Cascades and Fluctuations in an Economy with an Endogenous Production Network

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- Production in modern economies involves a complex network of producers supplying and demanding goods from each other
- The structure of this network
  - is an important determinant of how micro shocks aggregate into macro fluctuations
  - is also constantly changing in response to micro shocks
    - For instance, after a severe shock a producer might shut down which might lead its neighbors to shut down as well, etc...
    - Cascade of shutdowns that spreads through the network

This paper proposes a

Theory of network formation and aggregate fluctuations

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Overview of the Model

Simple framework

- Set of firms that use inputs from connected suppliers
- Fixed cost to operate
    - Firms operate or not depending on economic conditions
    - → Links between firms are active or not
    - → Endogenously shape the network

Modeling choice motivated by the data

- U.S.: $\approx 40\%$ of link destructions occur with exit of supplier or customer
- Theory also applies to link formation by thinking of links as special firms

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Overview of the Results

Key economic force: Complementarities in operation decisions of nearby firms

Efficient organization of production

- Create tightly connected clusters centered around productive firms
- Small changes can trigger large reorganization of the network

Cascades of firm shutdowns

- Well-connected firms are hard to topple but create big cascades
- Elasticities of substitution matter for size and propagation of cascades

Aggregate fluctuations

- Recessions feature fewer well-connected firms and less clustering
- Allowing the network to adjust yields substantially smaller fluctuations

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Along the way...

Difficulty in solving the planner's problem

- The Karush-Kuhn-Tucker conditions do not apply
  1. Discrete choice about network formation
     - Constraint set is not convex
  2. Complementarities in decisions of nearby firms
     - Objective function is not concave
- Novel approach that involves *reshaping* the problem

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Literature Review

- Endogenous network formation
  - Atalay et al (2011), Oberfield (2018), Carvalho and Voigtländer (2014), Acemoglu and Azar (2018), Tintelnot et al (2018), Lim (2018)
- Network and fluctuations
  - Long and Plosser (1983), Horvath (1998), Dupor (1999), Acemoglu et al (2012), Baqaee (2018), Acemoglu et al (2016), Baqaee and Farhi (2018)
- Non-convex adjustments in networks
  - Bak, Chen, Woodford and Scheinkman (1993), Elliott, Golub and Jackson (2014)
- Measuring the propagation of shocks through networks
  - Barrot and Sauvagnat (2016), Carvalho et al (2017)
- Macro fluctuations from micro shocks
  - Jovanovic (1987), Gabaix (2011)

I. Model

Model

- There are n units of production (firm) indexed by $j \in \mathcal{N} = \{1, \dots, n\}$
  - Each unit produces a differentiated good
  - Differentiated goods can be used to
    - produce a final good
$$Y \equiv \left(\sum_{j \in \mathcal{N}} \beta_j^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$
    - produce other differentiated goods
- Representative household
  - Consumes the final good
  - Supplies L units of labor inelastically

- Firm j produces good j

$$y_j = \frac{A}{\alpha_j^{\alpha_j}(1-\alpha_j)^{1-\alpha_j}} z_j \theta_j \left(\sum_{i \in \mathcal{N}} \Omega_{ij}^{\frac{1}{\varepsilon_j}} x_{ij}^{\frac{\varepsilon_j - 1}{\varepsilon_j}} \right)^{\frac{\varepsilon_j}{\varepsilon_j - 1} \alpha_j} l_j^{1-\alpha_j}$$

- Firm j can only use good i as input if there is a *connection* from firm i to j
  - $\Omega_{ij} > 0$ if connection and $\Omega_{ij} = 0$ otherwise
  - A connection can be *active* or *inactive*
  - Matrix Ω is *exogenous*
- A firm can only produce if it pays a fixed cost f_j in units of labor
  - $\theta_j = 1$ if j is operating and $\theta_j = 0$ otherwise
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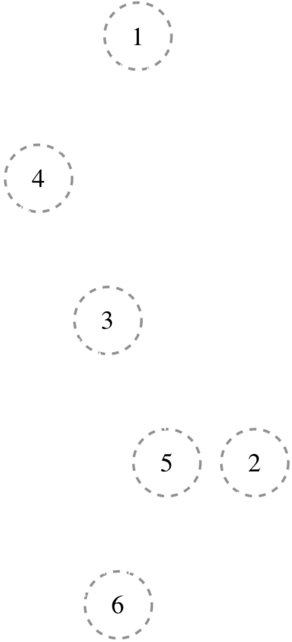
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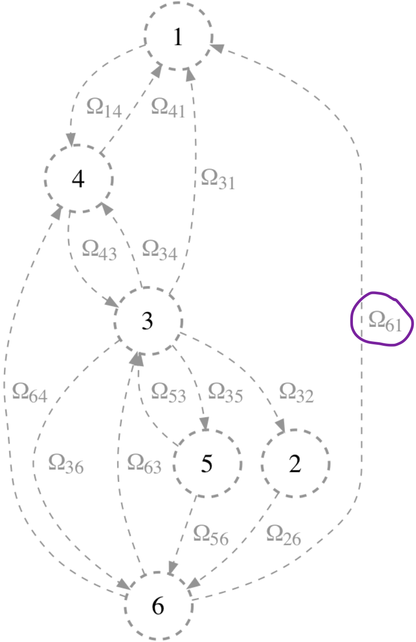
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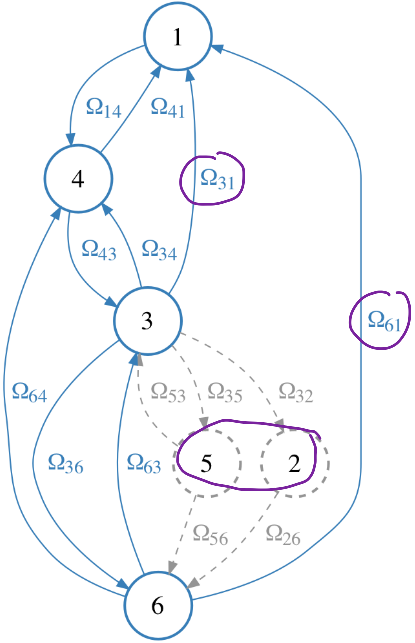
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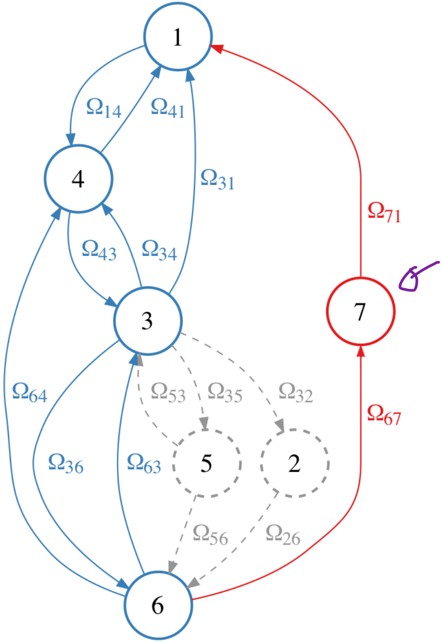
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Efficient Allocation and Equilibrium

Focus on the problem of a social planner, but...

Proposition

Every equilibrium is efficient.

Key equilibrium concept is *stability* (Hatfield et al. 2013, Oberfield 2018).

- An allocation is *stable* if there exist no coalition of firms that wishes to deviate.

▸ Equilibrium Definition

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Social Planner

Problem $\mathcal{P}_{SP}$ of a social planner

$$\max_{\substack{c,x,l \\ \theta \in \{0,1\}^n}} \left(\sum_{j \in \mathcal{N}} \beta_j^{\frac{1}{\sigma}} c_j^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to

1. a resource constraint for each good j

$$c_j + \sum_{k \in \mathcal{N}} x_{jk} \leq \frac{A}{\alpha_j^{\alpha_j} (1-\alpha_j)^{1-\alpha_j}} z_j \theta_j \left(\sum_{i \in \mathcal{N}} \Omega_{ij}^{\frac{1}{\varepsilon_j}} x_{ij}^{\frac{\varepsilon_j - 1}{\varepsilon_j}} \right)^{\alpha_j \frac{\varepsilon_j}{\varepsilon_j - 1}} l_j^{1-\alpha_j}$$

2. a resource constraint for labor

$$\sum_{j \in \mathcal{N}} l_j + \sum_{j \in \mathcal{N}} f_j \theta_j \leq L$$

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1. a resource constraint for each good j **LM:** λ_j

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2. a resource constraint for labor **LM:** w

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II. Social Planner with Exogenous θ

Social Planner with Exogenous θ

Define $q_j = w/\lambda_j$

- From the FOCs, output is $(1-\alpha_j)\, y_j = q_j l_j$
- q_j is the *labor productivity* of firm j

Proposition

In the efficient allocation,

$$q_j = z_j \theta_j A \left(\sum_{i \in \mathcal{N}} \Omega_{ij} q_i^{\varepsilon_j - 1} \right)^{\frac{\alpha_j}{\varepsilon_j - 1}} \tag{1}$$

for all $j \in \mathcal{N}$. Furthermore, there is a unique vector q that satisfies (1) such that $q_j > 0$ if firm j has access to a closed loop of active suppliers.

$$q_j = z_j \theta_j A \left(\sum_{i \in \mathcal{N}} \Omega_{ij} q_i^{\varepsilon_j - 1} \right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$

- Access to a larger set of inputs increases productivity q_j
- Access to cheaper inputs (lower $1/q_i$) leads to a cheaper output
- Gains in productivity propagate downstream in the supply chain

Key Economic Force: Gains from input variety

$$q_j = z_j \theta_j A \left(\sum_{i \in \mathcal{N}} \Omega_{ij} q_i^{\varepsilon_j - 1} \right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$

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Social Planner with Exogenous θ

Knowing q, we can solve for all other quantities easily.

Lemma

Aggregate output is

$$Y = Q\overbrace{\left(L - \sum_{j \in \mathcal{N}} f_j \theta_j\right)}$$

where $\underline{Q \equiv \left(\sum_{j \in \mathcal{N}} \beta_j q_j^{\sigma-1}\right)^{\frac{1}{\sigma-1}}}$ *is aggregate labor productivity.*

▸ Other quantities

III. Social Planner with Endogenous θ

Social Planner with Endogenous θ

Planner's problem is now

$$\max_{\theta \in \{0,1\}^n} Q\left(L - \sum_{j \in \mathcal{N}} f_j \theta_j\right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i \in \mathcal{N}} \Omega_{ij} q_i^{\varepsilon_j - 1}\right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$

Trade-off: making firm j produce ($\theta_j = 1$)

- increases labor productivity of the network (Q)
- reduces the amount of labor into production $\left(L - \sum_{j \in \mathcal{N}} f_j \theta_j\right)$

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Social Planner with Endogenous θ

$u(C - \sum f_j \theta_j)$

"Very hard problem" (MINLP — NP Hard)

1. The set $\theta \in \{0,1\}^n$ is not convex
2. Objective function is not concave

Naive approach: Exhaustive search

- For any vector $\theta \in \{0,1\}^n$ iterate on q and evaluate the objective function
- 2^n vectors θ to try ($\approx 10^6$ configurations for 20 firms)
- Guaranteed to find correct solution but infeasible for n large

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New solution approach: Find an alternative problem such that

- P1 The alternative problem is easy to solve
- P2 A solution to the alternative problem also solves $\mathcal{P}_{SP}$

Reshaping $\mathcal{P}_{SP}$

Consider the relaxed and reshaped problem $\mathcal{P}_{RR}$

$$\max_{\theta \in \{0,1\}^n} Q\left(L - \sum_{j \in \mathcal{N}} f_j \theta_j\right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i \in \mathcal{N}} \Omega_{ij} q_i^{\varepsilon_j - 1}\right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$

Parameters $a_j > 0$ and $b_{ij} \geq 0$ are *reshaping constants*

- Reshape the objective function *away* from optimum (i.e. when $0 < \theta_j < 1$)
  - For a_j: if $\theta_j \in \{0,1\}$ then $\theta_j^{a_j} = \theta_j$
  - For b_{ij}: $\{\theta_i = 0\} \Rightarrow \{q_i = 0\}$ and $\{\theta_i = 1\} \Rightarrow \left\{\theta_i^{b_{ij}} q_i^{\varepsilon_j - 1} = q_i^{\varepsilon_j - 1}\right\}$

Reshaping constants:

$$\boxed{a_j = \frac{1}{\sigma - 1} \quad \text{and} \quad b_{ij} = 1 - \frac{\varepsilon_j - 1}{\sigma - 1}} \qquad (\star)$$

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Parameters $a_j > 0$ and $b_{ij} \geq 0$ are *reshaping constants*

- Reshape the objective function *away* from optimum (i.e. when $0 < \theta_j < 1$)
  - For a_j: if $\theta_j \in \{0, 1\}$ then $\theta_j^{a_j} = \theta_j$
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Reshaping constants:

$$\boxed{a_j = \frac{1}{\sigma - 1} \quad \text{and} \quad b_{ij} = 1 - \frac{\varepsilon_j - 1}{\sigma - 1}} \qquad (\star)$$

Reshaping $\mathcal{P}_{SP}$

Consider the relaxed and reshaped problem $\mathcal{P}_{RR}$

$$\max_{\theta \in [0,1]^n} Q\left(L - \sum_{j \in \mathcal{N}} f_j \theta_j\right)$$

with

$$q_j = z_j \theta_j^{a_j} A \left(\sum_{i \in \mathcal{N}} \Omega_{ij} \theta_i^{b_{ij}} q_i^{\varepsilon_j - 1}\right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$

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Sufficiency of first-order conditions

P1 The alternative problem $\mathcal{P}_{RR}$ is easy to solve

Proposition

Let $\varepsilon_j = \varepsilon$ and $\alpha_j = \alpha$. If $\Omega_{ij} = c_i d_j$ for some vectors c and d then the KKT conditions are necessary and sufficient to characterize a solution to $\mathcal{P}_{RR}$.

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Define $\bar{\Omega} = \omega(\mathbb{1} - I)$ where $\mathbb{1}$ is the all-one matrix.

Proposition

Let $\sigma = \varepsilon_j$ for all j. Suppose that the $\{\beta_j\}$ are not too far from each other and that the fixed costs $f_j > 0$ are not too big. If Ω is close enough to $\bar{\Omega}$, then the KKT conditions are necessary and sufficient to characterize a solution to $\mathcal{P}_{RR}$.

These propositions

- Only provides *sufficient* conditions
- Later: robustness of this approach

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Equivalence between $\mathcal{P}_{RR}$ and $\mathcal{P}_{SP}$

P2 A solution to the alternative problem also solves $\mathcal{P}_{SP}$

Proposition

If a solution θ^* to $\mathcal{P}_{RR}$ is such that $\theta_j^* \in \{0, 1\}$ for all j, then θ^* also solves $\mathcal{P}_{SP}$.

We can check that this is verified, but...

Lemma

The first-order conditions for the operating decision of firm j only depends on θ_j through aggregates.

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Intuition

First-order condition on θ_j:

$$\text{Marginal Benefit}\left(\theta_j, F_j(\theta)\right) - \text{Marginal Cost}\left(\theta_j, G_j(\theta)\right) = \bar{\mu}_j - \underline{\mu}_j$$

where $\bar{\mu}_j$ is the LM on $\theta_j \leq 1$ and $\underline{\mu}_j$ is the LM on $\theta_j \geq 0$.

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Intuition

$$\Sigma(\theta)\, Q^{(\theta)}\, \underbrace{\theta_j^{\,a_j - \frac{1}{1-\sigma}}} = \bar{\mu}_j - \underline{\mu}_j$$

$$a_j = \frac{1}{1-\sigma}$$

First-order condition on θ_j:

$$\theta_j \uparrow \to \theta_j = 1$$

$$\left[\text{Marginal Benefit}\,(\cancel{\theta_j}, \cancel{F_j(\theta)}) - \text{Marginal Cost}\left(\cancel{\theta_j}, \cancel{G_j(\theta)}\right)\right] = \bar{\mu}_j - \underline{\mu}_j$$

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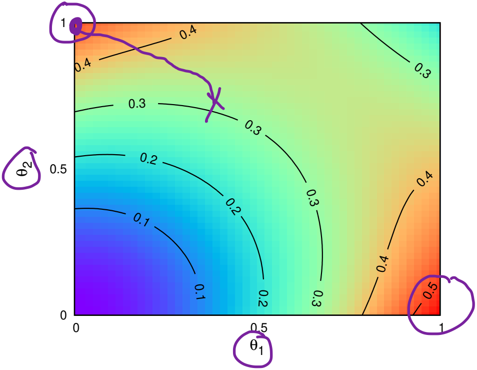
Example with two firms

Relaxed problem **without** reshaping

$$V(\theta) = Q(\theta)\left(L - \sum_{j \in \mathcal{N}} f_j \theta_j\right) \text{ with } q_j = z_j \theta_j A \left(\sum_{i \in \mathcal{N}} \Omega_{ij} q_i^{\varepsilon_j - 1}\right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$



Problem: V is not concave

⇒ First-order conditions are not sufficient

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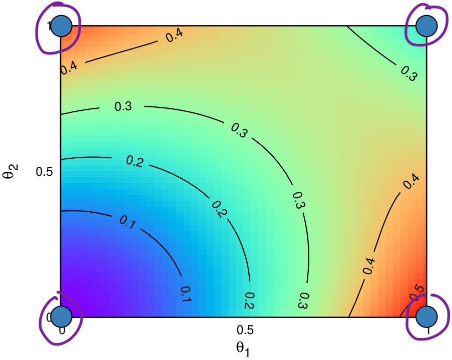

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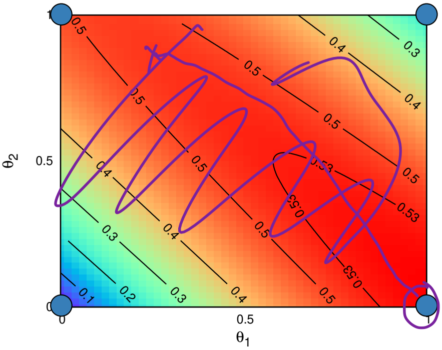
Example with two firms

Relaxed problem **with** reshaping

$$V(\theta) = Q(\theta)\left(L - \sum_{j \in \mathcal{N}} f_j \theta_j\right) \text{ with } q_j = z_j \theta_j^{\frac{1}{\sigma-1}} A \left(\sum_{i \in \mathcal{N}} \Omega_{ij} \theta_i^{1-\frac{\varepsilon_j - 1}{\sigma - 1}} q_i^{\varepsilon_j - 1}\right)^{\frac{\alpha_j}{\varepsilon_j - 1}}$$



~~Problem~~: V is now (quasi) concave

$\Rightarrow$ First-order conditions are necessary and sufficient

$\Rightarrow$ Numerical algorithm converges to global maximum

Tests on Small Networks

For small networks we can solve $\mathcal{P}_{SP}$ directly using exhaustive search

- Comparing solutions to $\mathcal{P}_{RR}$ and $\mathcal{P}_{SP}$:

	With reshaping		Without reshaping	
n	Correct θ	Error in C	Correct θ	Error in C
8	99.9%	0.001%	86.5%	0.791%
10	99.9%	0.001%	85.2%	0.855%
12	99.9%	0.001%	84.5%	0.903%
14	99.9%	0.001%	84.0%	0.926%

▸ Notes ▸ Break. by Ω ▸ Homo. firms ▸ Link by link ▸ Large networks
▸ Link by link large ▸ Error FOCs

The errors come from

- firms that are particularly isolated
- two θ configurations with almost same output

Same parameters as calibration

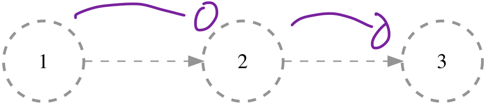
	With reshaping		Without reshaping	
n	Correct θ	Error in C	Correct θ	Error in C
8	98.2%	0.009%	89.6%	0.229%
10	98.9%	0.008%	87.7%	0.274%
12	98.8%	0.008%	86.8%	0.289%
14	98.8%	0.008%	85.3%	0.322%
16	98.8%	0.008%	84.5%	0.339%
18	98.9%	0.007%	84.2%	0.348%
20	98.8%	0.007%	83.3%	0.367%

IV. Economic Forces at Work

Gains From Input Diversity Create Complementarities

Proposition

Operating a firm increases the incentives to operate its direct and indirect neighbors in Ω.



- Impact of operating 2 on the incentives to operate 1 and 3
  - $\theta_2 = 1 \rightarrow q_3$ is larger if 3 operates
  - $\theta_2 = 1 \rightarrow q_2$ is larger if 1 operates
- Upstream and downstream complementarities in operating decisions
  $\rightarrow$ Cascades of firm shutdowns

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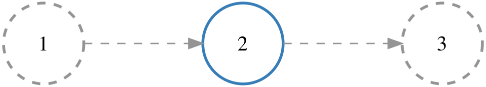

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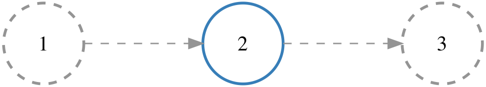

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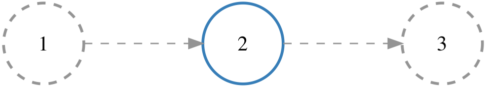
q_3



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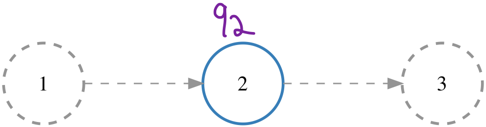

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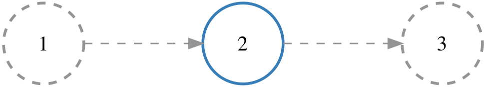

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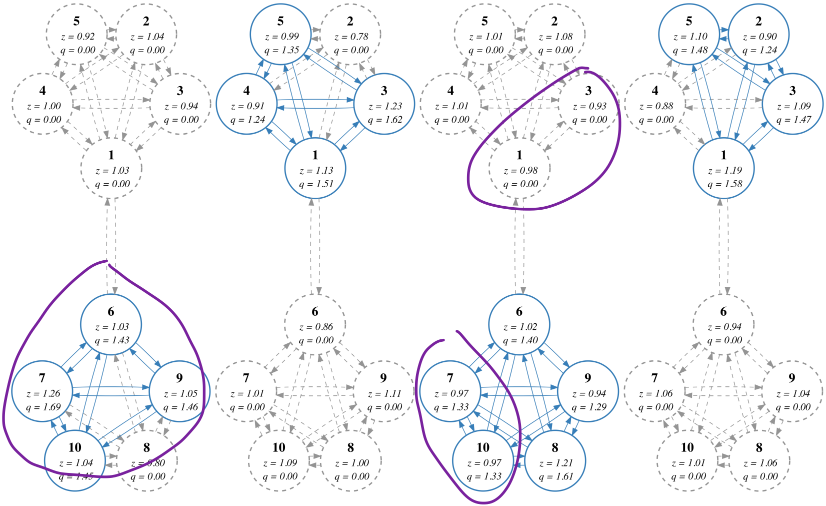
The incentives of the planner to operate a group of firms increase with additional potential connections between them.



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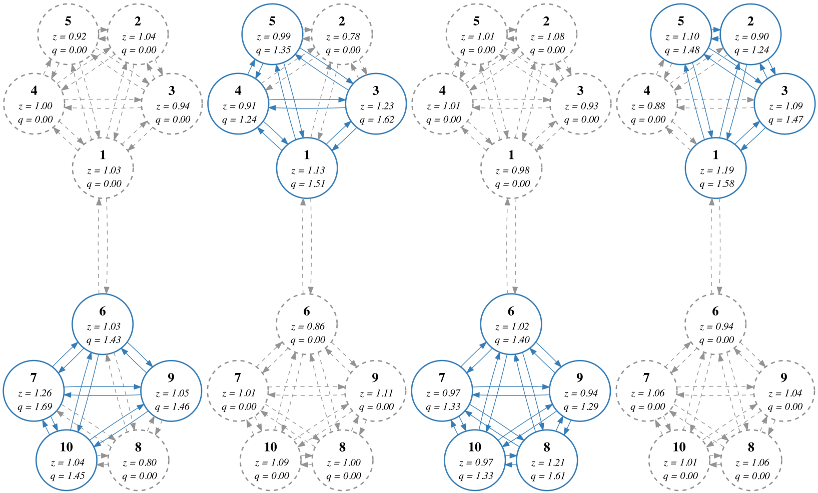
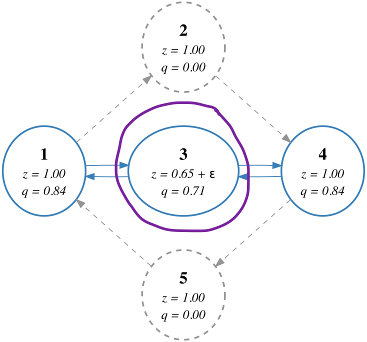
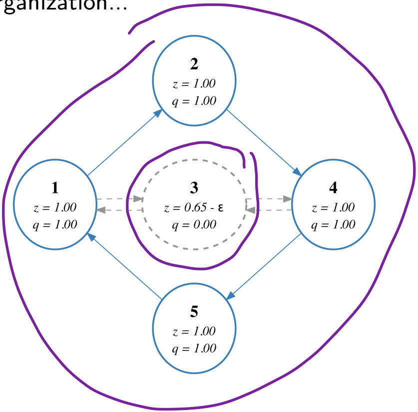


Large Impact of Small Shock

Non-convex nature of the economy:

- A small shock can lead to a large reorganization...





V. Quantitative Exploration

Network Data

Two datasets that cover the U.S. economy

- Compustat
  - Public firms must self-report important customers (>10% of sales)
  - Cohen and Frazzini (2008) and Atalay et al (2011) use fuzzy-text matching algorithms to build the network
- Factset Revere
  - Includes public and private firms, and less important relationships
  - Analysts gather data from 10-K, 10-Q, annual reports, investor presentations, websites, press releases, etc

	Year	Firms/year	Links/year
Compustat			
Atalay et al (2001)	1976 - 2009	1,300	1,500
Cohen and Frazzini (2006)	1980 - 2004	950	1,100
Factset	2003 - 2016	13,000	46,000

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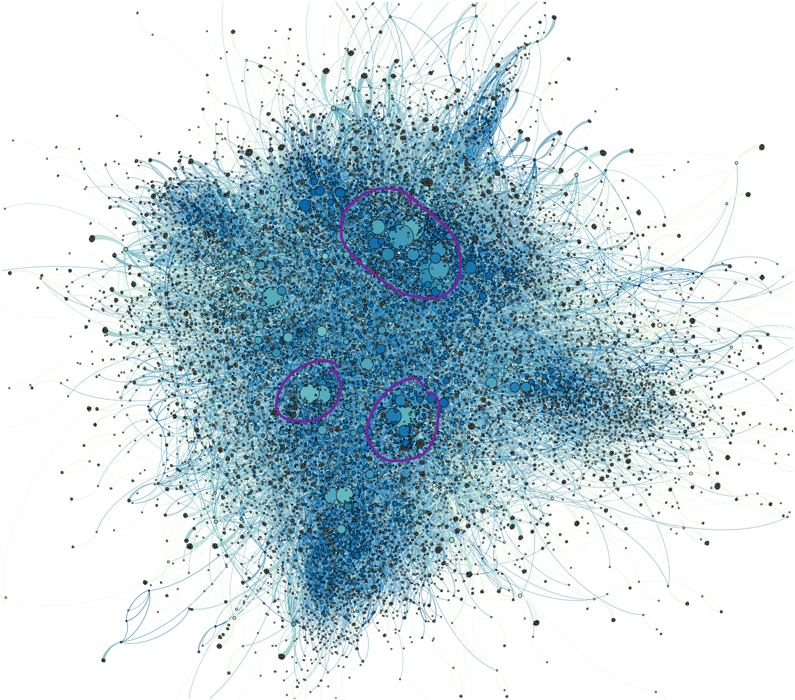

Parameters

Focus on the shape of the network and limit heterogeneity across firms

Parameters from the literature

- $\beta_j = 1$
- $\alpha_j = 0.5$ to fit share of intermediate (Jorgenson et al 1987, Jones 2011)
- $\sigma = \varepsilon_j = 5$ average of estimates (Broda et al 2006)
- Firm productivity follows AR1
  - $\log(z_{it}) \sim \text{iid } \mathcal{N}\left(0, 0.39^2\right)$ from Bartelsman et al (2013)
  - $\rho_z = 0.81$ from Foster et al (2008)
- $f_j \times n = 5\%$ to fit employment in management occupations
- Set $n = 1000$ for high precision while limiting computations

Unobserved matrix Ω:

- Picked to match the *observed* in-degree distribution
- Generate thousands of such Ω's and report averages
- All non-zero Ω_{ij} are set to 1

▸ Ω ▸ Cal. econ.

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Ω Cal. econ.

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What does an optimally designed network looks like?

- Compare **optimal networks** to completely **random networks**
- Differences highlights how efficient allocation shapes the network

	Power law exponents		Clustering coefficient
Network	In-degree	Out-degree	
Efficient	1.00	0.96	3.31
Neutral	1.16	1.15	2.25

Notes: Clustering coeff. multiplied by the square roots of number of nodes for better comparison.

Efficient network features

- More highly connected firms
- More clustering of firms

Def. clust. coeff.

Cascades of Shutdowns

For each firm in each year:

- Look at all neighbors upstream and downstream
- Regress the fraction of these neighbors that exits on whether the original firm exits and some controls

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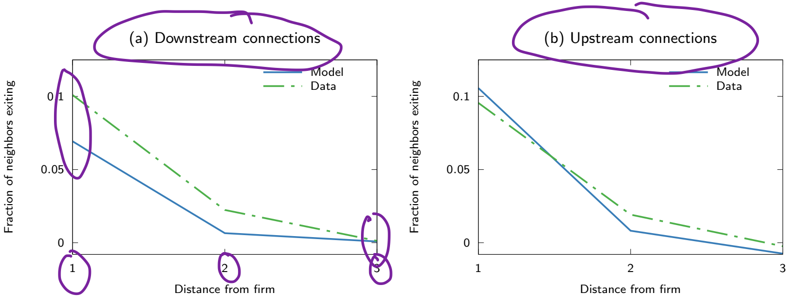

Resilience of Firms

Size of cascades and probability of exit by degree of firm

	Size of cascades		Probability of exit	
	Data	Model	Data	Model
Average firm	0.9	0.9	11.8%	16.6%
High-degree firm	3.1	3.1	2.0%	0.6%

Notes: Size of cascades refers to firm exits up to and including the third neighbors. High degree means above the 90th percentile.

- Highly-connected firms are hard to topple but upon shutting down they create large cascades

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Static theory but z shocks move output and the shape of network together

Table: Correlations with aggregate output

	Model	Datasets		
		Factset	Compustat	
			AHRS	CF
Power law exponents				
In-degree distribution	−0.59	−0.87	−0.35	−0.12
Out-degree distribution	−0.71	−0.97	−0.31	−0.11
Global clustering coefficient	0.54	0.76	0.18	0.11

- Recessions are periods with fewer highly-connected firms and in which clustering activity around most productive firms is costly

Aggregate Fluctuations

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Out-degree distribution	−0.71	−0.97	−0.31	−0.11
Global clustering coefficient	0.54	0.76	0.18	0.11

- Recessions are periods with fewer highly-connected firms and in which clustering activity around most productive firms is costly

Size of fluctuations

$$Y = Q\left(L - \sum_j f_j\theta_j\right)$$

Table: Standard deviations of aggregates

	Output Y	$\approx$	Labor Prod. Q	$+$	Prod. labor $L - \sum_j f_j\theta_j$
Optimal network	0.10		0.10		0.009
Fixed network	0.12		0.12		0

- Fluctuations are $\approx$ 20% smaller when network evolves endogenously
- The difference comes from changes in the shape of the network
- The mean of output is also 11% lower

Intuition

Aggregate Fluctuations

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▸ Intuition

Conclusion

Summary

- Theory of endogenous network formation and aggregate fluctuations
- The optimal network features complementarities between operating decisions of firms that lead to
  - clustering of activity
  - large impact of small changes
  - cascades of shutdowns/restarts
- Compared to U.S. data the model is able to replicate
  - intensity and occurrence of cascades of shutdowns
  - correlation between shape of network and business cycles
- The endogenous reorganization of the network limits the size of fluctuation
- Methodological contribution: approach to easily solve certain non-convex optimization problems

Appendix

Equilibrium

- Definitions
  - A *contract* between i and j is a quantity shipped x_{ij} and a payment T_{ij}.
  - An *arrangement* is a contract between all possible pairs of firms.
  - A *coalition* is a set of firms J.
  - A *deviation* for a coalition J consists of
    1. dropping any contracts with firms not in J and,
    2. altering any contract involving two firms in J.
  - A *dominating deviation* is a deviation such that no firm is worse off and one firm is better off.
  - An allocation is *feasible* if $c_j + \sum_{k \in \mathcal{N}} x_{jk} \leq y_j$ and $\sum_j l_j + f_j \theta_j \leq L$.

Equilibrium

- Firm j maximize profits

$$\pi_j = p_j c_j - wl_j + \sum_{i\in\mathcal{N}} T_{ji} - \sum_{i\in\mathcal{N}} T_{ij} - wf_j\theta_j,$$

subject to $c_j + \sum_{k\in\mathcal{N}} x_{jk} \leq y_j$ and $c_j = \beta_j C\left(p_j/P\right)^{-\sigma}$.

Definition 1

A stable equilibrium is an arrangement $\{x_{ij}, T_{ij}\}_{i,j\in\mathcal{N}^2}$, firms' choices $\{p_j, c_j, l_j, \theta_j\}_{j\in\mathcal{N}}$ and a wage w such that:

1. the household maximizes,
2. firms maximize,
3. markets clear,
4. there are no dominating deviations by any coalition, and
5. the equilibrium allocation is feasible.

Return

Equilibrium

- Firm j maximize profits

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Return

Other quantities

- Labor allocation

$$l = \left[(I_n - \Gamma) \operatorname{diag}\left(\frac{1}{1-\alpha} \right) \right]^{-1} \left(\beta \circ \left(\frac{q}{Q} \right)^{\circ(\sigma-1)} \frac{Y}{Q} \right)$$

- Output

$$(1-\alpha_j)\, y_j = q_j l_j$$

- Consumption

$$c_j = \beta_j \left(\frac{q_j}{w} \right)^{\sigma} Y$$

- Intermediate goods flows

$$x_{ij} \lambda_i^{\varepsilon_j} = \lambda_j^{\varepsilon_j} \alpha_j \left(A z_j \theta_j \left(\frac{\lambda_j}{w} \right)^{1-\alpha_j} \right)^{\frac{\varepsilon_j - 1}{\alpha_j}} \delta_{ij} \Omega_{ij}^{\varepsilon_j} y_j .$$

Return

Tests Details

Aggregates parameters

- $\sigma \in \{4, 6, 8\}$
- $\log(z_k) \sim$ iid $\mathcal{N}(0, 0.25^2)$
- Ω randomly drawn such that firms have on average 3,4,5,6,7 or 8 *potential* incoming connections
  - The corresponding average number of *active* incoming connections is 2.1, 3.0, 3.8, 4.5, 5.3, and 5.8, respectively.
  - For each non-zero: $\Omega_{ij} \sim$ iid $U([0, 1])$

Individual parameters

- $f_j \sim$ iid $U([0, 0.2/n])$
- $\alpha_j \sim$ iid $U([0.25, 0.75])$
- $\varepsilon_j \sim$ iid $U([4, \sigma])$
- $\beta_j \sim$ iid $U([0, 1])$

For each possible combination of aggregate parameters, 200 networks Ω and productivity vectors z are drawn. An economy is kept in the sample only if the first-order conditions yield a solution for which θ hits the bounds $\{0, 1\}$. More than 90% of the economies are kept in the sample.

Return

Breakdown by Ω

n	Reshaping?	Firms with correct θ		
		All Ω's	More connected Ω's	Less connected Ω's
8	Yes	99.8%	99.9%	99.6%
	No	88.2%	89.1%	87.4%
10	Yes	99.7%	99.9%	99.5%
	No	86.5%	87.3%	85.8%
12	Yes	99.7%	99.9%	99.5%
	No	86.2%	87.0%	85.5%
14	Yes	99.7%	99.9%	99.4%
	No	85.5%	86.1%	85.1%

- Less connected Ω: firms have 3, 4 or 5 potential incoming connections
- More connected Ω: firms have 6, 7 or 8 potential incoming connections

◂ Return

Homogeneous Firms

	Number of firms n			
	8	10	12	14
A. With reshaping				
Firms with correct θ	99.9%	99.8%	99.8%	99.8%
Error in output Y	0.001%	0.002%	0.002%	0.002%
B. Without reshaping				
Firms with correct θ	87.2%	85.8%	84.7%	83.8%
Error in output Y	0.71%	0.79%	0.85%	0.89%

Notes: Random networks with parameters $f \in \{0.05/n, 0.1/n, 0.15/n\}$, $\sigma_z = 0.25$, $\alpha \in \{0.45, 0.5, 0.55\}$, $\sigma \in \{4, 6, 8\}$, $\varepsilon \in \{4, 6, 8\}$ and networks Ω randomly drawn such that firms have on average 2, 4, 5, 6, 7 to 8 *potential* incoming connections. Each non-zero Ω_{ij} is set to 1. For each combination of the parameters, 200 different economies are created. For each economy, productivity is drawn from $\log(z_k) \sim$ iid $\mathcal{N}\left(0, \sigma_z^2\right)$. An economy is kept in the sample only if the first-order conditions yield a solution for which θ hits the bounds. More than 90% of the economies are kept in the sample.

◂ Return

Link by link

- Real firms: $f_j = 0$, $\alpha_j = 0.5$, $\sigma = \varepsilon_j = 6$ and $\sigma_z = 0.25$
- Link firms: $\beta_j = 0$, only one input and one output, $f_j \sim$ iid $U([0, 0.1/n])$, $\alpha_j \sim$ iid $U([0.5, 1])$, $\sigma_z = 0.25$
- Ω: between any two real firm, there is a link firm with probability $p \in \{0.7, 0.8, 0.9\}$

Number of firms		With reshaping		Without reshaping	
Real firms m	Link firms $n - m$	Correct θ	Error in C	Correct θ	Error in C
3	up to 6	99.9%	0.001%	94.1%	0.17%
4	up to 12	99.7%	0.003%	91.3%	0.25%
5	up to 20	99.7%	0.006%	89.2%	0.31%

Return

Large Networks

For large networks we cannot solve $\mathcal{P}_{SP}$ directly by trying all possible vectors θ

- After all the welfare-improving 1-deviations θ are exhausted:

	With reshaping		Without reshaping	
n	Correct θ	Error in C	Correct θ	Error in C
1000	$> 99.9\%$	$< 0.001\%$	68.9%	0.58%

Notes: 200 different Ω and z that satisfy the properties of the calibrated economy.

- No guarantee that the solution has been found but very few "obvious errors"

Return

Link by link

- Same parameters as before
- After all the welfare-improving 1-deviation in θ are exhausted:

Number of firms		With reshaping		Without reshaping	
Real firms m	Link firms $n - m$	Correct θ	Error in C	Correct θ	Error in C
10	up to 90	99.7%	0.005%	83.8%	0.46%
25	up to 600	99.9%	0.001%	80.5%	0.55%
40	up to 1560	$<$ 99.9%	$<$ 0.001%	79.5%	0.57%

- θ_j converges on $\{0, 1\}$ for all j in about 60-85% of the tests
  - Even without convergence small error in output and few errors in θ

Return

Solution away from corners

- Sometimes the first-order conditions do not converge on a corner.
- Without excluding these simulations:

		Error in C		
n	Reshaping?	All Ω's	More connected Ω's	Less connected Ω's
8	Yes	0.007%	$<$ 0.001%	0.014%
	No	0.683%	0.640%	0.726%
10	Yes	0.013%	$<$ 0.001%	0.027%
	No	0.781%	0.739%	0.823%
12	Yes	0.008%	$<$ 0.001%	0.016%
	No	0.799%	0.744%	0.853%
14	Yes	0.008%	0.001%	0.016%
	No	0.831%	0.801%	0.862%

Return

Clustering coefficient

- Ω is drawn randomly so that joint distribution of in-degree and out-degree is a bivariate power law of the first kind

$$f\left(x_{in}, x_{out}\right) = \xi\left(\xi - 1\right)\left(x_{in} + x_{out} - 1\right)^{-(\xi+1)}$$

  where ξ is calibrated to 1.85. The marginals for x_{in} and x_{out} follow power law with exponent ξ.

- Correlation between observed in-degree and out-degree
  - Model: 0.67
  - Data: 0.43

◂ Return

Calibrated Network

	Model	Datasets		
		Factset	Compustat	
			AHRS	CF
Power law exponents				
In-degree distribution	1.00	0.97	1.13	1.3
Out-degree distribution	0.96	0.83	2.24	2.2
Global clustering coefficient (normalized)	3.31	3.46	0.08	0.0

Notes: Global clustering coefficients are multiplied by the square roots of the number of nodes for better comparison.

Return

Shape of Network

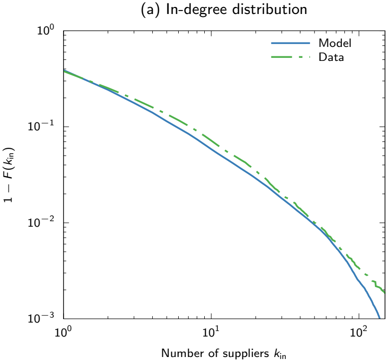


Figure: Model and Factset data for 2016

◂ Return

Clustering coefficient

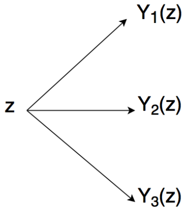
- Triplet: three connected nodes (might be overlapping)
- Triangles: three fully connected nodes (3 triplets)

$$\text{Clustering coefficient} = \frac{3 \times \text{number of triangles}}{\text{number of triplets}}$$

Return

Intuition

A given network θ^k is a function that maps $z \to Y_k(z)$



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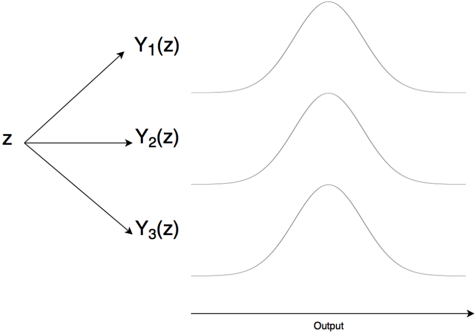

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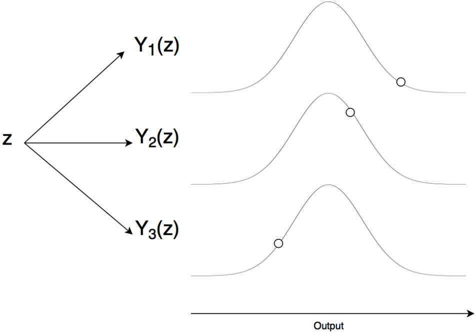

From extreme value theory

$$\mathrm{Var}(Y) = \mathrm{Var}\left(\max_{k \in \{1,\dots,2^n\}} Y_k\right)$$

declines rapidly with n

◂ Return

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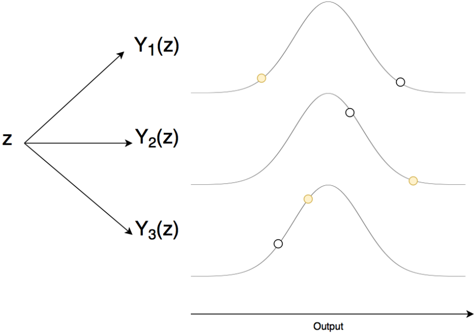

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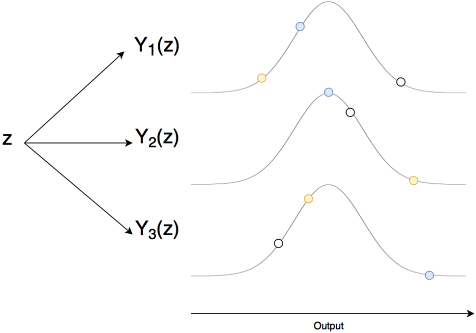

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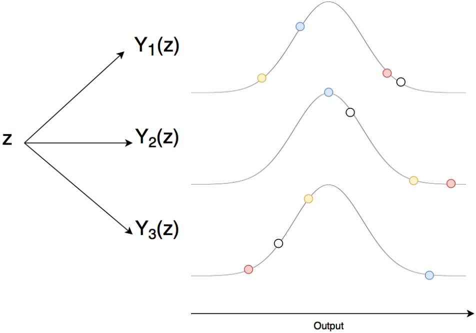

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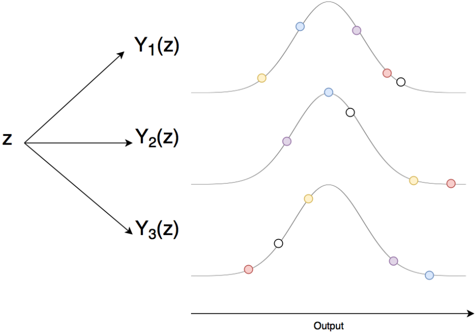

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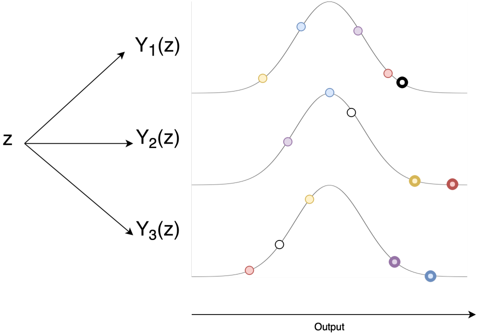

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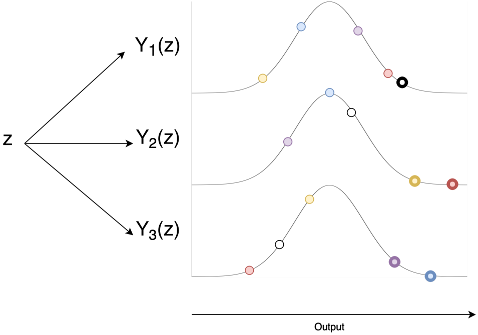

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