Principle

The electric constant ε_0 is determined by measuring the charge of a plate capacitor to which a voltage is applied. The dielectric constant ε is determined in the same way, with plastic or glass filling the space between the plates.

Related topics

Maxwell's equations, electric constant, capacitance of a plate capacitor, real charges, free charges, dielectric displacement, dielectric polarisation, dielectric constant.

Equipment

1	Plate capacitor, d = 260 mm	06220-00
1	Plastic plate 283×283 mm	06233-01
1	Glass plates f. current conductors	06406-00
1	High-value resistor, 10 MOhm	07160-00
1	Universal measuring amplifier	13626-93
1	High voltage supply unit, 0-10 kV	13670-93
1	Capacitor/case 1/0.22 µF	39105-19
1	Voltmeter, 0.3-300 VDC, 10-300 VAC	07035-00
1	Connecting cord, l = 100 mm, green-yellow	07359-15
1	Connecting cord, l = 500 mm, red	07361-01
1	Connecting cord, l = 500 mm, blue	07361-04
1	Connecting cord, l = 1000 mm, yellow	07363-02
1	Connecting cord, 30 kV, l = 500 mm	07366-00
1	Screened cable, BNC, l = 750 mm	07542-11
1	Adapter, BNC socket - 4 mm plug	07542-20
1	Connector, T type, BNC	07542-21
1	Adapter, BNC-plug/socket 4 mm	07542-26

Figure 1: Measurement set-up: Dielectric constant of different materials.

Tasks

1. The relation between charge Q and voltage U is to be measured using a plate capacitor.

2. The electric constant ε_0 is to be determined from the relation measured under point 1.

3. The charge of a plate capacitor is to be measured as a function of the inverse of the distance between the plates, under constant voltage.

4. The relation between charge Q and voltage U is to be measured by means of a plate capacitor, between the plates of which different solid dielectric media are introduced.

The corresponding dielectric constants are determined by comparison with measurements performed with air between the capacitor plates.

Set-up and procedure

The experimental set-up is shown in fig. 1 and the corresponding wiring diagram in fig. 2. The highly insulated capacitor plate is connected to the upper connector of the high voltage power supply over the 10 MΩ protective resistors. Both the middle connector of the high voltage power supply and the opposite capacitor plate are grounded over the 220 nF capacitor. Correct measurement of the initial voltage is to be assured by the corresponding adjustment of the toggle switch on the unit. The electrostatic induction charge on the plate capacitor can be measured over the voltage on the 220 nF capacitor, according to equation (4). The measurement amplifier is set to high input resistance, to amplification factor 1 and to time constant 0.

In a first step the plate capacitor is charged with the high voltage power supply. In a second step (with the high voltage power supply disconnected!) the charge of the plate capacitor is measured.
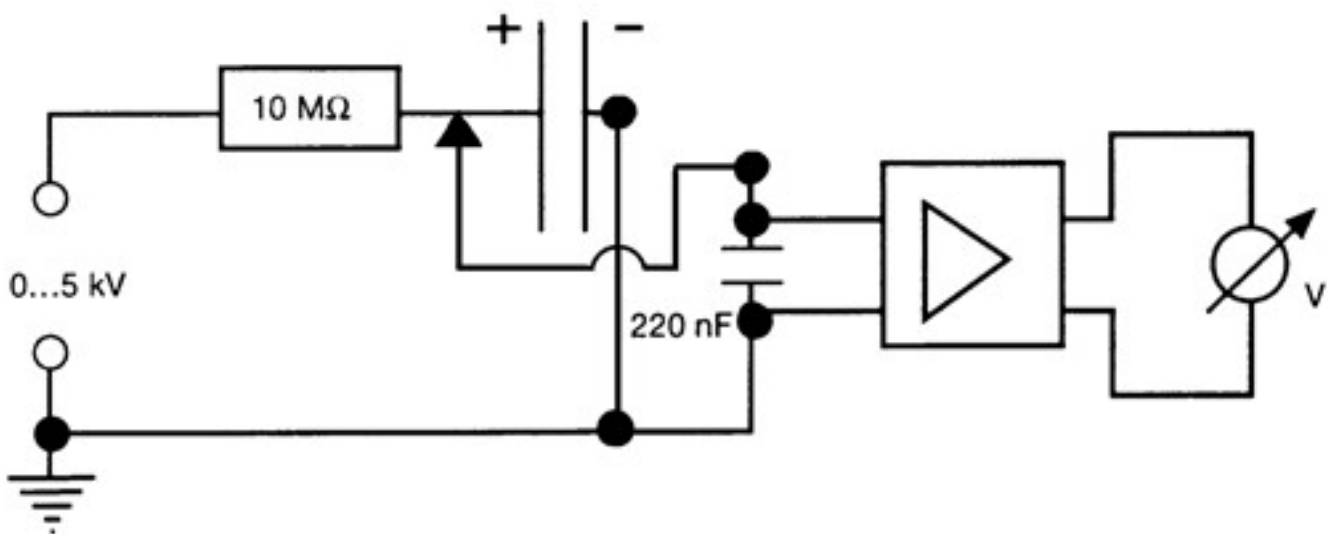

Figure 2: Wiring diagram.

To start with, the surface of the capacitor plates is determined by means of their radius. The experiment is carried out in two parts:

1. In the first part, the distance between the plates of the plate capacitor is varied under constant voltage, and the charge on the capacitor plates is measured. The linear relation between charge and plate capacitor voltage is then verified.

   Measurement data allow to determine the electric constant ε_0, using equation (4).

   Be sure not to be near the capacitor during measurements, as otherwise the electric field of the capacitor might be distorted.

2. In the second part, the dependence of the electrostatic induction charge from voltage, with and without plastic plate (without air gap!), is examined in the space between the plates, with the same distance between the plates. The ratio between the electrostatic induction charges allows to determine the dielectric constant ε_0 of plastic. The dielectric constant of the glass plate is determined in the same way.
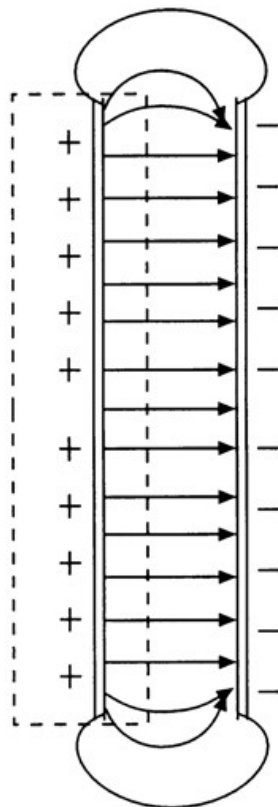

Figure 3: Electric field of a plate capacitor with small distance between the plates, as compared to the diameter of the plates. The dotted lines indicate the volume of integration.

Theory and evaluation

Electrostatic processes in vacuum (and with a good degree of approximation in air) are described by the following integral form of Maxwell's equations:

$$\vec{D} = \varepsilon_0 \cdot \vec{E} + \vec{P} = \varepsilon \cdot \varepsilon_0 \cdot \vec{E} \tag{1}$$
$$\tag{2}$$

where $\vec{E}$ is the electric field intensity, Q the charge enclosed by the closed surface A, ε_0 the electric constant and s a closed path.

If a voltage U_c is applied between two capacitor plates, an electric field $U_C = \frac{U_{vac}}{\varepsilon}$ will prevail between the plates, which is defined by:

$$U_C = \int_1^2 \vec{E} \, \mathrm{d}\vec{r}$$

(cf. figure 3). Due to the electric field, electrostatic charges of the opposite sign are drawn towards the surfaces of the capacitor. As voltage sources do not generate charges, but only can separate them, the absolute values of the opposite electrostatic induction charges must be equal.

Assuming the field lines of the electric field always to be perpendicular to the capacitor surfaces of surface A, due to symmetry, which can be experimentally verified for small distances d between the capacitor plates, one obtains from equation (1):
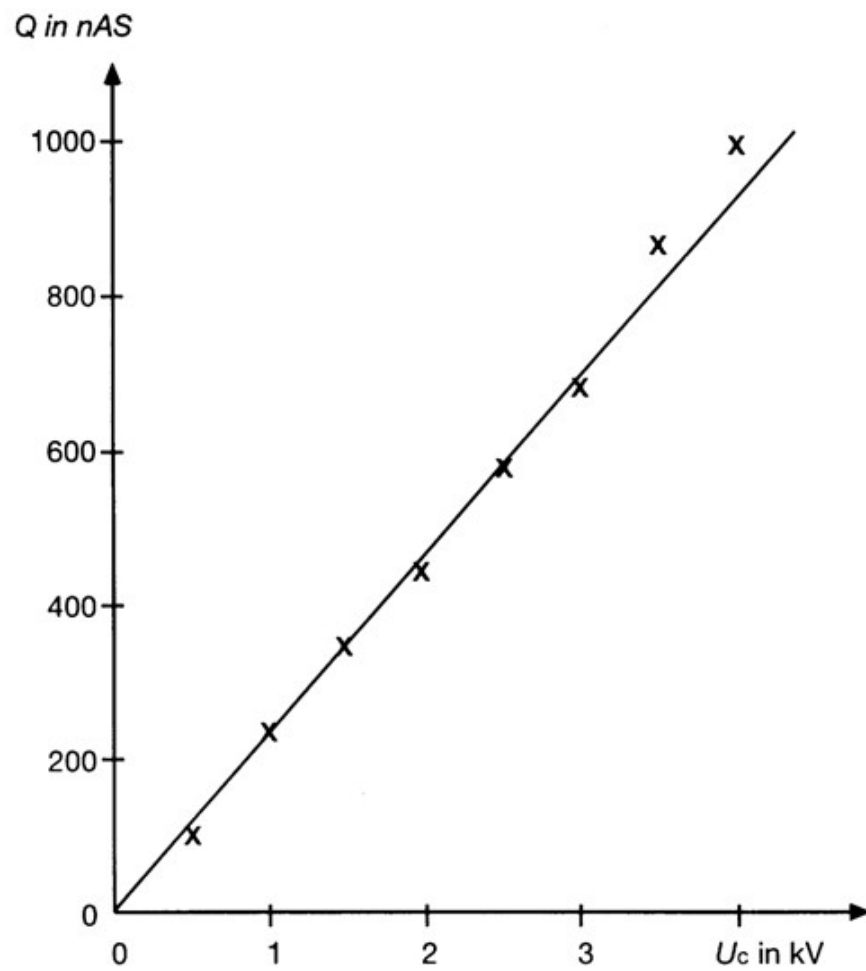
$$C = \varepsilon \cdot C_{\mathrm{vac}} \tag{3}$$

Fig 4: Electrostatic charge Q of a plate capacitor as a function of the applied voltage U_C (d = 0.2 cm).
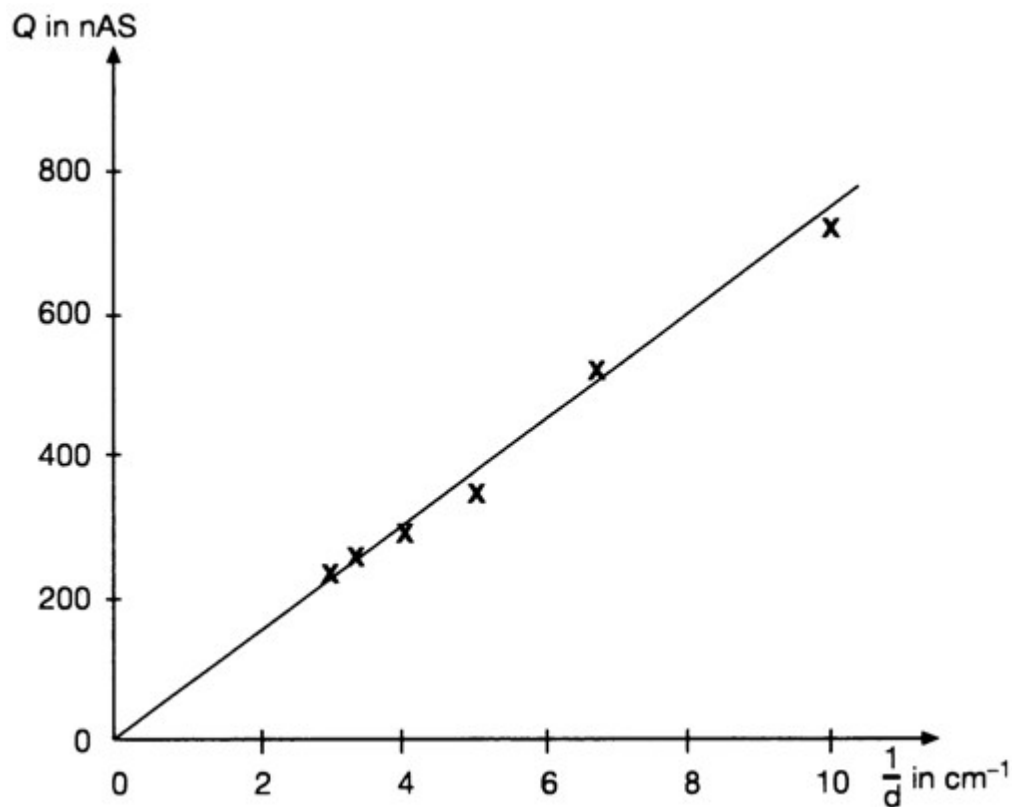
Fig 5: Electrostatic charge Q of a plate capacitor as a function of the inverse distance between the capacitor plates d^{-1} (U_c = 1.5 kV).

The volume indicated in fig. 3, which only encloses one capacitor plate, was taken as volume of integration. As the surface within the capacitor may be displaced without changing the flux, the capacitor field is homogeneous. Both the flow and the electric field E outside the capacitor are zero, because for arbitrary volumes which enclose both capacitor plates, the total enclosed charge is zero.

The charge Q of the capacitor is thus proportional to voltage; the proportionality constant C is called the capacitance of the capacitor.

$$Q = CU_C = \varepsilon_0 \frac{A}{d} \cdot U_C \qquad (4)$$

The linear relation between charge Q and voltage U applied to the otherwise unchanged capacitor is represented in fig. 4.

Equation (4) further shows that the capacitance C of the capacitor is inversely proportional to the distance d between the plates:

$$Q = \varepsilon \cdot \varepsilon_0 \cdot \frac{A}{d} \cdot U_C \qquad (5)$$

For constant voltage, the inverse distance between the plates, and thus the capacitance, are a measure for the amount of charge a capacitor can take (cf. fig. 5). If inversely U, Q, d and A were measured, these measurement data allow to calculate the electric constant ε_0:

$$\varepsilon_0 = \frac{d}{A} \cdot \frac{Q}{U_C} \qquad (6)$$

In this example of measurement, one obtains $\varepsilon_0 = 8.8 \cdot 10^{-12}$ As/(Vm), as compared to the exact value of
$\varepsilon_0 = 8.8542 \cdot 10^{-12}$ As/(Vm)

Equations (4), (5) and (6) are valid only approximately, due to the assumption that field lines are parallel. By too large distances the approximation of the homogeneous field is not working sufficiently any more, which in turn systematically yields a too large electric constant from equation (6). This is why the value of the electric constant should be determined for a small and constant distance between the plates (cf. fig. 4).
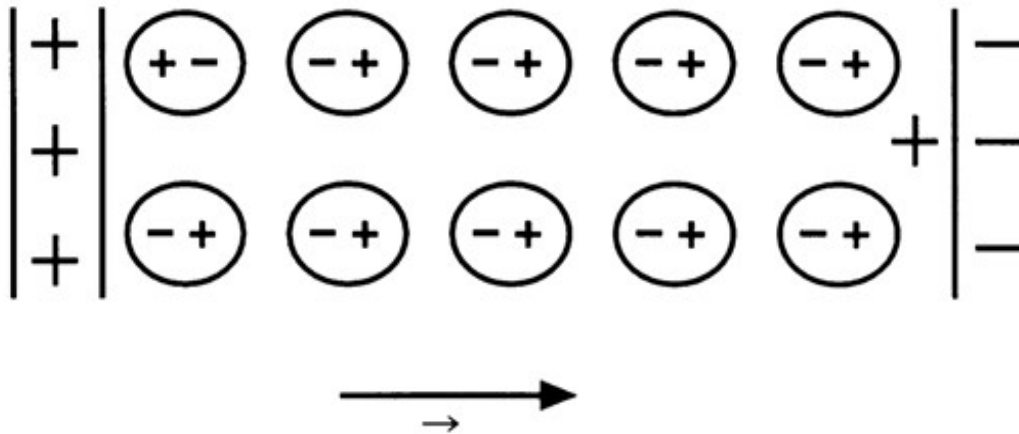

Fig. 6: Generation of free charges in a dielectric through polarisation of the molecules in the electric field of a plate capacitor.

Things change once insulating material (dielectrics) are inserted between the plates. Dielectrics have no free moving charge carriers, as metals have, but they do have positive nuclei and negative electrons. These may be arranged along the lines of an electric field. Formerly nonpolar molecules thus behave as locally stationary dipoles. As can be seen in fig. 6, the effects of the single dipoles cancel each other macroscopically inside the dielectric. However, no partners with opposite charges are present on the surfaces; these thus have a stationary charge, called a free charge.

The free charges in turn weaken the electric field $\vec{E}$ of the real charges Q, which are on the capacitor plates, within the dielectric.

The weakening of the electric field $\vec{E}$ within the dielectric is expressed by the dimensionless, material specific dielectric constant ε ($\varepsilon = 1$ in vacuum):

$$\vec{E} = \frac{\vec{E}_0}{\varepsilon} \tag{7}$$

where $\vec{E}_0$ is the electric field generated only by the real charges Q. Thus, the opposite field generated by the free charges must be

$$\vec{E}_f = \vec{E}_0 - \vec{E} = \frac{\varepsilon - 1}{\varepsilon} \vec{E}_0 \tag{8}$$

Neglecting the charges within the volume of the dielectric macroscopically, only the free surface charges ($\pm Q_f$) generate effectively the opposite field:

$$E_f = \frac{Q_f}{A\varepsilon_0} = \frac{Q_f \cdot d}{V\varepsilon_0} = \frac{p}{V\varepsilon_0} \tag{9}$$

where p is the total dipole moment of the surface charges. In the general case of an inhomogeneous dielectric, equation (9) becomes:

$$\vec{E}_f = \frac{1}{\varepsilon_0} \int \frac{d\vec{P}}{dV} = \frac{1}{\varepsilon_0}\vec{P} \tag{10}$$

where $\vec{P}$ -total dipole moment per unit volume – is called dielectric polarisation.
If additionally a $\vec{D}$ -field (dielectric displacement) is defined:

$$\vec{D} = \varepsilon \cdot \varepsilon_0 \cdot \vec{E} \tag{11}$$

whose field lines only begin or end in real (directly measurable) charges, the three electric magnitudes, field intensity $\vec{E}$, dielectric displacement $\vec{D}$ and dielectric polarization $\vec{P}$ are related to one another through the following equation:

$$\vec{D} = \varepsilon_0 \cdot \vec{E} + \vec{P} = \varepsilon \cdot \varepsilon_0 \cdot \vec{E}$$
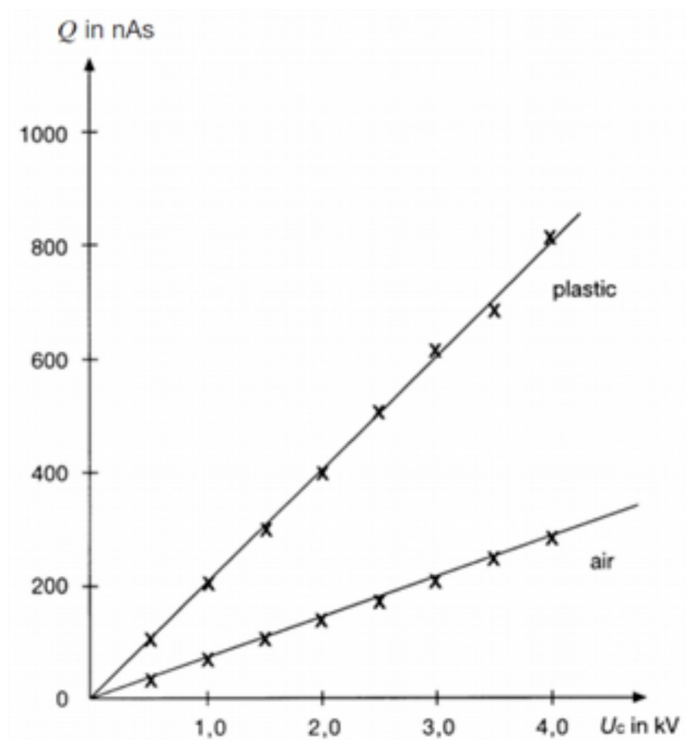

Figure 7: Electrostatic charge Q of a plate capacitor as a function of the applied voltage U_c, with and without dielectric (plastic) between the plates (d = 0.98 cm)

If the real charge Q remains on the capacitor, whilst a dielectric is inserted between the plates, according to definition (3), voltage U_c between the plates is reduced as compared to voltage U_{vac} in vacuum (or to a good approximation, in air) by the dielectric constant:

$$U_C = \frac{U_{vac}}{\varepsilon} \tag{12}$$

Similarly, one obtains from the definition of capacitance (4):

$$C = \varepsilon \cdot C_{vac} \tag{13}$$

The general form of equation (4) is thus:

$$Q = \varepsilon \cdot \varepsilon_0 \cdot \frac{A}{d} \cdot U_C \tag{14}$$

In fig. 7, charge Q on the capacitor is plotted against the applied plate voltage U_c for comparison to the situation with and without plastic plate between the capacitor plates, all other conditions remaining unchanged: thus, for the same voltage, the amount of charge of the capacitor is significantly increased by the dielectric, in this example by a factor of 2.9.
If the charges obtained with and without plastic (equations [4] and [14]) are divided by each other:

$$\frac{Q_{plastic}}{Q_{vacuum}} = \varepsilon \tag{15}$$

the obtained numerical value is the dielectric constant of the plastic.

For the glass plates, a value of $\varepsilon = 9.1$ is obtained similarly.
In order to take into consideration the above described influence of free charges, Maxwell's equation (1) is generally completed by the dielectric constant ε of the dielectric which fills the corresponding volume:

$$\oiint_A \varepsilon \cdot \varepsilon_0 \cdot \vec{E} \, d\vec{A} = \oiint \vec{D} \, d\vec{A} = Q \tag{16}$$

Thus, equation (14) becomes equation (4).

Measurement results

Measurement of the electric constant:

$A = 0.0531\ \text{m}^2$ $\qquad U_c = 1.5\ \text{kV}$ $\qquad C = 218\ \text{nF}$

U [V]	3.3	2.4	1.6	1.35	1.2	1.1
d [cm]	0.10	0.15	0.20	0.25	0.30	0.35
$1/d$ [cm^{-1}]	10.0	6.7	5.0	4.0	3.3	2.9
Q [nAs]	719	523	350	294	262	240
ε_0 [pAs/Vm]	9.00	9.85	8.75	9.25	9.85	10.50

$A = 0.0531\ \text{m}^2$ $\qquad d = 0.2\ \text{cm}$ $\qquad C = 218\ \text{nF}$

U_c [kV]	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
U [V]	0.5	1.1	1.6	2.05	2.65	3.15	4.0	4.6
Q [nAs]	109	240	348	447	578	687	872	1003
ε_0 [pAs/Vm]	8.2	9.0	8,7	8.4	8.7	8.6	9.4	9.5

Measurement of dielectric constant

Plastic: $\qquad A = 0.0531\ \text{m}^2 \quad d = 0.98\ \text{cm} \quad C = 218\ \text{nF}$

U_c [kV]	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0
U [V]	0.5	0.92	1.35	1.8	2.3	2.8	3.1	3.7
Q [nAs]	109	201	294	392	501	610	676	807
$Q \frac{d}{A\varepsilon_0}\frac{1}{U_c}$	4.6	4.2	4.1	4.1	4.2	4.3	4.0	4.2
U_{vac} [V]	0.16	0.32	0.51	0.62	0.78	0.95	1.12	1.3
Q_{vac} [nAs]	35	70	111	135	170	207	244	283
Q/Q_{vac}	3.1	2.9	2.6	2.9	2.9	2.9	2.9	2.9

Glass: $d = 0.17\ \text{cm} \quad \bar{U} = 5.8\ \text{V} \quad Q = 1.264\ \mu\text{As} \quad U_c = 500\ \text{V}$
$\qquad \varepsilon_{glass} = 9.1$