Definition of Burst Error

Before we begin, we need to define the definition of a burst error. The common definition of a burst error can be defined as a consecutive error preceded with and followed by correct bits. Note that a burst error definition in Wikipedia is slightly different, but I believe this definition is rarely used in wireline communication.

PRE-FEC BER:

Since the burst error length is uniformly distributed between 1 and 20, and the system does not include a DFE in its receiver path, the PRE-FEC BER computation is relatively easy:

$$BER_{PRE-FEC} = \frac{p_{eb} \cdot \frac{1}{20} \cdot \sum_{i=1}^{20} i}{(1 - p_{eb}) + p_{eb} \cdot \frac{1}{20} \cdot \sum_{i=1}^{20} i}, \qquad where \; p_{eb} = 0.0005$$

The numerator represents the expected length of burst error whenever a bit error occurs (with a chance of 0.0005), and the denominator represents the expected length of burst error followed by a correct bit, indicating the end of a burst error.

POST-FEC BER:

[1] presents a method to compute a post-FEC BER using a Markov-chain and Trellis diagram. Some of the computation methods are useful for this problem (especially Eq. (28)), but some methods cannot be directly applicable to this problem. This is because the burst error length is determined by uniformly distributed, making the model a non-Markovian process. Sweeping the entire bit (5280 for KR4 and 5440 for KP4) and computing the symbol and bit error probability in a brute-force fashion is also difficult since each bit has 21 different possible branches (1 for correct, 20 for burst errors).

Instead, one possible way is using the following assumption that sacrifices the accuracy but makes the computation simple and feasible:
- We assume that having 2+ burst error events in a single symbol have extremely rare chance to occur.
  - Example: A bit error occurred on the 2nd bit of the 'i'-th symbol with a burst error length of 20. This burst error ends on the 1st bit of the '$i+2$'-th symbol, and the 2nd bit of the '$i + 2$'-th symbol must be correct by definition of the burst error. With the above assumption, we exclude the event of having another burst error between the 3rd and the 10th bit of the '$i + 2$'-th symbol.

For the rest of this report, I assume that KR4 FEC (528, 514, 7) is used.

Below defines few important parameters used throughout this report:

$$p_{eb} = 0.0005, \qquad p_{es} = 1 - (1 - p_{eb})^{10}, \qquad p_{cs} = (1 - p_{eb})^{10}$$

p_{es} and p_{cs} represent the chance of getting and avoiding a symbol error, respectively.

Below table summarizes the burst error combinations for each index in a symbol to get a certain symbol errors (1~3).

index	1	2	3	4	5	6	7	8	9	10	total
1 symbol error	1~10	1~9	1~8	1~7	1~6	1~5	1~4	1~3	1~2	1	55
2 symbol error	11~20	10~19	9~18	8~17	7~16	6~15	5~14	4~13	3~12	2~11	100
3 symbol err	X	20	19~20	18~20	17~20	16~20	15~20	14~20	13~20	12~20	45

When a bit error occurs in a symbol, the probability of having 1~3 symbol errors are:

$$p_{es1} = p_{es} \cdot \frac{55}{55 + 100 + 45}, \qquad p_{es1} = p_{es} \cdot \frac{100}{55 + 100 + 45}, \qquad p_{es1} = p_{es} \cdot \frac{45}{55 + 100 + 45}$$

We can also compute the expected length of burst error for 1~3 symbol errors:

$$E_{es1} = \frac{\sum_{i=1}^{10} \sum_{j=1}^{i} j}{55} = 4, \qquad E_{es2} = \frac{\sum_{i=1}^{10} \sum_{j=1+i}^{10+i} j}{100} = 11, \qquad E_{es3} = \frac{\sum_{i=1}^{10} \sum_{j=11+i}^{20} j}{45} = 17.33$$

As the next step, let's find all the possible symbol error combinations leading to ≤7 symbol errors:

Total # of symbol error per FEC block	1	2	3	4	5	6	7
combinations	1	11 2	111 12 3	1111 112 13 22 ~~4~~	11111 1112 113 122 ~~14~~ 23 ~~5~~	111111 11112 1113 1122 ~~114~~ 123 ~~15~~ ~~24~~ 33 ~~6~~	1 1 1 1 1 1 1 1 1 1 1 1 2 1 1 1 1 3 1 1 1 2 2 ~~1 1 1 4~~ 1 1 2 3 ~~1 1 5~~ ~~1 2 4~~ 1 3 3 ~~1 6~~ ~~2 5~~ ~~3 4~~ ~~7~~

Note that we cannot compute the probability for the combinations with strikethrough and red font. This is because of the assumption made in the beginning.

Let's assume $n = 528, m = 10$. The BER for a single symbol error per FEC block is:

$$BER_{js=1} = \binom{n}{1} \cdot p_{es1} \cdot p_{cs}^{n-1} \cdot \frac{E_{es1}}{n \cdot m}$$

Similarly, the BER for double symbol errors per FEC block is:

$$BER_{js=2,\{1,1\}} = \binom{n}{2} \cdot p_{es1}^2 \cdot p_{cs}^{n-2} \cdot \frac{2 \cdot E_{es1}}{n \cdot m}$$

$$BER_{js=2,\{2\}} = \binom{n-1}{1} \cdot p_{es2} \cdot p_{cs}^{(n-1)-1} \cdot \frac{E_{es2}}{n \cdot m}$$

$$BER_{js=2} = BER_{js=2,\{11\}} + BER_{js=2,\{2\}}$$

BER for triple symbol errors per FEC block:

$$BER_{js=3,\{1,1,1\}} = \binom{n}{3} \cdot p_{es1}^3 \cdot p_{cs}^{n-3} \cdot \frac{3 \cdot E_{es1}}{n \cdot m}$$

$$BER_{js=3,\{1,2\}} = \binom{n-1}{1} \cdot p_{es2} \cdot \binom{n-2}{1} \cdot p_{es1} \cdot p_{cs}^{(n-2)-1} \cdot \frac{2 \cdot E_{es1} + E_{es2}}{n \cdot m}$$

$$BER_{js=3,\{3\}} = \binom{n-2}{1} \cdot p_{es3} \cdot p_{cs}^{(n-2)-1} \cdot \frac{E_{es3}}{n \cdot m}$$

$$BER_{js=3} = BER_{js=3,\{1,1,1\}} + BER_{js=3,\{1,2\}} + BER_{js=3,\{3\}}$$

We repeat this process up to 7 symbol errors per FEC block. Then we get:

$$BER_{POST-FEC} = BER_{PRE-FEC} - \sum_{i=1}^{7} BER_{js=i}$$

REFERENCE:

[1]   M. Yang, S. Shahramian, H. Shakiba, H. Wong, P. Krotnev, and A. C. Carusone, "Statistical BER Analysis of Wireline Links With Non-Binary Linear Block Codes Subject to DFE Error Propagation," *IEEE Trans. Circuits Syst. I Regul. Pap.*, vol. 67, no. 1, pp. 284–297, Jan. 2020.