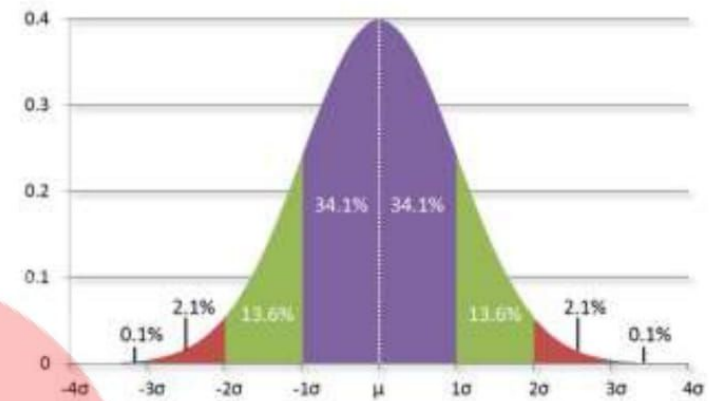# Distribution
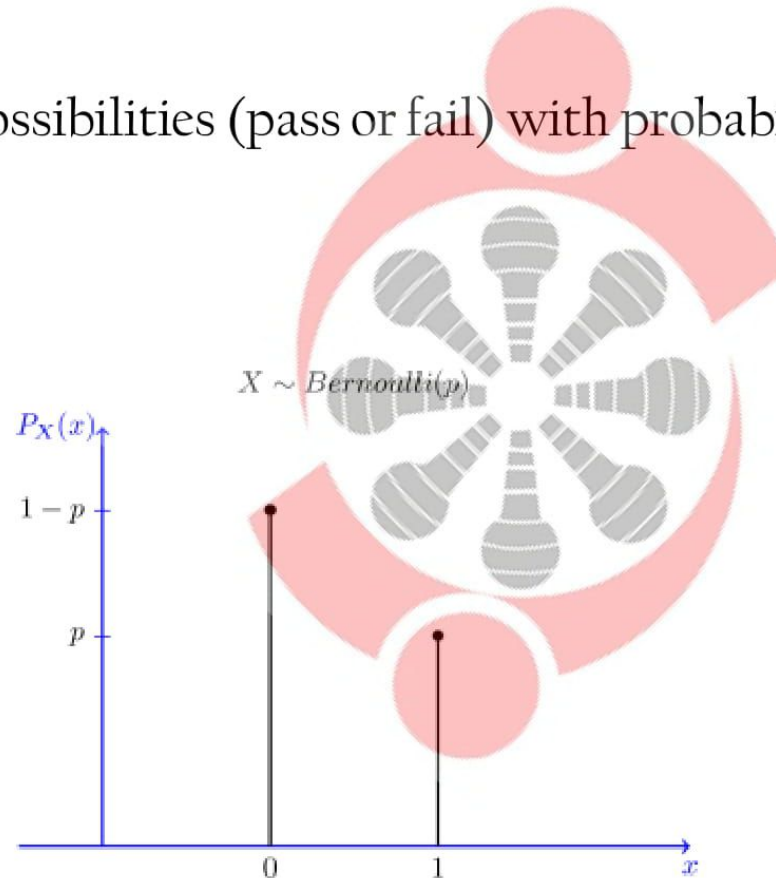
Bernoulli Distribution

Bernouli
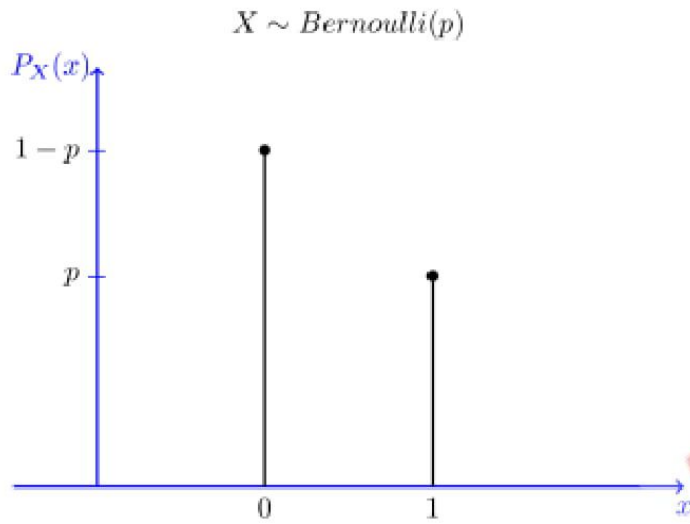
- There are two possibilities (pass or fail) with probability p of success and

  $q = 1\text{-}p$ of failure..

$X \sim Bernoulli(p)$

$P_X(x)$

$1-p$

p

0   1 x

Expectation : p
Variance :   pq

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Bernouli

$$X \sim Bernoulli(p)$$

$$Expectation, E(x) = \sum x_i\, P(x_i)$$

$$= 1 * p + 0 * q$$

$$= p$$

$$Variance, Var = \sum (x_i - \mu)^2\, P(x_i)$$

$$= (1 - p)^2 * p + (0 - p)^2 * (1 - p)$$
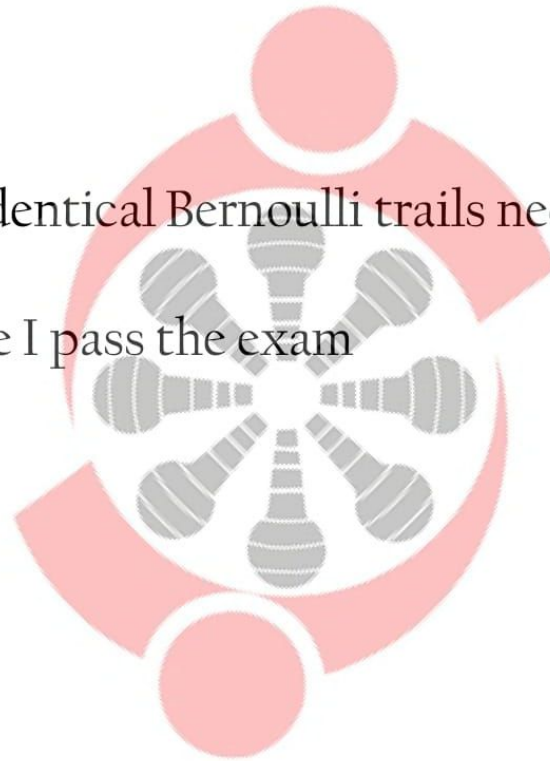
$$= p\,(1 - p)$$

$$= pq$$

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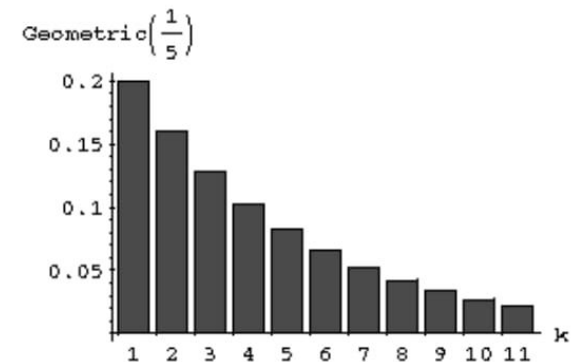
Geometric Distribution

Number of independent and identical Bernoulli trails needed to get ONE success.

Eg. Number of attempts before I pass the exam

Geometric Distribution

- You run a series of independent trail.

- There can be either a success or failure for each trail, and the probability of success is the same for each trail.

- How many trails are needed in order to get the first successful outcome.

$$Geometric\left(\frac{1}{5}\right)$$

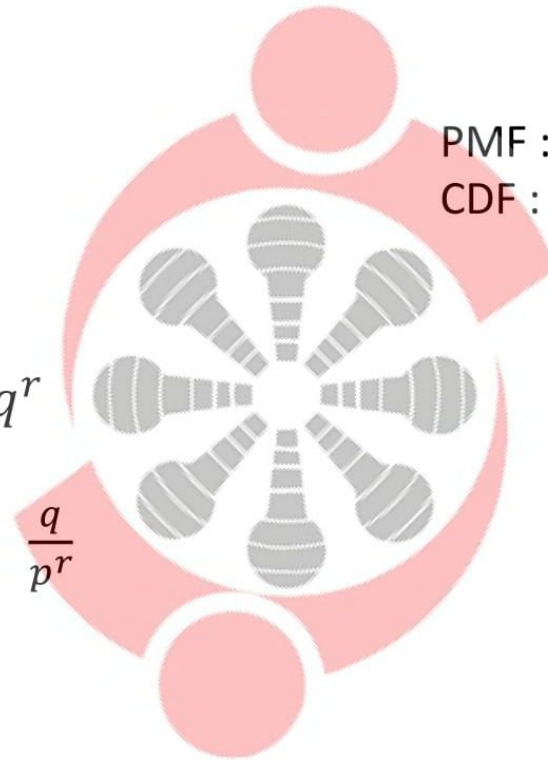# Geometric Distribution

- $\text{PMF*} = P(X = r) = q^{r-1}p$

- $P(X > r) = q^r$

- $CDF**, P(X \leq r) = 1 - q^r$

- $E(X) = \frac{1}{p}$ $var(x) = \frac{q}{p^r}$

PMF : Probaility Mass Funciton
CDF : Cumulative Distribution Function

7

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Binomial Experiment

- The process consists of a sequence of n trials.

- Only two exclusive outcomes are possible in each trail. One outcome is called "Success" and other a "failure".

- The probability of a success denotes p, does not change from trail to trail. The probability of failure is $1\text{-}p$ and is also fixed from trail to trail.

- The trails are independent; the outcome of previous trail not influence future trail.
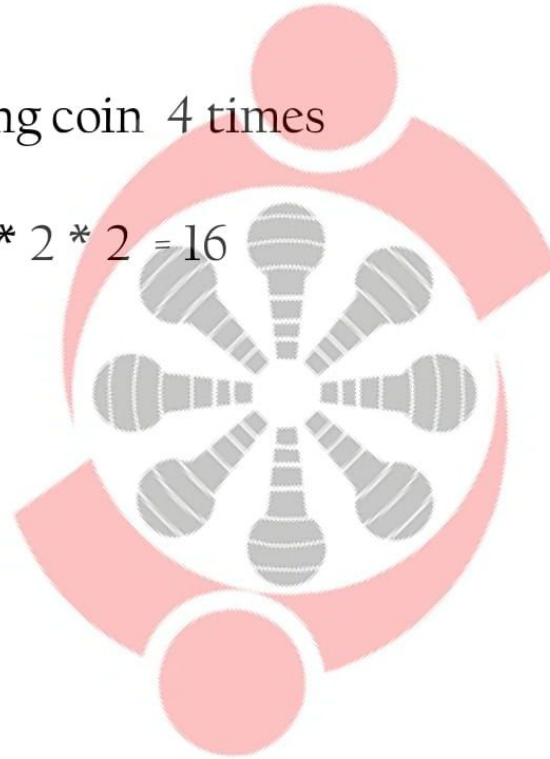
Binomial Variable

- Let's consider a coin $P(H) = 0.6$

- $P(T) = 0.4$

- $X = \#$ of heads after 10 flip of my coin

- Made up of independent trails

- Each trail can be classified as either success or failure

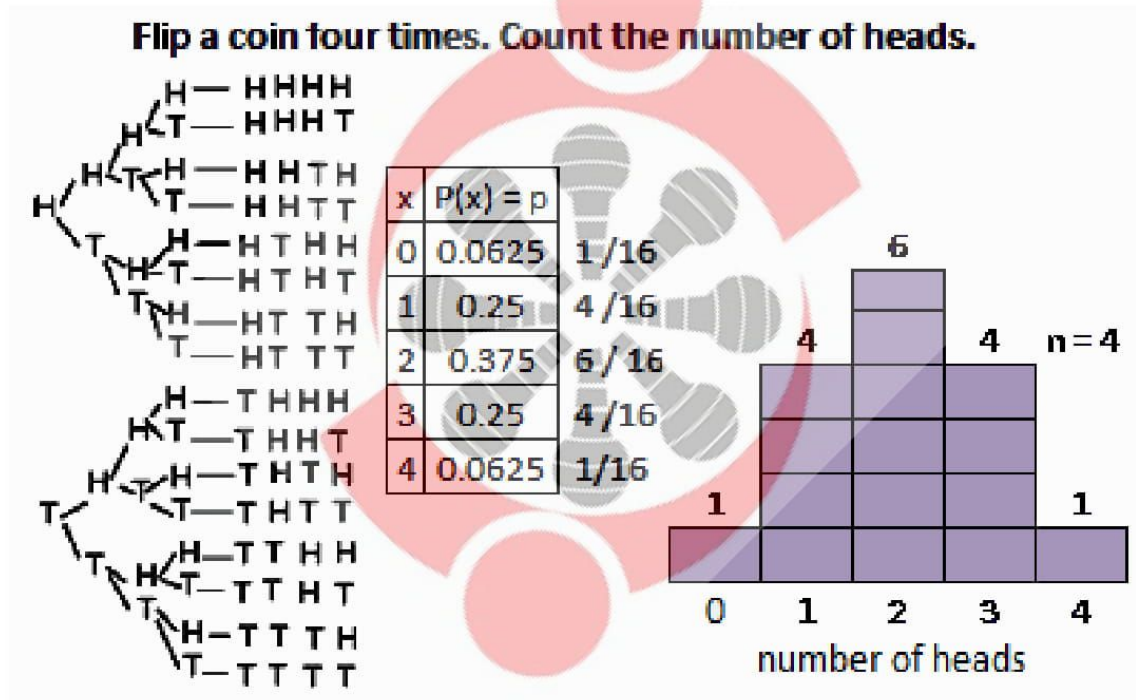- Probability of success on each trail is constant
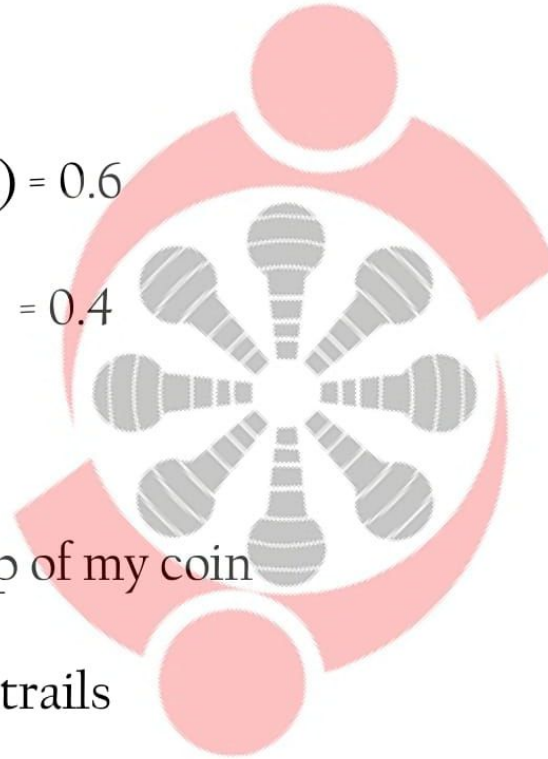
Binomial distribution
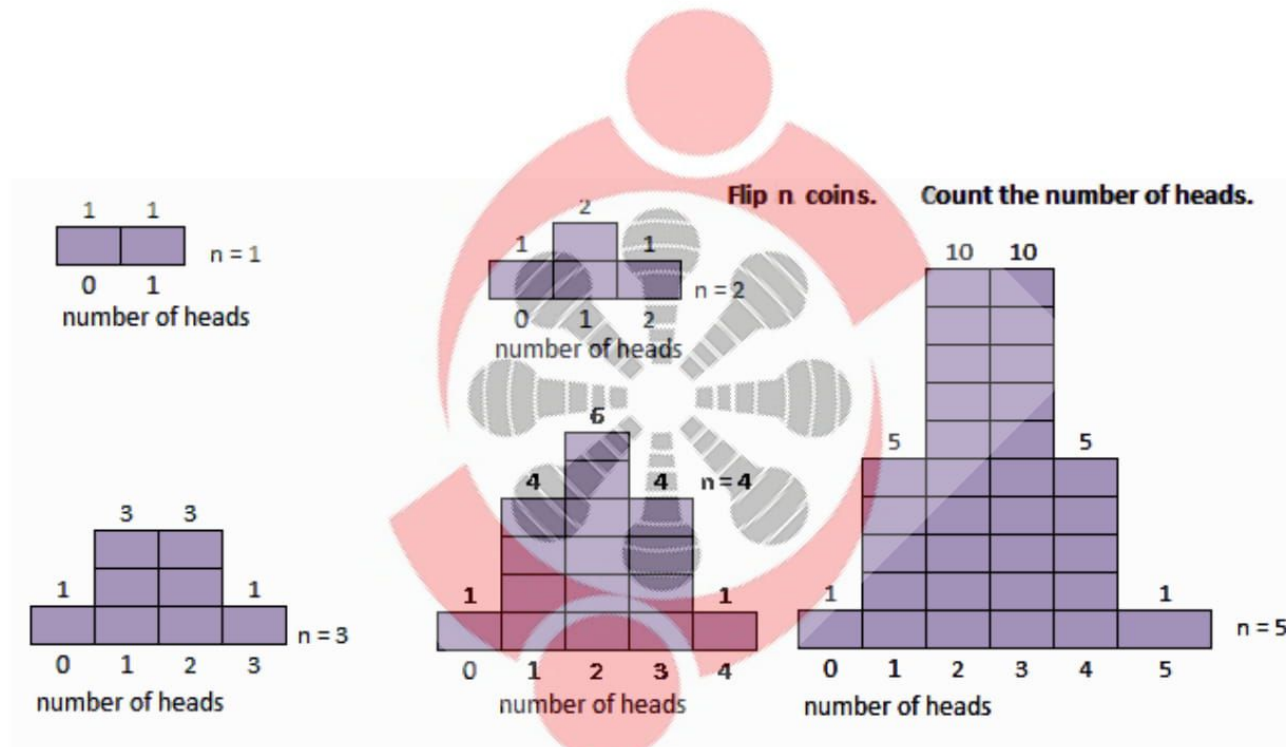
- $X = \#$ of heads after flipping coin 4 times

- Possible outcomes = 2 * 2 * 2 * 2 = 16

- $P(X = 0) = \dfrac{C_0^4}{32} = \dfrac{1}{32}$

- $P(X = 1) = \dfrac{C_1^4}{32} = \dfrac{5}{32}$

- $P(X = 2) = \dfrac{C_2^4}{32} = \dfrac{6}{32}$

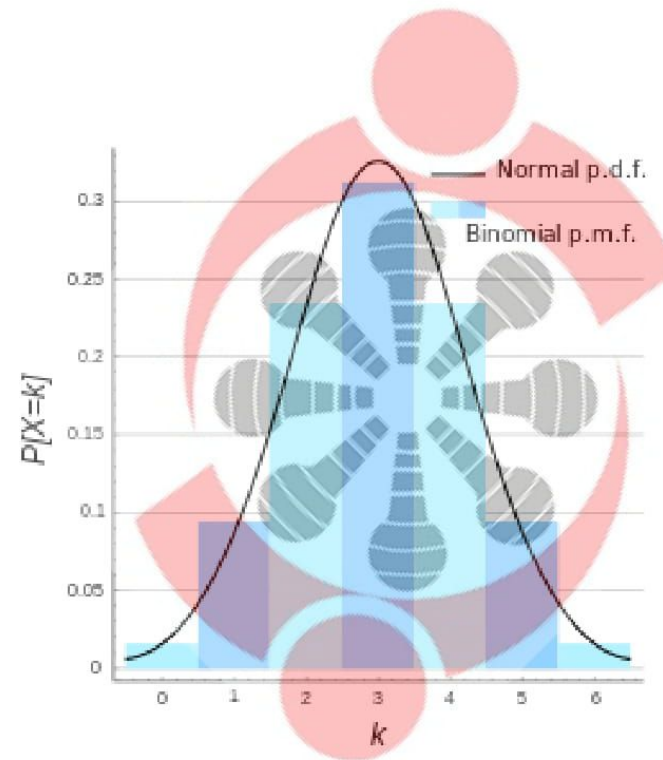# Binomial distribution

Flip a coin four times. Count the number of heads.

x	P(x) = p	
0	0.0625	1/16
1	0.25	4/16
2	0.375	6/16
3	0.25	4/16
4	0.0625	1/16

number of heads

Binomial distribution

Binomial distribution

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Binomial Probability example

P(Score) = 70 % or 0.7
P(miss) = 30 % or 0.3

P(Exactly 2 scores in 6 attempts) =

$$= C_2^6 \, (0.7)^2 \, (0.3)^4$$

P(Exactly k scores in n attempts) =

$$= C_k^n \, (p)^k \, (1-p)^{n-k}$$

Question

In this area it is expected that 9 cars are passing in a hours. What is the probability that there are exactly 6 cars are passing in this hour

$E(X) = \lambda = n * p = 9$ cars/hr

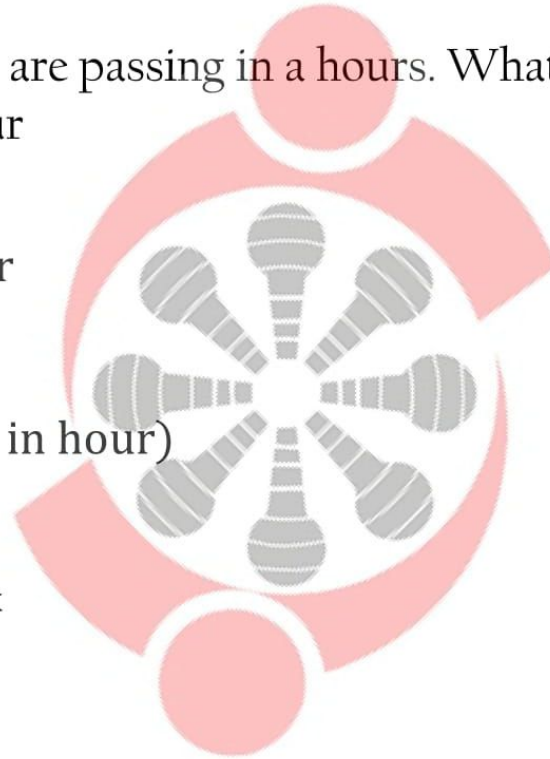Since , 1 hours = 60 minutes

So, n = 60

K = 6 (number of expected cars in hour)

$P(x = k) = C_k^n \, (p)^k \, (1 - p)^{n-k}$

$p = \lambda/n = 9 / 60$

Question

In this area it is expected that 9 cars are passing in a hours. What is the probability that there are exactly 6 cars are passing in this hour

$$p \quad = \quad \lambda / n = 9 / 60$$
$$1 - p = 51 / 60$$

Therefore, probability of exactly 6 cars are passing in an hours is

$$P(x = 6) = C_6^{60} \left(\frac{9}{60}\right)^6 \left(\frac{51}{60}\right)^{54}$$

$$n = 3600 \text{ (no. of seconds in an hour)}$$

Poisson Distribution

Poisson

E(X) = λ $= n * p$

P(X = k) $= \lim\limits_{n \to \infty} C_k^n \, (p)^k \, (1 - p)^{n-k}$

$$= \quad \frac{\lambda^k}{k\,!} \, e^{-\lambda}$$

λ = 9 cars pass

P(X = 2)
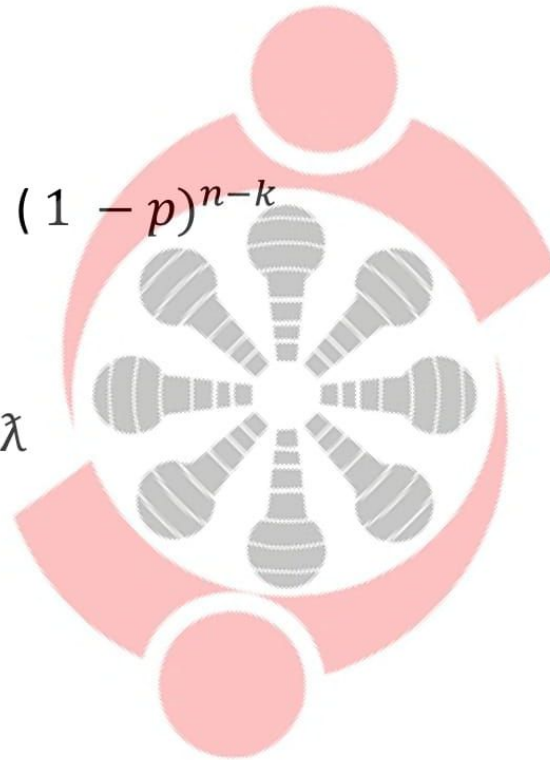
Question

Q. Probability that a cars will not pass in 6 min

$$= \frac{\lambda^k}{k!} e^{-\lambda}$$

For 1st second:

$$P(x = 0) = \frac{\lambda^0}{0!} e^{-\lambda} = e^{-\lambda} \quad (\text{for 1 sec})$$

For n second:

Probability that a cars will pass for n sec.

$$= e^{-n\lambda}$$

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Exponential Distribution

Q. Probability that a cars will pass in n sec

$$1 - e^{-n\lambda}$$

$$\text{CDF} = 1 - e^{-n\lambda}, n \geq 0$$

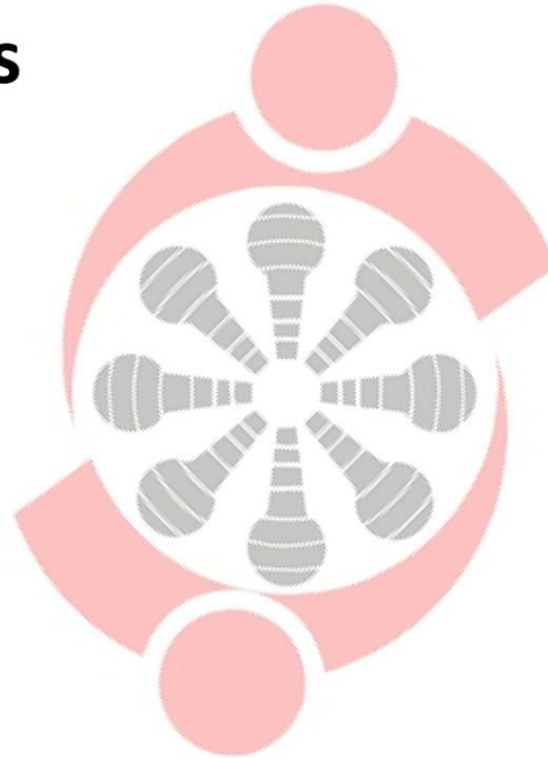$$\text{PDF} = \lambda e^{-n\lambda}, n \geq 0$$

Exponential Distribution

- Poisson process

- Continuous analog of Geometric distribution

$$E(X) = \frac{1}{\lambda} \qquad var(X) = \frac{1}{\lambda^2}$$

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Reference

- https://www.mathsisfun.com/data/index.html
- **Head First: Statistics**
- **KhanAcademy**

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