

Endogenous Production Networks Under Supply Chain Uncertainty

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How does uncertainty affect an economy's production network and, through that channel, macroeconomic aggregates?

Approach and results

We construct a model of **endogenous network formation** under **uncertainty**

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- Network flexibility has large impact on welfare
- Sizable role for uncertainty during high-volatility events like the Great Recession

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Reduced-form evidence for the model mechanisms

- Links with riskier suppliers are more likely to be destroyed
- Riskier firms have lower Domar weights

Model

Static model with two types of agents

1. **Representative household**: owns the firms, supplies labor and consumes
2. **Firms**: produce differentiated goods using labor and intermediate inputs
 - There are n industries/goods, indexed by $i \in \{1, \dots, n\}$
 - Representative firm that behaves **competitively**

Production technique

Each firm i has access to a set of production techniques \mathcal{A}_i .

A technique $\alpha_i \in \mathcal{A}_i$ specifies

- The set of intermediate inputs to be used in production
- The proportion in which these inputs are combined
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These techniques are **Cobb-Douglas production functions**

- We identify $\alpha_i = (\alpha_{i1}, \dots, \alpha_{in})$ with the input shares

$$F(\alpha_i, L_i, X_i) = e^{\varepsilon_i} \zeta(\alpha_i) A_i(\alpha_i) L_i^{1 - \sum_{j=1}^n \alpha_{ij}} \prod_{j=1}^n x_{ij}^{\alpha_{ij}},$$

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Allow adjustment along **intensive** and **extensive** margins: $\mathcal{A}_i = \left\{ \alpha_i \in [0, 1]^n : \sum_{j=1}^n \alpha_{ij} \leq \bar{\alpha}_i < 1 \right\}$.

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Example: A car manufacturer can use **only steel** or **only carbon fiber**, or a **combination** of both.

Assumption

$A_i(\alpha_i)$ is smooth and strictly log-concave.

Implication: There are **ideal input shares** α_{ij}° that maximize A_i

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Example

$$\log A_i(\alpha_i) = - \sum_{j=1}^n \kappa_{ij} (\alpha_{ij} - \alpha_{ij}^\circ)^2 - \kappa_{i0} \left(\sum_{j=1}^n \alpha_{ij} - \sum_{j=1}^n \alpha_{ij}^\circ \right)^2,$$

Source of uncertainty and timing

Firms are subject to **productivity shocks** $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n) \sim \mathcal{N}(\mu, \Sigma)$

- Vector μ captures **optimism/pessimism** about productivity
- Covariance matrix Σ captures **uncertainty** and **correlations**

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Timing

1. **Before ε is realized:** Production techniques are chosen
 - Beliefs (μ, Σ) affect technique choice \rightarrow production network $\alpha \in \mathcal{A}$ is **endogenous**
2. **After ε is realized:** All other decisions are taken
Only impact of uncertainty on decisions is through technique choice

The representative household makes decisions after ε is realized

- Owns the firms
- Supplies one unit of labor inelastically
- Chooses *state-contingent consumption* (C_1, \dots, C_n) to maximize

$$u \left(\left(\frac{C_1}{\beta_1} \right)^{\beta_1} \times \dots \times \left(\frac{C_n}{\beta_n} \right)^{\beta_n} \right),$$

subject to the *state-by-state* budget constraint

$$\sum_{i=1}^n P_i C_i \leq 1,$$

where u is CRRA with relative risk aversion $\rho \geq 1$.

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► Details

- We refer to aggregate consumption $Y = \prod_{i=1}^n (\beta_i^{-1} C_i)^{\beta_i}$ as *GDP*.

Firms solve a two-stage problem

1. Before ε is drawn: Choose production technique α_i
 - ex ante decision **under uncertainty**
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Problem of the firm: Labor and intermediate inputs

For a given technique α_i , the **cost minimization** problem of the firm is

$$K_i(\alpha_i, P) = \min_{L_i, X_i} \left(L_i + \sum_{j=1}^n P_j X_{ij} \right), \text{ subject to } F(\alpha_i, L_i, X_i) \geq 1$$

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1. **Constant returns to scale** $\rightarrow K_i$ does not depend on firm size
2. Given that each technique is Cobb-Douglas,

$$K_i(\alpha_i, P) = \frac{1}{e^{\varepsilon_i} A_i(\alpha_i)} \prod_{j=1}^n P_j^{\alpha_{ij}}.$$

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Problem of the firm: Production technique

Firm i chooses a technique $\alpha_i \in \mathcal{A}_i$ to maximize profits

$$\alpha_i^* \in \arg \max_{\alpha_i \in \mathcal{A}_i} \mathbb{E} [\Lambda Q_i (P_i - K_i (\alpha_i, P))]$$

where Q_i is the equilibrium demand for good i and Λ is the SDF.

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Lemma

In equilibrium, $\lambda (\alpha^*)$, $k_i (\alpha_i, \alpha^*)$ and $q_i (\alpha^*)$ are normally distributed, and the technique choice of the representative firm in sector i solves

$$\alpha_i^* \in \arg \min_{\alpha_i \in \mathcal{A}_i} \mathbb{E} [k_i (\alpha_i, \alpha^*)] + \text{Cov} [\lambda (\alpha^*), k_i (\alpha_i, \alpha^*)]. \quad (1)$$

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The firm prefers techniques with low

1. expected unit cost
2. unit cost when marg. utility is high \rightarrow firm “inherits” the household’s risk aversion through λ

Problem of the firm: Production technique

We can expand the two terms to minimize

$$\mathbb{E} [k_i(\alpha_i, \alpha^*)] = -a_i(\alpha_i) + \sum_{j=1}^n \alpha_{ij} \mathbb{E} [p_j]$$

Firm prefers techniques with high TFP and low average input prices.

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$$\text{Cov} [\lambda, k_i] = \sqrt{V [\lambda]} \times \text{Corr} [\lambda, k_i] \sqrt{V [k_i]}$$

Firm prefers techniques with low correlation with SDF.

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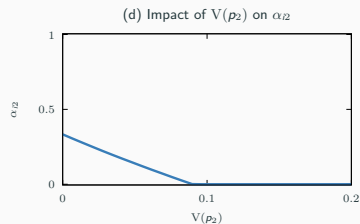
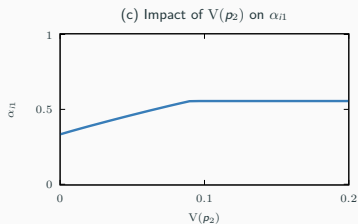
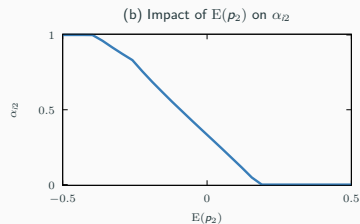
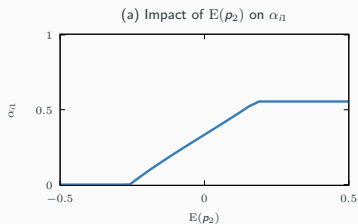
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In general $\text{Corr}[\lambda, k_i] > 0 \rightarrow$ Minimize variance of k_i

$$V[k_i] = \text{cte} + \underbrace{\sum_{j=1}^n \alpha_{ij}^2 V[p_j]}_{\text{stable prices}} + \underbrace{\sum_{j \neq k} \alpha_{ij} \alpha_{ik} \text{Cov}[p_j, p_k]}_{\text{uncorrelated prices}} + \underbrace{2 \text{Cov}\left[-\varepsilon_i, \sum_{j=1}^n \alpha_{ij} p_j\right]}_{\text{uncorrelated with own } \varepsilon_i}$$

Back to our example

- Car manufacturer i can use **steel** (input 1) or **carbon fiber** (input 2)
- Look at impact of $E p_2$ and $V p_2$ on the shares α_{i1} and α_{i2}



Definition

An equilibrium is a technique for every firm α^* and a stochastic tuple $(P^*, C^*, L^*, X^*, Q^*, \Lambda^*)$ such that

1. (Unit cost pricing) For each $i \in \{1, \dots, n\}$, $P_i^* = K_i(\alpha_i^*, P^*)$.
2. (Optimal technique choice) For each $i \in \{1, \dots, n\}$, factor demand L_i^* and X_i^* , and the technology choice $\alpha_i^* \in \mathcal{A}_i$ solves the firm's problem.
3. (Consumer maximization) The consumption vector C^* solves the household's problem.
4. (Market clearing) For each $i \in \{1, \dots, n\}$,

$$Q_i^* = C_i^* + \sum_{j=1}^n X_{ji}^*,$$

$$Q_i^* = F_i(\alpha_i^*, L_i^*, X_i^*),$$

$$\sum_{i=1}^n L_i^* = 1.$$

Fixed-network economy

GDP in a fixed-network economy

Define a firm's **Domar weight** ω_i as its sales share

$$\omega_i(\alpha) := \frac{P_i Q_i}{PC}$$

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$$\omega_i(\alpha) := \frac{P_i Q_i}{PC} = \beta' \mathcal{L}(\alpha) 1_i$$

Domar weights depend on

1. Demand from the household through β
2. Demand from intermediate good producers through $\mathcal{L}(\alpha) = (I - \alpha)^{-1} = I + \alpha + \alpha^2 + \dots$

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→ Domar weights capture the **importance of a firm as a supplier**

→ Domar weights are **constant** for a fixed network

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Lemma (Hulten's Theorem)

Under a given network α , the log of GDP $y = \log Y$ is given by

$$y = \omega(\alpha)' (\varepsilon + \mathbf{a}(\alpha)).$$

Proposition (Hulten's Theorem in expectation)

For a fixed network α ,

1. The impact of μ_i on expected log GDP is given by

$$\frac{\partial E[y]}{\partial \mu_i} = \omega_i.$$

2. The impact of Σ_{ij} on the variance of log GDP is given by

$$\frac{\partial V[y]}{\partial \Sigma_{ij}} = \begin{cases} \omega_i^2 & i = j, \\ 2\omega_i\omega_j & i \neq j. \end{cases}$$

3. μ does not affect $V[y]$ and Σ does not affect $E[y]$.

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3. μ does not affect $V[y]$ and Σ does not affect $E[y]$.

For a **fixed network**

1. Domar weights ω are enough to understand log GDP
2. Since $\omega_i > 0$ shocks have intuitive impact.

Flexible-network economy

The economy is **fully competitive** and **undistorted** by frictions or externalities.

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Proposition

1. There exists an efficient equilibrium
2. That equilibrium production network solves

$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} \mathbb{E} [y(\alpha)] - \frac{1}{2} (\rho - 1) \mathbb{V} [y(\alpha)]$$

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Implications

1. The planner prefers networks that balance high $\mathbb{E} [y(\alpha)]$ with low $\mathbb{V} [y(\alpha)]$
2. Complicated network formation problem \rightarrow simpler **optimization problem**.

Economic forces at work

Domar weights are constant when the network is fixed. But when it is flexible...

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Intuition

1. **Equilibrium:** Firms rely more on high- μ_i and low- Σ_{ii} firms as suppliers.
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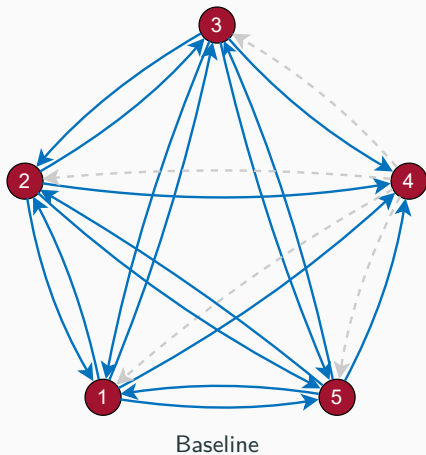
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Flexible network \rightarrow beneficial changes are amplified while adverse changes are mitigated.

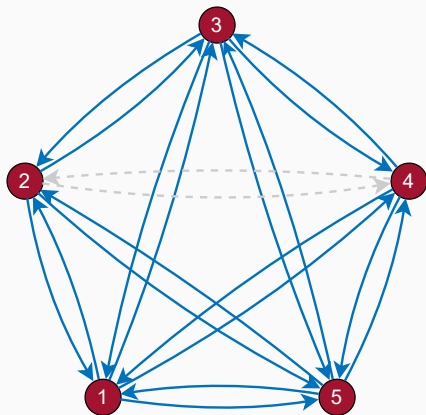
Example: Impact of beliefs on the network

Simple example of possible **substitution patterns**



Example: Impact of beliefs on the network

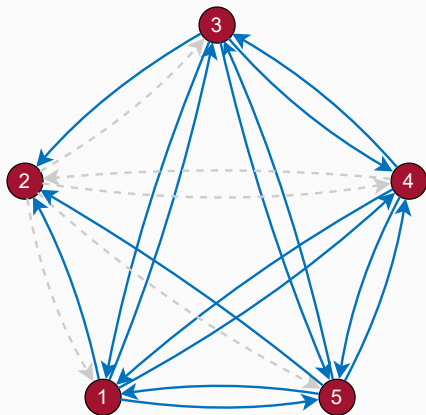
Simple example of possible **substitution patterns**



Small increase in $\Sigma_{22} \rightarrow$ Firms also purchase from 4 to diversify

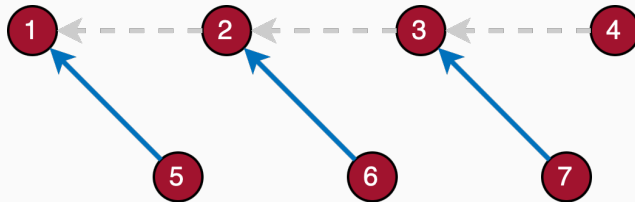
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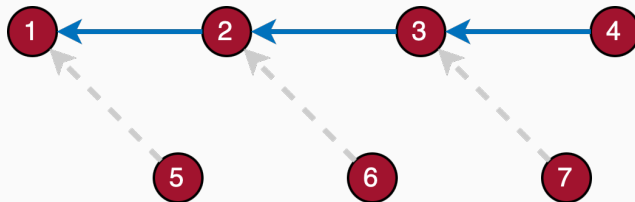


Large increase in $\Sigma_{22} \rightarrow$ Firms drop 2 as a supplier

Example: Cascading effect of uncertainty



(a) High uncertainty about ε_4



(b) Low uncertainty about ε_4

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Uncertainty lowers expected GDP in equilibrium, in the sense that $E[y]$ is largest when $\Sigma = 0_{n \times n}$.

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Intuition

1. **Equilibrium:** With uncertainty, firms seek stability at the cost of efficiency.
2. **Planner:** Only objective is to maximize $E[y]$.

$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} E[y(\alpha)] - \frac{1}{2} (\rho - 1) V[y(\alpha)]$$

Proposition

1. The impact of μ_i on welfare is given by

$$\frac{d\mathcal{W}}{d\mu_i} = \omega_i.$$

2. The impact of Σ_{ij} on welfare is given by

$$\frac{d\mathcal{W}}{d\Sigma_{ij}} = \begin{cases} -\frac{1}{2} (\rho - 1) \omega_i^2 & i = j, \\ -(\rho - 1) \omega_i \omega_j & i \neq j. \end{cases}$$

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The impact of beliefs on welfare is **intuitive**

1. Higher expected productivity increases welfare
2. Higher correlation or uncertainty lowers welfare

Effect of beliefs on GDP

Impact of shocks on

- Welfare: intuitive
- GDP when the network is fixed: intuitive
- GDP when the network is flexible: ???

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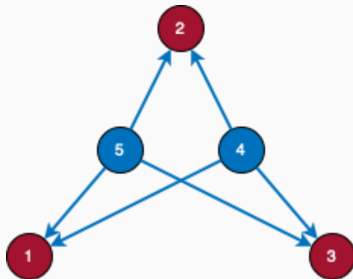
Decompose a shock to, say, μ_i as

$$\frac{dE[y]}{d\mu_i} = \underbrace{\frac{\partial E[y]}{\partial \mu_i}}_{\text{direct impact with fixed network}} + \underbrace{\frac{\partial E[y]}{\partial \alpha} \frac{d\alpha}{d\mu_i}}_{\text{network adjustment}}$$

Two effects

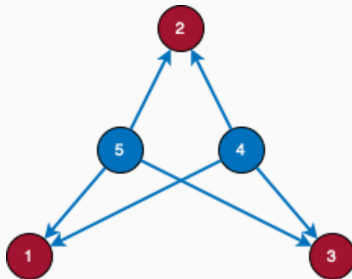
1. **Direct impact** keeping the network fixed = Domar weight
2. **Indirect impact** that take into account the network adjustment = ???

Example: Counterintuitive impact of a change in (μ, Σ)



- Firm 4 is **risky** (high Σ_{44}) but **productive** (high μ_4)
- Firm 5 is **safe** (low Σ_{55}) but **unproductive** (low μ_5)

Example: Counterintuitive impact of a change in (μ, Σ)



- Firm 4 is **risky** (high Σ_{44}) but **productive** (high μ_4)
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Consider two shocks

1. **Increase μ_5**
 - Move away from high- μ firm 4 toward low- μ firm 5 $\Rightarrow E[y]$ falls
2. **Increase Σ_{44}**
 - Move away from high- Σ firm 4 toward low- Σ firm 5 $\Rightarrow V[y]$ falls

Quantitative exploration

Data

- Annual **United States** data from 1947 to 2020 about 37 sectors

Calibration

- Consumption shares β and ideal shares α° taken from the data
- Risk-aversion ρ and cost of deviating κ are **estimated**
- ε_t is random walk with drift and **time-varying uncertainty** and is **estimated**

► Data details

► Estimation details

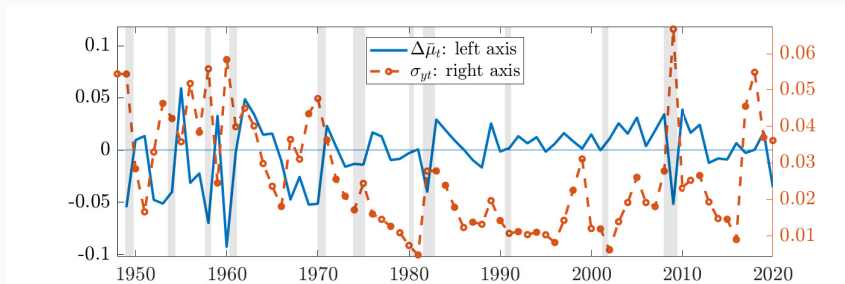
Calibrated economy

Estimated risk aversion: $\rho = 4.27$

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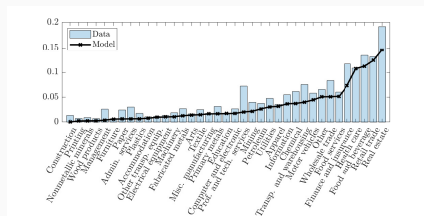
Estimated evolution of beliefs



$$\Delta\bar{\mu}_t = \sum_{j=1}^n \omega_{jt} \Delta\mu_{jt} \text{ and } \sigma_{yt} = \sqrt{V[y]} = \sqrt{\omega_t' \Sigma_t \omega_t}.$$

Calibrated economy: Domar weights

The calibrated **Domar weights** fit the data reasonably well



Beliefs have the expected impact on **Domar weights**

	Statistic	Data	Model
(1)	Average Domar weight $\bar{\omega}_j$	0.047	0.032
(2)	Standard deviation $\sigma(\omega_j)$	0.0050	0.0021
(3)	Coefficient of variation $\sigma(\omega_j) / \bar{\omega}_j$	0.11	0.07
(4)	Corr (ω_{jt}, μ_{jt})	0.08	0.08
(5)	Corr $(\omega_{jt}, \Sigma_{jjt})$	-0.37	-0.31

Two useful counterfactuals

1. Fixed-network economy
 - No change in network \rightarrow capture the full effect of network adjustments
2. “Risk-neutral” economy ($\rho = 1$)
 - Uncertainty has no impact on network \rightarrow capture the impact of uncertainty
 - Recall: only impact of uncertainty on expected GDP is through the network

Two useful counterfactuals

1. Fixed-network economy

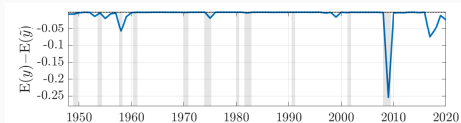
- No change in network \rightarrow capture the full effect of network adjustments

2. “Risk-neutral” economy ($\rho = 1$)

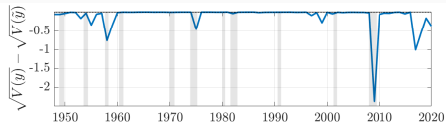
- Uncertainty has no impact on network \rightarrow capture the impact of uncertainty
- Recall: only impact of uncertainty on expected GDP is through the network

	Baseline model compared to...	
	Fixed network	Risk neutral
Expected GDP $E[y(\alpha)]$	+2.122%	−0.008%
Std. dev. of GDP $\sqrt{V[y(\alpha)]}$	+0.131%	−0.105%
Welfare \mathcal{W}	+2.109%	+0.010%

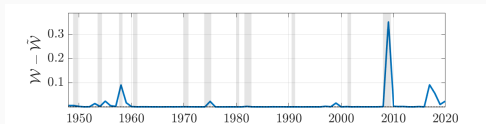
Calibrated model vs risk-neutral alternative



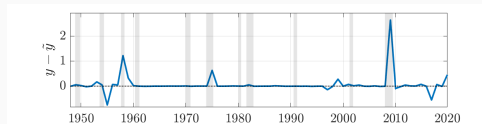
(a) Difference in expected GDP



(b) Difference in expected std. dev. of GDP



(c) Difference in expected welfare



(d) Difference in realized GDP

- During periods of high volatility, uncertainty matters.

Reduced-form evidence for the model mechanisms

Links with riskier suppliers are more likely to be destroyed

Use detailed U.S. data on **firm-to-firm relationship** (Factset 2003–2016)

Regress a dummy for **link destruction** on supplier **uncertainty measures**

- **Instruments** from Alfaro, Bloom and Lin (2019)

[► Details](#)

	Dummy for last year of supply relationship		
	(1) OLS	(2) IV	(3) IV
ΔVol_{t-1} of supp.	0.026** (0.010)	0.097*** (0.029)	0.1494** (0.064)
1st moment of IVs	No	Yes	Yes
Type of volatility	Realized	Realized	Implied
Fixed effects	Yes	Yes	Yes
Observations	35,629	35,620	26,195
F-statistic	—	39.0	23.2

All specifications include year \times customer \times supplier industry (2SIC) fixed effects. Standard errors are two-way clustered at the customer and the supplier levels. *F*-statistics are Kleibergen-Paap. *, **, *** indicate significance at the 10%, 5%, and 1% levels, respectively.

- Doubling volatility \rightarrow 12 p.p. increase in probability link destroyed (IV)

Domar weights and uncertainty in the data

Firms with **higher uncertainty** have **lower Domar weights**, in line with the model

- Specifications, uncertainty measures and instruments from Alfaro, Bloom and Lin (2019)

	Change in Domar weight		
	(1) OLS	(2) IV	(3) IV
$\Delta \text{Volatility}_{i,t-1}$	-0.043*** (0.004)	-0.250*** (0.076)	-0.672*** (0.185)
1st moment of IVs	No	Yes	Yes
Type of volatility	Realized	Realized	Implied
Fixed effects	Yes	Yes	Yes
Observations	111,587	26,962	16,862
F-statistic	—	17.0	9.8

All specifications include year and firm fixed effects. Standard errors are clustered at the industry (3SIC) level. *F*-statistics are Kleibergen-Paap.

*, **, *** indicate significance at the 10%, 5%, and 1% levels, respectively.

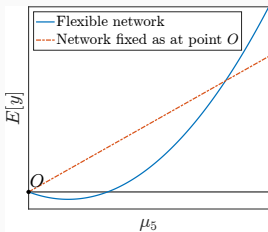
Conclusion

Main contributions

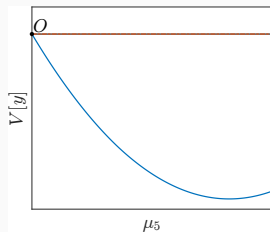
- We construct a model in which **beliefs**, and in particular uncertainty, affect the **production network**.
- During periods of high **uncertainty** firms purchase from safer but less productive suppliers which leads to a **decline in GDP**.
- Mechanism might be **quantitatively** important during periods of **high uncertainty**.

Future research

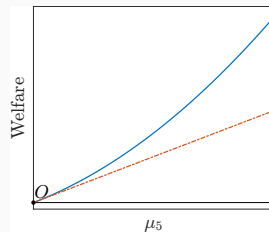
- Use firm-level data to calibrate the model — firm-to-firm network is more sparse and links are often broken.
- Use the model to evaluate the impact of uncertainty on **global supply chains**.



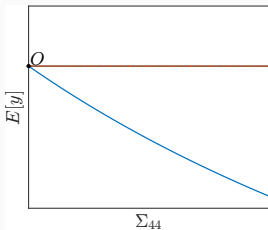
(a) $E[y]$ as a function of μ_5



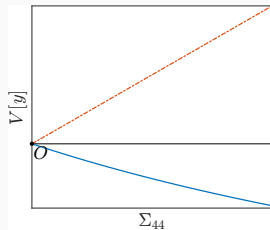
(b) $V[y]$ as a function of μ_5



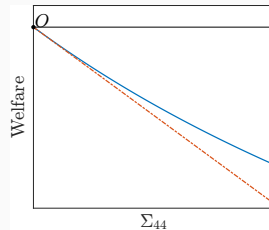
(c) Welfare as a function of μ_5



(d) $E[y]$ as a function of Σ_{44}



(e) $V[y]$ as a function of Σ_{44}



(f) Welfare as a function of Σ_{44}

United States data from vom Lehn and Winberry (2021)

- Input-output tables, sectoral total factor productivity, consumption shares

Mining	Utilities	Construction
Wood products	Nonmetallic minerals	Primary metals
Fabricated metals	Machinery	Computer and electronic manuf.
Electrical equipment manufacturing	Motor vehicles manufacturing	Other transportation equipment
Furniture and related manufacturing	Misc. manufacturing	Food and beverage manufacturing
Textile manufacturing	Apparel manufacturing	Paper manufacturing
Printing products manufacturing	Petroleum and coal manufacturing	Chemical manufacturing
Plastics manufacturing	Wholesale trade	Retail trade
Transportation and warehousing	Information	Finance and insurance
Real estate and rental services	Professional and technical services	Mgmt. of companies and enterprises
Admin. and waste mgmt. services	Educational services	Health care and social assistance
Arts and entertainment services	Accommodation	Food services
Other services		

- Average share of 1.4% with standard deviation of 0.5% over time

More about the estimation

Preferences

- Consumption shares β are taken directly from the data
- Relative risk aversion ρ is **estimated**

Production technique productivity shifters

- Function A_i as described earlier
- Set ideal shares α_{ij}° to their data average
- Costs κ_{ij} of deviating from α_{ij}° are **estimated**

Process for exogenous shocks ε_t

- Random walk with drift $\varepsilon_t = \gamma + \varepsilon_{t-1} + u_t^\varepsilon$, with $u_t^\varepsilon \sim \text{iid } \mathcal{N}(0, \Sigma_t)$.
- Drift vec. γ and cov. mat. Σ_t are **backed out from the data given** (ρ, κ) .

Loss function: Target the full set of shares α_{ijt} and the GDP growth.

- Random walk with drift $\varepsilon_t = \gamma + \varepsilon_{t-1} + u_t$, with $u_t \sim \text{iid } \mathcal{N}(0, \Sigma_t)$.
 - We estimate the vector γ by averaging $\Delta\varepsilon_t = \varepsilon_t - \varepsilon_{t-1}$ over time
 - We estimate Σ_t as

$$\hat{\Sigma}_{ijt} = \sum_{s=1}^{t-1} \lambda^{t-s-1} u_{is} u_{js}$$

where $\hat{\lambda} = 0.47$ is set to the sectoral average of the corresponding parameters of a GARCH(1,1) model estimated on each sector's productivity innovation u_{it}

The function $\zeta(\alpha_i)$ is

$$\zeta(\alpha_i) = \left[\left(1 - \sum_{j=1}^n \alpha_{ij} \right)^{1 - \sum_{j=1}^n \alpha_{ij}} \prod_{j=1}^n \alpha_{ij}^{\alpha_{ij}} \right]^{-1}$$

This functional form allows for a simple expression for the unit cost K

Microfoundation for "one technique" restriction and cost minimization

Key restriction

Each firm/industry i can only adopt one production technique.

- Each industry $i \in \{1, \dots, n\}$ has a continuum of firms $l \in [0, 1]$.
- Buyers use *shoppers* to purchase goods
 - Shoppers face an *information problem* and cannot differentiate between producers within an industry
 - Uniform allocation: each producer gets mass $Q_i dl$ of shoppers
 - Shoppers from firm m in industry j faces average price $\tilde{P}_i^{jm} = \int_0^1 \tilde{P}_{il}^{jm} dl$ for good i .
- When a shopper m from j meets a producer l from $i \rightarrow$ Nash bargaining

$$\tilde{P}_{il}^{jm} - K_i \left(\alpha_i^l, \left\{ \tilde{P}_k^{jl} \right\}_k \right) = \gamma \left(B_i^{jm} - K_i \left(\alpha_i^l, \left\{ \tilde{P}_k^{jl} \right\}_k \right) \right)$$

- Technique choice problem

$$\max_{\alpha_i^l \in \mathcal{A}_i} \mathbb{E} \left[\Lambda \sum_{j=0}^n Q_{ji} dl \int_0^1 \gamma \left(B_i^{jm} - K_i \left(\alpha_i^l, \left\{ \tilde{P}_k^{jl} \right\}_k \right) \right) dm \right] \longrightarrow \min_{\alpha_i^l \in \mathcal{A}_i} \mathbb{E} \left[\Lambda Q_i K_i \left(\alpha_i^l, \left\{ \tilde{P}_k^{jl} \right\}_k \right) \right]$$

- Take limit $\gamma \rightarrow 0$

- Nash bargaining implies $\tilde{P}_{il}^{jm} = K_i \left(\alpha_i^l, \left\{ \tilde{P}_k^{il} \right\}_k \right) \rightarrow \tilde{P}_{il}^{jm}$ does not depend on $j, m \rightarrow \tilde{P}_i^{jm} \equiv P_i$.
- $K_i \left(\alpha_i^l, \left\{ \tilde{P}_k^{il} \right\}_k \right) \rightarrow K_i \left(\alpha_i^l, P \right)$
- Cost minimization problem

$$\min_{\alpha_i^l \in \mathcal{A}_i} E \left[\Lambda Q_i K_i \left(\alpha_i^l, \left\{ \tilde{P}_k^{il} \right\}_k \right) \right] \longrightarrow \min_{\alpha_i^l \in \mathcal{A}_i} E \left[\Lambda Q_i K_i \left(\alpha_i^l, P \right) \right]$$

- We have the same pricing equation as in benchmark model with all firms in i choosing same technique

Given the log-normal nature of uncertainty $\rho \leq 1$ determines whether the agent is risk-averse or not. To see this, note that when $\log C$ normally distributed, maximizing

$$\mathbb{E} [C^{1-\rho}]$$

amounts to maximizing

$$\mathbb{E} [\log C] - \frac{1}{2} (\rho - 1) \text{V} [\log C] .$$

Assumption (Weak complementarity)

For all $i \in \mathcal{N}$, the function a_i is such that $\frac{\partial^2 a_i(\alpha_i)}{\partial \alpha_{ij} \partial \alpha_{ik}} \geq 0$ for all $j \neq k$.

Lemma

Let $\alpha^* \in \text{int}(\mathcal{A})$ be the equilibrium network and suppose that the assumption holds. There exists a $\bar{\Sigma} > 0$ such that if $|\Sigma_{ij}| < \bar{\Sigma}$ for all i, j , there is a neighborhood around α^* in which

1. an increase in μ_j leads to an increase in the shares α_{kl}^* for all k, l ;
2. an increase in Σ_{jj} leads to a decline in the shares α_{kl}^* for all k, l ;
3. an increase in Σ_{ij} leads to a decline in the shares α_{kl}^* for all k, l .

Details of the simulation:

1. a function: κ equal to 1, except $\kappa_{ii} = \infty$, α° are 1/10 except $\alpha_{ii}^\circ = 0$.
2. $\rho = 5$, $\beta = 0.2$. $\mu = 0.1$ except for $\mu_4 = 0.0571$. $\Sigma = 0.3 \times I_{n \times n}$ in Panel (a).
3. Panel (b): same as Panel (a) except $\text{Corr}(\varepsilon_2, \varepsilon_4) = 1$.
4. Panel (c): same in Panel (a) except $\Sigma_{22} = 1$.

We assume that $\kappa = \kappa^i \times \kappa^j$ where κ^i is an $n \times 1$ column vector and κ^j is an $1 \times (n + 1)$ row vector.

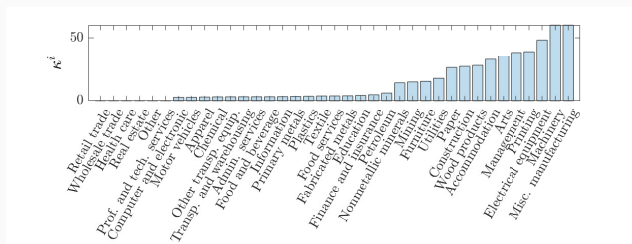
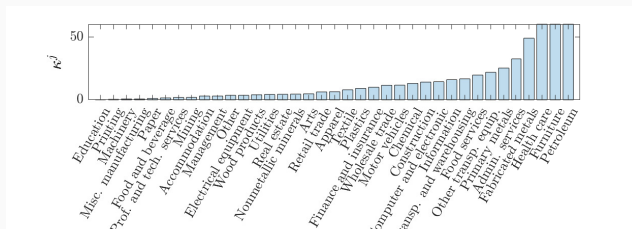


Figure 1: Vector of costs κ^i



Volatility measures

- Supplier ΔVol_{t-1} is the 1-year lagged change in supplier-level volatility.
- Realized volatility is the 12-month standard deviation of daily stock returns from CRSP.
- Implied volatility is the 12-month average of daily (365-day horizon) implied volatility of at-the-money-forward call options from OptionMetrics.

Instrument

- As in Alfaro et al. 2019 “we address endogeneity concerns on firm-level volatility by instrumenting with industry-level (3SIC) non-directional exposure to 10 aggregate sources of uncertainty shocks. These include the lagged exposure to annual changes in expected volatility of energy, currencies, and 10-year treasuries (as proxied by at-the-money forward-looking implied volatilities of oil, 7 widely traded currencies, and TYVIX) and economic policy uncertainty from Baker et al 2016.. [...] To tease out the impact of 2nd moment uncertainty shocks from 1st moment aggregate shocks we also include as controls the lagged directional industry 3SIC exposure to changes in the price of each of the 10 aggregate instruments (i.e., 1st moment return shocks). These are labeled 1st moment 1st moment of IVs.”