

Cascades and Fluctuations in an Economy with an Endogenous Production Network

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- Production in modern economies involves a complex network of producers supplying and demanding goods from each other
- The shape of this network
 - ▶ is an important determinant of how micro shocks aggregate into macro fluctuations
 - ▶ is also constantly changing in response to micro shocks
 - For instance, after a severe shock a producer might shut down which might lead its neighbors to shut down as well, etc...
 - Cascade of shutdowns that spreads through the network

This paper proposes a

Theory of network formation and aggregate fluctuations

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Theory of network formation and aggregate fluctuations

- Endogenous network formation
 - ▶ Atalay et al (2011), Oberfield (2013), Carvalho and Voigtländer (2014)
- Network of sectors and fluctuations
 - ▶ Horvath (1998), Dupor (1999), Acemoglu et al (2012), Baqaee (2016), Acemoglu et al (2016), Lim (2017)
- Non-convex adjustments in networks
 - ▶ Bak, Chen, Woodford and Scheinkman (1993), Elliott, Golub and Jackson (2014)

I. Model

- There are n units of production (firm) indexed by $j \in \{1, \dots, n\}$
 - ▶ Each unit produces a differentiated good
 - ▶ Differentiated goods can be used to
 - produce a final good

$$Y \equiv \left(\sum_{j=1}^n (y_j^0)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

- produce other differentiated goods
- Representative household
 - ▶ Consumes the final good
 - ▶ Supplies L units of labor inelastically

- Firm j produces good j

$$y_j = \frac{A}{\alpha^\alpha (1 - \alpha)^{1-\alpha}} z_j \left(\sum_{i=1}^n x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

- Firm j can only use good i as input if there is a *connection* between firms i and j
 - ▶ $\Omega_{ij} = 1$ if connection and $\Omega_{ij} = 0$ otherwise
 - ▶ A connection can be *active* or *inactive*
 - ▶ Matrix Ω is *exogenous*
- A firm can only produce if it pays a fixed cost f in units of labor
 - ▶ $\theta_j = 1$ if j is operating and $\theta_j = 0$ otherwise
 - ▶ Vector θ is *endogenous*

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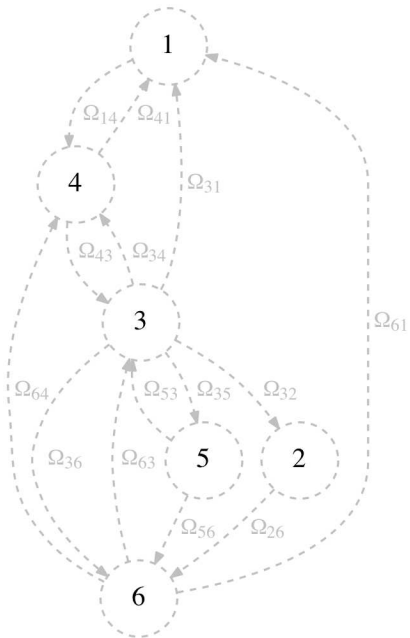
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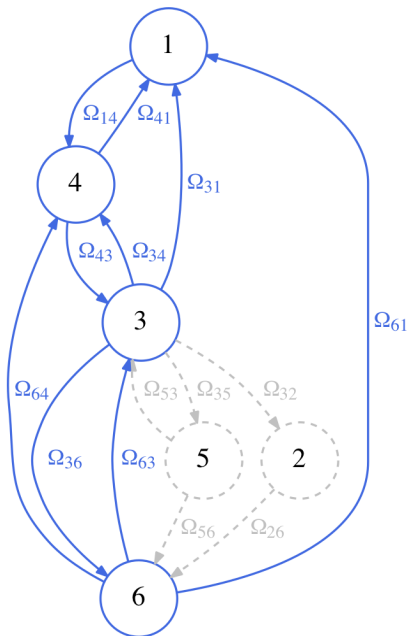
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Problem \mathcal{P}_{SP} of a social planner

$$\max_{\substack{y^0, x, l \\ \theta \in \{0,1\}^n}} \left(\sum_{j=1}^n (y_j^0)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to

1. a resource constraint for each good j

$$y_j^0 + \sum_{k=1}^n x_{jk} \leq \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \theta_j \left(\sum_{i=1}^n \Omega_{ij} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor

$$\sum_{j=1}^n l_j + f \sum_{j=1}^n \theta_j \leq L$$

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1. a resource constraint for each good j (**Lagrange multiplier: λ_j**)

$$y_j^0 + \sum_{k=1}^n x_{jk} \leq \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \theta_j \left(\sum_{i=1}^n \Omega_{ij} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor (**Lagrange multiplier: w**)

$$\sum_{j=1}^n l_j + f \sum_{j=1}^n \theta_j \leq L$$

II. Social Planner with Exogenous θ

Define $q_j = w/\lambda_j$

- From the FOCs, output is $(1 - \alpha) y_j = q_j l_j$
- q_j is the *labor productivity* of firm j

Proposition 1

In the efficient allocation,

$$q_j = z_j \theta_j A \left(\sum_{i=1}^n \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}} \quad (1)$$

Furthermore, there is a unique vector q that satisfies (1).

Knowing q we can solve for all other quantities easily.

Lemma 1

Aggregate output is

$$Y = Q \left(L - f \sum_{j=1}^n \theta_j \right)$$

where $Q \equiv \left(\sum_{j=1}^n q_j^{\sigma-1} \right)^{\frac{1}{\sigma-1}}$ is aggregate labor productivity.

III. Social Planner with Endogenous θ

Social Planner with Endogenous θ

$$\max_{\theta \in \{0,1\}^n} Q \left(L - f \sum_{j=1}^n \theta_j \right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i=1}^n \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}}$$

Very hard problem (MINLP — NP Hard)

- The set $\theta \in \{0,1\}^n$ is not convex
- Objective function is not concave

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Alternative approach

Solution approach: Find an alternative problem such that

- P1 The alternative problem is easy to solve
- P2 A solution to the alternative problem also solves \mathcal{P}_{SP}

Social Planner with Endogenous θ

Consider the relaxed and reshaped problem \mathcal{P}_{RR}

$$\max_{\theta \in \{0,1\}^n} Q \left(L - f \sum_{j=1}^n \theta_j \right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i=1}^n \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}}$$

Parameters $a > 0$ and $b \geq 0$ are *reshaping constants*

- Reshape the objective function away from optimum (i.e. when $0 < \theta_j < 1$)
 - For a : if $\theta_j \in \{0,1\}$ then $\theta_j^a = \theta_j$
 - For b : $\{\theta_i = 0\} \Rightarrow \{q_i = 0\}$ and $\{\theta_i = 1\} \Rightarrow \{\theta_i^b q_i^{\epsilon-1} = q_i^{\epsilon-1}\}$
- Parameters such that P1 and P2 are satisfied:

$$\boxed{a = \frac{1}{\sigma - 1} \quad \text{and} \quad b = 1 - \frac{\epsilon - 1}{\sigma - 1}} \quad (\star)$$

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P1 The alternative problem \mathcal{P}_{RR} is easy to solve

Proposition 2

If $\Omega_{ij} = c_i d_j$ for some vectors c and d then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to \mathcal{P}_{RR} .

Proposition 3

Let $\sigma = \epsilon$ and suppose that $f > 0$ and $\bar{z} - \underline{z} > 0$ are not too big. If Ω is sufficiently connected, then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to \mathcal{P}_{RR} .

These propositions

- Only provides *sufficient* conditions
- In the paper: Test the approach on thousands of economies

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P2 A solution to the alternative problem \mathcal{P}_{RR} also solves \mathcal{P}_{SP}

Proposition 4

If θ^ solves \mathcal{P}_{RR} and that $\theta_j^* \in \{0, 1\}$ for all j , then θ^* also solves \mathcal{P}_{SP} .*

Solution θ^* to \mathcal{P}_{RR} is such that $\theta_j^* \in \{0, 1\}$ for all j (**P2**) if

- the (\star) condition is satisfied
- there are many firms
- the network is sufficiently connected

► Details

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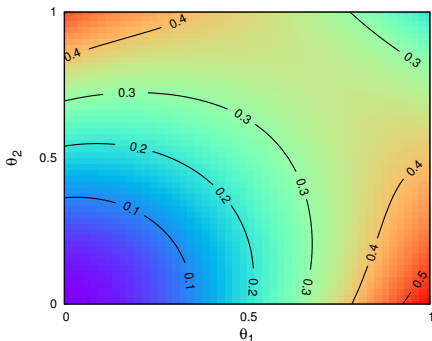
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► Details

Example with $n = 2$

Relaxed problem **without** reshaping

$$V(\theta) = Q(\theta) \left(L - f \sum_{j=1}^n \theta_j \right) \text{ with } q_j = z_j \theta_j A \left(\sum_{i=1}^n \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}}$$



Problem: V is not concave

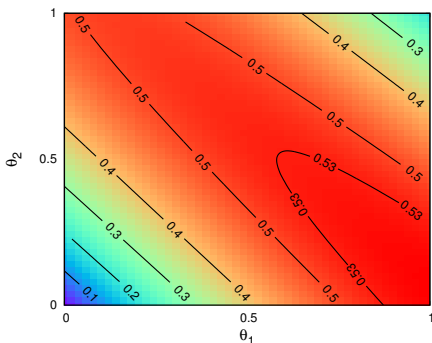
⇒ First-order conditions are not sufficient

⇒ Numerical algorithm can get stuck in local maxima

Example with $n = 2$

Relaxed problem **with** reshaping

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~~Problem:~~ V is now (quasi) concave

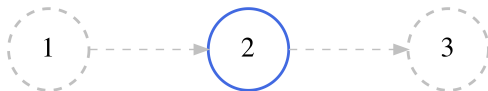
- ⇒ First-order conditions are necessary and sufficient
- ⇒ Numerical algorithm converges to global maximum

IV. Economic Forces at Work

Complementarities

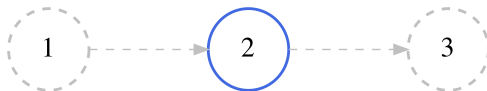


- Impact of operating 2 on the incentives to operate 1 and 3
 - ▶ Operating 3 leads to a larger q_3 because 2 is operating
 - ▶ Operating 1 increases q_2 because 2 is operating
- Complementarity between operating decisions of nearby firms



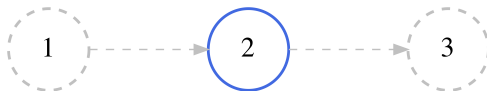
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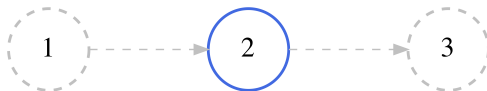


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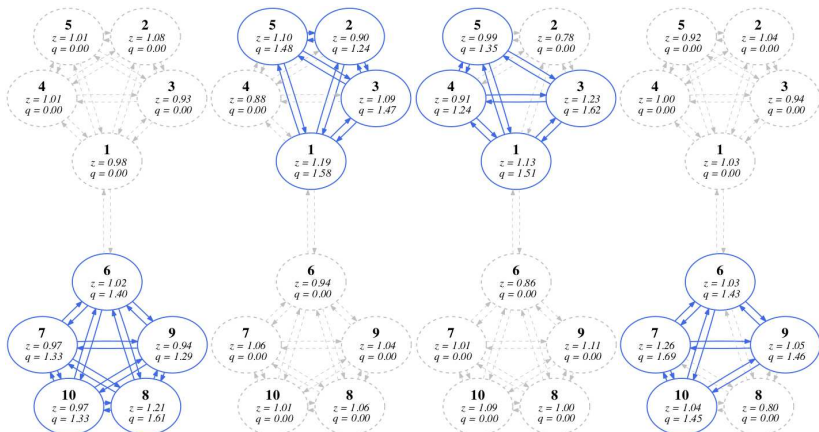


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Complementarities lead to clustering



V. Quantitative Exploration

- Two datasets that cover the U.S. economy
 - ▶ Cohen and Frazzini (2008) and Atalay et al (2011)
 - ▶ Both rely on Compustat data
 - Public firms must self-report customers that purchase more than 10% of sales
 - Use fuzzy-text matching algorithms and manual matching to build networks
 - ▶ Cover 1980 to 2004 and 1976 to 2009 respectively

Parameters from the literature

- $\alpha = 0.5$ to fit the share of intermediate (Jorgenson et al 1987, Jones 2011)
- $\sigma = \epsilon = 6$ average of estimates (Broda et al 2006)
 - ▶ Robustness with smaller ϵ in the paper
- $\log(z_{it}) \sim \mathcal{N}(0, 0.39^2)$ from Bartelsman et al (2013)
- $f \times n = 5\%$ to fit employment in management occupations
- Calibrate $n = 3000$ to match number of active firms in Atalay et al (2011)

Unobserved network Ω :

- Pick to match the *observed* in-degree distribution
- Generate thousands of such Ω 's and report averages

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Shape of the network

What types of network does the planner choose?

- Compare **optimal networks** to completely **random networks**
- Differences highlights how efficient allocation shapes the network

| | Optimal networks | Random networks |
|--------------------------------|------------------|-----------------|
| A. Pareto shape parameters | | |
| In-degree | 1.43 | 1.48 |
| Out-degree | 1.37 | 1.48 |
| B. Measures of proximity | | |
| Clustering coefficient | 0.027 | 0.018 |
| Average distance between firms | 2.26 | 2.64 |

Efficient allocation features

- More highly connected firms
- More clustering of firms

► Def. clust. coeff.

Cascades of shutdowns

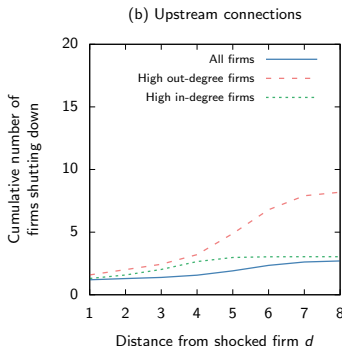
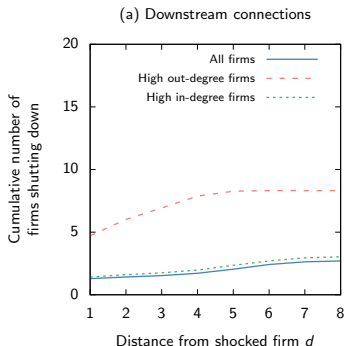
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- Exit of a firm makes it more likely that its neighbors exit as well ...
- ... which incentivizes the second neighbors to exit as well ...
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- Highly-connected firms are hard to topple but upon shutting down they create large cascades

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The shape of the network changes with the business cycle

| | Correlation with output | | |
|-------------------------------|-------------------------|-----------|-------------|
| | Model | Data | |
| | | CF (2008) | AHRS (2011) |
| A. Power law shape parameters | | | |
| In-degree | -0.10 | -0.10 | -0.21 |
| Out-degree | -0.31 | -0.24 | -0.13 |
| B. Clustering coefficient | 0.47 | 0.70 | 0.15 |

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Aggregate fluctuations

Size of fluctuations

$$Y = Q \left(L - f \sum_j \theta_j \right)$$

Table: Standard deviation of aggregates

| | Output Y | Labor Prod. Q | Prod. labor $L - f \sum_j \theta_j$ |
|-----------------|---------------|--------------------|--|
| Optimal network | 0.039 | 0.039 | 0.0014 |
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Implications:

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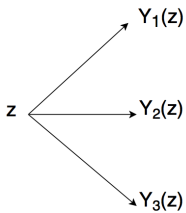
► Intuition

Intuition

A given network θ^k is a function that maps $z \rightarrow Y_k(z)$

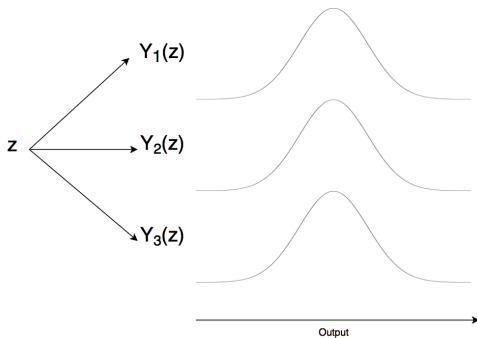
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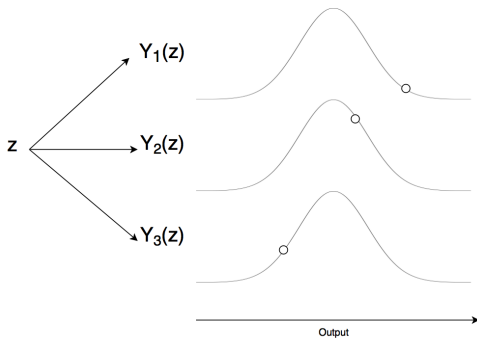
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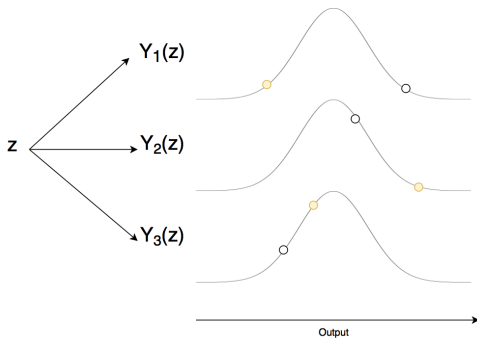
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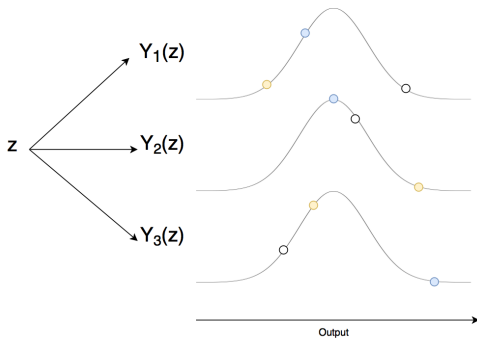
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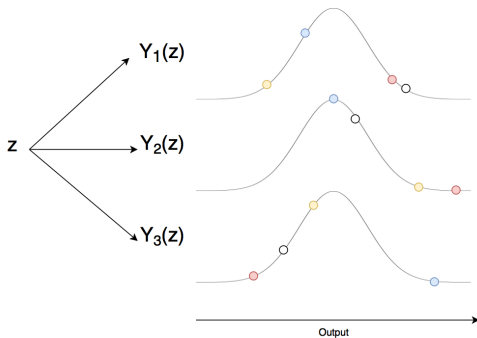
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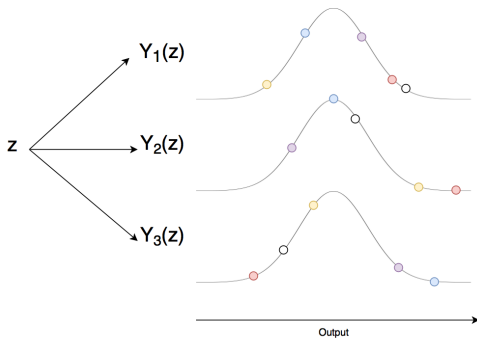
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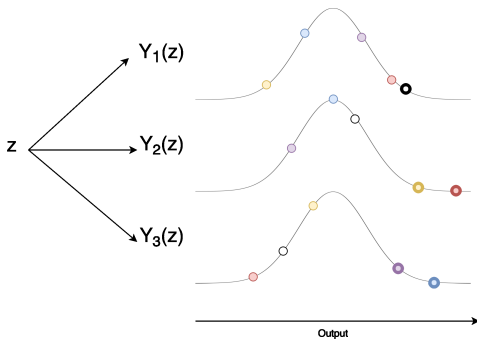
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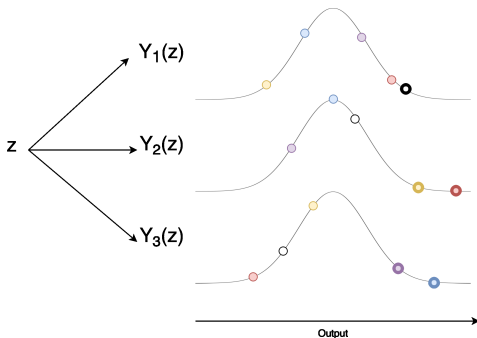
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From extreme value theory

$$\text{Var}(Y) = \text{Var}\left(\max_{k \in \{1, \dots, 2^n\}} Y_k\right)$$

declines rapidly with n

Conclusion

Additional results in the paper:

- Impact of position in the network on firm-level characteristics
- Endogenous skewness in distribution of employment, productivity, output

Summary

- Theory of network formation and aggregate fluctuations
- Propose an approach to solve these hard problems easily
- The optimal allocation features
 - ▶ Clustering of activity
 - ▶ Cascades of shutdowns/restarts
- Optimal network substantially limit the size of fluctuations

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Lemma 2

The optimal labor allocation satisfies

$$l = (1 - \alpha) \underbrace{[I_n - \alpha\Gamma]^{-1}}_{(1)} \underbrace{\left(\frac{q}{Q}\right)^{\circ(\sigma-1)}}_{(2)} \left(L - f \sum_{j=1}^n \theta_j\right)$$

where I_n is the identity matrix and where Γ is an $n \times n$ matrix where $\Gamma_{jk} = \frac{\Omega_{jk} q_j^{\epsilon-1}}{\sum_{i=1}^n \Omega_{ik} q_i^{\epsilon-1}}$ captures the importance of j as a supplier to k .

Determinants of l_j

(1) Importance of j as a supplier

- ▶ Leontief inverse $\left([I_n - \alpha\Gamma]^{-1} = I_n + \alpha\Gamma + (\alpha\Gamma)^2 + \dots\right)$

(2) Relative efficiency

Reshaping

Intuition:

- First-order condition on θ_j :

$$\text{Marginal Benefit}(\theta_j, F(\theta)) - \text{Marginal Cost}(\theta_j, G(\theta)) = \bar{\mu}_j - \underline{\mu}_j$$

- Under (\star) the marginal benefit of θ_j only depends on θ_j through aggregates
- For large connected network F and G are independent of θ_j

• Return

• FOCs

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Details of reshaping

Simpler to consider

$$\mathcal{P}'_{RD}: \max_{\theta \in [0,1]^n, q} \left(\sum_{j=1}^n q_j^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \left(L - f \sum_{j=1}^n \theta_j \right)$$
$$q_j \leq A z_j \theta_j^a A B_j^\alpha \quad (\text{LM: } \beta_j)$$

where $B_j = \left(\sum_{i=1}^n \Omega_{ij} \theta_i^b q_i^{\epsilon-1} \right)^{\frac{1}{\epsilon-1}}$.

First order condition with respect to θ_k :

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial Q}{\partial q_k} \left(L - f \sum_{j=1}^n \theta_j \right) - fQ + \sum_{j=1}^n \beta_j \left(\frac{\partial q_k}{\partial \theta_k} \frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} \right) \frac{\partial q_j}{\partial B_j} = \bar{\mu}_k - \underline{\mu}_k$$

The terms are

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial Q}{\partial q_k} = z_k a \theta_k^{a-1} A B_k^\alpha \times (z_k \theta_k^a A B_k^\alpha)^{\sigma-2} Q^{2-\sigma}$$
$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} = B_j \theta_k^{b-1} \Omega_{kj} \left(\frac{z_k \theta_k^a A B_k^\alpha}{B_j} \right)^{\epsilon-1} \left(a + \frac{b}{\epsilon-1} \right)$$

Testing the approach on small networks

For small networks we can solve \mathcal{P}_{SP} directly by trying all possible vectors θ

- Comparing approaches for a million different economies:

| | Number of firms n | | | |
|-------------------------------|---------------------|----------|----------|----------|
| | 8 | 10 | 12 | 14 |
| A. With reshaping | | | | |
| Firms with correct θ_j | 99.9% | 99.9% | 99.9% | 99.8% |
| Error in output Y | 0.00039% | 0.00081% | 0.00174% | 0.00171% |
| B. Without reshaping | | | | |
| Firms with correct θ_j | 84.3% | 83.2% | 82.3% | 81.3% |
| Error in output Y | 0.84% | 0.89% | 0.93% | 0.98% |

Notes: Parameters $f \in \{0.05/n, 0.1/n, 0.15/n\}$, $\sigma_z \in \{0.34, 0.39, 0.44\}$, $\alpha \in \{0.45, 0.5, 0.55\}$, $\sigma \in \{4, 6, 8\}$ and $\epsilon \in \{4, 6, 8\}$. For each combination of parameters 1000 different economies are created. For each economy, productivity is drawn from $\log(z_k) \sim \text{iid } \mathcal{N}(0, \sigma_z)$ and Ω is drawn randomly such that each link Ω_{ij} exists with some probability such that a firm has on average five possible incoming connections. A network is kept in the sample only if the first-order conditions give a solution in which θ hits the bounds.

The errors come from

- firms that are particularly isolated
- two θ configurations with almost same output

Testing the approach on large networks

For large networks we cannot solve \mathcal{P}_{SP} directly by trying all possible vectors θ

- After all the 1-deviations θ are exhausted:

| | With reshaping | Without reshaping |
|-------------------------------|----------------|-------------------|
| Firms with correct θ_j | 99.8% | 72.1% |
| Error in output Y | 0.00028% | 0.69647% |

Notes: Simulations of 200 different networks Ω and productivity vectors z that satisfy the properties of the calibrated economy.

- Very few “obvious errors” in the allocation found by the approach

◀ Return

Distribution of in-degree

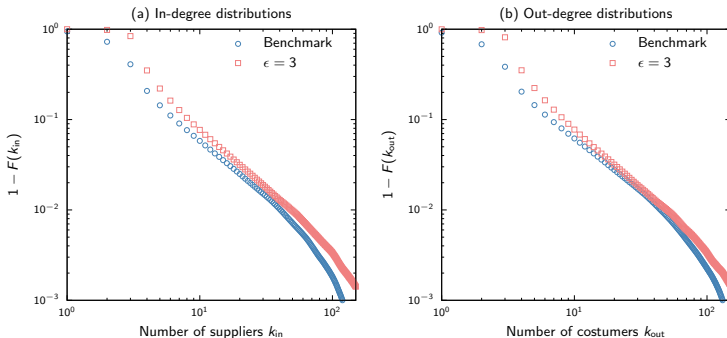


Figure: Distribution of the number of suppliers and the number of customers

In-degree power law shape parameter

- Calibration: 1.43
- Data: 1.37 (Cohen and Frazzini, 2008) and 1.3 (Atalay et al, 2011)

Figure 2: In-degree and Out-degree CDFs

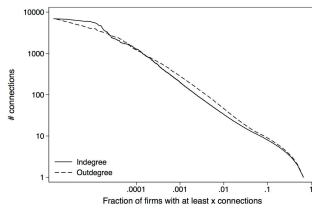


Figure: Distribution of in-degree and out-degree in Bernard et al (2015)

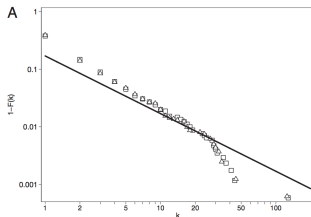


Figure: Distribution of in-degree in Atalay et al (2011)

Clustering coefficient

- Triplet: three connected nodes (might be overlapping)
- Triangles: three fully connected nodes (3 triplets)

$$\text{Clustering coefficient} = \frac{3 \times \text{number of triangles}}{\text{number of triplets}}$$

◀ Return

Firm-level distributions

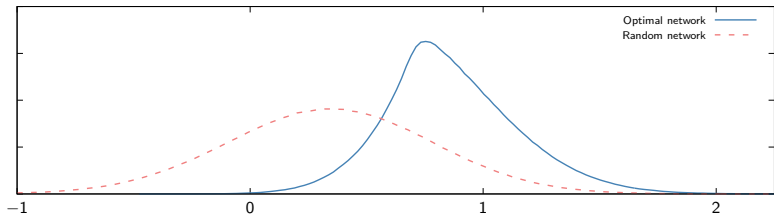


Figure: Distributions of $\log(q)$

◀ return

Cascades of shutdowns

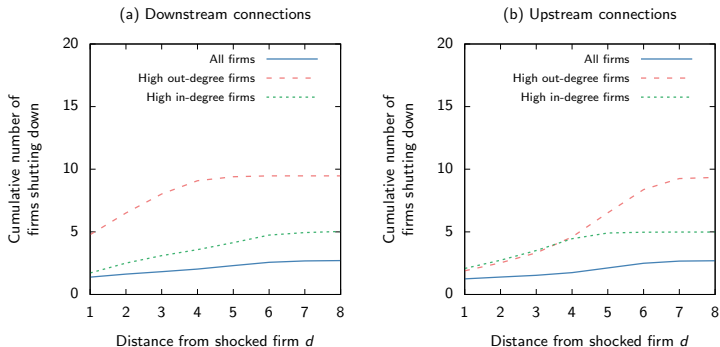


Figure: $\alpha = 0.75$

Cascades of shutdowns

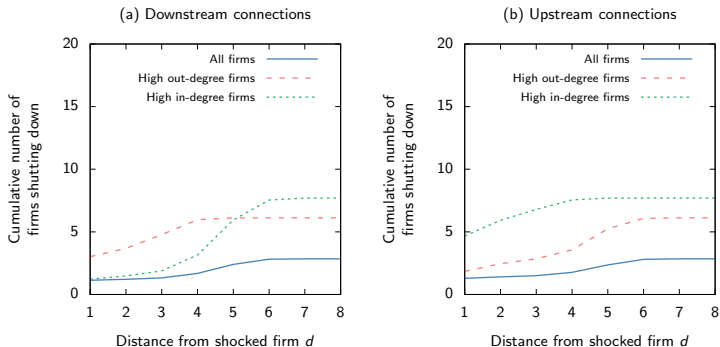


Figure: $\epsilon = 3$

| | Probability of firm shutdown | | |
|-----------------------|------------------------------|-----------------|----------------|
| | Benchmark | $\alpha = 0.75$ | $\epsilon = 3$ |
| All firms | 92% | 82% | 32% |
| High out-degree firms | 20% | 8% | 0% |
| High in-degree firms | 56% | 19% | 15% |

[◀ return](#)