Preface

This book on numerical analysis grew out of an ever expanding set of lecture notes that, over the years, the authors developed, corrected, used, and misused, while teaching the year-long sequence on this topic at the introductory graduate level at the University of Tennessee, Knoxville (UTK). The purpose of this sequence can be simply stated: prepare students for the PhD subject examination in numerical analysis and equip them for research in a rich, active, and expanding field.

The prerequisites for the book are (i) a solid understanding of linear algebra, at the level of Horn and Johnson [44], for example, and (ii) a working knowledge of advanced calculus, at the level of Rudin [76] or Bartel and Sherbert [6]. Both of these topics are thoroughly reviewed in the appendices. Those comfortable with the material in Appendices A and B should be well prepared for the book. Some important topics from differential equations, functional analysis, and measure and integration theory are also used. But, these are reviewed, as needed, and are not treated as prerequisites.

Our mission while writing this book was to present a spartan, but thoroughly understandable text in numerical analysis that students can use to pass PhD exams and get quickly started on research, similar to the philosophy behind the well-received graduate-level analysis text by Bass [7]. As a competing goal, we have designed the text so that no important topic should be left out; the student using this book should have all the details at their fingertips, to the extent possible. We make a concerted effort to use good notation; make definitions clear and concise; make hypotheses of theorems, lemmas, etc., apparent, perhaps to the point of being too pedantic; use the simplest versions of proofs; and keep proofs of theorems, lemmas, etc., where they are most natural for quick reference, after the respective results are stated. Important facts that students will need later are never buried deep in the exercises; they are front and center in the presentation. We may have fallen short in some or all of these goals, but they were our goals, nonetheless.

The end result is, we believe, and hope, a text that has a very broad coverage, but with a simple, modern, and easy-to-read style, and, importantly, with notation made as clear as possible, even if that means breaking with tradition. We deliberately choose not to be overly expository or conversational with our readers. The reader will find few long paragraphs of explanation. While some instructors will lament the the loss of those paragraphs, to be honest, in our experience, most students do not read them. They want core principles, easily locatable facts, and clear proofs. Thus, we focus on illustrative examples, good problems, and clean presentation.

Some other texts do a much better job of using eloquent language to present the ideas and their historical significance. Specifically, the books by Trefethen [95], Scott [83], and Süli and Mayers [89] include fascinating historical accounts of the development of numerical analysis and are must-reads. In our book, whenever a named concept or result is first introduced, we have included, as a footnote, the name and some minimal background on this person. This is done to convey that mathematics in general, and numerical analysis in particular, is a lively subject, made by and *for* people. We have relied on `Wikipedia` [101] and the `MacTutor History of Mathematics archive` [66] as references.

We emphasize theory over implementation in the book, and there are many good, classroom-tested problems that involve proofs and theoretical insight. This is a book that uses a lot of linear algebra and analysis, subjects that are near and dear to our hearts. The disciplined student will sharpen their theoretical abilities in those subjects with this book. It is our realization, however, that, at least in universities in the United States, advanced undergraduate or beginning graduate students sometimes do not have an adequate preparation in linear algebra and basic analysis. For this reason, we have included rather substantial reviews of the necessary results from linear algebra and analysis, in Appendices A and B, respectively. In addition, many of the ideas and techniques of Part V rely on the theory of partial differential equations. Chapter 23 and Appendix D provide a cooking-recipe list of facts. Some other background material is recalled as it is needed, and references to the literature are provided. To the extent possible, the presentation was designed to be self-contained.

We believe that the subject of numerical analysis does not need to be hard — or harder than it actually is — but, on the other hand, there is deep, beautiful, and, yes, sometimes difficult mathematics under the hood. One of our goals is to dispel the myth that numerical analysis is an *ad hoc* mixture of whatever seems to work, requiring little to no theory, proofs, or analysis. Our subject is a fundamental discipline of mathematics, a core sub-field of analysis, and many of the great mathematicians, from antiquity to the present — including Erdős and von Neumann, for example — have contributed to its advance. The biographical footnotes that we have added reflect this fact.

This book is not merely a catalog of numerical methods, though most of the important ones are contained herein. In almost every case, methods are developed and supported by rigorous theory. We give plenty of examples with hand calculations and output from numerical simulations. We include several sample codes, so that students can get their hands dirty. The codes are written in `MATLAB`® and were tested in its open source counterpart `GNU Octave` for compatibility. These codes are designed to help students learn the subject. To this end, we emphasize clarity over efficiency in our programming. Still, we adopt the point of view, shared by many others, that performing computational experiments can be both fun and deeply enlightening. Students are encouraged to code the methods and try them out. The interested student could use this text as a starting point for learning implementation issues which, we admit, are highly nontrivial and almost completely ignored in this text. The active researcher could use it to learn how to design a better algorithm or to understand why a particular algorithm works, or does not work. The codes listed in the text — in addition to those not listed but used to generate examples and figures — can be obtained from GitHub:

`https://github.com/stevenmwise/ClassicalNumericalAnalysis`

While this text covers more than what can be presented in a year-long course, we have undoubtedly omitted several topics that can be found in other books on the subject. Most of these omissions were deliberate, made either because we believed that the topics were more advanced or because we deemed the topics to be nonessential. For instance, many numerical analysis books begin the discussion by addressing rounding errors and floating point arithmetic. We chose not to discuss these points, as we believe that, while important, they skew the student's perception about what is numerical analysis. This, then, begs the question: *What is numerical analysis?* For an answer, we defer to one of the classics [41]:

> . . . we shall mean by numerical analysis the *theory of constructive methods of mathematical analysis*. The emphasis is on the word "constructive."

We urge the reader to examine the Introduction to this reference for a very insightful definition of what numerical analysis is, and *what it is not*, one that was established in the early days of our beloved discipline. For an update on this viewpoint, which only reinforces our beliefs, we refer to the appendix of [96].

Regarding usage, the graduate-level numerical analysis sequence at UTK focuses heavily on numerical linear algebra, nonlinear equations, and numerical differential equations. These topics are well represented in the text, specifically in Parts I, III, IV, and V, and students using this book will get a thorough and modern introduction in those areas. However, we are well aware that other universities choose to emphasize other topics of numerical analysis. While the subjects of interpolation, constructive approximation, and quadrature theory, the focus of Part II, are not specifically in the catalog description of the introductory graduate sequence at UTK, these are included to support the theory needed to analyze numerical methods in the other parts of the book, to broaden the background knowledge of our students when they study these subjects in other classes, and for audiences outside of our university. We have taught the material from Part II in a separate, single-semester, graduate course on classical approximation theory.

We envision, and hope, that this text will be used in several year-long numerical analysis sequences. For this reason the dependency of all chapters is linear, with the exception of the appendices. Some sample course plans and ways we have used this text to teach are as follows:

- *UTK, numerical algebra:* (1 semester) Part I and Chapter 15.
- *UTK, numerical differential equations:* (1 semester) Part IV and Chapters 23 — 25, 28 and 29, referencing results from Part II, as needed.
- *UTK, classical approximation theory:* (1 semester) Part II, with Chapters 26 and 27 as applications.

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- *Classical numerical analysis:* (1 semester) Chapters 3, 4, 6, 7, 9, 12, 14, 15, and Part IV.
- *Topics in numerical analysis:* (1 semester) Part II, Chapters 2 and 5, Part IV, Chapters 6 and 7 (using Chapter 24 as motivation), and Chapters 15 and 16.