Bouncer Challenge

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DE1 Solid Mechanics (Group 21)

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2 SECTION ONE

2.1 INTRODUCTION

2.1.1 Experiment Aim

The bouncer challenge experiment set as its main aim to determine and predict the horizontal displacement of a stainless-steel ball from the point of leaving an inclined PVC tube to the point of the second bounce of the ball. The tube was set to a given height above the ground and a given angle.

2.1.2 Assumptions Made

To simplify the computation of the aforementioned displacement, several assumptions about the conditions of the experiment were made. These are:

1. The ball was said to have been released from rest.
2. It was assumed that air resistance was negligible throughout the entire motion of the ball.
3. The tube through which the ball rolls initially (see Figure 1) was said to be smooth.
4. It was said that the ball does not slip while rolling down the tube.
5. It was assumed that all the gravitational potential energy is transferred to kinetic energy with a translational and a rotary component.
6. For the calculation of inertia, the ball was considered to have a solid sphere shape.
7. The trajectory of the ball was said to follow projectile motion.
8. The collision between the ball and the ground was assumed to be frictionless, hence the coefficient of restitution was only applied to the vertical component of the velocity.

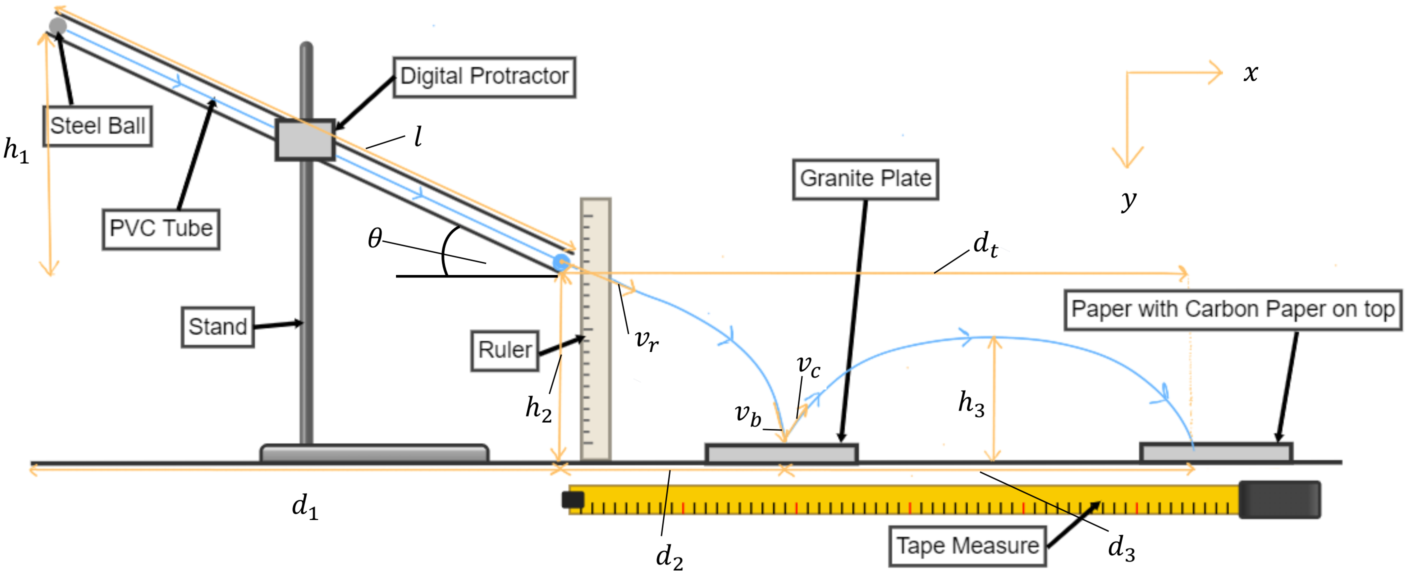


Figure 1 - Bouncer Challenge experiment setup (adapted from DRAW bouncer challenge notes) (1)

2.2 METHODS

The following calculations were used to derive an expression for the horizontal distance from the location of the second bounce of the stainless-steel ball to the lower end of the tube, d_t. This distance was defined as the following:

$$d_t = d_2 + d_3 \tag{1}$$

2.2.1 d_2 derivation

Based on assumption number 5, which talks about the total energy of the ball at the top of the tube, E_{top} being the same as the total energy at the bottom , E_{bottom}, it can be said that:

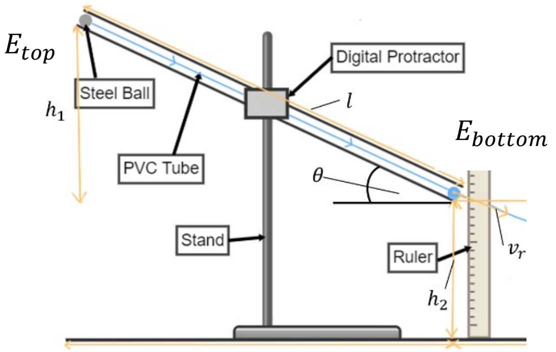


Figure 2 - Diagram showing how the ball initially moves through the tube.

$$E_{top} = E_{bottom} \tag{2}$$

The energy at the top of the tube can be said to only consist of the gravitational potential energy of the ball, as assumption number 1 states that the ball is released from rest. Therefore:

$$E_{top} = mg(h_1 + h_2) \tag{3}$$

where: m is the mass of the ball, g is the acceleration due to gravity, h_1 is the vertical distance from the bottom to the top of the tube and h_2 is the distance from the ground to the bottom of the tube. As shown in Figure 1.

Based on assumptions 1-5, the energy at the bottom of the tube can then be defined as follows (1):

$$E_{bottom} = mgh_2 + \frac{1}{2}mv_r{}^2 + \frac{1}{2}I\omega^2 \tag{4}$$

where: v_r is the velocity of the ball when leaving the tube, I is the moment of inertia and ω is the angular velocity of the ball.

The ball was assumed to be a solid sphere (assumption 6) and for such shape, I and ω can be defined as (1) (2):

$$I = \frac{2}{5}mr^2, \qquad \omega = \frac{v}{r} \tag{5}$$

which in turn means that the energy at the bottom of the tube can be expressed as:

$$E_{bottom} = mgh_2 + \frac{1}{2}mv_r{}^2 + \frac{1}{2} \times \frac{2}{5}mr^2 \times \left(\frac{v}{r}\right)^2 \tag{6}$$

Using conservation of energy, we can say:

$$mg(h_1 + h_2) = mgh_2 + \frac{1}{2}mv_r{}^2 + \frac{1}{2} \times \frac{2}{5}mr^2 \times \left(\frac{v}{r}\right)^2 \tag{7}$$

simplifying:

$$gh_1 = \frac{7}{10}v_r{}^2 \tag{8}$$

$$v_r = \sqrt{\frac{10}{7}gh_1} \tag{9}$$

The height, h_1 can then be expressed as:

$$h_1 = l \times sin\theta \tag{10}$$

where l is the length of the tube and θ is the angle between the tube and the ground (see Figure 2). Substituting this in we get:

$$v_r = \sqrt{\frac{10}{7} g \times l \times sin\theta} \tag{11}$$

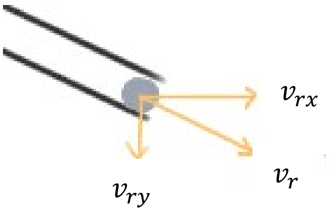


Figure 3 - A diagram showing the velocity of the ball when leaving the tube alongside each of the components of the velocity.

This equation produces the velocity of the ball as it leaves the PVC tube. Then, using assumptions 2 and 7 we can model the further motion of the ball as a projectile by considering the vertical and horizontal components of v_r (see Figure 3):

$$v_{rx} = v_r cos\theta \tag{12}$$

$$v_{ry} = v_r sin\theta \tag{13}$$

Solving in the vertical direction we can one of the kinematic motion equations.

$$s = ut + \frac{1}{2}at^2 \tag{14}$$

where s is the displacement, a is the acceleration and u is the initial velocity.

This equation can then be applied to our system such that:

$$u = v_{ry} \tag{15}$$

$$a = \boldsymbol{g} \tag{16}$$

$$s = h_2 \tag{17}$$

where $\boldsymbol{g}$ is the acceleration due to gravity. Substituting in:

$$h_2 = v_{ry}t_2 + \frac{1}{2}\boldsymbol{g}{t_2}^2 \tag{18}$$

Solving for t_2:

$$t_2 = \frac{\sqrt{2\boldsymbol{g}h_2 + {v_{ry}}^2} - v_{ry}}{\boldsymbol{g}} \tag{19}$$

Solving horizontally, we use a different kinematic equation and assume that there is no air resistance (assumption 2).

$$s = ut \tag{20}$$

$$d_2 = v_{rx}t_2 \tag{21}$$

Therefore, the distance d_2 can be expressed as:

$$d_2 = v_{rx}\frac{\sqrt{2\boldsymbol{g}h_2 + {v_{ry}}^2} - v_{ry}}{\boldsymbol{g}} \tag{22}$$

Horizontally, no forces are acting on the ball. Hence v_{rx} stays constant during motion. However, the ball is accelerating downwards – therefore v_{ry} only holds its value instantaneously after leaving the tube. The velocity just before the ball hits the ground, v_{by} (see Figure 4) must then be found using another kinematic equation:

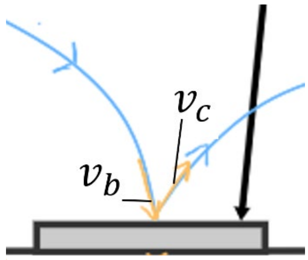


Figure 4 - A diagram showing the velocities of the ball before and after bouncing off the plate.

$$v^2 = u^2 + 2as \tag{23}$$

where v is the final velocity. Substituting values from the ball system:

$$v_{by}{}^2 = v_{ry}{}^2 + 2\boldsymbol{g}h_2 \tag{24}$$

Rearranging for v_{by}:

$$v_{by} = \sqrt{v_{ry}{}^2 + 2\boldsymbol{g}h_2} \tag{25}$$

2.2.2 $\boldsymbol{d_3}$ derivation

To derive this distance, assumptions must be considered. It is said that the collision between the ball and the ground is frictionless and in turn the coefficient of restitution, e applies only to the vertical component of velocity.

$$e = \frac{velocity\ after\ collision}{velocity\ before\ collision} \tag{26}$$

Substituting the values of the ball system and rearranging:

$$e = \frac{v_{cy}}{v_{by}} \tag{27}$$

$$\Longrightarrow v_{cy} = ev_{by} \tag{28}$$

The ball now moves in the opposite direction vertically; hence the equation is modified to:

$$v_{cy} = -ev_{by} \tag{29}$$

After the collision with the ground, the ball again follows projectile motion. This means that again assumptions 2 and 7 must be considered. Considering vertical motion:

$$s = ut + \frac{1}{2}at^2 \tag{30}$$

Substituting in values from the ball system:

$$0 = v_{cy}t_3 + \frac{1}{2}\boldsymbol{g}t_3{}^2 \tag{31}$$

Rearranging:

$$t_3 = -\frac{2v_{cy}}{\boldsymbol{g}} \tag{32}$$

As per assumption 8, the horizontal velocity, v_{rx} has remained constant. This means the following equation can be used:

$$s = ut \tag{33}$$

Substituting in values of the ball system:

$$d_3 = v_{rx}t_3 \tag{34}$$

Therefore distance d_3 may be expressed as:

$$d_3 = -\frac{2v_{rx}v_{cy}}{\boldsymbol{g}} \tag{35}$$

or substituting previously found variables:

$$d_3 = \frac{2ev_{rx}\sqrt{v_{ry}{}^2 + 2\boldsymbol{g}h_2}}{\boldsymbol{g}} \tag{36}$$

2.2.3 $\boldsymbol{d_t}$ expression

It was said that:

$$d_t = d_2 + d_3 \tag{1}$$

Based on the above derivation, this can now be substituted into such that:

$$d_t = v_{rx}\frac{\sqrt{2\boldsymbol{g}h_2 + v_{ry}{}^2} - v_{ry}}{\boldsymbol{g}} + \frac{2ev_{rx}\sqrt{v_{ry}{}^2 + 2\boldsymbol{g}h_2}}{\boldsymbol{g}} \tag{37}$$

Rearranging:

$$d_t = \frac{v_{rx}}{\boldsymbol{g}}\left(\sqrt{v_{ry}{}^2 + 2\boldsymbol{g}h_2} - v_{ry} + 2e\sqrt{v_{ry}{}^2 + 2\boldsymbol{g}h_2}\right) \tag{38}$$

$$d_t = \frac{v_{rx}}{\boldsymbol{g}}\left(\sqrt{v_{ry}{}^2 + 2\boldsymbol{g}h_2}(1 + 2e) - v_{ry}\right) \tag{39}$$

v_{rx} and v_{ry} were previously defined as follows:

$$v_{rx} = v_r cos\theta \tag{12}$$

$$v_{ry} = v_r sin\theta \tag{13}$$

Substituting these in:

$$d_t = \frac{v_r cos\theta}{\boldsymbol{g}}\left(\sqrt{(v_r sin\theta)^2 + 2\boldsymbol{g}h_2}(1 + 2e) - v_r sin\theta\right) \tag{40}$$

Lastly, v_r was previously shown to be:

$$v_r = \sqrt{\frac{10}{7}\boldsymbol{g}lsin\theta} \tag{9}$$

Therefore, the distance d_t can be found by using the following system of equations:

$$v_r = \sqrt{\frac{10}{7}\boldsymbol{g}lsin\theta} \tag{9}$$

$$d_t = \frac{v_r cos\theta}{\boldsymbol{g}}\left(\sqrt{(v_r sin\theta)^2 + 2\boldsymbol{g}h_2}(1 + 2e) - v_r sin\theta\right) \tag{40}$$

2.3 Results

2.3.1 d_t prediction

The equations derived in section 1.2 can now be used to predict the distance travelled by the steel ball. Shown below in Table 1 are the values that were used to compute the predicted distance.

Table 1 - The value of each variable with its respective uncertainty and lower and upper bounds

Variable	Uncertainty	Standard	Lower bound	Upper bound
Height, h_2	$\pm 0.005\ m$	$0.55\ m$	$0.545\ m$	$0.555\ m$
Angle, θ	$\pm 0.5°$	$25°$	$24.5°$	$25.5°$
Tube length, l	$\pm 0.005\ m$	$1.50\ m$	$1.495\ m$	$1.505\ m$
Coefficient of restitution, e	± 0.0087	0.8279	0.8192	0.8366
Acceleration due to gravity, g	$\pm 0.005\ ms^{-2}$	$9.81\ ms^{-2}$	$9.805\ ms^{-2}$	$9.815\ ms^{-2}$

In order to automate the process of calculation, the system of equations was transferred into Desmos. All the variables can be changed and a new value for d_t will be calculated automatically. Show below (Figure 5) is a screenshot showing the tool.

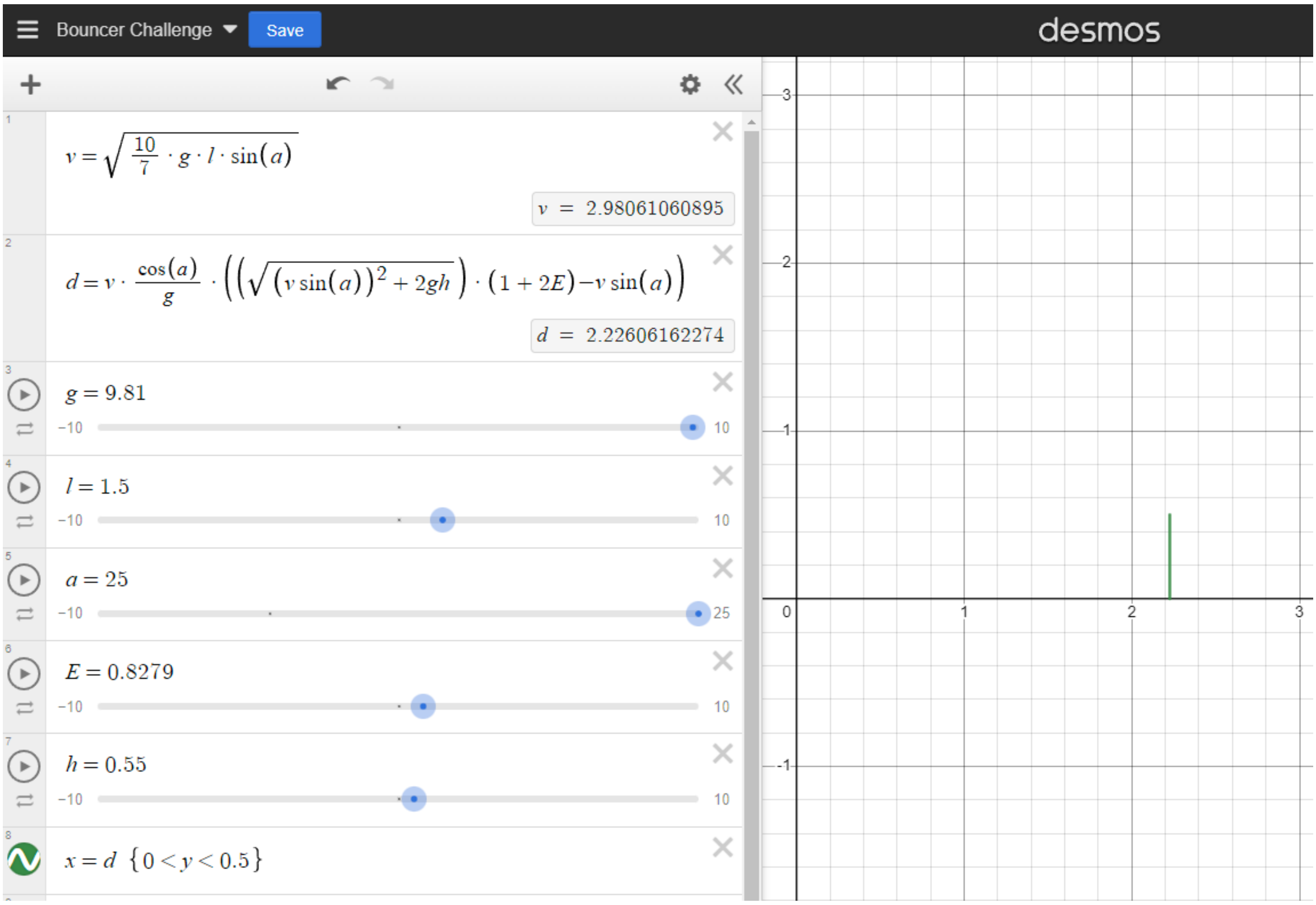


Figure 5 – Desmos tool setup to calculate the distance travelled with all the variables being easily editable.

Using the calculations shown above, the calculated value of d_t was $2.23\ m$ or $223\ cm$.

2.3.2 Sensitivity Analysis

A sensitivity analysis was performed to determine the biggest sources of error in the experiment. Shown below are the results of the analysis.

Table 2 - Sensitivity analysis results

Variable	d_t using lower bound (cm)	d_t using upper bound (cm)	Range (cm)
Height, h_2	221.6	223.6	2
Angle, θ	221.5	223.7	2.2
Tube length, l	222.2	223.0	0.8
Coefficient of restitution, e	220.9	224.3	3.4
Acceleration due to gravity, g	222.6	222.6	0

As can be seen, for all of the variables, using the lower bound of the variable's value produced a lower distance and the same is true the other way around. The coefficient of restitution has the biggest effect on the distance travelled by the steel ball, with the difference being $3.4\ cm$. This is then followed closely by the angle with a $2.2\ cm$ difference in d_t and by the height with a $2\ cm$ difference between the upper and lower bounds. Lastly, the uncertainty in the tube length produces minimal variation in the value of d_t of only $0.8\ cm$ and any variation in the value of $\boldsymbol{g}$ produces no change in the distance travelled by the ball.

Consequently, predictions for the lower and upper bounds of the distance travelled by the ball were calculated.

Table 3 - Final predictions after sensitivity analysis (to 3 s.f.)

Name	Value
d_t prediction using lower bound for all variables	$218\ cm$
d_t prediction using upper bound for all variables	$227\ cm$
d_t prediction using average of lower and upper bounds	$223\ cm$
Range	$9\ cm$

Therefore, the final prediction for d_t is $223 \pm 5\ cm$ to 3 significant figures. This is a value produced by considering all the variables shown above. The actual distance is likely to vary even more due to the assumptions that were made about the motion of the ball in section 2.2.

3 SECTION TWO

3.1 EXPERIMENT

The picture below (Figure 6) shows the results gathered by my group. The centre line (label 0 in the image) was placed at a distance of $223\ cm$ from the end of the tube as all members of the group agreed on the predicted distance being $223\ cm$.

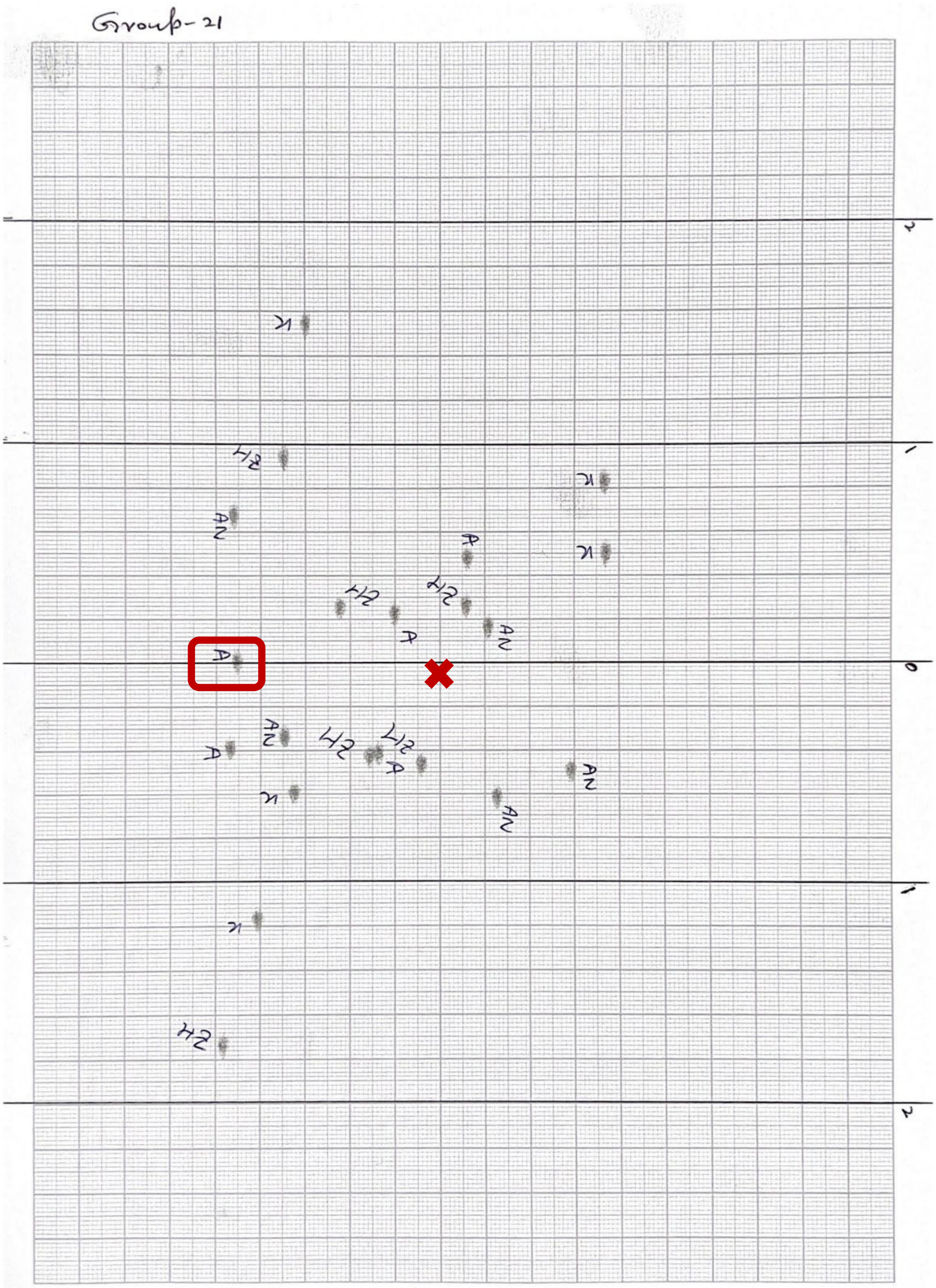
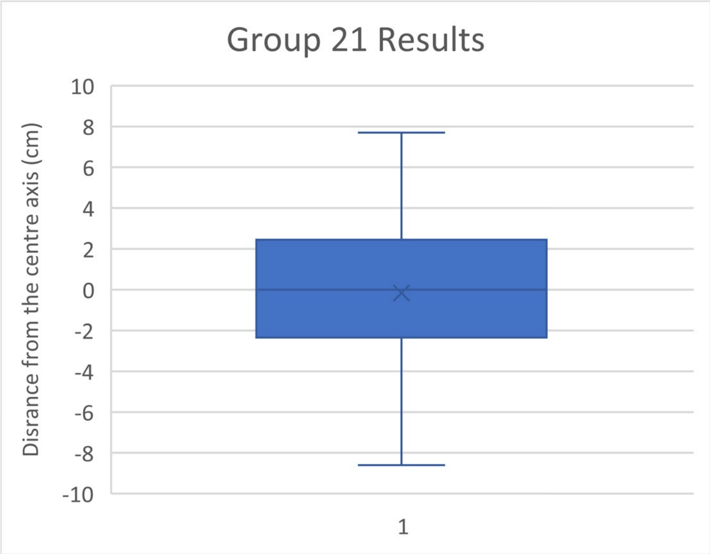


Figure 6 - Group 21 results

The median value has been marked on the sheet, within the A region of the sheet using a red rectangle. The mean has been marked using a red cross. The value of the mean is $0.16\ cm$ down from the centre line and the value of the median is on the line.



The plot on the left was used to visualise the results collected. Its y-axis uses the distance from the centreline, with positive being up from it and negative being down.

3.2 Discussion

After conducting the experiment, the assumptions made initially were reflected upon. This discussion can be found below.

3.2.1 The collision between the ball and the plate was frictionless (Assumption 8)

After closely analysing the results sheet, the ball leaves a waterdrop-shaped mark on the paper. This would suggest that the ball had travelled across it while bouncing off. During sliding, it can be assumed that there was friction between the ball and the paper. This can be extrapolated to the first bounce which would suggest that the collision was not frictionless and that the coefficient of restitution should also be applied to the horizontal component of the ball's velocity.

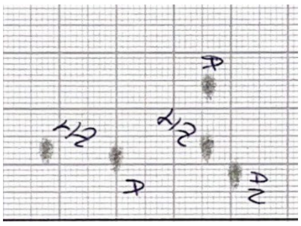


Figure 7 - An extract from the target sheet showing the droplet shaped points.

3.2.2 The ball rolls down the PVC tube without slipping (Assumption 4)

During the experiment it could be heard that the sound of the ball changes around when the ball passes the middle of the tube. After analysing the footage of the ball rolling, the ball starts slipping down the pipe at the point where the sound changes. This would suggest that not much of the kinetic energy gets converted to the rotary element. The equation for the energy of the ball at the bottom of the tube can now be changed into:

Figure 8 - A frame from the video which showed the ball slipping in the tube.

$$E_{bottom} = mgh_2 + \frac{1}{2}mv_r{}^2 \tag{41}$$

or rearranging for the velocity of the ball at the end of the tube:

$$v_r = \sqrt{2\boldsymbol{g}lsin\theta} \tag{42}$$

Using this modified form of the equation, the prediction of d_t changes to $262\ cm$. This value is significantly greater than the observed distance d_t. Therefore, it can be concluded that the original assumption was valid.

3.2.3 Negligible air resistance (Assumption 2)

While it is apparent that the ball will have experienced air resistance during flight, the actual magnitude of it can be concluded to have been very small. The discrepancy of the results is much more likely to be caused by other factors like the one talked about below in point 3.2.4, due to the random nature of the differences in d_t. If air resistance had a significant effect, then all of the results would be skewed towards the end of the tube which was not the case. Therefore, looking at the complexity such an addition would bring to the calculation, the decision to not include air resistance in the calculations was a right one.

3.2.4 The ball was released from rest (Assumption 1)

While this assumption was mostly down to the person dropping the ball, it proved surprisingly difficult to control the release of the ball in such a way that it would leave without being pushed down. This caused the velocity of the ball while leaving the tube to be greater than the predicted value and therefore overshoot the target. This can be seen on one of my attempts (K) in Figure 6 which has travelled much further than the other attempts. On the other hand, another member of the group (A) was able to control the ball's release more accurately because his landings all fall within the A area of the sheet.

3.2.5 Experiment setup issues

Additionally, some problems with the setup were found:

3.2.5.1 Challenging setup

The mechanism used to hold the tube in place proved to be rather unsteady. Initially, it was very difficult to setup as it was required to measure both the angle of the tube and the height from the ground simultaneously. This led to a situation where when one of the parameters was set perfectly, the other required adjustment which would alter the one that was already set. This caused a loop of continuous trial and error and eventually it was decided to settle with an angle of $25.69°$, which was outside of the initially assumed uncertainty range. This higher angle results in value of d_t of $224\ cm$.

Figure 9 - The digital protractor showing the angle to which the tube was set.

3.2.5.2 Unsteady equipment

While carrying out the experiment it became apparent that the mounting mechanism for the tube was rather unsteady and while the ball was rolling the whole setup shifted and rocked around. This motion is likely to change the parameters of the setup and to affect the accuracy of the prediction.

3.2.5.3 Target Sheet placement

The target sheet was placed on a slippery surface which caused it to slightly shift after every landing of the ball. This caused the centre line on sheet to move away from the target distance. This phenomenon is likely what caused some of the landings to happen before the centre line.

3.2.6 Other factors

It is likely that the results of the experiment were influenced by other factors which were beyond the control of the group carrying out the experiment. These might include things such as airflow caused by a person walking past the experiment or wind gusts from an open window.

Overall, the prediction made turned out to be very close to the mean and median values of where the ball landed. It can be concluded that the assumptions made, and the complexity level of calculations were effective and accurate.

4 REFERENCES

1. Nanayakkara PT. DE1 Solid Mechanics: DRAW week assignment – Bouncer.

2. LectureNotes - OneDrive [Internet]. [cited 2023 Feb 22]. Available from: https://imperiallondon-my.sharepoint.com/personal/tnanayak_ic_ac_uk/_layouts/15/onedrive.aspx?ga=1&id=%2Fpersonal%2Ftnanayak%5Fic%5Fac%5Fuk%2FDocuments%2FLectures%2FMechanics%2FMechanics2023%2FLecturesAndTutorials%2FLectureNotes%2FMechanics%5Flecture%5Fnotes%2Epdf&parent=%2Fpersonal%2Ftnanayak%5Fic%5Fac%5Fuk%2FDocuments%2FLectures%2FMechanics%2FMechanics2023%2FLecturesAndTutorials%2FLectureNotes