Cascades and Fluctuations in an Economy with an Endogenous Production Network

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Introduction

- Production in modern economies involves a complex network of producers supplying and demanding goods from each other
- The shape of this network
  - is an important determinant of how micro shocks aggregate into macro fluctuations
  - is also constantly changing in response to micro shocks
    - For instance, after a severe shock a producer might shut down which might lead its neighbors to shut down as well, etc...
    - Cascade of shutdowns that spreads through the network

This paper proposes a

Theory of network formation and aggregate fluctuations

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Theory of network formation and aggregate fluctuations

Literature Review

- Endogenous network formation
  - Atalay et al (2011), Oberfield (2013), Carvalho and Voigtländer (2014)
- Network of sectors and fluctuations
  - Horvath (1998), Dupor (1999), Acemoglu et al (2012), Baqaee (2016), Acemoglu et al (2016), Lim (2017)
- Non-convex adjustments in networks
  - Bak, Chen, Woodford and Scheinkman (1993), Elliott, Golub and Jackson (2014)

I. Model

Model

- There are n units of production (firm) indexed by $j \in \{1, \ldots, n\}$
  - Each unit produces a differentiated good
  - Differentiated goods can be used to
    - produce a final good
$$Y \equiv \left(\sum_{j=1}^{n} \left(y_j^0 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$
    - produce other differentiated goods
- Representative household
  - Consumes the final good
  - Supplies L units of labor inelastically

Model

- Firm j produces good j

$$y_j = \frac{A}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}} z_j \left(\sum_{i=1}^{n} x_{ij}^{\frac{\epsilon-1}{\epsilon}}\right)^{\alpha\frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

- Firm j can only use good i as input if there is a *connection* from firm i to j
  - $\Omega_{ij} = 1$ if connection and $\Omega_{ij} = 0$ otherwise
  - A connection can be *active* or *inactive*
  - Matrix Ω is *exogenous*
- A firm can only produce if it pays a fixed cost f in units of labor
  - $\theta_j = 1$ if j is operating and $\theta_j = 0$ otherwise
  - Vector θ is *endogenous*

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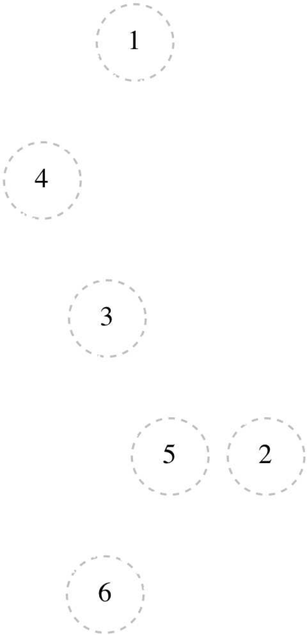
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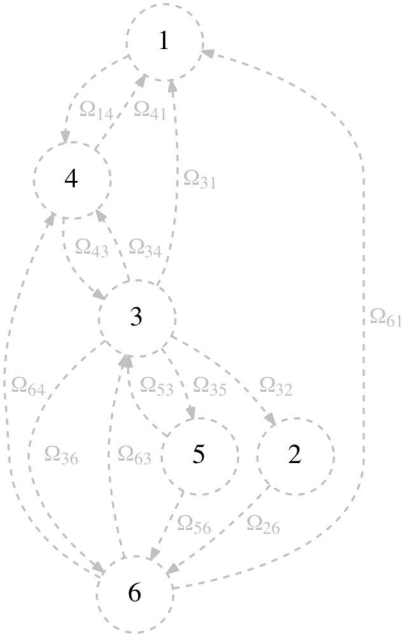
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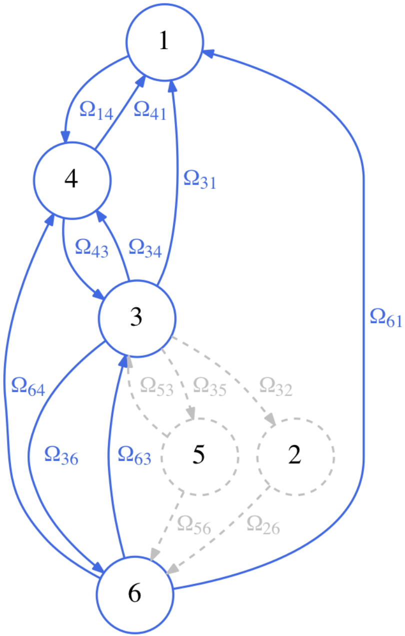
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Social Planner

Problem $\mathcal{P}_{SP}$ of a social planner

$$\max_{\substack{y^0, x, l \\ \theta \in \{0,1\}^n}} \left(\sum_{j=1}^{n} \left(y_j^0 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to

1. a resource constraint for each good j

$$y_j^0 + \sum_{k=1}^{n} x_{jk} \leq \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \theta_j \left(\sum_{i=1}^{n} \Omega_{ij} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor

$$\sum_{j=1}^{n} l_j + f \sum_{j=1}^{n} \theta_j \leq L$$

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1. a resource constraint for each good j **(Lagrange multiplier: λ_j)**

$$y_j^0+\sum_{k=1}^{n}x_{jk}\leq\frac{A}{\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}}z_j\theta_j\left(\sum_{i=1}^{n}\Omega_{ij}x_{ij}^{\frac{\epsilon-1}{\epsilon}}\right)^{\alpha\frac{\epsilon}{\epsilon-1}}l_j^{1-\alpha}$$

2. a resource constraint on labor **(Lagrange multiplier: w)**

$$\sum_{j=1}^{n}l_j+f\sum_{j=1}^{n}\theta_j\leq L$$

II. Social Planner with Exogenous θ

Social Planner with Exogenous θ

Define $q_j = w/\lambda_j$

- From the FOCs, output is $(1-\alpha)\, y_j = q_j l_j$
- q_j is the *labor productivity* of firm j

Proposition 1

In the efficient allocation,

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\epsilon - 1} \right)^{\frac{\alpha}{\epsilon - 1}} \tag{1}$$

Furthermore, there is a unique vector q that satisfies (1).

Social Planner with Exogenous θ

Knowing q we can solve for all other quantities easily.

Lemma 1

Aggregate output is

$$Y = Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

where $Q \equiv \left(\sum_{j=1}^{n} q_j^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ *is aggregate labor productivity.*

Labor allocation

III. Social Planner with Endogenous θ

Social Planner with Endogenous θ

Planner's problem is now

$$\max_{\theta \in \{0,1\}^n} Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

with

$$q_j = z_j \theta_j A\left(\sum_{i=1}^{n}\Omega_{ij} q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$

Trade-off: making firm j produce ($\theta_j = 1$)

- increases labor productivity of the network (Q)
- reduces the amount of labor into production $\left(L - f\sum_{j=1}^{n}\theta_j\right)$

Social Planner with Endogenous θ

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Social Planner with Endogenous θ

"Very hard problem" (MINLP — NP Hard)

- The set $\theta \in \{0, 1\}^n$ is not convex
- Objective function is not concave

Naive approach

- For any vector $\theta \in \{0, 1\}^n$ iterate on q and evaluate the objective function
- 2^n vectors θ to try ($\approx 10^6$ configurations for 20 firms)
- Impossible for n large

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Alternative approach

Solution approach: Find an alternative problem such that

- P1 The alternative problem is easy to solve
- P2 A solution to the alternative problem also solves $\mathcal{P}_{SP}$

Social Planner with Endogenous θ

Consider the relaxed and reshaped problem $\mathcal{P}_{RR}$

$$\max_{\theta \in \{0,1\}^n} Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$

Parameters $a > 0$ and $b \geq 0$ are *reshaping constants*

- Reshape the objective function *away* from optimum (i.e. when $0 < \theta_j < 1$)
  - For a: if $\theta_j \in \{0,1\}$ then $\theta_j^a = \theta_j$
  - For b: $\{\theta_i = 0\} \Rightarrow \{q_i = 0\}$ and $\{\theta_i = 1\} \Rightarrow \left\{\theta_i^b q_i^{\epsilon-1} = q_i^{\epsilon-1}\right\}$
- Parameters such that P1 and P2 are satisfied:

$$\boxed{a = \frac{1}{\sigma-1} \quad \text{and} \quad b = 1 - \frac{\epsilon-1}{\sigma-1}} \qquad (\star)$$

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Social Planner with Endogenous θ

P1 The alternative problem $\mathcal{P}_{RR}$ is easy to solve

Proposition 2

If $\Omega_{ij} = c_i d_j$ for some vectors c and d then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to $\mathcal{P}_{RR}$.

Proposition 3

Let $\sigma = \epsilon$ and suppose that $f > 0$ and $\overline{z} - \underline{z} > 0$ are not too big. If Ω is sufficiently connected, then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to $\mathcal{P}_{RR}$.

These propositions

- Only provides *sufficient* conditions
- Later: Test the approach on thousands of economies

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Social Planner with Endogenous θ

P2 A solution to the alternative problem $\mathcal{P}_{RR}$ also solves $\mathcal{P}_{SP}$

Proposition 4

If θ^* solves $\mathcal{P}_{RR}$ and that $\theta_j^* \in \{0,1\}$ for all j, then θ^* also solves $\mathcal{P}_{SP}$.

Solution θ^* to $\mathcal{P}_{RR}$ is such that $\theta_j^* \in \{0,1\}$ for all j (P2) if

- the $(\star)$ condition is satisfied
- there are many firms
- the network is sufficiently connected

▸ Details

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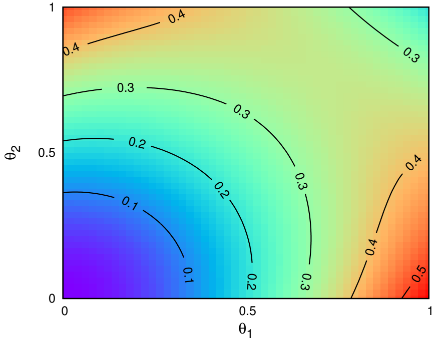
Details

Example with $n = 2$

Relaxed problem **without** reshaping

$$V(\theta) = Q(\theta)\left(L - f\sum_{j=1}^{n}\theta_j\right) \text{ with } q_j = z_j\theta_j A\left(\sum_{i=1}^{n}\Omega_{ij}q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$



Problem: V is not concave

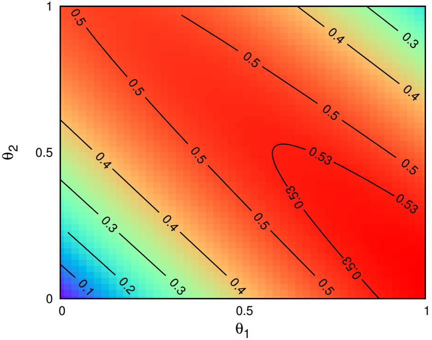
$\Rightarrow$ First-order conditions are not sufficient

$\Rightarrow$ Numerical algorithm can get stuck in local maxima

Example with $n = 2$

Relaxed problem **with** reshaping

$$V(\theta) = Q(\theta)\left(L - f\sum_{j=1}^{n}\theta_j\right) \text{ with } q_j = z_j\theta_j^{\frac{1}{\sigma-1}} A\left(\sum_{i=1}^{n}\Omega_{ij}\theta_i^{1-\frac{\epsilon-1}{\sigma-1}} q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$



~~Problem~~: V is now (quasi) concave

⇒ First-order conditions are necessary and sufficient

⇒ Numerical algorithm converges to global maximum

Testing the approach on small networks

For small networks we can solve $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

- Comparing approaches for a million different economies:

	Number of firms n			
	8	10	12	14
A. With reshaping				
Firms with correct θ_j	99.9%	99.9%	99.9%	99.8%
Error in output Y	0.00039%	0.00081%	0.00174%	0.00171%
B. Without reshaping				
Firms with correct θ_j	84.3%	83.2%	82.3%	81.3%
Error in output Y	0.84%	0.89%	0.93%	0.98%

Notes: Parameters $f \in \{0.05/n, 0.1/n, 0.15/n\}$, $\sigma_z \in \{0.34, 0.39, 0.44\}$, $\alpha \in \{0.45, 0.5, 0.55\}$, $\sigma \in \{4, 6, 8\}$ and $\epsilon \in \{4, 6, 8\}$. For each combination of parameters 1000 different economies are created. For each economy, productivity is drawn from $\log(z_k) \sim$ iid $\mathcal{N}(0, \sigma_z)$ and Ω is drawn randomly such that each link Ω_{ij} exists with some probability such that a firm has on average five possible incoming connections. A network is kept in the sample only if the first-order conditions give a solution in which θ hits the bounds.

The errors come from

- firms that are particularly isolated
- two θ configurations with almost same output

Testing the approach on large networks

For large networks we cannot solve $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

- After all the 1-deviations θ are exhausted:

	With reshaping	Without reshaping
Firms with correct θ_j	99.8%	72.1%
Error in output Y	0.00028%	0.69647%

Notes: Simulations of 200 different networks Ω and productivity vectors z that satisfy the properties of the calibrated economy.

- Very few “obvious errors” in the allocation found by the approach

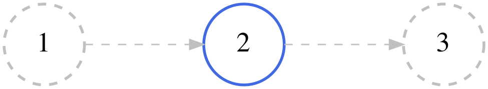
Return

IV. Economic Forces at Work

Complementarities

- Impact of operating 2 on the incentives to operate 1 and 3
  - Operating 3 leads to a larger q_3 because 2 is operating
  - Operating 1 increases q_2 because 2 is operating
- Complementarity between operating decisions of nearby firms

Complementarities



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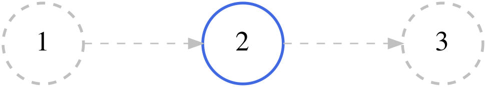

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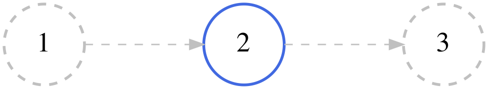

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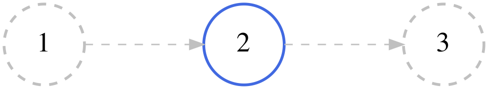

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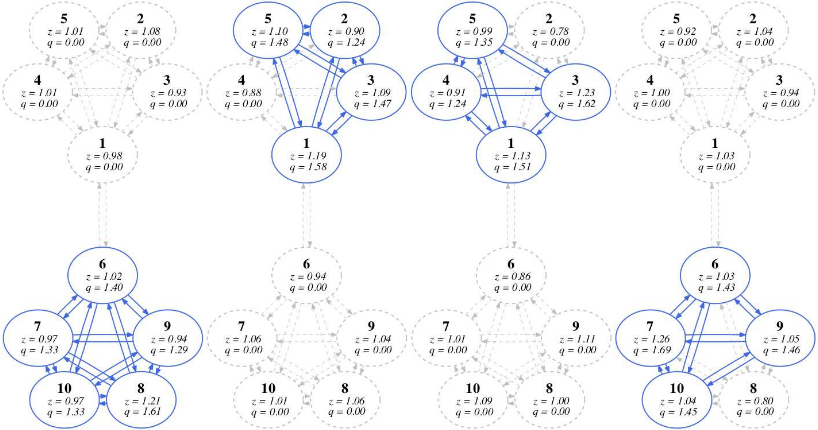
Complementarities lead to clustering



V. Quantitative Exploration

Network data

- Two datasets that cover the U.S. economy
  - Cohen and Frazzini (2008) and Atalay et al (2011)
  - Both rely on Compustat data
    - Public firms must self-report customers that purchase more than 10% of sales
    - Use fuzzy-text matching algorithms and manual matching to build networks
  - Cover 1980 to 2004 and 1976 to 2009 respectively

Parameters

Parameters from the literature

- $\alpha = 0.5$ to fit the share of intermediate (Jorgenson et al 1987, Jones 2011)
- $\sigma = \epsilon = 6$ average of estimates (Broda et al 2006)
  - Robustness with smaller ϵ in the paper
- $\log(z_{it}) \sim \mathcal{N}\left(0, 0.39^2\right)$ from Bartelsman et al (2013)
- $f \times n = 5\%$ to fit employment in management occupations
- Calibrate $n = 3000$ to match number of active firms in Atalay et al (2011)

Unobserved network Ω:

- Pick to match the *observed* in-degree distribution
- Generate thousands of such Ω's and report averages

In-degree

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Shape of the network

What types of network does the planner choose?

- Compare **optimal networks** to completely **random networks**
- Differences highlights how efficient allocation shapes the network

	Optimal networks	Random networks
A. Power law shape parameters		
In-degree	1.43	1.48
Out-degree	1.37	1.48
B. Measures of proximity		
Clustering coefficient	0.027	0.018
Average distance between firms	2.26	2.64

Efficient allocation features

- More highly connected firms
- More clustering of firms

▸ Def. clust. coeff.

Firm-level distributions

In the efficient allocation:

- **Selection:** Low productivity firms do not operate
- **Magnification:** High productivity firms benefit from clustering

	Labor prod. q	Employment l
A. Optimal network		
Standard deviation	0.29	1.24
Skewness	0.39	0.85
Excess kurtosis	0.57	0.39
B. Random network		
Standard deviation	0.44	2.21
Skewness	-0.03	-0.05
Excess kurtosis	0.01	-0.06

Because of the optimal organization of the network

- Distributions are positively skewed ...
- ... and have fatter tails

Dist. of q

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Cascades of shutdowns

Because of the complementarities between firms

- Exit of a firm makes it more likely that its neighbors exit as well ...
- ... which incentivizes the second neighbors to exit as well ...
- ...

Cascades of shutdowns

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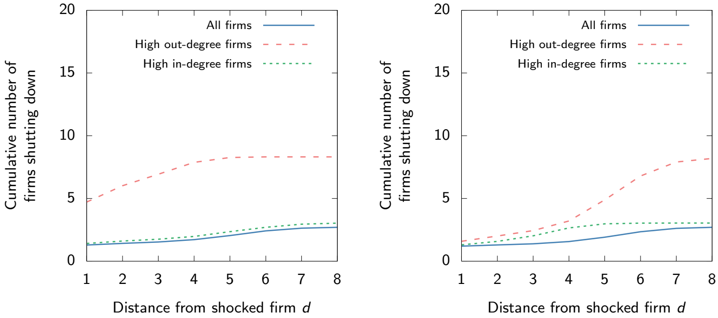
- ...

(a) Downstream connections (b) Upstream connections



▸ Robustness

Resilience of firms

Magnitude of shock necessary to make a firm exit varies

	Probability of firm shut down after 1 std shock
All firms	92%
High out-degree firms	20%
High in-degree firms	56%

Implications:

- Highly-connected firms are hard to topple but upon shutting down they create large cascades

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Robustness

Aggregate fluctuations

The shape of the network changes with the business cycle

	Correlation with output		
	Model	Data	
		CF (2008)	AHRS (2011)
A. Power law shape parameters			
In-degree	-0.10	-0.10	-0.21
Out-degree	-0.31	-0.24	-0.13
B. Clustering coefficient	0.47	0.70	0.15

Implications:

- Recessions are periods with fewer highly-connected firms and in which clustering activity around most productive firms is costly

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Aggregate fluctuations

Size of fluctuations

$$Y = Q\left(L - f\sum_j \theta_j\right)$$

Table: Standard deviation of aggregates

	Output Y	Labor Prod. Q	Prod. labor $L - f\sum_j \theta_j$
Optimal network	**0.039**	0.039	0.0014
Fixed network	**0.054**	0.054	0

Implications:

- Substantially smaller fluctuations in optimal network economy comes from the reorganization of network after shocks

Intuition

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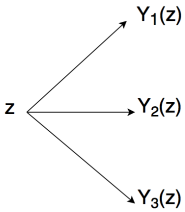

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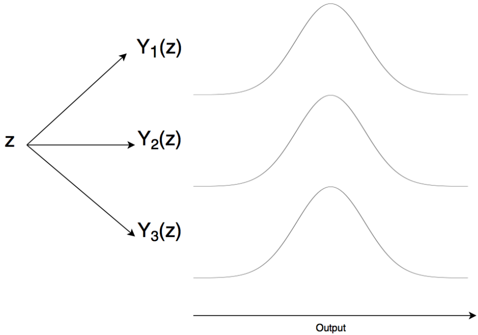

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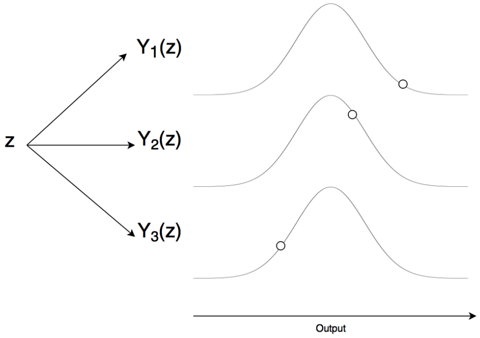

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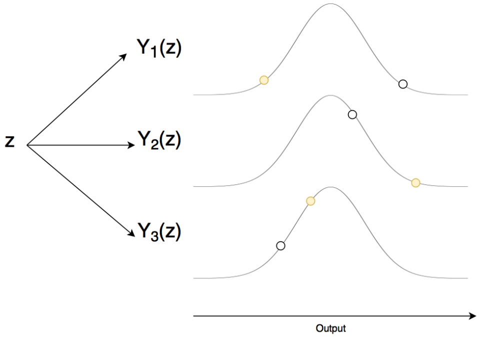

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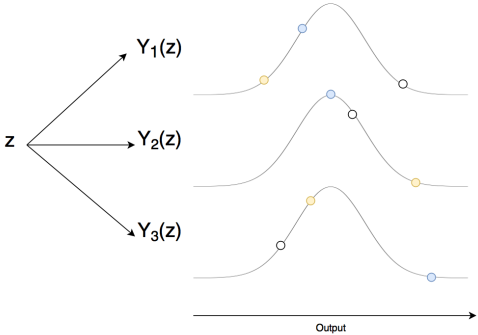

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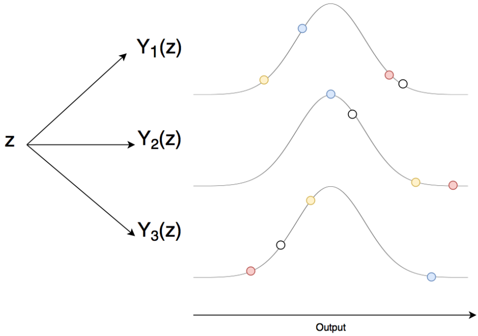

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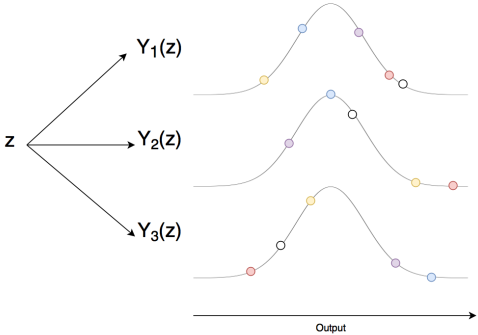

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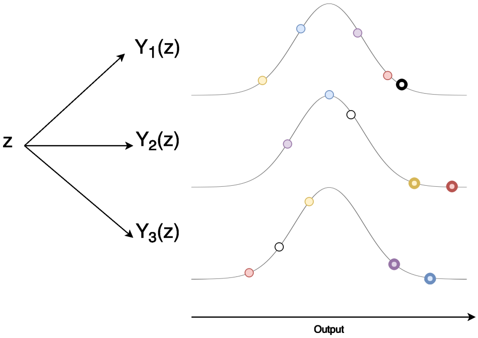

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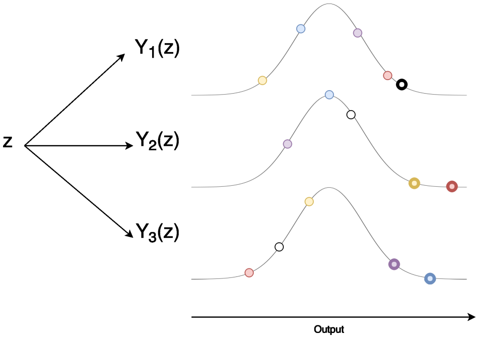

From extreme value theory

$$\mathrm{Var}(Y) = \mathrm{Var}\left(\max_{k \in \{1,\ldots,2^n\}} Y_k\right)$$

declines rapidly with n

Conclusion

Additional results in the paper:

- Impact of position in the network on firm-level characteristics
- Endogenous skewness in distribution of employment, productivity, output

Summary

- Theory of network formation and aggregate fluctuations
- Propose an approach to solve these hard problems easily
- The optimal allocation features
  - Clustering of activity
  - Cascades of shutdowns/restarts
- Optimal network substantially limit the size of fluctuations

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Labor allocation

Lemma 2

The optimal labor allocation satisfies

$$l = (1-\alpha)\underbrace{\left[I_n - \alpha\Gamma\right]^{-1}}_{(1)} \underbrace{\left(\frac{q}{Q}\right)}_{(2)}^{\circ(\sigma-1)} \left(L - f\sum_{j=1}^{n}\theta_j\right)$$

where I_n is the identity matrix and where Γ is an $n \times n$ matrix where $\Gamma_{jk} = \frac{\Omega_{jk} q_j^{\epsilon-1}}{\sum_{i=1}^{n} \Omega_{ik} q_i^{\epsilon-1}}$ captures the importance of j as a supplier to k.

Determinants of l_j

(1) Importance of j as a supplier

- Leontief inverse $\left([I_n - \alpha\Gamma]^{-1} = I_n + \alpha\Gamma + (\alpha\Gamma)^2 + \dots\right)$

(2) Relative efficiency

Return

Reshaping

Intuition:

- First-order condition on θ_j:

$$\text{Marginal Benefit}\left(\theta_j, F\left(\theta\right)\right) - \text{Marginal Cost}\left(\theta_j, G\left(\theta\right)\right) = \bar{\mu}_j - \underline{\mu}_j$$

- Under $(\star)$ the marginal benefit of θ_j only depends on θ_j through aggregates
- For large connected network F and G are independent of θ_j

Return FOCs

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Return FOCs

Details of reshaping

Simpler to consider

$$\mathcal{P}'_{RD}: \quad \max_{\theta \in [0,1]^n, q} \left(\sum_{j=1}^{n} q_j^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \left(L - f \sum_{j=1}^{n} \theta_j \right)$$

$$q_j \leq A z_j \theta_j^a A B_j^\alpha \qquad \text{(LM: } \beta_j\text{)}$$

where $B_j = \left(\sum_{i=1}^{n} \Omega_{ij} \theta_i^b q_i^{\epsilon-1} \right)^{\frac{1}{\epsilon-1}}$.
First order condition with respect to θ_k:

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial Q}{\partial q_k} \left(L - f \sum_{j=1}^{n} \theta_j \right) - fQ + \sum_{j=1}^{n} \beta_j \left(\frac{\partial q_k}{\partial \theta_k} \frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} \right) \frac{\partial q_j}{\partial B_j} = \overline{\mu}_k - \underline{\mu}_k$$

The terms are

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial Q}{\partial q_k} = z_k a \theta_k^{a-1} A B_k^\alpha \times (z_k \theta_k^a A B_k^\alpha)^{\sigma-2} \, Q^{2-\sigma}$$

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} = B_j \theta_k^{b-1} \Omega_{kj} \left(\frac{z_k \theta_k^a A B_k^\alpha}{B_j} \right)^{\epsilon-1} \left(a + \frac{b}{\epsilon - 1} \right)$$

Return

Distribution of in-degree

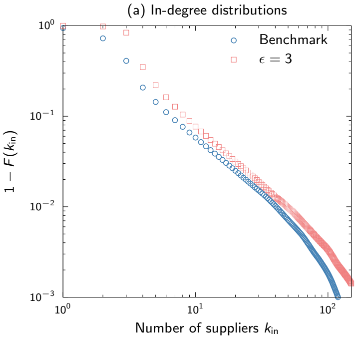

Figure: Distribution of the number of suppliers and the number of customers

In-degree power law shape parameter

- Calibration: 1.43
- Data: 1.37 (Cohen and Frazzini, 2008) and 1.3 (Atalay et al, 2011)

Return

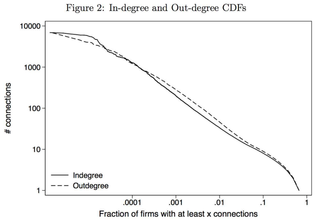

Figure: Distribution of in-degree and out-degree in Bernard et al (2015)

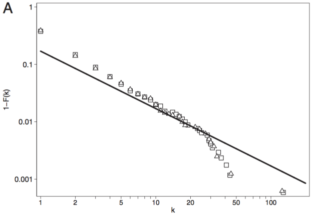

Figure: Distribution of in-degree in Atalay et al (2011)

Return

Clustering coefficient

- Triplet: three connected nodes (might be overlapping)
- Triangles: three fully connected nodes (3 triplets)

$$\text{Clustering coefficient} = \frac{3 \times \text{number of triangles}}{\text{number of triplets}}$$

Return

Firm-level distributions

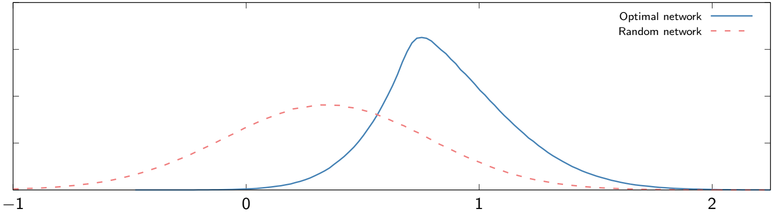

Figure: Distributions of $\log(q)$

return

Cascades of shutdowns

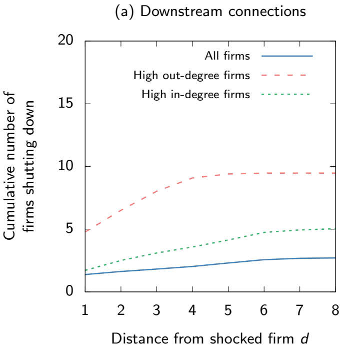

Figure: $\alpha = 0.75$

return

Cascades of shutdowns

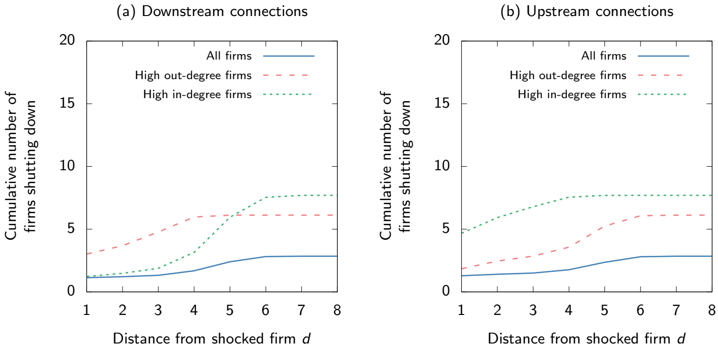

Figure: $\epsilon = 3$

return

Resilience

	Probability of firm shutdown		
	Benchmark	$\alpha = 0.75$	$\epsilon = 3$
All firms	92%	82%	32%
High out-degree firms	20%	8%	0%
High in-degree firms	56%	19%	15%

return