Echoes and Delays: Time-to-Build in Production Networks

Edouard Schaal
CREI, ICREA, UPF, BSE and CEPR

Mathieu Taschereau-Dumouchel
Cornell University

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Motivation

- Firms in a modern economy rely on a complex network of suppliers, whose time-to-build varies significantly
  - Ex.: $<$ 1 month for furniture assembly, 1-2 years for container ships
- Most of the literature on production networks ignores time-to-build and possibilty of delays
  - Acemoglu et al. (2012), Baqaee and Farhi (2019, 2020),... essentially static
  - Roundabout production: disruptions are resolved within period
- How does the introduction of time-to-build or delivery lags affect dynamics of production networks?

What We Do

- We propose a simple model to introduce time-to-build (T2B) in production networks
  - Long and Plosser (1983) (one period delay) + heterogeneous T2B
- We analyze how T2B contributes to propagation of shocks:
  1. Persistence, delays and bottlenecks
  2. Echoes and endogenous fluctuations
  3. Dynamic sectoral comovements and aggregation
- Empirical evidence (in progress)

Related Literature

- Shock propagation in production networks
  - Acemoglu et al. (2012), Barrot and Sauvagnat (2016), Carvalho et al. (2016), Acemoglu et al. (2017), Baqaee and Farhi (2019), Ghassibe (2024),...
- Time-to-build
  - Kydland and Prescott (1982), Schwartzman (2014),...
  - In production networks:
    - Long and Plosser (1983), Liu and Tsyvinski (2023), Bizarri (2024), Carvalho and Reischer (2025), Bizarri, Pangallo and Queirós (2025), Leng, Liu, Ren and Tsyvinski (2025)
- Endogenous fluctuations in multi-sector economies:
  - Benhabib and Nishimura (1979, 1985, ...)
- Delays in supply chains:
  - Djankov et al. (2010), Hummels and Schaur (2013), Meier (2020), Alessandria et al. (2023), Carreras-Valle and Ferrari (2025)
- Sequential production in trade
  - Dixit and Grossman (1982), Antràs and Chor (2013), Costinot et al. (2013), etc.

Data

Input-Output

- IO-Use tables from BEA for 2017
  - 402 6-digit NAICS industries

Time-to-Build

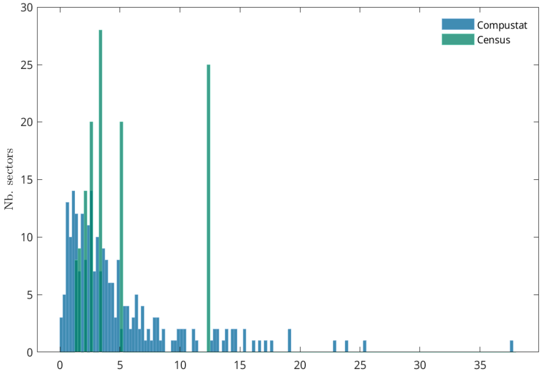
- Measure:

$$\text{backlog ratio} = \frac{\text{stock value of unfilled orders}}{\text{flow value of goods delivered}}$$

  - Includes production time + waiting and delivery times, some potential biases
- US Census M3 survey on "Shipments, Inventories and Orders" (monthly)
  - All manufacturing, aggregated to $\sim$ 10 subsectors for 1992-2024
- Compustat "Order Backlog" variable (annual)
  - Publicly listed firms but firm level & broader sectoral coverage for 1970-2024

Intuition

Backlog Distribution



Note: weighted mean = 5.80 (Compustat) / 6.28 (Census) in months, average $\sigma\left(\widehat{\text{backlog}}_t\right) = 0.23$

Figure 1: Distribution of backlog ratios (months) across 6-digit sectors

Model

Model

- Time is discrete
- Representative household with inelastic labor supply
- Sectors $i = 1, \ldots, N$ with production

$$y_{it} = A_{it} F_i \left(l_{it}, x_{i1,t}, \ldots, x_{iN,t} \right)$$

- Time-to-build modeled as delivery lags:
  - Goods in sector i take τ_i periods to be delivered
  - Denote $X_{i\tau} \equiv$ agg. stock of i scheduled for delivery in τ periods
- No capital, no storage between periods for now

Planning Problem

$$V\left(\{A_i\}, \{X_{1\tau}\}_{\tau=0}^{\tau_1-1}, ..., \{X_{N\tau}\}_{\tau=0}^{\tau_N-1}\right) = \max_{c_i, l_i, x_{ij}, y_i} U(c_1, ..., c_N)$$
$$+\beta E\left[V\left(\{A_i'\}, \{X_{1\tau}'\}_{\tau=0}^{\tau_1-1}, ..., \{X_{N\tau}'\}_{\tau=0}^{\tau_N-1}\right)\right]$$

subject to:

$$1 \geq \sum_{i=1}^{N} l_i$$

and for all $i = 1..N$:

$$X_{i\tau}' = X_{i\tau+1} \text{ for } 0 \leq \tau < \tau_i - 1$$
$$X_{i\tau_i-1}' = y_i$$
$$X_{i0} \geq c_i + \sum_j x_{ji}$$
$$y_i = A_i F_i(l_{it}, x_{i1}, ..., x_{iN})$$

Planning Problem

$$V\left(\{A_i\},\{X_{1\tau}\}_{t=0}^{\tau_1-1},\ldots,\{X_{N\tau}\}_{\tau=0}^{\tau_N-1}\right)=\max_{c_i,l_i,x_{ij},y_i} U\left(c_1,\ldots,c_N\right)$$
$$+\beta E\left[V\left(\{A_i'\},\{X_{1\tau}'\}_{t=0}^{\tau_1-1},\ldots,\{X_{N\tau}'\}_{\tau=0}^{\tau_N-1}\right)\right]$$

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$$y_i=A_iF_i\left(l_{it},x_{i1},\ldots,x_{iN}\right)$$

Planning Problem

$$V\left(\{A_i\}, \{X_{1\tau}\}_{t=0}^{\tau_1 - 1}, ..., \{X_{N\tau}\}_{\tau=0}^{\tau_N - 1}\right) = \max_{c_i, l_i, x_{ij}, y_i} U(c_1, ..., c_N) + \beta E\left[V\left(\{A_i'\}, \{X_{1\tau}'\}_{t=0}^{\tau_1 - 1}, ..., \{X_{N\tau}'\}_{\tau=0}^{\tau_N - 1}\right)\right]$$

subject to:

$$1 \geq \sum_{i=1}^{N} l_i$$

and for all $i = 1..N$:

$$X_{i\tau}' = X_{i\tau+1} \text{ for } 0 \leq \tau < \tau_i - 1$$

$$X_{i\tau_i - 1}' = y_i$$

$$X_{i0} \geq c_i + \sum_j x_{ji}$$

$$y_i = A_i F_i(l_{it}, x_{i1}, ..., x_{iN})$$

Planning Problem

$$V\left(\{A_i\}, \{X_{1\tau}\}_{t=0}^{\tau_1-1}, \ldots, \{X_{N\tau}\}_{\tau=0}^{\tau_N-1}\right) = \max_{c_i, l_i, x_{ij}, y_i} U(c_1, \ldots, c_N) + \beta E\left[V\left(\{A_i'\}, \{X_{1\tau}'\}_{t=0}^{\tau_1-1}, \ldots, \{X_{N\tau}'\}_{\tau=0}^{\tau_N-1}\right)\right]$$

subject to:

$$1 \geq \sum_{i=1}^{N} l_i$$

and for all $i = 1..N$:

$$X_{i\tau}' = X_{i\tau+1} \text{ for } 0 \leq \tau < \tau_i - 1$$

$$X_{i\tau_i-1}' = y_i$$

$$X_{i0} \geq c_i + \sum_j x_{ji}$$

$$y_i = A_i F_i\left(l_{it}, x_{i1}, \ldots, x_{iN}\right)$$

Solution

- High dimensional state space: 402 sectors $\times$ # lags !

Proposition

For $F_i\left(l, x_{i1}, \ldots, x_{iN}\right) = l^{\alpha_i} \prod_{j=1}^{N} x_{ij}^{\omega_{ij}}$ *with* $\alpha_i + \sum_j \omega_{ij} = 1$ *and* $U\left(c_1, \ldots, c_N\right) = \sum_1^N \gamma_i \log c_i$, *the economy can be solved analytically and*

$$
\begin{aligned}
c_i &= \overline{c_i} X_{i0} \\
x_{ji} &= \overline{x_{ji}} X_{i0} \\
l_i &= \overline{l_i}
\end{aligned}
$$

where $\overline{c_i}$, $\overline{x_{ij}}$ *and* $\overline{l_i}$ *are constants, and*

$$
V\left(\mathbf{A}, \mathbf{X}_1, \ldots\right) = \sum_{i=1}^{N} \sum_{\tau=0}^{\tau_i} \beta^{\tau} \zeta_i \log X_{i\tau} + G\left(\mathbf{A}\right)
$$

where

$$
\begin{aligned}
\boldsymbol{\zeta} &= \left(I - \left[\boldsymbol{\Omega} \cdot \beta^{\boldsymbol{\tau}}\right]'\right)^{-1} \boldsymbol{\gamma} \quad (\textit{time-adjusted Domar weights}) \\
G\left(\mathbf{A}\right) &= \sum_i \beta^{\tau_i} \zeta_i \log A_i + \beta E\left[G\left(\mathbf{A}'\right)\right]
\end{aligned}
$$

Output

- In log-deviation from steady state:

$$\boxed{\hat{y}_{it} = \hat{A}_{it} + \sum_j \omega_{ij} \hat{y}_{jt-\tau_j}}$$

- VAR(τ_{max}) representation:

$$\hat{\mathbf{y}}_t = \mathbf{\Omega}_1 \hat{\mathbf{y}}_{t-1} + \ldots + \mathbf{\Omega}_{\tau_{max}} \hat{\mathbf{y}}_{t-\tau_{max}} + \hat{\mathbf{A}}_t$$

  where $\mathbf{\Omega}_\tau = \mathbf{\Omega} \cdot \mathbf{1}\{\tau = \tau_i\}$

- Nested cases:
  - Roundabout production: no time-to-build

$$\hat{\mathbf{y}}_t = \hat{\mathbf{A}}_t + \mathbf{\Omega}\hat{\mathbf{y}}_t \Rightarrow \hat{\mathbf{y}}_t = \hat{\mathbf{A}}_t + \mathbf{\Omega}\hat{\mathbf{A}}_t + \mathbf{\Omega}^2\hat{\mathbf{A}}_t + \ldots$$
$$= [\mathbf{I} - \mathbf{\Omega}]^{-1}\hat{\mathbf{A}}_t \quad \text{(Leontieff inverse)}$$

  - Long and Plosser (1983): one-period time-to-build

$$\hat{\mathbf{y}}_t = \hat{\mathbf{A}}_t + \mathbf{\Omega}\hat{\mathbf{y}}_{t-1} \Rightarrow \hat{\mathbf{y}}_t = \hat{\mathbf{A}}_t + \mathbf{\Omega}\hat{\mathbf{A}}_{t-1} + \mathbf{\Omega}^2\hat{\mathbf{A}}_{t-2} + \ldots$$

Persistence and Delays

Additional results:

- Persistence statistics ▸ Go

$$\mathcal{T}\left(\hat{\mathbf{A}}\right) = \frac{1}{CIR\left(\hat{\mathbf{A}}\right)} \sum_{\tau=0}^{\infty} \sum_{i} \tau w_i \hat{y}_{i\tau}\left(\hat{\mathbf{A}}\right)$$

$\Rightarrow$ Substantial additional persistence even for iid shocks, highly heterogeneous

- Delay shocks ▸ Go
  - Delayed arrival of intermediate goods to later date

  $\Rightarrow$ Sizeable impact of delays, some leading to oscillations

- Bottlenecks ▸ Go
  - To which sectors do delay shocks contribute most to the persistence of shocks?

  $\Rightarrow$ Bottlenecks identified by (weighted) supplier $\times$ buyer centrality measures

Echoes and Endogenous Fluctuations

VAR(1) Representation

- The *VAR* (τ_{max}) system can be put into VAR(1) form

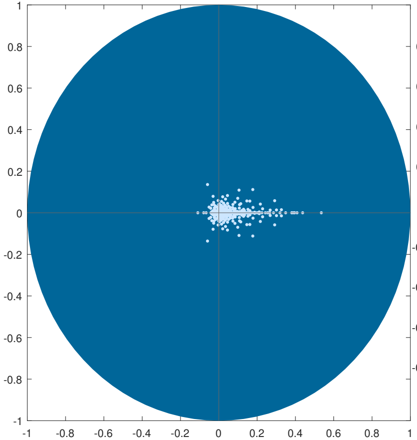
$$\underbrace{\begin{pmatrix} \hat{\mathbf{y}}_t \\ \hat{\mathbf{y}}_{t-1} \\ \\ \hat{\mathbf{y}}_{t-\tau_{max}+1} \end{pmatrix}}_{\equiv \mathbf{Y}_t} = \underbrace{\begin{pmatrix} \mathbf{\Omega}_1 & \mathbf{\Omega}_2 & \dots & \mathbf{\Omega}_{\tau_{max}} \\ I_n & & & \\ & I_n & & \\ & & \ddots & \\ & & I_n & \end{pmatrix}}_{\equiv \mathbb{O}} \underbrace{\begin{pmatrix} \hat{\mathbf{y}}_{t-1} \\ \hat{\mathbf{y}}_{t-2} \\ \\ \hat{\mathbf{y}}_{t-\tau_{max}} \end{pmatrix}}_{\equiv \mathbf{Y}_{t-1}} + \underbrace{\begin{pmatrix} \hat{\mathbf{A}}_t \\ 0 \\ \vdots \\ 0 \end{pmatrix}}_{\equiv \mathbf{e}_t}$$

- *VAR* (1) representation:

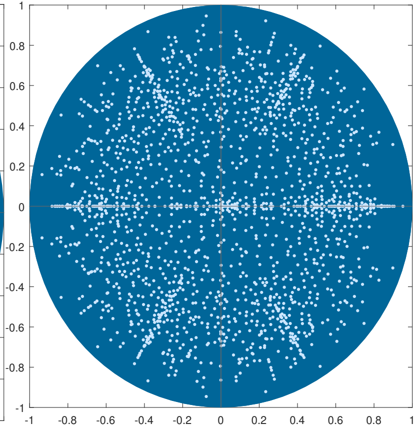
$$\mathbf{Y}_t = \mathbb{O}\mathbf{Y}_{t-1} + \mathbf{e}_t$$

- The system can oscillate if $\mathbb{O}$ has complex eigenvalues
  - Only true with time-to-build
  - In roundabout case, oscillations absent because collapsed within period

Spectrum

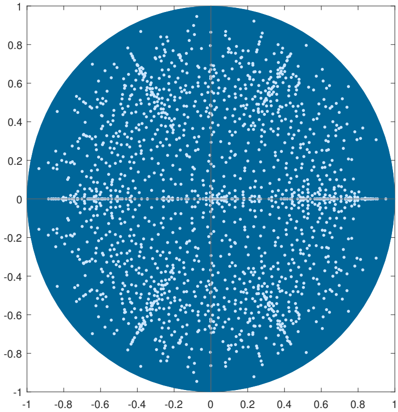


(a) LP83

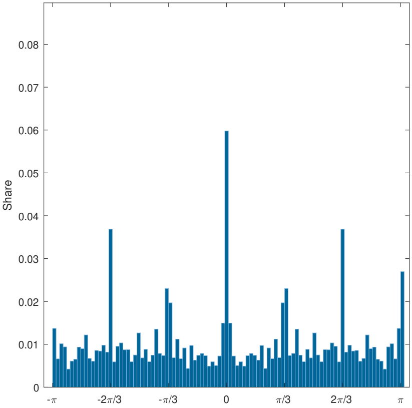


(b) Heterogeneous T2B (Compustat)

Frequencies with Heterogeneous T2B



(a) Spectrum (T2B Compustat)



(b) Angular Frequency ω

$\Rightarrow$ Rich spectrum with peaks at periods of 2, 3 and 6 months

- Period $= \frac{1}{f} = \frac{2\pi}{\omega}$

Oscillations and Network Cycles

- Oscillations are a consequence of cycles (loops) in the network
- A simple result:

Proposition

A vertical production network (i.e. acyclical) displays no oscillations.

Proof.

- There exists an ordering of sectors in which Ω is lower triangular with 0s on the diagonal
- All eigenvalues of $\mathbb{O}$ are 0 (requires a few steps)
- Note: shocks vanish after a finite number of iterations (at most $N \times \tau_{max}$)

- Eigenvalues in the general case are too complicated
  - Algebraic graph theory: at most characterize 1st and 2nd largest eigenvalues...
  - ... but we can characterize the **Fourier spectrum**!

Refresher: Discrete-Time Fourier Transform (DTFT)

- Any discrete-time 0-mean stationary process x_t can be represented by

$$x_t = \int_{-\pi}^{\pi} \delta(\omega) e^{i\omega t} d\omega$$

where $E[\delta(\omega)] = 0$, $E[\delta(\omega)\delta(\omega')] = 0$ for $\omega \neq \omega'$

- The Discrete Time Fourier Transform (DTFT) is

$$\delta(\omega) = \frac{1}{2\pi} \sum_{t=-\infty}^{\infty} x_t e^{-i\omega t}$$

- The spectral density is

$$f(\omega) \equiv E\left[\delta(\omega)\overline{\delta(\omega)}\right]$$

provides information about which frequencies ω are important for x_t.

DTFT and Autocorrelation Function

- Autocorrelation function (ACF)

$$\gamma_k = E\left[x_t x_{t-k}\right] \text{ for } k = -\infty, ..., \infty$$

- **Key property**: Fourier spectrum is the DTFT of the ACF

$$f(\omega) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} \gamma_k e^{-i\omega k}$$

$\Rightarrow$ The ACF can be characterized analytically & using network topology

ACMF of a VAR(1)

- Recall the VAR(1) representation

$$\mathbf{Y}_t = \mathbb{O}\mathbf{Y}_{t-1} + \mathbf{e}_t$$

and $\boldsymbol{\Sigma} = E\left[\mathbf{ee}'\right]$ and $\mathbf{e}$ *iid*

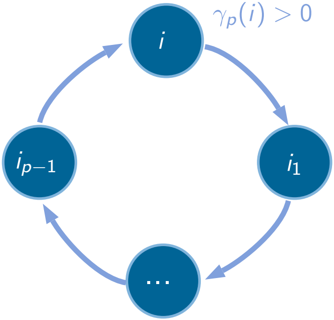
- The Autocovariance Matrix Function $\boldsymbol{\Gamma}_k = E\left[\mathbf{Y}_t\mathbf{Y}'_{t-k}\right]$ is

$$\boldsymbol{\Gamma}_0 = \sum_{k=0}^{\infty} \mathbb{O}^k \boldsymbol{\Sigma} \left(\mathbb{O}'\right)^k$$

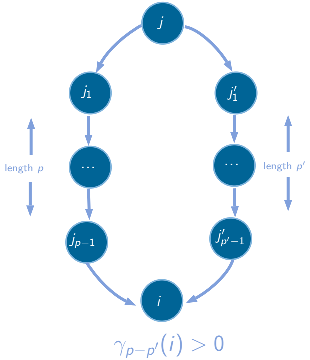
$$\boldsymbol{\Gamma}_k = \mathbb{O}^k \boldsymbol{\Gamma}_0$$

- We can extract the relevant $\gamma_k(i) = E\left[\hat{y}_{it}\hat{y}_{it-k}\right]$ and construct spectrum
  - ... but provides little understanding

Serial correlation for sector i happens for only 2 reasons:

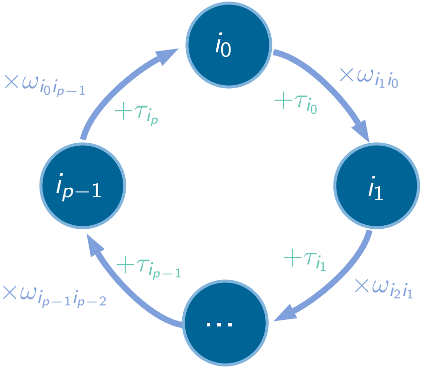


(a) Directed cycle



(b) Undirected cycle

$\Rightarrow$ shocks **echoe** in the production network through **cycles**



$p-$cycle $\varsigma = (i_0, i_1, \dots, i_{p-1}, i_p = i_0)$

- Duration of cycle:
  - $\tau(\varsigma) = \sum_{k=0}^{p-1} \tau_k$
- Weight of cycle:
  - $w(\varsigma) = \prod_{k=0}^{p-1} \omega_{i_{k+1} i_k}$

Proposition

A p−cycle $\varsigma = (i_0, i_1, \ldots, i_{p-1}, i_p = i_0)$ *contributes (at least) to the ACF*

$$\gamma_{k\tau(\varsigma)}(i_0) = w(\varsigma)^k \sigma^2(\hat{y}_{i_0 t})$$

for $k = 1, \ldots, \infty$ *and to the Fourier spectrum*

$$f_{i_0}(\omega) = \frac{\sigma^2(\hat{y}_{i_0 t})}{2\pi} \frac{1 - w(\varsigma)^2}{1 + w(\varsigma)^2 - 2w(\varsigma)\cos(\omega\tau(\varsigma))}.$$

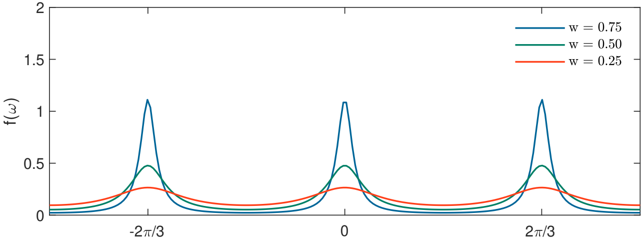
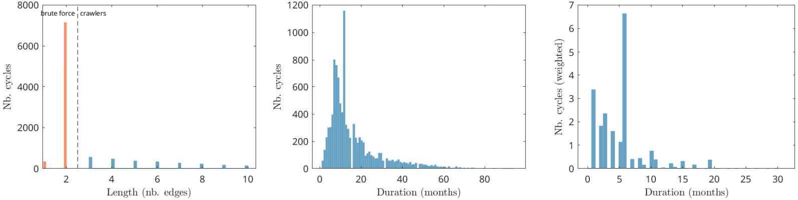


Figure 2: Spectrum of a cycle of duration $\tau = 3$ for different weights

Harmonics Poisson

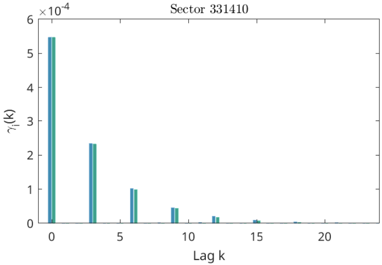
Identifying Cycles

- Finding cycles in a network is a highly combinatorial problem
  - Cannot by brute force for length > 2-3
- We use a population of crawlers that travel the network randomly
  - Record cycles, their weights and durations whenever encountered
  - Not exhaustive, but cycles of length > 3 have low weights

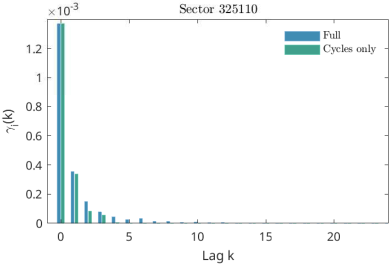


(a) By length **(b)** By duration **(c)** By duration (weighted)

ACF Full vs. Directed Cycles only



(a) Nonferrous metal (top 1-cycle)



(b) Petrochemical manufacturing (top 2-cycle)

- Directed cycles account for virtually all the ACF
  - $R^2 = 0.9995$

Sectoral Comovements and Aggregation

Aggregation

- Oscillations survive aggregation
  - Large networks cycles appear in conditional GDP response
  - Depends how sectoral shocks spread to other sectors and involve other cycles/paths

- Real GDP $y_t = \sum \overline{p}_i \alpha_i y_{it}$ has ACF

$$\begin{aligned} E\left[\hat{y}_t \hat{y}_{t-k}\right] &= E\left[\boldsymbol{\mu}' \hat{\mathbf{y}}_t \hat{\mathbf{y}}_{t-k}' \boldsymbol{\mu}\right] \\ &= \boldsymbol{\mu}' \boldsymbol{\Gamma}_k \boldsymbol{\mu} \end{aligned}$$

where $\mu_i = \overline{p}_i \alpha_i \overline{y}_i / \sum_j \overline{p}_j \alpha_j \overline{y}_j$

Spectrum of GDP

Proposition

The spectrum of real GDP is given by

$$f_y(\omega) = \underbrace{\sum_{i=1}^{N} \mu_i^2 f_i(\omega)}_{\textit{sum of sectoral spectra}} + \underbrace{\frac{1}{2\pi} \sum_{i \neq j} \sum_{k} \mu_i \mu_j [\boldsymbol{\Gamma}_k]_{ij} e^{-i\omega k}}_{\textit{sectoral comovement term}}$$

- The spectrum of GDP is the sum of two terms:
  - Sum of individual sectoral spectra implied by dominant cycles
  - Sum of spectra implied by sectoral comovements due to dominant paths

Spectrum of GDP

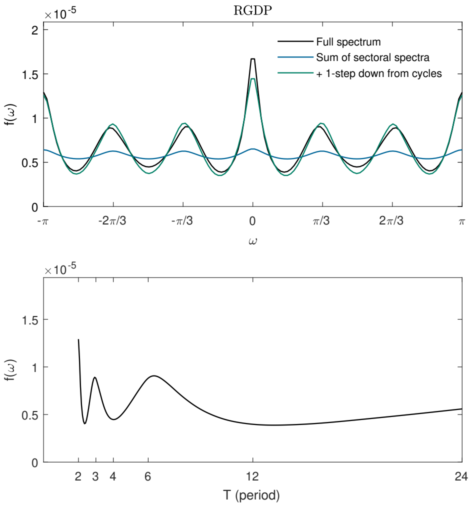


Figure 3: Spectrum of Real GDP

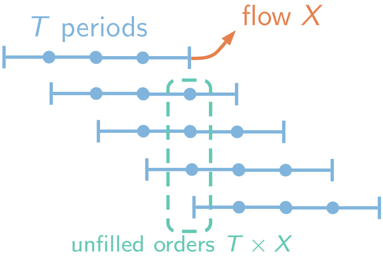
- Dominant 2-cycles
  - #298 Insurance carriers
  - #225 Petroleum refineries
  - #233 Organic chemical manuf.
- Dominant 3-cycles
  - #214 Leather and allied prod.
  - #213 Apparel manuf.
  - #43 Iron and steel mills
- Dominant 6-cycles
  - #299 Insurance, brokerage
  - #213 Hospitals
  - #14 Oil and gas

Conclusion

- Heterogeneous T2B significantly affects the propagation of shocks in network
  - Adds substantial & heterogeneous persistence across sectors
  - Can study impact of delay shocks & bottlenecks in time
- The economy fluctuates at frequencies implied by dominant cycles
  - Rich Fourier spectrum for aggregate GDP
- Complex dynamic sectoral comovements
  - Role of dominant paths to be further explored
- **Coming next:**
  - Empirical evidence
  - Robustness to inventories & other modeling assumptions

Backlog Ratio

- In steady state, backlog $= \frac{T \times X}{X} = T$



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Domar Weights and Hulten Theorem

- A horizon-adjusted version of Hulten theorem applies:

$$\frac{\partial V}{\partial \log A_i} = \beta^{\tau_i} \zeta_i$$

  - V is welfare, not real GDP
  - β^{τ_i} is time adjustment for delayed delivery

- ζ corresponds to the Domar weights: for $VA_t = \sum p_{it} c_{it}$,

$$\zeta_i = \frac{p_{it} X_{i0}(t)}{VA_t} = \frac{p_{it} y_{it-\tau_i}}{VA_t} = \frac{p_{it} \left(c_{it} + \sum_j x_{ji,t} \right)}{VA_t}$$
$$= \gamma_i + \sum_j \omega_{ji} \beta^{\tau_j} \zeta_j$$

$$\Rightarrow \boxed{\boldsymbol{\zeta} = \left(I - [\boldsymbol{\Omega} \cdot \beta^{\boldsymbol{\tau}}]' \right)^{-1} \boldsymbol{\gamma}}$$

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Domar Weights

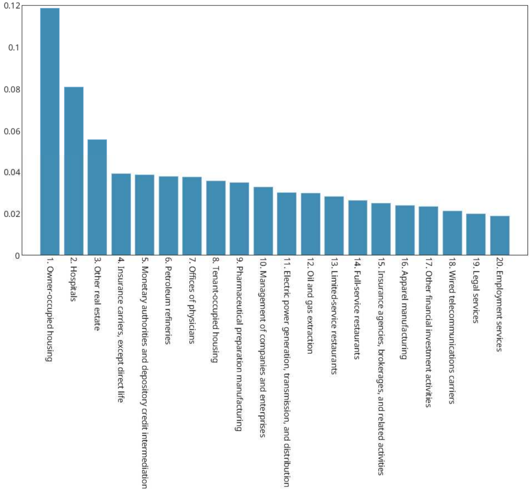

Figure 4: Top-20 sectors by Domar weight (Compustat)

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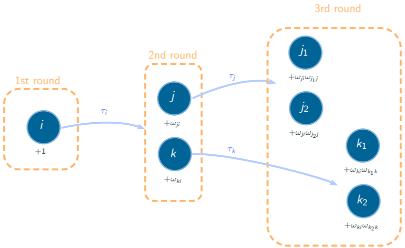
Persistence Statistics

- Consider a shock to i at time t:



- Define the average duration of a shock:

$$\mathcal{T}\left(\hat{\mathbf{A}}\right) = \frac{1}{CIR\left(\hat{\mathbf{A}}\right)} \sum_{\tau=0}^{\infty} \sum_{i} \tau w_i \hat{y}_{i\tau}\left(\hat{\mathbf{A}}\right)$$

where w_i a weighting vector and $\hat{y}_{i\tau}\left(\hat{\mathbf{A}}\right)$ the IRF to shock $\hat{\mathbf{A}}$ and

$$CIR\left(\hat{\mathbf{A}}\right) = \sum_{\tau=0}^{\infty} \sum_{i} w_i \hat{y}_{i\tau}\left(\hat{\mathbf{A}}\right)$$

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Average Duration

Proposition

The average duration $\mathcal{T}\left(\hat{\mathbf{A}}\right)$ for weighting vector $\mathbf{w}$ is equal to

$$\mathcal{T}\left(\hat{\mathbf{A}}\right) = \frac{1}{CIR\left(\hat{\mathbf{A}}\right)} \mathbf{w}' \boldsymbol{\Omega} \left[\mathbf{I} - \boldsymbol{\Omega}\right]^{-1} diag\left(\boldsymbol{\tau}\right) \left[\mathbf{I} - \boldsymbol{\Omega}\right]^{-1} \hat{\mathbf{A}}$$

where $CIR\left(\hat{\mathbf{A}}\right) = \mathbf{w}' \left[\mathbf{I} - \boldsymbol{\Omega}\right]^{-1} \hat{\mathbf{A}}$.

Intuition: Consider single shock $\boldsymbol{\delta}_i = \begin{pmatrix} 0 & \dots & 1 & \dots & 0 \end{pmatrix}'$ to sector i:

$$\mathcal{T}\left(\boldsymbol{\delta}_i\right) = \frac{1}{CIR\left(\hat{\mathbf{A}}\right)} \mathbf{w}' \underbrace{\boldsymbol{\Omega}}_{\substack{\text{duration } \tau \text{ only} \\ \text{contributes after 1 round}}} \underbrace{\left[\sum_{k=0}^{\infty} \boldsymbol{\Omega}^k\right]}_{\substack{\text{contribution of } \tau_j \text{ to} \\ \text{later rounds of production}}} \text{diag}\left(\boldsymbol{\tau}\right) \underbrace{\left[\sum_{k=0}^{\infty} \boldsymbol{\Omega}^k\right] \boldsymbol{\delta}_\mathbf{i}}_{\substack{\#\text{ of walks from sector } i \\ \text{to other sector} j \text{ of any length}}}$$

The rest follows by linearity to any shock $\hat{\mathbf{A}}$.

Intuition Back

Average Duration

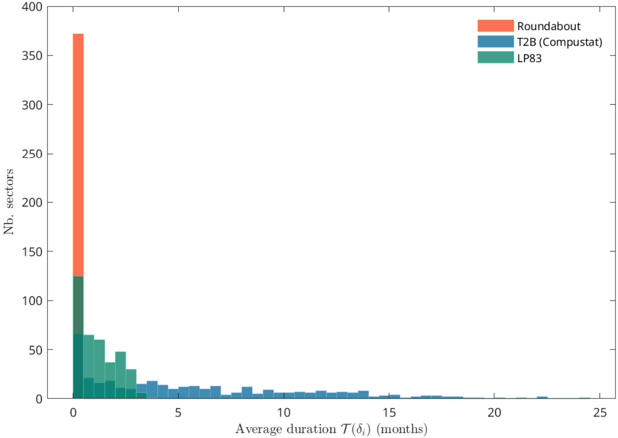

Figure 5: Comparison of average durations of *iid* sectoral shocks

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Delay Shocks

- Consider a $T-$period delay shock in sector i

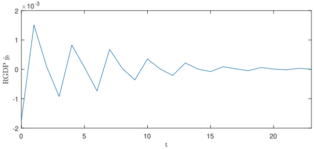
$$\hat{X}_{i\tau} = -\varepsilon \text{ for } \tau = 0, ..., T-1$$
$$\hat{X}_{i\tau} = +\varepsilon \text{ for } \tau = T, ..., 2T-1$$

- Plot the response of aggregate real GDP $y_t = \sum \overline{p}_i \alpha_i y_{it}$
  - -1% of deliveries for 1 and 3 months

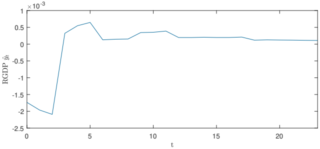
Back

Delay Shocks IRFs

Figure 6: Nonferrous metal smelting and refining (bottleneck #2, $\tau = 3$ months)

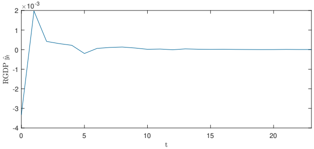


(a) 1 month

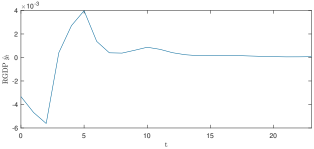


(b) 3 months

Figure 7: Plastics material and resin manuf. (bottleneck #3, $\tau = 5$ months)



(a) 1 month



(b) 3 months

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Bottlenecks

Which sector's T2B contributes the most to the persistence of shocks?

$$\frac{\partial \mathcal{T}\left(\hat{\mathbf{A}}\right)}{\partial \tau_i} = \frac{1}{CIR\left(\hat{\mathbf{A}}\right)} \mathbf{w}' \mathbf{\Omega} \left[\mathbf{I} - \mathbf{\Omega}\right]^{-1} \frac{\partial \mathrm{diag}\left(\boldsymbol{\tau}\right)}{\partial \tau_i} \left[\mathbf{I} - \mathbf{\Omega}\right]^{-1} \hat{\mathbf{A}}$$

Bottlenecks

Which sector's T2B contributes the most to the persistence of shocks?

$$\frac{\partial \mathcal{T}\left(\hat{\mathbf{A}}\right)}{\partial \tau_i} = \frac{1}{CIR\left(\hat{\mathbf{A}}\right)} \mathbf{w}'\boldsymbol{\Omega}\left[\mathbf{I}-\boldsymbol{\Omega}\right]^{-1} \frac{\partial \text{diag}\left(\boldsymbol{\tau}\right)}{\partial \tau_i} \left[\mathbf{I}-\boldsymbol{\Omega}\right]^{-1} \hat{\mathbf{A}}$$

Proposition

The marginal impact of a delay $\partial \tau_n$ on the persistence of shock $\hat{\mathbf{A}}$ is given by

$$\frac{\partial \mathcal{T}\left(\hat{\mathbf{A}}\right)}{\partial \tau_i} = \frac{1}{CIR\left(\hat{\mathbf{A}}\right)} s_i \times b_i$$

where

$$s_i = \mathbf{w}'\boldsymbol{\Omega}\left[\mathbf{I}-\boldsymbol{\Omega}\right]^{-1} \boldsymbol{\delta}_i = \underbrace{\sum_j \mathbf{w}'\boldsymbol{\Omega}\left[\sum_{k=0}^{\infty} \boldsymbol{\Omega}^k\right]_{ji}}_{\substack{\#\text{ of walks from } i \text{ to all sectors} \\ \text{of any length (weighted by } \Omega' w)}} \qquad \text{(supplier centrality)}$$

$$b_i = \hat{\mathbf{A}}'\left(\mathbf{I}-\boldsymbol{\Omega}'\right)^{-1} \boldsymbol{\delta}_i = \underbrace{\hat{\mathbf{A}}' \sum_j \left[\sum_{k=0}^{\infty} (\boldsymbol{\Omega}')^k\right]_{ij}}_{\substack{\#\text{ of walks from all sectors } j \\ \text{hit by } \hat{\mathbf{A}} \text{ to } i \text{ of any length}}} \qquad \text{(buyer centrality)}$$

Bottlenecks

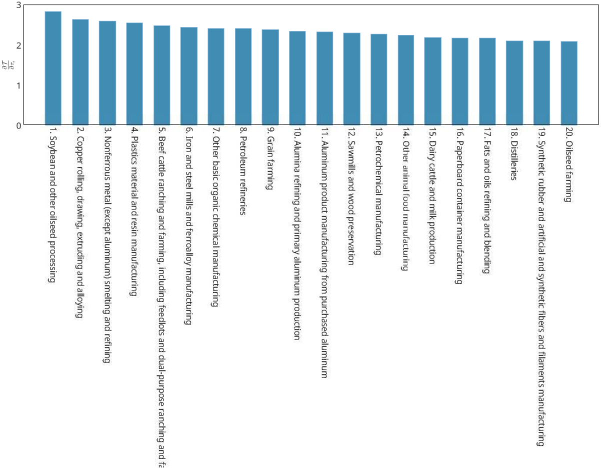

Figure 8: Top-20 bottleneck sectors

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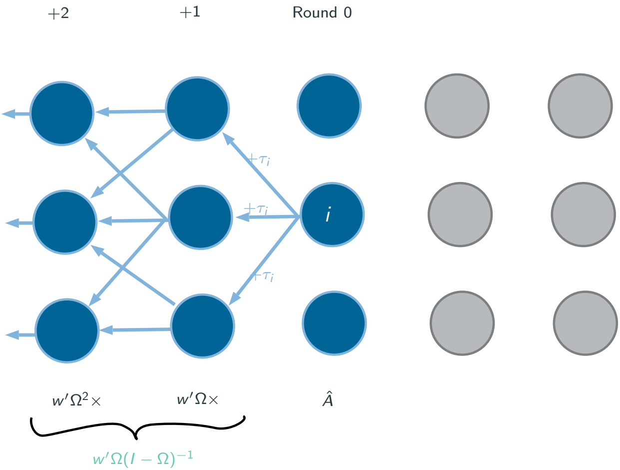
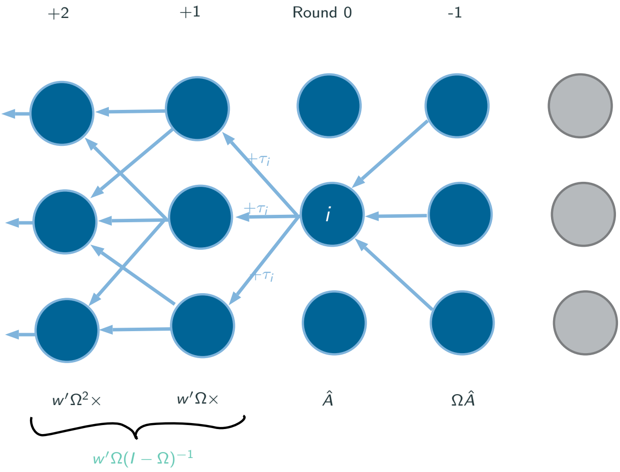
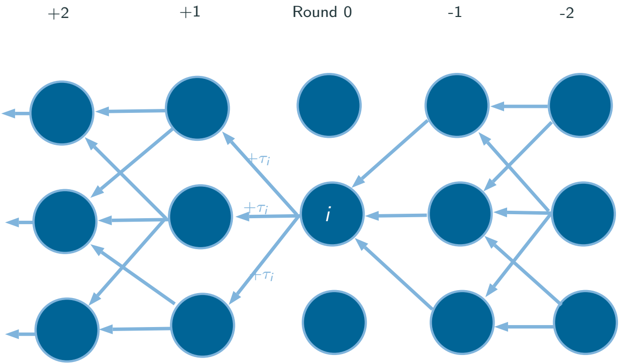
Average Duration



Average Duration



Average Duration



$$\underbrace{w'\Omega^2\times \quad w'\Omega\times}_{w'\Omega(I-\Omega)^{-1}} \quad \times \quad diag(\tau) \quad \times \quad \underbrace{\hat{A} \quad \Omega\hat{A} \quad \Omega^2\hat{A}}_{(I-\Omega)^{-1}\hat{A}}$$

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Fundamentals and Harmonics

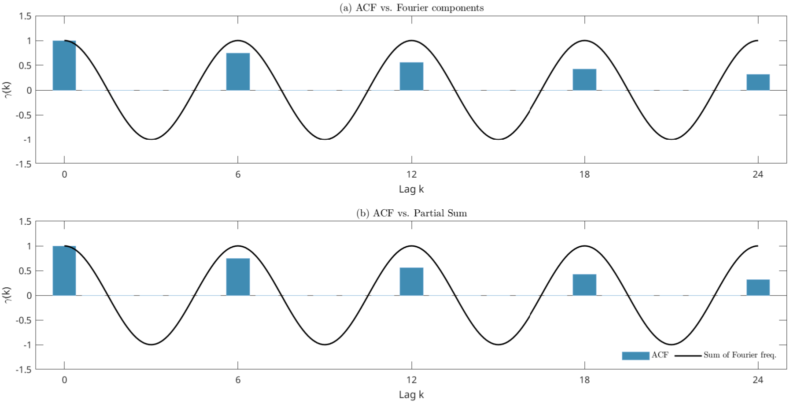

Figure: Fourier decomposition of ACF for $\tau = 6$ and $w = 0.75$

Fundamentals and Harmonics

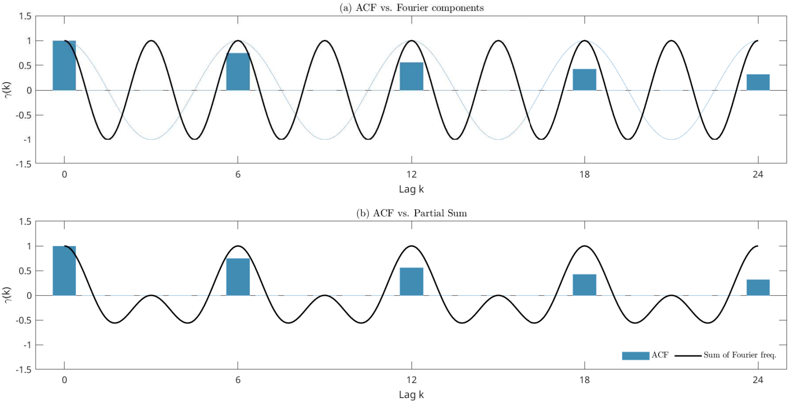

Figure: Fourier decomposition of ACF for $\tau = 6$ and $w = 0.75$

Fundamentals and Harmonics

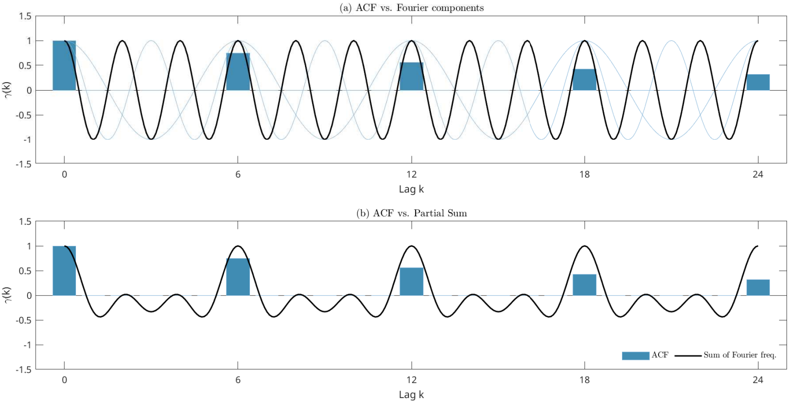

Figure: Fourier decomposition of ACF for $\tau = 6$ and $w = 0.75$

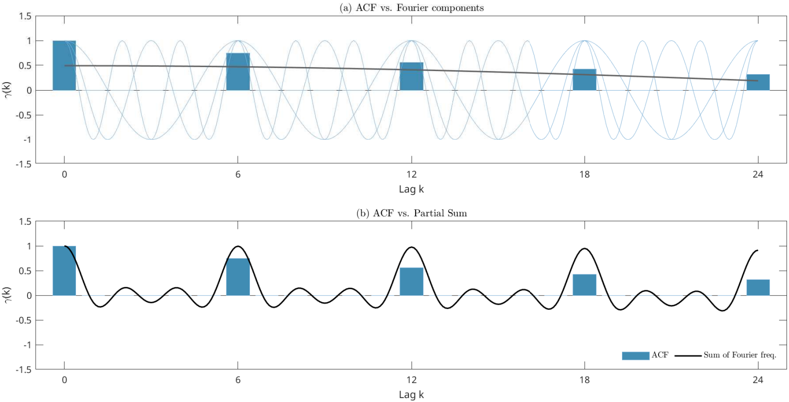

Figure: Fourier decomposition of ACF for $\tau = 6$ and $w = 0.75$

Fundamentals and Harmonics

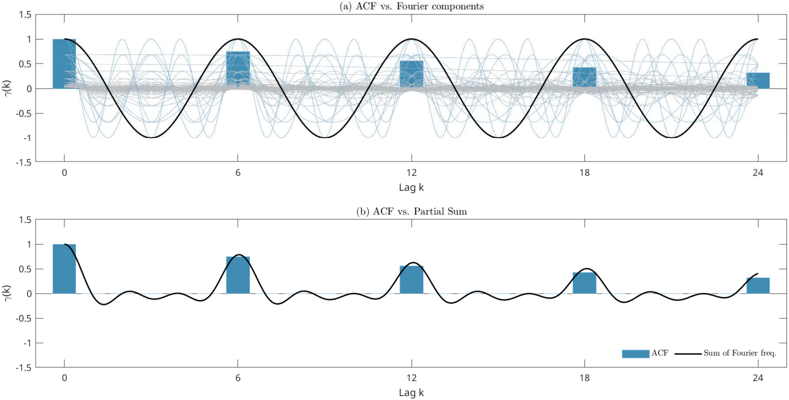

Figure: Fourier decomposition of ACF for $\tau = 6$ and $w = 0.75$

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Poisson Model

- A common trick to model delays is to assume Poisson arrival:
  - For delivery lag τ, assume delivery with probability $1/\tau$ each period
- Example: suppose i_0 has a self-loop of weight w

$$\gamma_k(i_0) = w\left(1 - \frac{1}{\tau}\right)^{k-1} \frac{1}{\tau}\,\sigma^2\left(\hat{y}_{i_0 t}\right) + \text{further iterations}$$

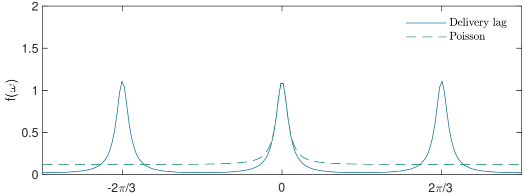

Figure 9: Spectrum of a Poisson model vs. delivery lag for $\tau = 3$

$\Rightarrow$ Poisson arrival heavily **distorts the spectrum**

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