Producers Shutdowns and Restarts, and the Network Structure of Production
(or a better title?)

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Introduction

- Large amount of establishment turnover in the data.
  - Firms shutdown and restart units of production in response to shocks
- When a unit stops or starts
  - It affects other units that *consume* the goods it *produces* (downstream)
  - It affects other units that *produce* the goods it *consumes* (upstream)
- Evidence that these shocks to supply chains have large and long-lasting impact on firms (Hendricks et al 2005)

- This paper proposes a joint theory of producers shutdowns and restarts and the network structure of production
- One main difficulty, the decision to operate is non-convex
  - If n producers: 2^n possible configurations $\rightarrow$ impossible to solve
- I show that there is an alternative optimization problem that
  - is convex $\rightarrow$ easy solution through standard algorithm
  - has the same solution as the non-convex problem

Preview of results

- Joint characterization of distributions of ...
  - firm-level outcome (productivity, output, employment) and...
  - the network structure of production (distribution of in-degrees and out-degree, clustering)
- Because of the non-convexity, small shocks to a single producer can lead to large reorganization of the network
- Clustering of turnover
  - The shutdown of a unit can trigger the shutdown of its neighbors
  - The restart of a unit can trigger the restart of its neighbors
- Units shutdowns and restarts is a powerful adjustment margin
  - Higher and less volatile output

Literature

- Endogenous network formation
  - Oberfield (2013), Lim (2016)
- Network propagation of shocks
  - Horvath (1998), Dupor (1999), Acemoglu et al (2012), Baqaee (2016), Acemoglu et al (2016)
- Non-convex adjustments in networks
  - Bak, Chen, Woodford and Scheinkman (1993)

I. Model

Model

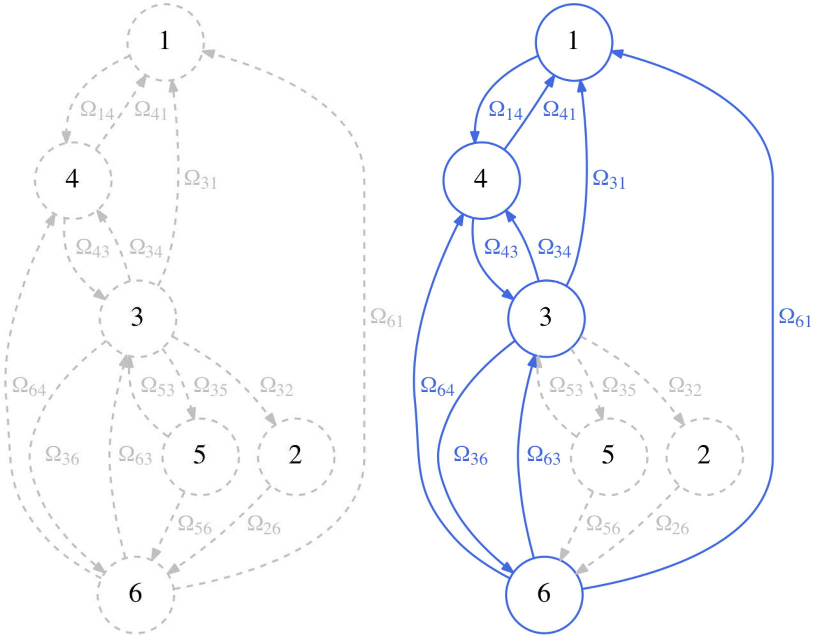
- There are *n* units of production
  - Each unit produces a different good
  - Goods can be consumed by the representative household or used as input into production of other goods
- The representative household
  - has CES preferences over the differentiated goods with elasticity of substitution $\sigma > 1$
  - supplies *L* units of labor unelasticaly

Model

- Firm j can convert l_j unit of labor and a vector x_j of intermediate inputs into units of good j

$$y_j = \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \left(\sum_{i=1}^{n} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

- A firm j can only use goods i in production if there is a *connection* between firms i and j
    - $\Omega_{ij} = 1$ if connection and $\Omega_{ij} = 0$ otherwise
    - A connection can be *active* or *inactive*
    - One network of interest: $\Omega_{ij} = 1$ for all i, j
- A firm can only produce if it pays a fixed cost f in units of labor
    - $\theta_j = 1$ if operating and $\theta_j = 0$ otherwise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Social Planner

Problem $\mathcal{P}_{SP}$ of a social planner

$$\max_{c,x,l,\theta} \left(\sum_{j=1}^{n} c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to:

1. resource constraints on each good j

$$c_j + \sum_{k=1}^{n} x_{jk} \leq \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \left(\sum_{i=1}^{n} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor

$$\sum_{j=1}^{n} l_j + f \sum_{j=1}^{n} \theta_j \leq L$$

3. operation constraints: $l_j > 0$ only if $\theta_j = 1$
4. connection constraints: $x_{ij} > 0$ only if $\Omega_{ij} = 1$

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subject to:

1. resource constraints on each good j **(Lagrange multiplier: λ_j)**

$$c_j + \sum_{k=1}^{n} x_{jk} \leq \frac{A}{\alpha^{\alpha}(1-\alpha)^{1-\alpha}} z_j \theta_j \left(\sum_{i=1}^{n} \Omega_{ij} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor **(Lagrange multiplier: w)**

$$\sum_{j=1}^{n} l_j + f \sum_{j=1}^{n} \theta_j \leq L$$

3. ~~operation constraints: $l_j > 0$ only if $\theta_j = 1$~~

4. ~~connection constraints: $x_{ij} > 0$ only if $\Omega_{ij} = 1$~~

II. Social Planner with Given θ

Social Planner with Exogenous θ

Consider first a planner problem in which θ is given.

Proposition 1

For a given θ, the the optimal allocation satisfy

$$q_j = z_j \theta_j A \left(\sum_{i=1}^{n} \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}} \tag{1}$$

where $q_j = w/\lambda_j$. Furthermore, there is a unique vector q that satisfies (1) such that $q_j > 0$ if firm j is active and part of a strongly connected set of active firms.

Note:

- (1) describes a concave mapping in $q_j^{\epsilon-1}$
  - We can solve for q by iterating on the mapping
- From the FOCs, output is $y_j = (1-\alpha)\, q_j l_j$
  - q_j is the *labor productivity* of firm j

Social Planner with Given θ

Lemma 1

For a given θ, the optimal consumption of the representative household is

$$C = Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

where $Q \equiv \left(\sum_{j=1}^{n} q_j^{\sigma-1}\right)^{\frac{1}{\sigma-1}}$ is aggregate labor productivity.

Social Planner with Given θ

Lemma 2

The optimal labor allocation satisfies

$$l = (1-\alpha)\underbrace{[I_n - \alpha\Gamma]^{-1}}_{(1)} \underbrace{\left(\frac{q}{Q}\right)}_{(2)}^{\circ(\sigma-1)} \left(L - f\sum_{j=1}^{n}\theta_j\right)$$

where Γ is an $n \times n$ matrix with elements $\Gamma_{jk} = \frac{\Omega_{jk} q_j^{\epsilon-1}}{\sum_{i=1}^{n} \Omega_{ik} q_i^{\epsilon-1}}$ and I_n is the identity matrix.

Determinants of l

(1) Importance of network connection

- Leontief inverse $\left([I_n - \alpha\Gamma]^{-1} = I_n + \alpha\Gamma + (\alpha\Gamma)^2 + \dots\right)$
- more labor to firms that are important suppliers

(2) Relative efficiency

III. Social Planner with Chosen θ

Social Planner with Chosen θ

$$\max_{\theta\in\{0,1\}^n, q\geq 0} Q\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

subject to

$$q_j = z_j\theta_j A\left(\sum_{i=1}^{n}\Omega_{ij}q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$

Very hard problem (MINLP - NP Hard)

- For any vector θ iterate on q and evaluate the objective function
- 2^n vectors θ to try ($2^{20} > 10^6$ configuration)

Social Planner with Chosen θ

Solution steps

1. Consider *relaxed problem*
   - Let $\theta_j \in [0, 1]$ for all j
2. Deformation
   - Modify the problem to create nice equivalent problem

Social Planner with Chosen θ

Consider the relaxed and deformed problem $\mathcal{P}_R$

$$\max_{\theta \in [0,1]^n, q \geq 0} Q\left(L - f \sum_{j=1}^{n} \theta_j\right)$$

subject to

$$q_j = z_j \theta_j^a A \left(\sum_{i=1}^{n} \Omega_{ij} \theta_i^b q_i^{\epsilon-1}\right)^{\frac{\alpha}{\epsilon-1}}$$

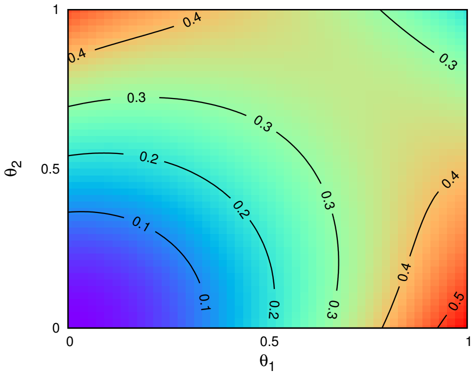
The constants $a > 0$ and $b \geq 0$ are *deformation constants*.

- They modify the shape of the problem *away* from the optimal allocation (i.e. when $0 < \theta_j < 1$)
- Pick a and b such that
  - $\mathcal{P}_R$ is well-behaved
  - $\mathcal{P}_R$ is easy to solve
  - $\mathcal{P}_R$ is equivalent to the original problem $\mathcal{P}_{SP}$

Example of the relaxed problem without deformation

$$T(\theta) = Q(\theta)\left(L - f\sum_{j=1}^{n}\theta_j\right)$$

where $Q(\theta)$ solves the recursion on q

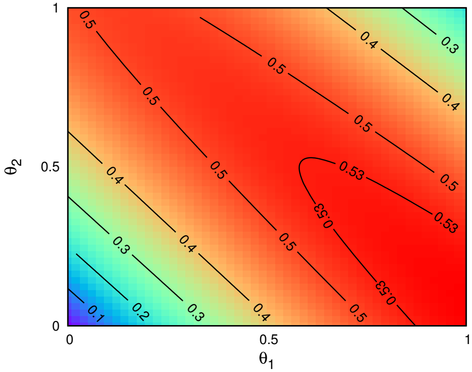


- Problem 1: T is not concave $\rightarrow$ first-order conditions are not sufficient
- Problem 2: Numerical algorithm can get stuck in local maxima

Deformation

Consider instead the following deformation parameters

$$\boxed{a = \frac{1}{\sigma - 1} \quad \text{and} \quad b = 1 - \frac{\epsilon - 1}{\sigma - 1}} \qquad (\star)$$



- ~~Problem 1:~~ T is now concave $\rightarrow$ first-order conditions are necessary and sufficient
- ~~Problem 2:~~ Numerical algorithm converges to global maximum

Deformation

Proposition 2

If the network is complete ($\Omega_{ij} = 1$ for all i, j), then T is log-concave under the deformation parameters $(\star)$ and the Karush-Kuhn-Tucker conditions are necessary and sufficient to describe an optimum. Furthermore, if a solution to the deformed problem $\mathcal{P}_R$ is such that $\theta_j \in \{0, 1\}$ for all j then this solution also solves the original problem $\mathcal{P}_{SP}$.

▸ Towards general result

How to solve the problem

- Solve problem $\mathcal{P}_R$: easy since the first-order conditions are necessary and sufficient
- Check if the solution is such that $\theta_j \in \{0, 1\}$ for all j. If so, we have solved $\mathcal{P}_{SP}$

Deformation

For this approach to be useful, we must hit the bounds on θ

- In large connected network, the first-order conditions in θ_k becomes independent of firm k endogenous variables.

First order condition with respect to θ_k:

$$\frac{q_k^{\sigma-2}}{Q^{\sigma-1}} \frac{dq_k}{d\theta_k} + \sum_{j=1}^{n} \lambda_j z_j \theta_j^a A \alpha B_j^{\alpha-1} \left(\frac{\partial B_j}{\partial \theta_k} + \frac{\partial B_j}{\partial q_k} \frac{dq_k}{d\theta_k} \right) - \frac{f}{L - f \sum_{j=1}^{n} \theta_j} = \bar{\mu}_j - \underline{\mu}_j$$

where $B_k = \left(\sum_{i=1}^{n} \theta_i^b \Omega_{ik} q_i^{\epsilon-1} \right)^{\frac{1}{\epsilon-1}}$ is composite input used by k.

$$\frac{q_k^{\sigma-2}}{Q^{\sigma-1}} \times \frac{dq_k}{d\theta_k} = \frac{(z_k \theta_k^a A B_k^\alpha)^{\sigma-2}}{Q^{\sigma-1}} \times a \frac{z_k \theta_k^a A B_k^\alpha}{\theta_k}$$

$$\frac{\partial B_j}{\partial \theta_k} + \frac{\partial B_j}{\partial q_k} \frac{dq_k}{d\theta_k} = B_j \theta_k^{b-1} \Omega_{kj} \left(\frac{z_k \theta_k^a A B_k^\alpha}{B_j} \right)^{\epsilon-1} \left(a + \frac{b}{\epsilon - 1} \right)$$

Testing the approach

For small networks $n < 15$ we can solve the problem $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

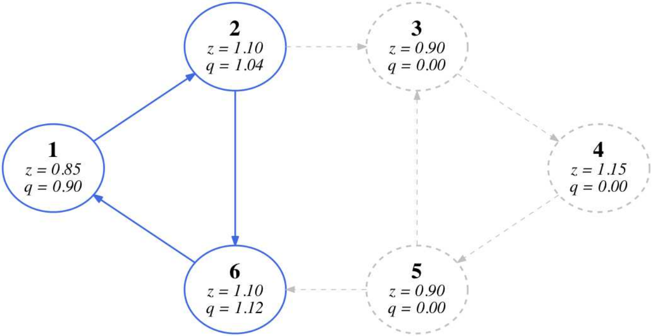
- Compare this solution with the solution of the deformation approach

	Number of firms n			
	8	10	12	14
Firms with wrong θ	0.047%	0.077%	0.078%	0.078%
Error in C	0.00015%	0.00022%	0.00026%	0.00015%

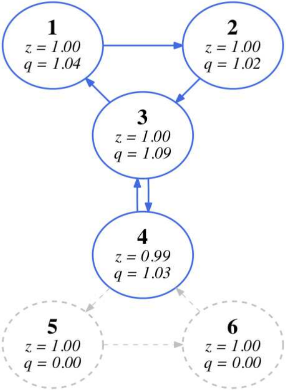
Notes: I construct networks with parameters $f \in \{0.18/n, 0.2/n, 0.22/n\}$, $\sigma_z \in \{0.35, 0.39, 0.44\}$, $\alpha \in \{0.35, 0.5, 0.65\}$, $\sigma \in \{4, 6, 8\}$ and $\epsilon \in \{4, 6, 8\}$. For each possible combination of the parameters I create 1000 different economies. For each economy, I draw $\log(z_k) \sim$ iid $\mathcal{N}(0, \sigma_z)$ and Ω where each link Ω_{ij} exists with some probability such that a firm has on average five possible connections. All draws are independent. The network is redrawn until it is strongly connected.

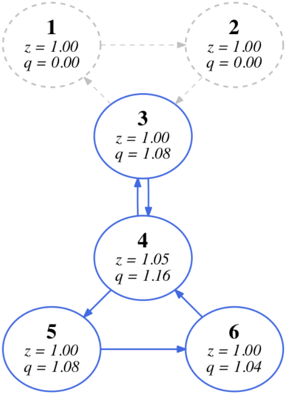
IV. Economic Forces at Work

Network structure matters for firm status

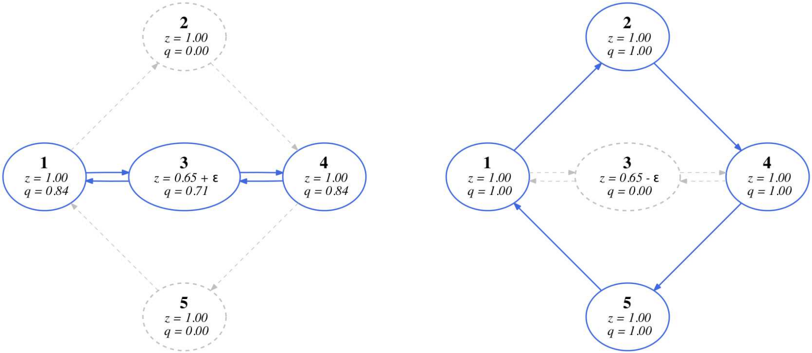


Clustering of changes in the network





Small shocks can lead to large reorganization



V. Quantitative Exploration

Network data

Network data from Compustat

- Firms must report consumers that purchase more than 10% of sales
- Since consumers are self-reported, use a fuzzy text matching algorithm (Cohen and Frazzini 2008, Ataly et al 2011, Kelly et al 2013, Wu 2016)
  - Linked-data covers 1979-2014, 1211 firms on average, set $n = 1600$ to get this number
- Set Ω to so that the distribution of in-degrees is a power law with same parameters as the data

Parameters

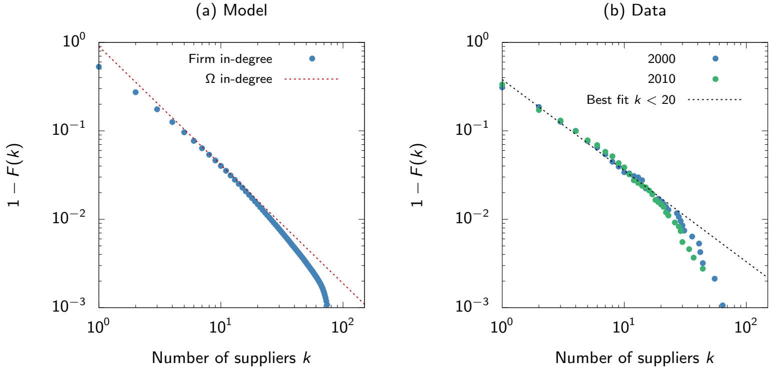
Parameters from the literature

- $\alpha = 1/2$ to fit the share of intermediate (Basu 1995, Jones 2011)
- $\sigma = \epsilon = 4.5$ average of estimates (Broda et al 2006)
- $\log(z_{it})$ follows an AR(1) with ergodic dispersion 0.39 (Bartelsman et al 2013) and persistence 0.81 (Lucia Foster et al 2008)
- $f = 0.2/n$ to match fraction of non-production workers to production workers in manufacturing (Ramey 1991)

I create 100 matrices Ω. For each of them I draw 500 vectors z. In each case I solve for the optimal allocation.

Distribution of in-degree

Distribution of number of suppliers



Impact of the number neighbors on producers

	Model		Data	
	Output	Labor prod.	Output	Labor prod.
# of suppliers	0.771	0.187	0.210***	0.095***
			(0.035)	(0.019)
# of consumers	0.805	0.087	0.120***	0.130***
			(0.042)	(0.030)
Firm fixed effects	yes	yes	yes	yes

Notes: Ordinary least-square regressions with robust standard errors in parenthesis: *** = 1%, ** = 5%, * = 10% significance levels. Standard errors are negligible in the model regressions. All variables are in logs.

Contagion of shocks

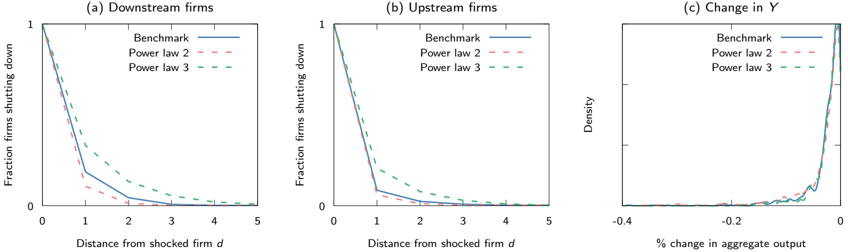
In the simulated data:

	Producer stops production	Producer starts production
# of suppliers stopping production	0.078	-0.054
# of suppliers starting production	-0.088	0.166
# of consumers stopping production	0.051	-0.042
# of consumers starting production	-0.070	0.170

Notes: Ordinary least-square regressions with robust standard errors in parenthesis: *** = 1%, ** = 5%, * = 10% significance levels. Standard errors are negligible in the model regressions. All variables are in logs.

Cascades

Impact of a 3 std. dev. shock on a single firm



Contagion in the data

In the data:

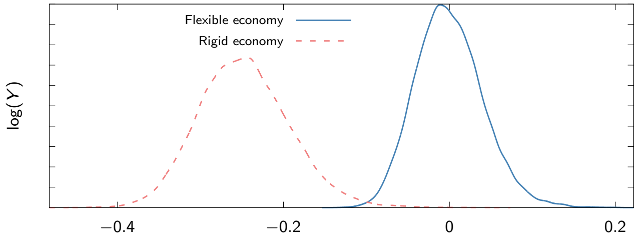
	Data
	Firm exits
# of suppliers	-0.000063*
	(0.000037)
# of consumers	-0.00054*
	(0.00031)

Notes: Ordinary least-square regressions with robust standard errors. Standard errors in parenthesis: *** = 1%, ** = 5%, * = 10% significance levels.

Aggregate fluctuations

With and without the adjustment mechanism

	mean Y	std Y
Flexible economies	2.76	0.041
Rigid economies	2.52	0.055



Suggests that frictions to producers shutdown/restart have large impact on the level of GDP and the size of fluctuations.

Conclusion

- Theory to understand the interaction of producers turnover and the network structure of production
- Propose an approach to solve these hard problems easily
- The model captures features of the data

Deformation

The result seems to hold for general networks but I don't have a full proof yet. Need to show that the system of n equations

$$f_j^2 = x_j^2 \left(\sum_{i=1}^{n} \Omega_{ij} f_i \right)$$

describes functions f_j's that are log-concave in the vector x.

Return