Cheap Thrills: the Price of Leisure and the Global Decline in Work Hours*

Alexandr Kopytov

Nikolai Roussanov

Mathieu Taschereau-Dumouchel

University of Hong Kong

Wharton and NBER

Cornell University

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Abstract

Recreation prices and hours worked have both fallen over the last century. We construct a macroeconomic model with general preferences that allows for trending recreation prices, wages, and work hours along a balanced-growth path. Estimating the model using aggregate data from OECD countries, we find that the fall in recreation prices can explain a large fraction of the decline in hours. We also use our model to show that the diverging prices of the recreation bundles consumed by different demographic groups can account for much of the increase in leisure inequality observed in the United States over the last decades.

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1 Introduction

Hours worked have declined substantially over the last hundred years. Nowadays, American workers spend on average two thousand hours a year at work, while their 1900 counterparts worked 50% more. Over the same period, technological progress has increased labor productivity and wages, and so the decline in hours is often attributed to an income effect through which richer households choose to enjoy more leisure time. Indeed, Keynes (1930) prophesized that "the economic problem may be solved [...] within a hundred years" and that therefore there would be no need to work long hours to satisfy one's desire for consumption. Another important change occurred over the same period, however. New technologies such as television and the internet have brought a virtually unlimited trove of cheap entertainment that occupies a growing portion of households' leisure time (Aguiar et al., 2021). The impact of these technologies is clearly visible in the price data. For instance, the Bureau of Labor Statistics (BLS) documents that the (real and quality-adjusted) price of a television set has fallen about 1000-fold since the 1950s, while computers are about fifty times cheaper than they were in the mid-1990s. Similarly, the inflation-adjusted price of admission to a (silent, black and white) movie in 1919 is roughly equal to the current monthly cost of a streaming service providing essentially unlimited access to movies and television shows. Overall, the aggregate price index tracking recreation goods and services in the U.S. has fallen by more than half in real terms since 1900.

It is natural to think that cheaper recreation might have contributed to the decline in work hours. Becker (1965) argued that complementarity between certain consumption goods and the time required to consume them is crucial for understanding how households allocate their time, in particular between market work and leisure activities. Accordingly, if recreation goods and services are complementary to leisure, a decline in their price would push households to work less. We incorporate this insight into a macroeconomic model by extending the framework of Boppart and Krusell (2020), which allows for hours worked to decline along a balanced-growth path. We estimate the model using data from 42 OECD countries and find that the fall in recreation prices is important to explain the cross-country variation in the fall of hours worked. We also show that recreation prices can help to account for the growing leisure inequality across demographic groups within the U.S., where we take advantage of more detailed disaggregated data to discipline the model.

We begin our analysis by reviewing key stylized facts. First, we show that hours per worker have been declining in the U.S. at a steady pace since 1900, with the exception of large movements around the Great Depression and the Second World War.¹ Hours per capita have also fallen over that period, although the decline is concentrated in the first part of the twentieth century. After

¹Similar evidence is presented in a number of studies, including Owen (1971), Lebergott (1993), Fogel (2000), Greenwood and Vandenbroucke (2005), and Boppart and Krusell (2020).

1950, the large increase in female labor force participation has kept that measure mostly flat. In contrast, the decline in male hours per capita has continued over that period. The American Time Use Survey shows that self-reported leisure time has also been increasing, for both men and women, since the 1960s (Aguiar and Hurst, 2007b; Robinson and Godbey, 2010).² This last piece of evidence shows that the increase in women's market hours is more than compensated by the decline in their non-market work hours, so that their leisure time has been on the rise. The trends observed in the U.S. are also visible in other developed countries. We look at the evolution of work hours in 42 OECD countries and find that hours per worker have declined virtually everywhere, while hours per capita have fallen in 33 countries.

This decline in work hours in the United States over the last 120 years was accompanied by a large, well-documented, increase in wages, as well as by a large decline in recreation prices. We extend early work by Owen (1970) using recent data from the BLS to show that the real price of recreation goods and services has been decreasing at a steady pace of about -0.75% per year since 1900. This trend is also clearly visible in our multi-country sample. Indeed, real recreation prices have fallen in *all* the countries that we consider, with an average annual decline of -1.48%. We conclude from these data that the simultaneous decline in work hours and real recreation prices is a widespread phenomenon that affected a broad array of developed countries.

In order to account for these facts, we construct a macroeconomic model in which both recreation prices and wages can affect labor supply decisions. At the heart of our analysis is a household that values recreation time and recreation goods and services, as well as standard (i.e., non-recreation) consumption goods. To be consistent with well-known long-run trends, we build on the standard macroeconomic framework of balanced growth and assume that all prices and quantities in the economy grow at constant, but potentially different, rates. Importantly, and in contrast to the standard balanced-growth assumptions, we do not assume that hours worked remain constant over time, but instead allow them to decline at a constant rate. For our analysis to be as general as possible, we follow the approach of Boppart and Krusell (2020) and keep the household's preferences mostly unrestricted, only requiring that they be consistent with balanced growth. We characterize the general form that a utility function must take in this setup, and show that it nests the standard balanced-growth preferences with constant hours of King et al. (1988), as well as the more general preferences of Boppart and Krusell (2020), which allow for hours to decline over time through the income effect of rising wages. In addition, we show that in the class of economies that we study the growth rates of hours, recreation consumption, and non-recreation consumption are log-linearly related to those of the wage rate and the real price of recreation items.

We use this theoretical framework to quantify the importance of falling recreation prices and

²Ramey and Francis (2009) also provide evidence that leisure time per capita has increased between 1900 and 2005. Their estimates are somewhat smaller than those of Aguiar and Hurst (2007b), mostly because of a different classification of child-related activities.

rising wages in explaining the decline in hours worked. Our model has several key advantages when it comes to making contact with the data. First, since we keep the household's preferences quite general, our empirical strategy does not hinge on a specific utility function, but instead remains valid under several functional forms that have been proposed in the literature. Second, there is no need to fully specify the production sector of the economy. We only need wages and recreation prices to grow at constant rates for our analysis to be well-grounded. Third, the system of equations derived from the model can be estimated using standard techniques and allows for a straightforward identification of the key structural parameters of the economy. Finally, the model provides a set of cross-equation restrictions that impose more structure on the estimation compared to standard reduced-form techniques. In particular, these restrictions allow us to use consumption data to discipline the estimation of the effect of recreation prices on work hours.

We estimate the structural relations implied by the model using our multi-country data. We find that a decline in recreation prices is associated with a large and statistically significant increase in leisure time. Specifically, a one percentage-point decline in the growth rate of real recreation prices is associated with about a 0.25 p.p. decline in the growth rate of hours per capita. Rising wages are also strongly associated with a decline in hours worked, such that the income effect dominates the substitution effect in the estimated model. These findings are robust to various changes in specifications and to the inclusion of additional controls in the estimation. Finally, we perform back-of-the-envelope calculations and find that the fall in the price of recreation goods and services, on its own, can explain a large fraction of the decline in hours worked observed in the cross-section of countries. Our favorite specification suggests that the recreation channel has been about a third as important as the income effect as a driver of the decline in work hours.

While our main focus is on aggregate variables, we also use our model to better understand changes in hours worked across U.S. households. The motivation for this inquiry comes from the large increase in leisure inequality that has been observed in the data since 1985 (Aguiar and Hurst, 2009). Indeed, leisure time has grown the most among groups that have experienced the slowest growth in wages (e.g., the less educated). This pattern is hard to reconcile with the dominating income effect of wages that we found in our cross-country analysis. Over the same period, however, the price of recreation goods and services consumed by less-educated households has declined significantly, which might have driven them to consume more leisure.³ In contrast, more-educated households consume a disproportionate amount of items that have become more expensive. As a result, their leisure time has been roughly stable during the last decades.

One advantage of using these disaggregated data is that they allow us to construct two instrumental variables to tackle potential endogeneity issues. In the spirit of Bartik (1991), we construct a first instrument, for wages, that uses location-specific industry employment shares to tease out

³Consistent with this interpretation, Aguiar et al. (2021) show that the increased leisure time of young men is strongly associated with their consumption of online streaming and video games.

fluctuations in local wages that are driven by national movements. We also construct a second instrument, for recreation prices, using variation in the type of recreation goods and services that are ex-ante consumed by different demographic groups. Using data from the Consumer Expenditure Survey, we document that, for instance, individuals without a high-school diploma consume a disproportionate amount of "Audio and video" items, while those with more than a college education consume relatively more of "Other services", which includes admissions, fees for lessons, club memberships, etc. Importantly, the national price of these items has diverged markedly in our sample, creating substantial variation in the price of the recreation bundles consumed by different demographics. We use a shift-share approach to construct an instrument that takes advantage of this variation.

Using these two instruments, we estimate the structural equations implied by our model on the household-level data. We find a strong positive effect of recreation prices on hours worked, suggesting not only that the relationship visible in the cross-country data survives at the individual level but might also have a causal interpretation. We also find a strong negative impact of wages on hours worked, so that the income effect dominates the substitution effect in this disaggregated sample as well. Overall, we find that the drop in recreation prices was a key driving force behind the increase in leisure inequality predicted by the estimated model, with wages actually pushing for less leisure inequality.

Literature

Our empirical results update and extend an early analysis by Owen (1971) who finds strong evidence of complementarity between leisure time and recreational goods and services in the United States (see also Gonzalez-Chapela, 2007). Owen attributes one quarter of the decline in hours worked over the 1900-1961 period to the declining price of recreation items, and the remaining three quarters to the income effect of rising wages. An important difference with our approach is that we build a general balanced-growth model that allows us to impose cross-equation restrictions on the joint evolution of hours and consumption in our empirical analysis. We also investigate the impact of recreation prices at the household level using instruments to handle endogeneity issues.

Our findings are also consistent with Aguiar et al. (2021) who show that the increased leisure time among young men is strongly associated with the consumption of leisure goods and services made available due to the advent of cheap new media technologies, such as online streaming and video games. Bick et al. (2018) find that the relationship between hours and labor productivity is strongly negative across developing countries, but that it is flat or even slightly positive across individuals in developed countries. Our findings that both the income effect and heterogeneous changes in the price of households' recreation bundles are at work can help make sense of that data.

Vandenbroucke (2009) evaluates the impact of recreation prices in a static model with worker heterogeneity. In a calibration exercise over the 1900-1950 period, he finds that 82% of the decline in hours worked can be attributed to the income effect and only 7% to the declining price of recreation goods. Kopecky (2011) focuses on the reduced labor market participation of older men and argues that retirement has become more attractive due to the decline in the price of leisure. In a recent paper, Fenton and Koenig (2018) argue that the introduction of televisions in the United States in the 1940s and 1950s had a substantial negative effect on labor supply decisions, especially for older men.

Our main theoretical result generalizes recent work by Boppart and Krusell (2020) who characterize the class of preferences that are consistent with balanced growth and declining work hours. We extend their preferences to include recreation goods that are complement with leisure time. As a result, we can jointly investigate the importance of wages and recreation prices as drivers of the decline in work hours.⁴

Greenwood and Vandenbroucke (2005) consider a static model of the impact of technological changes in the long-run evolution of work hours through three channels: rising marginal product of labor (the income effect), the introduction of new time-saving goods (the home production channel) and the introduction of time-using goods (the leisure channel). The second effect, in particular, is important for accounting for the entry of women into the labor force, which makes the long-run decline of work hours per person (rather than hours per worker) less pronounced.

Ngai and Pissarides (2008) construct a model in which leisure time rises on a balanced-growth path due to a complementarity between leisure and "capital goods" (such as entertainment durables), as well as marketization of home production. Building on this, Boppart and Ngai (2017) provide a model where both leisure time and leisure inequality increase along a balanced-growth path due to the growing dispersion in labor market productivity. Boerma and Karabarbounis (2020) argue that the rising productivity of leisure time combined with cross-sectional heterogeneity in preferences (or "non-market productivity") is responsible for these trends.

Two recent papers have studied the impact of free entertainment on labor supply decisions. Greenwood et al. (2020) construct a model in which digital advertisement finances the provision of free leisure goods. In the model of Rachel (2021) hours fall along the balanced-growth path as the quality of "free" leisure improves due to technological innovation driven by producers' demand for consumer attention.

Our work departs from the existing literature in several ways. On the theoretical side, we keep the preferences of the household as general as possible. On the empirical side, we investigate the impact of recreation prices in both aggregate data in a broad cross-section of countries and in disaggregated data in the U.S. We also use instruments to tease out the causal impact of recreation

⁴Boppart (2014) builds a model in which changes in relative prices can differentially affect consumption expenditure shares of heterogeneous households along a balanced-growth path of the aggregate economy.

prices and wages.

The next section provides an overview of the data. We then introduce the model and provide our main theoretical result in Section 3. Section 4 estimates the structural relationships derived from the model. The implications of the model for rising leisure inequality are discussed in Section 5. The last section concludes.

2 Trends in working hours, recreation prices and wages

We begin by presenting aggregate data for the United States and a cross-section of countries. We document three important trends that hold in almost all the countries in our sample over the last decades: 1) hours worked have fallen, 2) the price of recreation goods and services has declined substantially in real terms, and 3) real wages have been increasing. These trends will serve as motivation for the model that we describe in the next section.⁵

2.1 Evidence from the United States

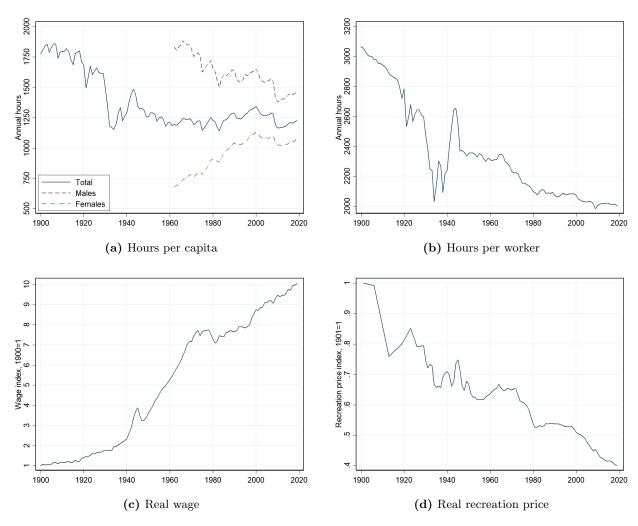
Figure 1 shows the evolution of work hours, wages and recreation prices in the United States. The solid blue line in panel (a) shows how hours per capita have evolved between 1900 and 2019. Over the whole period, hours have fallen significantly from about 1750 annual hours per adult person in 1900 to about 1200 hours per person today. While the figure shows an overall reduction in hours, all of the decline actually took place before 1960. But these aggregate statistics are somewhat misleading as they conceal substantial heterogeneity between men and women, whose hours are shown in red and green in panel (a). As we can see, the second half of the twentieth century saw a large increase in women's hours, presumably due to a rise in labor force participation, which clearly contributed to the stagnation of the aggregate hours per capita series. At the same time, male hours per capita have kept declining. In the more recent period, between 2000 and 2019, hours have declined for both men and women.

The evidence in panel (a) might suggest that women are working much more in 2019 than in 1960, but the figure only reports hours worked in the marketplace. Total work hours, which also include home production, have been declining since at least the 1960s for both men and women. To show this, we follow Aguiar and Hurst (2007b) and Aguiar et al. (2021) and use the American

⁵To avoid burdening the text, we keep the precise data sources and the steps taken to construct the datasets in Appendix A.

⁶Here, we define adults as individuals above 20 years old. The trends are similar if we divide total hours by the population older than 15 or by the working age population (25-64 years old) instead.

⁷This increase in female labor force participation is well documented and was likely driven by several factors. Many women were probably kept away from market work because of discriminatory social norms. As these norms evolved, the stigma of women in the labor force faded and female participation increased. In addition, technological improvements made it easier to perform nonmarket work—mostly done by women—leaving more time for market work (Greenwood et al., 2005). Goldin and Katz (2002) also document that the adoption of contraceptives might have affected women's decisions to pursue higher education.



Panel (a): Annual hours worked over population (20 years and older). Source: Kendrick et al., 1961 (hours, 1990-1947); Kendrick et al., 1973 (hours, 1948-1961); U.S. Census (population, 1900-1961); ASEC (total, male and female hours per capita, 1962-2019). Panel (b): Annual hours worked over number of employed. Source: Bureau of the Census, 1975 (1900-1947); BLS (1947-2019; retrieved from FRED). Panel (c): Real labor productivity. Source: Kendrick et al., 1961 (real gross national product divided by hours, 1900-1929); BEA and BLS (real compensation of employees, divided by hours and CPI, 1929-2019; retrieved from FRED). Panel (d): Real price of recreation goods and services. Source: Owen, 1970 (real recreation price, 1900-1934); Bureau of the Census, 1975 (real price of category 'Reading and recreation', 1935-1966); BLS (real price of category 'Retreation', 1993-2019). Series coming from different sources are continuously pasted.

Figure 1: Hours, wages and recreation price in the U.S.

Time Use Survey to construct measures of market work, total work (including market work, home production and non-recreational childcare), and leisure for men and women between 16 and 64 years old (excluding full-time students). These series are presented in Figure 2. Between 1965 and 2017, total annual work hours have declined by 416 (8.0 hours per week) for women and by 502 (9.7 hours per week) for men. According to that metric, women work substantially less now than fifty years ago.⁸

The decline in hours worked is also clearly visible when looking at hours per worker, instead of per capita. This series is presented in panel (b) of Figure 1. Except for large fluctuations around the Great Depression and the Second World War, that measure has been on a steady decline, from more than 3000 annual hours per worker in 1900 to about 2000 today.⁹

What are the drivers behind this long-run decline in hours? Clearly, people are now richer than in 1900 and it might be that at higher income levels they prefer enjoying leisure to working. Indeed, panel (c) of Figure 1 shows that real hourly wages have gone up ten-fold since 1900. Theoretically, this large increase in wages could lead to an increase in labor supply, if the standard substitution effect dominates, or to its decline, if the income effect dominates instead.

Like the benefit of working, the cost of enjoying leisure has also undergone a massive change over the last century. To show this, we plot in panel (d) of Figure 1 the *real* price of recreation goods and services since 1901.¹⁰ Items in that category generally follow the BLS classification and include goods and services that are associated with leisure time, such as video and audio equipment, pet products and services, sporting goods, photography, toys, games, recreational reading materials, and recreation services (such as admission to movies, theaters, concerts, sporting events, etc.).¹¹ As we can see, these prices have experienced a steep decline, falling by about 60% in real terms since 1901. If these goods and services are complementary to leisure time, a decline in their price would incentivize households to consume more leisure. As a result, they could play an important role in the decline in hours worked.

2.2 Evidence from a cross-section of countries

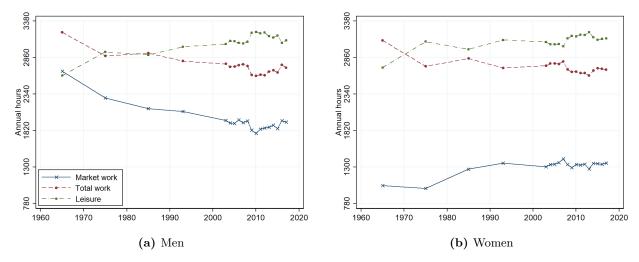
The trends observed in the U.S. economy are also visible in international data. To show this, we gather data on hours worked, real recreation prices and wages from a variety of sources, such as the OECD, Eurostat and national statistical agencies. The OECD and Eurostat track the price

⁸Classifying all time spent with children, such as playing games and going to a zoo, as childcare "work" rather than "leisure" moderates this trend somewhat. See discussion in Ramey and Francis (2009) and Aguiar and Hurst (2007a) for more details.

⁹Using decennial data from the Census, McGrattan et al. (2004) also find that hours per worker have declined and hours per capita have increased in the U.S. since 1950. Kendrick et al. (1961) and Whaples (1991) document a decline in work hours since 1830 (see also Figure 1 in Vandenbroucke, 2009). Kendrick et al. (1961) show that this decline has happened in all industries.

¹⁰We use the price of all consumption goods and services as deflator for all nominal variables.

¹¹The BLS price data is not available before 1967. We rely on Owen (1971) and data from the Bureau of the Census (1975) for the earlier years. See Appendix A for more details.



Annual hours spent on market work, total work and leisure. Market work includes any work-related activities, travel related to work, and job search activities. Total work includes market work, home production, shopping, and non-recreational childcare. Leisure is any time not allocated to market and nonmarket work, net of time required for fulfilling biological necessities (8 hours per day). Sample includes people between 16 and 64 years old who are not full-time students. Source: ATUS, Aguiar and Hurst (2007b) and Aguiar et al. (2021).

Figure 2: Market work, total work, and leisure in the U.S.

of "Recreation and culture" items which we use as our main recreation price index. This category includes items such as audio-visual, photographic and information processing equipment, reading materials, package holidays, various other recreation goods (such as musical instruments, toys, sporting goods, pet and garden products, etc.), and recreation and cultural services. For several countries, we are able to augment these data using price series from national statistical agencies. We restrict the sample to countries with at least 15 years of data for recreation prices. Our final sample covers 42 countries and 1,215 country-year observations.¹²

Figure 3 shows the evolution of hours worked (both per capita and per worker), recreation prices and wages for a selected group of countries in our sample.¹³ The black curves represent the global movements in these quantities, for all countries in our sample, estimated as year fixed effects from regressing each variable on a set of country and year fixed effects. While there is some heterogeneity across countries, the figure shows a clear overall decline in both hours and recreation prices, and an increase in real wages. Across the full sample, we find that per capita hours have been declining at an average rate of 0.44% per year and hours per worker have been declining at a rate of 0.45% per year.¹⁴ At the same time, real wages have been increasing by 2.00% per year,

¹²Data on hours worked comes from the Total Economy Database of the Conference Board. We compute hours per capita by dividing total hours worked by population between 20 and 74 years old, and similarly for hours per worker. Population and labor force statistics by age and sex are from the OECD. We use the OECD and Eurostat compensation of employees divided by hours as our main measure of wages. We adjust all prices for inflation using country-specific all-item consumer price indices. More information about how the dataset is constructed is provided in Appendix A.

¹³See Figure 8 in Appendix B for the same graphs with all the countries in our sample.

¹⁴Table 1 in Huberman and Minns (2007) shows that the decline in hours per worker goes back to at least 1870 in Australia, Canada, the United States and Western Europe.

and real recreation prices have been declining by 1.09% per year. 15

To show how widespread these patterns are, Table 4 in Appendix A.2 provides the list of countries in our sample along with their individual average growth rates for hours, wages and recreation prices. We observe, first, that there has been a broad decline in hours worked throughout our sample. Hours per capita have had a negative growth rate in 33 countries out of 42. The decline is even more pronounced when looking at hours per worker, which have declined in all but three countries (Lithuania, Luxembourg and Turkey). Second, the growth in real wages is positive for all countries except Mexico, which experienced a large decline in real wages in the 1980's due to very high inflation rates.

Real recreation prices have also been declining worldwide. As the table shows, we find a negative growth rate for *all* countries in our sample, and these growth rates are statistically different from zero at the 1% level in all cases. The coefficients are also economically large. Even for the country with the slowest decline (Ireland), recreation prices have gone down by 0.4% per year. Compared to the other countries in our sample, the United States experienced a relatively slow decline in real recreation prices (-0.7% per year). Only four countries (Ireland, Japan, Luxembourg and Norway) went through slower declines.

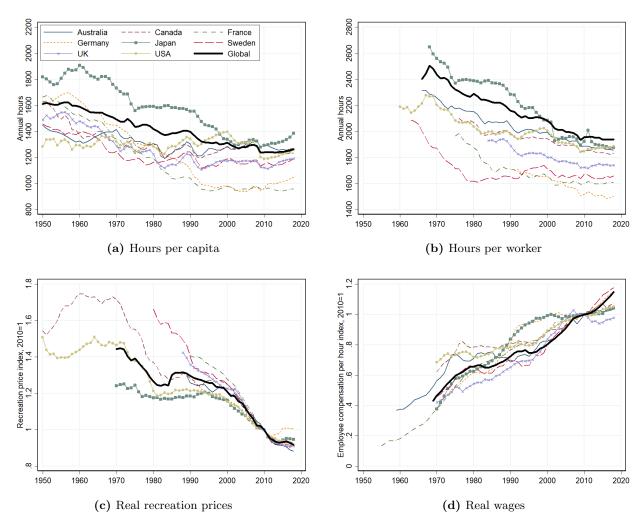
2.3 Balanced-growth-path facts for consumption

To better understand the relation between these trends, we develop in the next section a labor supply model in which recreation prices can affect hours worked. Since our goal is to explain economic changes that occur over long time horizons, we adopt the standard macroeconomic framework for this type of analysis, namely that of balanced growth. This framework implies that all prices and quantities grow at constant, but perhaps different, rates. We make however one important departure from standard BGP assumptions and allow hours worked to decline over time in contrast to the usual requirement that they remain constant.

In a recent paper, Boppart and Krusell (2020) show that—except for the evolution of hours—stylized balanced-growth facts, as outlined by Kaldor (1961), remain valid for the United States today. However, these facts do not distinguish between different types of consumption. Since our modeling strategy will assume that the consumption of recreation and non-recreation items evolve in such a way that their ratio remains constant over time, we therefore provide some evidence to show that this assumption is justified for the United States and our sample of countries.

For the United States, we use consumption data from the NIPA tables and construct a measure of recreation consumption that follows the BLS classification and includes items such as video and audio equipment, sports goods, memberships and admissions, gambling, recreational reading

¹⁵We compute these growth rates by running a pooled regression of a given variable of interest x_{lt} in country l at time t on the time trend and a set of country fixed effects α_l , so that $\log x_{lt} = \alpha_l + \gamma^x t + \varepsilon_{lt}$. The coefficient γ^x therefore provides a measure of average growth rates for variable x across countries.



The black lines show the year fixed effects from regressions of the corresponding variables on a set of country and year fixed effects, with all countries included. Regressions are weighted by country-specific total hours. For panels (a) and (b), the levels of the lines are normalized to match the all-country weighted average in 2015. Panel (a): Annual hours worked over population between 20 and 74 years old. Source: Total Economy Database and OECD. Panel (b): Annual hours worked over number of employed between 20 and 74 years old. Source: Total Economy Database and OECD. Panel (c): Price of consumption for OECD category "Recreation and culture", normalized by price index for all consumption items. Base year = 2010. Source: OECD, Eurostat, national statistical agencies. Panel (d): Real compensation of employees divided by hours worked. Base year = 2010. Source: OECD, Eurostat and Total Economy Database.

Figure 3: Hours, wages and recreation prices for a selected group of countries.

materials, pet products, photographic goods and services, and package tours (see Appendix A for the details of that exercise). We then compute the share of recreation in total consumption expenditure and plot that measure as the blue solid line in panel (a) of Figure 4. As we can see, this share has remained roughly constant over the last hundred years, moving from about six percent in 1929 to seven percent today.¹⁶

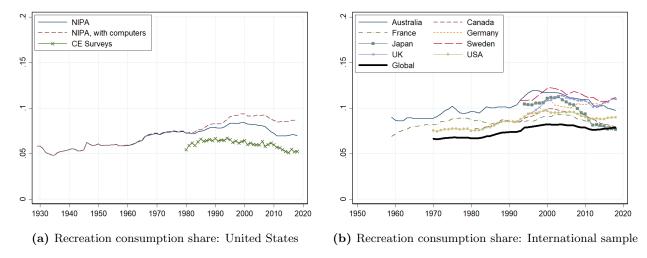
When constructing our measure of recreation consumption, we follow the classification used by the BLS and exclude information processing equipment (i.e., computers), which might also be used for work or education. The NIPA tables, however, classify those expenditures as part of recreation consumption. We therefore provide an alternative measure, displayed in red in panel (a), that follows that classification. In this case, the share of recreation expenditure increases slightly over our sample.¹⁷ To further emphasize that the share of recreation consumption has remained constant, we also construct expenditures on recreation goods and services using data from the Consumer Expenditure (CE) Survey, as in Aguiar and Bils (2015). That measure is also shown, in green, in panel (a). Although it is only available since 1980, it has remained fairly stable since then.

Since our analysis is not limited to the U.S. economy, we also compute the recreation consumption shares for other countries in our sample, using data from the OECD and Eurostat. Our measure of recreation consumption corresponds to the 'Recreation and culture' category and includes the same goods and services as the recreation price data that was discussed in Section 2.2. ¹⁸ Panel (b) shows that measure for a selected group of countries together with the all-country average in black. We include the same figure for all countries in Appendix B. While there is some variation across countries, the recreation shares stay fairly constant over time, in line with our modeling assumption.

¹⁶Our finding that the share of recreation consumption has been roughly constant is in contrast with earlier work by Kopecky (2011) who uses data from Lebergott (2014) and finds an increasing recreation share over the twentieth century. Two important differences between the datasets are responsible for the different conclusions. First, our sample includes additional data from 2000 to 2019, a period over which the recreation share has declined by more than one percentage point. Second, Lebergott (2014) finds a large increase (from three to six percentage points) in the recreation share between 1900 and 1929 (see Figure 3 in Kopecky, 2011). Unlike the rest of the time series, these data are not from NIPA, but are instead imputed from a variety of sources. For instance, adjusted sectoral wages are used as a proxy for the consumption of recreation services. While we cannot rule out a small increase in the recreation share since 1900, we view the data available starting in 1930 as more reliable for estimating its overall trend.

¹⁷Kopecky (2011) argues that up to 30% of transportation expenses are related to social and recreational trips. The transportation expenditure share has been slowly declining starting from 1980. Including transportation expenditures in the recreation consumption category would largely undo the impact of computers.

¹⁸Since the consumption categories are not as fine as the ones available from the NIPA tables or the CE Surveys, we cannot exclude information processing equipment and computers are therefore counted as recreation in this measure.



Panel (a): Share of recreation consumption in total consumption for the United States. Source: NIPA and CE Surveys. Panel (b): Share of recreation consumption in total consumption for a selected group of countries. The black line shows the year fixed effects from a regression of the recreation consumption share on a set of country and year fixed effects, with all countries included. The regression is weighted by country-specific total hours. The level of the line is normalized to match the all-country weighted average in 2015. Source: OECD and Eurostat.

Figure 4: Recreation consumption share.

3 Model

To better understand the relation between recreation prices, wages and hours worked, we construct a labor supply model that is general, microfounded, and that can be easily brought to the data. To do so, we adopt a standard balanced-growth framework. In what follows, we therefore assume that prices and quantities grow at constant, but perhaps different, rates. As we have discussed, that framework offers a good description of the evolution of the U.S. economy over the long run, so that we can be sure that our model economy does not clash with important regularities in the data.

3.1 Problem of the household

At the heart of our analysis is a household—representative or else—that maximizes some period utility function u. Our main mechanism operates through the impact of cheaper recreation goods and services on labor supply decisions. We therefore include these items, denoted by d, directly into u. The utility function also depends on the consumption of other (non-recreation) goods and services c, and on the amount of time worked h. Since it plays a central role, we keep the utility function as general as possible, only assuming that it be consistent with balanced growth. We will show below that this assumption imposes some structure on the shape of the utility function. Importantly for our mechanism, the utility function is free to feature some complementarity between leisure time and recreation consumption, such that, for instance, the purchase of a subscription to an online streaming service can make leisure time more enjoyable, which can then push the household

to work less. It follows that with such a complementarity a decline in recreation prices can lead to a decline in work hours.

The household maximizes its lifetime discounted utility

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t, d_t\right),\tag{1}$$

subject to a budget constraint

$$c_t + p_{dt}d_t + b_{t+1} = w_t h_t + b_t (1 + r_t), \qquad (2)$$

where w_t denotes the wage, p_{dt} the price of recreation goods, r_t the interest rate, and b_{t+1} the asset position of the household at the end of period t.¹⁹ Since time worked h_t is constrained by the size of the (normalized) time endowment, we assume $h_t \leq 1$, but we focus on interior solutions, so this inequality never binds.

The household chooses the sequence $\{c_t, d_t, h_t, b_{t+1}\}$ while taking the prices $\{w_t, p_{dt}, r_t\}$ as given. On a balanced-growth path the prices $\{w_t, p_{dt}\}$ grow at constant rates, and the interest rate $r_t > 0$ remains constant. We therefore assume that $p_{dt} = \gamma_{p_d}^t p_{d0}$ and $w_t = \gamma_w^t w_0$, where $\gamma_{p_d} > 0$ and $\gamma_w > 0$ are exogenous growth rates, and p_{d0} and w_0 are initial values. In Appendix C.1, we provide a potential microfoundation for the growth rates γ_w and γ_{p_d} that involves the production sector of the economy.

On a balanced-growth path, c_t , d_t and h_t also grow at constant (endogenous) rates, which we denote by g_c , g_d and g_h .²⁰ These growth rates might depend, in turn, on the growth rates of the fundamentals γ_w and γ_{p_d} , and perhaps on other features of the economy. The budget constraint of the household imposes some restrictions on these endogenous growth rates. For (2) to be satisfied in every period, each term must grow at the same rate and it must therefore be that

$$g_c = \gamma_{p_d} g_d = \gamma_w g_h. \tag{3}$$

3.2 Balanced-growth preferences

Another set of restrictions on the endogenous growth rates comes from the preferences of the household. For instance, under the utility function introduced by King et al. (1988), hours worked h_t must remain constant over time which, with (3), implies that consumption and the wage grow at the same rate: $g_c = \gamma_w$. Boppart and Krusell (2020) generalize these preferences to let hours

¹⁹The model uses non-recreation consumption as the numeraire. However, a price index for these items is not readily available for all the countries in our sample, so in our empirical exercises we normalize nominal terms by allitem price indices. The discrepancy between the two is unlikely to be large because recreation expenditures typically account for less than 10% of the overall consumption spending. In the U.S., where this data is available, the all-item and non-recreation price series follow each other closely.

²⁰Since hours worked are naturally bounded by the time endowment, we focus on the case in which $g_h \leq 1$.

worked decline on a balanced-growth path and the growth rate of consumption can take the more general form $g_c = \gamma_w^{1-\nu}$, where ν is a parameter of the utility function. In our case, the growth rate of consumption might also be affected by the growth rate of recreation prices, γ_{p_d} , and we therefore consider the more general form

$$g_c = \gamma_w^{\eta} \gamma_{p_d}^{\tau},\tag{4}$$

where η and τ are constants that have to be determined.

We can combine equations (3) and (4) to characterize the growth rates of all the endogenous quantities in terms of the constants η and τ such that

$$g_c = \gamma_w^{\eta} \gamma_{p_d}^{\tau},$$

$$g_h = \gamma_w^{\eta - 1} \gamma_{p_d}^{\tau},$$

$$g_d = \gamma_w^{\eta} \gamma_{p_d}^{\tau - 1}.$$
(5)

Given these restrictions, we can formally define the properties of a utility function that is consistent with balanced growth in this economy.²¹

Definition 1 (Balanced-growth preferences). The utility function u is consistent with balanced growth if it is twice continuously differentiable and has the following properties: for any w > 0, p > 0, c > 0, $\gamma_w > 0$ and $\gamma_p > 0$, there exist h > 0, d > 0 and r > -1 such that for any t

$$-\frac{u_h \left(c \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau}\right)^t, h \left(\gamma_w^{\eta-1} \gamma_{p_d}^{\tau}\right)^t, d \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau-1}\right)^t\right)}{u_c \left(c \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau}\right)^t, h \left(\gamma_w^{\eta-1} \gamma_{p_d}^{\tau}\right)^t, d \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau-1}\right)^t\right)} = w \gamma_w^t, \tag{6}$$

$$\frac{u_d \left(c \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau} \right)^t, h \left(\gamma_w^{\eta - 1} \gamma_{p_d}^{\tau} \right)^t, d \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau - 1} \right)^t \right)}{u_c \left(c \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau} \right)^t, h \left(\gamma_w^{\eta - 1} \gamma_{p_d}^{\tau} \right)^t, d \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau - 1} \right)^t \right)} = p_d \gamma_{p_d}^t, \tag{7}$$

and

$$\frac{u_c \left(c \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau} \right)^t, h \left(\gamma_w^{\eta-1} \gamma_{p_d}^{\tau} \right)^t, d \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau-1} \right)^t \right)}{u_c \left(c \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau} \right)^{t+1}, h \left(\gamma_w^{\eta-1} \gamma_{p_d}^{\tau} \right)^{t+1}, d \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau-1} \right)^{t+1} \right)} = \beta \left(1 + r \right),$$
(8)

where $\eta > 0$ and $\tau > 0$.

These equations are the usual first-order conditions of the household. The first one states that the marginal rate of substitution between hours h_t and consumption c_t must equal the wage

²¹The following definition is a generalization of Assumption 1 in Boppart and Krusell (2020). Notice from (5) that when $\eta < 0$ higher wage growth leads to lower consumption growth. Also, when $\tau < 0$ higher growth in the price of recreation goods leads to smaller growth in work hours. We focus on the more empirically plausible economies with $\eta > 0$ and $\tau > 0$.

 w_t , the second equation implies that the marginal rate of substitution between leisure goods d_t and consumption c_t must equal the relative price of leisure goods p_{dt} , and the third equation is the intertemporal Euler equation. Definition 1 imposes that these optimality conditions must be satisfied in every period t, starting from some initial point $\{c, h, d, p_d, w\}$ and taking into account the respective growth rates of each variable provided by (5).

The following proposition describes the class of utility functions that are consistent with balanced growth.

Proposition 1. The utility function u(c, h, d) is consistent with balanced growth (Definition 1) if and only if (save for additive and multiplicative constants) it is of the form

$$u(c, h, d) = \frac{\left(c^{1-\varepsilon} d^{\varepsilon} v \left(c^{1-\eta-\tau} h^{\eta} d^{\tau}\right)\right)^{1-\sigma} - 1}{1-\sigma},\tag{9}$$

for $\sigma \neq 1$,

$$u(c, h, d) = \log \left(c^{1-\varepsilon} d^{\varepsilon} \right) + \log \left(v \left(c^{1-\eta-\tau} h^{\eta} d^{\tau} \right) \right), \tag{10}$$

for $\sigma = 1$, and where v is an arbitrary twice continuously differentiable function and where $\eta > 0$ and $\tau > 0$.

Proof. The proof is in Appendix F.
$$\Box$$

This proposition establishes necessary and sufficient conditions on the shape of u so that it is consistent with balanced growth. They are the only restrictions that we impose on the utility function, such that our empirical analysis remains general and does not hinge on a particular choice of u. Of course, several utility functions that satisfy (9)-(10) make little economic sense. Additional restrictions would need to be imposed so that, for instance, u is increasing in c and decreasing in d. But we do not need to explicitly specify these restrictions. For our analysis to hold, we only need that the household maximizes some version of (9)-(10), and that the first-order conditions are necessary to characterize its optimal choice.

Several utility functions that have been used in the literature are nested in (9)–(10). For instance, the standard balanced-growth preferences of King et al. (1988) in which labor remains constant can be obtained by setting $\varepsilon = 0$, $\tau = 0$ and $\eta = 1$. To allow for a nonzero income effect of rising wages on the labor supply, we can instead set $\varepsilon = 0$, $\tau = 0$ and $\eta \neq 1$ to get the preferences of Boppart and Krusell (2020). The functional form (9)–(10), however, does not nest some other utility functions that have recreation goods and services as an input. For instance, the

²²Our analysis goes through even if the utility function (9)–(10) is not concave. In this case, the first-order conditions are not sufficient to characterize a solution to the household's optimization problem but they are still necessary. As a result, they are satisfied at the household's optimal decision and we can use them to characterize the balanced-growth path and derive the system of equations that we estimate.

preferences used by Vandenbroucke (2009) and Kopecky (2011) do not allow for balanced growth and are therefore not a special case of (9)–(10).²³

3.3 The impact of wages and recreation prices

Proposition 1 shows that the constants η and τ introduced as placeholders in (4) come directly from the utility function. As such, they are structural parameters and do not depend on other (perhaps endogenous) economic variables whose presence might lead to endogeneity issues in our estimation. Taking the log of (5), we can therefore write the system of three equations

$$\log g_c = \eta \log \gamma_w + \tau \log \gamma_{p_d},$$

$$\log g_d = \eta \log \gamma_w + (\tau - 1) \log \gamma_{p_d},$$

$$\log g_h = (\eta - 1) \log \gamma_w + \tau \log \gamma_{p_d},$$
(11)

to be estimated in the following section.

These equations show that the log of the growth rates of the endogenous variables c_t , d_t and h_t are linear functions of the log of the growth rates of the exogenous variables w_t and p_{dt} , and that the preference parameters η and τ characterize these relationships. These parameters therefore capture the intensity of standard income and substitution effects, triggered by changes in prices, that are at work in the model.

The third equation in (11) plays a central role in our exploration of the causes behind the decline in work hours. The first term on the right-hand side captures how rising wages affect the supply of labor. When $\eta - 1 < 0$, higher wage growth leads to more leisure growth through a standard income effect: richer households substitute consumption with leisure. When instead $\eta - 1 > 0$, the substitution effect dominates, and the household takes advantage of the higher wage rate to work more and earn additional income. The second term on the right-hand side of the equation captures the impact of recreation prices on labor supply decisions. For instance, when $\tau > 0$, a decline in the price of recreation goods and services incentivizes the household to enjoy more leisure and work less.

Overall, the results of this section provide a clear path to empirically evaluate the importance of the decline in recreation prices on hours worked. From (11), we know that g_c , g_d and g_h are related log-linearly to γ_w and γ_{p_d} , so that we can estimate these relationships readily through standard linear techniques. Furthermore, these relationships are structural, so that we can be sure that our estimation captures deep parameters that are unaffected by changes in policy. The system of equations (11) also shows that the relationship between hours worked and leisure prices is invariant to various features of the utility function, such as the function v and the parameters ε and σ . As a

²³We can compute the Frisch elasticity of labor supply associated with the utility function (9)–(10) and show that it is constant along a balanced-growth path, although it is not, in general, only a function of the parameters of the utility function. As such, it can vary for example across countries with different factor endowments or growth rates.

result, we can be confident that our empirical strategy is robust to a broad class of utility functions. Finally, our analysis does not hinge on a particular set of assumptions about the production sector of the economy, as long as w_t and p_{dt} grow at constant rates. As such, it is robust to different production technologies, market structures, etc.²⁴

4 Estimating the model on cross-country data

We now estimate the model on the cross section of OECD countries. To do so, we use the data on hours, wages and recreation prices introduced in Section 2, as well as consumption data from the OECD and Eurostat.²⁵

4.1 Data and specification

Denote by $\Delta \log c_i$, $\Delta \log d_i$ and $\Delta \log h_i$ the average annual growth rates of non-recreation consumption, recreation consumption and hours worked in country i.²⁶ We use a generalized method of moments (GMM, Hansen, 1982) that allows us to impose the key cross-equation restrictions implied by (11) without the need to make any additional assumptions about the distribution of the shocks, etc. Specifically, our benchmark specification is

$$\Delta \log c_i = \alpha^c + \eta \Delta \log w_i + \tau \Delta \log p_i + \varepsilon_i^c,$$

$$\Delta \log d_i = \alpha^d + \eta \Delta \log w_i + (\tau - 1) \Delta \log p_i + \varepsilon_i^d,$$

$$\Delta \log h_i = \alpha^h + (\eta - 1) \Delta \log w_i + \tau \Delta \log p_i + \varepsilon_i^h,$$
(12)

where $\Delta \log w_i$ and $\Delta \log p_i$ are the growth rates of real wages and real recreation prices, and where the ε 's are error terms. We also include the constants α^c , α^d and α^h to absorb potential aggregate changes in the data that were not explicitly included in the model.

The results of the estimation are presented in Table 1.²⁷ The first column shows the estimated coefficients τ and $\eta - 1$ when wages are measured as GDP per hour. In column (2), we use real compensation per hour instead. We can see that the results are similar across columns. Overall, we find a significantly positive coefficient τ , which, from (12), is consistent with cheaper recreation items having a negative impact on hours per capita.

We also find a negative and strongly significant value for $\eta - 1$, which is consistent with a

²⁴See Appendix C.1 for an example of a production structure that provides a microfoundation in which the constant growth rates γ_w and γ_{p_d} depend on underlying productivity growth in the non-recreation and recreation sectors.

²⁵See Appendix A.2 for details about the consumption data. Since the model does not feature population growth, we normalize consumption variables and hours by the population between 20 and 74 years old. We show robustness of our results to this normalization in Appendix D.2.

²⁶We remove the Great Recession years (2008 and 2009) from the sample as they are clear outliers that can substantially change the estimate of steady-state growth rates given the small number of years available for some countries. See Appendix D.2 for the results without excluding the Great Recession.

²⁷We only have 41 countries in that sample because recreation consumption data is not available for Brazil.

dominant income effect of rising wages on hours worked. We note that estimating the full system of equations (12) is important for this last result, since a simple regression of hours growth on the growth of wages and recreation prices does not find a significant coefficient for wages.²⁸ When jointly estimating the three equations, the consumption data together with the model restrictions impose enough discipline to make the income effect visible. To understand why, notice from (12) that a dominating substitution effect $(\eta - 1 > 0)$ implies that consumption growth reacts more than one for one to a change in wage growth. Intuitively, when $\eta - 1 > 0$, higher wages not only lead to additional income keeping hours fixed, but they also raise hours worked leading to an extra increase in income. That additional income then leads to a larger increase in recreation and non-recreation consumption, as the first and second equations in (12) show. The data rejects such a strong effect of wage growth on consumption, and so the estimation finds that the income effect dominates.

Finally, Table 1 also shows that the constant α^h is positive and significantly different from zero. This constant absorbs aggregate changes such as the secular increase in female labor force participation and other demographic trends that are not explicitly included in the model.²⁹

	(1)	(2)
au	0.290*** (0.090)	0.194** (0.087)
$\eta - 1$	-0.459^{***} (0.069)	-0.420^{***} (0.053)
α^h	0.013^{***} (0.002)	0.010^{***} (0.002)
Wages J-test: p-value Observations	GDP/hour 0.038 41	Empl. comp./hour 0.026 41

Results of iterative GMM estimation of (12). Robust standard errors in parentheses. *,***,**** indicate significance at the 10%, 5%, and 1% levels, respectively. Variables are constructed using all years except for 2008 and 2009. Work hours are measured in per capita terms. Population includes individuals between 20 and 74 years old. The "J-test: p-value" row reports p-values of Hansen's J-test of overidentifying restrictions.

Table 1: GMM estimation of the structural model (12).

4.2 Robustness

We provide robustness for the results of Table 1 in Appendix D.2, where we investigate the following changes in specifications: 1) using working age population (between 25 and 64 years old) to define per capita variables; 2) restricting the sample to countries with at least 20 years

²⁸See Table 7 in Appendix D for this exercise.

 $^{^{29}}$ Notice that the system (12) is over-identified due to cross-equation restrictions. We can test the validity of these restrictions via Hansen's J-test. Table 1 reports the corresponding p-values. They imply that the restrictions cannot be rejected at the 1% level but can be rejected at the 5% level. One reason for that might be the simplicity of the benchmark model, as it incorporates only two types of consumption. In Section 4.2, we allow for three types of consumption and find that this extended model cannot be rejected at any conventional significance level.

of recreation price data; 3) using hours per worker instead of per capita; and, 4) including the Great Recession years in the sample. In almost all cases, the results are essentially unchanged with cheaper recreation items and higher wages still associated with fewer hours worked.

Home production

We can also extend our theoretical framework to allow for other types of consumption to influence how households allocate their time. The existing literature (e.g. Greenwood et al., 2005) argues that the decline in the price of household durable goods, such as household appliances, has had a substantial impact on labor supply decisions by making housework less time intensive. In this subsection we evaluate the importance of that mechanism for our results by augmenting the utility function to also depend on the consumption of household items, which we denote by a, with its associated relative price p^a . Following the same steps as in our main model (see Appendix C.2 for details), we can derive the system of equations

$$\Delta \log c_{i} = \alpha^{c} + \eta \Delta \log w_{i} + \tau \Delta \log p_{i}^{d} + \delta \Delta \log p_{i}^{a} + \varepsilon_{i}^{c},$$

$$\Delta \log d_{i} = \alpha^{d} + \eta \Delta \log w_{i} + (\tau - 1) \Delta \log p_{i}^{d} + \delta \Delta \log p_{i}^{a} + \varepsilon_{i}^{d},$$

$$\Delta \log a_{i} = \alpha^{a} + \eta \Delta \log w_{i} + \tau \Delta \log p_{i}^{d} + (\delta - 1) \Delta \log p_{i}^{a} + \varepsilon_{i}^{a},$$

$$\Delta \log h_{i} = \alpha^{h} + (\eta - 1) \Delta \log w_{i} + \tau \Delta \log p_{i}^{d} + \delta \Delta \log p_{i}^{a} + \varepsilon_{i}^{h},$$

$$(13)$$

which takes into account that cheaper household items might affect labor supply decisions, as we can see from the last equation. The preference parameter δ captures the importance of that mechanism.

We estimate the system (13) via GMM. The price and consumption data for household items comes from the OECD and Eurostat. This group of goods and services includes such items as household appliances, furniture, household textiles and utensils, garden tools and equipment, and goods and services for routine household maintenance. The results are presented in Table 2 below. Reassuringly, we still find a significantly positive coefficient τ , implying a positive association between growth in recreation prices and hours worked. The magnitude of the coefficient is somewhat smaller than what is reported in Table 1, such that the effect of recreation prices is attenuated by the presence of household items. While the estimate for δ is at most only marginally significant its negative sign is in line with the idea that cheaper household items push toward less housework and more market work hours, in line with the literature. Finally, we again find that the income effect of wages dominates the substitution effect, as the coefficient $\eta - 1$ is significantly negative.³⁰

 $^{^{30}}$ Note that the model with three types of consumption provides a better description of the data, in the sense that the over-identification tests cannot reject the model, as can be seen from the p-values of the J-test.

	(1)	(2)
$\overline{\tau}$	0.137**	0.169***
δ	(0.063) -0.173^*	(0.058) -0.123
	(0.095)	(0.088)
$\eta - 1$	-0.406^{***} (0.067)	-0.266^{***} (0.055)
α^h	0.008*** (0.002)	0.006*** (0.001)
Wages J -test: p -value	GDP/hour 0.212	Empl. comp./hour 0.200
Observations	41	41

Results of iterative GMM estimation of (13). Robust standard errors in parentheses. *,***,*** indicate significance at the 10%, 5%, and 1% levels, respectively. Variables are constructed using all years except for 2008 and 2009. Work hours are measured in per capita terms. Population includes individuals between 20 and 74 years old. The "J-test: p-value" row reports p-values of Hansen's J-test of overidentifying restrictions.

Table 2: GMM estimation of the system of equations (13).

Reduced-form estimation

While it is straightforward to extend the model to include additional types of consumptions, we cannot control for some other mechanisms without deeper changes to the model that might break the balanced-growth assumption. In Appendix D, we however provide results from reduced-form exercises that involve estimating only the last equation of (12)—the one that links hours growth with the growth in wage and recreation prices—using ordinary least-squares. In this reduced-form exercise, we can easily control for additional mechanisms that might have affected labor supply decisions, such as the increase in female labor force participation and variations in the share of young men in the population. Controlling for these changes, we still find a strong and significant association between recreation prices and work hours. In contrast, the coefficient that captures the impact of wage growth on hours is close to zero and statistically insignificant in the majority of the specifications.

4.3 Economic impact

What do our estimates imply for the importance of wage growth and the fall in recreation prices in driving the global decline in work hours? In order to answer this question, we can perform a back-of-the-envelope calculation using the average values of the estimated coefficients in Table 1, which are $\tau = 0.24$ and $\eta - 1 = -0.44$. From Table 4 in Appendix A.2, we see that the annual growth rate of wages has been 2.45% across the countries in our sample, and that the equivalent number for recreation prices is -1.48%. Our results therefore suggest that wage growth has pushed for a decline in the growth rate of hours of about $2.45\% \times 0.44 \approx 1.08\%$ per year. Similarly, the decline

in recreation prices can account for a decline in the growth of hours of about $1.48\% \times 0.24 \approx 0.36\%$ per year. Based on these calculations, the recreation channel has been about a third as important as the income effect as a driver of the decline in work hours.

Put together, these two channels would suggest that the average annual growth rate of work hours should be about -1.4%, more than the actual annual movement in hours per capita (-0.32%) observed since 1950 and reported in Table 4. What explains this discrepancy? Clearly, the intercept α_h reported in Table 1 plays a non-trivial role, capturing for instance the entry of women into the labor force. We can filter out that effect by looking at male employment in the United States, for which data is readily available. From panel (a) of Figure 1, we see that male hours per capita have gone down by about 0.25% per year since 1979. From the CPS, we find that the median real weekly earnings for males have been essentially unchanged over the same period, so that wage growth had approximately no impact on male labor supply decisions over that period. Since recreation prices have gone down by 0.70% a year in the last forty years in the U.S., the predicted impact of the decline in recreation prices ($-0.70 \times 0.24 = 0.17\%$ per year) can explain about 70% of the decline in male work hours.³¹

5 Implications for cross-household trends and leisure inequality

While our main focus is on the relation between hours, recreation prices and wages at the aggregate level, our general preference specification also makes quantitative predictions for the labor supply decisions of individual households. In this section, we employ our structural model to investigate the role of recreation prices in driving work hours in the cross-section of households. This exercise is motivated by the marked increase in U.S. leisure inequality recent decades—with less-educated individuals working fewer and fewer hours compared to their more-educated counterparts (see, among others, Aguiar and Hurst, 2009 and Attanasio et al., 2014). This fact is documented in Figure 5. Panel (a) shows the evolution of work hours for individuals with a high-school diploma or less, and for those with at least a college degree. Between 1965 and 1985 the hours of these two groups have gone down by almost exactly the same amount. After 1985, however, individuals that have at most a high school education have seen their work hours go down relative to their college-educated counterparts. Panel (b) shows similar patterns for total leisure time. Since this increase in leisure inequality was accompanied by a growing skill premium (see panel d), these trends are

 $^{^{31}}$ We can extend this analysis to the richer model that includes consumption of household items involved in home production. By taking averages across the columns of Table 2, we get $\tau=0.15$, $\delta=-0.15$ and $\eta-1=-0.34$. The cross-country average growth rates of recreation prices, household items prices and wages are, respectively, -1.48%, -1.55% and 2.45%. These values imply an average growth rate in hours worked of $-1.48\% \times 0.15 + 1.55\% \times 0.15 - 2.45\% \times 0.34 = -0.82\%$. This number is closer to what is observed in the data than the magnitudes above. The intercept α_h is still important here but its estimate is substantially lower than that in the model with only two types of consumption.

hard to reconcile with the dominating income effect that we found in our county-level estimation.³² However, as we show in this section, the price of recreation items that less-educated individuals tend to consume has declined significantly over the recent decades, making leisure effectively more attractive for them. Therefore, our mechanism can potentially reconcile the simultaneous increase in leisure inequality and in the skill premium.

One key advantage of investigating the impact of recreation prices on work hours at the individual level is that this disaggregated data is sufficiently rich to allow us to construct instrumental variables for wages and recreation prices, and to therefore alleviate potential endogeneity concerns. Below, we first describe the data and the instruments. We then estimate the model-implied three-equation system (11) applied to synthetic households.

5.1 Data

Our instrumental strategy, described below, requires detailed data at the locality-demographic-industry level. To construct the needed measures of hours and earnings at that level, we use data from the U.S. Census (years 1980 and 1990) and the Census' American Community Surveys (2014-2018 five-year sample). In what follows, we denote the years 1980 and 1990 as t=0 and t=1, respectively, while the 2014-2018 period is t=2. Later on, we will use the data from t=0 (the "pre-period") to construct the instruments, while the data from t=1 and t=2 will be used to compute the growth rates of the variables of interest. One key advantage of the Census data is that they cover a large sample of the U.S. population, which allows us to exploit variation across 741 commuting zones, defined as in Dorn et al. (2019). As in Aguiar and Bils (2015), we limit our analysis to individuals between the ages of 25 and 64, and split them into 15 demographic groups based on age and education.³³ Overall, such demographic-locality split implies 11115 groups. We exclude groups with less than 50 individual observations, leaving us with 10469 groups.

Data on recreation and non-recreation consumption come from the interview part of the CE Survey. We follow Aguiar and Bils (2015) in constructing and cleaning the sample.³⁴

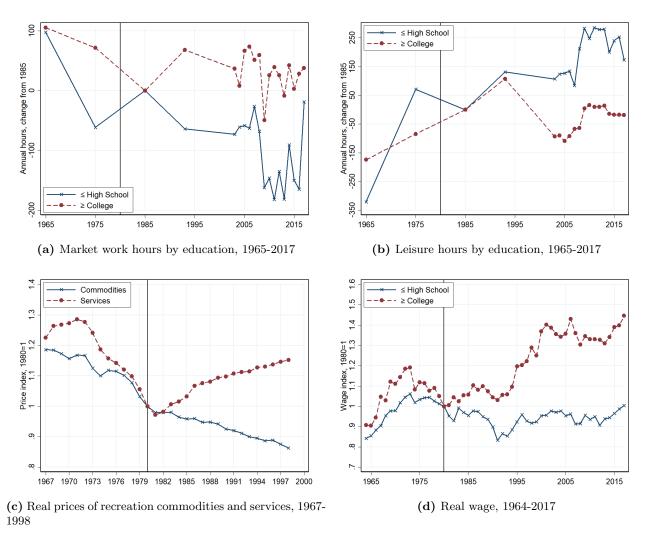
5.2 Specification

We adapt the three-equation system (12), implied by the model, to the household-level data. Our main specification takes the form

³²Bick et al. (2018) find that high-income adults tend to work more than low-income adults in rich countries, even though the income effect appears to dominate in the cross-section of countries as well as in the evolution of hours over longer time periods.

³³The age groups are "25-34 years old", "35-49 years old", "50-64 years old". The education groups are "less than high school", "high school", "some college", "four years of college" and "more than college". We exclude individuals serving in the armed forces and institutional inmates.

 $^{^{34}}$ We use the CE data between 1980 and 1988 as the t=0 period, and the 1989-1991 and 2014-2018 periods serve as t=1 and t=2, respectively. We pool observations between 1980 and 1988 to reduce noise, since the CE has on average only 1484 annual observations. The results are largely unchanged if we use a shorter pooling period instead.



The vertical black lines denote the start of the detailed consumption and price data. Panels (a) and (b): Evolution of work and leisure annual hours for individuals with no more than high school diploma and at least four years of college. Market work includes any work-related activities, travel related to work, and job search activities. Leisure is any time not allocated to market and nonmarket work (home production, shopping, non-recreational childcare), net of time required for fulfilling biological necessities (8 hours per day). Sample includes people between 25 and 64 years old who are not full-time students. Source: ATUS, Aguiar and Hurst (2007b) and Aguiar et al. (2021). Panel (c): Real U.S.-wide price of recreation commodities and services. Source: BLS. Panel (d): Real hourly wage for individuals with no more than high school diploma and at least four years of college. Sample includes people between 25 and 64 years old who are not full-time students. Source: ASEC.

Figure 5: Work hours, leisure hours, recreation prices and wages.

$$\Delta \log c_g = \alpha^c + \eta \Delta \log w_{gl} + \tau \Delta \log p_g + \varepsilon_{gl}^c,$$

$$\Delta \log d_g = \alpha^d + \eta \Delta \log w_{gl} + (\tau - 1) \Delta \log p_g + \varepsilon_{gl}^d,$$

$$\Delta \log h_{gl} = \alpha^h + (\eta - 1) \Delta \log w_{gl} + \tau \Delta \log p_g + \varepsilon_{gl}^h,$$
(14)

where $\Delta \log x_{gl}$ denotes the log growth rate of a variable x for households in an age-education group g in location l between 1990 and the 2014-2018 period. As before, c is non-recreation consumption, d is recreation consumption, h is hours worked, w is the real wage and p is the real price of recreation items. All variables in (22) are demographic- and location-specific except for the consumption data, which is not rich enough at the local level, and recreation prices which are not available at the local level. We instead construct demographic-specific prices by using the demographic-specific consumption shares of various types of recreation items together with the aggregate prices of these items.³⁵ Note that the system (14) is purely cross-sectional, with no time dimension. The identification therefore comes from variations across localities and demographic groups, and aggregate trends are absorbed by the constants.

5.3 Identification

A potential issue, which is more acute in this setting compared to the cross-country analysis above, is the endogeneity of the variables that enter both the left-hand and the right-hand sides of our structural equations (14). One set of concerns comes from the fact that the recreation price index is constructed as a consumption-share-weighted average of underlying recreation items, and that the weights might be affected by other economic variables that are not directly accounted for by our model. As a result, shocks that affect the consumption shares might lead to spurious correlations that could bias our estimates of the structural parameters. For instance, if a productivity shock makes televisions cheaper, households might substitute towards TV watching and away from relatively more expensive recreation, such as live sports events. This would lead to a larger decline in the recreation price index that we use in the estimation than warranted by the productivity shock alone. Similarly, if changes in income push households to consume cheaper recreation items, a sudden decline in employment opportunities might show up as a decline in recreation prices and interfere with the estimation. Additional issues can arise if certain recreation goods behave as "leisure luxuries," such that their consumption increases with leisure time (Aguiar et al., 2021). In that case, growing leisure time, perhaps because of falling labor demand, might be increasingly allocated to these items, and as a result we would observe a decline in the price of the leisure basket together with a fall in work hours. Another set of endogeneity concerns might arise from labor

The series $p_{g,t}$ according to the formula $\frac{p_{g,t+1}}{p_{g,t}} = \sum_j \frac{c_{jg,t+1}}{\sum_i c_{ig,t+1}} \frac{p_{j,t+1}^{US}}{p_{j,t}^{US}}$, where $c_{jg,t}$ is the consumption by group g of recreation items of type j at time t and $p_{j,t}^{US}$ is the national price of these items.

supply shocks, which are outside of our model but might be present in the data. For instance, a preference shock that makes households enjoy leisure more would lead to a drop in hours and an increase in wages, leading to reverse causality in our estimation of the relationship between wages and hours. Since we are not modeling these effects explicitly, we want to ensure that they do not bias our estimates of η and τ , and invalidate the interpretation of these coefficients as structural parameters.

In this subsection, we describe how we construct two instrumental variables for wages and recreation prices that allow us to identify these structural parameters by imposing additional orthogonality conditions in the GMM estimation. Our wage instrument relies on the differences in sector-level employment across U.S. localities and across demographic groups, as is relatively standard in the literature (Bartik, 1991). In the same spirit, we construct a novel instrument for recreation prices that takes advantage of differences in recreation consumption bundles, across households with different demographic characteristics, that predate the sample period used in our estimation (i.e., in the pre-period). We split all recreation consumption expenditures into the seven subcategories used by the BLS to build price indices: Audio-video, Sports, Pets, Photo, Reading, Other goods (including toys and musical instruments), Other services (including admissions, fees for lessons and instructions, club memberships, etc.). ³⁶

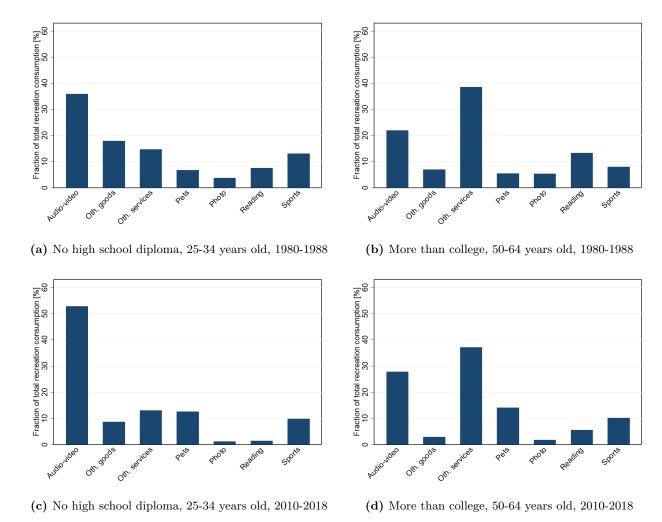
Instrument for recreation prices

To motivate our instrument for recreation prices, we first show that, in the United States, there are large differences in the types of recreation goods and services that are consumed by households with different demographic characteristics, such as education and age. For instance, panel (a) of Figure 6 shows how households whose heads are between 25 and 34 years old and do not have a high school diploma allocated their recreation spending in the period between 1980 and 1988. Panel (b) provides the same information for households whose heads have more than a college degree and who are between 50 and 64. We see that the consumption baskets vary substantially across these demographics. In particular, young and less-educated households spend disproportionally more on "Audio-video" items, while older and more educated households spend more on "Other services". ³⁷ Panels (c) and (d), which provide the same shares over the 2010-2018 period, show that these differences remain in the most recent decade and, if anything, have become starker.

While Figure 6 shows that different households consume different baskets of recreation items, the prices of these items have also evolved very differently over the last three decades. As we can

³⁶When constructing recreation consumption baskets across demographic groups, we use the demographic characteristics of the household's reference person. Our measures of wages and hours from the Census are at the individual level. Our results are similar if we instead use hours and wage data for the household heads only (see Appendix E.3.1).

³⁷Figure 10 in Appendix E.1 shows that education alone can account for large variations in spending habits on recreation items.



Shares of different items in total recreation consumption, constructed by pooling observations for the two periods, 1980-1988 and 2010-2018. Source: Consumer Expenditure Survey.

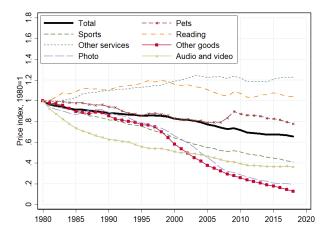
Figure 6: Share of recreation spending across education and age groups.

see from Figure 7, the real price of "Audio-video" items, disproportionately consumed by young and less-educated households, has declined by 60% since 1980. In contrast, the average price of items in the "Other services" category, mostly consumed by older and more-educated households, has increased by about 20%. As a result, the price of the typical recreation basket has evolved very differently across demographic groups.

We use this variation to construct the shift-share instrument

$$\Delta \log p_g^{IV} = \sum_j \frac{c_{jg}^0}{\sum_i c_{ig}^0} \Delta \log p_j^{US}, \tag{15}$$

where $\Delta \log p_j^{US}$ denotes the change in the nation-wide price of recreation items of type j between the periods t=1 and t=2. The quantity c_{jg}^0 denotes the consumption expenditure on recreation



Real U.S.-wide price of various recreation goods and services. Source: BLS.

Figure 7: Real prices of different recreation goods and services.

items of type j by individuals in demographic group g in period t = 0. As (15) shows, the instrument captures how nationwide changes in prices $\Delta \log p_j^{US}$ affect the price of the recreation bundle for a household of given demographic characteristics.³⁸

For this instrument to be relevant, it must be that growth in the demographic-specific recreation prices $\Delta \log p_g$ in (14) is correlated with the *initial* composition of the basket of recreation consumption, as captured by the shares $c_{jg}^0/\sum_i c_{ig}^0$ in (15). Figure 6 suggests that this is indeed the case. As the figure shows, these shares are quite persistent over time and, as a result the initial basket should be a good predictor of the growth in the price of the basket going forward. Since there are large differences in the growth of the price of different recreation items (as shown in Figure 7), the instrument (15) should vary substantially across demographic groups and be strong.

For that instrument to be valid, it must be that the consumption shares $c_{jg}^0/\sum_i c_{ig}^0$ are exogenous, i.e. uncorrelated with the error terms in the reduced-form equations (14) (Goldsmith-Pinkham et al., 2018). We view this assumption as reasonable for several reasons. First, we make sure to compute the shares in a pre-period (t=0) to minimize their correlation with any potential omitted variables at t=1 and t=2, the period over which the growth rates are computed. Second, we view the consumption shares as being largely driven by differences in preferences (which, in particular, explains their persistence over time, as shown in Figure 6). For instance, college might introduce students to the theater, leading some of them to consume theater plays after graduation.

³⁸A similar approach is used by Acemoglu and Linn (2004) to instrument for changes in demand for new drugs, as they interact expenditure shares of individual goods with demographic changes in order to capture shifts in market sizes over time. As shown by Goldsmith-Pinkham et al. (2018) in the context of the standard Bartik instrument, this construction is essentially equivalent to a differences-in-differences research design. Goldsmith-Pinkham et al. (2018) also discuss the implicit assumptions under which the exclusion restriction is satisfied.

 $^{^{39}}$ We confirm this formally by showing that the first-stage F-statistics are large. As it is unclear how to compute F-statistics when doing the structural estimation of the system (14), we report them for the one-equation reduced-form estimation in Appendix E.3.

These deep-seated preferences are unlikely to be related to random shocks that would also affect the error terms in (14). Of course, other economic outcomes such as the prices of different recreation items and household income might also affect the shares. However, in that case the shares would be mainly affected by the *levels* of these variables at time t = 0 and not by the *changes* in these variables between t = 1 and t = 2, which would be more likely correlated with the error terms.

To further check the validity of our instrument, we can look at pre-trends in the data since the 1960s. We do so in Figure 5 where we show in panel (c) the separate evolution of the real price of recreation commodities, mostly consumed by younger less-educated individuals, and services, mostly consumed by older more-educated individuals, between 1967 and 1998.⁴⁰ Interestingly, we see that the time series follow each other closely until about 1980 and then diverge markedly afterward. From 1982 on, the real price of recreation commodities has been on a steady decline while the real price of recreation services has been increasing. This pattern is reassuring for the exogeneity of our instrument: work hours for both college- and high-school-educated workers declined by the same amount between 1965 and 1985 (panel a), when the prices of their recreation bundles moved together. The fact that the prices of recreation goods and services and the wages of higher- and lower-educated individuals start to diverge only in the 1980s also alleviates the concern that different recreation consumption shares in the pre-period might reflect prior trends in recreation prices or wages.

The patterns in Figure 5 suggest a potential explanation for the recent rise in leisure inequality. As we can see from panel (b), leisure time has grown the most among the less educated. These individuals have also faced the slowest growth in wages over that period so that the income effect alone would be unable to explain their relative rise in leisure time (in fact, it might suggest that the substitution effect rather than the income effect dominates). At the same time, the price of recreation items that these households tend to consume has declined significantly, making leisure effectively more attractive for them. In contrast, the real price of recreation items consumed by more-educated households has been increasing, but so have their wages. If the two effects roughly offset each other, that would explain why their leisure time has been stable over the last decades.

Instrument for wages

Another potential concern is that technological progress over the past decades has moved manufacturing jobs overseas or made them obsolete and, at the same time, made recreation goods cheaper (e.g., Autor et al., 2006, Autor and Dorn, 2013, Bloom et al., 2019, Jaimovich and Siu, 2020). These changes might have affected different demographic groups in different ways thereby creating a correlation with the consumption shares. In particular, less-educated workers in the

⁴⁰Disaggregated price data for the various recreation items are not available prior to the late 1970s. The series shown in panel (c) of Figure 5 were discontinued in 1998 due to changes in the classification scheme. But importantly, and as evident from Figure 7, the diverging trends of real prices of recreation commodities and services are also present during the two latest decades.

manufacturing sector have been disproportionately affected. In principle, the presence of wages on the right-hand sides of the equations in (14) would take care of any potential endogeneity having to do with employment opportunities, as long as they are indeed capturing exogenous shocks.⁴¹ In order to account for the potential endogeneity in wages themselves we construct our second instrument. Here, we directly follow the approach of Bartik (1991) that is now standard in the literature.

We use initial variation in industrial employment across localities and demographic groups together with nation-wide changes in wages by industry to construct a measure of changes in wages that are driven by factors independent of local labor market conditions, such as technological growth, etc.⁴² To be precise, we compute

$$\Delta \log w_{gl}^{IV} = \sum_{i} \frac{e_{igl}^{0}}{\sum_{j} e_{jgl}^{0}} \Delta \log e_{ig}^{US} - \sum_{i} \frac{h_{igl}^{0}}{\sum_{j} h_{jgl}^{0}} \Delta \log h_{ig}^{US}, \tag{16}$$

where i denotes an industry, g is a demographic group, and l is a locality. As before, the operator Δ denotes the total growth rate between t=1 and t=2. The variable $e_{igl}=w_{igl}\times h_{igl}$ refers to labor earnings and h_{igl} is total hours worked. As (16) shows, we construct $\Delta \log w_{gl}^{IV}$ by first computing the fraction of earnings and hours worked that can be attributed to an industry i in a given locality-demographic unit (g,l) in the pre-period t=0. Since these shares provide a measure of how sensitive local earnings and hours are to aggregate changes in industry i, we can then compute $\Delta \log w_{gl}^{IV}$ as the growth rate in local wages that can be attributed to changes in the national factors $\Delta \log e_{ig}^{US}$ and $\Delta \log h_{ig}^{US}$.

5.4 Estimating the effect of recreation prices on individual labor supply

The estimated coefficients τ and $\eta-1$ are presented in Table 3. Column 1 shows the estimates without the instruments while column 2 shows the outcome of the instrumental variable estimation. In both cases, we find that the τ coefficients are significantly above zero, suggesting that the decline in recreation prices makes leisure time more attractive and, thus, leads to a reduction in work hours. We also find in both columns that $\eta-1$ is estimated to be significantly negative, although its value is somewhat smaller in absolute terms with the instruments. As in our cross-country analysis above, this implies that higher wage growth leads to smaller growth in work hours. In other words, the

⁴¹In Appendix E.3, we estimate the third equation of (14) alone. In this reduced-form exercise, we add many additional controls, including a rich set of demographic variables and the share of each demographic-locality group employed in manufacturing in 1980, well before the relevant movements in technology occurred. We show that our results are robust to including these controls.

 $^{^{42}\}mathrm{Our}$ industry classification includes 34 industries. See Appendix A.3 for details.

⁴³We show in Appendix E.2 that equation (16) can be derived from the definition of labor earnings $e_{iglt} = w_{iglt} \times h_{iglt}$ together with replacing the local growth rates $\frac{x_{iglt+1}}{x_{iglt}}$, for some variable x, by their nation-wide equivalent $\left(\frac{x_{igt+1}}{x_{igt}}\right)^{US}$.

preferences of the households are such that the income effect dominates.

	(1)	(2)
τ	0.361*** (0.045)	0.397*** (0.047)
$\eta - 1$	-0.629^{***} (0.009)	-0.281^{***} (0.080)
α^h	0.008*** (0.001)	0.008^{***} (0.001)
Instruments J -test: p -value Observations	N 0.006 10,469	Y 0.360 10,469

Results of iterative GMM estimation of (14). Whenever iterative procedure does not converge, two-step procedure is used. Standard errors account for an arbitrary correlation within education-age groups and regions. They are reported in parentheses. *, **, *** indicate significance at the 10%, 5%, and 1% levels, respectively. Column (2) uses Bartik-like instruments for wages and recreation prices. The "J-test: p-value" row reports p-values of Hansen's J-test of overidentifying restrictions.

Table 3: GMM estimation of the system (14).

Overall, the household-level results of Table 3 are similar to the cross-country estimates presented in Table 1. In both cases, the income effect of wages dominates the substitution effect, and cheaper recreation items are associated with fewer hours worked. These results also hold using instrumental variables. We therefore find that the joint impact of wages and recreation prices on work hours is consistent across levels of aggregation and identification strategies, and helps account for the recent evolution of leisure hours across demographic groups in the U.S.

We can use the estimates in Table 3 to quantify the importance of recreation prices for the increase in leisure inequality between two extreme groups in our sample: young individuals without a high-school degree and their older counterpart with more than a college degree. In 1990, young less-educated individuals worked 1153 hours per year, while older more-educated individuals worked 1718 annual hours, or 49% more. By 2016, the gap between these two groups had grown to 57% in the data, while the estimated model implies a gap of 59%. We find that recreation prices played a crucial role in driving this large increase in inequality. Without their impact, the model-implied gap would have declined from 49% in 1990 to 44% in 2016, so that according to the model recreation prices have been the key contributor to leisure inequality over the last decades, with wages actually pushing for more equal hours across demographic groups.⁴⁴

⁴⁴In 1990, the young and less-educated worked 1153 annual hours while the older and more-educated worked 1718 hours, or 49% more. In 2016, these numbers were 1111 and 1743, respectively, with a difference of 57%. The growths of wages and recreation prices over that period were $\Delta \log w_g = 0.013$ and $\Delta \log p_g = -0.547$ for the young less-educated workers, and $\Delta \log w_g = 0.133$ and $\Delta \log p_g = -0.297$ for old more-educated ones. Using the IV coefficients from Table 3, we find that the model-implied 2016 levels of hours are 1153 × exp (0.013 × (-0.281) – 0.547 × 0.397 + 0.189) = 1117, for young less-educated workers, where 0.189 is the accumulated impact of the constant over 27 years. The same number for the older educated workers is 1718 × exp (0.133 × (-0.281) – 0.297 × 0.397 + 0.189) = 1777, or 59% more. Repeating the same exercise without taking recreation prices into account leads to a 44% difference in 2016.

As in the multi-country exercise, we also provide, in Appendix E.3, a set of ordinary least-squares regressions that focus on the third equation of (14)—the one that captures the impact of wages and recreation prices on hours worked—to control for additional mechanisms. We also find that the income effect dominates and that cheaper recreation prices are associated with fewer hours. These findings are statistically and economically significant, and are robust to using our two instruments. We also show that the results are robust to including additional variables to control for the increase of offshoring and in disability over our sample. Overall, our results in this section confirm the importance of the rapid decline in recreation prices for the evolution of hours worked over the last decades.

6 Conclusion

In this paper we evaluate the contribution of the rapid fall in recreation prices observed in the data in recent decades toward the concurrent decline in work hours. To do so, we build a macroeconomic model in which recreation prices and wages affect labor supply decisions. So that our results do not hinge on a specific utility function, we provide a general specification of preferences that are consistent with balanced growth, and show that they imply a set of crossequation restrictions on the growth rates of wages, recreation prices, hours worked, and consumption of recreation and non-recreation goods and services. We estimate these relationships using country-level data and find that a large fraction of the decline in work hours can be attributed to the falling price of recreation goods and services. We conduct a similar exercise using household-level data in the United States and find that the impact of recreation prices is also visible at that level of aggregation. In addition, we find that the differential change in the price of recreation items consumed by different demographic groups is largely responsible for the increase in leisure inequality observed in the United States over the last decades. Our results are robust to various changes in specification, the inclusion of additional controls, and to using instruments.

One advantage of our modeling strategy is that it imposes few restrictions on the preferences of the household and instead leverages the discipline imposed by the balanced-growth assumption. But balanced-growth restrictions prevent us from modeling one-time changes in the environment, such as the entry of women into the labor force. An alternative modeling strategy would be to deviate from balanced growth and to explicitly include these one-time changes in the environment. It would be interesting to see if estimating such a model yields results similar to ours.

Another related direction for future inquiry involves more carefully modeling the allocation of time within the household. Recent evidence points to the growing importance of spending time with children, primarily among highly-educated households (Guryan et al., 2008; Ramey and Ramey, 2010; Dotti Sani and Treas, 2016). Accounting for these mechanisms should provide a more complete picture of the forces affecting labor supply.

Finally, recent evidence by Aguiar et al. (2021) shows that young men, in particular, increasingly devote the bulk of their time to recreational activities such as video games instead of working or attending school. Our evidence together with theirs suggests that declining recreation prices might disincentivize human capital accumulation, and thus slow down the movement towards a more highly-skilled workforce. Introducing this mechanism into macroeconomic models of skill acquisition, such as Kopytov et al. (2018), might improve their performance in matching the employment data. Exploring these forces in detail is an exciting avenue for future research.

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Appendix

A Data

This appendix lists the data sources and the steps taken to construct the datasets.

A.1 Aggregate time series for the United States

This appendix describes the aggregate data for the United States.

Prices Early data on real recreation prices comes from Owen (1970) (Table 4-B, pages 85-86, the data covers the period between 1901 and 1961). The data between 1935 and 1970 is from the Bureau of the Census (1975) (page 210, column 'Reading and recreation' divided by column 'All items'). Between 1967 and 1992, data on recreation prices comes from the BLS (series 'MUUR0000SA6'). Starting from 1993, the BLS provides a new series on recreation prices, encoded as 'CUUR0000SA6'. The BLS data is deflated using the all-item CPI series, encoded as 'CUUR0000SA0'.

Hours, wages and population Early data on average weekly hours is from the Bureau of the Census (1975) (series 'D765' and 'D803'). For the postwar sample, the data is available from the FRED website of the St. Louis Fed (series 'PRS85006023').

Early data on total hours worked is from Kendrick et al. (1961) (table A-X) and Kendrick et al. (1973) (table A-10). Early data on population by age comes from the U.S. Census (available at https://www.census.gov/data/tables/time-series/demo/popest/pre-1980-national.html). Recent data on hours worked and population are from ASEC. Following Cociuba et al. (2018), we compute average weighted annual hours worked using the variable 'ahrsworkt'. Population is constructed by summing 'asecwt'. For panel (d) of Figure 5, we construct wages by dividing total pre-tax wage and salary income ('incwage') by hours and aggregating them across individuals with different education levels.

Early data on labor productivity (wages) is from Kendrick et al. (1961) (table A-I; real gross national product, normalized by hours worked). From 1929, FRED provides data on compensation of employees (series 'A033RC1A027NBEA'), which we normalize by total hours worked and CPI (FRED series 'CPIAUCNS'), which is the same as reported by the BLS in 'CUUR0000SA0'.

Consumption and labor income To construct the consumption shares in Section 2.3, we use data from the NIPA tables. The consumption data is from Table 2.5.5 "Personal Consumption Expenditures by Function". Recreation consumption is the sum of rows 75, 77, 78, 82, 90, 91, 92, 93, 94. We subtract $\frac{\text{row } 76}{\text{row } 75 + \text{row} 76} \times \text{row } 77$ to exclude a computer-related component from row 77

("Services related to video and audio goods and computers"). Total consumption expenditures is row 1. Data on personal income is from Table 2.1 "Personal Income and Its Disposition". We use row 1 (total personal income) and row 2 (compensation of employees).

American Time Use Survey We use the American Time Use Survey for Figures 2 and 5. The variables are constructed as in Aguiar and Hurst (2007b) and Aguiar et al. (2021). We refer the reader to their extensive data description for further details.

A.2 Country-level data

Our international data comes primarily from the OECD and Eurostat. As mentioned in the main text, we restrict our sample to countries with at least 15 years of data on real recreation prices. Our final sample includes the 42 countries shown in Table 4. Below, we describe how the variables are constructed in more details.

Table 4 provides summary statistics for the countries in our sample.

Prices For the majority of countries, the price data is from the OECD database, category 'Prices and Purchasing Power Parities'. We use the series 'All items', 'Recreation and culture' and 'Furnishing, household equipment and routine household maintenance' for our analysis. For a few countries, longer price series are obtained from different sources. The U.S. price data is in Section A.3 below. For Australia, the data comes from the Australian Bureau of Statistics, Catalogue Number 6401.0. For Canada, the data comes from Statistics Canada, Table 18-10-0005-01. For a few European countries, the data comes from the Eurostat's Harmonized Index of Consumer Prices (HICP) dataset (which is also available at the OECD website). When several different series are available for the same variable (e.g., one from the OECD and one from Eurostat), we select the longer one.

Hours Hours data is from the Conference Board Total Economy Database.

Population and labor force statistics Population by age and sex is from the OECD database ('Demography and population' category). Labor force statistics by age and sex are from the OECD database ('Labour'-'Labour Force Statistics'-'LFS by sex and age').

Consumption, employee compensation and GDP These data are from the OECD and Eurostat. In the OECD database, it is available in the 'National Accounts' category. Total consumption expenditure is encoded as 'P31S14', recreation consumption is encoded as 'P31CP090', consumption of household items is encoded as 'P31CP050'. To obtain non-recreation consumption, we subtract recreation consumption from total consumption. Compensation of employees is

encoded as 'D1' and GDP is encoded as 'B1_GE'. Eurostat follows the same naming convention. When several different series are available for the same variable (one from the OECD and one from Eurostat), we select the longer one. All nominal series are deflated by all-item CPIs.

A.3 Cross-household data for the United States

We rely on three datasets to conduct our cross-household exercises. The price data comes from the Bureau of Labor Statistics. Data on hours worked and wages are from the U.S. Census and the American Community Surveys. The consumption data is from the Consumer Expenditure Survey.

Bureau of Labor Statistics The price data is from the Bureau of Labor Statistics (BLS). The all-item Consumer Price Index (CPI) series are encoded as 'CUUR0000SA0'. This series is used as deflator for all nominal variables. The recreation CPI series are encoded as 'CUUR0000SAR' and are available starting from 1993. Before 1993, we use the price index for the 'Entertainment' group, encoded as 'MUUR0000SA6', which is available between 1967 and 1997.

For our cross-household analysis in Section 5, we construct price indices for seven subcategories of recreation goods and services. The BLS changed their classification of goods and services in 1993; we try to map pre- and post-1993 price series as closely as possible to ensure consistency over time. Table 5 shows the price items that we use in the pre- and post-1993 periods. For a few subcategories (Other goods, Pets, Photo, Reading, Sports), the price series were not changed in 1993 and are available for the entire sample.⁴⁵ While there does not seem to be any major change in the "Other services" subcategory in 1993, there is no unique price series that covers the entire sample. We therefore smoothly paste the price indices 'SE62' (pre-1993) and 'SERF' (post-1993). For 'Audio-video' in the pre-1993 sample, we aggregate 'SE31' (video and audio products) with 'SE2703' (cable television) using the corresponding consumption shares from the Consumer Expenditure Surveys. We smoothly paste the resulting series with 'SERA' (post-1993) to get a price series over the entire sample.

Consumer Expenditure Survey For consumption categories, we follow Aguiar and Bils (2015) as closely as possible, so we refer the reader to their data construction section for a detailed description. One difference however is that we construct recreation consumption for seven different subcategories. In the CE, the consumption categories are coded using Universal Classification Codes, UCCs. Table 6 shows the UCCs corresponding to the seven recreation consumption subcategories.

Similarly to Aguiar and Bils (2015), we consider only households with reference persons of ages

⁴⁵In the post-1993 period, some of these subcategories feature a few new items (for example, veterinary services were added to the 'Pets' subcategory, encoded by 'SERB02'). We do not include these new additions to make the price indices as comparable across the pre- and post-1993 periods as possible.

between 25 and 64 that completed 4 quarterly interviews within a year. We exclude households with extremely large expenditure shares on generally small consumption categories. We exclude households with nonzero wage and salary income ('FSALARYX') and zero hours ('INC_HRS1' multiplied by 'INCWEEK1' plus 'INC_HRS2' multiplied by 'INCWEEK2'). We also exclude households with zero wage and salary income and nonzero hours. To construct consumption baskets across age-education groups, we use age and education of reference persons.

United States Census and American Community Survey Hours are measured as 'UHR-SWORK' multiplied by 'WKSWORK1'. When 'WKSROWK1' is unavailable (the ASC sample of 2014-2018), we use projected values of 'WKSWORK2' on 'WKSWORK1'. The measure of wage is 'INCWAGE'. Geographic regions are constructed using the cross-walk files from David Dorn's website (https://www.ddorn.net/data.htm). Industry classification is based on 'IND1990' and includes 34 industries: Agriculture, Forestry, and Fisheries; Mining; Construction; Manufacturing (19 subcategories); Transportation; Communications; Utilities and Sanitary Services; Wholesale Trade (2 subcategories); Retail Trade; Finance, Insurance, and Real Estate; Business and Repair Services; Personal Services; Entertainment and Recreation Services; Professional and Related Services; Public Administration.

For our cross-household regressions in Section 5, we also use disability indicators. The U.S. Census does not provide a consistent disability measure throughout our sample. For 1980, we use 'DISABWRK' that indicates whether respondents have any lasting condition that causes difficulty working. For 1990 and 2016, we use 'DIFFCARE' that indicates whether respondents have any lasting condition that causes difficulty to take care of their own personal needs, and 'DIFFMOB' that indicates whether respondents have any lasting condition that causes difficulty to perform basic activities outside the home alone.

	Hours	per capita	Hours p	er worker	Rea	l wages	Real rec	reation price
	Growth	Starting	Growth	Starting	Growth	Starting	Growth	Starting
	rate, $\%$	year	rate, $\%$	year	rate, $\%$	year	rate, $\%$	year
Australia	-0.15	1950	-0.41	1966	1.53	1959	-1.41	1989
Austria	-0.37	1950	-0.45	1994	1.61	1970	-1.17	1996
Belgium	-0.49	1950	-0.43	1983	1.54	1970	-1.20	1996
Brazil	-0.32	1950	-0.18	2001	4.14	2000	-2.28	2002
Bulgaria	0.66	1995	-0.27	2000	4.51	1997	-2.36	1997
Canada	-0.29	1950	-0.30	1976	0.90	1970	-0.95	1950
Costa Rica	-0.29	1987	-0.54	1987	2.31	1991	-3.56	1995
Croatia	0.39	1995	-0.66	2007	1.36	1998	-0.81	1998
Cyprus	-0.95	1995	-0.61	2000	1.86	1996	-1.43	1996
Czechia	-0.46	1993	-0.48	1993	2.55	1993	-1.56	1995
Denmark	-0.63	1950	-0.25	1983	1.85	1967	-1.29	1996
Estonia	-0.08	1995	-0.66	1995	4.92	1996	-1.79	1996
Finland	-0.86	1950	-0.70	1963	2.43	1970	-1.05	1996
France	-0.96	1950	-0.44	1975	3.02	1955	-1.81	1990
Germany	-0.98	1950	-0.55	1991	1.98	1970	-1.02	1991
Greece	-0.47	1950	-0.31	1983	1.78	1970	-1.34	1996
Hungary	-0.97	1980	-0.57	1992	1.84	1995	-1.72	1996
Iceland	-0.57	1964	-0.58	1991	2.46	1976	-0.94	1996
Ireland	-0.67	1950	-1.20	1961	2.70	1976	-0.40	1983
Israel	0.45	1981	-0.08	1985	0.99	1995	-1.74	1985
Italy	-0.52	1950	-0.35	1970	1.28	1970	-0.96	1996
Japan	-0.62	1950	-0.70	1968	1.74	1970	-0.57	1970
Korea	-0.49	1960	-1.35	1980	6.57	1970	-2.57	1985
Latvia	0.38	1995	-0.25	2000	5.43	1995	-2.04	1995
Lithuania	0.69	1995	0.28	2000	4.91	1995	-2.28	1993
Luxembourg	0.03 0.94	1970	0.90	1983	2.18	1970	-0.47	1995
Malta	-0.48	1994	-1.29	2000	$\frac{2.16}{2.16}$	1996	-1.57	1996
Mexico	0.22	1954 1950	-0.54	1991	-0.82	1970	-1.30	2003
Netherlands	-0.43	1950 1950	-0.34 -0.14	1971	$\frac{-0.32}{1.01}$	1969	-1.30 -1.37	1996
Norway	-0.45 -0.45	1950	-0.14 -0.38	1971 1972	$\frac{1.01}{2.35}$	1909	-1.57 -0.51	1979
Poland	$-0.45 \\ 0.04$	1930 1993	-0.38 -0.10	1972	$\frac{2.33}{2.67}$	1970	-0.51 -1.54	1979
Portugal	-0.23	1950	-0.10 -0.42	1993 1974	$\frac{2.07}{1.63}$	1993 1970	-1.04 -1.03	1955
0		1995	-0.42 -0.50	2000	6.54	1976		1996
Romania Russia	-0.96						-2.02	
	0.28	1992	-0.19	1992	4.27	1992	-1.81	2004
Slovakia	-0.65	1990	-0.51	1994	2.11	1993	-1.48	1996
Slovenia	-0.21	1995	-0.17	2000	1.81	1995	-0.92	1996
Spain	-0.72	1950	-0.72	1972	1.73	1970	-1.95	1996
Sweden	-0.32	1950	-0.26	1963	1.72	1970	-1.68	1980
Switzerland	-0.37	1950	-0.25	1991	1.28	1970	-0.86	1983
Turkey	-1.01	1970	0.07	1988	3.37	1998	-3.00	1996
United Kingdom United States	-0.46	1950	-0.35	1984	1.89	1970	-1.64	1988
	-0.06	1950	-0.32	1960	0.90	1970	-0.70	1950
Average	-0.32		-0.41		2.45		-1.48	

Columns "Growth rate [%]" report log-linear trend coefficients. The series are available between the starting year given in the "Starting year" column and 2018. The earliest starting year is 1950—the first year for hours worked in the Total Economy Database.

 ${\bf Table~4:~Summary~statistics~for~multi-country~sample.}$

	Pre-1993 code	Post-1993 code	Notes
Audio-video	SE31 and SE2703	SERA	SE31: Video and audio products SE2703: Cable television
Other goods	SE6101	SERE01	
Other services	SE62	SERF	
Pets	SE6103	SERB01	
Photo	SE6102	SERD01	
Reading	SE59	SERG	
Sports	SE60	SERC	

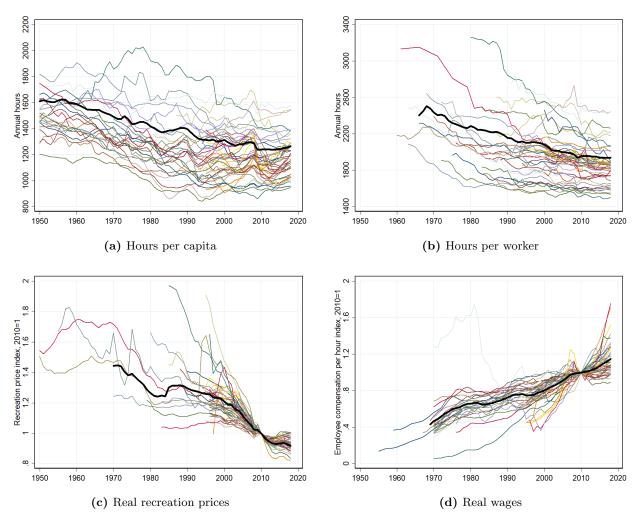
Table 5: Prices of recreation goods and services before and after 1993.

	Universal Classification Codes
Audio-video	270310, 270311, 310110-310350, 310400, 340610, 340902, 340905, 610130, 620904, 620912, 620930, 620916-620918
Other goods	610110, 610140, 610120, 610130
Other services	$610900\text{-}620111,\ 620121\text{-}620310,\ 620903$
Pets	610320,620410,620420
Photo	610210, 620330, 620906, 610230, 620320
Reading	660310, 590110-590230, 590310, 590410, 690118
Sports	$520901,520904,520907,600131,600132,600141,600142,\\600110-600122,600210-609999,620906-620909,620919-620922,620902,\\600127,600128,600137,600138$

 Table 6: Recreation consumption subcategories.

B Appendix for Section 2

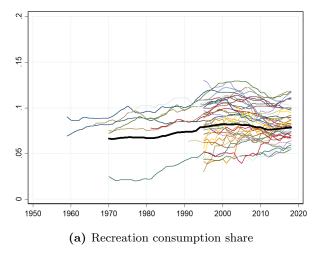
In the main text, we present the time series of hours worked, recreation prices and wages for a selected group of countries. Figure 8 shows the same graphs for the entire cross-section of 42 countries.



The bold black lines show the year fixed effects from regressions of the corresponding variable on a set of country and year fixed effects, with all countries included. Regressions are weighted by country-specific total hours. For panels (a) and (b), the levels of the lines are normalized to all-country weighted averages in 2015. Panel (a): Annual hours worked over population between 25 and 64 years old. Source: Total Economy Database and OECD. Panel (b): Annual hours worked over number of employed between 25 and 64 years old. Source: Total Economy Database and OECD. Panel (c): Price of consumption for OECD category "Recreation and culture", normalized by price index for all consumption items. Base year = 2010. Source: OECD, Eurostat, national statistical agencies. Panel (d): Real compensation of employees divided by hours worked. Base year = 2010. Source: OECD, Eurostat and Total Economy Database.

Figure 8: Hours, wages and recreation prices in the international sample.

Figure 9 shows the recreation consumption shares for the entire cross-section of countries. This share is fairly stable over time, consistent with the assumption behind our model. In the earlier part of the sample, the slight increase is mostly driven by South Korea quickly growing in the aftermath of the Korean War.



Fraction of recreation consumption in total consumption. Source: OECD and Eurostat. The bold black line shows the year fixed effects from regressions of the corresponding variable on a set of country and year fixed effects, with all countries included. Regression is weighted by country-specific total hours. The level of the lines is normalized to all-country weighted average in 2015.

Figure 9: Recreation consumption share in the international sample.

C Appendix for Section 3

This appendix contains additional exercises and derivations related to the model.

C.1 Production side of the economy

Our empirical analysis relies on the system of equations (11). As such, it is agnostic about how prices are determined in equilibrium as long as they grow at constant rates. In this section, we provide one example of a production structure that delivers these constant rates, and show how they depend on underlying productivity processes.

There are two competitive industries producing non-recreation and recreation goods c and d using Cobb-Douglas technologies

$$y_{jt} = A_{jt} l_{jt}^{\alpha} k_{jt}^{1-\alpha}, \tag{17}$$

where $j \in \{c, d\}$ denotes the industry, l_{jt} is labor, k_{jt} is capital and A_{jt} is Harrod-neutral total factor productivity. Consistent with our balanced-growth framework, we assume that A_{jt} grows at an exogenous rate $\gamma_{A_j} > 0$ for $j \in \{c, d\}$. Labor and capital are perfectly mobile across industries and their respective prices are w_t and R_t . Firms maximize profits

$$\Pi_{jt} = p_{jt}y_{jt} - w_t l_{jt} - R_t k_{jt},$$

where p_{jt} is the price of good j at time t. As before, we use non-leisure consumption as the numeraire so that $p_{ct} = 1$ for all t, and the price of leisure goods p_{dt} , the wage w_t and the interest rate R_t are measured in units of non-leisure goods.

Investment goods are produced by a competitive industry using the production function $y_{it} = Ak_{it}$. Since these goods trade at a price p_{it} , the investment sector maximizes profits

$$\Pi_{it} = p_{it}Ak_{jt} - R_tk_{jt}.$$

That sector is competitive such that $p_{it}A = R_t$ in equilibrium.

Market clearing implies that the demand for leisure and non-leisure goods is equal to their supply $y_{jt} = c_{jt}$ for $j \in \{c, d\}$. Similarly, the labor market clears, $h_t = l_{ct} + l_{dt}$, and so does the asset market $a_t = K_t$. The total stock of capital $K_t = k_{ct} + k_{lt} + k_{it}$ must also follow the law of motion

$$K_{t+1} = y_{it} + (1 - \delta) K_t,$$

where $0 < \delta < 1$ is the depreciation rate. Finally, the market rate of returns on assets has to equal the rental rate of capital net of depreciation, such that $r_t = R_t - \delta$.

We can now define an equilibrium in this economy.

Definition 2. A dynamic competitive equilibrium, is a time path of household's consumption, hours worked and asset position $\{c_t, d_t, h_t, a_t\}$; a time path for prices, wages, returns on asset and returns on capital $\{p_{dt}, p_{it}, w_t, r_t, R_t\}$ and a time path of factor allocations $\{l_{ct}, l_{dt}, k_{ct}, k_{dt}, k_{it}\}$ which satisfies household and firm optimization, perfect competition, resources constraints and market clearing.

The following proposition shows that, on a balanced-growth path, the growth rates of the leisure price p_{dt} and the wage w_t are constant and linked to the growth rates of the productivity processes A_c and A_d .

Proposition 2. On a balanced-growth path, the growth rates of p_{dt} and w_t are

$$\log \gamma_{p_d} = \log \gamma_{A_c} - \log \gamma_{A_d},$$

$$\log \gamma_w = \alpha \log \gamma_{A_c}.$$
(18)

This proposition shows that, since p_d is denominated in units of non-leisure goods, its growth rate captures how fast technological improvements occur in the leisure sector compared to the non-leisure sector. Similarly, productivity growth in the non-leisure sectors push wages higher.⁴⁶

Combining (18) with (11) provides the growth rates of c, d and h has functions of the primitives γ_{A_c} and γ_{A_d} .

⁴⁶While γ_{A_d} does not show up in the equation for γ_w , improvements in the leisure technology still lower p_d which increases the purchasing power of each unit of the wage.

C.2 Model with many types of consumption

While our main analysis focuses on two types of consumption goods (recreation and non-recreation), it is straightforward to extend our empirical framework to an arbitrary number of goods. To that end, denote by d_t an $n \times 1$ vector whose element d_{it} is the consumption of type i of goods and services at time t. There is another category of consumption, which we denote by c, that we use as numeraire and therefore handle separately from d for tractability. The maximization problem of the household is therefore

$$\sum_{t=0}^{\infty} \beta^t u\left(c_t, d_t, h_t\right),\,$$

subject to a budget constraint

$$c_t + \sum_{i} p_{it}d_{it} + b_{t+1} = w_t h_t + b_t (1 + r_t).$$

where p_{it} is the relative price of consumption item i, which is assumed to grow at a constant rate γ_{p_i} , to be consistent with our balanced growth framework. For this budget constraint to be satisfied at each point in time, it must be that the growth rates satisfy the restrictions

$$g_c = \gamma_{p_i} g_{d_i} = \gamma_w g_h, \tag{19}$$

for all i. In line with the discussion in Section 3.2, the preferences of the household also impose some restriction of the form

$$g_c = \gamma_w^{\eta} \prod_i \gamma_{p_i}^{\tau_i} \tag{20}$$

on the growth rates, where η and (τ_1, \ldots, τ_n) are preferences of the utility function. Taking together, (19) and (20) imply the following relationships between endogenous and exogenous growth rates in a balanced growth path

$$\log g_c = \eta \log \gamma_w + \sum_i \tau_i \log \gamma_{p_i},$$

$$\log g_{d_i} = \eta \log \gamma_w + \sum_{j \neq i} \tau_j \log \gamma_{p_j} + (\tau_i - 1) \log \gamma_{p_i},$$

$$\log g_h = (\eta - 1) \log \gamma_w + \sum_i \tau_i \log \gamma_{p_i},$$

for all i. We use these relationships in Section 4.2 to control for household items in our structural estimation.

D Appendix for Section 4

This appendix contains additional exercises related to the cross-country estimation of Section 4

D.1 Ordinary least-square estimation of the last equation in (12)

In this appendix, we provide a series of cross-country regressions to better highlight the relationship between hours worked and recreation prices. For each country i in our sample, we focus on a time period for which data on wages, hours and prices are available simultaneously and then compute the average annual growth rate in hours per capita $\Delta \log h_i$, wages $\Delta \log w_i$ and recreation prices $\Delta \log p_i$. As in the main text, we construct hours per capita using population between 20 and 74 years old. We also remove Great Recession years. We then estimate cross-sectional specification of the form

$$\Delta \log h_i = \beta_0 + \beta_p \Delta \log p_i + \beta_w \Delta \log w_i + \gamma X_i + \varepsilon_i, \tag{21}$$

where X_i includes some additional controls, ε_i is an error term and β_0 is a constant to absorb any aggregate changes. Note that this equation is an augmented version of the third equation in our model-derived system of structural equations (12).

The results are presented in Table 7. In the first column, we use real GDP per hour as a proxy for wages. That data is widely available and allows us to compute growth rates over longer time periods. In column (2), we use real employee compensation per hour for wages instead. In both cases, we see a positive association between the growth rates of hours per capita and recreation prices, which is consistent with individuals reducing their work hours to enjoy more leisure in the face of cheaper recreations goods and services. In the third column, we control for (the average annual growth of) female labor force participation which, as noted earlier, has been been an important driver of movements in hours per capita over the last century. In column (4), we also control for (the average annual growth of) the share of young men in the population to account for potential changes in behavior documented by Aguiar et al. (2021). In both cases, recreation prices remain significantly and positively associated with hours per capita. In contrast, we find small point estimates and no significant association between wages and hours per capita, which would be consistent with the substitution and income effects roughly offsetting each other.

D.2 Robustness regarding the structural estimation

In this appendix, we provide several robustness checks to our three-equation estimation exercise conducted in Section 4. The results are reported in Table 8. In columns (1) and (2), we construct hours per capita using only working age population (between 25 and 64 years old). In columns (3) and (4), we restrict the sample by focusing on countries with at least twenty years of data available. In columns (5) and (6) we use hours per worker as the measure of hours worked. Note

	(1)	(2)	(3)	(4)
	Dependent	variable: Gro	owth in hours p	per capita $\Delta \log h$
$\Delta \log p$	0.234**	0.240**	0.247**	0.229**
	(0.109)	(0.109)	(0.112)	(0.110)
$\Delta \log w$				
GDP per hour	0.071		0.078	0.069
	(0.074)		(0.070)	(0.075)
Empl. comp. per hour		0.051		
		(0.066)		
Female labor force part.			0.140	
1			(0.168)	
Share of young male in pop.				0.039
				(0.222)
R^2	0.110	0.096	0.144	0.111
Observations	42	42	42	42

Robust standard errors are in parentheses. *, ***, **** indicate significance at the 10%, 5%, and 1% levels, respectively. All variables are in growth rates. Growth rates are annual averages over all years except for 2008 and 2009. Population includes individuals between 20 and 74 years old.

Table 7: Cross-country regressions of hours per capita on recreation prices and wages.

that hours per worker only reflects the intensive adjustment margin of hours while hours in our model captures both the intensive and extensive margins. Finally, in columns (7) and (8) we add the Great Recession years when constructing growth rates. We find that in the majority of specifications, the association between growth rates of hours and recreation prices is significantly positive. We also find a dominating income effects in all specifications. These results are largely in line with the benchmark results given in Table 1.

	Working ago	Working age population	At least 20 y	At least 20 years of data	Hours pe	Hours per worker	With Great	With Great Recession
	(1)	(2)	(3)	(4)	(5)	(9)	(7)	(8)
Τ	0.307***	0.186** (0.079)	0.314^{***} (0.090)	0.191* (0.117)	0.580*** (0.197)	0.138 (0.128)	0.151^{**} (0.076)	0.125 (0.084)
$\eta - 1$	-0.467^{***} (0.072)	-0.407^{***} (0.053)	-0.571^{***} (0.057)	-0.757^{***} (0.066)	-0.411^{***} (0.060)	-0.588*** (0.059)	-0.181^{***} (0.042)	-0.273^{***} (0.036)
α^h	0.013^{***} (0.002)	0.009*** (0.002)	0.015^{***} (0.002)	0.015^{***} (0.002)	0.010^{***} (0.002)	0.007^{***} (0.002)	0.005^{***} (0.001)	0.006^{***} (0.001)
Wages J test: p -val. Obs.	$\begin{array}{c} \mathrm{GDP/hour} \\ 0.038 \\ \end{array}$	$\begin{array}{c} \text{Comp./hour} \\ 0.027 \\ 41 \end{array}$	GDP/hour 0.056 39	Comp./hour 0.035 39	GDP/hour 0.011 40	Comp./hour 0.011 40	GDP/hour 0.032 41	Comp./hour 0.045 41

Results of iterative GMM estimation of (12). Robust standard errors in parentheses. *, **, *** indicate significance at the 10%, 5%, and 1% levels, respectively. In the benchamrk analysis we focus on hours per capita for population between 20 and 74 years old and exclude 2008 and 2009 when constructing the variables. In columns (1) and (2), working age population (25-64 years old) is used. In columns (3) and (4), only countries with at least twenty years of data are included in the sample. In columns (5) and (6), hours per worker is used as the measure of hours worked. In columns (7) and (8), the Great Recession years of 2008 and 2009 are not excluded. The "J-test: p-value" row reports p-values of Hansen's J-test of overidentifying restrictions.

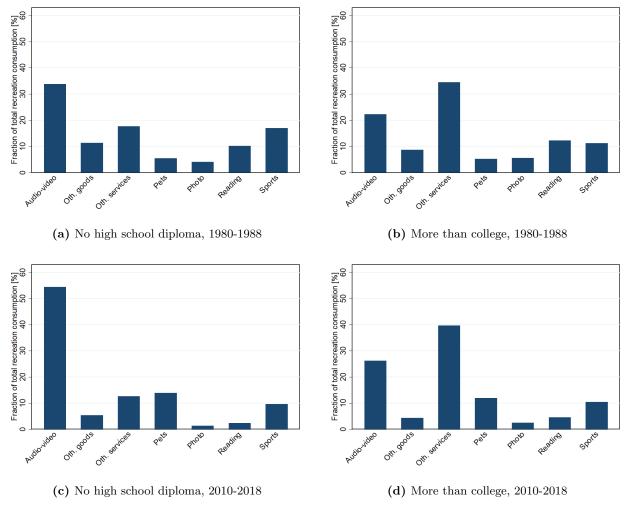
Table 8: GMM estimation of (12): Robustness.

E Appendix for Section 5

This appendix contains additional information and exercises related to the cross-household estimation of Section 5

E.1 Recreation consumption shares across education levels

Figure 10 shows how recreation consumption baskets vary by the level of education attainment of household heads. We do observe substantial variation, with households with low-educated heads consuming disproportionally more of "Audio-video" items, and households with highly-educated heads consuming disproportionally more of "Other services" items.



Shares of different items in total recreation consumption, constructed by pooling observations for the two periods, 1980-1988 and 2010-2018. Source: Consumer Expenditure Survey.

Figure 10: Share of recreation spending across education groups.

E.2 Derivation of equation (16)

We show here how to derive equation (16) in Section 5. We start from the definition of wages in a locality c for a demographic group d at time t:

$$w_{glt} = \frac{\sum_{i} e_{iglt}}{\sum_{i} h_{iglt}}.$$

It follows that we can write the growth rate of wages as

$$\frac{w_{glt+1}}{w_{glt}} = \frac{\frac{\sum_{i} e_{iglt+1}}{\sum_{i} e_{iglt}}}{\frac{\sum_{i} h_{iglt+1}}{\sum_{i} h_{iglt}}} = \frac{\sum_{i} \frac{e_{iglt}}{\sum_{j} e_{jglt}} \frac{e_{iglt+1}}{e_{iglt}}}{\sum_{i} \frac{h_{iglt}}{\sum_{j} h_{jglt}} \frac{h_{iglt+1}}{h_{iglt}}}.$$

The key idea behind our instrumental strategy is to replace the *local* growth in earnings and hours in the equation above by their national equivalent. We therefore write, after taking the log,

$$\Delta \log w_{glt}^{IV} = \log \left(\frac{w_{glt+1}}{w_{glt}}\right)^{IV} = \log \left(\sum_{i} \frac{e_{iglt}}{\sum_{j} e_{jglt}} \frac{e_{igt+1}}{e_{igt}}\right) - \log \left(\sum_{i} \frac{h_{iglt}}{\sum_{j} h_{jglt}} \frac{h_{igt+1}}{h_{igt}}\right)$$

We can also write that expression as

$$\begin{split} \Delta \log w_{glt}^{IV} &= \log \left(1 + \sum_{i} \frac{e_{iglt}}{\sum_{j} e_{jglt}} \frac{e_{igt+1} - e_{igt}}{e_{igt}} \right) - \log \left(1 + \sum_{i} \frac{h_{iglt}}{\sum_{j} h_{jglt}} \frac{h_{igt+1} - h_{igt}}{h_{igt}} \right) \\ &\approx \sum_{i} \frac{e_{iglt}}{\sum_{j} e_{jglt}} \frac{e_{igt+1} - e_{igt}}{e_{igt}} - \sum_{i} \frac{h_{iglt}}{\sum_{j} h_{jglt}} \frac{h_{igt+1} - h_{igt}}{h_{igt}} \\ &\approx \sum_{i} \frac{e_{iglt}}{\sum_{j} e_{jglt}} \Delta \log e_{igt+1} - \sum_{i} \frac{h_{iglt}}{\sum_{j} h_{jglt}} \Delta \log h_{igt+1} \end{split}$$

where we have used the fact that $\log(1+x) \approx x$ and so

$$\Delta \log x_{it+1} = \log x_{it+1} - \log x_{it} = \log \frac{x_{it+1}}{x_{it}} = \log \left(1 + \frac{x_{it+1} - x_{it}}{x_{it}} \right) \approx \frac{x_{it+1} - x_{it}}{x_{it}}.$$

E.3 Cross-household regressions

In this appendix, we provide OLS and IV estimates of the relation between the growth in hours per capita h and the growth in real recreation prices p and real wages w. The advantage of this reduced-form approach is that we can use flexible specifications and add various sets of control variables. Our main specification is

$$\Delta \log h_{gl} = \beta_0 + \beta_p \Delta \log p_g + \beta_w \Delta \log w_{gl} + \gamma X_{gl} + \varepsilon_{gl}, \tag{22}$$

where the subscripts g and l denote, respectively, demographic groups and localities. Note that this equation nests the last equation of the system (14) in isolation. We allow for a set of control variables X_{ql} that are specified below.

Over the past decades, manufacturing jobs have been moving overseas at the same time as technological improvements have led to cheaper recreation goods. These changes might have affected different demographic groups in different ways thereby creating a correlation with the consumption shares (e.g., Autor et al., 2006, Autor and Dorn, 2013, Bloom et al., 2019, Jaimovich and Siu, 2020). In particular, less-educated workers in the manufacturing sector have been disproportionately affected. While we believe that the presence of wages in equation (22) largely takes care of any potential endogeneity, in some specifications we also control for the share of each demographic-locality group employed in manufacturing in 1980, well before the relevant movements in technology occurred, to make sure that these trends are not driving our results.

In addition, it is possible that certain changes in demographic attributes between t=1 and t=2 might affect both hours worked and, at the same time, be correlated with the consumption shares. For instance, the rise in disability benefits over the last decades might have had a negative impact on hours worked, be correlated with the consumption shares of low-education people, and not be controlled for by the other covariates on the right-hand side of (22) (see Abraham and Kearney, 2020 for an overview). For that reason, we also include a set of additional demographic controls in some specifications. These controls are, for each location-demographic group, the fractions of males, whites, married and people with disabilities (see Section A.1 for the description of the disability control construction). We control for the 1980 values of these fractions, as well as for their growth rates between 1990 and 2016.⁴⁷

The outcome of the estimation is presented in Table 9, where the first three columns refer to ordinary-least square regressions and the last three columns take advantage of our two instruments. In all cases, the F-statistics are large, suggesting that the instruments are strong. In columns (2)-(3) and (5)-(6) we allow for additional demographic controls.⁴⁸ Columns (3) and (6) also control for the share of manufacturing hours in each demographic group in 1980.⁴⁹

In all specifications, an increase in recreation prices is associated with an increase in work hours. The coefficients are strongly statistically and economically significant with a decline in real

⁴⁷Recall that to construct recreation prices for different demographic groups we use the household-level CE data, while our measures of wages and hours from the Census are at the individual level. In Appendix E.3.1, we redo the same regressions using hours and wage data for all household heads and married household heads, with additional controls for the number of kids, and find very similar results.

⁴⁸Changes in some of demographic variables between t = 1 and t = 2 (such as fractions of married individuals and people with disabilities) might be affected by the treatment themselves and so might be "bad controls". To address this issue, we also run the same regressions by only including the 1980 values of demographic controls. The results are largely unchanged.

⁴⁹Standard errors are clustered at the locality level. The results for recreation prices and wages stay significant at the 5% level if we cluster standard errors at both the locality and demographic group levels. We prefer not to perform double clustering in our main analysis because we have only 15 demographic groups.

	(1): OLS	(2): OLS	(3): OLS	(4): IV	(5): IV	(6): IV
	Γ	Dependent var	riable: Growt	h in hours pe	er capita $\Delta \log$	gh
$\Delta \log p$	0.427^{***} (0.025)	0.474^{***} (0.036)	0.204*** (0.041)	0.763*** (0.047)	0.761*** (0.062)	0.466*** (0.066)
$\Delta \log w$	-0.048^{***} (0.015)	-0.093^{***} (0.013)	-0.094*** (0.013)	-0.713^{***} (0.074)	-0.539*** (0.070)	-0.529^{***} (0.068)
1980 manuf. hours			-0.285^{***} (0.023)			-0.286^{***} (0.025)
Locality F.E.	Y	Y	Y	Y	Y	Y
Addtl. dem. cont.	N	Y	Y	N	Y	Y
F-statistics		_	_	145.1	124.7	124.8
R^2	0.304	0.452	0.469			_
Observations	10,469	10,469	10,469	10,469	10,469	10,469

Standard errors clustered at the locality level in parentheses. *,**,*** indicate significance at the 10%, 5%, and 1% levels, respectively. F-statistics are Kleibergen-Paap. The regressions are across people sorted by locality/education-age group. Columns marked by "IV" use Bartik-like instruments for wages and recreation prices. Controls include manufacturing hours share in 1980 and a rich set of additional demographic controls (see text for details).

Table 9: Regressions of hours per capita on recreation prices and wages across locality/demographic-sorted households.

recreation prices of 1 percent associated with a 0.47 percent decline in hours worked in our preferred specification (column 6). Importantly, this effect is even stronger in the IV regressions, which are less subject to the endogeneity issues. We also find a significant effect of wages on hours worked. In all the specifications that we consider, the income effect dominates, so that an increase in wages is associated with a decline in work hours. The magnitude of this effect, however, changes markedly across specifications. In the OLS regressions (columns 1 to 3), the impact of wages is quite weak while the IV estimation finds a stronger effect. This is consistent with the fact that both measured recreation prices and wages are endogenous. For instance, a technological improvement in leisure goods drives down the effective price of a unit of leisure, but by incentivizing workers to supply less labor in order to enjoy more leisure time it puts upward pressure on observed wages. This might mute the initial reduction in hours, thus pushing the coefficient on wages towards zero. Using the shift-share instrument allows us to isolate the impact of exogenous changes in wages on labor supply that is not contaminated by such equilibrium effects.

Additional demographic controls have only a limited impact on the coefficients in Table 9, but increase the explanatory power significantly, suggesting that their impact on work hours might be orthogonal to that of wages and recreations prices. Adding 1980 manufacturing employment shares as a control somewhat lowers the recreation price coefficient while leaving the wage coefficient unaffected.

E.3.1 Using household heads instead of all individuals

In the baseline analysis, our measures of wages and hours from the Census are at the individual level. The CE data, however, is at the household level, and we use the demographic characteristics of reference persons to construct demographic-specific consumption baskets. In this Appendix, we construct hours and wages using the Census data on the household heads only (variable 'RE-LATE'=1). To control for potentially very different consumption and labor supply choices across married and non-married household heads, we run regressions 22 for all and married only household heads separately. Demographic controls include the 1980 shares of male, white, household heads with disabilities within each demographic-locality bin, as well as the 1990-2016 changes in these variables. In addition, we also control for the number of co-living children by computing the 1980 shares and the 1990-2016 changes in shares of household heads co-living with one, two, or more children below 18 years old. Table 10 shows the results.

We also redo GMM estimation of system 14 for all and married only household heads. Results are given in Table 11. Crucially, the sign and magnitude of τ are quite similar to our baseline findings.

	(1): OLS	(2): OLS	(3): IV	(4): IV	(5): OLS	(e): OLS	(7): IV	(8): IV
			Dependent	Dependent variable: Growth in hours per capita $\Delta \log h$	ı in hours per ca	pita $\Delta \log h$		
		All house	ll household heads			Marrie	Married only	
$\Delta \log p$	0.440*** (0.030)	0.350***	0.563***	0.533***	0.564***	0.491*** (0.028)	0.640***	0.593***
$\Delta \log w$	-0.074^{***} (0.013)	-0.080^{***} (0.013)	-0.218^{***} (0.060)	-0.343^{***} (0.067)	-0.061^{***} (0.012)	-0.066*** (0.012)	-0.152^{***} (0.046)	-0.232^{***} (0.051)
1980 manuf. hours		-0.126^{***} (0.020)		-0.169*** (0.025)		-0.099*** (0.019)		-0.124^{***} (0.022)
Locality F.E. Addtl. dem. cont. F -statistics R^2 $\#$ observations	Y Y — 0.405 9,458	Y Y — 0.409 9,458	Y Y 105.4 — 9,458	Y Y 82.8 — 9,458	Y Y — 0.390 8,233	Y Y — 0.393 8,233	Y Y 125.6 — 8,233	Y Y 103.2 — 8,233

Standard errors clustered at the locality level are in parentheses. *, *, *, *, *, * indicate significance at the 10%, 5%, and 1% levels, respectively. F-statistics are Kleibergen-Paap. The regressions are across people sorted by locality/education-age group. Columns marked by "IV" use Bartik-like instruments for wages and recreation prices. Controls include manufacturing hours share in 1980 and a rich set of additional demographic controls (see text for details).

Table 10: Regressions of hours per capita on recreation prices and wages across locality/demographic-sorted households.

	All househ	old heads	Marrie	d only
	(1)	(2)	(3)	(4)
$\overline{\tau}$	0.424*** (0.100)	0.376*** (0.102)	0.489*** (0.104)	0.492*** (0.106)
$\eta - 1$	-0.634^{***} (0.007)	0.204** (0.097)	-0.667^{***} (0.014)	-0.583^{***} (0.099)
$lpha^h$	0.003 (0.002)	0.003 (0.002)	0.002 (0.002)	0.002 (0.002)
Instruments J -test: p -value Observations	N 0.009 9,458	Y 0.141 9,458	N 0.009 8,233	Y 0.039 8,233

Results of iterative GMM estimation of (14). Whenever iterative procedure does not converge, two-step procedure is used. Standard errors account for an arbitrary correlation within education-age groups and regions. They are reported in parentheses. *, **, *** indicate significance at the 10%, 5%, and 1% levels, respectively. Columns (2) and (4) use Bartik-like instruments for wages and recreation prices. The "J-test: p-value" row reports p-values of Hansen's J-test of overidentifying restrictions.

Table 11: GMM estimation of the system (14).

F Proofs

This section contains the formal results establishing restrictions on the shape of the utility function so that it be consistent with a balanced-growth path. The proofs follow mostly the same steps as Boppart and Krusell (2020) but must take care of an additional variable in the utility function.

The proof of Proposition 1 relies on the following two lemmata.

Lemma 1. If u(c, h, d) satisfies (6) and (7) for all t > 0, $\gamma_w > 0$ and $\gamma_{p_d} > 0$, and for arbitrary c > 0, w > 0 and $p_d > 0$, then its marginal rate of substitution functions, defined by $u_h(c, h, d) / u_c(c, h, d)$ and $u_d(c, h, d) / u_c(c, h, d)$ must be of the form

$$\frac{u_h\left(c,h,d\right)}{u_c\left(c,h,d\right)} = \frac{c}{h}x\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right) \tag{23}$$

and

$$\frac{u_d\left(c,h,d\right)}{u_c\left(c,h,d\right)} = \frac{c}{d}y\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right) \tag{24}$$

where x and y are arbitrary functions, and η and τ are arbitrary numbers.

Proof. We beginning by showing how to derive (23). Set t = 0 in (6) to find $-u_h(c, h, d) / u_c(c, h, d) = w$. Using that equation with (6) yields

$$\frac{u_h\left(c\lambda^{\eta}\mu^{\tau}, h\lambda^{\eta-1}\mu^{\tau}, d\lambda^{\eta}\mu^{\tau-1}\right)}{u_c\left(c\lambda^{\eta}\mu^{\tau}, h\lambda^{\eta-1}\mu^{\tau}, d\lambda^{\eta}\mu^{\tau-1}\right)} = \lambda \frac{u_h\left(c, h, d\right)}{u_c\left(c, h, d\right)}.$$
(25)

where we denote $\lambda = \gamma_w^t$ and $\mu = \gamma_{p_d}^t$ to simplify the expression. This equation must hold for every

 λ and μ .⁵⁰ For any given c and h, set $\lambda = h/c$ and $\mu = (c^{1-\eta}h^{\eta})^{-1/\tau}$. These imply that $c\lambda^{\eta}\mu^{\tau} = 1$, $h\lambda^{\eta-1}\mu^{\tau} = 1$ and $d\lambda^{\eta}\mu^{\tau-1} = dh^{\frac{\eta}{\tau}}c^{-1+\frac{1}{\tau}(1-\eta)}$. From (25), we can therefore write

$$\frac{u_h\left(1,1,dh^{\frac{\eta}{\tau}}c^{-1+\frac{1}{\tau}(1-\eta)}\right)}{u_c\left(1,1,dh^{\frac{\eta}{\tau}}c^{-1+\frac{1}{\tau}(1-\eta)}\right)} = \frac{h}{c}\frac{u_h\left(c,h,d\right)}{u_c\left(c,h,d\right)}.$$

Now, define the function $x(t) = \frac{u_h(1,1,t^{1/\tau})}{u_c(1,1,t^{1/\tau})}$. We can rewrite this last equation as (23) which is the result.

We now turn to (24). Set t = 0 in (7) to find $u_d(c, h, d) / u_c(c, h, d) = p_d$. Combining with (7) yields

$$\frac{u_d\left(c\lambda^{\eta}\mu^{\tau}, h\lambda^{\eta-1}\mu^{\tau}, d\lambda^{\eta}\mu^{\tau-1}\right)}{u_c\left(c\lambda^{\eta}\mu^{\tau}, h\lambda^{\eta-1}\mu^{\tau}, d\lambda^{\eta}\mu^{\tau-1}\right)} = \mu \frac{u_d\left(c, h, d\right)}{u_c\left(c, h, d\right)}$$

$$(26)$$

where again $\lambda = \gamma_w^t$ and $\mu = \gamma_{p_d}^t$. Since this most old for any λ and μ , Set $\mu = d/c$ and $\lambda = (d^{\tau}c^{1-\tau})^{-1/\eta}$ to find that $c\lambda^{\eta}\mu^{\tau} = 1$, $d\lambda^{\eta}\mu^{\tau-1} = 1$ and $h\lambda^{\eta-1}\mu^{\tau} = hd^{\frac{\tau}{\eta}}c^{-1+(1-\tau)\frac{1}{\eta}}$. We can therefore write (26) as

$$\frac{u_d\left(1, hd^{\frac{\tau}{\eta}}c^{-1+(1-\tau)\frac{1}{\eta}}, 1\right)}{u_c\left(1, hd^{\frac{\tau}{\eta}}c^{-1+(1-\tau)\frac{1}{\eta}}, 1\right)} = \mu \frac{u_d\left(c, h, d\right)}{u_c\left(c, h, d\right)}$$

Now, define the function $y(t) = \frac{u_h(1,t^{1/\eta},1)}{u_c(1,t^{1/\eta},1)}$. We can rewrite this last equation as (24) which completes the proof.

We now turn to a Lemma that characterizes the second derivatives of u.

Lemma 2. Under Definition 1, the second derivative of u must satisfy

$$-\frac{cu_{cc}(c, h, d)}{u_c(c, h, d)} = z_1 \left(c^{1 - \eta - \tau} h^{\eta} d^{\tau} \right)$$
 (27)

$$-\frac{hu_{ch}(c, h, d)}{u_c(c, h, d)} = z_2 \left(c^{1-\eta-\tau} h^{\eta} d^{\tau}\right)$$
 (28)

$$-\frac{du_{cd}(c, h, d)}{u_{c}(c, h, d)} = z_{3} \left(c^{1-\eta-\tau} h^{\eta} d^{\tau}\right)$$
 (29)

for arbitrary functions z_1 , z_2 and z_3 .

Proof. Since (8) must hold for all t, we can differentiate it with respect to t, divide the differentiated equation by (8) and set t = 0. Doing so we find

Thanging μ and λ involves changing a mixture of t, γ_w and γ_p . Changing t is innocuous as Definition 1 must hold for every t. Changing γ_w or γ_p would affect the interest rate r, but r does not show up here.

$$\frac{u_{cc}\left(c,h,d\right)c\log\left(\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau}\right)+u_{ch}\left(c,h,d\right)h\log\left(\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau}\right)+u_{cd}\left(c,h,d\right)d\log\left(\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}{u_{c}\left(c,h,d\right)}=\\\frac{u_{cc}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau}\log\left(\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau}\right)}{u_{c}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}\\+\frac{u_{ch}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau}\log\left(\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau}\right)}{u_{c}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}\\+\frac{u_{cd}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\log\left(\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}{u_{c}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}.$$

Now differentiating (23) and (24) with respect to c, we find that $h \frac{u_{hc}(c,h,d)}{u_c(c,h,d)}$ and $d \frac{u_{dc}(c,h,d)}{u_c(c,h,d)}$ are functions of $c^{1-\eta-\tau}h^{\eta}d^{\tau}$ and $\frac{u_{cc}(c,h,d)}{u_c(c,h,d)}c$ only. We can write

$$h \frac{u_{hc}(c, h, d)}{u_{c}(c, h, d)} = f_{1} \left(c^{1-\eta-\tau} h^{\eta} d^{\tau}, \frac{u_{cc}(c, h, d)}{u_{c}(c, h, d)} c \right)$$
$$d \frac{u_{dc}(c, h, d)}{u_{c}(c, h, d)} = f_{2} \left(c^{1-\eta-\tau} h^{\eta} d^{\tau}, \frac{u_{cc}(c, h, d)}{u_{c}(c, h, d)} c \right)$$

and, since these equations holds for any c, h and d,

$$\begin{split} &h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau}\frac{u_{hc}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}{u_{c}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)} = f_{1}\left(c^{1-\eta-\tau}h^{\eta}d^{\tau},\frac{u_{cc}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}{u_{c}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau}\right)\\ &d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\frac{u_{dc}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}{u_{c}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}=f_{2}\left(c^{1-\eta-\tau}h^{\eta}d^{\tau},\frac{u_{cc}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}{u_{c}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau},h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau},d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau}\right). \end{split}$$

Plugging into (30) implies that

$$\frac{u_{cc}\left(c,h,d\right)c}{u_{c}\left(c,h,d\right)} = f_{3}\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}, \frac{u_{cc}\left(c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau}, h\gamma_{w}^{\eta-1}\gamma_{p_{d}}^{\tau}, d\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau-1}\right)}{u_{c}\left(c\gamma_{w}^{1-\nu}, h\gamma_{w}^{-\nu}, d\gamma_{w}^{-\tilde{\gamma}g(\nu)}\right)}c\gamma_{w}^{\eta}\gamma_{p_{d}}^{\tau}\right), \tag{31}$$

where f_3 is an arbitrary function. This equation must hold for every γ_w and γ_p (r would also need to be adjusted, but r does not show up here). We can therefore set $\gamma_w = 1$ and $\gamma_p = 1$, and we find that $\frac{u_{cc}(c,h,d)c}{u_c(c,h,d)}$ only depends on $c^{1-\eta-\tau}h^{\eta}d^{\tau}$.

Proposition 1. The utility function u(c, h, d) is consistent with a balanced-growth path (Definition

1) if and only if (save for additive and multiplicative constants) it is of the form

$$u(c, h, d) = \frac{\left(c^{1-\varepsilon} d^{\varepsilon} v \left(c^{1-\eta-\tau} h^{\eta} d^{\tau}\right)\right)^{1-\sigma} - 1}{1-\sigma}$$

$$\tag{9}$$

for $\sigma \neq 1$, or

$$u(c, h, d) = \log\left(c^{1-\varepsilon}d^{\varepsilon}\right) + \log\left(v\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)\right) \tag{10}$$

for $\sigma = 1$, and where v is an arbitrary twice continuously differentiable function and where $\eta > 0$ and $\tau > 0$.

Proof. We first consider the "if" direction of the proof and then turn to the "only if" part. Consider the case with $1 - \eta - \tau \neq 0$. From Lemma 2 we have

$$\frac{\partial \log \left(u_c\left(c,h,d\right)\right)}{\partial \log \left(c\right)} = -z_1 \left(\exp \left(\left(1-\eta-\tau\right)\log \left(c\right)+\eta \log \left(h\right)+\tau \log \left(d\right)\right)\right). \tag{32}$$

Integrating with respect to $\log c$ we find that

$$u_{c}(c, h, d) = f_{4}\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)m_{1}(h, d)$$
(33)

where f_4 is a new function of $c^{1-\eta-\tau}h^{\eta}d^{\tau}$, and m_1 is an arbitrary function of h and d.

Now we can restrict m_1 since, from Lemma 2, $\frac{hu_{hc}(c,h,d)}{u_c(c,h,d)}$ and $\frac{du_{dc}(c,h,d)}{u_c(c,h,d)}$ are also only functions of $c^{1-\eta-\tau}h^{\eta}d^{\tau}$. Taking the derivative of (32) with respect to h, multiplying by h and dividing by u_c we obtain

$$\frac{hu_{hc}(c,h,d)}{u_{c}(c,h,d)} = \frac{f_{4}'\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)c^{1-\eta-\tau}h^{\eta}d^{\tau}\eta}{f_{4}\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)} + \frac{hm_{1,h}(h,d)}{m_{1}(h,d)}.$$

Similarly, we can take the derivative of (32) with respect to d, multiplying by d and dividing by u_c to find

$$\frac{du_{dc}(c,h,d)}{u_{c}(c,h,d)} = \frac{f_{4}'\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)c^{1-\eta-\tau}h^{\eta}d^{\tau}\tau}{f_{4}\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)} + \frac{dm_{1,d}(h,d)}{m_{1}(h,d)}.$$

So that $\frac{hu_{hc}(c,h,d)}{u_c(c,h,d)}$ and $\frac{du_{dc}(c,h,d)}{u_c(c,h,d)}$ only depend on $c^{1-\eta-\tau}h^{\eta}d^{\tau}$, it must be that $\frac{hm_{1,h}(h,d)}{m_1(h,d)}$ and $\frac{dm_{1,d}(h,d)}{m_1(h,d)}$ are constants and therefore $m_1(h,d) = A_2h^{\kappa}d^{\iota}$. We can rewrite (33) as

$$u_c(c, h, d) = f_4\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)A_2h^{\kappa}d^{\iota}. \tag{34}$$

Since $1 - \eta - \tau \neq 0$ we can rewrite that equation as

$$u_c(c, h, d) = f_5\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right)A_2h^{\kappa}d^{\iota}.$$

We can integrate this equation with respect to c to find

$$u(c, h, d) = f_6 \left(ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\iota - \frac{\tau}{1 - \eta - \tau}} + m_2(h, d)$$
 (35)

where f_6 is another arbitrary function.

To further restrict $m_2(h, d)$, we combine Lemma 1 together with (34) to find

$$u_{h}(c, h, d) = f_{7}\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right)A_{2}h^{\kappa-1-\frac{\eta}{1-\eta-\tau}}d^{\nu-\frac{\tau}{1-\eta-\tau}}$$
(36)

and

$$u_d(c, h, d) = f_8 \left(c h^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) A_2 h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\iota - 1 - \frac{\tau}{1 - \eta - \tau}}$$
(37)

where f_7 and f_8 are appropriately defined functions.

We can now compare the derivatives of u, from (35), to these last two expressions. First, taking the derivative of (35) with respect to h we find

$$\begin{split} u_{h}\left(c,h,d\right) &= f_{9}\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right)h^{\kappa-\frac{\eta}{1-\eta-\tau}-1}d^{\iota-\frac{\tau}{1-\eta-\tau}}\frac{\eta}{1-\eta-\tau} \\ &+ f_{6}\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right)h^{\kappa-\frac{\eta}{1-\eta-\tau}-1}d^{\iota-\frac{\tau}{1-\eta-\tau}}\left(\kappa-\frac{\eta}{1-\eta-\tau}\right) + m_{2,1}\left(h,d\right) \\ &= f_{10}\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right)h^{\kappa-\frac{\eta}{1-\eta-\tau}-1}d^{\iota-\frac{\tau}{1-\eta-\tau}} + m_{2,1}\left(h,d\right) \end{split}$$

For this to work with (36) for all c, h and d, it must be that $m_{2,1}(h,d) = A_3 h^{\kappa - \frac{\eta}{1-\eta-\tau}-1} d^{\iota - \frac{\tau}{1-\eta-\tau}}$. Similarly, taking the derivative of (35) with respect to d we find

$$u_{d}\left(c,h,d\right) = f_{11}\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right)h^{\kappa-\frac{\eta}{1-\eta-\tau}}d^{\iota-\frac{\tau}{1-\eta-\tau}-1} + m_{2,2}\left(h,d\right)$$

For this to work with (37), it must be that $m_{2,2}(h,d) = A_4 h^{\kappa - \frac{\eta}{1-\eta-\tau}} d^{\iota - \frac{\tau}{1-\eta-\tau}-1}$.

We can integrate $m_{2,1}$ and $m_{2,2}$ to find m. Let us first handle the case with $\kappa \neq \frac{\eta}{1-\eta-\tau}$ and $\iota \neq \frac{\tau}{1-\eta-\tau}$. Integrating, we find

$$m_{2}(h,d) = A_{5}h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\iota - \frac{\tau}{1 - \eta - \tau}} + g_{3}(d)$$

$$m_{2}(h,d) = A_{6}h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\iota - \frac{\tau}{1 - \eta - \tau}} + g_{4}(h)$$
(38)

For these two equations to be jointly true it must be that $A_5 = A_6$, and that g_3 and g_4 are the same constant. That constant can be set arbitrarily as it does not affect choices. In this case, we can merge m_2 in (35) and find

$$u(c, h, d) = f_{12} \left(ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) h^{\kappa - \frac{\eta}{1 - \eta - \tau}} d^{\iota - \frac{\tau}{1 - \eta - \tau}} + A_7.$$
(39)

Since $\eta \neq 0$, we can write

$$u(c, h, d) = f_{13} \left(ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) c^{1 - \kappa \frac{1 - \eta - \tau}{\eta}} d^{\iota - \frac{\tau}{\eta} \kappa} + A_7.$$

which is equivalent to

$$u(c, h, d) = \frac{\left(c^{1-\varepsilon} d^{\varepsilon} v \left(c^{1-\eta-\tau} h^{\eta} d^{\tau}\right)\right)^{1-\sigma} - 1}{1-\sigma}$$

$$\tag{40}$$

where

$$(1 - \sigma)(1 - \varepsilon) = 1 - \kappa \frac{1 - \eta - \tau}{\eta}$$
$$(1 - \sigma)\varepsilon = \iota - \frac{\tau}{\eta}\kappa$$

If instead $\kappa = \frac{\eta}{1-\eta-\tau}$, integrating $m_{2,1}(h,d) = A_3 h^{\kappa-\frac{\eta}{1-\eta-\tau}-1} d^{\iota-\frac{\tau}{1-\eta-\tau}}$ yields

$$m_2(h,d) = A_5 d^{l-\frac{\tau}{1-\eta-\tau}} \log h + g_3(d),$$
 (41)

and if $\iota = \frac{\tau}{1-\eta-\tau}$, integrating $m_{2,2}(h,d) = A_4 h^{\kappa-\frac{\eta}{1-\eta-\tau}} d^{\iota-\frac{\tau}{1-\eta-\tau}-1}$ yields

$$m_2(h,d) = A_6 h^{\kappa - \frac{\eta}{1 - \eta - \tau}} \log d + g_4(h).$$
 (42)

If only one of $\kappa = \frac{\eta}{1-\eta-\tau}$ or $\iota = \frac{\tau}{1-\eta-\tau}$ is true, it must be that $m_2 = A_7$, where A_7 is a constant. Suppose that only $\kappa = \frac{\eta}{1-\eta-\tau}$, (35) becomes

$$u\left(c,h,d\right) = f_{6}\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right)d^{\iota-\frac{\tau}{1-\eta-\tau}} + A_{7}$$

so we find (40) with

$$\varepsilon = 1$$

$$1 - \sigma = \iota - \frac{\tau}{1 - \eta - \tau}.$$

If only $\iota = \frac{\tau}{1-\eta-\tau}$, (35) becomes

$$u\left(c,h,d\right) = f_{6}\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right)h^{\kappa-\frac{\eta}{1-\eta-\tau}} + m_{2}\left(h,d\right)$$

which we can rewrite as

$$u(c, h, d) = f_{14} \left(ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) c^{1 - \kappa \frac{1 - \eta - \tau}{\eta}} d^{\frac{\tau}{1 - \eta - \tau} - \frac{\tau}{\eta} \kappa} + m_2(h, d)$$

so we find (40) with

$$(1 - \sigma)(1 - \varepsilon) = 1 - \kappa \frac{1 - \eta - \tau}{\eta}$$
$$(1 - \sigma)\varepsilon = \frac{\tau}{1 - \eta - \tau} - \frac{\tau}{\eta}\kappa$$

If both $\kappa = \frac{\eta}{1-\eta-\tau}$ and $\iota = \frac{\tau}{1-\eta-\tau}$ it must be, from (41) and (42), that

$$m_2(h,d) = A_8 \log h + A_9 \log d + A_7,$$

in which case we can write (35) as

$$u(c, h, d) = f_6\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right) + A_8\log h + A_9\log d + A_7.$$

We can use

$$\log\left(ch^{\frac{\eta}{1-\eta-\tau}}d^{\frac{\tau}{1-\eta-\tau}}\right) = \log c + \frac{\eta}{1-\eta-\tau}\log h + \frac{\tau}{1-\eta-\tau}\log d,$$

to write

$$u(c, h, d) = f_{15} \left(ch^{\frac{\eta}{1 - \eta - \tau}} d^{\frac{\tau}{1 - \eta - \tau}} \right) + A_8 \frac{1 - \eta - \tau}{\eta} \log c + \left(A_9 - A_8 \frac{\tau}{\eta} \right) \log d + A_7.$$

Since the utility function is invariant to multiplication by a constant we can normalize the sum of the powers on c and d to 1, and get

$$u(c, h, d) = (1 - \varepsilon) \log c + \varepsilon \log d + \log v \left(c^{1 - \eta - \tau} h^{\eta} d^{\tau}\right). \tag{43}$$

We now turn to the case in which $1 - \eta - \tau = 0$.

We now turn to the case with $1 - \eta - \tau = 0$. The characterization of u_{cc} in Lemma 2 can be written as

$$\frac{\partial \log \left(u_{c}\left(c,h,d\right)\right)}{\partial \log \left(c\right)}=-z_{1}\left(h^{\eta}d^{\tau}\right).$$

Integrating with respect to $\log c$ we find that

$$\log(u_c(c, h, d)) = -\log(c) z(h^{\eta} d^{\tau}) + m_3(h, d)$$
(44)

where m_3 is an arbitrary function of h and d. Differentiating with respect to h and multiplying by h yields

$$\frac{hu_{ch}(c, h, d)}{u_{c}(c, h, d)} = -\log(c) z'(h^{\eta} d^{\tau}) \eta h^{\eta} + hm_{3,1}(h, d).$$
(45)

Similarly, differentiating with respect to d and multiplying by d yields

$$\frac{du_{cd}(c, h, d)}{u_c(c, h, d)} = -\log(c) z'(h^{\eta} d^{\tau}) \tau d^{\tau} + dm_{3,2}(h, d).$$
(46)

From Lemma 2 we know that $\frac{hu_{hc}(c,h,d)}{u_c(c,h,d)}$ and $\frac{du_{dc}(c,h,d)}{u_c(c,h,d)}$ are only functions of $h^{\eta}d^{\tau}$. For (45) and (46) to hold true for every c it must therefore be that $z'(h^{\eta}d^{\tau}) = 0$ (note that a and b cannot both be equal to 0 since $1 - \eta - \tau = 0$) so that $z = -\sigma$ is a constant. Similarly, it must be that $hm_{3,1}(h,d) = g_5(h^{\eta}d^{\tau})$ and $dm_{3,2}(h,d) = g_6(h^{\eta}d^{\tau})$. Integrating, we find that $m_3(h,d) = f_{16}(h^{\eta}d^{\tau})$ for some function f_{16} . By exponentiating on both sides of (44), we can therefore rewrite

$$u_c(c, h, d) = c^{-\sigma} m_4(h^{\eta} d^{\tau}). \tag{47}$$

We can integrate this equation with respect to c to find

$$u(c, h, d) = \frac{(cv(h^{\eta}d^{\tau}))^{1-\sigma} - 1}{1 - \sigma} + m_5(h, d)$$
(48)

if $\sigma \neq 1$, or

$$u(c, h, d) = m_4(h^{\eta} d^{\tau}) \log(c) + \log(v(h^{\eta} d^{\tau}))$$
(49)

otherwise.

For the case with $\sigma \neq 1$, combine (47) with Lemma 1 that

$$u_h(c, h, d) = \frac{1}{h} x (h^{\eta} d^{\tau}) c^{1-\sigma} m_4 (h^{\eta} d^{\tau})$$

and

$$u_{d}\left(c,h,d\right)=\frac{1}{d}y\left(h^{\eta}d^{\tau}\right)c^{1-\sigma}m_{4}\left(h^{\eta}d^{\tau}\right).$$

Differentiating (48) yields

$$u_h(c, h, d) = (cv(h^{\eta}d^{\tau}))^{-\sigma} cv'(h^{\eta}d^{\tau}) a \frac{h^{\eta}d^{\tau}}{h} + m_{5,1}(h, d)$$

and

$$u_d(c, h, d) = (cv(h^{\eta}d^{\tau}))^{-\sigma} cv'(h^{\eta}d^{\tau}) b \frac{h^{\eta}d^{\tau}}{d} + m_{5,2}(h, d).$$

Since $\sigma \neq 1$ it must be that m_5 is a constant that can be set to 0 as it does not affect decisions. (48) is therefore a special case of (40).

For the case with $\sigma = 1$, we can again combine (47) with Lemma 1 to find the two equations

$$u_h\left(c,h,d\right) = \frac{1}{h}x\left(h^{\eta}d^{\tau}\right)m_4\left(h^{\eta}d^{\tau}\right)$$
$$u_d\left(c,h,d\right) = \frac{1}{d}y\left(h^{\eta}d^{\tau}\right)m_4\left(h^{\eta}d^{\tau}\right).$$

Differentiating (49) yields

$$\begin{split} u_h\left(c,h,d\right) &= m_4'\left(h^{\eta}d^{\tau}\right)a\frac{h^{\eta}d^{\tau}}{h}\log\left(c\right) + \frac{v'\left(h^{\eta}d^{\tau}\right)}{v\left(h^{\eta}d^{\tau}\right)}a\frac{h^{\eta}d^{\tau}}{h}\\ u_d\left(c,h,d\right) &= m_4'\left(h^{\eta}d^{\tau}\right)b\frac{h^{\eta}d^{\tau}}{d}\log\left(c\right) + \frac{v'\left(h^{\eta}d^{\tau}\right)}{v\left(h^{\eta}d^{\tau}\right)}b\frac{h^{\eta}d^{\tau}}{d}. \end{split}$$

For these equations to be consistent it must be that m_4 is a constant so we find (43) again.

This completes the proofs that if u satisfies Definition 1 then it must be of the form (9)–(10). We now show that if u is defined as (9)–(10) then Definition 1 is also satisfied.

First notice that if we evaluate the function $c^{1-\eta-\tau}h^{\eta}d^{\tau}$ along a balanced-growth path, i.e. at a point $\left(c_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau}\right)^t,h_0\left(\gamma_w^{\eta-1}\gamma_{p_d}^{\tau}\right)^t,d_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau-1}\right)^t\right)$, we get

$$\left(c_0 \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau} \right)^t \right)^{1 - \eta - \tau} \left(h_0 \left(\gamma_w^{\eta - 1} \gamma_{p_d}^{\tau} \right)^t \right)^{\eta} \left(d_0 \left(\gamma_w^{\eta} \gamma_{p_d}^{\tau - 1} \right)^t \right)^{\tau} = c_0^{1 - \eta - \tau} h_0^{\eta} d_0^{\tau}.$$

In other words, $c^{1-\eta-\tau}h^{\eta}d^{\tau}$ is invariant along a balanced-growth path.

The derivatives of u are

$$\begin{split} u_h &= \left(c^{1-\varepsilon}d^{\varepsilon}v\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)\right)^{-\sigma}c^{1-\varepsilon}d^{\varepsilon}v'\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)\eta\frac{c^{1-\eta-\tau}h^{\eta}d^{\tau}}{h} \\ u_d &= \left(c^{1-\varepsilon}d^{\varepsilon}v\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)\right)^{-\sigma}\left(\varepsilon\frac{c^{1-\varepsilon}d^{\varepsilon}}{d}v\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right) + c^{1-\varepsilon}d^{\varepsilon}v'\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)\tau\frac{c^{1-\eta-\tau}h^{\eta}d^{\tau}}{d}\right) \\ u_c &= \left(c^{1-\varepsilon}d^{\varepsilon}v\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)\right)^{-\sigma}\times \\ &\left(\left(1-\varepsilon\right)\frac{c^{1-\varepsilon}d^{\varepsilon}}{c}v\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right) + c^{1-\varepsilon}d^{\varepsilon}v'\left(c^{1-\eta-\tau}h^{\eta}d^{\tau}\right)\left(1-\eta-\tau\right)\frac{c^{1-\eta-\tau}h^{\eta}d^{\tau}}{c}\right) \end{split}$$

Taking the ratio of u_h and u_c and evaluating the expression at a point on a balanced-growth path, $\left(c_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau}\right)^t, h_0\left(\gamma_w^{\eta-1}\gamma_{p_d}^{\tau}\right)^t, d_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau-1}\right)^t\right)$, we find that

$$\frac{u_h}{u_c} = \frac{v'\left(c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right)\eta c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}}{\left(1-\varepsilon\right)v\left(c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right) + v'\left(c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right)\left(1-\eta-\tau\right)c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}}\frac{c_0}{h_0}\gamma_w^t$$

so that u_h/u_c grows at rate γ_w and so (6) is satisfied.⁵¹

Similarly, taking the ratio of u_d and u_c and evaluating the expression at $\left(c_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau}\right)^t, h_0\left(\gamma_w^{\eta-1}\gamma_{p_d}^{\tau}\right)^t, d_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau-1}\right)^t\right)$ we find

$$\frac{u_d}{u_c} = \frac{\left(\varepsilon v \left(c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right) + v'\left(c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right)\tau c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right)}{\left(\left(1-\varepsilon\right)v \left(c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right) + v'\left(c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right)\left(1-\eta-\tau\right)c_0^{1-\eta-\tau}h_0^{\eta}d_0^{\tau}\right)} \frac{c_0}{d_0}\gamma_{p_d}^t$$

so that u_d/u_c grows at rate γ_{p_d} and (7) is satisfied.

Finally, dividing u_c evaluated at $\left(c_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau}\right)^t, h_0\left(\gamma_w^{\eta-1}\gamma_{p_d}^{\tau}\right)^t, d_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau-1}\right)^t\right)$ by u_c evaluated at $\left(c_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau}\right)^{t+1}, h_0\left(\gamma_w^{\eta-1}\gamma_{p_d}^{\tau}\right)^{t+1}, d_0\left(\gamma_w^{\eta}\gamma_{p_d}^{\tau-1}\right)^{t+1}\right)$ we find

$$\frac{u_c}{u_c'} = \gamma_w^{\eta\sigma} \gamma_{p_d}^{\tau - (1 - \sigma)(\tau - \varepsilon)}$$

which is an expression independent of c, d and h, as required by 8, and that defines r.

Proposition 2. On a balanced-growth path, the growth rates of p_{dt} and w_t are

$$\log \gamma_{p_d} = \log \gamma_{A_c} - \log \gamma_{A_d},$$

$$\log \gamma_w = \alpha \log \gamma_{A_c}.$$
(18)

Proof. The first-order conditions of the firms are

$$\alpha p_{jt} y_{jt} = w_t l_{jt} \tag{50}$$

and

$$(1 - \alpha) p_{it} y_{it} = R_t k_{it} \tag{51}$$

so that

$$\frac{\alpha}{1-\alpha}R_t\left(k_{ct}+k_{dt}\right) = w_t\left(l_{ct}+l_{dt}\right) \tag{52}$$

and

$$\frac{l_{ct}}{k_{ct}} = \frac{l_{dt}}{k_{dt}} = \frac{l_{ct} + l_{dt}}{k_{ct} + k_{dt}}.$$

Combining (51) for j = c with $p_{it}A = R_t$, the production function (17) and using the fact that $p_{ct} = 1$ yields the price of investment

$$p_{it} = (1 - \alpha) \frac{A_{ct}}{A} \left(\frac{l_{ct} + l_{dt}}{k_{ct} + k_{dt}} \right)^{\alpha}.$$

$$(53)$$

Note that by Definition 1 we can adjust h_0 to match the wage so that $-u_h/u_c$ matches the arbitrary wage w. This requires $v' \neq 0$, but if v' = 0 hours does not enter the utility function and the only possible wage is w = 0.

With $p_{it}A = R_t$, this equation also pins down the interest rate

$$R_t = (1 - \alpha) A_{ct} \left(\frac{l_{ct} + l_{dt}}{k_{ct} + k_{dt}} \right)^{\alpha}.$$

$$(54)$$

Doing the same operations with j = d instead, and combining with (53) we find that the price of recreation goods and services, measured in units of non-recreation prices, is the ratio of sector c and sector d productivities:

$$p_{dt} = \frac{A_{ct}}{A_{dt}}.$$

It follows that the growth rate γ_{p_d} of p_{dt} is such that $\log \gamma_{p_d} = \log \gamma_{A_c} - \log \gamma_{A_d}$.

Combining (54) with (52) yields

$$R_t^{1-\alpha} = (1-\alpha) A_{ct} \left(\frac{\alpha}{1-\alpha} \frac{1}{w_t} \right)^{\alpha}.$$

Since the first-order conditions of the household imply a constant R_t , this last equation yields that

$$\log \gamma_w = \alpha \log \gamma_{A_c},$$

which completes the proof.