Endogenous Production Networks Under Uncertainty

Alexandr Kopytov
University of Hong Kong

Bineet Mishra
Cornell University

Kristoffer Nimark
Cornell University

Mathieu Taschereau-Dumouchel
Cornell University

Motivation

- Firms rely on complex supply chains to get intermediate inputs.

Motivation

- Firms rely on complex supply chains to get intermediate inputs.
- These chains are often disrupted by natural disasters, trade barriers, changes in regulations, congestion in transportation links, etc.

Motivation

- Firms rely on complex supply chains to get intermediate inputs.
- These chains are often disrupted by natural disasters, trade barriers, changes in regulations, congestion in transportation links, etc.
- These shocks propagate through input-output link. $\Rightarrow$ agg. fluctuations.

Motivation

- Firms rely on complex supply chains to get intermediate inputs.
- These chains are often disrupted by natural disasters, trade barriers, changes in regulations, congestion in transportation links, etc.
- These shocks propagate through input-output link. $\Rightarrow$ agg. fluctuations.
- But individual firms may mitigate such propagation by reducing their reliance on risky suppliers.

Motivation

- Firms rely on complex supply chains to get intermediate inputs.
- These chains are often disrupted by natural disasters, trade barriers, changes in regulations, congestion in transportation links, etc.
- These shocks propagate through input-output link. $\Rightarrow$ agg. fluctuations.
- But individual firms may mitigate such propagation by reducing their reliance on risky suppliers.

How does risk affect an economy's production network and, through that channel, macroeconomic aggregates?

Approach and results

We construct a model of endogenous network formation under uncertainty

- Firms create links with suppliers to acquire intermediate inputs
- Sourcing decisions are taken under uncertainty
  - Tradeoff between buying goods whose prices are low vs stable

Approach and results

We construct a model of endogenous network formation under uncertainty

- Firms create links with suppliers to acquire intermediate inputs
- Sourcing decisions are taken under uncertainty
  - Tradeoff between buying goods whose prices are low vs stable
- We show that there exists an efficient equilibrium in this economy

Approach and results

We construct a model of endogenous network formation under uncertainty

- Firms create links with suppliers to acquire intermediate inputs
- Sourcing decisions are taken under uncertainty
    - Tradeoff between buying goods whose prices are low vs stable
- We show that there exists an efficient equilibrium in this economy

We characterize the impact of the mechanism on the economy

- More productive/stable firms $\Rightarrow$ more important role in the equilibrium network.

Approach and results

We construct a model of endogenous network formation under uncertainty

- Firms create links with suppliers to acquire intermediate inputs
- Sourcing decisions are taken under uncertainty
  - Tradeoff between buying goods whose prices are low vs stable
- We show that there exists an efficient equilibrium in this economy

We characterize the impact of the mechanism on the economy

- More productive/stable firms $\Rightarrow$ more important role in the equilibrium network.
- Uncertainty lowers expected GDP
  - Mechanism operates through the endogenous response of the network
  - Firms seek stability at the cost of lower efficiency

Approach and results

We construct a model of endogenous network formation under uncertainty

- Firms create links with suppliers to acquire intermediate inputs
- Sourcing decisions are taken under uncertainty
  - Tradeoff between buying goods whose prices are low vs stable
- We show that there exists an efficient equilibrium in this economy

We characterize the impact of the mechanism on the economy

- More productive/stable firms $\Rightarrow$ more important role in the equilibrium network.
- Uncertainty lowers expected GDP
  - Mechanism operates through the endogenous response of the network
  - Firms seek stability at the cost of lower efficiency
- Shocks can have counterintuitive effects
  - Higher firm-level expected productivity can lead to lower expected GDP
  - Higher firm-level volatility can lead to more stable GDP

Approach and results

We calibrate the model to the United States economy

- The model is able to replicate the relationship between shocks and the structure of the network well.
- Letting the network adjust to shocks has large impact on welfare
- The impact of uncertainty on the network is small on average but can be substantial during high-volatility events like the Great Recession

Survey evidence

Surveys of business executives

- German executives: supply chains issues were responsible for significant disruption to production (Wagner and Bode, 2008)

Survey evidence

Surveys of business executives

- German executives: supply chains issues were responsible for significant disruption to production (Wagner and Bode, 2008)
- Global survey of small and medium firms: 39% report that losing their main supplier would adversely affect their operation, and 14% report that they would need to significantly downsize their business, require emergency support or shut down (Zurich Insurance Group, 2015)

Survey evidence

Surveys of business executives

- German executives: supply chains issues were responsible for significant disruption to production (Wagner and Bode, 2008)
- Global survey of small and medium firms: 39% report that losing their main supplier would adversely affect their operation, and 14% report that they would need to significantly downsize their business, require emergency support or shut down (Zurich Insurance Group, 2015)
- COVID-19 pandemic: 70% agreed that the pandemic pushed companies to favor higher supply chain resiliency instead of purchasing from the lowest-cost supplier (Foley & Lardner, 2020)

Slightly less anecdotal evidence

Use detailed U.S. data on firm-to-firm relationship (Factset 2003–2016)

Regress a dummy for link destruction on supplier uncertainty measures

- Instruments from Alfaro, Bloom and Lin (2019)

Details

	Dummy for last year of supply relationship		
	(1) OLS	(2) IV	(3) IV
ΔVol_{t-1} of supp.	0.023**	0.113***	0.149**
	(0.011)	(0.032)	(0.067)
1st moment of IVs	No	Yes	Yes
Type of volatility	Realized	Realized	Implied
Fixed effects	Yes	Yes	Yes
Observations	28,687	28,687	21,124
F-statistic	—	67.0	30.6

All specifications include year × customer × supplier industry (3SIC) fixed effects. Standard errors are two-way clustered at the customer and the supplier levels. *F*-statistics are Kleibergen-Paap. *,** ,*** indicate significance at the 10%, 5%, and 1% levels, respectively.

- Doubling volatility ⇒ 13 p.p. increase in probability link destroyed (IV)

Related literature

Uncertainty

- Bloom (2009); Fernandez-Villaverde et al (2011); Bloom (2014); Bloom et al (2018); and many others ...

Exogenous production networks

- Long and Plosser (1983); Dupor (1999); Horvath (2000); Acemoglu et al (2012); Carvalho and Gabaix (2013); and many others ...

Endogenous production networks

- Oberfield (2018); Acemoglu and Azar (2020); Boehm and Oberfield (2020); Taschereau-Dumouchel (2021); Acemoglu and Tahbaz-Salehi (2021).

Model

Model

Static model with two types of agents

1. Representative household: supplies labor and consumes
2. Firms: produce differentiated goods using labor and intermediate inputs
   - There are n industries/goods, indexed by $i \in \{1, \ldots, n\}$
   - Representative firm that behaves competitively

Production technique

Each firm i has access to a set of production techniques $\mathcal{A}_i$.

A technique $\alpha_i \in \mathcal{A}_i$ specifies

- The set of intermediate inputs to be used in production
- The proportion in which these inputs are combined
- A productivity shifter $A_i(\alpha_i)$ for the firm

Production technique

Each firm i has access to a set of production techniques $\mathcal{A}_i$.

A technique $\alpha_i \in \mathcal{A}_i$ specifies

- The set of intermediate inputs to be used in production
- The proportion in which these inputs are combined
- A productivity shifter $A_i(\alpha_i)$ for the firm

These techniques are Cobb-Douglas production functions

- We identify $\alpha_i = (\alpha_{i1}, \dots, \alpha_{in})$ with the input shares

$$F(\alpha_i, L_i, X_i) = e^{\varepsilon_i} \zeta(\alpha_i) A_i(\alpha_i) L_i^{1-\sum_{j=1}^n \alpha_{ij}} \prod_{j=1}^n X_{ij}^{\alpha_{ij}},$$

Production technique

Each firm i has access to a set of production techniques $\mathcal{A}_i$.

A technique $\alpha_i \in \mathcal{A}_i$ specifies

- The set of intermediate inputs to be used in production
- The proportion in which these inputs are combined
- A productivity shifter $A_i(\alpha_i)$ for the firm

These techniques are Cobb-Douglas production functions

- We identify $\alpha_i = (\alpha_{i1}, \ldots, \alpha_{in})$ with the input shares

$$F(\alpha_i, L_i, X_i) = e^{\varepsilon_i} \zeta(\alpha_i) A_i(\alpha_i) L_i^{1-\sum_{j=1}^n \alpha_{ij}} \prod_{j=1}^n X_{ij}^{\alpha_{ij}},$$

Set $\mathcal{A}_i$ allows adjustment along the intensive and extensive margins

$$\mathcal{A}_i = \left\{ \alpha \in [0,1]^n : \sum_{j=1}^n \alpha_j \le \overline{\alpha}_i < 1 \right\}.$$

Production technique

Assumption

$A_i(\alpha_i)$ is smooth and strictly log-concave.

Implications:

- There are ideal input shares α_{ij}° that maximize A_i
- Deviating from these ideal shares reduces productivity

Production technique

Assumption

$A_i(\alpha_i)$ is smooth and strictly log-concave.

Implications:

- There are ideal input shares α_{ij}° that maximize A_i
- Deviating from these ideal shares reduces productivity

Example

$$\log A_i(\alpha_i) = -\sum_{j=1}^{n} \kappa_{ij} \left(\alpha_{ij} - \alpha_{ij}^{\circ}\right)^2 - \kappa_{i0} \left(\sum_{j=1}^{n} \alpha_{ij} - \sum_{j=1}^{n} \alpha_{ij}^{\circ}\right)^2,$$

Source of uncertainty

Firms are subject to productivity shocks ε

- Shocks are jointly normal $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n) \sim \mathcal{N}(\mu, \Sigma)$
  - μ captures optimism/pessimism about productivity
  - Σ captures uncertainty and correlations

Source of uncertainty

Firms are subject to productivity shocks ε

- Shocks are jointly normal $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n) \sim \mathcal{N}(\mu, \Sigma)$
  - μ captures optimism/pessimism about productivity
  - Σ captures uncertainty and correlations

- In equilibrium, these shocks will affect input prices
  - High $\mu_i \Rightarrow$ low expected price
  - High $\Sigma_{ii} \Rightarrow$ uncertain price

- These shocks are the only source of randomness in the model

Source of uncertainty

Firms are subject to productivity shocks ε

- Shocks are jointly normal $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n) \sim \mathcal{N}(\mu, \Sigma)$
  - μ captures optimism/pessimism about productivity
  - Σ captures uncertainty and correlations
- In equilibrium, these shocks will affect input prices
  - High $\mu_i \Rightarrow$ low expected price
  - High $\Sigma_{ii} \Rightarrow$ uncertain price
- These shocks are the only source of randomness in the model
- Production techniques are chosen before ε is realized
  - Beliefs (μ, Σ) affect technique choice
  - All other decisions are taken, and markets clear, after ε is drawn

Production technique

Example

- A car manufacturer can use steel or carbon fiber for certain parts
- All else equal the manufacturer prefers carbon fiber
- If carbon fiber is expensive ($\mu_{\text{carbon fiber}}$ small) or its price is volatile ($\Sigma_{\text{carbon fiber}}$ large), the manufacturer switches to steel.

Production technique

Example

- A car manufacturer can use steel or carbon fiber for certain parts
- All else equal the manufacturer prefers carbon fiber
- If carbon fiber is expensive ($\mu_{\text{carbon fiber}}$ small) or its price is volatile ($\Sigma_{\text{carbon fiber}}$ large), the manufacturer switches to steel.

A production network

- Together the techniques $\alpha \in \mathcal{A}$ of the firms form a production network.
- Beliefs (μ, Σ) affect the structure of the network.
- Model allows for intensive and extensive adjustments in the network.

Household

The representative household makes decisions after ε is realized

- Owns the firms
- Supplies one unit of labor inelastically
- Chooses *state-contingent* consumption $(C_1, \ldots, C_n)$ to maximize

$$u\left(\left(\frac{C_1}{\beta_1}\right)^{\beta_1} \times \cdots \times \left(\frac{C_n}{\beta_n}\right)^{\beta_n}\right),$$

subject to the *state-by-state* budget constraint

$$\sum_{i=1}^{n} P_i C_i \leq 1,$$

where u is CRRA with relative risk aversion $(W = 1)$ $\rho \geq 1$.

Details

Household

Two key quantities from the household's problem

1. The stochastic discount factor of the household is

$$\Lambda = u'(Y)/\overline{P}$$

where $Y = \prod_{i=1}^{n} \left(\beta_i^{-1} C_i\right)^{\beta_i}$ is consumption and GDP and $\overline{P} = \prod_{i=1}^{n} P_i^{\beta_i}$.

Household

Two key quantities from the household's problem

1. The stochastic discount factor of the household is

$$\Lambda = u'(Y)/\overline{P}$$

where $Y = \prod_{i=1}^{n} \left(\beta_i^{-1} C_i\right)^{\beta_i}$ is consumption and GDP and $\overline{P} = \prod_{i=1}^{n} P_i^{\beta_i}$.

   - Firms use SDF to value profits in different states of the world

Household

Two key quantities from the household's problem

1. The stochastic discount factor of the household is

$$\Lambda = u'(Y)/\overline{P}$$

where $Y = \prod_{i=1}^{n} \left(\beta_i^{-1} C_i\right)^{\beta_i}$ is consumption and GDP and $\overline{P} = \prod_{i=1}^{n} P_i^{\beta_i}$.

- Firms use SDF to value profits in different states of the world

2. GDP as a function of prices

$$y = -\beta' p,$$

where $y = \log Y$, $p = (\log(P_1), \ldots, \log(P_n))$ and $\beta = (\beta_1, \ldots, \beta_n)$.

Household

Two key quantities from the household's problem

1. The stochastic discount factor of the household is

$$\Lambda = u'(Y)/\overline{P}$$

   where $Y = \prod_{i=1}^{n} \left(\beta_i^{-1} C_i\right)^{\beta_i}$ is consumption and GDP and $\overline{P} = \prod_{i=1}^{n} P_i^{\beta_i}$.

   - Firms use SDF to value profits in different states of the world

2. GDP as a function of prices

$$y = -\beta' p,$$

   where $y = \log Y$, $p = (\log(P_1), \ldots, \log(P_n))$ and $\beta = (\beta_1, \ldots, \beta_n)$.

   $\Rightarrow$ We only need prices to compute GDP

Problem of the firm: Labor and intermediate inputs

For a given technique α_i, the cost minimization problem of the firm is

$$K_i\left(\alpha_i, P\right) = \min_{L_i, X_i} \left(L_i + \sum_{j=1}^{n} P_j X_{ij} \right), \text{ subject to } F\left(\alpha_i, L_i, X_i\right) \geq 1$$

where $K_i\left(\alpha_i, P\right)$ is the unit cost of production.

Problem of the firm: Labor and intermediate inputs

For a given technique α_i, the cost minimization problem of the firm is

$$K_i(\alpha_i, P) = \min_{L_i, X_i} \left(L_i + \sum_{j=1}^{n} P_j X_{ij} \right), \text{ subject to } F(\alpha_i, L_i, X_i) \geq 1$$

where $K_i(\alpha_i, P)$ is the unit cost of production.

Note:

1. Constant returns to scale $\Rightarrow$ K_i does not depend on the size of the firm.
2. Given that each technique is Cobb-Douglas,

$$K_i(\alpha_i, P) = \frac{1}{e^{\varepsilon_i} A_i(\alpha_i)} \prod_{j=1}^{n} P_j^{\alpha_{ij}}.$$

3. Since we have perfect competition, it must be that in equilibrium.

$$P_i = K_i(\alpha_i, P) \text{ for all } i \in \{1, \ldots, n\}.$$

Problem of the firm: Labor and intermediate inputs

For a given technique α_i, the cost minimization problem of the firm is

$$K_i(\alpha_i, P) = \min_{L_i, X_i} \left(L_i + \sum_{j=1}^{n} P_j X_{ij} \right), \text{ subject to } F(\alpha_i, L_i, X_i) \geq 1$$

where $K_i(\alpha_i, P)$ is the unit cost of production.

Note:

1. Constant returns to scale $\Rightarrow$ K_i does not depend on the size of the firm.
2. Given that each technique is Cobb-Douglas,

$$K_i(\alpha_i, P) = \frac{1}{e^{\varepsilon_i} A_i(\alpha_i)} \prod_{j=1}^{n} P_j^{\alpha_{ij}}.$$

3. Since we have perfect competition, it must be that in equilibrium.

$$P_i = K_i(\alpha_i, P) \text{ for all } i \in \{1, \ldots, n\}.$$

$\Rightarrow$ For a given network α we can compute equilibrium prices

Problem of the firm: Production technique

Firm i chooses a technique $\alpha_i \in \mathcal{A}_i$ to solve

$$\alpha_i^* \in \arg\max_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i \left(P_i - K_i\left(\alpha_i, P\right)\right)\right]$$

where Q_i is the equilibrium demand for good i.

Problem of the firm: Production technique

Firm i chooses a technique $\alpha_i \in \mathcal{A}_i$ to solve

$$\alpha_i^* \in \arg\max_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i \left(P_i - K_i\left(\alpha_i, P\right)\right)\right]$$

where Q_i is the equilibrium demand for good i.

Lemma

$\lambda = \log\left(\Lambda\right)$, $k_i = \log\left(K_i\right)$, $q_i = \log\left(Q_i\right)$ are normally distributed so that

$$\alpha_i^* \in \arg\min_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[k_i\left(\alpha_i, \alpha^*\right)\right] + \frac{1}{2}\mathrm{V}\left[k_i\left(\alpha_i, \alpha^*\right)\right]$$
$$+ \mathrm{Cov}\left[k_i\left(\alpha_i, \alpha^*\right), \lambda\left(\alpha^*\right) + q_i\left(\alpha^*\right)\right].$$

Problem of the firm: Production technique

$$\alpha_i^* \in \arg\min_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[k_i\right] + \frac{1}{2}\mathrm{V}\left[k_i\right] + \mathrm{Cov}\left[k_i, \lambda + q_i\right].$$

Problem of the firm: Production technique

$$\alpha_i^* \in \arg\min_{\alpha_i \in \mathcal{A}_i} \mathrm{E}[k_i] + \frac{1}{2}\mathrm{V}[k_i] + \mathrm{Cov}[k_i, \lambda + q_i].$$

Three channels:

1. Minimize expectation $\mathrm{E}[k_i]$ of unit cost
   - Use technique with cheap inputs (low p) and high productivity (high a)

Problem of the firm: Production technique

$$\alpha_i^* \in \arg\min_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[k_i\right] + \frac{1}{2}\mathrm{V}\left[k_i\right] + \mathrm{Cov}\left[k_i, \lambda + q_i\right].$$

Three channels:

1. Minimize expectation $\mathrm{E}\left[k_i\right]$ of unit cost
   - Use technique with cheap inputs (low p) and high productivity (high a)
2. Minimize variance $\mathrm{V}\left[k_i\right]$ of unit cost

$$\mathrm{V}\left[k_i\right] = \mathsf{cte} + \underbrace{\sum_{j=1}^{n} \alpha_{ij}^2 \mathrm{V}\left[p_j\right]}_{\text{stable prices}} + \underbrace{\sum_{j \neq k} \alpha_{ij}\alpha_{ik} \mathrm{Cov}\left[p_j, p_k\right]}_{\text{uncorrelated prices}} + \underbrace{2\,\mathrm{Cov}\left[-\varepsilon_i, \sum_{j=1}^{n} \alpha_{ij} p_j\right]}_{\text{uncorrelated with own TFP}}$$

Problem of the firm: Production technique

$$\alpha_i^* \in \arg\min_{\alpha_i \in \mathcal{A}_i} \mathrm{E}[k_i] + \frac{1}{2}\mathrm{V}[k_i] + \mathrm{Cov}[k_i, \lambda + q_i].$$

Three channels:

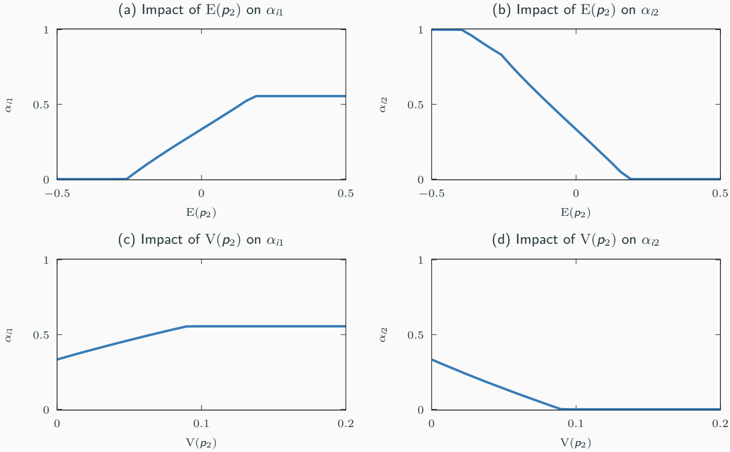
1. Minimize expectation $\mathrm{E}[k_i]$ of unit cost
   - Use technique with cheap inputs (low p) and high productivity (high a)
2. Minimize variance $\mathrm{V}[k_i]$ of unit cost

$$\mathrm{V}[k_i] = \mathsf{cte} + \underbrace{\sum_{j=1}^{n} \alpha_{ij}^2 \mathrm{V}[p_j]}_{\text{stable prices}} + \underbrace{\sum_{j \neq k} \alpha_{ij}\alpha_{ik} \mathrm{Cov}[p_j, p_k]}_{\text{uncorrelated prices}} + \underbrace{2\,\mathrm{Cov}\left[-\varepsilon_i, \sum_{j=1}^{n} \alpha_{ij} p_j\right]}_{\text{uncorrelated with own TFP}}$$

3. Importance of aggregate conditions through $\mathrm{Cov}[k_i, \lambda + q_i]$
   - Seek low unit costs when high demand (q_i) and high marginal utility (λ).
   - Because of the SDF the firm inherits the risk aversion of the household.

Back to our example

- Firm i can use steel (input 1) or carbon fiber (input 2)
- Look at impact of $\mathrm{E}\, p_2$ and $\mathrm{V}\, p_2$ on the shares α_{i1} and α_{i2}



Definition

An equilibrium is a choice of technique for every firm α^* and a stochastic tuple $(P^*, C^*, L^*, X^*, Q^*, \Lambda^*)$ such that:

1. (Unit cost pricing) For each $i \in \{1, \dots, n\}$, $P_i^* = K_i(\alpha_i^*, P^*)$.
2. (Optimal technique choice) For each $i \in \{1, \dots, n\}$, factor demand L_i^* and X_i^*, and the technology choice $\alpha_i^* \in \mathcal{A}_i$ solves the firm's problem.
3. (Consumer maximization) The consumption vector C^* solves the household's problem.
4. (Market clearing) For each $i \in \{1, \dots, n\}$,

$$Q_i^* = C_i^* + \sum_{j=1}^{n} X_{ji}^*,$$

$$Q_i^* = F_i(\alpha_i^*, L_i^*, X_i^*),$$

$$\sum_{i=1}^{n} L_i^* = 1.$$

Fixed-network economy

GDP in a fixed-network economy

Lemma

Under a given network α, the log of GDP $y = \log Y$ is given by

$$y = \beta' \mathcal{L}(\alpha)(\varepsilon + a(\alpha)),$$

where $a(\alpha) = (\log A_i(\alpha_i), \ldots, \log A_n(\alpha_n))$ and $\mathcal{L}(\alpha) = (I - \alpha)^{-1}$ is the Leontief inverse.

GDP in a fixed-network economy

Lemma

Under a given network α, the log of GDP $y = \log Y$ is given by

$$y = \beta' \mathcal{L}(\alpha)(\varepsilon + a(\alpha)),$$

where $a(\alpha) = (\log A_i(\alpha_i), \ldots, \log A_n(\alpha_n))$ and $\mathcal{L}(\alpha) = (I - \alpha)^{-1}$ is the Leontief inverse.

The importance of a firm's shock for GDP is given by its Domar weight

$$\omega_i := \beta' \mathcal{L}(\alpha) 1_i = \underbrace{\frac{P_i Q_i}{PC}}_{\substack{\text{sales share in} \\ \text{value added}}}$$

Domar weights capture the importance of a firm as a supplier

1. Role of β: Goods in high demand have larger impact on GDP
2. Role of $\mathcal{L} = I + \alpha + \alpha^2 + \ldots$: Important suppliers matter more for GDP

Impact of shocks on GDP

Moments of y: $\mathrm{E}\left[y\right] = \beta' \mathcal{L}\left(\alpha\right)\left(\mu + a\left(\alpha\right)\right)$ and $\mathrm{V}\left[y\right] = \beta' \mathcal{L}\left(\alpha\right) \Sigma \mathcal{L}\left(\alpha\right)' \beta$

Impact of shocks on GDP

Moments of y: $\mathrm{E}[y] = \beta' \mathcal{L}(\alpha)(\mu + a(\alpha))$ and $\mathrm{V}[y] = \beta' \mathcal{L}(\alpha) \Sigma \mathcal{L}(\alpha)' \beta$

Proposition (Hulten's Theorem in expectation)

For a fixed network α,

1. The impact of μ_i on expected GDP $\mathrm{E}[y]$ is given by
$$\frac{\partial \mathrm{E}[y]}{\partial \mu_i} = \omega_i.$$
2. The impact of Σ_{ij} on the variance of GDP $\mathrm{V}[y]$ is given by
$$\frac{\partial \mathrm{V}[y]}{\partial \Sigma_{ij}} = \begin{cases} \omega_i^2 & i = j, \\ 2\omega_i \omega_j & i \neq j. \end{cases}$$

Impact of shocks on GDP

Moments of y: $\mathrm{E}[y] = \beta' \mathcal{L}(\alpha)(\mu + a(\alpha))$ and $\mathrm{V}[y] = \beta' \mathcal{L}(\alpha) \Sigma \mathcal{L}(\alpha)' \beta$

Proposition (Hulten's Theorem in expectation)

For a fixed network α,

1. The impact of μ_i on expected GDP $\mathrm{E}[y]$ is given by

$$\frac{\partial \, \mathrm{E}[y]}{\partial \mu_i} = \omega_i.$$

2. The impact of Σ_{ij} on the variance of GDP $\mathrm{V}[y]$ is given by

$$\frac{\partial \, \mathrm{V}[y]}{\partial \Sigma_{ij}} = \begin{cases} \omega_i^2 & i = j, \\ 2\omega_i \omega_j & i \neq j. \end{cases}$$

Under a fixed network:

1. Sales shares ω are enough to understand GDP (Hulten's Theorem).
2. Since $\omega_i > 0$ shocks have intuitive impact.

Flexible-network economy

Equilibrium and efficiency

Proposition

There exists an efficient equilibrium. Furthermore, the equilibrium production network α^* solves

$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} \mathrm{E}\left[y(\alpha)\right] - \frac{1}{2}(\rho - 1)\,\mathrm{V}\left[y(\alpha)\right]$$

Equilibrium and efficiency

Proposition

There exists an efficient equilibrium. Furthermore, the equilibrium production network α^* solves

$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} \mathrm{E}\left[y(\alpha)\right] - \frac{1}{2}(\rho - 1)\,\mathrm{V}\left[y(\alpha)\right]$$

Implications:

1. The economy is undistorted by externalities or imperfections.
2. Complicated network formation problem $\Rightarrow$ simple optimization problem.

Economic forces at work

Impact of beliefs on the network

Domar weights are constant when the network is fixed. When it is flexible...

Proposition

The Domar weight ω_i of firm i is increasing in μ_i and decreasing in Σ_{ii}.

Impact of beliefs on the network

Domar weights are constant when the network is fixed. When it is flexible...

Proposition

The Domar weight ω_i of firm i is increasing in μ_i and decreasing in Σ_{ii}.

This result is intuitive

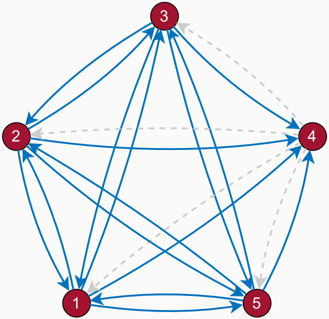
1. Equilibrium's perspective: Firms rely more on high-μ_i and low-Σ_{ii} firms as suppliers.
2. Planner's perspective: The importance of high-μ_i and low-Σ_{ii} firms for welfare is magnified if they are important suppliers.

▸ Impact on α

Example: Impact of beliefs on the network

Simple economy

- Five firms with uncorrelated shocks.
- Firms are identical except that $i = 4$ is less productive (μ_4 low).



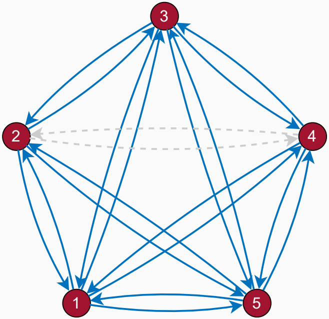
Baseline

Details

Example: Impact of beliefs on the network

Simple economy

- Five firms with uncorrelated shocks.
- Firms are identical except that $i = 4$ is less productive (μ_4 low).



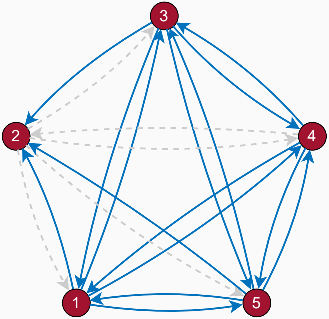
Small increase in Σ_{22}

Details

Example: Impact of beliefs on the network

Simple economy

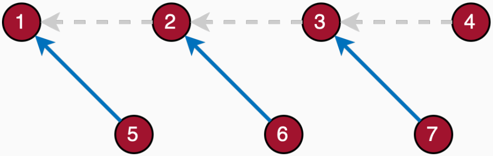
- Five firms with uncorrelated shocks.
- Firms are identical except that $i = 4$ is less productive (μ_4 low).



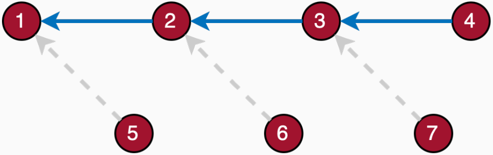
Large increase in Σ_{22}

Details

Example: Cascading effect of uncertainty



(a) High uncertainty about ε_4



(b) Low uncertainty about ε_4

Effect of uncertainty on GDP

Proposition

Uncertainty lowers the expected value of GDP in equilibrium, such that $\mathrm{E}[y]$ is largest when $\Sigma = 0_{n \times n}$.

Effect of uncertainty on GDP

Proposition

Uncertainty lowers the expected value of GDP in equilibrium, such that $\mathrm{E}[y]$ is largest when $\Sigma = 0_{n\times n}$.

This result is intuitive

1. Equilibrium's perspective: When there is no uncertainty firms purchase from the lowest expected price (highest expected utility) supplier. This maximizes expected GDP.
2. Planner's perspective: When $\Sigma = 0_{n\times n}$ the variance of GDP is 0. Only objective is to maximize $\mathrm{E}[y]$.

Effect of shocks on welfare

Proposition

1. The impact of an increase in μ_i on expected welfare is given by

$$\frac{d\mathcal{W}}{d\mu_i} = \frac{\partial \, \mathrm{E}\left[y\right]}{\partial \mu_i} = \omega_i.$$

2. The impact of an increase in Σ_{ij} on expected welfare is given by

$$\frac{d\mathcal{W}}{d\Sigma_{ij}} = \begin{cases} -\frac{1}{2}\left(\rho - 1\right)\left(\frac{\partial \, \mathrm{E}[y]}{\partial \mu_i}\right)^2 = -\frac{1}{2}\left(\rho - 1\right)\omega_i^2 & i = j, \\ -\left(\rho - 1\right)\frac{\partial \, \mathrm{E}[y]}{\partial \mu_i}\frac{\partial \, \mathrm{E}[y]}{\partial \mu_j} = -\left(\rho - 1\right)\omega_i \omega_j & i \neq j. \end{cases}$$

Effect of shocks on welfare

Proposition

1. The impact of an increase in μ_i on expected welfare is given by

$$\frac{d\mathcal{W}}{d\mu_i} = \frac{\partial\,\mathrm{E}\,[y]}{\partial\mu_i} = \omega_i.$$

2. The impact of an increase in Σ_{ij} on expected welfare is given by

$$\frac{d\mathcal{W}}{d\Sigma_{ij}} = \begin{cases} -\frac{1}{2}\left(\rho-1\right)\left(\frac{\partial\,\mathrm{E}[y]}{\partial\mu_i}\right)^2 = -\frac{1}{2}\left(\rho-1\right)\omega_i^2 & i=j, \\ -\left(\rho-1\right)\frac{\partial\,\mathrm{E}[y]}{\partial\mu_i}\frac{\partial\,\mathrm{E}[y]}{\partial\mu_j} = -\left(\rho-1\right)\omega_i\omega_j & i\neq j. \end{cases}$$

The impact of shocks on welfare is intuitive

1. Higher productivity leads to higher welfare.
2. Higher correlation or uncertainty leads to lower welfare.

Effect of shocks on GDP

Impact of shocks on

- Welfare: intuitive
- GDP when the network is fixed: intuitive
- GDP when the network is flexible: ???

Effect of shocks on GDP

Impact of shocks on

- Welfare: intuitive
- GDP when the network is fixed: intuitive
- GDP when the network is flexible: ???

Decompose a shock to, say, μ_i as

$$\frac{d\,\mathrm{E}[y]}{d\mu_i} = \underbrace{\frac{\partial\,\mathrm{E}[y]}{\partial\mu_i}}_{\substack{\text{direct impact with} \\ \text{fixed network}}} + \underbrace{\frac{\partial\,\mathrm{E}[y]}{\partial\alpha}\frac{d\alpha}{d\mu_i}}_{\text{network adjustment}}$$

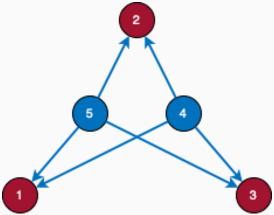
Two effects:

1. Direct impact keeping the network fixed = Domar weight
2. Indirect impact that take into account the network adjustment = ???

Example: Surprising impact of a shock

Simple economy:

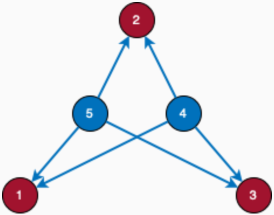
- 5 firms with uncorrelated shocks
- Firms are identical except that
  - Firm 4 is risky (high Σ_{44})
  - Firm 5 is safe (low Σ_{55}) but unproductive (low μ_5)
  - $\beta_4 = \beta_5$ are very small



Example: Surprising impact of a shock

Simple economy:

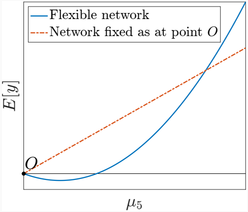
- 5 firms with uncorrelated shocks
- Firms are identical except that
  - Firm 4 is risky (high Σ_{44})
  - Firm 5 is safe (low Σ_{55}) but unproductive (low μ_5)
  - $\beta_4 = \beta_5$ are very small



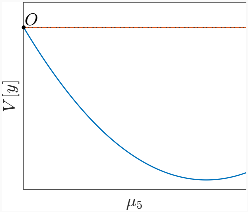
We are going to consider two shocks

1. An increase in μ_5
2. An increase in Σ_{44}

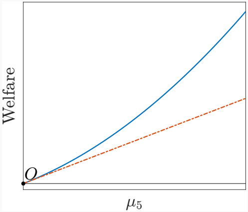
Example: Surprising impact of a shock



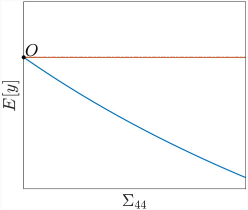
(a) $\mathrm{E}[y]$ as a function of μ_5



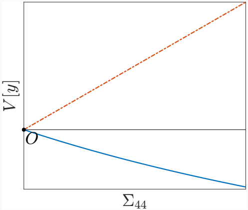
(b) $\mathrm{V}[y]$ as a function of μ_5



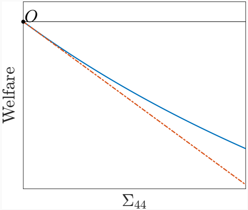
(c) Welfare as a function of μ_5



(d) $\mathrm{E}[y]$ as a function of Σ_{44}



(e) $\mathrm{V}[y]$ as a function of Σ_{44}



(f) Welfare as a function of Σ_{44}

Quantitative exploration

Data

Data about the United States from vom Lehn and Winberry (2021)

- Input-output tables, sectoral total factor productivity, consumption shares
- 37 sectors, from 1947 to 2018:

Mining	Utilities	Construction
Wood products	Nonmetallic minerals	Primary metals
Fabricated metals	Machinery	Computer and electronic manuf.
Electrical equipment manufacturing	Motor vehicles manufacturing	Other transportation equipment
Furniture and related manufacturing	Misc. manufacturing	Food and beverage manufacturing
Textile manufacturing	Apparel manufacturing	Paper manufacturing
Printing products manufacturing	Petroleum and coal manufacturing	Chemical manufacturing
Plastics manufacturing	Wholesale trade	Retail trade
Transportation and warehousing	Information	Finance and insurance
Real estate and rental services	Professional and technical services	Mgmt. of companies and enterprises
Admin. and waste mgmt. services	Educational services	Health care and social assistance
Arts and entertainment services	Accommodation	Food services
Other services		

Data

Data about the United States from vom Lehn and Winberry (2021)

- Input-output tables, sectoral total factor productivity, consumption shares
- 37 sectors, from 1947 to 2018:

Mining	Utilities	Construction
Wood products	Nonmetallic minerals	Primary metals
Fabricated metals	Machinery	Computer and electronic manuf.
Electrical equipment manufacturing	Motor vehicles manufacturing	Other transportation equipment
Furniture and related manufacturing	Misc. manufacturing	Food and beverage manufacturing
Textile manufacturing	Apparel manufacturing	Paper manufacturing
Printing products manufacturing	Petroleum and coal manufacturing	Chemical manufacturing
Plastics manufacturing	Wholesale trade	Retail trade
Transportation and warehousing	Information	Finance and insurance
Real estate and rental services	Professional and technical services	Mgmt. of companies and enterprises
Admin. and waste mgmt. services	Educational services	Health care and social assistance
Arts and entertainment services	Accommodation	Food services
Other services		

- Average share of 1.4% with standard deviation of 0.5% over time

Calibration

Preferences

- Consumption shares β are taken directly from the data
- Relative risk aversion ρ is **estimated**

Production technique productivity shifters

- Function A_i as described earlier
- Set ideal shares α_{ij}° to their data average
- Costs κ_{ij} of deviating from α_{ij}° are **estimated**

Process for exogenous shocks ε_t

- Random walk with drift $\varepsilon_t = \gamma + \varepsilon_{t-1} + u_t^{\varepsilon}$, with $u_t^{\varepsilon} \sim$ iid $\mathcal{N}(0, \Sigma_t)$.
- Drift vec. γ and cov. mat. Σ_t are **backed out from the data given** (ρ, κ)**.**

Loss function: Target the full set of shares α_{ijt} and the variance of GDP.

Calibrated economy

Estimated risk aversion: $\rho = 5.8$

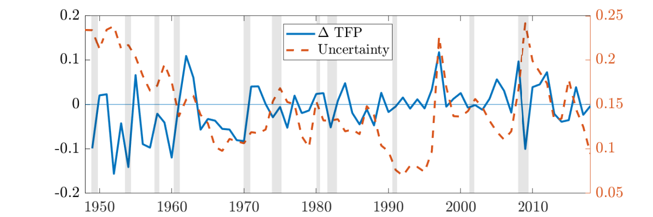
- High by macro standards, but in most models ρ is pinned down by IES
- Studies that separate IES from RA often find ρ between 5 and 10

Calibrated economy

Estimated risk aversion: $\rho = 5.8$

- High by macro standards, but in most models ρ is pinned down by IES
- Studies that separate IES from RA often find ρ between 5 and 10

Estimated covariance process Σ_t



▸ κ's

Calibrated economy: Domar weights

The calibrated Domar weights fit the data well in terms of

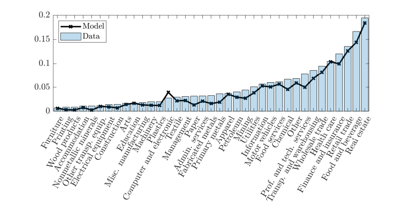


Figure 1: Average Domar weights

The calibrated Domar weights fit the data well in terms of

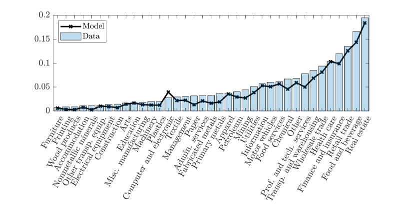


Figure 1: Average Domar weights

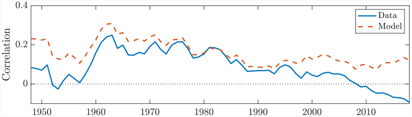


Figure 2: Correlation between Domar weight ω_j and μ_j

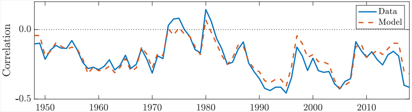


Figure 3: Correlation between Domar weight ω_j and Σ_{jj}

Isolating the mechanism

Two useful counterfactuals

1. Fixed-network economy
   - to capture the full effect of network adjustments
2. Risk-neutral economy ($\rho = 1$)
   - to capture the impact of uncertainty

Isolating the mechanism

Two useful counterfactuals

1. Fixed-network economy
   - to capture the full effect of network adjustments
2. Risk-neutral economy ($\rho = 1$)
   - to capture the impact of uncertainty

	Baseline model compared to...	
	Fixed network	Risk neutral
Expected GDP $\mathrm{E}[y(\alpha)]$	$+2.55\%$	-0.02%
Std. dev. of GDP $\sqrt{\mathrm{V}[y(\alpha)]}$	$+0.07\%$	-0.08%
Welfare $\mathcal{W}$	$+2.52\%$	$+0.02\%$

Isolating the mechanism

Two useful counterfactuals

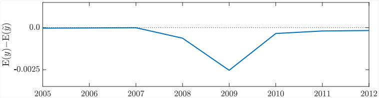
1. Fixed-network economy
   - to capture the full effect of network adjustments
2. Risk-neutral economy ($\rho = 1$)
   - to capture the impact of uncertainty

	Baseline model compared to...	
	Fixed network	Risk neutral
Expected GDP $\mathrm{E}[y(\alpha)]$	$+2.55\%$	-0.02%
Std. dev. of GDP $\sqrt{\mathrm{V}[y(\alpha)]}$	$+0.07\%$	-0.08%
Welfare $\mathcal{W}$	$+2.52\%$	$+0.02\%$

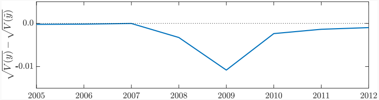
Recall: only impact of uncertainty on expected GDP is through the network

The Great Recession

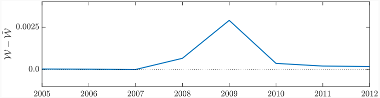
Great recession in the calibrated model vs risk-neutral alternative



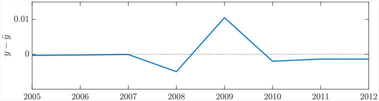
(a) Difference in expected GDP



(b) Difference in expected std. dev. of GDP



(c) Difference in expected welfare



(d) Difference in realized GDP

- During periods of high volatility, uncertainty matters.

Conclusion

Conclusion

Main contributions

- We construct a model in which beliefs, and in particular uncertainty, affect the production network.
- During periods of high uncertainty firms purchase from safer but less productive suppliers which leads to a decline in GDP.
- The calibrated model suggests that this mechanism was important during the Great Recession.

Ongoing work

- Include the COVID-19 pandemic in the dataset

Future research

- Use firm-level data to estimate the model
- Use the model to evaluate the impact of uncertainty on global supply chains

Details of regressions

Volatility measures

- Supplier ΔVol_{t-1} is the 1-year lagged change in supplier-level volatility.
- Realized volatility is the 12-month standard deviation of daily stock returns from CRSP.
- Implied volatility is the 12-month average of daily (365-day horizon) implied volatility of at-the-money-forward call options from OptionMetrics.

Instrument

- As in Alfaro et al. 2019 "we address endogeneity concerns on firm-level volatility by instrumenting with industry-level (3SIC) non-directional exposure to 10 aggregate sources of uncertainty shocks. These include the lagged exposure to annual changes in expected volatility of energy, currencies, and 10-year treasuries (as proxied by at-the-money forward-looking implied volatilities of oil, 7 widely traded currencies, and TYVIX) and economic policy uncertainty from Baker et al 2016.. [...] To tease out the impact of 2nd moment uncertainty shocks from 1st moment aggregate shocks we also include as controls the lagged directional industry 3SIC exposure to changes in the price of each of the 10 aggregate instruments (i.e., 1st moment return shocks). These are labeled 1st moment 1st moment of IVs."

Back

Risk aversion and ρ

Given the log-normal nature of uncertainty $\rho \lesseqgtr 1$ determines whether the agent is risk-averse or not. To see this, note that when $\log C$ normally distributed, maximizing

$$\mathrm{E}\left[C^{1-\rho}\right]$$

amounts to maximizing

$$\mathrm{E}\left[\log C\right] - \frac{1}{2}\left(\rho - 1\right)\mathrm{V}\left[\log C\right].$$

◂ Back

Impact of μ and Σ for α

Assumption (Weak complementarity)

For all $i \in \mathcal{N}$, the function a_i is such that $\frac{\partial^2 a_i(\alpha_i)}{\partial \alpha_{ij} \partial \alpha_{ik}} \geq 0$ for all $j \neq k$.

Lemma

Let $\alpha^* \in \text{int}\,(\mathcal{A})$ be the equilibrium network and suppose that the assumption holds. There exists a $\overline{\Sigma} > 0$ such that if $|\Sigma_{ij}| < \overline{\Sigma}$ for all i, j, there is a neighborhood around α^* in which

1. an increase in μ_j leads to an increase in the shares α^*_{kl} for all k, l;
2. an increase in Σ_{jj} leads to a decline in the shares α^*_{kl} for all k, l;
3. an increase in Σ_{ij} leads to a decline in the shares α^*_{kl} for all k, l.

Back

Pentagon example: parameter value

Details of the simulation:

1. a function: κ equal to 1, except $\kappa_{ii} = \infty$, α° are 1/10 except $\alpha^{\circ}_{ii} = 0$.
2. $\rho = 5$, $\beta = 0.2$. $\mu = 0.1$ except for $\mu_4 = 0.0571$. $\Sigma = 0.3 \times I_{n \times n}$ in Panel (a).
3. Panel (b): same as Panel (a) except $\mathrm{Corr}\left(\varepsilon_2, \varepsilon_4\right) = 1$.
4. Panel (c): same in Panel (a) except $\Sigma_{22} = 1$.

Back

Calibrated κ

We assume that $\kappa = \kappa^i \times \kappa^j$ where κ^i is an $n \times 1$ column vector and κ^j is an $1 \times (n+1)$ row vector.

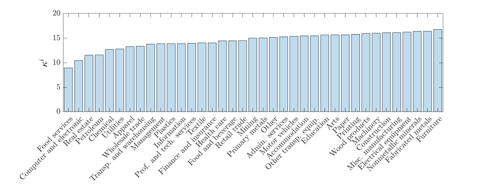


Figure 4: Vector of costs κ^i

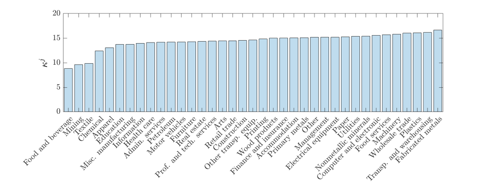


Figure 5: Vector of costs κ^j

Back