Asset Allocation for Italian S.p.a.

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May 2023

1 Introduction

We start our work computing an asset allocation exercise following different techniques, like Mean-Variance optimization, the Black-Litterman approach for the computation of mean and variance-covariance matrix, GMV optimization, and Bayesian approaches. We solved these different allocation problems utilizing MATLAB and its Add-Ons "Financial Toolbox", "Statistics and Machine Leaning Toolbox", and "Optimization Toolbox".

For the sake of simplicity and to avoid redundance, our following discussion of the methodologies applied will refer only to one single frequency. It is to be obviously intended that we have applied the same processes and elaborations to both frequencies, daily and monthly, employing the same exact procedures, but changing the data.

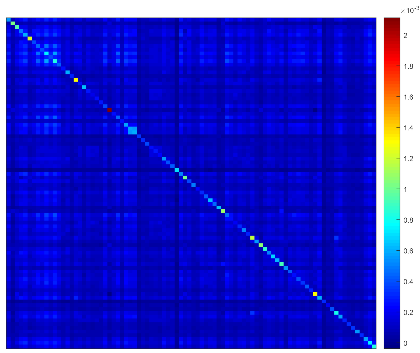
1.1 Summary statistics of returns

After an initial handling of the data to properly import it from the Excel file, we computed the percentage returns of each of the 88 stocks employing the "tick2ret( )" function. We then computed summary statistics for each of these returns, i.e., mean, variance, standard deviation, skewness and kurtosis, employing corrective measures for missing values (NaNs) whenever needed. Results can be seen in Appendix A.

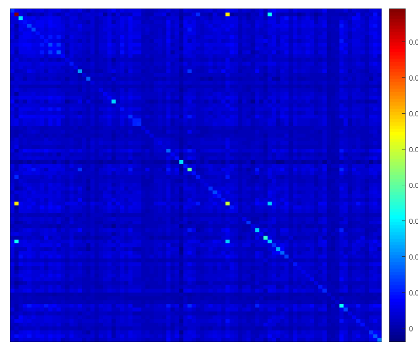
From a simple glance of the table, it is easy to see that returns do not follow a normal distribution, especially by looking at skewness and kurtosis which are very different from the ones of a gaussian. In order to confirm this, we ran some normality tests (Jarque-Bera), visible in our code, which again confirmed this fact.

2 Covariance and Correlation matrix of returns

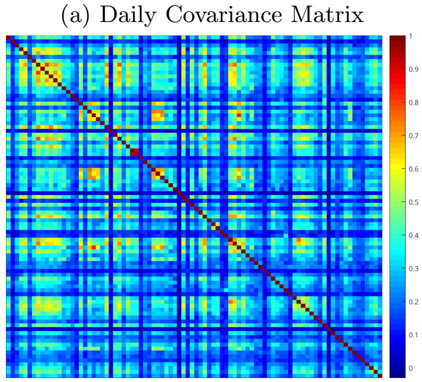
We first computed the Variance-Covariance matrix, which, due to the very large dimensions they possess (they are indeed 88 x 88 matrices, that is 7744 values), we decided to represent through a heatmap. They are consultable in figure (a) and (b). Then, we proceeded to calculate the Correlation matrix, which again we displayed though a heatmap as it makes really evident the intensity of correlation between the different companies. These matrices are undeniably the fundaments of our work, since based on the correlation between the different assets we both made our decision of the 12 preferred stocks and performed all the following calculations for optimal asset allocation.



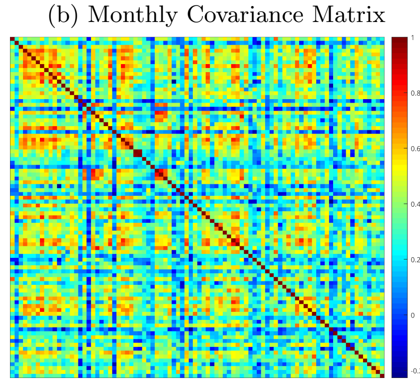
(a) Daily Covariance Matrix



(b) Monthly Covariance Matrix



(c) Daily Correlation Matrix



(d) Monthly Correlation Matrix

There is a notable difference between daily and monthly correlations: in particular, monthly returns are much more correlated than daily ones. This is due to the nature of the time horizon on which we measure these correlations: monthly returns are higher since the larger time horizon allows for shocks and news to influence stocks more homogeneously, whilst daily returns are influenced way more by noise and randomness.

3 Selection of a sample of 12 securities

Before making our choice of the 12 preferred assets, we first took an important step: we removed from our dataset all the currently delisted companies. Though this might seem unnecessary at first, we wanted to make sure not to pick delisted companies, since their stock won't move in price, providing effectively null returns and no potential hedging capabilities. Including such assets in our allocation would indeed defy the very purpose of building a portfolio, that is to obtain a positive return. We therefore removed Astaldi, DeA Capital, Banca Intermobiliare, Cattolica Assicurazioni, and Borgosesia RSP from our set of possible choices. Now we proceeded to find the 12 assets. Our strategy was to firstly find the assets with minimal correlation amongst them in order to reduce as much as possible idiosyncratic risk, and then, based on this first result, modify it by hand also considering each company's historical returns and standard deviation as well as views on the future. In order to obtain the first set of 12 assets, we ran the algorithm explained below.

Algorithm Explanation:

1. The algorithm identifies the pair of variables with the lowest correlation value in the original correlation matrix previously computed. Then it records the positions (row and column) of these variables, that is the two least correlated companies in our dataset.

2. Now, we need to find the third variable that has the minimum correlation with the two already chosen variables. The algorithm scans all the variables (except the already selected two) and finds the one which adds the overall lower amount of correlation with respect to the previously selected. It then creates a triplet with this asset and the previous ones.

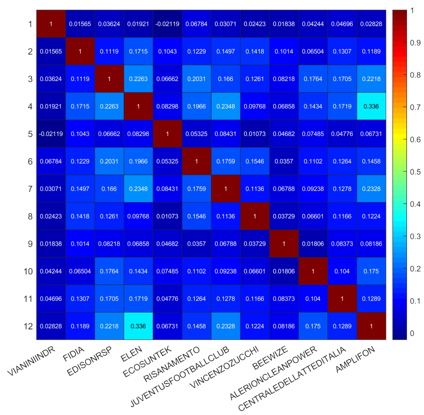
3. The algorithm continues this iterative process, now with the triplet instead of the couple, and so on, until it selects 12 assets, always seeking the new variable which adds the lowest amount of correlation with respect to all the already selected variables. Having obtained a sample of companies from the algorithm, we then proceeded to modify it to remove eventually unsatisfying securities present and check for the soundness of the algorithm.

The final stocks selected after our elaboration are: "Ecosuntek", "Alerion Clean Power", "Centrale Del Latte D'Italia", "El.En.", "Amplifon", "Fidia", "Risanamento", "Beewize", "Juventus Football Club", "Vincenzo Zucchi", "Vianini", "Edison".

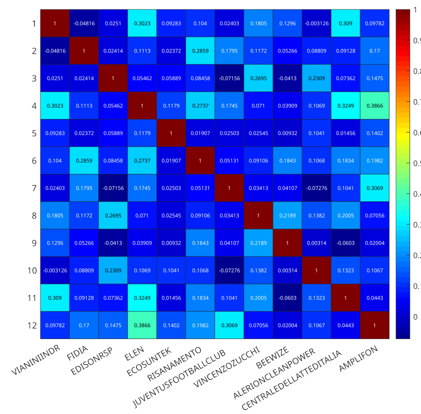
As we can clearly see from the tables, our algorithm worked correctly and our objective was respected: the correlation matrices are extremely low (there are also some couples with negative correlation), allowing us to reduce drastically the risk of our portfolios. This is also confirmed by the p-values of each coefficient, which in most cases turn out to be significant.

4 Plot of security prices

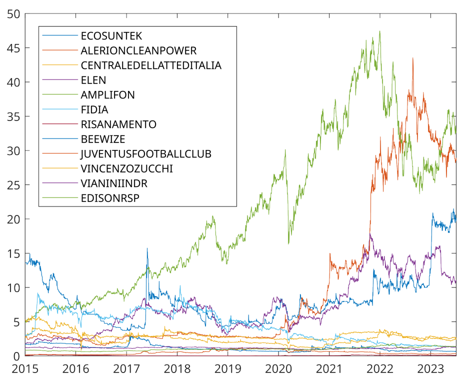
We start the analysis of the selected 12 securities by plotting prices of the securities in both daily and monthly frequency.



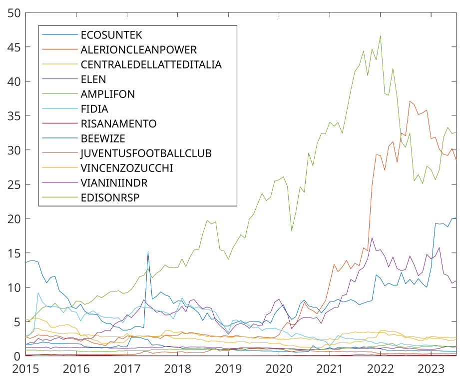
(a) Daily Correlation Matrix



(b) Monthly Correlation Matrix



(a) Daily Prices



(b) Monthly Prices

5 Mean-Variance optimal portfolio allocation

5.1 Short selling allowed

Before stating the results of our allocation problem, it is important to briefly summarize the most notable characteristics of Mean-Variance optimization (MV from now on), in order to properly understand what is being done and what our final result represents. We all are familiar with the fundamental idea of the trade-off between risk and return: the more risk we take, obviously, the more we want to be compensated. But how can we move from this simple intuitive concept to the definition of an optimal portfolio? We first have to make some assumptions. The main idea behind the MV model is that the first two moments of the distribution of returns are the only two elements required to evaluate an investment, and as such the only two needed to solve a portfolio optimization problem. This is a sustainable idea as long as we assume that investors have a quadratic utility function and returns are normal. This is helpful in many ways, mainly as it simplifies the portfolio optimization problem by reducing it to a simple linear algebra optimization problem: we indeed ultimately want to find the weights that maximize the utility function. There are though some drawbacks: quadratic utility functions have limitations, like increasing absolute risk aversion relative to wealth or the impossibility of describing particular behaviors of investors, and assuming normally distributed returns is never a good choice since both empirical and theoretical data point against it (recall how we tested for normality at the beginning). Finally, we have to assume that historical data is a consistent estimator of expected returns and variance of returns. While these might seem a set of too strong assumptions, and they are in some way, MV optimization is a necessary starting point to establish some form of benchmark for an optimal portfolio. We set up our problem such that we identified as optimal the portfolio on the efficient frontier which maximized the Sharpe ratio, an indicator of risk-weighted return on investment, given an initial risk-free rate of 0, i.e., no risk-free asset exists. Initially we assume that short selling is allowed. To perform our computations, we utilized the "estimateMaxSharpeRatio( )" function to find the weights that maximize the Sharpe ratio.

	Weights daily	Weights monthly
ECOSUNTEK	0.078912	0.024561
ALERIONCLEANPOWER	0.3201	0.32429
CENTRALEDELLATTEDITALIA	-0.043258	-0.046017
ELEN	0.2394	0.23463
AMPLIFON	0.32176	0.40292
FIDIA	-0.054444	-0.072462
RISANAMENTO	-0.0007112	0.002314
BEEWIZE	-0.016359	0.032598
JUVENTUSFOOTBALLCLUB	0.064326	0.081859
VINCENZOZUCCHI	-0.0052346	-0.26625
VIANINIINDR	0.091847	0.12263
EDISONRSP	0.0036576	0.15893

Figure 4: MV optimal weights (Short-selling allowed)

The MV optimal weights reflect clearly what historical data says to us: the better performing companies, as long as they don't have extreme value of standard deviation, are the preferred ones with the biggest shares of the portfolio. Notice indeed how "Amplifon" (daily returns 0.11%, monthly returns 2.24%), "Alerion Clean Power" (daily returns 0.14%, monthly returns 2.98%), and "El.En." (daily returns 0.11%, monthly returns 2.72%) alone have a bit more than 88% of the portfolio. Assets with instead negative mean returns are the ones which are shorted, as one might guess. It's important to notice also how the final weights are influenced by the companies' standard deviation: stocks with a lower one are preferred to others, even if they have on average lower returns (see “Amplifon” and “Alerion Clean Power”).

5.2 Short selling NOT allowed

Now we adopt the same technique but we impose non-negativity constraint on portfolio weights. We again found the weights which maximized the Sharpe ratio: Notice how here, since there is no more the possibility of shorting, the

	Weights Daily	Weights Monthly
ECOSUNTEK	0.068525	0.021315
ALERIONCLEANPOWER	0.2954	0.27579
CENTRALEDELLATTEDITALIA	6.5615e-20	8.9992e-20
ELEN	0.21022	0.13753
AMPLIFON	0.29292	0.38261
FIDIA	7.5214e-21	1.4759e-20
RISANAMENTO	3.5352e-18	5.6053e-19
BEEWIZE	2.4858e-19	0.003134
JUVENTUSFOOTBALLCLUB	0.0503	0.062999
VINCENZOZUCCHI	1.5698e-18	1.8164e-20
VIANINIINDR	0.082635	0.053237
EDISONRSP	3.7764e-17	0.063389

Figure 5: MV optimal weights (Short-selling NOT allowed)

assets with negative average returns loose all their attractiveness: since they can no more be shorted to profit from their decrease in price, they are reduced to insignificant percentages, effectively allowing us to ignore them.

6 Statistics of the MV portfolios

	Mean	Standard deviation	Variance	Skewness	Kurtosis	Sharpe Ratio
Daily, no constraints	0.001147	0.014787	0.00021867	-0.19904	9.0332	0.077581
Monthly, no constraints	0.028921	0.074827	0.005599	0.089376	5.4716	0.38841
Daily, constrained	0.0010499	0.013673	0.00018694	-0.29006	10.049	0.076806
Monthly, constrained	0.022736	0.063425	0.0040227	-0.06695	4.5558	0.36024

Figure 6: Descriptive statistics of the MV portfolios

On average, when short selling is allowed, we obtain higher expected returns with respect to the constrained portfolio: 0.11% against 0.10% for the daily portfolios and 2.89% against 2.27% for the monthly ones. It goes without saying though that the unconstrained portfolios have an embedded level of riskiness which is higher: this is reflected in the higher levers of standard deviation. The returns of our portfolios are in line with the best performers stocks, but with a much lower level of standard deviation: this is undeniably due to our voluntary choice of picking the most negatively correlated assets, or at least the less correlated ones, with the clear intent of ultimately avoiding for the standard deviation of the portfolio to skyrocket. The skewness represents the "asymmetry" of the distribution: negative skewness implies a fatter left tail, whilst a positive one the exact opposite. We can see that daily portfolios have fatter left tails (those of negative returns), whilst the monthly unconstrained portfolio has a positive one. Note how as we move from daily to monthly frequency our empirical distribution of returns moves closer and closer to a gaussian: the skewness moves towards 0 and the kurtosis towards 3. We also calculated the Sharpe ratio for each portfolio. It is evident both how monthly portfolios have a higher ratio due to less volatility in returns and how unconstrained portfolios have higher ratios since they are allowed to make an "extra" profit by short selling.

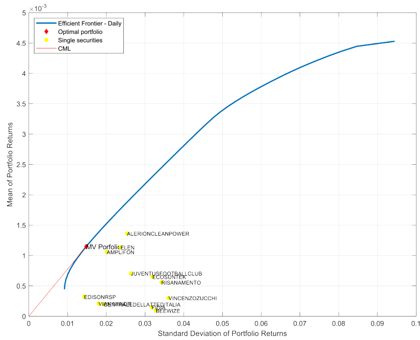
7 Efficient frontier

We then proceeded to define the more general efficient frontier originating from both daily and monthly data, that is all the points which are considered to be the most efficient as they offer the highest return for a given level of risk or the lowest risk for a given level of return.

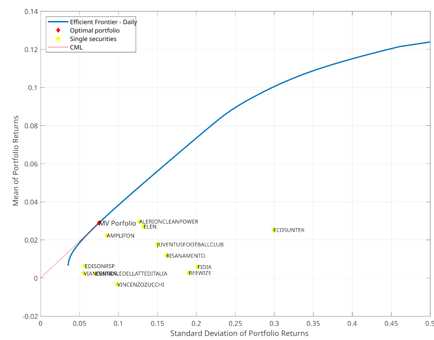
In our plot we highlighted the tangent line to the efficient frontier originating from the origin (the Capital Market Line), as we assumed a risk-free rate of 0. As expected, the tangency point is the MV optimal portfolio, as it's the one that maximizes the Sharpe ratio. We also plotted all the securities, which are in yellow. It's interesting to notice how "Alerion Clean Power" lies exactly on the frontier in the constrained portfolios. This might seem strange, but it's perfectly logical: since we cannot short sell, the more we move up the efficient frontier, the more our returns are given only by the best performing assets, up until the point of maximum returns, which will be a portfolio containing only by the highest performing asset.

8 FTSE Italia All Market index analysis

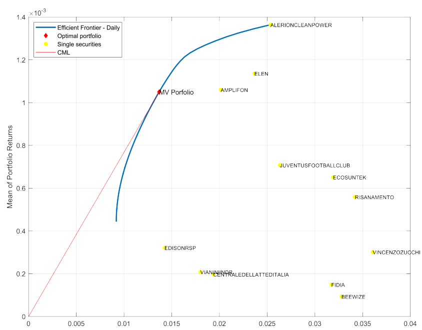
The FTSE Italia All Market is a stock market index that represents the performance of all companies listed on the Borsa Italiana Exchange, which is the stock exchange of Italy. We utilize this index in its Total Return version since our prices are adjusted for dividends. This index provides a sort of measure of the "market's wellbeing" and serves as a reference for evaluating the performance of individual stocks, mutual funds, and other investment products. We computed the returns both in daily and monthly frequency for the index and all the descriptive statistics such as mean, standard deviation, variance, skewness and kurtosis. Since the index does not consider negative weights in its construction,



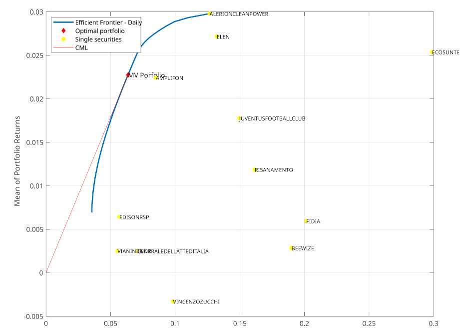
(a) Daily efficient frontier NO constraints



(b) Monthly efficient frontier NO constraints



(c) Daily efficient frontier constraints



(d) Monthly frontier constraints

	Mean	Variance	Standard Deviation	Skewness	Kurtosis
Daily, FTSE All Share	0.00041825	0.00018985	0.013779	-1.3311	18.184
Monthly, FTSE All Share	0.0087539	0.0033768	0.058111	-0.55079	5.7895

Figure 8: Descriptive statistics of FTSE Italia All Market index

we decided to compare it only with the constrained portfolios. Immediately we can see how the mean returns are much lower if compared to those of the portfolio: this is because the FTSE All Share averages the returns of all the shares on the Italian market, whilst our portfolio was a precise selection of performing stocks. Standard deviation values are similar, though slightly smaller for the index. Again, as we move from daily to monthly frequency the returns start to assume a more gaussian behavior (skewness and kurtosis decrease), though less if compared to the portfolio: for example, skewness values are much smaller than those of the portfolio, implying that the left-heaviness of the distribution of returns is much more accentuated.

9 Beta of securities of the portfolios

The beta of a security is, in short, the measure of its correlation to movements in the overall market. It quantifies the relationship between the return of a security and the return of a benchmark index representative of the market. In our case, the role of the market index will be taken by the above-mentioned FTSE Italia All Market.

The formula for beta for a general i-th security is:

$$\beta_i = \frac{\mathrm{Cov}(r_i, r_m)}{\mathrm{Var}(r_m)} \tag{1}$$

Where $\mathrm{Cov}(r_i, r_m)$ is the historical covariance between the returns of the i-th security and the returns of the market (in our case of the index), and $\mathrm{Var}(r_m)$ is the historical variance of the returns of the market, which acts as a sort of standardization factor.

From what we just said, we can definitely say that β_i has a very simple and intuitive interpretation:

- If $|\beta_i| > 1$, the security is expected to have stronger reactions to fluctuations than the market itself (the asset is considered riskier or "aggressive").
- If $|\beta_i| = 1$, the security is expected to react to fluctuations in line with the market (the asset is considered "neutral").
- If $|\beta_i| < 1$, the security is expected to have smaller reactions than the market to fluctuations (the asset is considered safer or "defensive").

Note that we used $|\beta_i|$ as these definitions are valid for both positive and negative betas. The difference in sign is that if $\beta_i < 0$, the asset has negative correlation with the market and as such reacts in an opposite way, therefore can be used as a hedger.

To compute the beta of each portfolio we used the above-mentioned formula with r_i equal to the weighted returns, with weights assigned respectively for each portfolio.

10 Security Market Line

The Security Market Line (SML) is a graphical representation of the Capital Asset Pricing Model (CAPM), which is a widely used framework for determining the expected return of an asset or portfolio based on its beta. The CAPM assumes that all investors are rational and therefore will diversify away all idiosyncratic risk, leaving only systematic risk to be priced, which is done through, indeed, beta. In simpler words, it provides a benchmark that helps us to assess whether a security is offering appropriate compensation for its level of risk.

The equation of the SML is:

$$E(r_i) = r_f + \beta_i(E(r_m) - r_f) \tag{2}$$

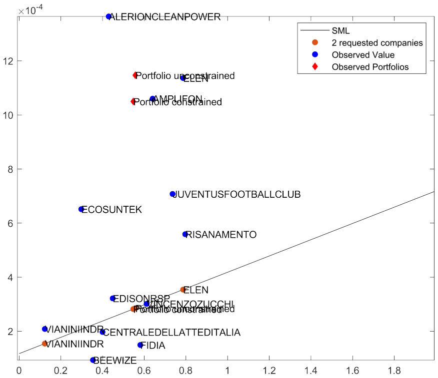
	Beta daily	Beta monthly
ECOSUNTEK	0.29794	0.8129
ALERIONCLEANPOWER	0.42969	0.5241
CENTRALEDELLATTEDITALIA	0.39977	0.47593
ELEN	0.78644	1.3205
AMPLIFON	0.64036	0.58063
FIDIA	0.58122	0.98261
RISANAMENTO	0.79676	0.90441
BEEWIZE	0.35407	0.31582
JUVENTUSFOOTBALLCLUB	0.7359	0.72723
VINCENZOZUCCHI	0.61125	0.75667
VIANINIINDR	0.12308	0.20225
EDISONRSP	0.44884	0.3791

	Beta daily	Beta monthly
MV unconstrained	0.55716	0.5961
MV constrained	0.54743	0.64722

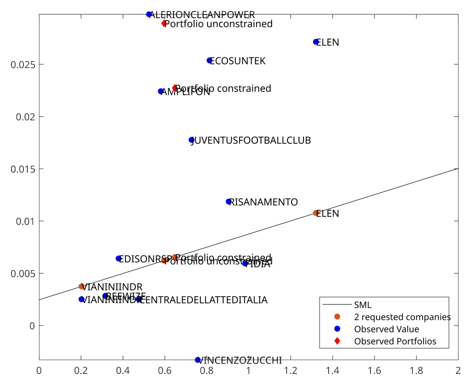
Figure 9: Betas of the companies and the MV portfolios

where $E(r_i)$ is the expected return of the asset or portfolio, r_f is the risk-free rate of return (in our case is given 3.00%), β_i is the beta of the asset or portfolio, and $E(r_m)$ is the expected return of the market (in our case of the FTSE Italia All Share). As we can see, the SML states that the expected return of an asset or portfolio should be equal to the risk-free rate plus a risk premium, which reflects the additional return investors require for taking on additional systematic risk, determined by the beta. On the SML graph, therefore, assets or portfolios that lie above the line are considered underpriced (attractive investments), since for a given amount of risk they return more. Conversely, assets or portfolios that fall below the line are overpriced (less desirable investment). We computed the SML return estimation for 2 securities of our portfolio, Vianini and El.En. (in orange the estimate, in blue the observed value), as well as for the portfolios we previously computed. The results are represented in the graphs (a) and (b).

It is clearly visible that, whilst in daily frequency the data is mostly above the SML, when switching to monthly some companies like Vianini fall below, suggesting that on a longer time-horizon of investment, companies such as that one might be, though slightly, overpriced. Still, the majority of them lies above the line, implying that the performance of our selected stocks is above the benchmark.



(a) SML Daily



(b) SML Monthly

11 Black-Litterman approach

The Black-Litterman model (BL henceforth) is an asset allocation framework which detaches itself from the traditional allocation problem as it drops some objectivity in favor of subjectivity. Indeed, in order to solve the input sensitivity problem, one of the main deficiencies of Markowitz optimization, the BL model enables its users to incorporate their personal views on the performance of some specific assets, resulting in the creation not only of more balanced portfolios compared to those generated by conventional mean-variance optimization methods, but also in portfolios oriented towards one's specific view.

The first step in the BL model is to define a prior, i.e., a vector of "default" estimates in absence of views. There are many ways to define a prior, and each of them might lead to different results. Some, however, tend to be better than others: historical returns, for example, tend to be too vague of a prior. We decided to stick with Black and Litterman's original suggestion of using the market's estimate of the return, which is embedded into the market capitalization of the assets.

We therefore calculated market implied returns with the following formula:

$$\pi = \delta \mathbf{\Sigma} \omega_{market} \tag{3}$$

where Σ is the variance-covariance matrix of the selected companies, ω_{market} is the weight of each company in terms of market capitalization and δ is the market-implied risk premium, calculated as:

$$\delta = \frac{r_m - r_f}{\mathrm{Var}(r_m)} = \frac{r_m}{\mathrm{Var}(r_m)} \tag{4}$$

since we reverted to the original assumption of $r_f = 0$. After having calculated the prior, we proceeded to define our views.

We formulated 5 views in total, 3 absolute and 2 relative:

1. Absolute view: "Alerion Clean Power" will have an annual return of 35% (that translates to 0,14% of daily returns and 2,92% of monthly returns)

with estimated uncertainty monthly $\omega_1 = 0,0013$ and daily $\omega_1 = 0,0536$. The motivation behind this view is that Alerion Clean Power is an eco-sustainable company that produces and sells electricity from renewable sources, a sector which has been rapidly growing in the last years. Whilst in the current year it lost about 12%, the general growth of the macro sector to which it belongs (note that green companies have become a strong trend in recent years) led us to forecast that in a year it will rebound and generate 35% of annual return (in line with the previous years). It is to be noted that another reason for the rapid growth of the electricity sector (especially renewables) is to be attributed to the willingness of EU countries to detach more and more from Russian dependency since the war in Ukraine started.

2. Absolute view: "Ecosuntek" will have an annual return of 30% (that translates to 0,12% of daily returns and 2,50% of monthly returns) with estimated uncertainty monthly $\omega_2 = 0,0074$ and daily $\omega_2 = 0,0848$. Being it another eco-sustainable company that produces and sells electricity from solar panels, this forecast is similar to the previous one in terms of reasons and motivations.

3. Absolute view: "Beewize" will have an annual return of 8% (that translates to 0,03% of daily returns and 0,67% of monthly returns) with estimated uncertainty monthly $\omega_3 = 0,0003$ and daily $\omega_3 = 0,0169$. Beewize is a digital marketing company in strong expansion. Note that since its macro sector is already more solid and not in such an expansion as the renewable one, expected growth is necessarily lower than the previous companies.

4. Relative view: "Alerion Clean Power" will overperform "Edison" by 15% in annual return terms with estimated uncertainty monthly $\omega_4 = 0,0013$ and daily $\omega_4 = 0,0599$. Whilst as said the electricity sector is predicted to expand, we predict that Alerion Clean Power will overperform traditional electricity companies due to their focus on renewable, a highly popular subject at the moment.

5. Relative view: "Beewize" will overperform "Vianini" by 2% in annual return terms with estimated uncertainty monthly $\omega_5 = 0,0061$ and $\omega_5 = 0,1558$. We formulated this relative view based on the different forecast of growth of the sectors to which they belong: Beewize is a digital marketing company while Vianini is a company in the construction sector, a lower yielding one.

Now that we defined our views, we were able to compute the remaining vectors needed for our evaluation: $\mathbf{P}$, which is the matrix which tells us to which a specific view is applied, q, the vector of the views, $\mathbf{\Omega}$, the matrix of uncertainty with respect to our views, and $\mathbf{C}$, the matrix of variance-covariance scaled by a factor τ.

We defined τ, $\mathbf{\Omega}$ and $\mathbf{C}$ as follows:

$$\tau = \frac{1}{\text{number of assets}} \tag{5}$$

$$\mathbf{\Omega} = \begin{bmatrix} \omega_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \omega_5 \end{bmatrix}, \quad \text{where } \omega_i = \text{Var}(r_i) \cdot \tau \tag{6}$$

$$\mathbf{C} = \tau \mathbf{\Sigma} \tag{7}$$

We now had all the parameters necessary to compute the posterior estimate for returns and variance-covariance matrix.

From theory, they are calculated by:

$$\boldsymbol{\mu_{BL}} = \left[\mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{P} + \mathbf{C}^{-1}\right]^{-1} \left[\mathbf{P}'\mathbf{\Omega}^{-1}q + \mathbf{C}^{-1}\pi\right] \tag{8}$$

$$\mathbf{\Sigma_{BL}} = \left[\mathbf{C}^{-1} + \mathbf{P}'\mathbf{\Omega}^{-1}\mathbf{P}\right]^{-1} \tag{9}$$

We then used $\boldsymbol{\mu_{BL}}$ and $\mathbf{\Sigma_{BL}}$ as our new input for a classical MV optimization, in the same way as explained above. The results are illustrated in Figure 11,12,13,14. The statistics of the BL optimal portfolio are below in this document, along with those of other portfolios.

12 Standard Bayesian approach

We then moved on to compute our allocation once more, now under a Standard Bayesian model. This approach is a very powerful one since again it allows investors to specify views and beliefs on data. In a Bayesian setting, indeed, the prior produces not a precise estimate of the parameter but a random variable which can be represented by a probability distribution function, whose density is called posterior. Prior density and posterior density are linked, quite obviously, through Bayesian probabilistic rules. In our specific case we followed the exercise's requests and defined our prior density as:

$$f_{\text{prior}} \sim \mathcal{N}(\mu + \sigma, 2 \cdot \mathbf{\Sigma}) = \mathcal{N}(\mu_{\text{prior}}, \mathbf{\Sigma}_{\text{prior}}) \tag{10}$$

where $\mu + \sigma$ is the sum between the historical mean return vector and the historical standard deviation of the same 12 selected companies, and $2 \cdot \mathbf{\Sigma}$ is the original variance-covariance matrix multiplied by 2. From this prior density we recovered the posterior one:

$$f_{\text{post}} \sim \mathcal{N}(\mu_{\text{post}}, \mathbf{\Sigma}_{\text{post}}) \tag{11}$$

Where:

$$\mu_{\text{post}} = \left[(T\mathbf{\Sigma})^{-1} + \mathbf{\Sigma}^{-1}_{\text{prior}}\right]^{-1} \left[(T\mathbf{\Sigma})^{-1}\mu + \mathbf{\Sigma}^{-1}_{\text{prior}}\ \mu_{\text{prior}}\right] \tag{12}$$

$$\mathbf{\Sigma}_{\text{post}} = \left[(T\mathbf{\Sigma})^{-1} + \mathbf{\Sigma}^{-1}_{\text{prior}}\right]^{-1} \tag{13}$$

AssetName	Weights MV constrain	Weights Black_Litterman
"ECOSUNTEK"	0.068525	0.13857
"ALERIONCLEANPOWER"	0.2954	0.29479
"CENTRALEDELLATTEDITALIA"	6.5615e-20	0.00085103
"ELEN"	0.21022	0.019593
"AMPLIFON"	0.29292	0.1533
"FIDIA"	7.5214e-21	0.20788
"RISANAMENTO"	3.5352e-18	0.0039113
"BEEWIZE"	2.4858e-19	0.011271
"JUVENTUSFOOTBALLCLUB"	0.0503	0.02031
"VINCENZOZUCCHI"	1.5698e-18	0.0013733
"VIANINIINDR"	0.082635	0.016541
"EDISONRSP"	3.7764e-17	0.13161

Figure 11: BL Weights for Daily frequency

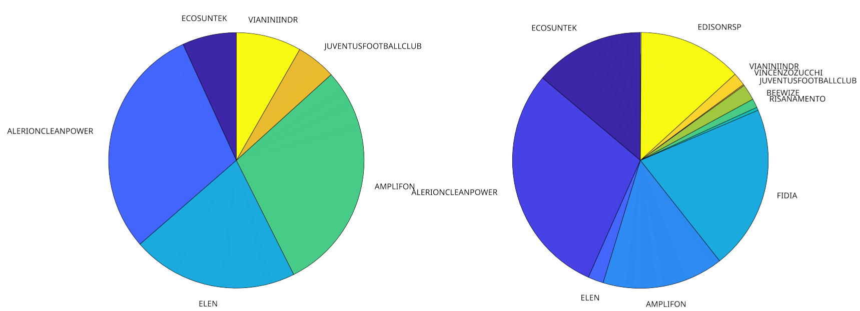


Figure 12: BL Pie chart for Daily frequency

We compute them both in daily and monthly frequency. We then again computed a standard MV optimization, with μ_{post} and $\mathbf{\Sigma}_{\text{post}}$ as our input. The results are presented below in Figure 15 and Figure 16, compared to the BL results.

13 Global Minimum Variance portfolio (GMV)

The Global Minimum Variance (GMV) is a simpler alternative to standard MV optimization as it focuses not on maximizing Sharpe's ratio, but on minimizing the level of risk in the portfolio. In other words, it sacrifices returns in favor of decreasing as much as possible the level of variance of the portfolio. GMV optimization therefore simply consists in finding the portfolio amongst all the feasible and efficient ones, that is the ones on the efficient frontier, that has the overall lower amount of variance (standard deviation). Graphically, it is the

AssetName	Weights MV constrain	Weights Black_Litterman
"ECOSUNTEK"	0.021315	0.029616
"ALERIONCLEANPOWER"	0.27579	0.24888
"CENTRALEDELLATTEDITALIA"	8.9992e-20	0.0010553
"ELEN"	0.13753	0.024296
"AMPLIFON"	0.38261	0.1901
"FIDIA"	1.4759e-20	0.25777
"RISANAMENTO"	5.6053e-19	0.0048501
"BEEWIZE"	0.003134	0.012489
"JUVENTUSFOOTBALLCLUB"	0.062999	0.025184
"VINCENZOZUCCHI"	1.8164e-20	0.001703
"VIANINIINDR"	0.053237	0.013443
"EDISONRSP"	0.063389	0.19062

Figure 13: BL Weights for Monthly frequency

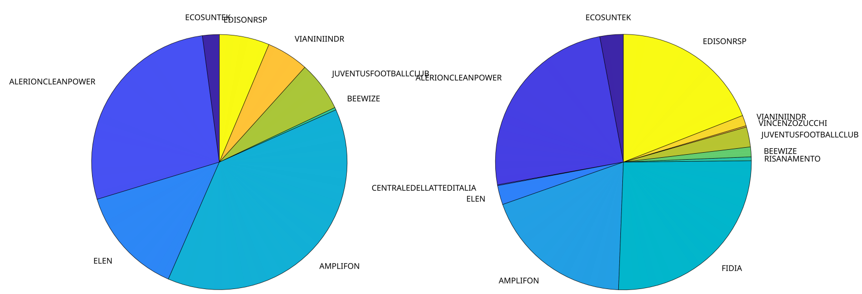


Figure 14: BL Pie chart for Monthly frequency

portfolio which lies on the farthest left point of the efficient frontier. We plotted it (in green) along with the standard MV approach to highlight the difference between the two portfolios [See Figure 17 and Figure 18].

14 Overall descriptive Statistics

We note from figure 18 that the BL portfolio looks rather disappointing: it has a lower mean return and higher standard deviation than Bayesian and MV portfolios. This is to be taken with great care however, since it can be explained by the initial choice of the prior: we indeed used priors calculated not on historical returns but on market capitalization. This technique gives much different weights to assets than our allocations and assumes that there are no risk-free returns, which might not be entirely true. Choosing a different prior, as said, has the ability to dramatically alter our results. The Bayesian optimal port-

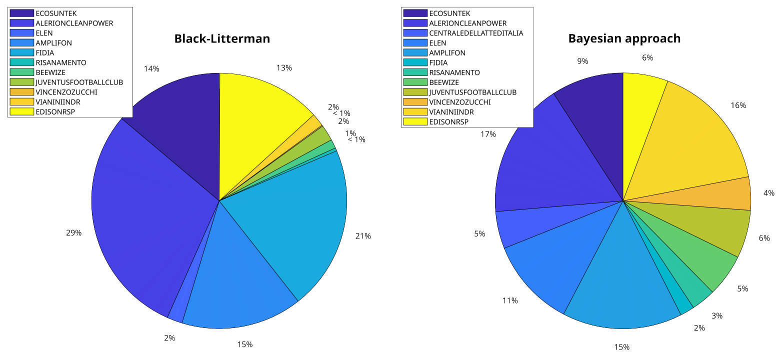


Figure 15: Bayes Pie chart for Daily frequency

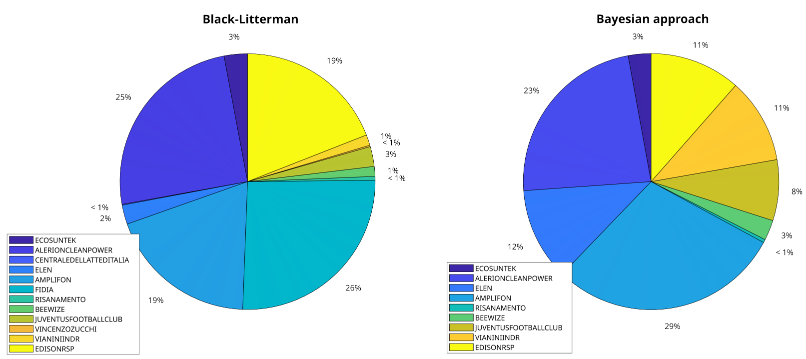
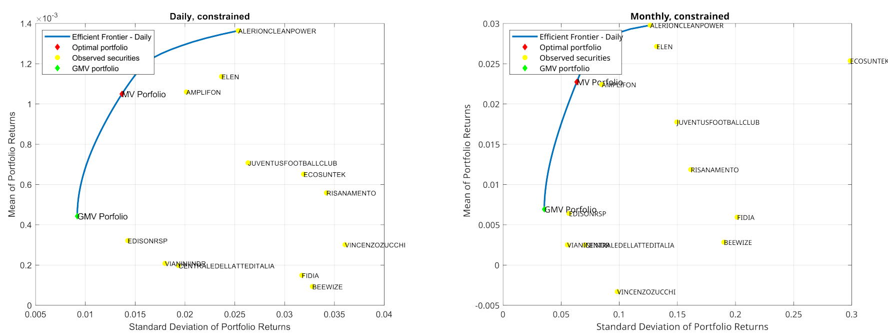


Figure 16: Bayes Pie chart for Monthly frequency

folio is the most equally balanced one, especially the daily where there are no particularly concentrated stocks. It presents a more diversified portfolio than any other method, whilst still providing a very satisfactory Sharpe ratio, just slightly below the MV ones. Also, in terms of general descriptive statistics the Bayesian behaves well, with mean returns and standard deviations again just slightly below the MV optimal portfolios. As far as Skewness is concerned, the BL portfolio behaves better since it always displays a positive one, implying that the right-tail of returns is heavier. This stems from the views we adopted, which are of strong growth, and hence require a great deal of positive returns in order to be achieved. This heavier right-tail is however mitigated by a very high kurtosis, which in the monthly one is the highest by far. The BL portfolios are also the only case where the switch from daily to monthly does not make re-

Asset Name	Weights GMV daily	Weights GMV monthly
"ECOSUNTEK"	0.057398	0
"ALERIONCLEANPOWER"	0.056649	0.028464
"CENTRALEDELLATTEDITALIA"	0.12739	0.13365
"ELEN"	0.028877	0
"AMPLIFON"	0.086343	0.081926
"FIDIA"	0.025835	0.014776
"RISANAMENTO"	0	0
"BEEWIZE"	0.045371	0.027048
"JUVENTUSFOOTBALLCLUB"	0.035622	0.038888
"VINCENZOZUCCHI"	0.021584	0
"VIANINIINDR"	0.23669	0.32673
"EDISONRSP"	0.27824	0.34852

Figure 17: GMV Weights



(a) GMV Daily frontier

(b) GMV Monthly frontier

turns more similar to a gaussian, but the opposite. Finally, the GMV portfolios are, rightfully, the ones with the overall lowest amount of variance and standard deviation. This, however, comes at a cost: returns both for daily and monthly are severely lower than all the previous portfolios. Indeed, the lower amount of variance is not enough to compensate for the lower returns, as we can see from the overall low Sharpe ratios (the lowest overall).

15 Linear Combination of all portfolios

As a last step, we combined all portfolios into a single one. We weighed each of the previous 4 portfolios with a 25% factor and obtained a linear combination of the weights, which we then used to compute all the statistics regarding the combined portfolio. The results obtained are illustrated below. Since we took a linear combination of all the previous portfolios with equal weights, we can expect a mixture of all the properties that we analyzed previously. Firstly, we can

	Mean	Standard deviation	Variance	Skewness	Kurtosis	Sharpe Ratio
Mean-Variance Daily	0.0010499	0.013673	0.00018694	-0.29006	10.049	0.076806
Mean-Variance Monthly	0.022736	0.063425	0.0040227	-0.06695	4.5558	0.36024
Black-Litterman Daily	0.0007722	0.013632	0.00018584	0.13173	10.589	0.056657
Black-Litterman Monthly	0.016411	0.074655	0.0055734	2.2776	14.068	0.22091
Bayesian Daily	0.00072522	0.010651	0.00011344	-0.78812	12.928	0.068105
Bayesian Monthly	0.019888	0.056094	0.0031466	-0.12377	4.3095	0.3563
GMV Daily	0.00044275	0.0091875	8.4411e-05	-0.80366	13.617	0.048202
GMV Monthly	0.0069293	0.035506	0.0012607	-0.056132	2.8916	0.19612

Figure 18: Overall statistcs

Asset Name	Weights Combined daily	Weights Combined monthly
"ECOSUNTEK"	0.088966	0.019952
"ALERIONCLEANPOWER"	0.20474	0.19639
"CENTRALEDELLATTEDITALIA"	0.043806	0.033676
"ELEN"	0.092822	0.06961
"AMPLIFON"	0.1711	0.23697
"FIDIA"	0.062941	0.068137
"RISANAMENTO"	0.0086245	0.00221
"BEEWIZE"	0.027648	0.01707
"JUVENTUSFOOTBALLCLUB"	0.04172	0.050968
"VINCENZOZUCCHI"	0.016278	0.00042574
"VIANINIINDR"	0.12465	0.12523
"EDISONRSP"	0.11671	0.17936

Figure 19: Weights of the COMBINED portfolio

	Mean	Standard deviation	Variance	Skewness	Kurtosis	Sharpe Ratio
Daily combined portfolio	0.00074752	0.010842	0.00011755	-0.63866	13.049	0.068963
Monthly combined porfolio	0.016491	0.051329	0.0026347	0.13617	4.2371	0.32287

Figure 20: Statistics of the COMBINED portfolio

see how mean returns are influenced by the GMV portfolio, which drags them below the MV and Bayesian on level with BL ones. However, GMV also makes the overall standard deviation lower, at about 1.1% daily and 5.1% monthly, very similar to those of the Bayesian model. Skewness behaves differently from daily to monthly, going from negative to positive, but decreasing towards 0 in absolute value. Kurtosis behaves very similarly to that of the Bayesian portfolio, ranging from about 13 in daily frequency to around 4 in monthly. Once again, we observe how increasing the frequency of returns makes its distribution more and more similar to a Gaussian (like in all our portfolios, except for BL).

Despite the fact that an equally weighted combination of our portfolios might seem strange, this has actually proved to be a very nice trade-off between the characteristics of our 4 previous portfolios. As a matter of fact, the results are still very present: this is highlighted by the Sharpe ratios, which are on par with those of our previous portfolios.