Discrete Random Expected Utility

Igor Kopylov* Kevin Peng†‡

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Abstract

We model stochastic choice rules via finitely many types θ that maximize distinct expected utility functions and use endogenous tie-breaking rules. In general, the likelihoods $\mu_A(\theta)$ of such types are allowed to depend on the feasible menu A, and we also derive representations where the distribution $\mu(\theta)$ is unaffected by A. This invariant case provides a discrete version for the *random expected utility* of Gul and Pesendorfer (2006), but we use distinct axioms and identification methods. More generally, we study representations where the menu-dependent type distribution $\mu_A(\theta)$ accommodates various kinds of *context dependence*. In particular, we show that the standard monotonicity principle imposed on stochastic choice data can be used to characterize *self-selection* in type likelihoods. In other words, type θ should not become more likely when new alternatives are added to a feasible menu, but do not improve the best choice for θ. Both the discrete type space Θ and the bivariate function μ can be identified uniquely in our model. Finally, we discuss applications to heterogeneous risk attitudes and beliefs, and to filtering measurement noise.

*Department of Economics, University of California, Irvine. E-mail: ikopylov@uci.edu.

†Department of Economics, University of California, Irvine. E-mail: kuangyup@uci.edu

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1 Introduction

Observed choices in consumption menus can be naturally *stochastic* when they are produced by distinct rational (i.e., utility maximizing) agents. Data is often presented as an aggregation of choices from such distinct agents. It can be seen in a wide range of empirical work, such as preferred modes of transportation (McFadden, 2001) and choice of fishing sites (Train, 1998). This heterogeneity may also result from more behavioral motivations where a single individual can exhibit distinct random responses in identical choice problems (see Agranov and Ortoleva (2017)).

The classic random utility model (RUM) by Block and Marschak (1960) aims to represent stochastic choice data by a stable probability distribution μ across *types* that include all possible utility functions. In the general case where types are unrestricted, Falmagne (1978) characterizes RUM via non-negativity of Block and Marschak's polynomials. Gul and Pesendorfer (2006) (henceforth, GP) refine this model to *random expected utility* (REU) where all types should maximize expected utility functions rather than arbitrary ones. Other refinements of RUM have restricted types by single-crossing conditions (Apesteguia, Ballester, and Lu, 2017), quasi-linearity (McFadden, 1973; Williams, 1977; Daly and Zachery, 1979; Yang and Kopylov, 2023), and other assumptions. Behavioral patterns, such as ambiguity aversion (Lu, 2021), have also been discussed within the context of random utility. All of these refinements preserve the assumption that μ is invariant of the consumption menu.

In this paper, we propose a novel version of random utility that has

- a discrete (i.e., finite) type space Θ where all types $\theta \in \Theta$ combine expected utility maximization with endogenous tie-breaking rules;
- a distribution of types μ_A that can depend on the menu A and capture context dependence and self-selection;
- a unique identification for both Θ and μ_A.

We still consider a representation where the distribution $\mu(\theta)$ is invariant of A. This case is a discrete version of the GP's REU (henceforth, DREU), but our axioms and identifications are more parsimonious than theirs and rely more on algebraic rather than analytical techniques. Moreover, the algebraic approach makes the consumption domain X more flexible than in GP, where X is a finite-dimensional simplex of lotteries. In our framework, X is convex, but need not be closed, or bounded, or finite dimensional. For example, X may consist of monetary gambles with payoffs on the real line, or X can be the non-negative orthant of consumption bundles.

More substantially, we depart from REU by allowing the distribution $\mu_A(\theta)$ to vary with the menu A. In general, it is only required that the support $\Theta = \{\theta : \mu_A(\theta) > 0\}$ be finite and invariant of A. This Θ is called a discrete type space. It is used to represent observable likelihoods $\rho(x, A)$ of choosing any element x in any finite consumption menu A. Our most general representation takes the form

$$\rho(x, A) = \mu_A\{\theta \in \Theta : x \text{ is the best element for } \theta \text{ in } A\} \tag{1}$$

where $\mu_A(\theta)$ is the probability of type θ making a choice in the menu A.

Each type θ is assumed to maximize some expected utility function u_θ in each menu A. However, in order for representation (1) to be well-defined, each type $\theta \in \Theta$ must also incorporate some endogenous tie-breaking rule that selects a unique maximizer for u_θ in A. Formally, each θ is associated with a total, complete, transitive binary relation that satisfies von Neumann-Morgenstern's Independence axiom as well. Such total types have been used in finite-dimensional contexts by Fishburn (1982), Myerson (1986), Blume, Branderburger, and Dekel (1991) and others. We argue that there is a rich family of total types even if X is not finite dimensional. Thus we provide a novel way to accommodate tie-breaking in random utility models. Without totality, Piermont (2022) relaxes GP's model to represent only those choices that are strict maximizers for corresponding types.

We show several characterization results for stochastic choice rules. First, Theorem **??** below characterizes representation where $\mu_A = \mu$ is menu-invariant and relates this result to GP's model. Our main result (Theorem **??** below) characterizes representation (1) and provides a convenient platform for various refinements that impose more structure on the menu-contingent distribution μ_A and/or types θ. We obtain several such refinements.

Theorem **??** summarizes the implications of the standard monotonicity principle that asserts

$$\rho(x, A) \geq \rho(x, A \cup B)$$

for all menus $A, B \in \mathcal{M}$ and elements $x \in A$. It turns out to be equivalent for the function μ_A to satisfy

$$\mu_A(\theta) \geq \mu_{A \cup B}(\theta)$$

for all types $\theta \in \Theta$ that have the same maximal element in A and in $A \cup B$. This finding can be interpreted in terms of *self-selection*: types can increase their participation in the menu $A \cup B$, but only if they find better choices in the larger menu. For example, it should be reasonable for gamblers to show up in higher numbers to bookmakers that offer them more favorable odds on some sporting event. Even the expansion of bet types with the rise of online gambling can incentivize new customers to enter the market (Hing et al., 2022). This can also be observed in the dining industry. Garnett et al. (2019) found that doubling the options of vegetarian meals increased sales of such meals by around 15%.

By contrast, self-selection cannot be detected in the general RUM examples because it can keep all Block-Marschak polynomials non-negative. In that example, the presence of self-selection makes the general RUM *misspecified* without rejecting this model outright. Our DREU model does not have this identification issue and disentangles self-selection from the composition of the type space Θ.

More broadly, there are some behavioral patterns, such as reasoned-based heuristics in Shafir, Simonson, and Tversky (1993), where Monotonicity does not hold and hence, self-selection is no longer a plausible explanation. We use *context dependence* as a blanket term to refer to all such patterns (see a literature review by Rabin (1998)). An example of such patterns is extremeness aversion. Sharpe et al. (2008) observed that the removal of the largest drink size (44-oz) from the menu prompted consumers to switch from the 32-oz drink size (which is the largest size in the new menu) to 21 ounces.

Finally, we discuss applications where the type space Θ captures heterogeneous *risk attitudes* and *beliefs*. For example, agents make more risk neutral choices when lotteries are presented in a binary format (i.e., only two states of the world have positive probabilities) instead of lotteries that are more complex or compound lotteries (Harrison et al., 2013). Changes in beliefs arise in the form of partition dependence. Ahn and Ergin (2010) observed that agents, when choosing deductible plans, place a higher weight on surgeries when they are individually listed than when they are bundled under the umbrella term "surgery". In such situations, the consumption domain X can be taken consists of lotteries or Anscombe-Aumann (1963) (henceforth, AA) uncertain prospects called acts.

References

M. Agranov and P. Ortoleva. Stochastic choice and preferences for randomization. *Journal of Political Economy*, 125:40–68, 2017.

D. S. Ahn and H. Ergin. Framing contingencies. 78(2):655–695, 2010.

F. Anscombe and R. Aumann. A definition of subjective probability. *Annals of Mathematical Statistics*, 34:199–205, 1963.

J. Apesteguia, M. Ballester, and J. Lu. Single-crossing random utility models. *Econometrica*, 85:661–674, 2017.

H. Block and J. Marschak. Random orderings and stochastic theories of response. In Olkin, Ghurye, Hoeffding, Madow, and Mann, editors, *Contributions to Probability and Statistics*, pages 97–132. Stanford University Press, Palo Alto, 1960.

L. Blume, A. Branderburger, and E. Dekel. Lexicographic probabilities and choice under uncertainty. *Econometrica*, pages 61–79, 1991.

A. Daly and S. Zachery. Improved multiple choice models. In D. Hensher and Q. Dalvi, editors, *Identifying and Measuring the Determinants of Mode Choice*. London: Teakfield, 1979.

J. Falmagne. A representation theorem for finite random scale systems. *Journal of Mathematical Psychology*, 18:52–72, 1978.

P. C. Fishburn. *The Foundations of Expected Utility*. D. Reidel Publishing Company, Dordrecht, 1982.

E. E. Garnett, A. Balmford, C. Sandbrook, M. A. Pilling, and T. M. Marteau. Impact of increasing vegetarian availability on meal selection and sales in cafeterias. *Proceedings of the National Academy of Sciences*, 116(42):20923–20929, 2019.

F. Gul and W. Pesendorfer. Random expected utility. *Econometrica*, 74:121–146, 2006.

G. W. Harrison, J. MartÃnez-Correa, and J. T. Swarthout. Inducing risk neutral preferences with binary lotteries: A reconsideration. *Journal of Economic Behavior & Organization*, 94:145–159, 2013.

N. Hing, M. Smith, M. Rockloff, H. Thorne, A. M. T. Russell, N. A. Dowling, and B. Breen. How structural changes in online gambling are shaping the contemporary experiences and behaviours of online gamblers: an interview study. *BMC Public Health*, 22(1620), 2022.

J. Lu. Random ambiguity. *Theoretical Economics*, 16(2):539–570, 2021.

D. McFadden. Conditional logit analysis of qualitative choice behavior. In P. Zarembka, editor, *Frontiers in Economics*. Academic Press, New York, 1973.

D. McFadden. Economic choices. *American Economic Review*, 91(3):351–378, 2001. doi: 10.1257/aer.91.3.351.

R. Myerson. Multistage games with communication. *Econometrica*, pages 323–358, 1986.

E. Piermont. Disentangling strict and weak choice in random expected utility models. *Journal of Economic Theory*, 202:201–230, 2022.

M. Rabin. Psychology and economics. *Journal of Economic Literature*, pages 11–46, 1998.

E. Shafir, I. Simonson, and A. Tversky. Reason-based choice. *Cognition*, 49:11–36, 1993.

K. M. Sharpe, R. Staelin, and J. Huber. Using extremeness aversion to fight obesity: Policy implications of context dependent demand. *Journal of Consumer Research*, 35(3):406–422, 2008.

K. E. Train. Recreation demand models with taste differences over people. *Land economics*, pages 230–239, 1998.

H. Williams. On the formulation of travel demand models and economic measures of user benefit. *Environment and Planning A*, 9:285–344, 1977.

E. Yang and I. Kopylov. Random quasi-linear utility. *Journal of Economic Theory*, 209, 2023. doi: 10.1016/j.jet.2023.105650.