Cascades and Fluctuations in an Economy with an Endogenous Production Network

Mathieu Taschereau-Dumouchel

Cornell University

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Introduction

- Production in modern economies involves a complex network of producers supplying and demanding goods from each other
- The shape of this network
  - is an important determinant of how micro shocks aggregate into macro fluctuations
  - is also constantly changing in response to micro shocks
    - For instance, after a severe shock a producer might shut down which might lead its neighbors to shut down as well, etc...
    - Cascade of shutdowns that spreads through the network

This paper proposes a

Theory of network formation and aggregate fluctuations

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Theory of network formation and aggregate fluctuations

Overview of the Model

Simple framework

- Set of n firms that use inputs from connected firms
- Fixed cost to operate
  - Firms operate or not depending on economic conditions
  - $\rightarrow$ Links between firms are active or not
  - $\rightarrow$ Endogenously shape the network

Modeling choice motivated by the data

- In the U.S., about 70% of link destructions occur with exit of supplier or customer

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Key economic force: Complementarities in operation decisions of nearby firms

Efficient organization of production

- Create tightly connected clusters centered around productive firms
- Small changes can trigger large reorganization of the network

Cascades of firm shutdowns

- Well-connected firms are hard to topple but create big cascades
- Elasticities of substitution matter for size and propagation of cascades

Aggregate fluctuations

- Recessions feature fewer well-connected firms and less clustering
- Allowing the network to adjust yields substantially smaller fluctuations

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Literature Review

- Endogenous network formation
  - Atalay et al (2011), Oberfield (2013), Carvalho and Voigtländer (2014), Acemoglu and Azar (2017)
- Network of sectors and fluctuations
  - Long and Plosser (1983), Horvath (1998), Dupor (1999), Acemoglu et al (2012), Baqaee (2016), Acemoglu et al (2016), Lim (2017), Baqaee and Farhi (2017)
- Non-convex adjustments in networks
  - Bak, Chen, Woodford and Scheinkman (1993), Elliott, Golub and Jackson (2014)
- Propagation of shocks through networks
  - Barrot and Sauvagnat (2016), Carvalho et al (2017)

I. Model

Model

- There are n units of production (firm) indexed by $j \in \{1, \ldots, n\}$
  - Each unit produces a differentiated good
  - Differentiated goods can be used to
    - produce a final good

$$Y \equiv \left(\sum_{j=1}^{n} c_j^{\frac{\sigma - 1}{\sigma}} \right)^{\frac{\sigma}{\sigma - 1}}$$

    - produce other differentiated goods
- Representative household
  - Consumes the final good
  - Supplies L units of labor inelastically

Model

- Firm j produces good j

$$y_j = \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \left(\sum_{i=1}^{n} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

- Firm j can only use good i as input if there is a *connection* from firm i to j
  - $\Omega_{ij} = 1$ if connection and $\Omega_{ij} = 0$ otherwise
  - A connection can be *active* or *inactive*
  - Matrix Ω is *exogenous*
- A firm can only produce if it pays a fixed cost f in units of labor
  - $\theta_j = 1$ if j is operating and $\theta_j = 0$ otherwise
  - Vector θ is *endogenous*

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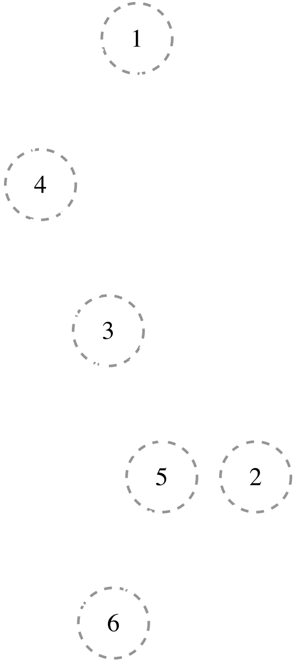
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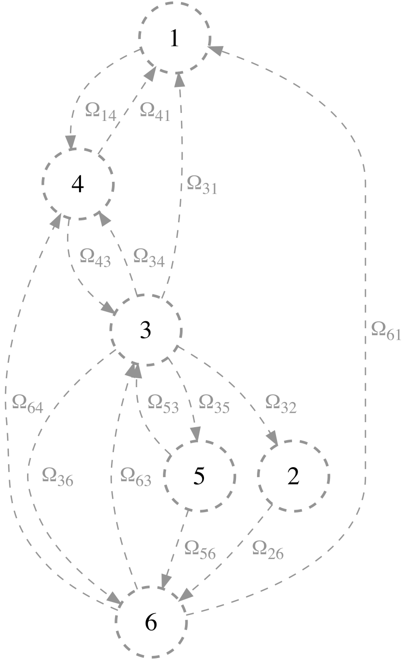
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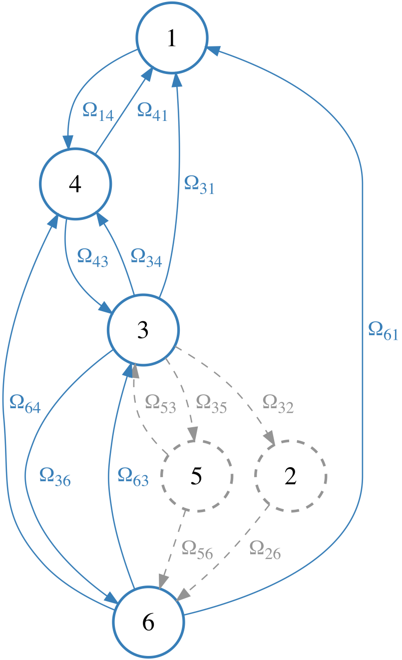
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Social Planner

Problem $\mathcal{P}_{SP}$ of a social planner

$$\max_{\substack{y^0, x, l \\ \theta \in \{0,1\}^n}} \left(\sum_{j=1}^{n} c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to

1. a resource constraint for each good j

$$c_j + \sum_{k=1}^{n} x_{jk} \leq \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \theta_j \left(\sum_{i=1}^{n} \Omega_{ij} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor

$$\sum_{j=1}^{n} l_j + f \sum_{j=1}^{n} \theta_j \leq L$$

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II. Social Planner with Exogenous θ

Social Planner with Exogenous θ

Define $q_j = w/\lambda_j$

- From the FOCs, output is $(1-\alpha)\, y_j = q_j l_j$
- q_j is the *labor productivity* of firm j

Proposition 1

In the efficient allocation,

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\epsilon - 1} \right)^{\frac{\alpha}{\epsilon - 1}} \tag{1}$$

Furthermore, there is a unique vector q that satisfies (1).

Knowing q we can solve for all other quantities easily.

Lemma 1

Aggregate output is

$$Y = Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

where $Q \equiv \left(\sum_{j=1}^{n} q_j^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ *is aggregate labor productivity.*

III. Social Planner with Endogenous θ

Social Planner with Endogenous θ

Planner's problem is now

$$\max_{\theta\in\{0,1\}^n} Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

with

$$q_j = z_j\theta_j A\left(\sum_{i=1}^{n}\Omega_{ij}q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$

Trade-off: making firm j produce ($\theta_j = 1$)

- increases labor productivity of the network (Q)
- reduces the amount of labor into production $\left(L - f\sum_{j=1}^{n}\theta_j\right)$

"Very hard problem" (MINLP — NP Hard)

- The set $\theta \in \{0,1\}^n$ is not convex
- Objective function is not concave

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Social Planner with Endogenous θ

Consider the relaxed and reshaped problem $\mathcal{P}_{RR}$

$$\max_{\theta \in \{0,1\}^n} Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n}\Omega_{ij} q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$

Parameters $a > 0$ and $b \geq 0$ are *reshaping constants*

- Reshape the objective function *away* from optimum (i.e. when $0 < \theta_j < 1$)
  - For a: if $\theta_j \in \{0,1\}$ then $\theta_j^a = \theta_j$
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Reshaping constants:

$$\boxed{a = \frac{1}{\sigma-1} \quad \text{and} \quad b = 1 - \frac{\epsilon-1}{\sigma-1}} \tag{$\star$}$$

Social Planner with Endogenous θ

Proposition 2

Under some parameter restrictions and if Ω is sufficiently connected then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to $\mathcal{P}_{RR}$. Furthermore, a solution $\theta^* \in \{0,1\}^n$ to $\mathcal{P}_{RR}$ also solves $\mathcal{P}_{SP}$.

Details

- This proposition only provides *sufficient* conditions
- In the paper: test the approach on thousands of economies Tests

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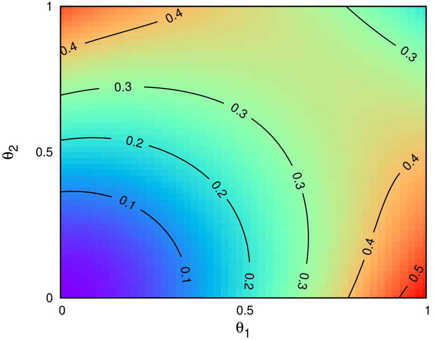
Example with $n = 2$

Relaxed problem **without** reshaping

$$V(\theta) = Q(\theta)\left(L - f\sum_{j=1}^{n}\theta_j\right) \text{ with } q_j = z_j\theta_j A\left(\sum_{i=1}^{n}\Omega_{ij}q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$



Problem: V is not concave

⇒ First-order conditions are not sufficient

⇒ Numerical algorithm can get stuck in local maxima

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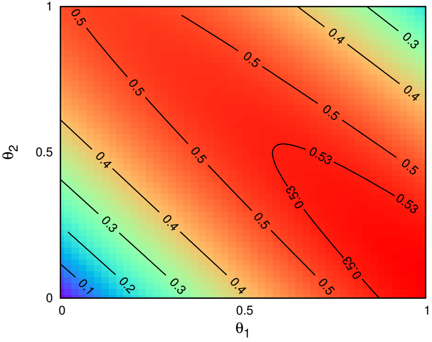
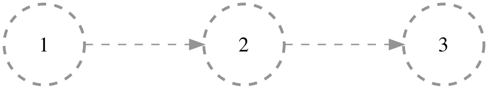


~~Problem~~: V is now (quasi) concave

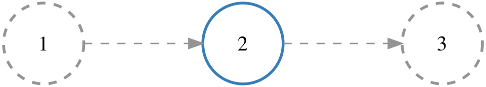
- ⇒ First-order conditions are necessary and sufficient
- ⇒ Numerical algorithm converges to global maximum

IV. Economic Forces at Work

Complementarities



- Impact of operating 2 on the incentives to operate 1 and 3
  - Operating 3 leads to a larger q_3 because 2 is operating
  - Operating 1 increases q_2 because 2 is operating
- Complementarity between operating decisions of nearby firms
  - → Cascades of firm shutdowns may arise



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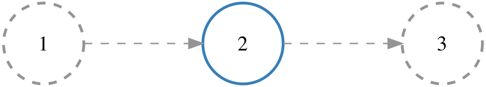

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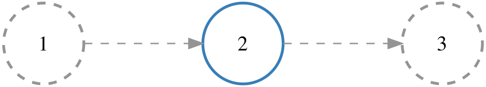
  - Operating 3 leads to a larger q_3 because 2 is operating
  - Operating 1 increases q_2 because 2 is operating
- Complementarity between operating decisions of nearby firms
  → Cascades of firm shutdowns may arise

Complementarities



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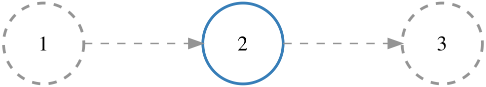

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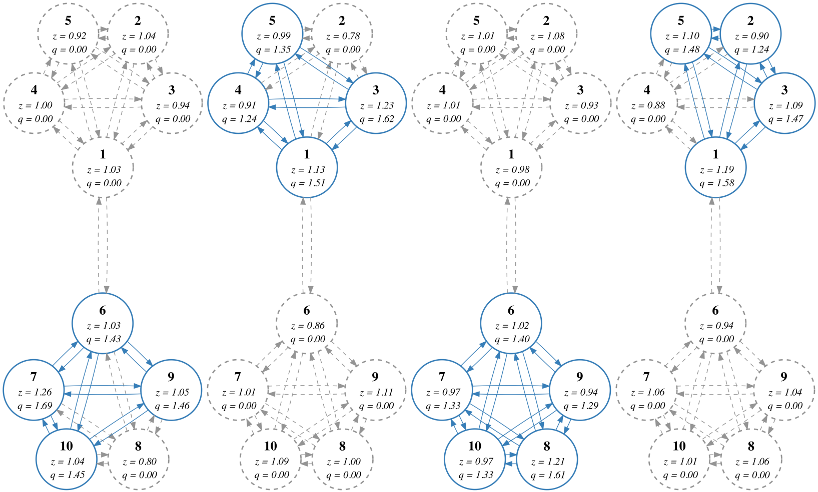
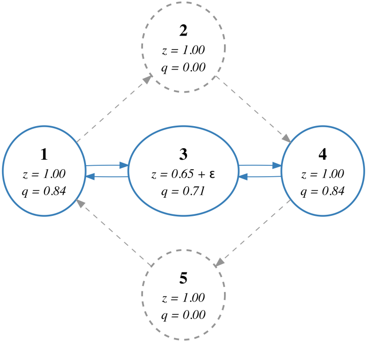
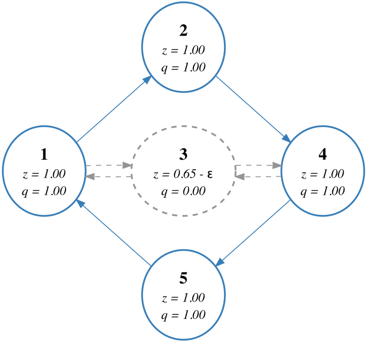
Complementarities Lead to Clustering



Non-convex nature of the economy:

- A small shock can lead to a large reorganization...





V. Quantitative Exploration

Network Data

Two datasets that cover the U.S. economy

- Compustat
  - Public firms must self-report important customers (>10% of sales)
  - Cohen and Frazzini (2008) and Atalay et al (2011) use fuzzy-text matching algorithms to build the network
- Factset Revere
  - Includes public and private firms, and less important relationships
  - Analysts gather data from 10-K, 10-Q, annual reports, investor presentations, websites, press releases, etc

	Year	Firms/year	Links/year
Compustat			
Atalay et al (2001)	1976 - 2009	1,300	1,500
Cohen and Frazzini (2006)	1980 - 2004	950	1,100
Factset	2003 - 2016	13,000	46,000

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Parameters

Parameters from the literature

- $\alpha = 0.5$ to fit the share of intermediate (Jorgenson et al 1987, Jones 2011)
- $\sigma = \epsilon = 5$ average of estimates (Broda et al 2006)
  - Robustness with smaller ϵ in the paper
- Firm productivity follows AR1
  - $\log(z_{it}) \sim \mathcal{N}\left(0, 0.39^2\right)$ from Bartelsman et al (2013)
  - $\rho_z = 0.81$ from Foster et al (2008)
- $f \times n = 5\%$ to fit employment in management occupations
- Set $n = 1000$ for high precision while limiting computations

Unobserved network Ω:

- Pick to match the *observed* in-degree distribution
- Generate thousands of such Ω's and report averages

Details

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Details

Shape of the Network

What does an optimally designed network looks like?

- Compare **optimal networks** to completely **random networks**
- Differences highlights how efficient allocation shapes the network

	Model	
	Optimal network	Random network
Power law exponents		
In-degree distribution	1.07	1.22
Out-degree distribution	1.02	1.21
Global clustering coefficient	0.51	0.30

Notes: Clustering coeff. multiplied by the square roots of number of nodes for better comparison.

Efficient network features

- More highly connected firms
- More clustering of firms

▸ Def. clust. coeff.

Cascades of Shutdowns

For each firm in each year:

- Look at all neighbors upstream and downstream
- Regress the fraction of these neighbors that exits on whether the original firm exits and some controls

▸ Causal

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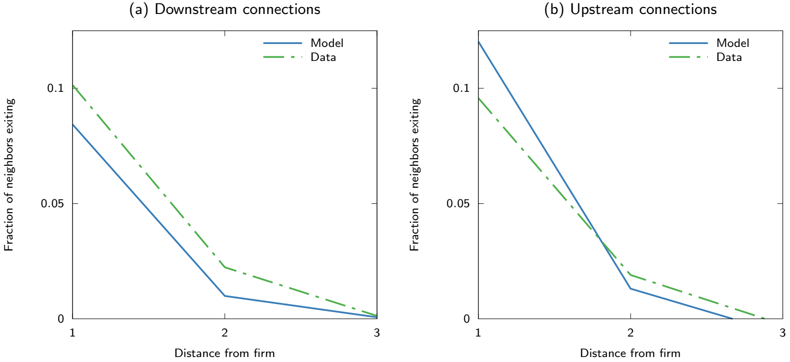

▸ Causal

Resilience of Firms

Size of cascades and probability of exit by degree of firm

	Size of cascades		Probability of exit	
	Model	Data	Model	Data
Average firm	0.4	0.9	16.3%	12.2%
High degree firm	5.9	7.9	2.3%	3.4%

Notes: Size of cascades refers to firm exits up to and including the third neighbors

Implications:

- Highly-connected firms are hard to topple but upon shutting down they create large cascades

Robustness

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Robustness

Aggregate Fluctuations

The shape of the network changes with the business cycle

Table: Correlations with aggregate output

	Model	Data		
		Factset	AHRS	CF
Power law exponents				
In-degree distribution	-0.57	-0.85	-0.35	-0.12
Out-degree distribution	-0.67	-0.94	-0.30	-0.11
Global clustering coefficient	0.46	0.68	0.17	0.20

Implications:

- Recessions are periods with fewer highly-connected firms and in which clustering activity around most productive firms is costly

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Size of fluctuations

$$Y = Q\left(L - f\sum_j \theta_j\right)$$

Table: Standard deviations of aggregates

	Output Y	$\approx$	Labor Prod. Q	$+$	Prod. labor $L - f\sum_j \theta_j$
Optimal network	0.10		0.10		0.009
Fixed network	0.13		0.13		0

Implications:

- Fluctuations are more than 30% smaller in optimal network economy
- The difference comes from changes in the production network

Intuition

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Conclusion

Summary

- Theory of endogenous network formation and aggregate fluctuations
- The optimal network features complementarities between operating decisions of firms that lead to
  - clustering of activity
  - large impact of small changes
  - cascades of shutdowns/restarts
- Compared to U.S. data the model is able to replicate
  - intensity and occurrence of cascades of shutdowns
  - correlation between shape of network and business cycles
- The endogenous reorganization of the network limits the size of fluctuation
- Methodological contribution: approach to easily solve certain non-convex optimization problems

Appendix

Proposition 3

If $\Omega_{ij} = c_i d_j$ *for some vectors* c *and* d *then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to* $\mathcal{P}_{RR}$.

Proposition 4

Let $\sigma = \epsilon$ *and suppose that* $f > 0$ *and* $\overline{z} - \underline{z} > 0$ *are not too big. If* Ω *is sufficiently connected, then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to* $\mathcal{P}_{RR}$.

◂ Return

Details of reshaping

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Proposition 5

If θ^* solves $\mathcal{P}_{RR}$ and that $\theta^*_j \in \{0,1\}$ for all j, then θ^* also solves $\mathcal{P}_{SP}$.

Solution θ^* to $\mathcal{P}_{RR}$ is such that $\theta^*_j \in \{0,1\}$ for all j (P2) if there are many firms and they are sufficiently connected

- In practice we verify that this is satisfied

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Details of reshaping

Intuition:

- First-order condition on θ_j:

$$\text{Marginal Benefit}\left(\theta_j, F\left(\theta\right)\right) - \text{Marginal Cost}\left(\theta_j, G\left(\theta\right)\right) = \bar{\mu}_j - \underline{\mu}_j$$

- Under $(\star)$ the marginal benefit of θ_j only depends on θ_j through aggregates
- For large connected network F and G are independent of θ_j

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Details of reshaping

Simpler to consider

$$\mathcal{P}'_{RD}: \quad \max_{\theta\in[0,1]^n, q} \left(\sum_{j=1}^{n} q_j^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \left(L - f\sum_{j=1}^{n}\theta_j\right)$$

$$q_j \leq A z_j \theta_j^a A B_j^\alpha \qquad (\text{LM: } \beta_j)$$

where $B_j = \left(\sum_{i=1}^{n} \Omega_{ij}\theta_i^b q_i^{\epsilon-1}\right)^{\frac{1}{\epsilon-1}}$.
First-order conditions with respect to θ_k:

$$\frac{\partial q_k}{\partial \theta_k}\frac{\partial Q}{\partial q_k}\left(L - f\sum_{j=1}^{n}\theta_j\right) - fQ + \sum_{j=1}^{n}\beta_j\left(\frac{\partial q_k}{\partial \theta_k}\frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k}\right)\frac{\partial q_j}{\partial B_j} = \overline{\mu}_k - \underline{\mu}_k$$

The terms are

$$\frac{\partial q_k}{\partial \theta_k}\frac{\partial Q}{\partial q_k} = z_k a\theta_k^{a-1} A B_k^\alpha \times \left(z_k\theta_k^a A B_k^\alpha\right)^{\sigma-2} Q^{2-\sigma}$$

$$\frac{\partial q_k}{\partial \theta_k}\frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} = B_j\theta_k^{b-1}\Omega_{kj}\left(\frac{z_k\theta_k^a A B_k^\alpha}{B_j}\right)^{\epsilon-1}\left(a + \frac{b}{\epsilon-1}\right)$$

Return

Testing the Approach on Small Networks

For small networks we can solve $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

- Comparing approaches for a million different economies:

	Number of firms n			
	8	10	12	14
A. With reshaping				
Firms with correct θ_j	99.9%	99.9%	99.9%	99.8%
Error in output Y	0.00039%	0.00081%	0.00174%	0.00171%
B. Without reshaping				
Firms with correct θ_j	84.3%	83.2%	82.3%	81.3%
Error in output Y	0.84%	0.89%	0.93%	0.98%

Notes: Parameters $f \in \{0.05/n, 0.1/n, 0.15/n\}$, $\sigma_z \in \{0.34, 0.39, 0.44\}$, $\alpha \in \{0.45, 0.5, 0.55\}$, $\sigma \in \{4, 6, 8\}$ and $\epsilon \in \{4, 6, 8\}$. For each combination of parameters 1000 different economies are created. For each economy, productivity is drawn from $\log(z_k) \sim$ iid $\mathcal{N}(0, \sigma_z)$ and Ω is drawn randomly such that each link Ω_{ij} exists with some probability such that a firm has on average five possible incoming connections. A network is kept in the sample only if the first-order conditions give a solution in which θ hits the bounds.

The errors come from

- firms that are particularly isolated
- two θ configurations with almost same output

Testing the Approach on Large Networks

For large networks we cannot solve $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

- After all the 1-deviations θ are exhausted:

	With reshaping	Without reshaping
Firms with correct θ	99.98%	69.3%
Error in output Y	0.00007%	0.696%

Notes: Simulations of 200 different networks Ω and productivity vectors z that satisfy the properties of the calibrated economy.

- Very few "obvious errors" in the allocation found by the approach

Return

Shape of the network

	Model	Data		
		Factset	AHRS	CF
Power law exponents				
In-degree distribution	1.07	0.95	1.13	1.32
Out-degree distribution	1.02	0.81	2.24	2.22
Global clustering coefficient	0.51	0.64	0.013	0.014

Notes: Global clustering coefficients are multiplied by the square roots of the number of nodes for better comparison.

Shape of the network

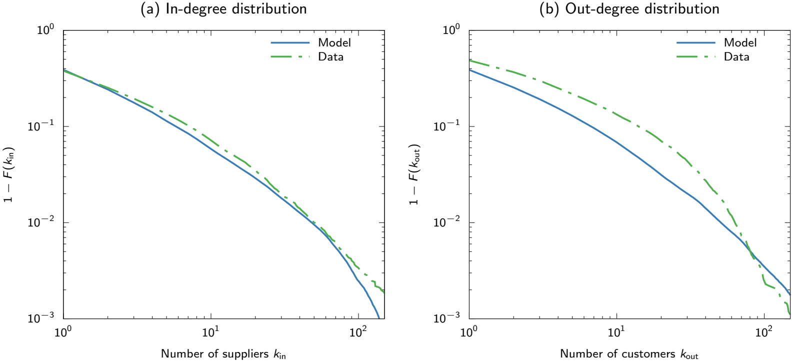


Figure: Model and Factset data for 2016

Return

Clustering coefficient

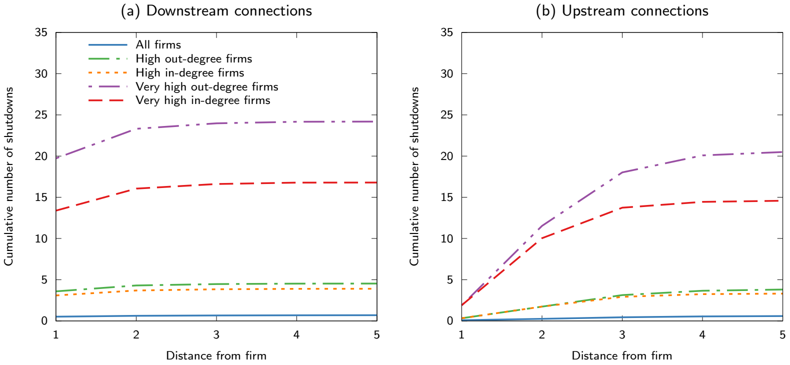
- Triplet: three connected nodes (might be overlapping)
- Triangles: three fully connected nodes (3 triplets)

$$\text{Clustering coefficient} = \frac{3 \times \text{number of triangles}}{\text{number of triplets}}$$

Return

Cascades of shutdowns

Causal impact of a firm exits on its neighbors



Implications:

- Cascades mostly propagate downstream
- Firms with higher degree create larger cascades

Robustness Return

Cascades of shutdowns

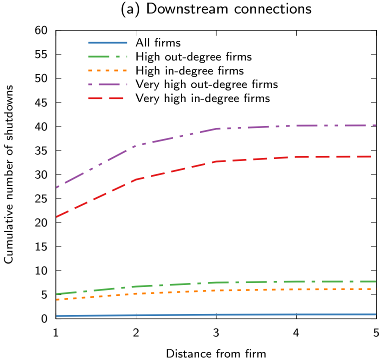


Figure: $\epsilon = 3$

return

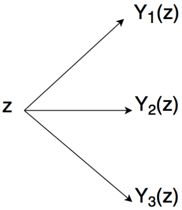
Resilience Robustness

	Probability of exit			
	Benchmark	$\alpha = 0.75$	$\sigma = 7$	$\varepsilon = 4$
Average firm	16.3%	17.0%	25.2%	15.0%
High degree firm	2.3%	1.2%	2.8%	1.0%

return

Intuition

A given network θ^k is a function that maps $z \rightarrow Y_k(z)$



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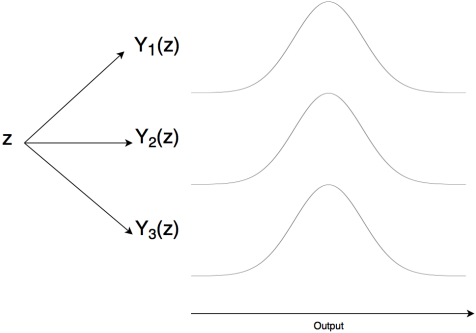

From extreme value theory

$$\mathrm{Var}(Y) = \mathrm{Var}\left(\max_{k \in \{1,\ldots,2^n\}} Y_k\right)$$

declines rapidly with n

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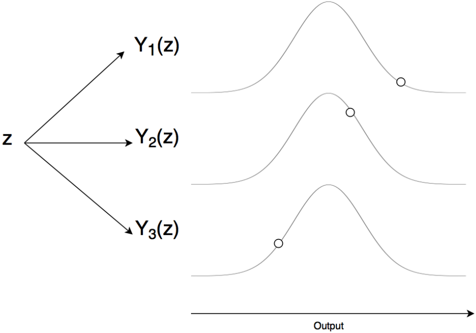

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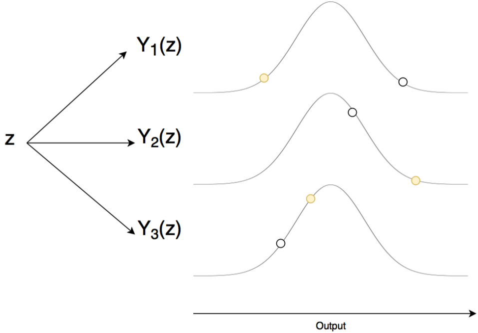

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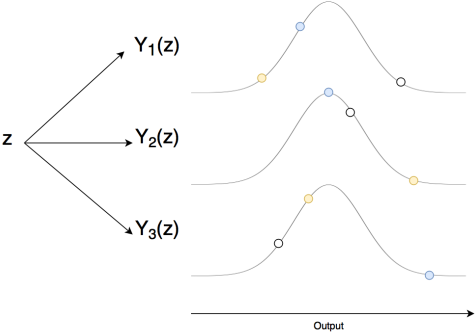

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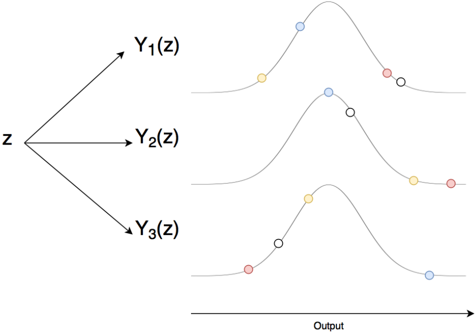

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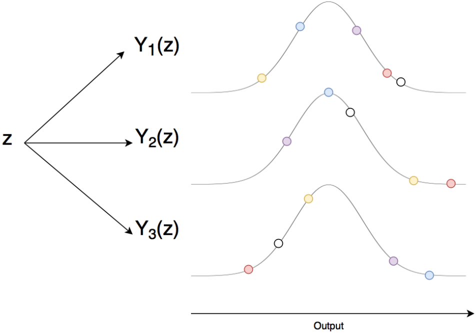

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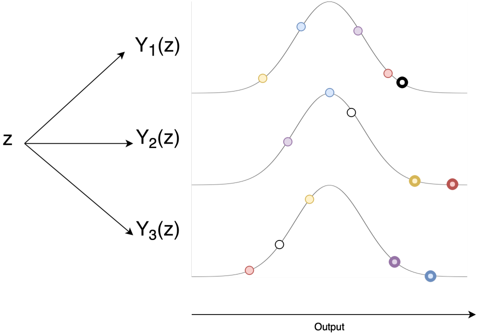

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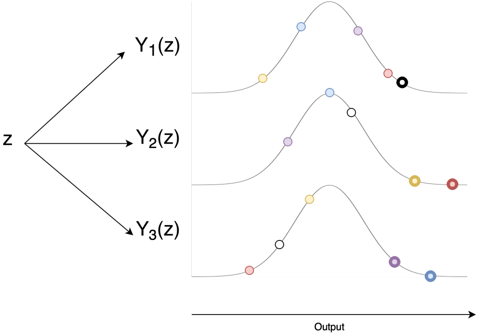

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