Symbols

This list describes several symbols that will be commonly used within the text. The page number indicates where its definition, or first appearance, is found.

$(\cdot,\cdot)_2$	The Euclidean inner product, page 849
$(\cdot,\cdot)_{L^2_h}$	The discrete inner product on $\mathcal{V}_0(\bar{\Omega}_h)$, page 673
$[\boldsymbol{x}]_i$	The ith component of the n-vector $\boldsymbol{x}$, page 840
$[\cdot,\cdot]_{L^2_h}$	The discrete inner product on $\mathcal{V}(\bar{\Omega}_h)$, page 673
$[\mathsf{A}]_{i,j}$	The element in the ith row and jth column of the matrix A, page 4
$\bar{z}$	The complex conjugate of $z \in \mathbb{C}$, page 838
$\bar{\delta}_h$	Backward difference operator, page 669
$\bar{\Omega}_h$	$[0,1]^d \cap \mathbb{Z}^d_h$, with $d \in \mathbb{N}$, page 666
$\boldsymbol{u} \otimes \boldsymbol{v}$	The exterior product of $\boldsymbol{u}$ and $\boldsymbol{v}$, also denoted $\boldsymbol{u}\boldsymbol{v}^{\mathsf{H}}$, page 27
$\mathcal{C}$	In the context of parabolic equations, this is the space–time cylinder, $\Omega \times (0,T)$, page 646
$\mathcal{D}^j_h v$	For a periodic grid function v and $j \in \{1,2\}$, this denotes its pseudo-spectral derivative of order j, page 732
$\mathcal{F}_n[\cdot]$	For $n \in \mathbb{N}$, this denotes the Discrete Fourier Transform (DFT), page 350
$\mathcal{G}^B_h$	For a grid domain $\mathcal{G}_h$, these are the boundary points (with respect to a finite difference operator), page 670
$\mathcal{G}^I_h$	For a grid domain $\mathcal{G}_h$, these are the interior points (with respect to a finite difference operator), page 670
$\mathcal{I}_X$	For a nodal set $X \subset [a,b] \subset \mathbb{R}$, this denotes the interpolation operator subordinate to X, page 234
$\mathcal{K}_m(\mathsf{A},\boldsymbol{q})$	The Krylov subspace of the matrix A of degree m, page 169
$\mathcal{L}_X$	For a nodal set X, this denotes the Lagrange nodal basis, page 235
$\mathcal{R}(a,b)$	The same as $\mathcal{R}([a,b])$, page 862
$\mathcal{R}(a,b;\mathbb{C})$	The collection of complex-valued Riemann integrable functions, page 865
$\mathcal{R}(I)$	For I a finite interval, this denotes the collection of functions that are Riemann integrable, page 862
$\mathcal{V}(\mathcal{G}_h)$	For a grid domain $\mathcal{G}_h \subseteq \mathbb{Z}^d_h$ with $h = 1/(N+1)$, this is the collection of grid functions, page 666
$\mathcal{V}(\mathbb{C})$	The space of functions $\mathbb{Z} \to \mathbb{C}$, page 348
$\mathcal{V}_0(\bar{\Omega}_h)$	The collection of functions in $\mathcal{V}(\bar{\Omega}_h)$ that vanish on the discrete boundary $\partial\Omega_h$, page 667

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$\mathcal{V}_{M,\mathrm{p}}(\mathbb{C})$	For $M \in \mathbb{N}$, this denotes the space of complex-valued grid functions that are, in addition, periodic, page 722
$\mathcal{Z}[\cdot]$	The Fourier-$\mathbb{Z}$, or Discrete Fourier, transform on grid functions in $L^2_h(\mathbb{Z}_h;\mathbb{C})$, page 794
χ_{A}	The characteristic polynomial of the matrix A, page 12
$\mathrm{clo}_s(K)$	For a subset K of a Hilbert space, this denotes its closure, page 455
$\mathrm{clo}_w(K)$	For a subset K of a Hilbert space, this denotes its weak closure, page 455
$\mathrm{col}(\mathsf{A})$	The column space of the matrix A, page 6
$\mathbb{C}$	The set of complex numbers, page 838
$\mathbb{C}^n$	The vector space of complex n-vectors, page 840
$\mathbb{C}^n_\star$	The collection of nonzero vectors in $\mathbb{C}^n$, page 9
$\mathbb{C}^{n\times n}_{\mathrm{Her}}$	The space of Hermitian matrices of size n, page 57
$\Delta\phi$	For a smooth scalar-valued function ϕ, this denotes its Laplacian, page 618
$\delta^{n,\mathrm{p}}$	The n-periodic grid delta function, page 349
Δ_h	The discrete Laplacian, page 669
δ_h	Forward difference operator, page 669
$\delta_h^{\diamondsuit}$	The discrete mixed derivative, page 669
$\Delta_h^{\square}$	The two-dimensional skew Laplacian, page 670
$\delta_{i,j}$	The Kronecker delta, page 842
$\ell^2(\mathbb{Z};\mathbb{C})$	The collection of all sequences $\{a_j\}_{j\in\mathbb{Z}} \subset \mathbb{C}$ that are square summable, page 330
$\mathfrak{B}(\mathbb{V})$	The same as $\mathfrak{B}(\mathbb{V},\mathbb{V})$, page 457
$\mathfrak{B}(\mathbb{V},\mathbb{W})$	For normed spaces $\mathbb{V}$ and $\mathbb{W}$, this denotes the vector space of bounded linear operators $\mathbb{V}\to\mathbb{W}$, page 457
$\mathfrak{L}(\mathbb{V})$	The same as $\mathfrak{L}(\mathbb{V},\mathbb{V})$, page 3
$\mathfrak{L}(\mathbb{V},\mathbb{W})$	The set of linear operators from $\mathbb{V}$ to $\mathbb{W}$, page 3
$\Im z$	The imaginary part of the complex number z, i.e., $\Im z = b$, if $z = a + \mathrm{i}b$, page 838
$\mathrm{im}(\mathsf{A})$	The image (or range) of the matrix A, also denoted $\mathcal{R}(\mathsf{A})$, page 6
i	The imaginary unit, page 838
$\kappa(\mathsf{A})$	The condition number of the matrix A, page 80
$\kappa_2(\mathsf{A})$	The spectral condition number of the matrix A, page 80
$\ker(\mathsf{A})$	The kernel (or null space) of the matrix A, also denoted $\mathcal{N}(\mathsf{A})$, page 6
$\langle\cdot,\cdot\rangle$	The duality pairing between a Hilbert space $\mathcal{H}$ and its dual $\mathcal{H}'$, page 464
$[u\star v]^{n,\mathrm{p}}$	For $u, v \in \mathcal{V}_{n,\mathrm{p}}(\mathbb{C})$, this denotes their discrete periodic convolution, page 352
$\leq$	For a vector space $\mathbb{V}$ and $\mathbb{W}\subseteq\mathbb{V}$, $\mathbb{W}\leq\mathbb{V}$ denotes that $\mathbb{W}$ is a subspace of $\mathbb{V}$. If $\mathbb{W}\neq\mathbb{V}$, then we denote $\mathbb{W}<\mathbb{V}$, page 841
$\mathcal{B}_\alpha(-1,1)$	For α the Chebyshev weight function, this denotes the subspace of $C^1([-1,1])$ of functions such that $\alpha(x)g(x)\to 0$ as $x\to\pm1$, page 743
$\mathcal{F}^m(S)$	For $m\in\mathbb{N}$, this is $F^1(S)\cap C^m(S;\mathbb{R}^d)$, page 517

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$\mathcal{V}_{n,\mathrm{p}}(\mathbb{C})$	For $n \in \mathbb{N}$, this denotes the space of n-periodic grid functions, page 348
$\mathring{\mathcal{V}}_{M,\mathrm{p}}(\mathbb{C})$	For $M \in \mathbb{N}$, this denotes the space of mean-zero, complex-valued, periodic grid functions, page 723
$\mathring{\delta}_h$	Centered difference operator, page 669
$\mathring{\mathscr{S}}_N(0,1;\mathbb{C})$	For $N \in \mathbb{N}$, this is the space of complex-valued, mean-zero, trigonometric polynomials of degree at most N, page 730
$\mathring{C}_{\mathrm{p}}^{\infty}(0,1;\mathbb{C})$	The space infinitely differentiable, complex-valued, periodic functions that have mean-zero, page 884
$\mathring{C}_{\mathrm{p}}^{m}(0,1;\mathbb{C})$	For $m \in \mathbb{N}_0$, this denotes the space functions in $C_{\mathrm{p}}^{m}(0,1;\mathbb{C})$ that have mean-zero, page 884
$\mathring{H}_{\mathrm{p}}^{m}(0,L;\mathbb{C})$	For $m \in \mathbb{N}_0$, this is the space of functions in $H_{\mathrm{p}}^{m}(0,L;\mathbb{C})$ that have mean-zero, page 892
$\mathsf{A} \asymp \mathsf{B}$	The matrix A is similar to the matrix B, page 13
$\mathsf{A}(S)$	For $\mathsf{A} \in \mathbb{C}^{n\times n}$ and $S \subseteq \{1,\ldots,n\}$, this denotes the sub-matrix obtained by deleting the rows and columns whose indices are not in S, page 36
A^{H}	The conjugate transpose of the matrix A, page 7
$\mathsf{A}^{\dagger}$	The Moore–Penrose pseudo-inverse of A, page 30
$\mathsf{A}^{\intercal}$	The transpose of the matrix A, page 7
A^{-1}	The inverse of the matrix A, page 8
∇v	For a smooth scalar-valued function, this denotes its gradient, page 612
$\nabla \cdot \boldsymbol{u}$	For a smooth vector-valued function $\boldsymbol{u}$, this denotes its divergence, page 612
$\|\mathsf{A}\|_{\max}$	The matrix max-norm of the matrix A, page 9
$\|\cdot\|_{H_h^1}$	The discrete H_h^1-norm on the space of grid functions on $\mathcal{V}(\bar{\Omega}_h)$, page 686
$\|\cdot\|_{L_h^p}$	For $p \in [1,\infty]$, this denotes the discrete L_h^p-norm on the spaces of grid functions on $(0,1)^d$, page 671
$\|\mathsf{A}\|_F$	The Frobenius norm of the matrix A, page 9
$\|\mathsf{A}\|_p$	The induced p-norm of the matrix A, page 10
$\|f\|_{L^p(\Omega;\mathbb{C})}$	For a function $f\colon \Omega \to \mathbb{C}$, this denotes its L^p-norm, $p \in [1,\infty]$, page 881
$\|f\|_{L_w^p(a,b;\mathbb{C})}$	For a function $f\colon [a,b] \to \mathbb{C}$, this denotes its weighted L^p-norm, $p \in [1,\infty)$, with weight w, page 882
Ω_h	$(0,1)^d \cap \mathbb{Z}_h^d$, with $d \in \mathbb{N}$, page 666
$\partial\Omega_h$	$\bar{\Omega}_h \backslash \Omega_h$, page 666
$\partial_p\mathcal{C}$	The parabolic boundary of $\mathcal{C}$, page 646
$\mathbb{H}^*$	For a complex Hilbert space $\mathbb{H}$, this denotes the anti-dual, page 736
$\mathbb{K}^n$	The vector space of n-vectors, page 840
$\mathbb{K}^{m\times n}$	The set of matrices with m rows and n columns with coefficients in $\mathbb{K}$, page 4
$\mathbb{P}_n$	This is, typically, $\mathbb{P}_n(\mathbb{R})$ or $\mathbb{P}_n(\mathbb{C})$, depending upon the context, page 840

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$\mathbb{P}_n(\mathbb{K})$	The vector space of polynomials of degree no larger than n with coefficients in $\mathbb{K}$, page 840
$\mathbb{P}_{m/n}$	For $m, n \in \mathbb{N}_0$, this is the set of rational polynomials whose numerator and denominator lie in $\mathbb{P}_m$ and $\mathbb{P}_n$, respectively, page 588
$\mathbb{Q}$	The set of rational numbers, page 838
$\mathbb{T}_n$	For $n \in \mathbb{N}_0$, this denotes the space of all one-periodic trigonometric polynomials, page 321
$\mathbb{Z}^d_h$	The collection of vectors in $\mathbb{R}^d$ of the form $h\boldsymbol{z}$ with $\boldsymbol{z} \in \mathbb{Z}^d$, page 665
$\Re z$	The real part of the complex number z, i.e., $\Re z = a$, if $z = a + \mathrm{i}b$, page 838
$\mathbb{R}$	The set of real numbers, page 838
$\mathbb{R}^n$	The vector space of real n-vectors, page 840
$\mathbb{R}^n_\star$	The collection of nonzero vectors in $\mathbb{R}^n$, page 9
$\mathbb{R}^{n\times n}_{\mathrm{sym}}$	The space of real symmetric matrices of size n, page 57
$\rho(\mathsf{A})$	The spectral radius of matrix $\mathsf{A} \in \mathbb{C}^{n\times n}$, page 73
$\mathrm{row}(\mathsf{A})$	The row space of the matrix A, page 6
$\sigma(\mathsf{A})$	The spectrum of the square matrix A, page 12
$\sigma(A)$	The spectrum of the linear operator A, page 15
$\mathscr{S}^{1,0}(\mathscr{T}_h)$	The space of continuous piecewise linear functions subject to the triangulation $\mathscr{T}_h$, page 705
$\mathscr{S}^{1,0}_0(\mathscr{T}_h)$	This is $\mathscr{S}^{1,0}(\mathscr{T}_h) \cap H^1_0(\Omega)$, page 705
$\mathscr{S}^{\boldsymbol{p},0}(\mathscr{T}_h)$	For a one-dimensional mesh $\mathscr{T}_h$, with $\#\mathscr{T}_h = N$ and $\boldsymbol{p} \in \mathbb{N}^{N+1}$, this is the space of functions that are continuous, and for every $I_i \in \mathscr{T}_h$ their restriction to I_i is a polynomial of degree p_{i+1}, page 710
$\mathscr{S}^{\boldsymbol{p},0}_0(\mathscr{T}_h)$	This is $\mathscr{S}^{\boldsymbol{p},0}(\mathscr{T}_h) \cap H^1_0(0,1)$, page 710
$\mathscr{S}^{p,-1}(\boldsymbol{\tau};\mathcal{H})$	For a Hilbert space $\mathcal{H}$, this is the space of $\mathcal{H}$-valued piecewise polynomials of degree at most p over the partition $\boldsymbol{\tau}$, page 599
$\mathscr{S}^{p,0}(\boldsymbol{\tau};\mathcal{H})$	This is $\mathscr{S}^{p,-1}(\boldsymbol{\tau};\mathcal{H}) \cap C([0,T];\mathcal{H})$, page 605
$\mathscr{S}^{p,0}(\mathscr{T}_h)$	For $p \in \mathbb{N}$, this is the space of functions that are continuous and piecewise polynomials, of degree p, subject to the triangulation $\mathscr{T}_h$, page 710
$\mathscr{S}^{p,0}_0(\mathscr{T}_h)$	This is $\mathscr{S}^{p,0}(\mathscr{T}_h) \cap H^1_0(\Omega)$, page 710
$\mathscr{S}^{p,r}(\boldsymbol{\tau};\mathcal{H})$	This is $\mathscr{S}^{p,-1}(\boldsymbol{\tau};\mathcal{H}) \cap C^r([0,T];\mathcal{H})$, page 605
$\mathscr{S}_{N,0}(-1,1)$	For $N \in \mathbb{N}$, this denotes the set of polynomials of degree at most N that vanish at $x = \pm 1$, page 747
$\mathrm{span}(S)$	The span of the set S, also denoted $\langle S \rangle$, page 840
$\mathrm{supp}\, g$	The support of the function g, page 705
$\mathrm{supp}(\phi)$	For a function ϕ, this denotes its support, page 887
$\tilde{\delta}^{n,\mathrm{p}}$	The singular n-periodic grid delta function, page 353
$\tilde{\mathsf{E}}^{\tau}_h(\xi)$	The symbol of a two-layer, matrix-valued, finite difference method, page 828
$\tilde{E}^{\tau}_h(\xi)$	The symbol of a two-layer finite difference method, page 795
$\mathscr{T}_h$	A mesh with mesh size $h > 0$, page 705
$\{\boldsymbol{x}_k\}_{k=1}^\infty$	A sequence of vectors in either $\mathbb{C}^d$ or $\mathbb{R}^d$, page 854
A^*	The adjoint of the linear operator A, page 7

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$C(A; B)$	The vector space of continuous functions with domain A and range in B, page 842
$C(I)$	For I and interval this denotes the set of functions $f\colon I \to \mathbb{R}$ that are continuous, page 858
$C^0(I)$	The same as $C(I)$, page 859
$C^m(I)$	For $m \in \mathbb{N}$ and I an interval, this denotes the collection of functions $f\colon I \to \mathbb{R}$ whose derivatives up to and including mth order exist and are continuous on I, page 859
$C^m_{\mathrm{p}}(0,1;\mathbb{C})$	For $m \in \mathbb{N}_0$, this denotes the space of complex-valued, m-times continuously differentiable periodic functions, page 884
$C^{0,1}(I)$	For I an interval, this denotes the collection of functions $f\colon I \to \mathbb{R}$ that are Lipschitz continuous, page 859
$C^{0,\alpha}([0,1])$	For $\alpha > 0$, this denotes the set of functions $v\colon [0,1] \to \mathbb{R}$ that are Hölder continuous of order α, page 895
$C^{0,\alpha}(I)$	For I an interval and $\alpha \in (0,1]$, this denotes the collection of functions $f\colon I \to \mathbb{R}$ that are Hölder continuous of order α, page 859
$C_b(\mathbb{R}^d)$	The space of continuous functions $\mathbb{R}^d \to \mathbb{R}$ that, in addition, are bounded on $\mathbb{R}^d$, page 639
$C^m_b(I)$	For $m \in \mathbb{N}$ and I an interval, this denotes the collection of functions in $C^m(I)$ such that, in addition, the function and all its derivatives up to and including order m are bounded on I, page 860
$f = \mathcal{O}(g)$	The Landau symbol. Whenever f and g are two related quantities, this is used to denote that f is, asymptotically, of the order of g, page 856
$F^1(S)$	The class of slope functions that are continuously differentiable on S and whose partial $\boldsymbol{u}$-derivatives are bounded, page 517
$H^1(\Omega)$	For a bounded domain $\Omega \in \mathbb{R}^d$, with $d \in \mathbb{N}$ this denotes the Sobolev space of functions $v \in L^2(\Omega)$ such that $\nabla v \in L^2(\Omega; \mathbb{R}^d)$, page 888
$H^1_0(\Omega)$	The subspace of $H^1(\Omega)$ of functions that vanish on the boundary, page 888
$H^1_{\alpha,0}(-1,1)$	The subspace of $H^1_\alpha(-1,1)$ of functions that vanish at $x = \pm 1$, page 744
$H^m_\alpha(-1,1)$	For $m \in \mathbb{N}_0$ and α the Chebyshev weight function, this denotes the Chebyshev weighted Sobolev space of order m, page 743
$H^m_{\mathrm{p}}(0,L;\mathbb{C})$	For $L > 0$ and $m \in \mathbb{N}_0$, this denotes the space of L-periodic Sobolev functions, page 892
$L^2_h(\mathbb{Z}_h)$	The collection of grid functions $\mathcal{V}(\mathbb{Z}_h)$ that are square summable, page 793
$L^2_{\mathrm{p}}(0,1;\mathbb{C})$	The set of all one-periodic, locally square integrable functions, page 886
L_ℓ	For a nodal set X of size $n+1$ and $0 \leq \ell \leq n$, this denotes the ℓth element of the Lagrange nodal basis, page 235
$S_1 + S_2$	For $S_1, S_2 \leq \mathbb{C}^n$, this denotes their sum, page 94
$S_1 \oplus S_2$	For $S_1, S_2 \leq \mathbb{C}^n$, this means that they are complementary subspaces, i.e., $S_1 + S_2 = \mathbb{C}^n$, page 94

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$S_1 \overset{\perp}{\oplus} S_2$	For $S_1, S_2 \leq \mathbb{C}^n$, this means that they are complementary, and orthogonal, subspaces, i.e., $S_1 + S_2 = \mathbb{C}^n$ and $\boldsymbol{s}_1 \in S_1$ $\boldsymbol{s}_2 \in S_2$ implies $\boldsymbol{s}_2^{\mathsf{H}}\boldsymbol{s}_1 = 0$, page 95
$W^\perp$	The orthogonal complement of the set W, page 849
$x \perp y$	The vector x is orthogonal to y, page 849
$X \hookrightarrow Y$	For normed spaces X and Y, this means that X is continuously embedded in Y, page 706
$\mathsf{A}^{(k)}$	The leading principal sub-matrix of order k of A, page 36
$\Vert\boldsymbol{x}\Vert_p$	The p-norm of a complex n-vector $\boldsymbol{x}$. Also denoted $\Vert\boldsymbol{x}\Vert_{\ell^p(\mathbb{C}^n)}$, page 844
$H^m(\Omega)$	For $m \in \mathbb{N}$, this denotes the collection of functions $v \in L^2(\Omega)$ whose weak derivatives up to order m belong to $L^2(\Omega)$ as well, page 890
$L^p(0, T; \mathbb{V})$	For a Banach space $\mathbb{V}$, this denotes the space of functions such that the mapping $t \mapsto \Vert v(t)\Vert_{\mathbb{V}}$ belongs to $L^p(0, T)$, page 644
$\#S$	The cardinality of the set S, page 841
$\vert z\vert$	The modulus of the complex number z, page 839

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