The Origin of Risk

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What drives individual risk-taking decisions and how do they affect aggregate risk?

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- Because of endogenous risk, distortions can make GDP more volatile

We calibrate the model to the Spanish economy

- Removing distortions leads to a large decline in aggregate volatility

A model of endogenous risk

Environment

Static model with two types of agents

1. A representative household owns the firms, supplies labor and risk management resources
   - Risk mgmt. resources: land, managers, raw materials, lobbyists, etc.
2. N firms produce differentiated goods using labor and intermediate inputs
   - Firms are competitive and take all prices and aggregate quantities as given.
   - Firm i has a constant returns to scale Cobb-Douglas production function

▸ ζ

$$F(\delta_i, L_i, X_i) = e^{a_i(\varepsilon, \delta_i)} \zeta_i L_i^{1-\sum_{j=1}^{N} \alpha_{ij}} \prod_{j=1}^{N} X_{ij}^{\alpha_{ij}}$$

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- Resource cost g_i: land, managers, raw materials, etc. (available at price W_R)

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$$b_i(\delta_i) = \frac{1}{2}(\delta_i - \delta_i^\circ)^\top B_i (\delta_i - \delta_i^\circ), \text{ and } g_i(\delta_i) = \frac{1}{2}(\delta_i - \delta_i^\circ)^\top G_i (\delta_i - \delta_i^\circ)$$

where δ_i° is the *natural* risk exposure ($b_i, g_i = 0$), and B_i and G_i are positive definite matrices

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Owns the firms, supplies one unit of labor inelastically, supplies risk management resources

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Maximizes King, Plosser, Rebelo (1988) preferences

$$\mathcal{U}(Y)\,\mathcal{V}(R)$$

where $\mathcal{U}$ is CRRA with risk aversion $\rho \geq 1$, and disutility of risk management $\mathcal{V}(R)$ is Details

$$\mathcal{V}(R) = \exp\left(-\eta\left(1-\rho\right)R\right)$$

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Budget constraint in each state of the world (set $W_L = 1$ from now on)

$$\sum_{i=1}^{N} P_i C_i \leq W_L + W_R R + \Pi$$

Timing and distortions

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$$\text{Unit cost} := K_i(\delta_i, P) = \frac{1}{e^{a_i(\varepsilon,\delta_i)}} \prod_{j=1}^{N} P_j^{\alpha_{ij}}$$

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   - Maximize expected discounted profits

$$\delta_i^* \in \arg\max_{\delta_i} \mathrm{E}\left[\Lambda\left[P_i Q_i - K_i\left(\delta_i, P\right) Q_i - g_i\left(\delta_i\right) W_R\right]\right]$$

   where Q_i is *equilibrium* demand and Λ is the stochastic discount factor of the household.

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- Prices are set at a constant wedge τ_i over marginal cost K_i: $P_i = \left(1 + \tau_i\right) K_i\left(\delta_i, P\right)$
  - Example: markups, taxes, or other distortions

Equilibrium definition

An *equilibrium* is a risk choice for every firm δ^* and a stochastic tuple $(P^*, W_R^*, C^*, L^*, R^*, X^*, Q^*)$ such that

1. (Optimal technique choice) For each i, factor demand L_i^*, X_i^* and R_i^*, and the risk exposure decision δ_i^* solves the firm's problem.
2. (Consumer maximization) The consumption vector C^* and the supply of risk managers R^* solve the household problem.
3. (Unit cost pricing) For each i, $P_i = (1 + \tau_i) K_i (\delta_i, P)$.
4. (Market clearing) For each i,

$$C_i^* + \sum_{j=1}^{N} X_{ji}^* = Q_i^* = F_i(\alpha_i^*, L_i^*, X_i^*), \quad \sum_{i=1}^{N} L_i^* = 1, \text{ and } \sum_{i=1}^{N} g_i(\delta_i^*) = R^*.$$

Domar weights and GDP

Two measures of supplier importance

Cost-based Domar weights:

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- Captures firm's importance as a supplier (share of production costs)

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Revenue-based Domar weights:

$$\omega^\top = \beta^\top \left(I - [\operatorname{diag}(1+\tau)]^{-1} \alpha\right)^{-1}$$

- Also captures importance as a supplier (share of revenues)
- Declines with wedges τ
- Are equal to the firm's sales share in nominal GDP

$$\omega_i = \frac{P_i Q_i}{PY}$$

Determinants of GDP

Define the aggregate risk exposure vector Δ as

$$\Delta := \delta^\top \tilde{\omega}$$

- Firms with high cost-based Domar weights contribute more to aggregate risk exposure

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Lemma

(log) real GDP $y := \log Y$ is given by

$$y = \Delta^\top \varepsilon - \tilde{\omega}^\top b(\delta) - \tilde{\omega}^\top \log(1+\tau) - \log(\text{Labor share}(\omega, \tau))$$

- log GDP y is normal; aggregate risk exposure Δ determines how risky GDP is
- Without distortions ($\tau = 0$) we have Hulten's theorem: $y = \omega^\top a(\varepsilon, \delta)$

Firm risk-taking decisions

Lemma

In equilibrium, the risk exposure decision δ_i of firm i solves

$$\underbrace{\mathcal{E} K_i Q_i}_{\substack{\text{marginal benefit} \\ \text{of exposure to } \varepsilon}} = \underbrace{\nabla b_i(\delta_i) K_i Q_i + \nabla g_i(\delta_i) W_R}_{\text{marginal cost of exposure to } \varepsilon},$$

where the marg. value of risk exposure per unit of size is defined as $\mathcal{E} := \mathrm{E}[\varepsilon] + \mathrm{Cov}[\lambda, \varepsilon]$.

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- Determinants of $\mathcal{E}$
  - High expected value factors and countercyclical factors have higher $\mathcal{E}$
  - We say that a risk factor is “good” if $\mathcal{E} > 0$ and “bad” if $\mathcal{E} < 0$

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  - We say that a risk factor is "good" if $\mathcal{E} > 0$ and "bad" if $\mathcal{E} < 0$
- Impact of firm size $K_i Q_i$
  - Marginal benefit and marginal productivity cost b_i of exposure scale one-for-one with size
  - The resource cost g_i is scale invariant $\Rightarrow$ Scale advantage in risk management
    - Data: larger firms are more likely to 1) have CRO, 2) implement Enterprise-wide Risk Management systems, etc.

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- We can rewrite the marginal cost

$$\nabla b_i(\delta_i) K_i Q_i + \nabla g_i(\delta_i) W_R = \nabla h_i(\delta_i) K_i Q_i$$

where the effective exposure cost h_i is defined as

$$h_i(\delta_i) := \frac{1}{2}(\delta_i - \delta_i^\circ)^\top H_i (\delta_i - \delta_i^\circ), \text{ with } H_i := B_i + G_i \frac{W_R}{K_i Q_i}.$$

Determinant of firm size

Cost of goods sold $K_i Q_i$ matters for risk decisions

$$K_i Q_i = \frac{P_i Q_i}{1 + \tau_i}$$

- Higher sales $P_i Q_i \Rightarrow$ Higher $K_i Q_i$
  - Pinned down by demand for goods from the household (β) and other firms (α) through ω_i
- Lower wedge $\tau_i \Rightarrow$ Lower price $P_i \Rightarrow$ Higher demand $Q_i \Rightarrow$ Higher $K_i Q_i$
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Corollary

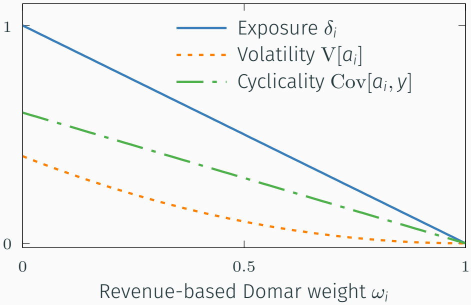
Firms with higher ω_i and lower τ_i manage risk more aggressively:

$$\frac{\partial \left[\mathcal{E}^\top \delta_i\right]}{\partial \omega_i} > 0 \quad \text{and} \quad \frac{\partial \left[\mathcal{E}^\top \delta_i\right]}{\partial \tau_i} < 0.$$

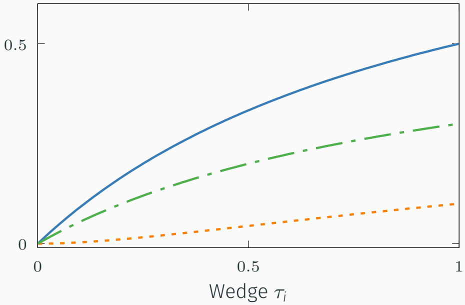
Example: sales, wedges, and risk exposure

Economy is positively exposed ($\Delta > 0$) to a unique bad ($\mathcal{E} < 0$) risk factor (business cycle risk)

(a) Impact of ω_i on risk



(b) Impact of τ_i on risk



Existence, uniqueness and efficiency

Planner's problem

Planner wants to achieve risk exposure Δ. What is the cheapest utility cost of doing so?

▸ Detail

$$\bar{h}_{SP}(\Delta) = \underbrace{\bar{b}_{SP}(\Delta)}_{\substack{\text{best aggregate} \\ \text{TFP cost}}} + \underbrace{\bar{g}_{SP}(\Delta)}_{\substack{\text{best util. loss from} \\ \text{risk resources}}}$$

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Proposition (Planner's problem)

$$\mathcal{W}_{SP} := \max_{\Delta} \underbrace{\Delta^{\top}\mu - \bar{b}_{SP}(\Delta)}_{\mathbb{E}[y_{SP}]} - \frac{1}{2}(\rho - 1)\underbrace{\Delta^{\top}\Sigma\Delta}_{\mathbb{V}[y_{SP}]} - \bar{g}_{SP}(\Delta),$$

The planner prefers aggregate risk exposure vectors Δ with

- 1) high expected GDP $\mathbb{E}[y_{SP}]$, 2) low GDP volatility $\mathbb{V}[y_{SP}]$, and 3) low risk mgmt. resource cost $\bar{g}_{SP}$

Equilibrium characterization through fictitious planner

Proposition (fictitious planner's problem)

There exists a unique equilibrium, and it solves

$$\mathcal{W}_{dist} := \max_{\Delta} \underbrace{\Delta^\top \mu - \bar{b}(\Delta) - \tilde{\omega}^\top \log(1+\tau) - \log \Gamma_L}_{\mathbf{E}[y]} - \frac{1}{2}(\rho - 1)\underbrace{\Delta^\top \Sigma \Delta}_{\mathbf{V}[y]} - \bar{g}(\Delta)$$

The equilibrium solves a distorted planning problem

- Still seeks to maximize $\mathbf{E}[y]$ and minimize $\mathbf{V}[y]$
- But higher perceived cost of managing risk ($\bar{g}$ instead of $\bar{g}_{SP}$)

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- Benefit of exposure $\mathcal{E}$ from firm problem coincides with social benefit
- Perceived cost $\bar{h}$ is weighted average of firm individual costs H_i

Impact of changes in risk factors

Corollary

1. An increase in μ_m raises Δ_m
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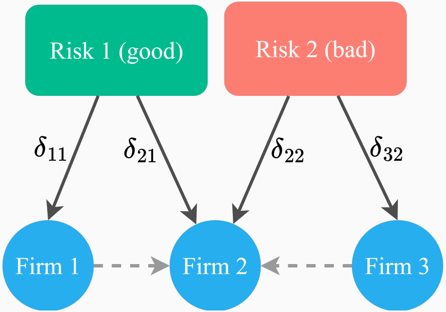
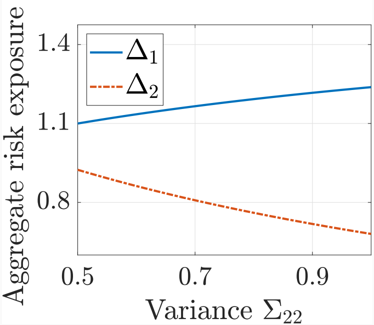
Firm 2 must decide where to locate plants: Region 1 (good risk) or Region 2 (bad risk)





▸ general case

Definition. An economy is diagonal if Σ and H_i are diagonal for every i

Impact of wedges

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In a diagonal economy, a higher wedge τ_i

1. increases Δ_m for all m such that $\mathcal{E}_m < 0$ (bad risks)
2. reduces Δ_m for all m such that $\mathcal{E}_m > 0$ (good risks)

- Higher wedges make firms shrink $\rightarrow$ manage risk less aggressively

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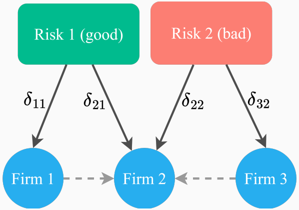

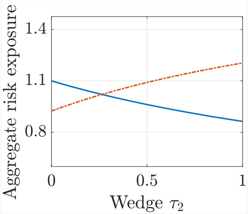

(Blue: good risk; Red: bad risk)

Implications for GDP and Welfare

Use ∂ to denote changes in the economy with exogenous risk

Proposition

In a diagonal economy:

$$\operatorname{sign}\left(\frac{d\,\mathrm{E}[y]}{d\mu_m} - \frac{\partial\,\mathrm{E}[y]}{\partial\mu_m}\right) = \operatorname{sign}(\mu_m) \quad \text{and} \quad \frac{d\,\mathrm{V}[y]}{d\Sigma_{mm}} - \frac{\partial\,\mathrm{V}[y]}{\partial\Sigma_{mm}} < 0.$$

- Increasing μ_m raises Δ_m $\rightarrow$ additional increase in $\mathrm{E}[y]$ if $\mu_m > 0$ compared to fixed risk
- Increasing Σ_{mm} decreases $|\Delta|$ $\rightarrow$ smaller increase in $\mathrm{V}[y]$ than with fixed risk

Distortions can increase aggregate volatility

Proposition (single risk factor)

$$\mathrm{sign}\left(\frac{d\,\mathrm{E}[y]}{d\tau_i} - \frac{\partial\,\mathrm{E}[y]}{\partial\tau_i}\right) = -\mathrm{sign}(\mu\mathcal{E}) \qquad \text{and} \qquad \mathrm{sign}\left(\frac{d\,\mathrm{V}[y]}{d\tau_i} - \frac{\partial\,\mathrm{V}[y]}{\partial\tau_i}\right) = -\mathrm{sign}(\Delta\mathcal{E})\,.$$

Suppose $\mathcal{E} < 0$ (bad risk, e.g. business cycle): increasing τ_i makes firms more exposed to risk factor

- if $\mu < 0$ this leads to a decline in $\mathrm{E}[y]$
- if $\Delta > 0$ the economy becomes even more exposed and $\mathrm{V}[y]$ increases

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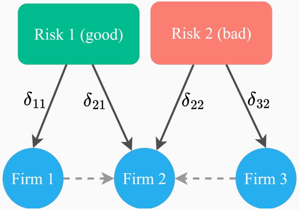

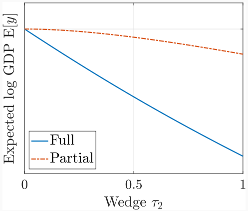

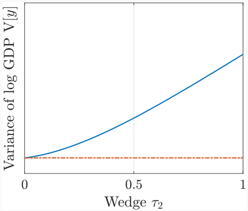

Implications for welfare

Proposition

In a diagonal economy, raising τ_i hurts welfare more than under exogenous risk.

- A higher τ_i increases exposure to bad risks and reduces exposure to good risks
- Additional exposure to bad risks hurts welfare, and vice-versa for good risks

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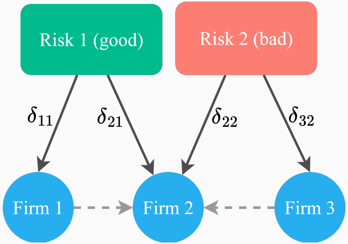
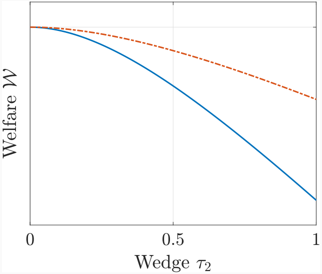




(Blue: flexible risk; Red: fixed risk)

Reduced-form evidence

Reduced-form evidence

Model: firms with large Domar weights and small markups are less volatile and less corr. with GDP

Details

Reduced-form evidence

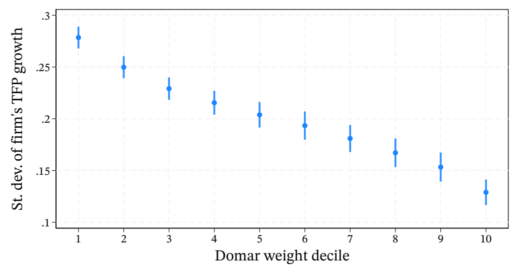
Model: firms with large Domar weights and small markups are less volatile and less corr. with GDP

▸ Details

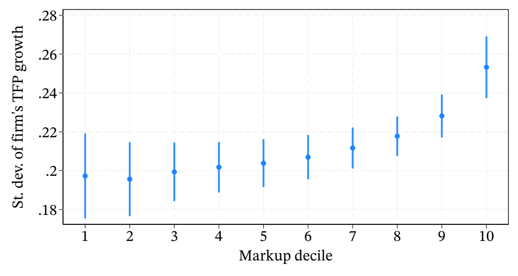
We test these predictions in the data

- Use detailed micro data from the near-universe of firms in Spain between 1995 and 2018 (Orbis) (7,513,081 firm-year observations)
- Compute markups using control function approach (De Loecker and Warzynski, 2012) ▸ Details
- Back out TFP growth as a residual

TFP growth volatility



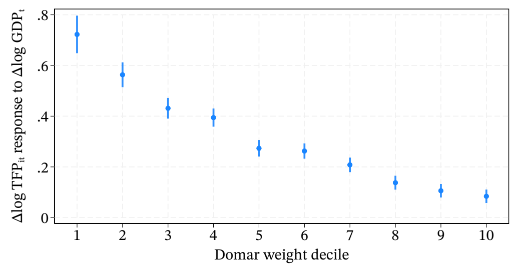
(a) TFP volatility by Domar weight decile



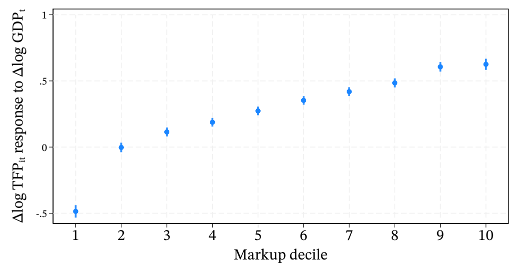
(b) TFP volatility by markup decile

Details

Covariance of TFP growth with GDP growth



(c) Sensitivity of firm TFP to GDP by Domar weight decile



(d) Sensitivity of firm TFP to GDP by markup decile

▸ Details

Calibration

Mapping to the data

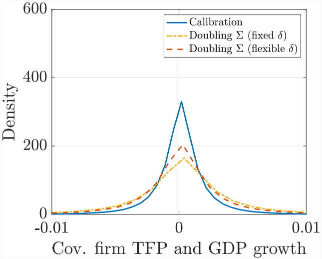
- We aim at replicating as much of the firm-level Spanish data as possible
- Our calibrated model has 62 sectors and 492,917 individual firms
- We invert parts of the model to exactly match some moments
  1. Sectoral consumption shares and input/output cost shares
  2. Firm shares in sectoral sales
  3. Variance of firm TFP growth
  4. Covariance of firm TFP growth and GDP growth
  5. Variance of GDP growth

▸ Model ▸ Details

Doubling Σ

What if we double the volatility Σ of the risk factor?

	Calibration	Doubling Σ	
		Fixed δ	Flexible δ
Agg. risk exposure Δ	0.014	0.014	0.011
Exposure value $\mathcal{E}$	-0.06	-0.11	-0.09
Std. Dev. of GDP growth	2.4%	3.1%	2.6%



- Fixed δ: Large increase in GDP variance; exposure to ε_t becomes more harmful ($\mathcal{E}$ declines)
- Flexible δ: Firms manage risk more aggressively which limits increase in $\mathbb{V}[y]$

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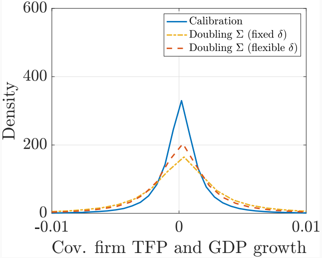


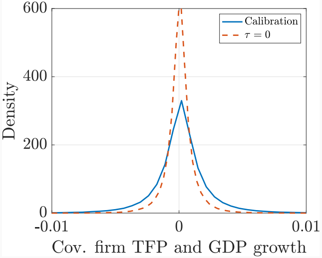
- Fixed δ: Large increase in GDP variance; exposure to ε_t becomes more harmful ($\mathcal{E}$ declines)
- Flexible δ: Firms manage risk more aggressively which limits increase in $\mathbb{V}[y]$

Impact of risk can be overestimated if reaction of agents is not taken into account

Removing distortions

What if we set wedges τ to zero?

	Calibration	No wedges	
		Fixed δ	Flexible δ
Agg. risk exposure Δ	0.014	0.014	0.007
Exposure value $\mathcal{E}$	-0.06	-0.06	-0.03
Std. Dev. of GDP growth	2.4%	2.4%	1.7%



- Fixed δ: Since only impact of τ is through δ, there is no change.
- Flexible δ: Firms manage risk more aggressively so $\mathbb{V}[y]$ declines

Removing distortions

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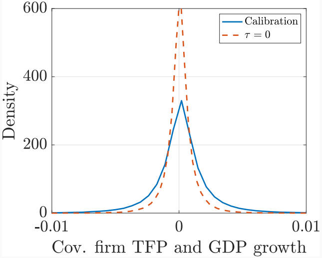

- Fixed δ: Since only impact of τ is through δ, there is no change.
- Flexible δ: Firms manage risk more aggressively so $\mathbb{V}[y]$ declines

Distortions make GDP more volatile

Conclusion

Conclusion

Main contributions

- We construct a model of endogenous risk, at both the micro and macro levels.
- Model predicts which firms are more volatile and covary more with business cycle.
- Distortions lead to less aggressive risk management and can increase GDP volatility.

More results in the paper

- Comparative static with general preferences and costs of risk exposure
- Explore substitution/complementarity patterns in risk exposure
- Model can explain patterns in stock market betas
- Full-fledged model with disaster risk
  - Changes in the environment (taxes, network, ...) affect the equity premium

Expression for $\zeta(\alpha_i)$

The function $\zeta\left(\alpha_i\right)$ is

$$\zeta\left(\alpha_i\right)=\left[\left(1-\sum_{j=1}^{n}\alpha_{ij}\right)^{1-\sum_{j=1}^{n}\alpha_{ij}}\prod_{j=1}^{n}\alpha_{ij}^{\alpha_{ij}}\right]^{-1}$$

This functional form allows for a simple expression for the unit cost K

Back

Given the log-normal nature of uncertainty $\rho \lesseqgtr 1$ determines whether the agent is risk-averse or not. To see this, note that when $\log C$ normally distributed, maximizing

$$\mathrm{E}\left[C^{1-\rho}\right]$$

amounts to maximizing

$$\mathrm{E}\left[\log C\right] - \frac{1}{2}\left(\rho - 1\right)\mathrm{V}\left[\log C\right].$$

Back

Planner's problem

Define $\bar{h}_{SP}(\Delta)$ as the smallest risk management utility cost needed to achieve Δ.

$$\bar{h}_{SP}(\Delta) := \min_{\delta} \tilde{\omega}^{\top} b(\delta) - \log V\left(\sum_{i=1}^{N} g_i(\delta_i)\right), \qquad \text{subject to } \Delta = \delta^{\top}\tilde{\omega}.$$

Planner's problem

Define $\bar{h}_{SP}(\Delta)$ as the smallest risk management utility cost needed to achieve Δ.

$$\bar{h}_{SP}(\Delta) = \underbrace{\tilde{\omega}^{\top} b\left(\delta_{SP}(\Delta)\right)}_{\bar{b}_{SP}(\Delta)} \underbrace{- \log V\left(\sum_{i=1}^{N} g_i\left(\delta_{SP,i}(\Delta)\right)\right)}_{\bar{g}_{SP}(\Delta)}$$

Replace minimizer $\delta_{SP}(\Delta)$ back in the function

Back

Equilibrium characterization through fictitious planner

Define $\bar{h}(\Delta)$ as the perceived smallest risk management utility cost needed to achieve Δ.

$$\bar{h}(\Delta) := \min_{\delta} \tilde{\omega}^{\top} b(\delta) - \log V\left(\sum_{i=1}^{N} \kappa_i g_i(\delta_i)\right), \qquad \text{subject to } \Delta = \delta^{\top}\tilde{\omega}.$$

where $\kappa_i = (1+\tau_i)\frac{\tilde{\omega}_i}{\omega_i} \propto \frac{(K_i Q_i)_{SP}}{(K_i Q_i)_{\tau \neq 0}}$ is the efficiency gap of firm i. If $\tau = 0$, then $\kappa_i = 1$ for all i.

Back

Proposition

The response of the equilibrium aggregate risk exposure Δ to a change in wedge τ_i is given by

$$\frac{d\Delta}{d\tau_i} = \mathcal{T}\left(\sum_{j=1}^{N} \frac{\partial\left[\nabla^2\bar{\kappa}\right]^{-1}}{\partial g_j}\frac{dg_j}{d\tau_i}\right)\mathcal{E}, \tag{1}$$

where the impact of g_j on $\left[\nabla^2\bar{\kappa}\right]^{-1}$ is given by $\frac{\partial\left[\nabla^2\bar{\kappa}\right]^{-1}}{\partial g_j} = -\frac{1}{\eta}\frac{\tilde{\omega}_j^2}{g_j^2}H_j^{-1}$, and where

$$\mathcal{T} := \left(I - \left[\nabla^2\bar{\kappa}\right]^{-1}\frac{\partial\mathcal{E}}{\partial\Delta}\right)^{-1}.$$

Back

Proposition

Let χ denote either μ_m, Σ_{mn}, or τ_i. Then the impact of a change in χ on the moments of log GDP are given by

$$\frac{d\,\mathrm{E}[y]}{d\chi} - \frac{\partial\,\mathrm{E}[y]}{\partial\chi} = \mu^\top \frac{d\Delta}{d\chi} \qquad \text{and} \qquad \frac{d\,\mathrm{V}[y]}{d\chi} - \frac{\partial\,\mathrm{V}[y]}{\partial\chi} = 2\Delta^\top\Sigma\frac{d\Delta}{d\chi},$$

where the use of a partial derivative indicates that Δ is kept fixed.

Back

Implications for GDP

Simplified model

Back

- Single risk factor $\varepsilon_t \sim \text{iid } \mathcal{N}(0, \Sigma)$
- Firm level TFP is $\log TFP_{it} = \delta_{it}\varepsilon_t + \gamma_i t + v_{it}$ where γ_i is deterministic trend and $v_{it} \sim \text{iid } \mathcal{N}(\mu_i^v, \Sigma_i^v)$

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Variance of firm-level TFP growth

$$\mathrm{V}\left[\log TFP_{it} - \log TFP_{it-1}\right] = 2\delta_i^2\Sigma + 2\Sigma_i^v$$

Simplified model

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Covariance of firm-level TFP growth with GDP growth

$$\mathrm{Cov}\left[\log TFP_{it} - \log TFP_{it-1}, y_t - y_{t-1}\right] = 2\Delta\Sigma\delta_i + 2\tilde{\omega}_i\Sigma_i^v.$$

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Model-implied firm risk exposure ($\mathcal{E} < 0$)

$$\delta_i = \delta_i^\circ + \frac{1}{\eta}\frac{\omega_i}{1+\tau_i}H_i^{-1}\mathcal{E}$$

$\Rightarrow$ Firms with large Domar weights and small markups are less volatile and less corr. with GDP

Markup estimation

Back

- Assume Cobb-Douglas production function

$$\log Q_{it} = \alpha_{Li} \log L_{it} + \alpha_{Mi} \log M_{it} + \alpha_{Ki} \log K_{it} + \varepsilon_{it},$$

- Elasticities estimated using Levinsohn and Petrin (2003) with the Ackberg et al. (2015) correction.
  - Capital is the "state" variable, labor is the "free" variable and materials is the "proxy" variable.
- Production function estimated at NACE 2-digit sector level. As in De Loecker et al. (2020), we control for markups using firms' sales shares in the production function estimation.
- Following De Loecker and Warzynski (2012), we compute the markup as $1 + \tau_{it} = \hat{\alpha}_{Li} \Big/ \left(\frac{\text{Wage Bill}_{it}}{\text{Sales}_{it}}\right)$.
- We compute TFP growth as

$$\begin{aligned}\Delta \log \text{TFP}_{it} =& \Delta \log Q_{it} - \alpha_{Li} \Delta \log L_{it} - \alpha_{Mi} \Delta \log M_{it} - \alpha_{Ki} \Delta \log K_{it} \\ & - \left(\Delta \log (1 + \tau_{it}) - \Delta \log \left(1 + \tau_{s(i)t}\right)\right).\end{aligned}$$

The term $\Delta \log (1 + \tau_{it}) - \Delta \log \left(1 + \tau_{s(i)t}\right)$ accounts for the firm-specific markup growth net of the sectoral markup growth. This adjustment allows us to remove the change in firm-specific nominal price that are not taken into account by the sector-level price deflator.

TFP growth volatility

- We compute the standard deviation of TFP growth for each firm, $\sigma_i\left(\Delta \log TFP_{it}\right)$, and the time-series average of its markup and Domar weight.
- We construct deciles based on average Domar weights and markups, and create dummy variables, FE_{ji}^{Domar} and FE_{ji}^{Markup}, such that $FE_{ji}^{Domar} = 1$ if firm i's Domar weight is in decile j, and analogously for markups.
- We run the cross-sectional regression

$$\sigma_i\left(\Delta \log TFP_{it}\right) = \alpha + \sum_{j=1}^{10} \beta_j^{Domar} FE_{ji}^{Domar} + \sum_{j=1}^{10} \beta_j^{Markup} FE_{ji}^{Markup} + \varepsilon_i,$$

and plot β_j^{Domar} in panel (a) and β_j^{Markup} in panel (b).

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TFP growth volatility

- We construct deciles based on firms' Domar weights and markups each year.
- We then construct a set of dummy variables, FE_{jit}^{Domar} and FE_{jit}^{Markup}, such that $FE_{jit}^{Domar} = 1$ if firm i's Domar weight is in decile j in year t, and analogously for markups.
- We then run the following panel regression,

$$\Delta \log TFP_{it} = \sum_{j=1}^{10} \beta_j^{Domar} \left(FE_{jit}^{Domar} \times \Delta \log GDP_t \right) + \sum_{j=1}^{10} \beta_j^{Markup} \left(FE_{jit}^{Markup} \times \Delta \log GDP_t \right)$$
$$+ \alpha + \beta_0 \Delta \log GDP_t + \sum_{j=1}^{10} FE_{jit}^{Domar} + \sum_{j=1}^{10} FE_{jit}^{Markup} + \varepsilon_{it},$$

  where $\Delta \log TFP_{it}$ is the annual growth of firm i's log TFP and $\Delta \log GDP_t$ is the annual growth of Spanish log GDP.
- The coefficients of interest, β_j^{Domar} and β_j^{Markup}, are reported in the figure.

Back

Model for the calibration

- Unique risk factor ε ($M = 1$)
- S sectors with sectoral shocks $z_s \sim$ iid $\mathcal{N}\left(\mu_s^z, \Sigma_s^z\right)$ and aggregator

$$Q_s = \prod_{i=1}^{N_s} e^{z_s} \left(\theta_{si}^{-1} Q_{si}\right)^{\theta_{si}}$$

- Firms have production function

$$Q_{si} = \exp\left(\delta_{sit}\varepsilon_t - b_i\left(\delta_{sit}\right) + \gamma_{si}t + v_{sit}\right) \zeta_{si} L_{si}^{1-\sum_{s'=1}^{S} \hat{\alpha}_{ss'}} \prod_{s'=1}^{S} X_{si,s'}^{\hat{\alpha}_{ss'}}$$

where $\hat{\alpha}_{ss'}$ are sectoral shares, $v_{sit} \sim$ iid $\mathcal{N}\left(\mu_{si}^v, \Sigma_{si}^v\right)$ and $\varepsilon_t \sim$ iid $\mathcal{N}\left(0, \Sigma\right)$

Back

Model for the calibration

- Risk exposure

$$\delta_{si} = \delta_{si}^{\circ} + \left(B_s + \eta \frac{1 + \tau_{si}}{\omega_{si}} G_s \right)^{-1} \mathcal{E}$$

- The variance of GDP growth is

$$\mathrm{V}\left[y_t - y_{t-1} \right] = 2\Sigma\Delta^2 + 2\tilde{\omega}_f^{\top} \Sigma^v \tilde{\omega}_f + 2\tilde{\omega}_s^{\top} \Sigma^z \tilde{\omega}_s.$$

- The variance of firm-level TFP growth is

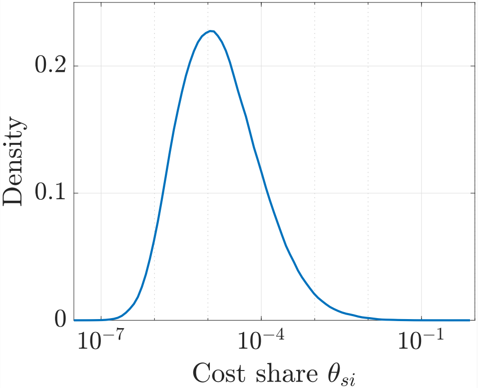
$$\mathrm{V}\left[\log TFP_{si,t} - \log TFP_{si,t-1} \right] = 2\delta_{si}^2 \Sigma + 2\Sigma_{si}^v.$$

- The covariance of firm-level TFP growth with GDP growth is

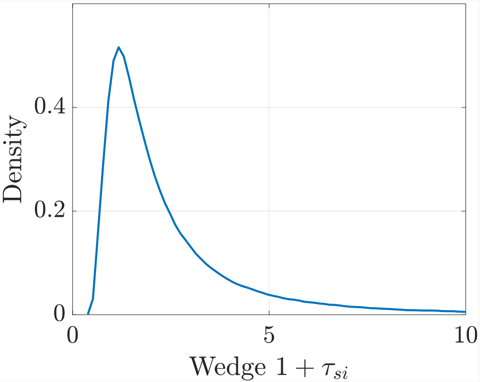
$$\mathrm{Cov}\left[y_t - y_{t-1}, \log TFP_{si,t} - \log TFP_{si,t-1} \right] = 2\Delta\Sigma\delta_{si} + 2\tilde{\omega}_{si}\Sigma_{si}^v.$$

Back

Figure 1: Data distributions that the calibration matches exactly



(a) Sales share θ_{si}



(b) Wedges $1 + \tau_i$

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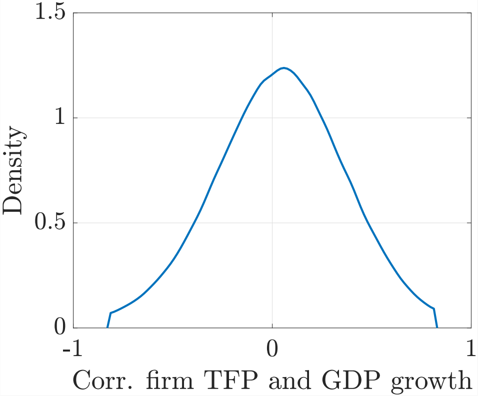
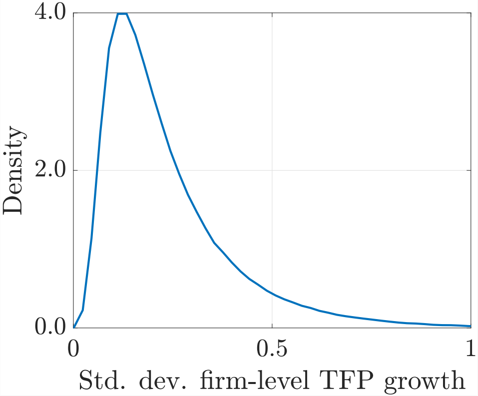
Calibrated model

Figure 2: Data distributions that the calibration matches exactly



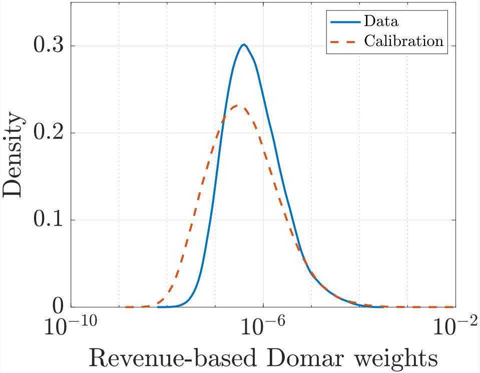
(a) Correlation firm-level TFP and GDP growth



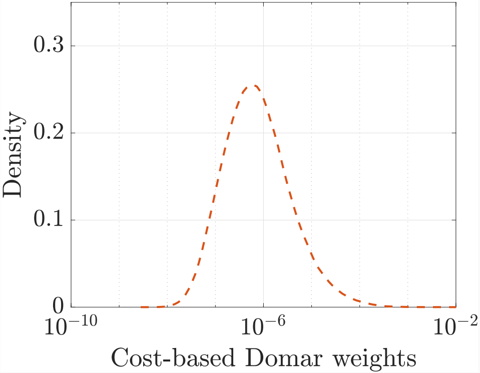
(b) Standard deviation of firm-level TFP growth

◂ Back

Figure 3: Domar weights of the firms in the data and in the model



(a) Revenue-based Domar weights



(b) Cost-based Domar weights

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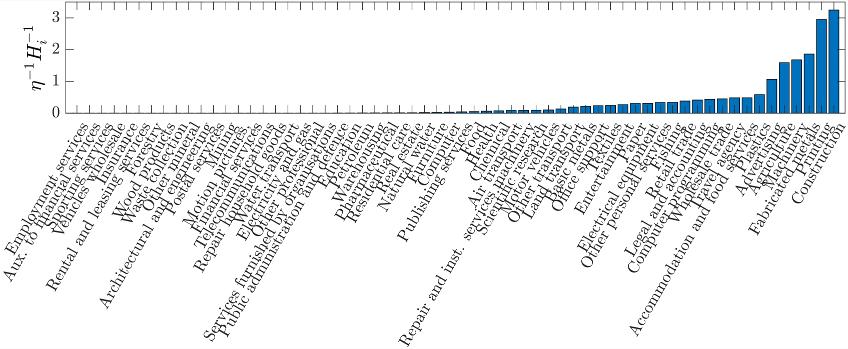
Figure 4: Estimated value of $\frac{1}{\eta} H_i^{-1}$ for each sector.



Notes. The scale of $\frac{1}{\eta} H_i^{-1}$ depends on our choice of ρ and Σ. We set $\rho = 5$ and $\Sigma = 1$ for this figure.

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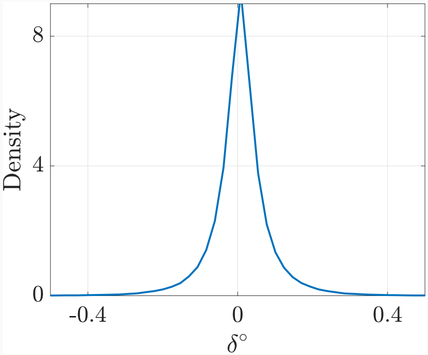
Figure 5: Distribution of the estimated firm-level natural risk exposure δ_i° /



Notes. The scale of δ_i° depends on our choice of ρ and Σ. We set $\rho = 5$ and $\Sigma = 1$ for this figure.

Back