Q1. What values of λ, does the system of equations [05]

$$3x + y - \lambda z = 0$$
$$4x - 2y - 3z = 0$$
$$2\lambda x + 4y + \lambda z = 0$$

has non-zero solution. Also solve them in each case.

(ii) Reduce the matrix [05]

$$A = \begin{pmatrix} 1 & 3 & 4 & 5 \\ 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \end{pmatrix}$$

into their normal form and find its rank.

Q2. (i) State Cayley-Hamilton Theorem and verify it for the matrix [05]

$$A = \begin{pmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

and hence find A^{-2}.

(ii) Define eigen values and eigen vectors of a square matrix and hence find the eigen values and eigen vector for the matrix [05]

$$A = \begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$$

Q3. (i) Diagonalize the matrix [05]

$$A = \begin{pmatrix} 2 & -2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$$

(ii) Define total differential of a function $u = f(x, y)$. [05]
Consider the implicit relation between two variables x and y as $f(x, y) = 0$ where y is a differentiable function of x. Then prove that

$$\frac{d^2y}{dx^2} = \frac{-1}{f_y^3}(f_{xx}f_y^2 - 2f_{xy}f_xf_y + f_{yy}f_x^2)$$

and hence evaluate $\frac{dy}{dx}$ of the relation $x^3 + y^3 - 3a^2xy = 0$.

$-1(0-1) - 2(0+1) - 2(1+2)$

$-2-2=-4$

$1-13+12$

$1 - 2 - 6$

$0-2=-2$

$-2=-4$

$8-4-20+7$

$2(-3-2)-2(-6+7)$

-10

$1-13+12$

47

$2 - (-2(2-\sqrt{5}))$ $-1-\sqrt{5}+2$

$2 + 4 - 2\sqrt{5}$ $(-1-\sqrt{5})(2-\sqrt{5}) - 2$

$-2 + 2\sqrt{5} - 2\sqrt{5} + 5 - 2$

$1 - \sqrt{5}$

Q4. (i) Define Beta and Gamma function and prove that [05]

$$\beta(m,n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$

Further show that

$$\beta(m,n) = \int_0^\infty \frac{y^{n-1}}{(1+y)^{m+n}}dy = \int_0^1 \frac{(x^{m-1}+x^{n-1})}{(1+x)^{m+n}}dx$$

(ii)Prove that [05]

$$\Gamma(m)\Gamma(m+\frac{1}{2}) = \frac{\sqrt{\pi}}{2^{2m-1}}\Gamma(2m)$$

OR

Prove that

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin n\pi},$$

given that $\int_0^\infty \frac{x^{n-1}}{1+x}dx = \frac{\pi}{\sin n\pi}$.

Q5. (i) Find the reduction formula for $\int \sin^n x$ and hence prove that [05]

$$\int_0^{2a} \frac{x^3}{\sqrt{2ax-x^2}}dx = \frac{5}{2}\pi a^2$$

(ii) Examine the function [05]

$$f(x,y) = x^3 + y^3 - 3axy$$

for the extreme values.

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(2) interpret the definite integral geometrically as the area under a curve.

(3) evaluate a definite integral using an anti-derivative (Fundamental Theorem of Calculus).

(4) understand the fundamentals of Integral calculus to understand their applications to length, area, volume, surface of revolution, moments and centre of gravity.

(5) understand the improper integrals and Beta and Gamma functions and their applications.

$\lambda^2 + \lambda - 12$

$\lambda^2 + 4\lambda - 3\lambda - 12$

$\lambda(\lambda+4) - 3(\lambda+4)$

$\lambda = 3$

$\lambda = -4$