The Origin of Risk

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When aggregated, these individual decisions matter for aggregate risk

- If everybody grows crops by the shore, a flood can lead to mass starvation

What drives individual risk-taking decisions and how do they affect aggregate risk?

Approach and results

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- Since TFP multiplies the input bundle, larger firms manage risk more aggressively

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- Since TFP multiplies the input bundle, larger firms manage risk more aggressively
- Larger firms and those with low markups are less volatile and covary less with GDP
- We find support for these predictions in detailed firm-level Spanish data

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- Because of endogenous risk, distortions can make GDP more volatile

We calibrate the model to the Spanish economy

- Removing distortions lead to a large decline in aggregate volatility

Literature review

- Most of macroeconomics takes risk as exogenous (at the micro and/or macro level)

- In models with individual firms, firm-level risk is generally exogenous but macro risk can be endogenous
    - Khan and Thomas (2008), Clementi and Palazzo (2016), Bloom et al. (2018), and many others
- In endogenous growth models, firms influence the growth rate of TFP but not its variance
    - Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Jones (1995)
- Corporate finance literature where managers influence how risky a project is
    - Jensen and Meckling (1976), Ross (1977)
- Wedges in production network economies
    - Jones (2011), Baqaee and Farhi (2019), Liu (2019) and Bigio and La'O (2020)
- Technique choice in production networks
    - Oberfield (2018), Acemoglu and Azar (2020), Kopytov et al. (2024)

A model of endogenous risk

Static model with two types of agents

1. A representative household owns the firms, supplies labor and risk management resources
2. N firms produce differentiated goods using labor and intermediate inputs
   - Firm i has constant returns to scale Cobb-Douglas production function

▸ ζ

$$F\left(\delta_i, L_i, X_i\right) = e^{a_i(\varepsilon,\delta_i)} \zeta_i L_i^{1-\sum_{j=1}^{N} \alpha_{ij}} \prod_{j=1}^{N} X_{ij}^{\alpha_{ij}}$$

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$$a_i(\varepsilon, \delta_i) = \delta_i^\top \varepsilon$$

Managing risk (picking δ_i) requires risk management resources R_i supplied by the household

$$R_i = \kappa_i(\delta_i) = \frac{1}{2}(\delta_i - \delta_i^\circ)^\top H_i (\delta_i - \delta_i^\circ)$$

where δ_i° is the *natural* risk exposure ($R_i = 0$), and H_i is a positive definite matrix

Representative household

Owns the firms, supplies one unit of labor inelastically, supplies risk management resources

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Values the consumption bundle (GDP)

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Maximizes King, Plosser, Rebelo (1988) preferences

$$\mathcal{U}(Y)\,\mathcal{V}(R)$$

where $\mathcal{U}$ is CRRA with risk aversion $\rho \geq 1$, and disutility of risk management $\mathcal{V}(R)$ is

Details

$$\mathcal{V}(R) = \exp\left(-\eta\left(1-\rho\right)R\right)$$

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Budget constraint in each state of the world (set $W_L = 1$ from now on)

$$\sum_{i=1}^{N} P_i C_i \leq W_L + W_R R + \Pi$$

Timing and distortions

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- Example: markups, taxes, or other distortions

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Cobb-Douglas unit cost is

$$K_i\,(\delta_i, P) = \frac{1}{e^{a_i(\varepsilon, \delta_i)}} \prod_{j=1}^{N} P_j^{\alpha_{ij}}$$

Risk-taking decision

Firm choose their risk exposure to maximize expected discounted profits

$$\delta_i^* \in \arg\max_{\delta_i \in \mathcal{A}_i} \mathrm{E}\left[\Lambda \left[P_i Q_i - K_i\left(\delta_i, P\right) Q_i - \kappa_i\left(\delta_i\right) W_R\right]\right]$$

where Q_i is *equilibrium* demand and Λ is the stochastic discount factor of the household.

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Firms prefer risk exposures with

1. low risk management expenses $\kappa_i(\delta)$
2. high expected TFP (low expected unit costs K_i)
3. low covariance with GDP

Equilibrium definition

An *equilibrium* is a risk choice for every firm δ^* and a stochastic tuple $(P^*, W_R^*, C^*, L^*, R^*, X^*, Q^*)$ such that

1. (Optimal technique choice) For each i, factor demand L_i^*, X_i^* and R_i^*, and the risk exposure decision δ_i^* solves the firm's problem.
2. (Consumer maximization) The consumption vector C^* and the supply of risk managers R^* solve the household problem.
3. (Unit cost pricing) For each i, $P_i = (1+\tau_i)\, K_i\,(\delta_i, P)$.
4. (Market clearing) For each i,

$$C_i^* + \sum_{j=1}^{N} X_{ji}^* = Q_i^* = F_i\left(\alpha_i^*, L_i^*, X_i^*\right), \quad \sum_{i=1}^{N} L_i^* = 1, \text{ and } \sum_{i=1}^{N} \kappa_i\left(\delta_i^*\right) = R^*.$$

Two measures of supplier importance

Cost-based Domar weight:

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- Captures firm's importance as a supplier (share of production costs)

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Revenue-based Domar weight:

$$\omega^\top = \beta^\top \mathcal{L} = \beta^\top \left(I - [\mathrm{diag}\,(1+\tau)]^{-1}\,\alpha\right)^{-1}$$

- Also captures importance as a supplier (share of revenues)
- Declines with wedges τ

Define aggregate risk exposure Δ as

$$\Delta := \delta^\top \tilde{\omega}$$

- Firms with high cost-based Domar weights contribute more to aggregate risk exposure

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Lemma

$$\log Y = y = \Delta^\top \varepsilon - \tilde{\omega}^\top \log(1+\tau) - \log(\text{Labor share}(\omega, \tau))$$

- Without distortions ($\tau = 0$) we have Hulten's theorem: $y = \Delta^\top \varepsilon = \omega^\top a(\varepsilon, \delta)$

Aggregate risk

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Impact of Σ

- A marginal increase in Σ_{mm} raises $\mathrm{V}[y]$ by Δ_m^2
  - Both $\Delta_m \gg 0$ and $\Delta_m \ll 0$ are bad for $\mathrm{V}[y]$
- If the economy is positively exposed to m and n, increasing Σ_{mn} raises $\mathrm{V}[y]$.
- If $\Delta_m > 0$ and $\Delta_n < 0$, the shocks offset each other. Higher Σ_{mn} reduces $\mathrm{V}[y]$.

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Impact of Δ

$$\frac{d\,\mathrm{V}[y]}{d\Delta_m} = 2\,\mathrm{Cov}[y, \varepsilon_m] = 2\sum_n \Delta_n \,\mathrm{Cov}[\varepsilon_n, \varepsilon_m]$$

- Extra exposure to ε_m increases volatility if ε_m is positively correlated with GDP

Firm risk-taking decision

Lemma

The equilibrium risk exposure decision δ_i solves

$$\mathcal{E} \underbrace{K_i Q_i}_{\substack{\text{cost of} \\ \text{goods sold}}} = \underbrace{W_R \nabla \kappa_i \left(\delta_i\right)}_{\substack{\text{marginal cost} \\ \text{of exposure}}},$$

where $\mathcal{E}$ is the value of exposure, given by $\mathcal{E} := \mathrm{E}\left[\varepsilon\right] + \mathrm{Cov}\left[\lambda, \varepsilon\right]$.

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Equation for $\mathcal{E}$ implies that firms prefer risk factors with

- high expected value $\mu = \mathrm{E}[\varepsilon_m]$
- negative covariance with GDP ($\mathrm{Cov}[\lambda, \varepsilon] > 0$)

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- negative covariance with GDP $\left(\mathrm{Cov}\left[\lambda, \varepsilon\right] > 0\right)$

Benefit of increasing δ_i grows with the size of the firm since TFP multiplies the input bundle

- Since $K_i Q_i = \omega_i \Gamma_L^{-1} / \left(1 + \tau_i\right)$ firms with high ω_i and low τ_i manage risk more aggressively

Existence, uniqueness and efficiency

Planner's problem

Define $\bar{\kappa}_{SP}(\Delta)$ as the smallest risk management utility cost needed to achieve Δ.

$$\bar{\kappa}_{SP}(\Delta) := \min_{\delta} -\log V\left(\sum_{i=1}^{N} \kappa_i(\delta_i)\right), \quad \text{subject to } \Delta = \delta^\top \tilde{\omega}$$

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$$\mathcal{W}_{SP} := \max_{\Delta} \underbrace{\Delta^\top \mu}_{\mathrm{E}[y_{SP}]} - \frac{1}{2}(\rho - 1)\underbrace{\Delta^\top \Sigma \Delta}_{\mathrm{V}[y_{SP}]} - \bar{\kappa}_{SP}(\Delta)$$

The planner prefers aggregate risk exposure vectors Δ with

- high expected GDP $\mathrm{E}[y_{SP}]$
- low GDP volatility $\mathrm{V}[y_{SP}]$
- low risk management cost $\bar{\kappa}_{SP}$

Equilibrium characterization through fictitious planner

Define $\bar{\kappa}\left(\Delta\right)$ as the perceived smallest risk management utility cost needed to achieve Δ.

$$\bar{\kappa}\left(\Delta\right) := \min_{\delta} - \log V\left(\sum_{i=1}^{N} g_i \kappa_i\left(\delta_i\right)\right), \quad \text{subject to } \Delta = \delta^{\top}\tilde{\omega}$$

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Proposition (fictitious planner's problem)

There exists a unique equilibrium, and it solves

$$\mathcal{W}_{dist} := \max_{\Delta} \underbrace{\Delta^{\top}\mu - \tilde{\omega}^{\top}\log(1+\tau) - \log\Gamma_L}_{\mathrm{E}[y]} - \frac{1}{2}(\rho-1)\underbrace{\Delta^{\top}\Sigma\Delta}_{\mathrm{V}[y]} - \bar{\kappa}(\Delta).$$

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The equilibrium solves a distorted planning problem

- Still seeks to maximize $\mathrm{E}[y]$ and minimize $\mathrm{V}[y]$
- But distorted perception of the cost of managing risk ($\bar{\kappa}$ instead of $\bar{\kappa}_{SP}$)

Determinants of equilibrium risk

Equilibrium risk exposure

First-order of fictitious planning problem

$$\underbrace{\mathcal{E}(\Delta)}_{\substack{\text{marginal}\\ \text{benefit of } \Delta}} = \underbrace{\nabla \bar{\kappa}(\Delta)}_{\substack{\text{marginal}\\ \text{cost of } \Delta}}$$

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Proposition

Let γ be either μ_m or Σ_{mn}. Then

$$\frac{d\Delta}{d\gamma} = \mathcal{H}^{-1}\frac{\partial\mathcal{E}}{\partial\gamma},$$

where $\mathcal{H}^{-1} := \left(\nabla^2\bar{\kappa} + (\rho - 1)\,\Sigma\right)^{-1}$ is an $M \times M$ positive definite matrix.

- The vector $\partial\mathcal{E}/\partial\gamma$ captures the *direct impact* of γ on the attractiveness of risk factors ▸ $\partial\mathcal{E}/\partial\gamma$
- The matrix $\mathcal{H}^{-1}$ propagates that impact to exposure vector Δ

Equilibrium risk exposure

Corollary

1. An increase in μ_m raises Δ_m
2. An increase in Σ_{mm} reduces Δ_m if $\Delta_m > 0$ and increases Δ_m if $\Delta_m < 0$

- A marginal increase in Σ_{mm} raises $\mathbb{V}[y]$ by $\Delta_m^2 \rightarrow$ When Σ_{mm} increases we want to reduce Δ_m^2

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What is the impact of μ_m or Σ_{mm} on Δ_n with $m \neq n$? Off-diagonal terms of $\mathcal{H}^{-1}$ are important.

- If $\left[\mathcal{H}^{-1}\right]_{mn} > 0$, m and n are *global complements* $\rightarrow$ an increase in $\mathcal{E}_m$ increases in Δ_n
- If $\left[\mathcal{H}^{-1}\right]_{mn} < 0$, m and n are *global substitutes* $\rightarrow$ an increase in $\mathcal{E}_m$ decreases Δ_n

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Global substitution patterns depend on $\mathcal{H}^{-1} := \left(\nabla^2 \bar{\kappa} + (\rho - 1)\,\Sigma\right)^{-1}$

- $\nabla^2 \bar{\kappa}$: global impact of the local substitution patterns embedded in $(\kappa_1, \ldots, \kappa_N)$
- Σ: if $\Sigma_{mn} > 0$ an increase in Δ_m makes the planner reduce Δ_n to avoid agg. risk

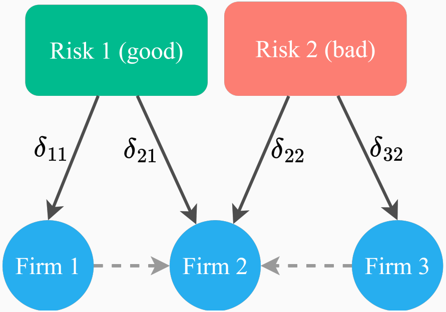
Example of substitution patterns

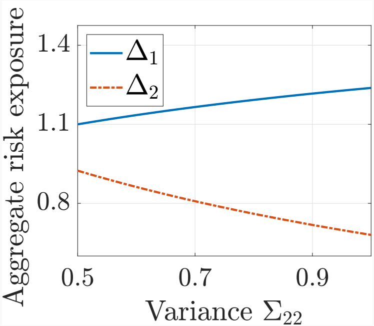
There are two regions both with their specific shocks

- Region 1: more productive in expectation (Risk 1 – good risk)
- Region 2: bigger shocks (Risk 2 – bad risk)

Firm 2 must decide where to locate plants

- Challenging to manage plants in different locations → risks are substitutes





Definition. An economy is diagonal if Σ and H_i are diagonal for every i

Impact of wedges

general case

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Corollary

In a diagonal economy, a higher wedge τ_i

1. increases Δ_m for all m such that $\mathcal{E}_m > 0$ (good risks)
2. reduces Δ_m for all m such that $\mathcal{E}_m < 0$ (bad risks)

- Higher wedges make firms shrink $\rightarrow$ manage risk less aggressively

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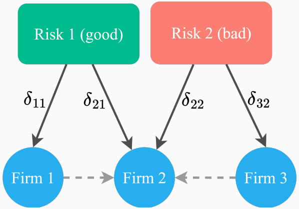

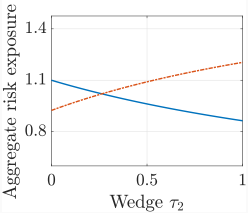

(Blue: good risk; Red: bad risk)

Equilibrium and efficient risk exposure

When all firms are at their natural exposure δ° we have $\mathcal{E}^\circ = \mu - (\rho - 1)\,\Sigma\Delta^\circ$

Lemma

Equilibrium risk exposure is distorted such that $(\Delta - \Delta_{SP})^\top \mathcal{E}^\circ < 0$.

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- Wedges make firms inefficiently small $\rightarrow$ less risk management
- Eqm. is on *average* overexposed to bad risks $(\mathcal{E}^\circ < 0)$ and underexposed to good risks $(\mathcal{E}^\circ > 0)$

Implications for GDP and Welfare

Implications for GDP

general case

Use ∂ to denote changes in the economy with exogenous risk

Proposition

In a diagonal economy:

$$\operatorname{sign}\left(\frac{d\,\mathrm{E}[y]}{d\mu_m} - \frac{\partial\,\mathrm{E}[y]}{\partial\mu_m}\right) = \operatorname{sign}(\mu_m) \quad \text{and} \quad \frac{d\,\mathrm{V}[y]}{d\Sigma_{mm}} - \frac{\partial\,\mathrm{V}[y]}{\partial\Sigma_{mm}} < 0.$$

- Increasing μ_m raises Δ_m $\rightarrow$ additional increase in $\mathrm{E}[y]$ if $\mu_m > 0$ compared to fixed risk
- Increasing Σ_{mm} decreases $|\Delta|$ $\rightarrow$ smaller increase in $\mathrm{V}[y]$ than with fixed risk

Distortions can increase aggregate volatility

Proposition (single risk factor)

$$\text{sign}\left(\frac{d\,\mathrm{E}[y]}{d\tau_i} - \frac{\partial\,\mathrm{E}[y]}{\partial\tau_i}\right) = -\text{sign}(\mu\mathcal{E}) \qquad \text{and} \qquad \text{sign}\left(\frac{d\,\mathrm{V}[y]}{d\tau_i} - \frac{\partial\,\mathrm{V}[y]}{\partial\tau_i}\right) = -\text{sign}(\Delta\mathcal{E}).$$

Suppose $\mathcal{E} < 0$ (bad risk): increasing τ_i makes firms more exposed to risk factor

- if $\mu < 0$ this leads to a decline in $\mathrm{E}[y]$
- if $\Delta > 0$ the economy becomes even more exposed and $\mathrm{V}[y]$ increases

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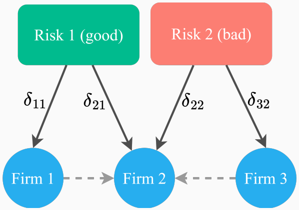

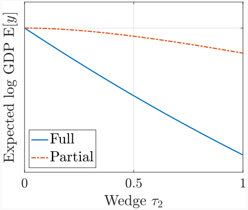

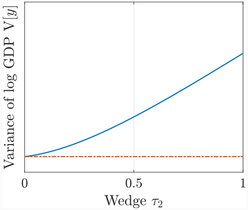

Implications for welfare

Proposition

In a diagonal economy, raising τ_i hurts welfare more than under exogenous risk.

- A higher τ_i increases exposure to bad risks and lower exposure to bad risks
- Additional exposure to bad risks hurts welfare, and vice-versa for good risks

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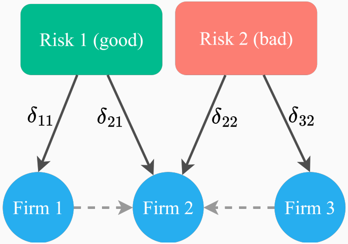
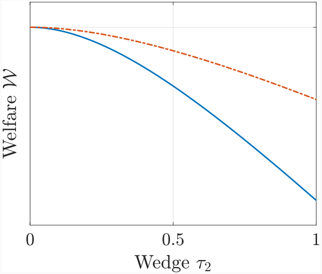




(Blue: flexible risk; Red: fixed risk)

Reduced-form evidence

Reduced-form evidence

Model: firms with large Domar weights and small markups are less volatile and less corr. with GDP

Details

Reduced-form evidence

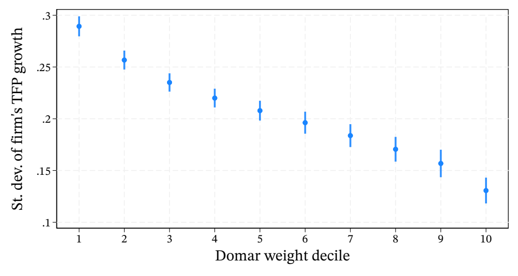
Model: firms with large Domar weights and small markups are less volatile and less corr. with GDP

▸ Details

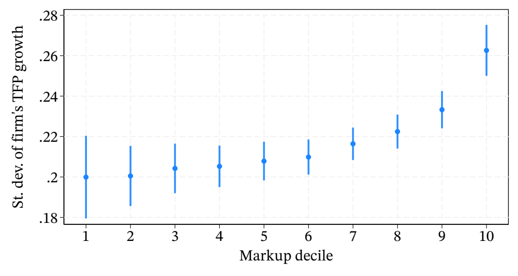
We test these predictions in the data

- Use detailed micro data from the near-universe of firms in Spain between 1995 and 2018 (Orbis) (7,513,081 firm-year observations)
- Compute markups using control function approach (De Loecker and Warzynski, 2012) ▸ Details
- Back out TFP growth as a residual

TFP growth volatility



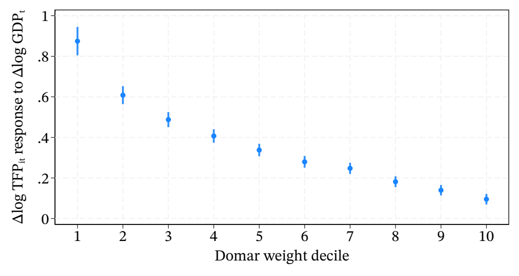
(a) TFP volatility by Domar weight decile



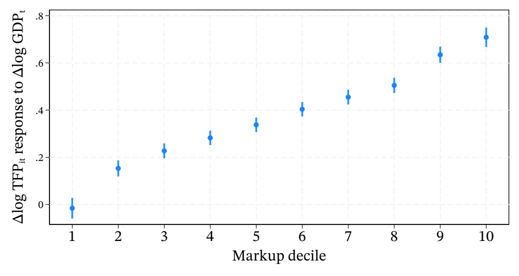
(b) TFP volatility by markup decile

▸ Details

Covariance of TFP growth with GDP growth



(c) Sensitivity of firm TFP to GDP by Domar weight decile



(d) Sensitivity of firm TFP to GDP by markup decile

Details

Calibration

A specialized model to map to the data

- S sectors with aggregator $Q_s = \prod_{i=1}^{N_s} e^{z_s} \left(\theta_{si}^{-1} Q_{si}\right)^{\theta_{si}}$ and sectoral shocks $z_s \sim \text{iid}\, \mathcal{N}\left(\mu_s^z, \Sigma_s^z\right)$
- Firms have production function

$$Q_{si} = e^{\delta_{sit}\varepsilon_t + \gamma_{si} t + v_{sit}} \zeta_{si} L_{si}^{1-\sum_{s'} \hat{\alpha}_{ss'}} \prod_{s'=1}^{S} X_{si,s'}^{\hat{\alpha}_{ss'}}$$

  where $\hat{\alpha}_{ss'}$ are sectoral shares, $v_{sit} \sim \text{iid}\, \mathcal{N}\left(\mu_{si}^v, \Sigma_{si}^v\right)$ and $\varepsilon_t \sim \text{iid}\, \mathcal{N}\left(0, \Sigma\right)$
- Risk management cost function is parametrized as

$$\frac{1}{\eta} H_{si}^{-1} = a_s \tilde{\omega}_{si}^{b_s} + c_s$$

  Allows for a size effect on risk management costs

▸ Details

Mapping to the data

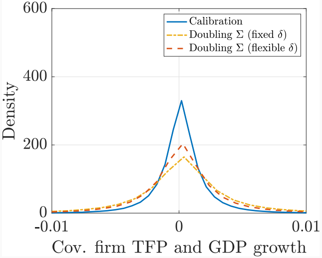
- We aim at replicating as much of the firm-level Spanish data as possible
- Our calibrated model has 62 sectors and 492,917 individual firms
- We invert parts of the model to exactly match some moments
    1. Sectoral consumption shares and input/output cost shares
    2. Firm shares in sectoral sales
    3. Variance of firm TFP growth
    4. Covariance of firm TFP growth and GDP growth
    5. Variance of GDP growth

▸ Details

Doubling Σ

What if we double the volatility Σ of the risk factor?

	Calibration	Doubling Σ	
		Fixed δ	Flexible δ
Agg. risk exposure Δ	0.014	0.014	0.011
Exposure value $\mathcal{E}$	-0.06	-0.11	-0.09
Std. Dev. of GDP growth	2.4%	3.1%	2.6%



- Fixed δ: Large increase in GDP variance; exposure to ε_t becomes more harmful ($\mathcal{E}$ declines)
- Flexible δ: Firms manage risk more aggressively which limits increase in $\mathbb{V}[y]$

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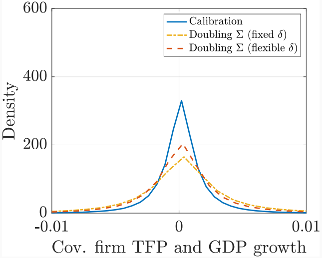

- Fixed δ: Large increase in GDP variance; exposure to ε_t becomes more harmful ($\mathcal{E}$ declines)
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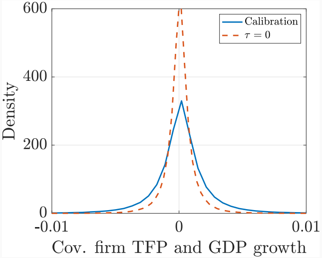
Impact of risk can be overestimated if reaction of agents is not taken into account

Removing distortions

What if we set wedges τ to zero?

	Calibration	No wedges	
		Fixed δ	Flexible δ
Agg. risk exposure Δ	0.014	0.014	0.007
Exposure value $\mathcal{E}$	-0.06	-0.06	-0.03
Std. Dev. of GDP growth	2.4%	2.4%	1.7%



- Fixed δ: Since only impact of τ is through δ, there is no change.
- Flexible δ: Firms manage risk more aggressively so $\mathbb{V}[y]$ declines

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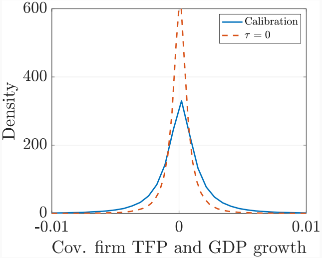

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Distortions can make GDP more volatile

Conclusion

Conclusion

Main contributions

- We construct a model of endogenous risk, at both the micro and macro levels.
- Model predicts which firms are more volatile and covary more with business cycle.
- Distortions lead to less aggressive risk management and can increase GDP volatility.

Future research

- What if there are entrepreneurs who cannot diversify their risk?
- Mechanisms would interact with capital/investment. Fully dynamic business cycle model.

Expression for $\zeta(\alpha_i)$

The function $\zeta\left(\alpha_i\right)$ is

$$\zeta\left(\alpha_i\right)=\left[\left(1-\sum_{j=1}^{n}\alpha_{ij}\right)^{1-\sum_{j=1}^{n}\alpha_{ij}}\prod_{j=1}^{n}\alpha_{ij}^{\alpha_{ij}}\right]^{-1}$$

This functional form allows for a simple expression for the unit cost K

Back

Risk aversion and ρ

Given the log-normal nature of uncertainty $\rho \lesseqgtr 1$ determines whether the agent is risk-averse or not. To see this, note that when $\log C$ normally distributed, maximizing

$$\mathrm{E}\left[C^{1-\rho}\right]$$

amounts to maximizing

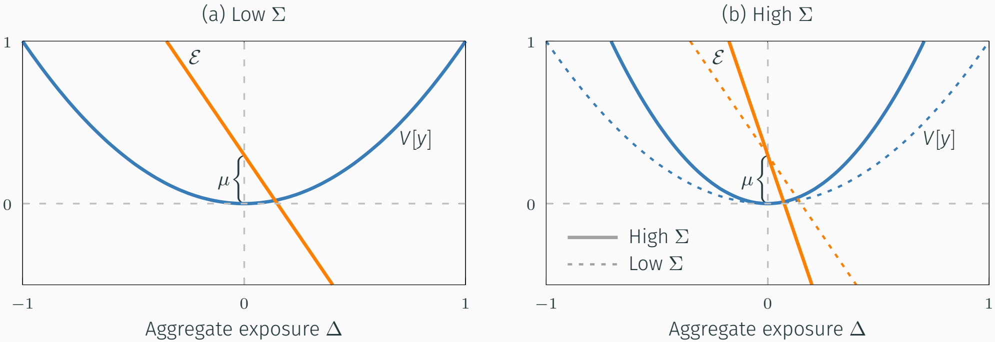
$$\mathrm{E}\left[\log C\right] - \frac{1}{2}\left(\rho - 1\right)\mathrm{V}\left[\log C\right].$$

◂ Back

Expressions for $\partial\mathcal{E}/\partial\gamma$

The direct impact of changes in (μ, Σ) is given by

$$\frac{\partial \mathcal{E}}{\partial \mu_m} = \mathbf{1}_m \qquad \text{and} \qquad \frac{\partial \mathcal{E}}{\partial \Sigma_{mn}} = -\frac{1}{2}(\rho - 1)\left(\Delta_m \mathbf{1}_n + \Delta_n \mathbf{1}_m\right).$$



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Impact of wedges

Proposition

The response of the equilibrium aggregate risk exposure Δ to a change in wedge τ_i is given by

$$\frac{d\Delta}{d\tau_i} = \mathcal{T}\left(\sum_{j=1}^{N} \frac{\partial\left[\nabla^2\bar{\kappa}\right]^{-1}}{\partial g_j}\frac{dg_j}{d\tau_i}\right)\mathcal{E}, \tag{1}$$

where the impact of g_j on $\left[\nabla^2\bar{\kappa}\right]^{-1}$ is given by $\frac{\partial\left[\nabla^2\bar{\kappa}\right]^{-1}}{\partial g_j} = -\frac{1}{\eta}\frac{\tilde{\omega}_j^2}{g_j^2}H_j^{-1}$, and where

$$\mathcal{T} := \left(I - \left[\nabla^2\bar{\kappa}\right]^{-1}\frac{\partial\mathcal{E}}{\partial\Delta}\right)^{-1}.$$

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Implications for GDP

Proposition

Let χ denote either μ_m, Σ_{mn}, or τ_i. Then the impact of a change in χ on the moments of log GDP are given by

$$\frac{d\,\mathrm{E}[y]}{d\chi} - \frac{\partial\,\mathrm{E}[y]}{\partial\chi} = \mu^\top \frac{d\Delta}{d\chi} \qquad \text{and} \qquad \frac{d\,\mathrm{V}[y]}{d\chi} - \frac{\partial\,\mathrm{V}[y]}{\partial\chi} = 2\Delta^\top \Sigma \frac{d\Delta}{d\chi},$$

where the use of a partial derivative indicates that Δ is kept fixed.

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Simplified model

Back

- Single risk factor $\varepsilon_t \sim \text{iid}\, \mathcal{N}(0, \Sigma)$
- Firm level TFP is $\log TFP_{it} = \delta_{it}\varepsilon_t + \gamma_i t + v_{it}$ where γ_i is deterministic trend and $v_{it} \sim \text{iid}\, \mathcal{N}(\mu_i^v, \Sigma_i^v)$

Simplified model

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Variance of firm-level TFP growth

$$\mathrm{V}\left[\log TFP_{it} - \log TFP_{it-1}\right] = 2\delta_i^2\Sigma + 2\Sigma_i^v$$

Implications for GDP

Simplified model

Back

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$$\mathrm{V}\left[\log TFP_{it} - \log TFP_{it-1}\right] = 2\delta_i^2\Sigma + 2\Sigma_i^v$$

Covariance of firm-level TFP growth with GDP growth

$$\mathrm{Cov}\left[\log TFP_{it} - \log TFP_{it-1}, y_t - y_{t-1}\right] = 2\Delta\Sigma\delta_i + 2\tilde{\omega}_i\Sigma_i^v.$$

Simplified model

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- Single risk factor $\varepsilon_t \sim$ iid $\mathcal{N}(0, \Sigma)$
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$$\mathrm{Cov}\left[\log TFP_{it} - \log TFP_{it-1}, y_t - y_{t-1}\right] = 2\Delta\Sigma\delta_i + 2\tilde{\omega}_i\Sigma_i^v.$$

Model-implied firm risk exposure ($\mathcal{E} < 0$)

$$\delta_i = \delta_i^\circ + \frac{1}{\eta}\frac{\omega_i}{1+\tau_i}H_i^{-1}\mathcal{E}$$

$\Rightarrow$ Firms with large Domar weights and small markups are less volatile and less corr. with GDP

Markup estimation

Back

- Assume Cobb-Douglas production function

$$\log Q_{it} = \alpha_{Li} \log L_{it} + \alpha_{Mi} \log M_{it} + \alpha_{Ki} \log K_{it} + \varepsilon_{it},$$

- Elasticities estimated using Levinsohn and Petrin (2003) with the Ackberg et al. (2015) correction.
    - Capital is the “state” variable, labor is the “free” variable and materials is the “proxy” variable.
- Production function estimated at NACE 2-digit sector level. As in De Loecker et al. (2020), we control for markups using firms’ sales shares in the production function estimation.
- Following De Loecker and Warzynski (2012), we compute the markup as $1 + \tau_{it} = \hat{\alpha}_{Li} \Big/ \left(\frac{\text{Wage Bill}_{it}}{\text{Sales}_{it}}\right)$.
- We compute TFP growth as

$$\begin{aligned}\Delta \log \text{TFP}_{it} =& \Delta \log Q_{it} - \alpha_{Li} \Delta \log L_{it} - \alpha_{Mi} \Delta \log M_{it} - \alpha_{Ki} \Delta \log K_{it} \\ & - \left(\Delta \log (1 + \tau_{it}) - \Delta \log \left(1 + \tau_{s(i)t}\right)\right).\end{aligned}$$

The term $\Delta \log (1 + \tau_{it}) - \Delta \log \left(1 + \tau_{s(i)t}\right)$ accounts for the firm-specific markup growth net of the sectoral markup growth. This adjustment allows us to remove the change in firm-specific nominal price that are not taken into account by the sector-level price deflator.

TFP growth volatility

- We compute the standard deviation of TFP growth for each firm, $\sigma_i\left(\Delta \log TFP_{it}\right)$, and the time-series average of its markup and Domar weight.
- We construct deciles based on average Domar weights and markups, and create dummy variables, FE_{ji}^{Domar} and FE_{ji}^{Markup}, such that $FE_{ji}^{Domar}=1$ if firm i's Domar weight is in decile j, and analogously for markups.
- We run the cross-sectional regression

$$\sigma_i\left(\Delta \log TFP_{it}\right)=\alpha+\sum_{j=1}^{10}\beta_j^{Domar} FE_{ji}^{Domar}+\sum_{j=1}^{10}\beta_j^{Markup} FE_{ji}^{Markup}+\varepsilon_i,$$

and plot β_j^{Domar} in panel (a) and β_j^{Markup} in panel (b).

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TFP growth volatility

- We construct deciles based on firms' Domar weights and markups each year.
- We then construct a set of dummy variables, FE_{jit}^{Domar} and FE_{jit}^{Markup}, such that $FE_{jit}^{Domar} = 1$ if firm i's Domar weight is in decile j in year t, and analogously for markups.
- We then run the following panel regression,

$$\Delta \log TFP_{it} = \sum_{j=1}^{10} \beta_j^{Domar} \left(FE_{jit}^{Domar} \times \Delta \log GDP_t \right) + \sum_{j=1}^{10} \beta_j^{Markup} \left(FE_{jit}^{Markup} \times \Delta \log GDP_t \right)$$
$$+ \alpha + \beta_0 \Delta \log GDP_t + \sum_{j=1}^{10} FE_{jit}^{Domar} + \sum_{j=1}^{10} FE_{jit}^{Markup} + \varepsilon_{it},$$

  where $\Delta \log TFP_{it}$ is the annual growth of firm i's log TFP and $\Delta \log GDP_t$ is the annual growth of Spanish log GDP.
- The coefficients of interest, β_j^{Domar} and β_j^{Markup}, are reported in the figure.

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Model for the calibration

- The variance of GDP growth is

$$\mathrm{V}\left[y_t - y_{t-1}\right] = 2\Sigma\Delta^2 + 2\tilde{\omega}_f^\top \Sigma^v \tilde{\omega}_f + 2\tilde{\omega}_s^\top \Sigma^z \tilde{\omega}_s.$$

- The variance of firm-level TFP growth is

$$\mathrm{V}\left[\log TFP_{si,t} - \log TFP_{si,t-1}\right] = 2\delta_{si}^2\Sigma + 2\Sigma_{si}^v.$$

- The covariance of firm-level TFP growth with GDP growth is

$$\mathrm{Cov}\left[y_t - y_{t-1}, \log TFP_{si,t} - \log TFP_{si,t-1}\right] = 2\Delta\Sigma\delta_{si} + 2\tilde{\omega}_{si}\Sigma_{si}^v.$$

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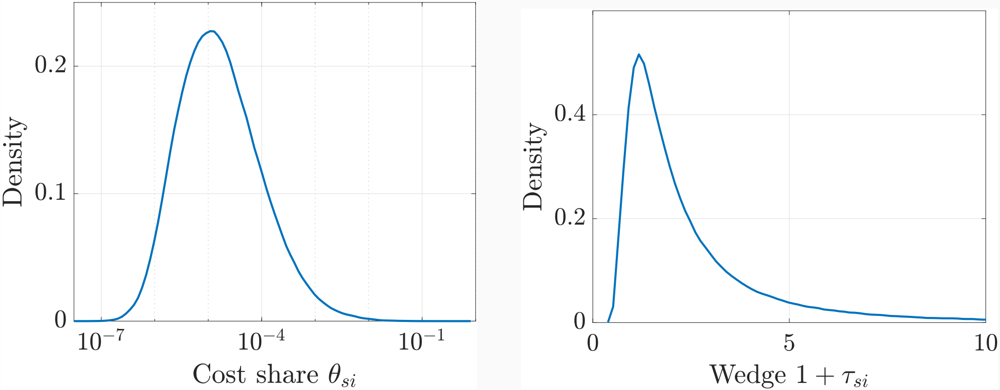
Calibrated model

Figure 1: Data distributions that the calibration matches exactly



(a) Sales share θ_{si} (b) Wedges $1 + \tau_i$

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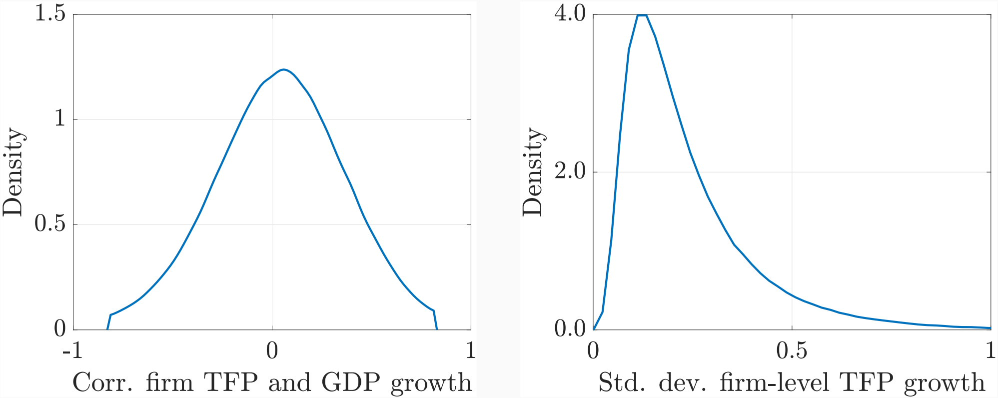
Calibrated model

Figure 2: Data distributions that the calibration matches exactly

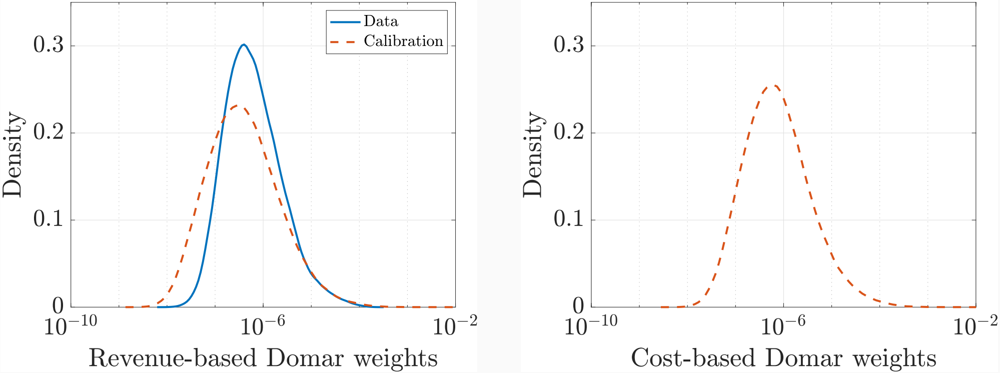


(a) Correlation firm-level TFP and GDP growth **(b)** Standard deviation of firm-level TFP growth

Back

Calibrated model

Figure 3: Domar weights of the firms in the data and in the model



(a) Revenue-based Domar weights

(b) Cost-based Domar weights

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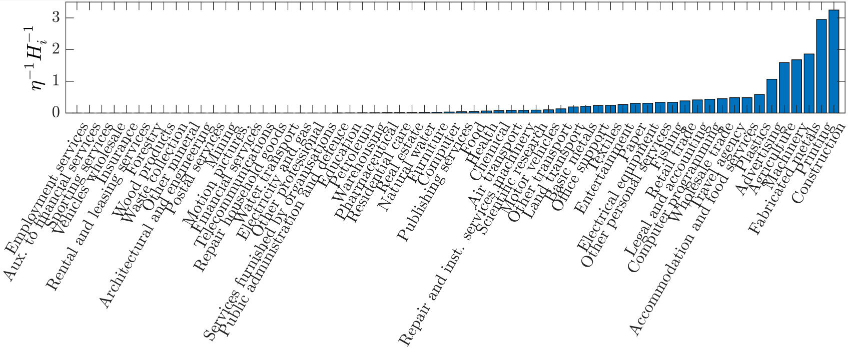
Figure 4: Estimated value of $\frac{1}{\eta}H_i^{-1}$ for each sector.



Notes. The scale of $\frac{1}{\eta}H_i^{-1}$ depends on our choice of ρ and Σ. We set $\rho = 5$ and $\Sigma = 1$ for this figure.

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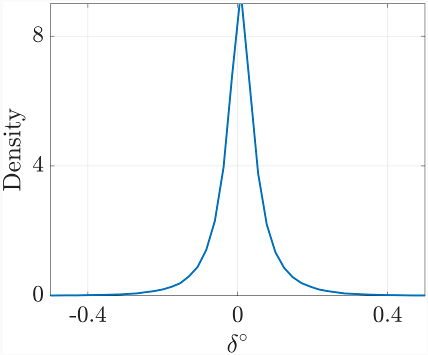
Calibrated model

Figure 5: Distribution of the estimated firm-level natural risk exposure δ_i° /



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