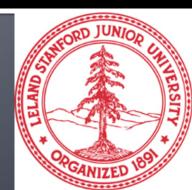
We need your help with our research on human interpretable machine learning. Please complete a survey at http://stanford.io/1WPOKCo. It should be fun and take about 15min to complete. Thanks a lot for your help!

Recommender Systems: Latent Factor Models

CS246: Mining Massive Datasets
Jure Leskovec, Stanford University
http://cs246.stanford.edu



The Netflix Prize: Movie Ratings

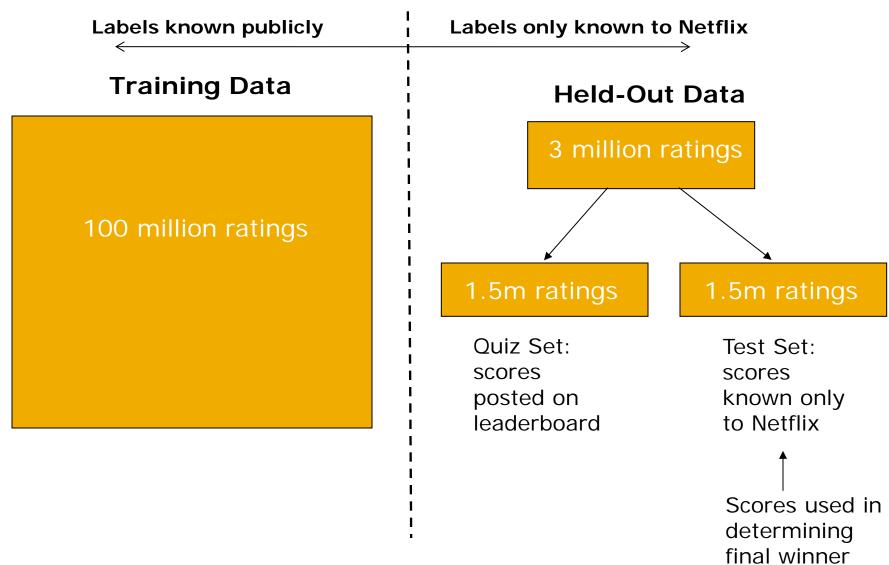
Training movie rating data

- 100 million ratings, 480,000 users, 17,770 movies
- 6 years of data: 2000-2005
- Test movie rating data data
 - Last few ratings of each user (2.8 million ratings)
 - Evaluation criterion: Root Mean Square Error (RMSE)

$$= \frac{1}{|R|} \sqrt{\sum_{(i,x) \in R} (\hat{r}_{xi} - r_{xi})^2}$$

- Netflix's system RMSE: 0.9514
- Competition:
 - 2,700+ teams
 - \$1 million prize for 10% improvement on Netflix

Competition Structure



The Netflix Utility Matrix R

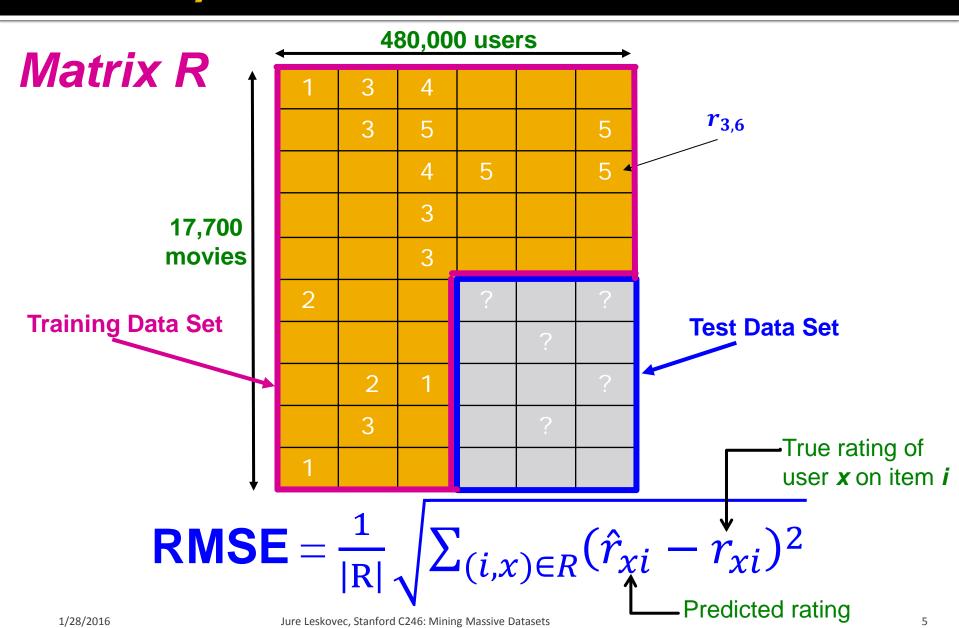
Matrix R

17,700 movies

5 5 5

480,000 users

Utility Matrix R: Evaluation



BellKor Recommender System

The winner of the Netflix Challenge

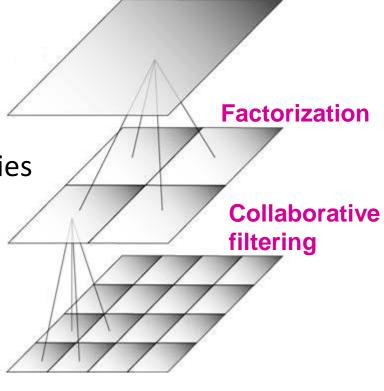
Multi-scale modeling of the data:

Combine top level, "regional" modeling of the data, with a refined, local view:

Global:

Overall deviations of users/movies

- Factorization:
 - Addressing "regional" effects
- Collaborative filtering:
 - Extract local patterns



Global effects

Modeling Local & Global Effects

Global:

- Mean movie rating: 3.7 stars
- The Sixth Sense is 0.5 stars above avg.
- Joe rates 0.2 stars below avg.
 - **⇒** Baseline estimation:

Joe will rate The Sixth Sense 4 stars

- Local neighborhood (CF/NN):
 - Joe didn't like related movie Signs
 - ⇒ Final estimate:
 Joe will rate The Sixth Sense 3.8 stars







Recap: Collaborative Filtering (CF)

- Earliest and most popular collaborative filtering method
- Derive unknown ratings from those of "similar" movies (item-item variant)
- Define similarity measure s_{ij} of items i and j
- Select k-nearest neighbors, compute the rating
 - N(i; x): items most similar to i that were rated by x

$$\hat{r}_{xi} = \frac{\sum_{j \in N(i;x)} s_{ij} \cdot r_{xj}}{\sum_{j \in N(i;x)} s_{ij}}$$

 s_{ij} ... similarity of items i and j r_{xj} ...rating of user x on item j N(i;x)... set of items similar to item i that were rated by x

Modeling Local & Global Effects

In practice we get better estimates if we model deviations:

$$\hat{r}_{xi} = b_{xi} + \frac{\sum_{j \in N(i;x)} s_{ij} \cdot (r_{xj} - b_{xj})}{\sum_{j \in N(i;x)} s_{ij}}$$

baseline estimate for r_{xi}

$$b_{xi} = \mu + b_x + b_i$$

 μ = overall mean rating

 b_x = rating deviation of user x

= $(avg. rating of user x) - \mu$

 $\mathbf{b}_{i} = (avg. \ rating \ of \ movie \ \mathbf{i}) - \boldsymbol{\mu}$

Problems/Issues with CF:

- 1) Similarity measures are "arbitrary"
- **2)** Pairwise similarities neglect interdependencies among users
- **3)** Taking a weighted average can be restricting

Solution: Instead of s_{ij} use w_{ij} that we estimate directly from data

Idea: Interpolation Weights w_{ij}

Use a weighted sum rather than weighted avg.:

$$\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj})$$

- A few notes:
 - N(i; x) ... set of movies rated by user x that are similar to movie i
 - \mathbf{w}_{ij} is the interpolation weight (some real number)
 - We allow: $\sum_{j \in N(i;x)} w_{ij} \neq 1$
 - w_{ij} models interaction between pairs of movies (it does not depend on user x)

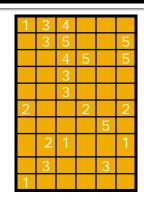
Idea: Interpolation Weights w_{ij}

- $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i,x)} w_{ij} (r_{xj} b_{xj})$
- How to set w_{ij} ?
 - Remember, error metric is: $\frac{1}{|R|} \sqrt{\sum_{(i,x) \in R} (\hat{r}_{xi} r_{xi})^2}$ or equivalently SSE: $\sum_{(i,x) \in R} (\hat{r}_{xi} r_{xi})^2$
 - Find w_{ii} that minimize SSE on training data!
 - Models relationships between item i and its neighbors j
 - w_{ij} can be learned/estimated based on x and all other users that rated i

Why is this a good idea?

Recommendations via Optimization

- Goal: Make good recommendations
 - Quantify goodness using RMSE:
 Lower RMSE ⇒ better recommendations



- Want to make good recommendations on items that user has not yet seen. Can't really do this!
- Let's set build a system such that it works well on known (user, item) ratings
 And hope the system will also predict well the unknown ratings

Recommendations via Optimization

- Idea: Let's set values w such that they work well on known (user, item) ratings
- How to find such values w?
- Idea: Define an objective function and solve the optimization problem
- Find w_{ij} that minimize SSE on training data!

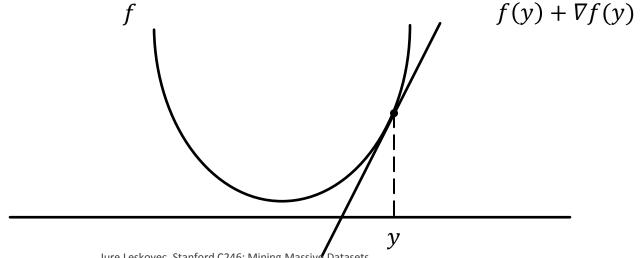
$$J(w) = \sum_{x,i \in R} \left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj}) \right] - r_{xi} \right)^{2}$$
Predicted rating

Predicted rating

Think of w as a vector of numbers

Detour: Minimizing a function

- A simple way to minimize a function f(x):
 - Compute the take a derivative ∇f
 - Start at some point y and evaluate $\nabla f(y)$
 - Make a step in the reverse direction of the gradient: $y = y - \nabla f(y)$
 - Repeat until converged



Interpolation Weights

• We have the optimization problem, now what?

$$J(w) = \sum_{x,i \in R} \left(\left[b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} - b_{xj}) \right] - r_{xi} \right)^{2}$$

- Gradient decent:
 - Iterate until convergence: $w \leftarrow w \eta \nabla_w J$ $\eta \dots$ learning rate
 - where $\nabla_w J$ is the gradient (derivative evaluated on data):

$$\nabla_{w}J = \left[\frac{\partial J(w)}{\partial w_{ij}}\right] = 2\sum_{x,i\in R} \left(\left[b_{xi} + \sum_{k\in N(i;x)} w_{ik}(r_{xk} - b_{xk})\right] - r_{xi}\right) (r_{xj} - b_{xj})$$

$$\text{for } \boldsymbol{j} \in \{\boldsymbol{N}(\boldsymbol{i};\boldsymbol{x}), \forall \boldsymbol{i}, \forall \boldsymbol{x}\}$$

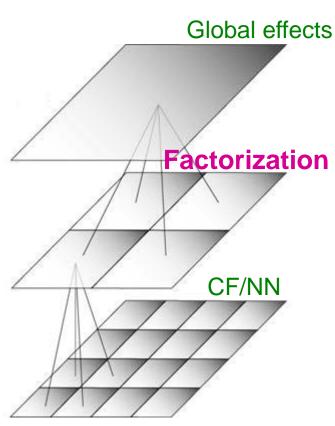
$$\text{else } \frac{\partial J(w)}{\partial w_{ij}} = \boldsymbol{0}$$

■ Note: We fix movie i, go over all r_{xi} , for every movie $j \in N(i; x)$, we compute $\frac{\partial J(w)}{\partial w_{ij}}$ while $|w_{now}|$ -

while
$$|w_{new} - w_{old}| > \varepsilon$$
:
 $w_{old} = w_{new}$
 $w_{new} = w_{old} - \eta \cdot \nabla w_{old}$

Interpolation Weights

- So far: $\widehat{r_{xi}} = b_{xi} + \sum_{j \in N(i;x)} w_{ij} (r_{xj} b_{xj})$
 - Weights w_{ij} derived based on their role; no use of an arbitrary similarity measure (w_{ii} ≠ s_{ii})
 - Explicitly account for interrelationships among the neighboring movies
- Next: Latent factor model
 - Extract "regional" correlations



Performance of Various Methods

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

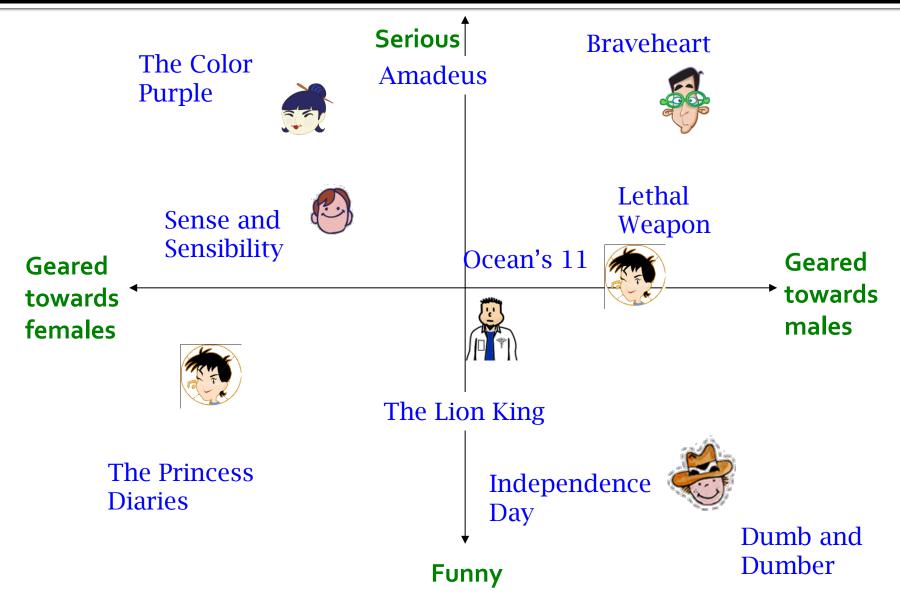
Netflix: 0.9514

Basic Collaborative filtering: 0.94

CF+Biases+learned weights: 0.91

Grand Prize: 0.8563

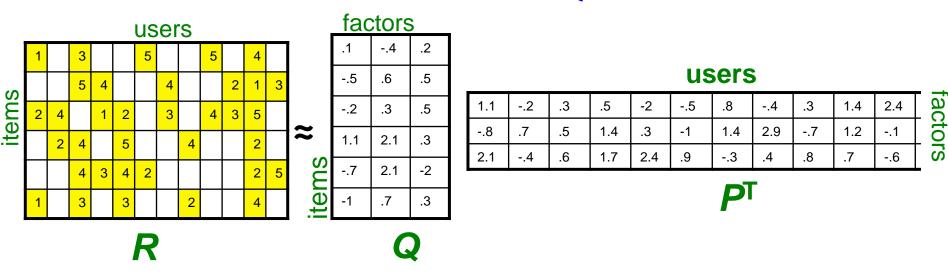
Latent Factor Models (e.g., SVD)



Latent Factor Models

SVD: $A = U \Sigma V^T$

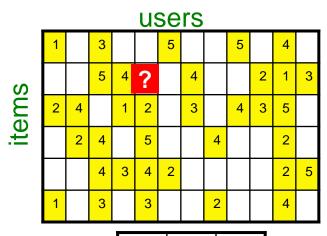
"SVD" on Netflix data: R ≈ Q · P^T



- For now let's assume we can approximate the rating matrix R as a product of "thin" $Q \cdot P^T$
 - R has missing entries but let's ignore that for now!
 - Basically, we will want the reconstruction error to be small on known ratings and we don't care about the values on the missing ones

Ratings as Products of Factors

■ How to estimate the missing rating of user x for item i?





\hat{r}_{x}	<u> </u>	q_i	• 1	o_x
=		q_{ij}	f •	p_{xf}
•	<i>f</i>	= row <i>i</i>	of C	0
		= colur		

	.1	4	.2
w	5	.6	.5
items	2	.3	.5
ite	1.1	2.1	.3
	7	2.1	-2
	-1	.7	.3

factors

users .3 .5 -.5 .3 -.2 -2 8. -.4 .7 .5 1.4 2.9 1.4 -1 -.7 -.4 1.7 2.4 -.3 .4 PT

2.4

-.1

-.6

-.9

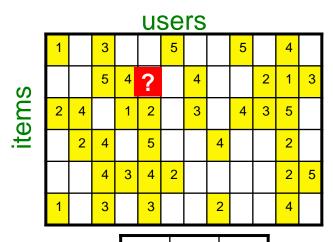
1.3

1.4

1.2

Ratings as Products of Factors

■ How to estimate the missing rating of user x for item i?





\hat{r}_{xi}	$= q_i \cdot p_x$
	$\sum q_{if} \cdot p_{xf}$
	f
	$q_i = \text{row } i \text{ of } Q$ $p_x = \text{column } x \text{ of } P^T$
	ρ_{x} – column x or r

	.1	4	.2	
(0	5	.6	.5	
items	2	.3	.5	
ite	1.1	2.1	.3	
	7	2.1	-2	
	-1	.7	.3	
factors				

_												
S	1.1	2	.3	.5	-2	5	.8	4	.3	1.4	2.4	9
	8	.7	.5	1.4	.3	-1	1.4	2.9	7	1.2	1	1.3
to the	2.1	4	.6	1.7	2.4	.9	3	.4	.8	.7	6	.1
•												

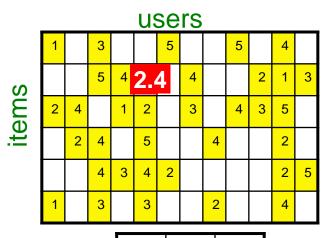
users

PT

G

Ratings as Products of Factors

■ How to estimate the missing rating of user x for item i?





$= \sum q_{if} \cdot p_{xj}$	f
f	
q_i = row i of Q p_x = column x of P^T	

	.1	4	.2
(0	5	.6	.5
items	2	.3	.5
ite	1.1	2.1	.3
	7	2.1	-2
	-1	.7	.3

f factors

users .3 .5 -.5 .3 -.2 1.4 1.1 -2 -.4 .7 .5 1.4 2.9 1.2 1.4 -1 -.7 -.4 1.7 2.4 -.3 .4 PT

Q

2.4

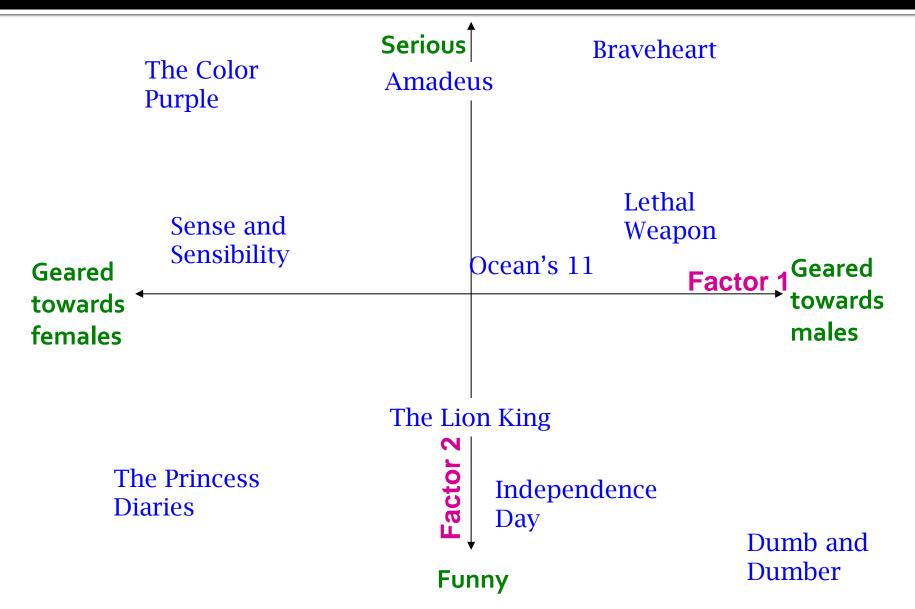
-.1

-.6

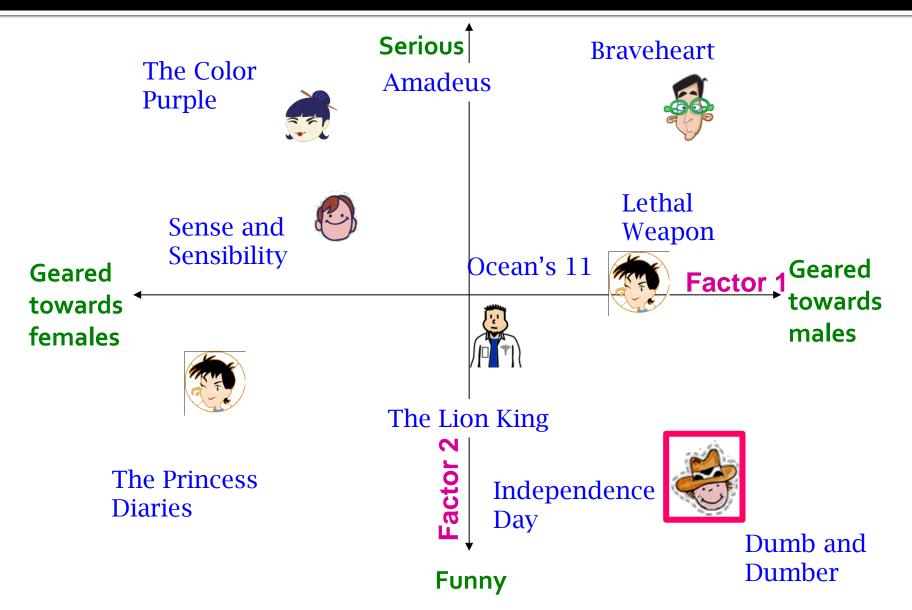
-.9

1.3

Latent Factor Models



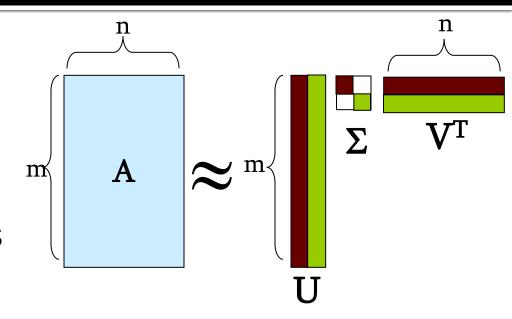
Latent Factor Models



Recap: SVD

Remember SVD:

- A: Input data matrix
- U: Left singular vecs
- V: Right singular vecs
- Σ: Singular values



So in our case:

"SVD" on Netflix data: $R \approx Q \cdot P^T$

$$A = R$$
, $Q = U$, $P^{T} = \sum V^{T}$

$$\hat{r}_{xi} = q_i \cdot p_x$$

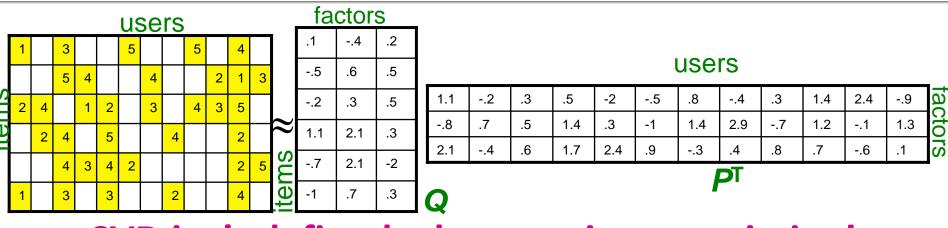
SVD: More good stuff

 We already know that SVD gives minimum reconstruction error (Sum of Squared Errors):

$$\min_{U,V,\Sigma} \sum_{ij\in A} \left(A_{ij} - [U\Sigma V^{\mathrm{T}}]_{ij} \right)^{2}$$

- Note two things:
 - SSE and RMSE are monotonically related:
 - $RMSE = \frac{1}{c}\sqrt{SSE}$ Great news: SVD is minimizing RMSE!
 - Complication: The sum in SVD error term is over all entries (no-rating in interpreted as zero-rating). But our R has missing entries!

Latent Factor Models



- SVD isn't defined when entries are missing!
- Use specialized methods to find P, Q

$$\min_{P,Q} \sum_{(i,x)\in \mathbf{R}} (r_{xi} - q_i \cdot p_x)^2$$

$$\hat{r}_{xi} = q_i \cdot p_x$$

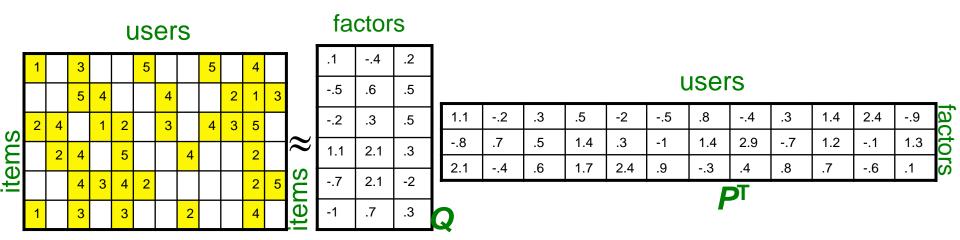
- Note:
 - We don't require cols of P, Q to be orthogonal/unit length
 - P, Q map users/movies to a latent space
 - The most popular model among Netflix contestants

Finding the Latent Factors

Latent Factor Models

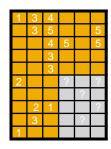
Our goal is to find P and Q such tat:

$$\min_{P,Q} \sum_{(i,x)\in R} (r_{xi} - q_i \cdot p_x)^2$$



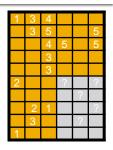
Back to Our Problem

- Want to minimize SSE for unseen test data
- Idea: Minimize SSE on training data
 - Want large k (# of factors) to capture all the signals
 - But, **SSE** on test data begins to rise for k > 2
- This is a classical example of overfitting:
 - With too much freedom (too many free parameters) the model starts fitting noise
 - That is, it fits too well the training data and thus not generalizing well to unseen test data



Dealing with Missing Entries

To solve overfitting we introduce regularization:

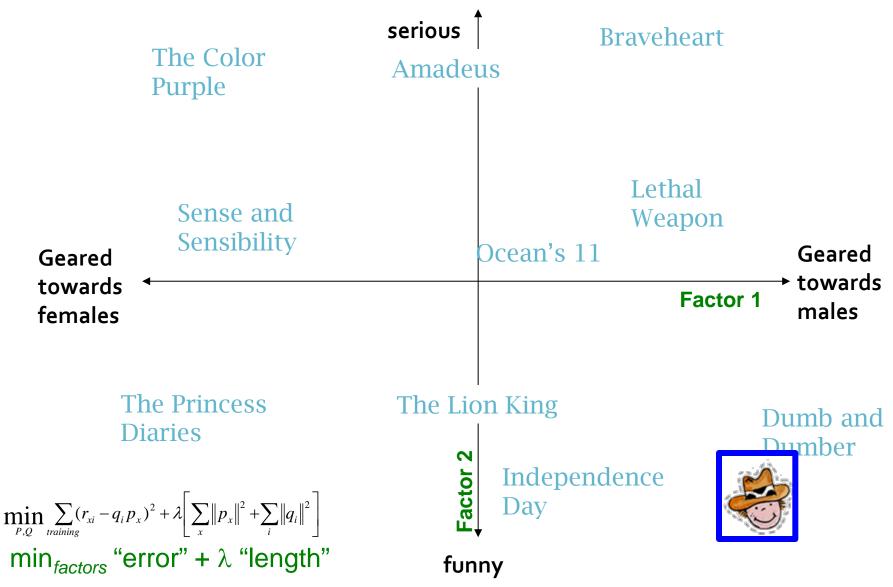


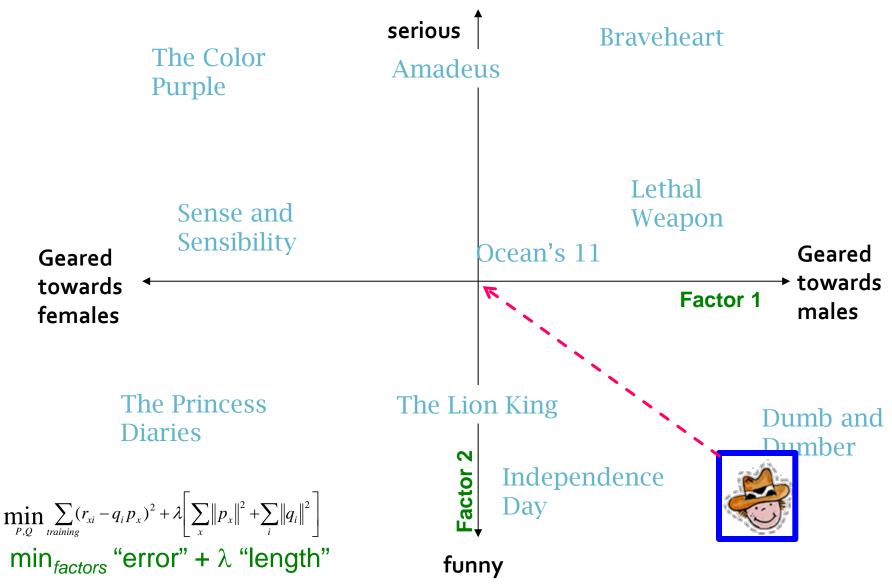
- Allow rich model where there are sufficient data
- Shrink aggressively where data are scarce

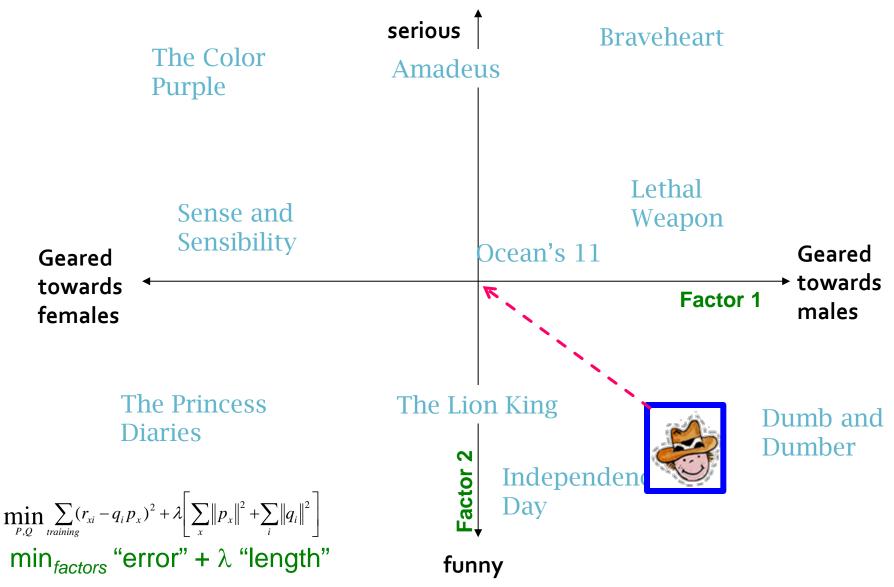
$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_{x} \|p_x\|^2 + \lambda_2 \sum_{i} \|q_i\|^2 \right]$$
"error"
"length"

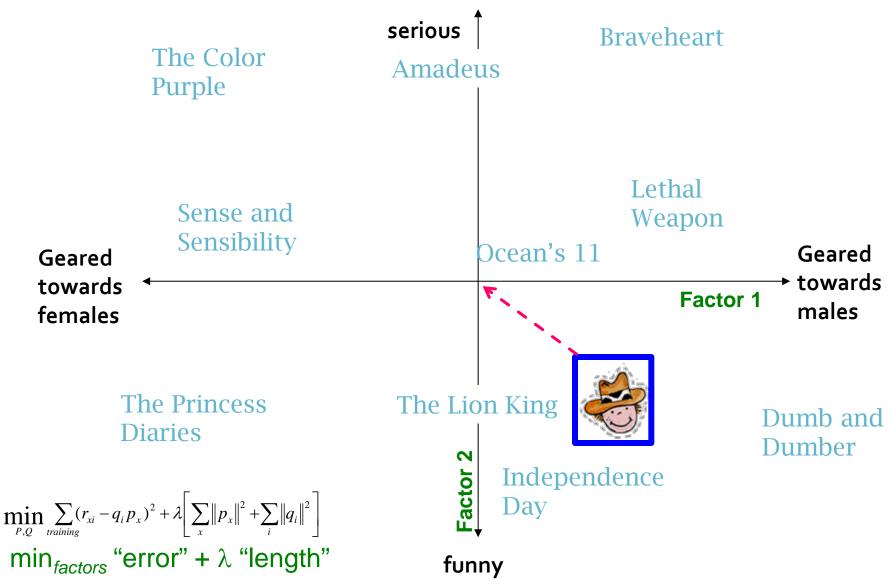
 $\lambda_1, \lambda_2 \dots$ user set regularization parameters

Note: We do not care about the "raw" value of the objective function, but we care in P,Q that achieve the minimum of the objective

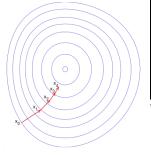








Stochastic Gradient Descent



Want to find matrices P and Q:

$$\min_{P,Q} \sum_{training} (r_{xi} - q_i p_x)^2 + \left[\lambda_1 \sum_{x} ||p_x||^2 + \lambda_2 \sum_{i} ||q_i||^2 \right]$$

- Gradient descent:
 - Initialize P and Q (using SVD, pretend missing ratings are 0)
 - Do gradient descent:

$$\blacksquare$$
 P ← *P* - η · ∇ P

•
$$Q \leftarrow Q - \eta \cdot \nabla Q$$

How to compute gradient of a matrix?

Compute gradient of every element independently!

• where ∇Q is gradient/derivative of matrix Q:

$$\nabla Q = [\nabla q_{if}]$$
 and $\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_i p_x)p_{xf} + 2\lambda_2 q_{if}$

- lacktriangle Here q_{if} is entry f of row $oldsymbol{q}_i$ of matrix $oldsymbol{Q}$
- Observation: Computing gradients is slow!

Stochastic Gradient Descent

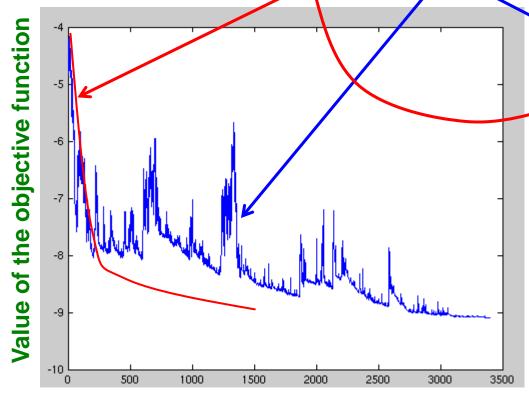
- Gradient Descent (GD) vs. Stochastic GD (SGD)
 - Observation: $\nabla Q = [\nabla q_{if}]$ where

$$\nabla q_{if} = \sum_{x,i} -2(r_{xi} - q_{if}p_{xf})p_{xf} + 2\lambda q_{if} = \sum_{x,i} \nabla Q(r_{xi})$$

- Here q_{if} is entry f of row q_i of matrix Q
- $\mathbf{Q} = \mathbf{Q} \mathbf{Q}$

SGD vs. GD

Convergence of GD vs. SGD



Iteration/step

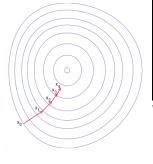
GD improves the value of the objective function at every step.

SGD improves the value but in a "noisy" way.

GD takes fewer steps to converge but each step takes much longer to compute.

In practice, **SGD** is much faster!

Stochastic Gradient Descent



Stochastic gradient descent:

- Initialize P and Q (using SVD, pretend missing ratings are 0)
- Then iterate over the ratings (multiple times if necessary) and update factors:

For each r_{xi} :

$$\bullet \ \varepsilon_{xi} = 2(r_{xi} - q_i \cdot p_x)$$

$$q_i \leftarrow q_i + \mu_1 \left(\varepsilon_{xi} \ p_x - \lambda_2 \ q_i \right)$$

$$p_x \leftarrow p_x + \mu_2 \left(\varepsilon_{xi} \ q_i - \lambda_1 \ p_x \right)$$

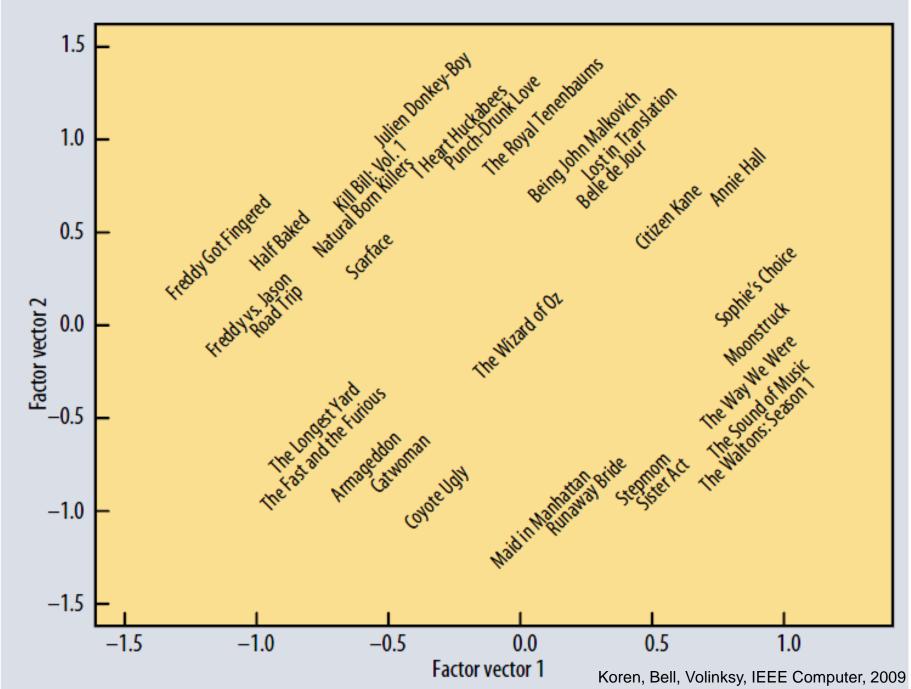
2 for loops:

- For until convergence:
 - For each r_{xi}
 - Compute gradient, do a "step" a above

(derivative of the "error")

(update equation)

(update equation) μ ... learning rate



Extending Latent Factor Model to Include Biases

Modeling Biases and Interactions

user bias



movie bias



user-movie interaction



Baseline predictor

- Separates users and movies
- Benefits from insights into user's behavior
- Among the main practical contributions of the competition
 - μ = overall mean rating
 - $\mathbf{b}_{\mathbf{x}}$ = bias of user \mathbf{x}
 - \mathbf{b}_{i}^{n} = bias of movie \mathbf{i}

User-Movie interaction

- Characterizes the matching between users and movies
- Attracts most research in the field
- Benefits from algorithmic and mathematical innovations

Baseline Predictor

We have expectations on the rating by user x of movie i, even without estimating x's attitude towards movies like i







- Rating scale of user x
- Values of other ratings user gave recently (day-specific mood, anchoring, multi-user accounts)

- (Recent) popularity of movie i
- Selection bias; related to number of ratings user gave on the same day ("frequency")

Putting It All Together

$$r_{\chi i} = \mu + b_{\chi} + b_{i} + q_{i} \cdot p_{\chi}$$

Overall

Mean rating

Overall

Sias for movie i

Movie i

Interaction

Example:

- Mean rating: $\mu = 3.7$
- You are a critical reviewer: your ratings are 1 star lower than the mean: $b_x = -1$
- Star Wars gets a mean rating of 0.5 higher than average movie: $b_i = +0.5$
- Predicted rating for you on Star Wars:

$$= 3.7 - 1 + 0.5 = 3.2$$

Fitting the New Model

Solve:

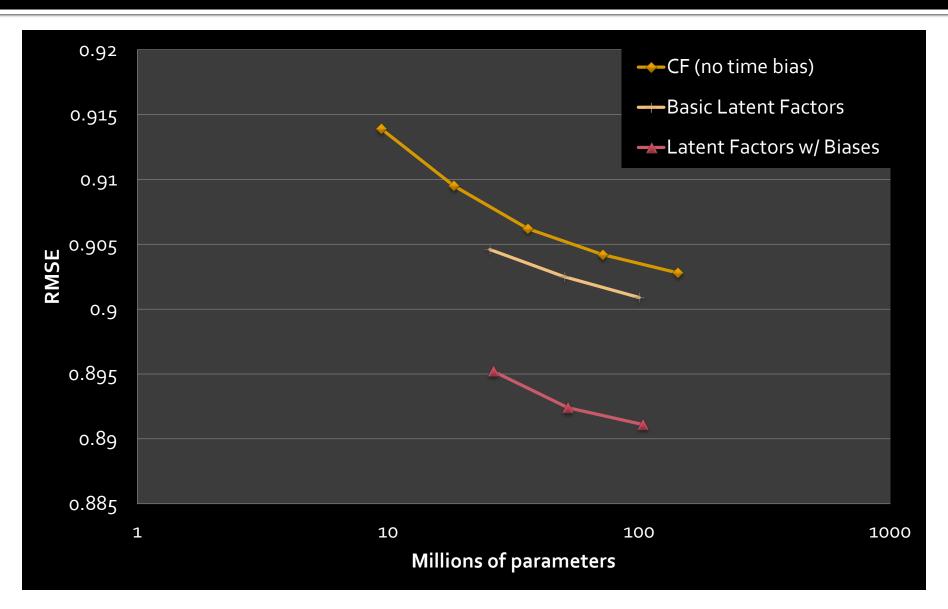
$$\min_{Q,P} \sum_{(x,i)\in R} (r_{xi} - (\mu + b_x + b_i + q_i p_x))^2$$
goodness of fit

$$+ \left(\frac{\lambda_{1}}{1} \sum_{i} \|q_{i}\|^{2} + \lambda_{2} \sum_{x} \|p_{x}\|^{2} + \lambda_{3} \sum_{x} \|b_{x}\|^{2} + \lambda_{4} \sum_{i} \|b_{i}\|^{2} \right)$$
regularization

 λ is selected via gridsearch on a validation set

- Stochastic gradient decent to find parameters
 - Note: Both biases b_x , b_i as well as interactions q_i , p_x are treated as parameters (we estimate them)

Performance of Various Methods



Performance of Various Methods

Global average: 1.1296

User average: 1.0651

Movie average: 1.0533

Netflix: 0.9514

Basic Collaborative filtering: 0.94

Collaborative filtering++: 0.91

Latent factors: 0.90

Latent factors+Biases: 0.89

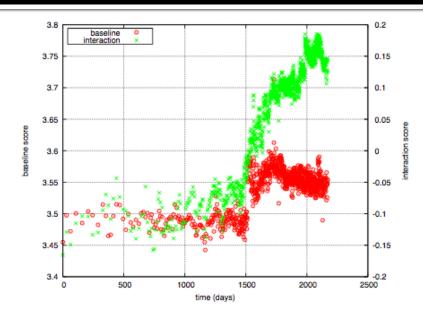
Grand Prize: 0.8563

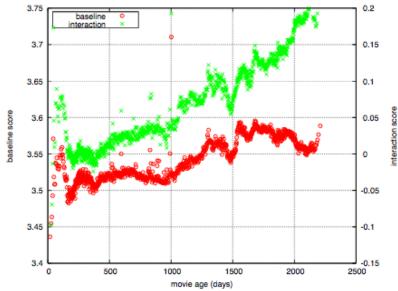
The Netflix Challenge: 2006-09

Temporal Biases Of Users

- Sudden rise in the average movie rating (early 2004)
 - Improvements in Netflix
 - GUI improvements
 - Meaning of rating changed
- Movie age
 - Users prefer new movies without any reasons
 - Older movies are just inherently better than newer ones

Y. Koren, Collaborative filtering with temporal dynamics, KDD '09





Temporal Biases & Factors

Original model:

$$r_{xi} = \mu + b_x + b_i + q_i \cdot p_x$$

Add time dependence to biases:

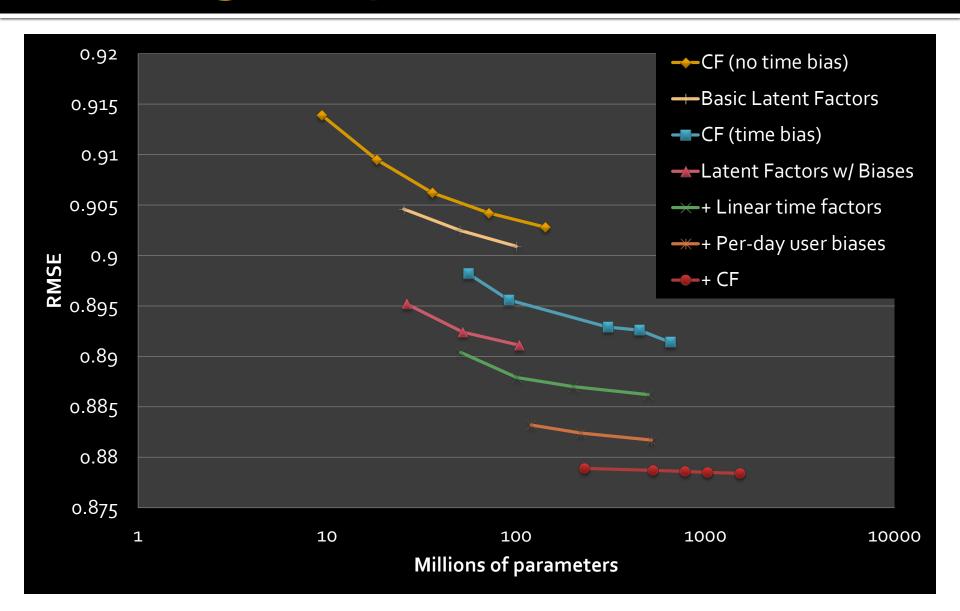
$$r_{xi} = \mu + b_x(t) + b_i(t) + q_i \cdot p_x$$

- Make parameters b_x and b_i to depend on time
- (1) Parameterize time-dependence by linear trends
 - (2) Each bin corresponds to 10 consecutive weeks

$$b_i(t) = b_i + b_{i,\operatorname{Bin}(t)}$$

- Add temporal dependence to factors
 - $p_x(t)$... user preference vector on day t

Adding Temporal Effects



Performance of Various Methods

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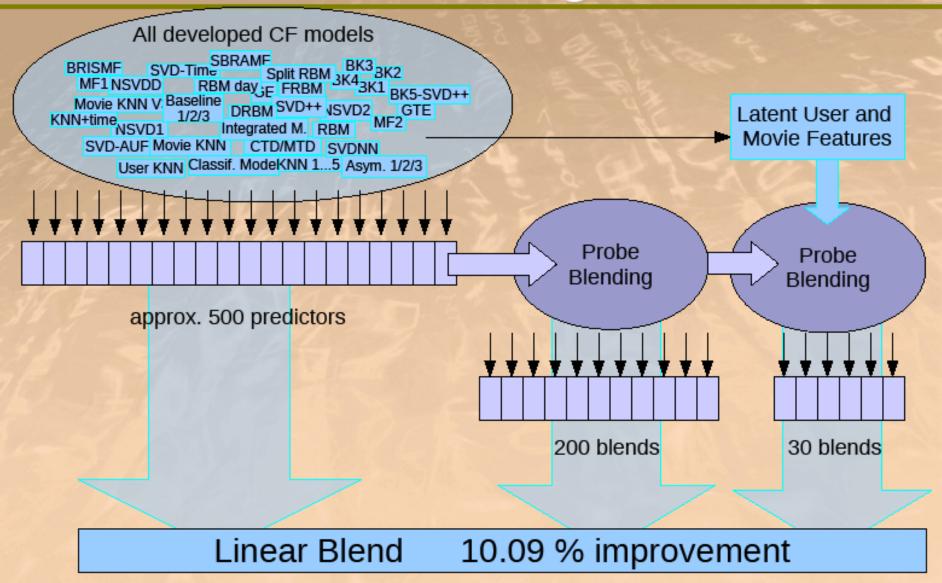
Latent factors+Biases+Time: 0.876

Still no prize! (2)
Getting desperate.
Try a "kitchen sink" approach!

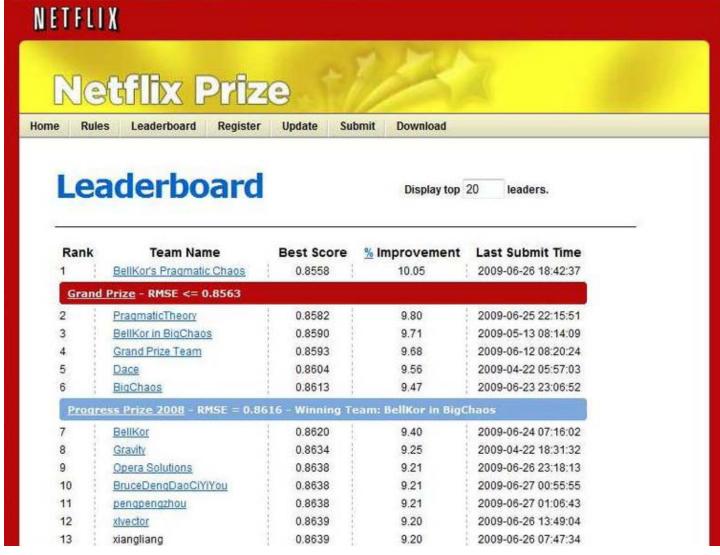
Grand Prize: 0.8563

The big picture

Solution of BellKor's Pragmatic Chaos



Standing on June 26th 2009



June 26th submission triggers 30-day "last call"

The Last 30 Days

Ensemble team formed

- Group of other teams on leaderboard forms a new team
- Relies on combining their models
- Quickly also get a qualifying score over 10%

BellKor

- Continue to get small improvements in their scores
- Realize they are in direct competition with team Ensemble

Strategy

- Both teams carefully monitoring the leaderboard
- Only sure way to check for improvement is to submit a set of predictions
 - This alerts the other team of your latest score

24 Hours from the Deadline

- Submissions limited to 1 a day
 - Only 1 final submission could be made in the last 24h
- 24 hours before deadline...
 - BellKor team member in Austria notices (by chance) that Ensemble posts a score that is slightly better than BellKor's
- Frantic last 24 hours for both teams
 - Much computer time on final optimization
 - Carefully calibrated to end about an hour before deadline
- Final submissions
 - BellKor submits a little early (on purpose), 40 mins before deadline
 - Ensemble submits their final entry 20 mins later
 -and everyone waits....

Netflix Prize



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Leaderboard

Showing Test Score. Click here to show quiz score

Display top 20 💠 leaders.

Rank	Team Name	Best Test Score	% Improvement	Best Submit Time
Grand Prize - RMSE = 0.8567 - Winning Team: BellKor's Progratic Chare				
1	BellKor's Pragmatic Chaos	0.8567	10.06	2009-07-26 18:18:28
2	The Ensemble	0.8567	10.06	2009-07-26 18:38:22
3	Grand Prize Team	0.0082	5.55	2000-07-10-21:24.40
4	Opera Solutions and Vandelay United	0.8588	9.84	2009-07-10 01:12:31
5	Vandelay Industries !	0.8591	9.81	2009-07-10 00:32:20
6	PragmaticTheory	0.8594	9.77	2009-06-24 12:06:56
7	BellKor in BigChaos	0.8601	9.70	2009-05-13 08:14:09
8	Dace_	0.8612	9.59	2009-07-24 17:18:43
9	Feeds2	0.8622	9.48	2009-07-12 13:11:51
10	BigChaos	0.8623	9.47	2009-04-07 12:33:59
11	Opera Solutions	0.8623	9.47	2009-07-24 00:34:07
12	BellKor	0.8624	9.46	2009-07-26 17:19:11
Progress Prize 2008 - RMSE = 0.8627 - Winning Team: BellKor in BigChaos				
13	xiangliang	0.8642	9.27	2009-07-15 14:53:22
14	Gravity	0.8643	9.26	2009-04-22 18:31:32
15	Ces	0.8651	9.18	2009-06-21 19:24:53
16	Invisible Ideas	0.8653	9.15	2009-07-15 15:53:04
17	Just a guy in a garage	0.8662	9.06	2009-05-24 10:02:54
18	J Dennis Su	0.8666	9.02	2009-03-07 17:16:17
19	Craig Carmichael	0.8666	9.02	2009-07-25 16:00:54
20	acmehill	0.8668	9.00	2009-03-21 16:20:50

Million \$ Awarded Sept 21st 2009



What's the moral of the story?

Submit early! ©

We need your help with our research on human interpretable machine learning. Please complete a survey at http://stanford.io/1WPOKCo It should be fun and take about 15min to complete.

Thanks a lot for your help!

Acknowledgments

- Some slides and plots borrowed from Yehuda Koren, Robert Bell and Padhraic Smyth
- Further reading:
 - Y. Koren, Collaborative filtering with temporal dynamics, KDD '09
 - http://www2.research.att.com/~volinsky/netflix/bpc.html
 - http://www.the-ensemble.com/