

# Herding Through Booms and Busts\*

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## Abstract

This paper explores whether rational herding can generate endogenous aggregate fluctuations. We embed a tractable model of rational herding into a business cycle framework. In the model, technological innovations arrive with unknown qualities, and agents have dispersed information about how productive the technology really is. Rational investors decide whether to invest based on their private information and the investment behavior of others. Herd-driven boom-bust cycles arise endogenously in this environment when the technology is unproductive but investors' initial information is overly optimistic. Their overoptimism leads to high investment rates, which investors mistakenly attribute to good fundamentals, leading to a self-reinforcing pattern of higher optimism and higher investment until the economy reaches a peak, followed by a crash when agents ultimately realize their mistake. We calibrate the model to the U.S. economy and show that it can broadly explain boom-and-bust cycles like the dot-com bubble of the 1990s.

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# 1 Introduction

Business cycle history is replete with examples in which new technologies led to periods of massive investment that ended in severe economic downturns. One salient example is the 1990s boom in information technologies that culminated in the stock market crash of 2001 (“dot-com bubble”). While the internet had been invented years earlier to connect academic and military networks, its commercial potential only became clear in the 1990s, when extreme enthusiasm for the new technology led to large investments in communication networks, software, and IT equipment. The high volume of investment and rising valuations of IT companies initially seemed to validate an optimistic outlook, but a crash eventually followed as some of the expected returns failed to materialize.<sup>1</sup> While the deep drivers that caused this sequence of events are still debated, a common view is that shifts in expectations played a key role in shaping the dot-com boom-bust cycle.

The idea that expectations contribute to aggregate economic fluctuations has a long tradition in macroeconomics. In seminal work, [Pigou \(1927\)](#) emphasized the importance of beliefs in shaping the business cycle. In his view, booms can be caused by waves of optimism among business executives, and crashes arise when their lofty expectations turn out to be mistaken. This hypothesis has been extensively studied in modern business cycle theory by the news-driven business cycle literature, pioneered by [Beaudry and Portier \(2004\)](#).<sup>2</sup> According to this view, agents receive news about future productivity, which sometimes turn out to be false. Boom-bust cycles arise after an initial sequence of positive news is later contradicted by experience.

These theories, however, remain mostly silent on the technological, social and psychological factors that drive the evolution of beliefs. In most of these studies, the belief process obeys an exogenous law of motion, and boom-bust cycles occur after a specific sequence of shocks—first positive, then negative. In other words, a large part of these cycles remains attributed to unexplained factors, precluding a deeper understanding of the key determinants of business cycles. What explains that beliefs follow this particular—and perhaps systematic—pattern which evolves from a phase of rising optimism to all-out pessimism? Is the growing optimism during the boom the consequence of luck, or the result of particular interactions between investors that lead to instability and inefficiencies? What causes precipitate the economy into a bust? Answering these questions is essential for our understanding of business cycles and for the design of stabilization policies.

This paper proposes one potential unifying explanation by exploring how herding in investors’ behavior can generate a full macroeconomic boom-bust cycle without relying on an exogenous

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<sup>1</sup>Other boom-bust episodes follow similar patterns. For instance, the Roaring Twenties, a period of massive economic growth fueled by technological innovations in many sectors such as car manufacturing, communication, aviation and the chemical industry, ended in the Great Depression. [Xiong \(2013\)](#) documents several instances of boom-bust episodes that follow the introduction of new technologies. [Arif and Lee \(2014\)](#) document that aggregate investment tends to peak during periods of optimism, and that these periods are followed by lower equity returns.

<sup>2</sup>See [Beaudry and Portier \(2014\)](#) for an overview of the empirical and theoretical research supporting the news view of the business cycle.

sequence of shocks. In our theory, entrepreneurs infer the quality of their investment opportunities by observing the decisions of others and can be tempted to invest when they see their competitors expand their operations. The introduction of a new technology of uncertain quality can trigger a slow-rising boom followed by a sudden crash, in line with the experience of the dot-com era. In the boom phase, the initial optimism of investors translates into high levels of aggregate investment, and high investment, in turn, leads to further increases in optimism. This self-reinforcing process can fuel a long-lasting expansion of the economy, which comes to an end when new observations no longer support an optimistic view of the technology. Agents stop investing and the economy rapidly collapses. *Herd*ing thus offers a potential explanation for the emergence of technology-driven boom-bust cycles.

### **A herd-driven theory of business cycles**

Our theory captures these ideas as follows. In the model, random technological innovations arrive over time and rational entrepreneurs decide whether to adopt the technology or not. The payoff from adoption is initially unknown, and agents use all available information to update their beliefs about the fundamental value of the technology. Information comes from both public and private sources. Importantly, to capture the idea that agents collect information from similar sources (news media, market reports, etc.), we assume that private signals feature some common noise. This assumption is key as it allows the distribution of beliefs across entrepreneurs to vary for reasons unrelated to the fundamental value of the technology. Entrepreneurs do not initially know the extent of that bias but progressively learn about it.

Agents also receive public signals. First, they learn by observing an exogenous public signal which stands in for the general information provided by public sources. They also learn from endogenous market outcomes such as aggregate quantities or prices. In the model, this amounts to observing, with some noise, the mass of agents who adopt the new technology. As the individual adoption decisions reflect the private information of the agents, this public signal operates as a *social learning* channel by aggregating, in a non-linear fashion, some of the information dispersed across agents.

How agents interpret this public signal is key for the emergence of boom-bust cycles. Such cycles are caused in our model by what we refer to as “false-positives”: bad realizations of the technology fundamental that are accompanied by unusually large and positive realizations of the common noise. False-positives may thus capture situations in which, for instance, excessively promising benchmark tests are widely advertised upon the introduction of the technology and lead to overly optimistic beliefs.

When observing the large rate of adoption induced by such false-positive shocks, agents infer that private signals are positive. These signals, in turn, can be positive either because the fun-

damental value of the technology is good, or because the common noise component of the private signals is high. Entrepreneurs cannot tell these stories apart, but if false-positive shocks are relatively rare, the high adoption rate is initially attributed to a high-value technology, whose posterior likelihood rises. More optimistic beliefs lead to further aggregate adoption next period, which, in turn, leads to even more positive beliefs about the fundamental and so on. It is in that sense that our model displays a form of *herding*: agents mimic the behavior of others and sometimes mistakenly follow the herd into an adoption boom, meanwhile a shrinking measure of agents use their private information to go against the crowd. Through this positive feedback loop, the arrival of a low-value technology can create a long-lasting boom as entrepreneurs are fooled by the initial adoption craze.

But agents are rational and understand the possibility that they can sometimes be mistaken in their assessment of the true state of the world. As a result, they keep track of the probability of being in a false-positive state, which appears increasingly likely over time, as adoption keeps falling short of the most optimistic predictions. At some point, the most pessimistic agents stop adopting the technology and aggregate adoption no longer supports a high-productivity scenario. This leads to a reversal in beliefs and a collapse in new technology adoption. We provide formal conditions under which these boom-and-bust episodes are guaranteed to arise in equilibrium.

A distinguishing feature of our approach is that the boom-and-bust cycle emerges *endogenously*. Standard practice in modern business cycle analysis often treats the booms and the busts as separate episodes, both driven by their own sequence of exogenous shocks. In contrast, our model generates an endogenous boom-and-bust cycle out of the single impulse shock that is the arrival of the new technology.<sup>3</sup> The crash, in particular, is not triggered by an exogenous shock but arises endogenously through the natural evolution of beliefs. As a consequence, the properties of the bust can be affected by what happened during the preceding boom, and government policies can have a large impact: policy interventions may affect the duration and magnitude of the boom as well as the timing and depth of the bust. They may also determine whether or not a cycle is to take place at all. This feature is absent from most standard models of the business cycle.

## Information cascades and government interventions

In the model, the mass of investing agents is a nonlinear aggregator of the dispersed information. As a result, the amount of information that agents receive is endogenous and varies with the cycle, which opens the door to a form of *information cascades*. When the public signals received up to a certain date are very positive, most agents invest regardless of their private signals so that

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<sup>3</sup>By “endogenous”, we mean that the entire boom-and-bust pattern is produced by the forces in the model. Our theory still relies on shocks, however, but only one-time shocks and does not rely on a particular sequence of positive then negative shocks. This approach is different from other theories of endogenous business cycles that generate deterministic periodic or chaotic dynamics (see [Boldrin and Woodford \(1990\)](#); [Benhabib \(1992\)](#); [Guesnerie and Woodford \(1992\)](#) for surveys).

their private information is not encoded into the mass of adopters. As a result, the model is able to generate sustained booms, when massive adoption restricts the flow of information, and rapid busts when slight downturns encourage enough entrepreneurs to use their private information, which suddenly reveals more information on the true state of the world.

Due to this variable flow of public information, the model features an information externality: agents do not internalize how their private adoption decisions affect the flow of public information. We characterize the solution of a social planning problem and show that the planner tends to lean against the wind by pushing for fewer agents to adopt the technology during booms and more during downturns so as to optimize the amount of information provided by aggregate adoption data.

## Quantitative exploration

To explore how the evolution of beliefs generated by our learning model can produce a general macroeconomic expansion followed by a recession, and to have a sense of the magnitude of the boom-bust cycles generated by the theory, we embed our main mechanism into a quantitative business cycle framework, which models the technology adoption decision of entrepreneurs after the arrival of a new technology. The model features two types of capital, “traditional” and “information technology” capital (IT), and we assume that the new technology is more intensive in IT capital. As in the basic model, social learning takes place as agents observe the measure of new-technology adopters.

We calibrate the model to match various moments of the data that relate to the dot-com period. In particular, we discipline the amount of private information—a key moment for our mechanism—using dispersion in forecasts from the Survey of Professional Forecasters (SPF). We also use data from the SPF to discipline investors’ beliefs about the true value of the technology. Under our calibration, the model is able to generate a boom-bust cycle with positive comovements in consumption, investment, hours worked and output. The overinvestment into IT capital during the boom period and the negative wealth effect associated to its sudden loss of value cause the economy to contract significantly when beliefs collapse as agents realize that resources were misallocated. Overall, our results suggest that rational herding among economic agents can generate realistic fluctuations in macroeconomic aggregates.

### 1.1 Literature Review

This paper builds on a long historical tradition in macroeconomics. The view that business cycles are shaped by expectations dates back at least to [Pigou \(1927\)](#), who also suggested a role of herding among investors.<sup>4</sup> While ignoring the financial aspects of booms and busts, our paper also echoes

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<sup>4</sup>In *Industrial Fluctuations* (1927), Pigou states that “the varying expectations of business men [...] and not anything else, constitute the immediate and direct causes or antecedents of industrial fluctuation”. He emphasized

parts of the narrative that describes the behavioral and psychological causes of cycles in [Minsky \(1977\)](#) and [Kindleberger \(1978\)](#) after an initial “displacement” (e.g., the introduction of a new technology).

Our paper is closely related to the literature on news or noise-driven business cycles ([Beaudry and Portier, 2004](#); [Lorenzoni, 2009](#); [Jaimovich and Rebelo, 2009](#)). Indeed, our model shares the view that boom-bust cycles may be due to false-positives. In the news-shock literature, beliefs are driven by the exogenous release of news at fixed dates. In contrast, in our approach, the rise and fall in beliefs are *endogenously* driven by model forces, allowing us to explore the model’s unique predictions on the frequency and timing of such cycles, and providing a greater role for stabilization policies. In addition, the news literature does not consider the role of herding in driving fluctuations, which is essential for our results: in our model, the gradually rising boom is the sole product of a positive feedback loop between investors under social learning, and the timing of the bust is determined by the time when the positive feedback disappears.

[Christiano et al. \(2008\)](#) consider the interaction of monetary policy and boom-bust cycles driven by news shocks. Closer to our work, [Benhima \(2019\)](#) builds a new-Keynesian model with dispersed private information in which news shock released at fixed exogenous dates can create a boom-bust episodes. In a recent paper, [Angeletos et al. \(2022\)](#) show that common noise shocks can generate excessive fluctuations in a model of start-up financing through complementary interactions between the entrepreneurial sector and financial markets. As in our setup, lean-against-the-wind policies can be beneficial. [Burnside et al. \(2016\)](#) propose an epidemiology-based model in which the transmission of optimistic beliefs in a population about the housing market can create a boom and bust. In a similar epidemiology-based variant of their framework, [Angeletos and La’O \(2013\)](#) propose a model in which a sentiment shock produces a hump-shaped cycle as it propagates through the population. [Gorton and Ordoñez \(2019\)](#) build a model in which the evolution of beliefs about the quality of collateral can lead to endogenous cycles. [Goldstein et al. \(2013\)](#) propose a model in which learning from prices creates strategic complementarities between financial markets and capital providers that give rise to trading frenzies.

Our paper also relates to the original work on herding and information cascades by [Banerjee \(1992\)](#), [Bikhchandani et al. \(1992\)](#) and [Chamley \(2004\)](#). It further relates to [Avery and Zemsky \(1998\)](#), who study herding in financial markets and introduce multidimensional uncertainty to allow for information cascades. Our model differs from these traditional models of herding in several dimensions. First, in previous herding models, agents make decisions sequentially and the dynamics of the model are governed by the gradual observation of these individual decisions. Both features do not sit well with standard macroeconomic models. In our setup, instead, agents act simultane-

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the importance of the herding process: “the pioneers, who thus undertake and expand enterprises, at once fill a social need and lay treasure for themselves. Gradually, as no disaster happens to them, other less bold spirits follow their example; then others and yet other.”

ously and learn by observing aggregates, which allows for a smoother integration of herding into macroeconomic frameworks. Second, the source of agents' confusion is different. In traditional herding models, people are confused between the fundamental return and the idiosyncratic shocks that stem from a particular ordering of the investors. As a consequence, boom-bust cycles arise only for specific sequences of idiosyncratic shocks. In our model instead, agents are confused between the fundamental and the common noise, which are drawn once and for all. Boom-bust patterns emerge endogenously through the natural evolution of beliefs and without any timing assumptions about shocks. This distinction with the existing literature is crucial to generate endogenous cycles.

To our knowledge, [Loisel et al. \(2012\)](#) is the only other macroeconomic model with herding. Their paper presents a simple general equilibrium model with overlapping generations of finitely-lived entrepreneurs who are endowed with private signals and must invest in a risky asset. As in traditional models of herding, entrepreneurs act sequentially and individual investment decisions are publicly observable. Our paper extends this approach by offering a novel herding model, based on contemporaneous decisions and the observation of aggregate actions, which, we believe, can be more easily integrated into traditional macroeconomic models.

Our learning model shares similarities with [Vives \(1997\)](#) who studies an environment in which agents with dispersed information learn by observing the average action across agents. Chapter 4 of [Chamley \(2004\)](#) briefly reviews a model in which privately informed agents learn from the average action. As in our model, the amount of information released by the public signal varies over the state space. As in [Caplin and Leahy \(1994\)](#) and [Veldkamp \(2005\)](#), the endogenous release of information in our model can generate sudden collapses in economic activity. Our work also contributes to a literature in which the aggregation of private information leads to nonlinear aggregate dynamics ([Fajgelbaum et al., 2017](#)). [Straub and Ulbricht \(2019\)](#) explore in a general setup the informativeness of nonlinear public signals and [Straub and Ulbricht \(2017\)](#) studies a particular application with financial constraints. None of these works consider the emergence of endogenous boom-bust cycles.

In contrast to our approach which maintains the assumption of rational expectations, a literature studies the emergence of boom-and-bust cycles in asset prices or in aggregate economic activity after departing from rationality or rational expectations. This includes the adaptive learning literature ([Carceles-Poveda and Giannitsarou, 2008](#); [Eusepi and Preston, 2011](#); [Adam et al., 2017](#)), the heterogeneous-belief literature with disagreement ([Harrison and Kreps, 1978](#); [Scheinkman and Xiong, 2003](#); [Simsek, 2013](#)) and, more recently, a literature that uses diagnostic expectations ([Bordalo et al., 2021](#)).

Our work also relates to a strand of literature that studies the role of bubbles in macroeconomic

environments,<sup>5</sup> the literature on endogenous deterministic cycles,<sup>6</sup> a literature that views endogenous cycles from the point of view of equilibrium indeterminacy and sunspots,<sup>7</sup> and a literature on learning from markets (Chen et al., 2007; Bond et al., 2012).

Section 2 introduces a simple learning model that conveys the intuition for the mechanism. The following section describes the forces at work in the model and discusses its welfare implications. Section 4 presents our business cycle model. We calibrate the model in Section 5 and show several empirical implications of the mechanism. The final section concludes.

## 2 Learning Model

We first present our mechanism in a simplified dynamic game of technology adoption. This allows us to provide intuition for why social learning can lead to an endogenous herd-driven boom-bust cycle out of a single impulse shock. We also use this simplified model to derive analytical results and discuss the policy implications.

### 2.1 Notation

In what follows, whenever  $F^x(\tilde{x}) = Pr(x \leq \tilde{x})$  denotes the cumulative distribution function (CDF) of some random variable  $x$ ,  $f^x$  refers to its associated probability density function and  $\overline{F}^x$  to its complementary CDF,  $\overline{F}^x(\tilde{x}) = Pr(x > \tilde{x})$ .

### 2.2 Environment

Time is discrete and goes on forever,  $t = 0, 1, \dots$ . The economy is populated by a unit measure of entrepreneurs indexed by  $j \in [0, 1]$ . Entrepreneurs are risk-neutral and discount future consumption at rate  $0 < \beta < 1$ . Each entrepreneur faces a technology adoption choice and must decide whether to use an *old* technology or adopt a *new* one. The old technology is known and provides a constant deterministic return  $A^o$ . The new technology, on the other hand, is characterized by an unknown stochastic return  $A_t^n$ . We assume that the new technology is not immediately productive. It initially delivers the same return as the old technology until it matures, which happens with some fixed

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<sup>5</sup>These include studies based on rational bubbles (Galí, 2014; Martin and Ventura, 2016; Asriyan et al., 2019; Guerrón-Quintana et al., 2020) as well as bubbles due to financial constraints and others (Kocherlakota, 1992; Miao and Wang, 2012; Barlevy, 2014; Hirano and Yanagawa, 2016).

<sup>6</sup>See, for instance, Grandmont (1985), Boldrin and Woodford (1990), Benhabib (1992), Benhabib et al. (2002) and Matsuyama (1999; 2013). More recent contributions include Beaudry et al. (2020), who provide empirical evidence in favor of endogenous cycles. In contrast to our setup, models in this literature feature deterministic limit cycles that can be periodic or chaotic.

<sup>7</sup>This literature includes Benhabib and Farmer (1994) and Wen (1998). More recent contributions include Benhabib et al. (2015), Kaplan and Menzio (2016), Eeckhout and Lindenlaub (2019) and Golosov and Menzio (2020). These studies typically feature multiple equilibria and aggregate fluctuations are due to shifts in expectation triggered by sunspot shocks. Our model, instead, features a unique equilibrium and boom-bust cycles result from agents' gradual learning about the technology and the common noise.



per-period probability  $\lambda > 0$ . After maturation, the true nature of the new technology is revealed and characterized by a constant return  $\theta \in \{\theta_H, \theta_L\}$ ,  $\theta_H > A^o > \theta_L$ . We refer to  $\theta$  as the technology *fundamental*. Maturation is a one-time event and all uncertainty is resolved afterwards. In other words,

$$A_t^n = \begin{cases} A^o & \text{before maturation} \\ \theta & \text{after maturation.} \end{cases}$$

Adopting the new technology is costless and, every period, agents must decide whether to use the new technology ( $i_{jt} = 1$ ) or not ( $i_{jt} = 0$ ). We assume that a fraction  $\mu$  of agents are “noise entrepreneurs”, that is, they are clueless regarding technological adoption and behave randomly. Specifically, we assume that a fraction  $\varepsilon_t$  of noise entrepreneurs adopt the new technology, where  $\varepsilon_t$  is *iid* distributed according to a CDF  $F^\varepsilon$ . The remaining  $1 - \mu$  entrepreneurs are rational and choose the best of the two technologies, based on public and private information. There is no cost of switching between technologies, so entrepreneur  $j$  solves

$$i_{jt} = \operatorname{argmax}_{i_{jt} \in \{0,1\}} i_{jt} E[A_t^n | \mathcal{I}_{jt}] + (1 - i_{jt}) A^o,$$

where  $\mathcal{I}_{jt}$  is its information set at time  $t$ .

## 2.3 Information

The technology fundamental  $\theta$  is randomly drawn once and for all at date 0. We denote by  $p_0$  the ex-ante probability that  $\theta = \theta_H$ . Entrepreneurs do not observe  $\theta$  directly but receive various private and public signals about its true value.

### Private signals

First, we assume that each agent receives a private signal  $s_j$  at date 0, upon the arrival of the new technology. Importantly, we allow these private signals to feature not only idiosyncratic noise but also common noise. This common noise might come, for instance, from sources of information shared by agents (mass media, internet) that may report noisy signals about the initial success of the technology (e.g., benchmark tests). Common noise is key to our mechanism as it introduces the possibility that the average belief about  $\theta$  varies for reasons that are orthogonal to the true value of the fundamental.

Common noise is captured by the random variable  $\xi$ , distributed according to the CDF  $F^\xi$ . Formally, we assume that the private signal  $s_j$  of agent  $j$  is drawn from the CDF  $F_{\theta+\xi}^s(s) = \Pr(s_j \leq s)$ , where  $\{F_x^s\}_{x \in I}$  is a family of distributions that admit the probability density functions  $\{f_x^s\}_{x \in I}$ . To prevent the possibility of trivial learning, we make the assumption that  $F_x^s$  has full support over  $\mathbb{R}$ , i.e.,  $f_{\theta+\xi}^s > 0$  everywhere. Finally, in order to guarantee monotonicity in learning,

we assume that the family  $\{F_x^s\}_{x \in I}$  satisfies the *monotone likelihood ratio property* (MLRP). That is, for  $x_1 < x_2 \in I$  and  $s_1 < s_2$ , we have

$$\frac{f_{x_2}^s(s_2)}{f_{x_1}^s(s_2)} \geq \frac{f_{x_2}^s(s_1)}{f_{x_1}^s(s_1)}. \quad (\text{MLRP})$$

Intuitively, the MLRP condition guarantees that a high signal  $s$  is more likely to be coming from a high realization of  $x = \theta + \xi$ . In other words, an entrepreneur observing a high private signal  $s_j$  becomes more optimistic and puts a higher probability on the value of the technology  $\theta$  and the common noise  $\xi$  being high.

**Example.** In most of our examples, we will use *additive private signals* so that

$$s_j = \theta + \xi + v_j, \text{ with } v_j \sim \text{iid CDF } F^v. \quad (1)$$

## Public signals

Before the new technology matures, entrepreneurs collect public information in addition to their initial private signal. We first assume that each period all agents observe a public signal  $S_t = \theta + u_t$ , centered around  $\theta$  with *iid* noise distributed according to CDF  $F^u$  and standard deviation  $\sigma_u$ , which stands in for all the information collected over time from exogenous public sources. After maturation, we assume that the fundamental  $\theta$  is observed and that all uncertainty is resolved.

Second, and more importantly for our mechanism, we introduce a form of *social learning* by allowing entrepreneurs to observe an endogenous signal which partially aggregates the private information of agents. This is to capture the type of information that agents learn by observing aggregate quantities or prices, which result from the aggregation of individual decisions.<sup>8</sup> Specifically, we assume that entrepreneurs observe the total measure of adopters of the new technology  $m_t$ , which, in the presence of noise entrepreneurs, can be written as<sup>9</sup>

$$m_t = \underbrace{\int_0^{1-\mu} i_{jt} dj}_{\text{rational entrepreneurs}} + \underbrace{\mu \varepsilon_t}_{\text{noise entrepreneurs}} \quad (2)$$

The presence of the noise  $\varepsilon_t$  is required in our setting to prevent agents from learning too quickly (or even immediately in some cases, as we discuss later).

In equilibrium, the decision to adopt  $i_{jt}$  is a nonlinear function of the agent's individual beliefs. In turn, these beliefs are a function of public information up to time  $t$ ,  $\{S_{t-1}, m_{t-1}, \dots, S_0, m_0\}$ , and of the private signal  $s_j$ . As a result, since public information is shared and can be filtered

<sup>8</sup>In our full business cycle model from Section 4, observing aggregate quantities or prices will provide a public signal of the same form.

<sup>9</sup>Alternatively, the noise  $\varepsilon_t$  in (2) can be interpreted as pure observational noise or measurement error in this simplified learning model.

out,  $m_t$  partially aggregates the private information across the population of entrepreneurs and therefore contains useful information regarding the fundamental  $\theta$  and the common noise  $\xi$ .

As we explain in more details below, it is the presence of this endogenous signal that will allow *herding* to occur in our environment. As  $m_t$  aggregates the dispersed information of private agents, a particularly high draw of  $m_t$  will be interpreted as being indicative of a strong fundamental  $\theta$  and a high common noise  $\xi$ . As a results, agents will update their beliefs in favor of these states, which might encourage technology adoption, increasing  $m_t$  further, and so on.

## 2.4 Belief Characterization

There are two aggregate shocks in this economy: the fundamental  $\theta$  and the common noise  $\xi$ . The beliefs of an individual entrepreneur  $j$  are described by a joint probability distribution that we denote by

$$\Lambda_{jt}(\tilde{\theta}, \tilde{\xi}) = Pr\left(\theta = \tilde{\theta}, \xi \in [\tilde{\xi}, \tilde{\xi} + d\tilde{\xi}] \mid \mathcal{I}_{jt}\right),$$

in which we explicitly allow for  $\xi$  to take a continuum of values and where  $\mathcal{I}_{jt}$  is agent  $j$ 's information set at date  $t$ . Since entrepreneurs receive different private signals, we should in principle keep track of the whole distribution of beliefs in the economy (i.e., a distribution over distributions). Fortunately, the information structure is simple enough that the model lends itself to a useful simplification. As in Chapter 3 of [Chamley \(2004\)](#), it is enough to keep track of only one set of time-varying beliefs, the *public beliefs*  $\Lambda_t(\tilde{\theta}, \tilde{\xi}) = Pr\left(\theta = \tilde{\theta}, \xi \in [\tilde{\xi}, \tilde{\xi} + d\tilde{\xi}] \mid \mathcal{I}_t\right)$ . These public beliefs correspond to the beliefs of an outside observer who only has access to public information  $\mathcal{I}_t$  at time  $t$ , which is the collection of past public signals on the technology and measures of entrepreneurs:  $\mathcal{I}_t = \{S_{t-1}, m_{t-1}, \dots, S_0, m_0\}$ . In comparison to this outside observer, an entrepreneur's information set also includes the private signal  $s_j$ , so that  $\mathcal{I}_{jt} = \mathcal{I}_t \cup \{s_j\}$ . Entrepreneurs' individual beliefs can easily be recovered from public beliefs using Bayes' rule and the private signal  $s_j$ , according to

$$\Lambda_{jt}(\tilde{\theta}, \tilde{\xi}) = \frac{\Lambda_t(\tilde{\theta}, \tilde{\xi}) f_{\tilde{\theta}+\tilde{\xi}}^s(s_j)}{\int \Lambda_t(\theta, \xi) f_{\theta+\xi}^s(s_j) d(\theta, \xi)}. \quad (3)$$

This simplification comes from the fact that only public information evolves over time. Indeed, since the private signal distribution  $f_{\theta+\xi}^s$  is constant and known up to the realization of  $\theta$  and  $\xi$ , it is easy to recover the entire distribution of private beliefs across agents for a given combination of  $(\theta, \xi)$  at any point in time. As a result, the only object whose evolution over time we need to keep track of is the public belief distribution  $\Lambda_t$ .

## 2.5 Timing and Adoption Decision

The timing is as follows. At date 0, the fundamental  $\theta$ , the common noise component  $\xi$  and the private signals  $s_j$  are drawn once and for all. At date  $t \geq 0$ ,

1. Agents choose whether to adopt the new technology or not based on their individual beliefs  $\Lambda_{jt}$ ;
2. If it has not matured yet, the new technology matures with probability  $\lambda$ ;
3. If the new technology matures,  $\theta$  is learned. Otherwise, agents observe  $\{S_t, m_t\}$  and update their beliefs. The economy moves to the next period.

Since the adoption decision is a simple static problem, it can be characterized in an easy way. Agent  $j$  adopts the technology in period  $t$  if and only if

$$E[A_t^n | \mathcal{I}_{jt}] = \lambda [p_{jt}\theta_H + (1 - p_{jt})\theta_L] + (1 - \lambda)A^o \geq A^o, \quad (4)$$

where we define

$$p_{jt} = \Pr(\theta = \theta_H | \mathcal{I}_{jt}) = \int \Lambda_{jt}(\theta_H, \xi) d\xi \quad (5)$$

as the probability that  $j$  puts on being in the good-technology state  $\theta_H$ , the adoption decision (4) is characterized by a cutoff rule  $p^*$  in the space of beliefs. That is, an agent adopts the technology if and only if  $p_{jt} \geq p^*$  where  $p^*$  is the belief of the marginal adopter such that

$$p^*\theta_H + (1 - p^*)\theta_L = A^o. \quad (6)$$

The total measure of adopting agents, including noise entrepreneurs, can then be expressed as

$$m_t = m^e(\Lambda_t, \theta, \xi) + \mu\varepsilon_t, \quad (7)$$

$$\text{where } m^e(\Lambda_t, \theta, \xi) = (1 - \mu) \int \mathbb{I}(p_j(\Lambda_t, s_j) \geq p^*) f_{\theta+\xi}^s(s_j) ds_j. \quad (8)$$

The variable  $m^e$  is the measure of adopting agents among rational entrepreneurs in a given state of the world  $(\theta, \xi)$ . Importantly for what follows,  $m^e$  is an object that any agent in the economy can compute. To see this, note that since they know the structure of the model and the public beliefs, all agents agree on the cutoff  $p^*$ . Second, thanks to the dichotomy between public beliefs and the fixed distribution of private signals  $f_{\theta+\xi}^s$ , agents understand the mapping from  $(\theta, \xi)$  to  $p_j$  and can therefore compute the *distribution* of beliefs  $p_j$  that would arise in a given state of the world  $(\theta, \xi)$ . This property is essential to tractably solve the inference problem from the endogenous public signal, to which we now turn.

## 2.6 Evolution of Beliefs

After characterizing the adoption decision, we can now describe how beliefs are updated over time. Before maturation, each end of period brings two new public signals for agents to process:  $S_t$  and  $m_t$ . The updating of information with  $S_t$  is straightforward as it is a simple exogenous signal. Applying Bayes' rule, we define the interim beliefs at the end of the period as

$$\Lambda_{t|S_t}(\tilde{\theta}, \tilde{\xi}) = \frac{\Lambda_t(\tilde{\theta}, \tilde{\xi}) f^u(S_t - \tilde{\theta})}{\int \Lambda_t(\theta, \xi) f^u(S_t - \theta) d(\theta, \xi)}. \quad (9)$$

We now turn to incorporating the information contained in  $m_t$ . Solving the inference problem from an endogenous signal like  $m_t$  can be complicated in general because individual decisions need to be inverted to back out their information content about  $\theta$  and  $\xi$ . Fortunately, and as highlighted at the end of the previous section, the inference problem is greatly simplified in our environment since the measure of rational adopters  $m^e$  in every state of the world is a simple function of the public beliefs  $\Lambda_t$  (known by everyone) and of the true realization of  $(\theta, \xi)$ . Entrepreneurs solely differ in their assessment of the probability of each state  $(\theta, \xi)$ , encoded in  $\Lambda_{jt}$ , but there is no *infinite regress* problem arising from the necessity to forecast the beliefs of agents after any history of shocks. Because of the equilibrium structure of the signal (7), Bayes' rule gives us the simple updating equation

$$\Lambda_{t+1}(\tilde{\theta}, \tilde{\xi}) = \frac{\Lambda_{t|S_t}(\tilde{\theta}, \tilde{\xi}) f^\varepsilon\left((m_t - m^e(\Lambda_t, \tilde{\theta}, \tilde{\xi})) / \mu\right)}{\int \Lambda_{t|S_t}(\theta, \xi) f^\varepsilon\left((m_t - m^e(\Lambda_t, \theta, \xi)) / \mu\right) d(\theta, \xi)}. \quad (10)$$

## 2.7 Equilibrium

We are now ready to define an equilibrium in this economy.

**Definition 1.** An equilibrium consists of history-contingent public beliefs  $\Lambda_t$ , a distribution of private beliefs  $\{\Lambda_{jt}\}_{j \in [0,1]}$  and a measure of technology adopters  $m_t$  for all  $t$ , such that: 1) the distribution of private beliefs is derived from the public beliefs through (3) and (5); 2) the measure of adopters is consistent with entrepreneurs decisions under their private beliefs as in (7); and 3) the public beliefs follow the laws of motion (9)–(10).

With that definition in hand, the following proposition characterizes the set of equilibria.

**Proposition 1.** *There exists a unique equilibrium.*

*Proof.* All proofs are in Appendix A.2. □

The proof of the proposition is straightforward. It shows that from a given distribution of public beliefs  $\Lambda_t$ , there is a unique mapping, given the realization of the shocks, to next period's public beliefs  $\Lambda_{t+1}$ . Starting from the initial  $\Lambda_0$  we can therefore reconstruct the unique equilibrium sequence  $\{\Lambda_0, \Lambda_1, \dots\}$ . All other equilibrium quantities such as the measure of adopters and the distribution of private beliefs can then be reconstructed from the public beliefs in a unique way.

### 3 Endogenous Booms and Busts

We are now fully equipped to analyze the dynamics implied by the model. To simplify the exposition, we focus on a simple special case that conveys the intuition about the emergence of i) a smooth form of information cascades and ii) endogenous booms and busts. We then show that these results extend to a more general setup. We also discuss the welfare properties of the model.

#### 3.1 The 3-state model

To simplify the exposition, we temporarily make the simplifying assumption that the pair  $(\theta, \xi)$  can only take three different values, the minimal number of states required for endogenous boom-bust cycles to emerge in our model. Specifically, we assume<sup>10</sup>

$$(\theta, \xi) \in \{(\theta_L, 0), (\theta_H, 0), (\theta_L, \bar{\xi})\} \text{ with } \theta_L < \theta_L + \bar{\xi} < \theta_H.$$

We refer to  $(\theta_L, 0)$  as the *bad-technology* state,  $(\theta_H, 0)$  as the *good-technology* state and  $(\theta_L, \bar{\xi})$  as the *false-positive* state. The latter is the state of interest as it is the one that will trigger a boom-and-bust cycle by having entrepreneurs mistakenly assess the technology to be of high quality before later realizing their mistake.

Having only three states reduces the number of state variables required to keep track of the belief distribution  $\Lambda_t$ . Public beliefs are now summarized by the two variables

$$p_t \equiv \Lambda_t(\theta_H, 0) \text{ and } q_t \equiv \Lambda_t(\theta_L, \bar{\xi}),$$

and the corresponding updating rules can be found in Appendix A.1.

We now establish a first result. Under our assumptions, the individual beliefs about the probability of the good technology,  $p_{jt} = \Lambda_{jt}(\theta_H, 0)$ , is increasing in the private signal  $s_j$ . As a result, the adoption decision can be further characterized by a cutoff rule  $s^*(p_t, q_t)$  in terms of private signals, which simplifies the expression of the measure of rational adopters  $m^e$  as the following

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<sup>10</sup>We only consider the case  $\theta_L < \theta_L + \bar{\xi} < \theta_H$  because it ensures that beliefs about the good state,  $p_{jt}$ , are nondecreasing in the signal  $s_j$ . This corresponds to the more general case from Section 3.2 where  $F^s$  satisfies the MLRP condition and  $\xi$  is normally distributed. In the case  $\theta_L + \bar{\xi} > \theta_H$ , beliefs are non-monotonic in  $s_j$  and other phenomena can be observed.

lemma shows.

**Lemma 1.** *In the three-state model, the optimal adoption strategy is characterized by a cutoff rule in the private signal  $s^*(p_t, q_t)$ , such that  $p_j(p_t, q_t, s^*(p_t, q_t)) = p^*$ , that is decreasing in  $p_t$ . That is, an agent adopts the technology if and only if  $s_j \geq s^*(p_t, q_t)$ . The measure of rational adopters is given by*

$$m^e(p_t, q_t, \theta, \xi) = (1 - \mu) \bar{F}_{\theta+\xi}^s(s^*(p_t, q_t)).$$

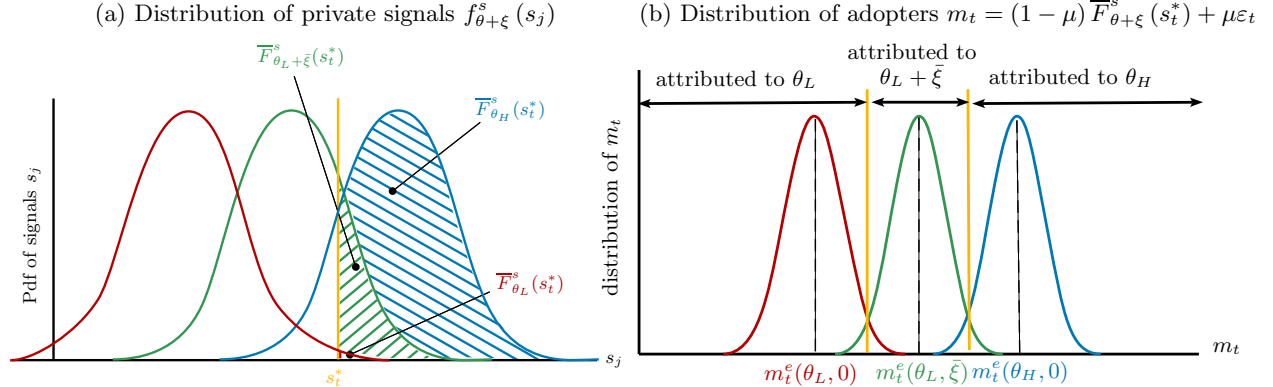
### Learning from $m_t$

To develop intuition on the way agents learn from the measure of adopters, we propose an example in Figure 1. Panel (a) displays the distribution of private signals  $s_j$  in the three states of the world. Due to the MLRP assumption, the three distributions are ordered in the sense of first-order stochastic dominance. The measure of rational adopters  $m^e$  is represented as the mass of agents located to the right of the cutoff  $s_t^*$ . We can see that  $m^e$  is small in the bad-technology state  $(\theta_L, 0)$  (in red), that agents expect more adoption in the false-positive state  $(\theta_L, \bar{\xi})$  (in green), and that it is at its largest in the good-technology state  $(\theta_H, 0)$  (in blue).

The three measures  $m^e$  being computed, we then present in panel (b) the three potential distributions of  $m_t$  in the three states of the world assuming that the noise  $\varepsilon$  is normally distributed with mean 0. As the graph illustrates, agents expect very different distributions of adopters  $m_t$ , each centered on their expected value  $m^e$  in the different states of the world  $(\theta, \xi)$ . We can split the  $m_t$ -space into three regions that indicate which state is attributed more probability after observing  $m_t$ . For instance, for low  $m_t$  the likelihood of the state  $\theta_L$  is greater than that of the other states, so information updating will attribute it a higher probability. The two other states,  $(\theta_L, \bar{\xi})$  and  $(\theta_H, 0)$ , have their own higher likelihood region that are also represented on the graph. Importantly for the emergence of boom-and-bust cycles, beliefs about the good state tend to increase after observing high realizations of  $m_t$ . It is in that sense that the model displays a form of “herding”: agents become more optimistic (resp. pessimistic) after seeing high (resp. low) patterns of adoption, leading them to make inefficient adoption decisions, as we will see in our welfare analysis.

### Signal-to-noise ratio and smooth information cascades

In the traditional herding literature (Banerjee, 1992; Bikhchandani et al., 1992), information cascades arise when public beliefs are so extreme ( $p_t$  extremely high or low, because of a particular history of public signals), that agents disregard their own private information. That is, agents adopt (or not) the technology no matter what their private information is. As a result, observing previous entrepreneurs’ decisions becomes uninformative and the economy may end up being stuck in a situation with mistaken beliefs forever.



Notes: Panel (a) on the left displays the distribution of private signals  $s_j$  across the population in the three possible states of the world along with the corresponding expected measures of adopters  $m_t^e = (1 - \mu) \bar{F}_{\theta+\xi}^s(s^*(p_t, q_t))$ , for some public beliefs  $(p_t, q_t)$ . Panel (b) on the right shows the distribution of  $m_t = m_t^e + \mu \varepsilon_t$  in the three states of the world assuming some Gaussian-like distribution  $F^\varepsilon$  with mean 0 and variance  $\sigma_\varepsilon^2$ .

Figure 1: Private beliefs and expected measure of adopters

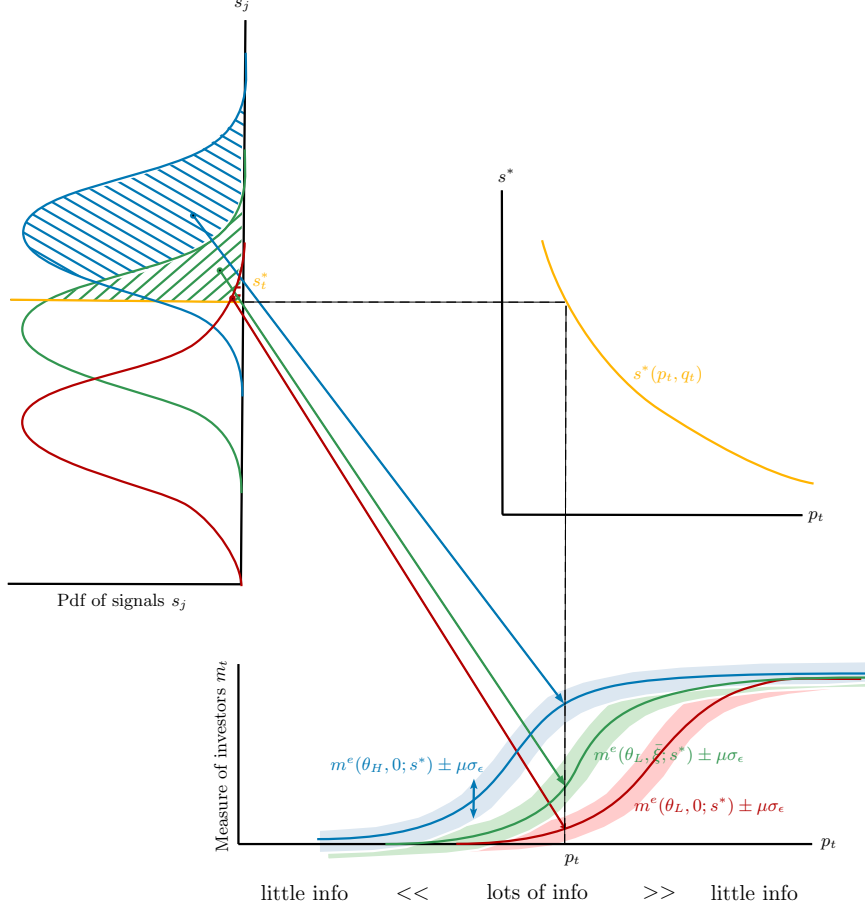
Because social learning takes place through the observation of the continuous variable  $m_t$ , rather than the sequence of binary decisions by previous entrepreneurs, the emergence of information cascades is somewhat different in our setup. We show nonetheless that a similar form of “smooth” information cascades may arise depending on assumptions about the distributions of signals.

The bottom-right panel of Figure 2 represents how the measure of adopters  $m_t$  varies in expectation, along with its  $\pm 1$ -standard deviation error bands, as a function of the public belief  $p_t$ , holding  $q_t$  constant. These curves are drawn by first connecting a given level of  $p_t$  in the bottom-right panel to the equilibrium signal threshold  $s^*(p_t, q_t)$  (upper-right panel), itself connected to the upper-left panel which shows how the measures  $m^e = (1 - \mu) \bar{F}_{\theta+\xi}^s(s^*)$  vary with the cutoff  $s^*$ . As the bottom panel shows, the expected measure of rational adopters  $m^e$  is a monotonic transformation of the CDF  $F_{\theta+\xi}^s$  in the three different states.

The key feature to take away from this graph is that the *signal-to-noise* ratio in  $m_t$  varies nonmonotonically with the public beliefs. For intermediate values of  $p_t$ , the three expected measures  $m^e$  are far apart so that despite the noise  $\varepsilon_t$ , observing  $m_t$  is highly informative about the underlying state  $\theta + \xi$  (i.e., the signal-to-noise ratio is high). For  $p_t$  large (resp. small), almost all (resp. no) agents adopt the technology, the three measures converge to  $\lim_{s^* \rightarrow -\infty} \bar{F}_{\theta+\xi}^s(s^*) = 1$  (resp. 0 when  $s^* \rightarrow \infty$ ), so that the signal  $m_t$  is dominated by noise and becomes uninformative about the underlying fundamentals (i.e., the signal-to-noise ratio is low). Note that this result is not an artifact of specific distributions or functional forms but is instead a general feature of the model as long as  $s^*$  varies sufficiently on the support of  $\bar{F}_{\theta+\xi}^s$ .<sup>11</sup>

<sup>11</sup>Figure 2 may give the wrong impression that the nonmonotonicity result highly depends on the sigmoidal shape of the CDFs  $F_{\theta+\xi}^s$ . While the regions with higher signal-to-noise ratio may change with the distribution, a robust prediction for any distribution is that the measure  $m_t$  is less informative for extreme public beliefs, when agents herd on the same action, since  $\bar{F}_{\theta+\xi}^s(s^*) \rightarrow 1$  (resp.  $\bar{F}_{\theta+\xi}^s(s^*) \rightarrow 0$ ) when  $p_t$  gets close to 1 (resp. 0) and the cutoff  $s^*$  goes to minus infinity (resp. infinity) for any signal distribution.





Notes: The top-left panel displays the distribution of private signals in the three states of the world along with the measure of rational technology adopters  $m_t^e = (1 - \mu) \bar{F}_{\theta+\xi}^s(s_t^*)$ , as previously represented in Figure 1 rotated by  $90^\circ$ . The top-right panel displays an example of equilibrium threshold  $s^*(p_t, q_t)$  as a function of public belief  $p_t$ . The bottom right panel shows how the measure of adopters  $m_t = m_t^e + \mu\varepsilon$  varies with public belief  $p_t$ , keeping  $q_t$  constant in the background, under the assumption that  $\varepsilon_t \sim \mathcal{N}(0, \sigma_\varepsilon)$ . The mean  $m_t^e$  is represented with a continuous line and the corresponding  $\pm 1$ -standard deviation  $\mu\sigma_\varepsilon$  error bands with dashed lines.

Figure 2: Measure of technology adopters  $m_t$  as a function of public belief  $p_t$

The model offers a smooth analog to informational cascades when the equilibrium  $s^*$  reaches the extreme regions of the state space where learning is slow. Suppose for instance that public beliefs are optimistic ( $p_t$  high) so that  $s^*$  is very low. In such a situation, almost all agents act in the same way and adopt the new technology. Only few agents use their private information to “go against the crowd” and do not adopt: the most pessimistic ones that have received particularly low private signals. Unfortunately, their measure is so small that they are hard to detect when looking at the aggregate adoption patterns. As a result, markets are nearly uninformative and beliefs can remain wrong for an extended period of time. The main difference with traditional herding models is that, under the assumption that private signals have full unbounded support, the information flow is never exactly 0 so that there is always some learning taking place through

$m_t$  and  $S_t$ . Such a smooth form of information cascades is of interest to us for two reasons: i) it explains why the economy may remain for an extended period of time in the booming region, where agents understand that they could be wrong in their assessment of the true state of the world but adopt the technology nonetheless, ii) it opens the door to the economy endogenously exiting the information cascade and crashing when some threshold in beliefs is reached, as we will now describe.

### Endogenous boom-and-bust cycle

We now turn to boom-and-bust cycles, which we define as a sequence of rising then declining public beliefs  $p_t$  about the good state. We present simulations of the model to illustrate its ability to generate endogenous boom-bust patterns out of a single impulse shock. We do not attempt to make a realistic calibration but merely pick parameters so as to highlight the model's properties. We will examine later under what general conditions one should expect the boom-bust cycles to occur.

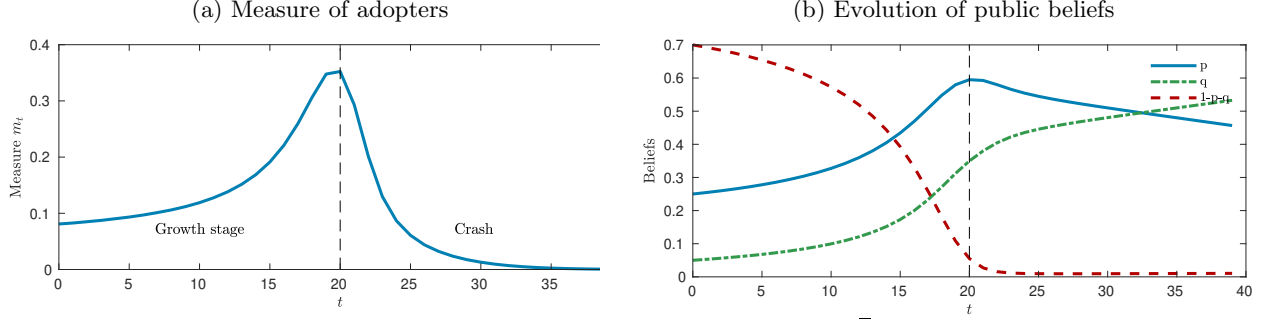
We present the impulse responses of the measure of adopters ( $m_t$ ) and the public beliefs ( $p_t$ ,  $q_t$ ), keeping all other shocks to their mean levels (e.g.,  $\varepsilon_t$ ,  $u_t = 0$ ), when the economy is in the false-positive state  $(\theta, \xi) = (\theta_L, \bar{\xi})$ , the case of interest for our purpose.<sup>12</sup>

Figures 3 and 4 present two examples of endogenous boom-and-bust patterns that may arise in the model, depending on whether or not the economy falls into an information cascade. In both examples, the emergence of boom-and-bust patterns hinges on three key assumptions: (i)  $\theta_L + \bar{\xi}$  needs to be sufficiently close to  $\theta_H$ , so that the two states are hard to distinguish; (ii)  $S_t$  is not too informative, to prevent agents from learning the truth too quickly, and (iii) the prior  $q_0$  on the false-positive state  $(\theta_L, \bar{\xi})$  needs to be sufficiently small relative to the true positive  $(\theta_H, 0)$  for agents to initially attribute most of the rising adoption pattern to the true positive state.

Figure 3 presents the evolution of an economy in which the prior about  $\theta$  is such that the measure of initial adopters is low. When the economy starts in period  $t = 0$ , the measure of adopters (panel a) is small but higher than agents expected. Seeing a surprisingly high adoption rate, agents understand that it is unlikely to come from the bad state and they reduce the probability assigned to it (red curve in panel b). Agents also understand that the high adoption rate could arise from either the good-technology or the false-positive states. As a result, they revise upward their probability assessments of both states ( $p_t$  and  $q_t$  rise). Importantly, however, given that agents start with a low prior on the false-positive state, the observed high level of adoption is mostly attributed to the good-technology state, so the rise in  $p_t$  dominates their expectation. Consequently, agents become more optimistic overall, adoption continues to grow, and the rising adoption pattern, in

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<sup>12</sup>Figures 9 and 10 in the appendix show the economy's response to the good-technology and bad-technology states. With our parametrization, these cases are relatively uninteresting: learning is fairly quick, and the dynamics are close to the full information case.



Notes: The simulation was performed with parameters:  $A^o = 1$ ,  $\theta_H = 1.25$ ,  $\theta_L = 0.6$ ,  $\bar{\xi} = 0.53$ ,  $\mu = 0.15$ . The priors are set to  $p_0 = 0.25$  and  $q_0 = 0.05$ . All the distributions are Gaussian:  $F_{\theta+\xi}^s \sim \mathcal{N}(\theta + \xi, \sigma_s)$ ,  $F^\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon)$  and  $F^u \sim \mathcal{N}(0, \sigma_u)$  with standard deviations  $\sigma_s = 0.6$ ,  $\sigma_\varepsilon = 1$ ,  $\sigma_u = 3$ .

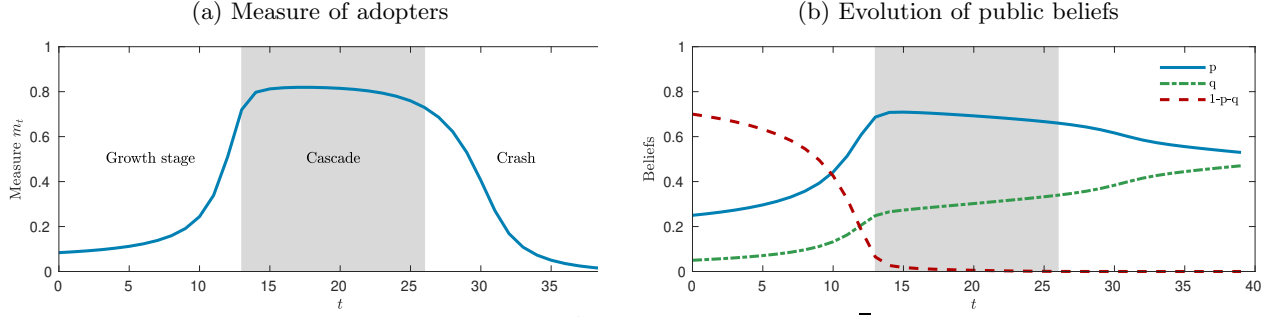
Figure 3: Slow boom, sudden crash

turn, leads to further upward revisions in expectations, seemingly confirming the assessment that the economy is in the good state. We refer to this first stage of the cycle, characterized by the joint increase of adoption rates and beliefs  $(p_t, q_t)$ , as the *growth stage*.

Being rational, agents do understand the possibility that they may be mistaken and keep track of the probability of the false-positive state  $q_t$  in the background, which also rises throughout the growth stage. Since signals are unbiased along the impulse response path, the belief  $q_t$  rises in fact faster than  $p_t$  despite starting from a lower prior. Therefore, a time comes when  $q_t$  is so high that agents become reluctant to adopt the technology and the aggregate adoption rate begins to decline. This is the start of the *crash* stage, which arises at an endogenous date without the need of an exogenous trigger. As adoption reaches a peak of about 35% given our parametrization, the measure of adopters  $m_t$  attains the intermediate region depicted in Figure 2 where it becomes more informative. As a consequence, agents learn the truth faster, adoption rates drop, and the probability  $p_t$  starts declining until a belief reversal occurs later when the belief  $q_t$  takes over. Note that the truth is always learned in the end because of the strictly positive information flow.

This example shows that the model is able to generate asymmetric cycles. The growth stage is slow due to the low information flow when  $m_t$  is close to 0. The crash, on the other hand, is more sudden because it occurs in the region where i) uncertainty between the good state and the false-positive state is high ( $p_t$  and  $q_t$  are close, so beliefs are more responsive to new information), and ii) the signal  $m_t$  is more informative at the peak.

To highlight the dynamics of the model, the simulations presented in Figure 3 assume that there are no shocks to the exogenous public signal ( $u_t = 0$ ) and no shock from the noise entrepreneurs ( $\varepsilon_t = 0$ ). But these random shocks, by influencing the signals that agents observe, can also play an important role in driving aggregate adoption rates. In Appendix A.4, we explore how they can affect the timing and intensity of the boom-bust cycle.



Notes: The simulation was performed with parameters:  $A^o = 1$ ,  $\theta_H = 1.25$ ,  $\theta_L = 0.6$ ,  $\bar{\xi} = 0.55$ ,  $\mu = 0.15$ . The priors are set to  $p_0 = 0.25$  and  $q_0 = 0.05$ . The distributions of all signals are Gaussian with standard deviations  $\sigma_s = 0.6$ ,  $\sigma_\varepsilon = 1$ ,  $\sigma_u = 3$ . The shaded area corresponds to the informational cascade period, loosely defined as periods when  $m_t \geq 70\%$  in which learning is markedly slower, given our calibration.

Figure 4: Endogenous boom and bust with information cascade

### Information cascades

Whether the growth stage gives way to a sudden collapse or not depends on the parametrization of the model. Figure 4 depicts an example of a cycle in which the economy grows so rapidly at first that it reaches the low-informativeness region associated with a high  $m_t$ . In that case, the economy goes through an information cascade before the crash. This simulation uses the same parametrization as in Figure 3, but with a higher common noise parameter  $\bar{\xi}$  so that the distribution of private signals in the false-positive and the good states are more similar. Adoption rises faster and reaches higher levels than in the previous example. As a result, there comes a time at the end of the growth stage when agents are so optimistic that they herd on adoption and  $m_t$  becomes uninformative. The economy thus enters a period akin to an information cascade, as described earlier, where almost all agents adopt the technology due to overly optimistic public beliefs, and in which markets almost cease to provide information. Through this mechanism, the economy may remain stuck for a long period of time with wrong beliefs and excessive adoption. Because the flow of information is never exactly zero, the economy eventually exits the cascade. This event occurs when the belief about the false-positive state  $q_t$  reaches a threshold at which a sufficient fraction of agents stop adopting, bringing back the economy to the region where  $m_t$  is informative. The crash takes place in a manner similar to the previous example: beliefs converge more quickly to their true values as the flow of information increases.<sup>13,14</sup>

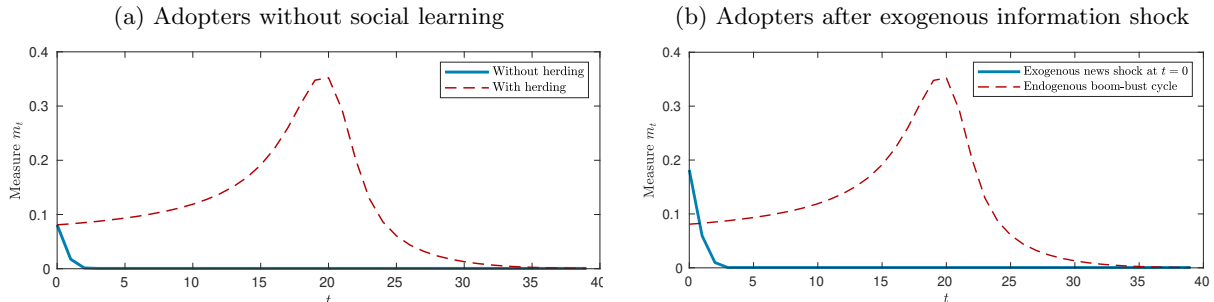
<sup>13</sup>The way the economy exits the cascade is reminiscent of the “wisdom after the fact” mechanism proposed by Caplin and Leahy (1994) and its reinterpretation in Chapter 4 of Chamley (2004).

<sup>14</sup>The model can also generate permanent information cascades. When private signals  $s_j$  are bounded to a set  $[\underline{s}, \bar{s}]$  and the exogenous public signal is uninformative ( $\sigma_u \rightarrow \infty$ ), the public beliefs might be so optimistic that the adoption threshold  $s^*$  reaches  $\underline{s}$ . In this case, all agents adopt regardless of their private signals. As a result, the endogenous public signal  $m_t$  does not reveal any of the dispersed information and is therefore completely uninformative. The economy is then trapped in a constant state of massive adoption, even though the true quality of the technology is bad. The opposite can happen when public beliefs are sufficiently pessimistic. Appendix A.5 shows how a permanent information cascade can arise in this sample economy.

## The importance of herding

To better understand the importance of the social learning channel for the dynamics of the economy, we provide a simulation in which that mechanism is turned off ( $\sigma_\varepsilon \rightarrow \infty$ ) so that agents no longer learn by observing  $m_t$  and, as a result, no herding takes place. That simulation is presented in panel (a) of Figure 5. As we can see, without the herding mechanism the economy does not initially grow into a boom, despite being in the false positive state and entrepreneurs having an unusually optimistic prior. In our previous examples, it was the slow diffusion of the information contained in private signals into the public beliefs that led to an economic boom, but since this channel is shut down here, the boom does not happen and the mass of adopters  $m_t$  quickly converges to zero.

To further highlight how our endogenous learning mechanism differs from more traditional exogenous noise shocks as in [Lorenzoni \(2009\)](#), we provide another simulation, in panel (b) of Figure 5, in which the economy is hit by an exogenous  $u_t$  shock immediately before  $t = 0$  and the social learning channel remains shut down ( $\sigma_\varepsilon \rightarrow \infty$ ). Upon impact, this shock leads to an increase in  $m_t$ , as expected, but the dynamics it triggers is qualitatively different from that generated by the herding mechanism (red dashed curve). As the figure illustrates, adoption peaks immediately then gradually fades out as more information is collected. The propagation is weak and does not lead to the positive feedback loop highlighted in the case of herding, which showed a slow rising pattern of self-reinforcing adoption and optimism.<sup>15</sup>



Notes: The simulations were performed with parameters identical to Figure 3 but with  $\sigma_\varepsilon$  multiplied by 20 (uninformative endogenous signal). Panel (b) has the same parameters and shows the impact of a shock to  $u_{-1}$  so that  $p_0 = 0.35$ . In both panels the dashed red lines represent  $m_t$  from (3).

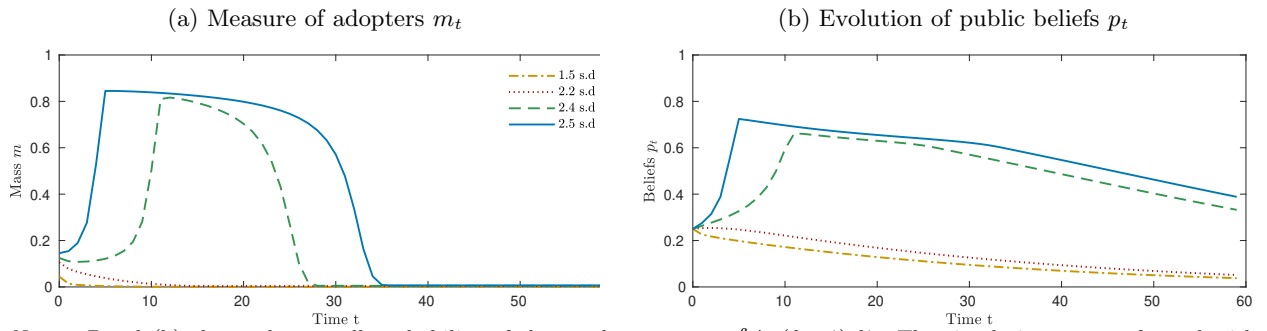
Figure 5: The role of endogenous learning and the impact of exogenous news shocks

## 3.2 Continuous Case

How general are the phenomena highlighted in the 3-state model? In this section, we discuss under what conditions endogenous boom-and-bust cycles arise in a less restrictive environment.

<sup>15</sup>Only a specific sequence of increasingly positive then negative exogenous shocks  $u_t$  can replicate the type of dynamics observed in Figure 3. In contrast, in our model, the boom and bust dynamic is the natural outcome of the economic forces at work. The two models also have very different policy implications, as we discuss in Section 3.3.

First, we relax the three-state assumption and return to the specification where  $\xi$  can take on a continuum of values. Second, we wish to understand how our two key conditions, (i)  $\theta_L + \xi$  close to  $\theta_H$  and (ii) low  $q_0$ , translate to the more general case. To build intuition on this issue, Figure 6 shows the impulse responses of the economy in the continuous- $\xi$  case assuming that  $\xi$  is independent of  $\theta$  and is normally distributed with mean 0 and standard deviation  $\sigma_\xi$ . As in the previous section, we present the response of the economy in the bad-technology state  $\theta = \theta_L$  but we vary the size of the  $\xi$  shock. Four shocks of various sizes are represented, with  $\xi$  expressed as a multiple of the standard deviation, namely  $\xi = k\sigma_\xi$ ,  $k \in \{1.5, 2.2, 2.4, 2.5\}$ . The figure shows very distinct behaviors depending on the size of the shock. When the shock is relatively small,  $\xi = 1.5\sigma_\xi$  (yellow dash-dotted line), the economy does not experience any herding behavior in which the high initial adoption rate leads to rising optimism. Agents put a sufficiently high likelihood on this  $\xi$  draw and are, consequently, able to detect it relatively quickly. Things start to differ as we increase the size of the shock. For an intermediate-sized shock,  $\xi = 2.4\sigma_\xi$  (green dashed line), the economy begins to experience a boom-bust cycle of the sort described earlier. Because of the low probability of experiencing a shock greater than two standard deviations, agents are initially fooled by the high adoption rates and the economy enters a growth stage with rising optimism and adoption. The growth stage is slow and the crash begins around date  $t = 12$ , as in Figure 3. When the size of the shock is larger,  $\xi > 2.5\sigma_\xi$  (blue continuous line), the rise in adoption is so large that the economy goes through an information cascade after experiencing a short growth stage, as in Figure 4. The economy exits the cascade endogenously at a date which is further delayed as the size of the shock increases.



Notes: Panel (b) shows the overall probability of the good state  $p_t = \int \Lambda_t(\theta_H, \xi) d\xi$ . The simulation was performed with parameters:  $A^o = 1$ ,  $\theta_H = 1.25$ ,  $\theta_L = 0.6$ ,  $\mu = 0.15$ . The priors are set to  $p_0 = 0.25$  and  $q_0 = 0.05$ . All the distributions are Gaussian as in Figure 3 with the additional assumption that  $\xi \sim \mathcal{N}(0, \sigma_\xi)$  with standard deviations  $\sigma_s = 0.6$ ,  $\sigma_\varepsilon = 1$ ,  $\sigma_u = 2.5$  and  $\sigma_\xi = 0.4$ .

Figure 6: Boom-and-bust cycles in the continuous case

These simulations show that the dynamics depicted in the examples of Figures 3 and 4, in the previous section, are not mere curiosities but regular fixtures of the more general model. They also emphasize the importance of *nonlinearities* in governing these dynamics. Indeed, the simulations show that the endogenous boom-and-bust phenomenon occurs whenever the shock to  $\xi$  is unusually

large, sufficiently so that agents underestimate its likelihood and initially attribute the observation of high adoption rates to the good-technology state.

We now show that there always exists a sufficiently large shock in  $\xi$  to trigger a boom-and-bust cycle in beliefs, as long as the exogenous signal coming from the public signal  $S_t$  is not too precise.

**Proposition 2.** *In the Gaussian case, i.e.,  $F^\xi \sim \mathcal{N}(0, \sigma_\xi^2)$ ,  $F^s | \theta, \xi \sim \mathcal{N}(\theta + \xi, \sigma_s^2)$ ,  $F^\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ ,  $F^u \sim \mathcal{N}(0, \sigma_u^2)$ , for  $\theta$  and  $\xi$  independent and signal  $S_t$  sufficiently uninformative ( $\sigma_u$  low), there exists a  $\underline{\xi}$  such that all shocks  $\xi \geq \underline{\xi}$  generate a boom-and-bust cycle in the impulse response of beliefs  $p_t$  to a false-positive shock  $(\theta_L, \xi)$ .*

Note that the above discussion shows a restriction imposed by the theory: because they are rational, agents cannot make systematic mistakes in their assessment of the probability of each state. Hence, boom-bust cycles can only arise for shocks that have a low enough probability of occurring. Our model thus offers a theory of *infrequent* booms-and-busts. Going beyond this limitation may require the introduction of deviations from rationality.

### 3.3 Welfare

We now turn to the analysis of welfare in this economy. Since entrepreneurs do not internalize that their adoption decisions affect the release of public information, the equilibrium is in general not efficient and policy interventions can be beneficial. To show this formally, we introduce a social planner that maximizes aggregate welfare under limited information. Specifically, we assume that the planner only observes signals that are publicly available and cannot rely on the private information of the entrepreneurs when making decisions.<sup>16</sup>

We follow Angeletos and Pavan (2007) in assuming that the planner seeks to maximize the sum of the entrepreneurs' expected utility, where the expectation is computed according to the agents' private beliefs. In each period, the planner picks an adoption threshold  $p_t^*$  such that agents with beliefs  $p_{jt} \geq p_t^*$  adopt the technology. Written in recursive form, the problem of the social planner is

$$V(\mathcal{I}) = \max_{p^*} E_{\theta, \xi} \left[ \int_{p_j \geq p^*} E[A^n | \mathcal{I}_j] dF_{\theta+\xi}^{p_j}(p_j) + A^o F_{\theta+\xi}^{p_j}(p^*) | \mathcal{I} \right] + \beta E_{\theta, \xi} [V(\mathcal{I}'(p^*)) | \mathcal{I}], \quad (11)$$

where  $\mathcal{I}'$  is public information next period, which evolves according to the law of motion (10), and where  $F_{\theta+\xi}^{p_j}(p_j)$  is the CDF of the agents' subjective probability that  $\theta = \theta_H$  when the true state of the world is  $\theta + \xi$ . The expectation  $E_{\theta, \xi}$  is then taking over these states using the public beliefs.

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<sup>16</sup>We impose these restrictions so that the problem of the planner is not trivial, and that it resembles that of a government trying to design policy under uncertainty about the true value of a new technology.

The first term in (11) captures the current-period returns from letting agents with private beliefs above  $p^*$  adopt the technology. To compute that term, the planner first uses the public beliefs  $\mathcal{I}$  to evaluate the likelihood of being in a given state  $\theta + \xi$ . Since the planner knows the structure of the economy, it can then reconstruct the distribution  $F_{\theta+\xi}^{p_j}(p_j)$  of private beliefs in that state, which is needed to compute the mass of entrepreneurs above and below  $p^*$ . The second term in (11), the continuation value, captures the impact of a given adoption threshold  $p^*$  on the future public information. It is this term that creates a gap between the equilibrium and the efficient allocation. In the competitive equilibrium, individual agents are atomistic, so they disregard the impact that their actions have on the release of public information. The planner, on the other hand, understands that by changing the cutoff  $p^*$ , the mass of adopters also changes, which affects the informativeness of public signals.

The first-order condition of the planner with respect to  $p^*$  can be written as

$$E_{\theta,\xi} \left[ \lambda (p^* \theta_H + (1 - p^*) \theta_L - A^o) f_{\theta+\xi}^p(p^*) \mid \mathcal{I} \right] = \beta \frac{\partial E_{\theta,\xi} [V(\mathcal{I}') \mid \mathcal{I}]}{\partial p^*}. \quad (12)$$

The left-hand side of this equation reflects the expected cost of increasing the threshold  $p^*$  at the margin. If the true state is  $\theta + \xi$ , increasing  $p^*$  slightly pushes a mass  $f_{\theta+\xi}^p(p^*)$  of agents away from adopting, each of which loses  $\lambda (p^* \theta_H + (1 - p^*) \theta_L - A^o)$  in expected returns. The planner takes the expectation of these losses over all the states  $\theta + \xi$ . The right-hand side of the equation reflects the impact of increasing  $p^*$  on the flow of public information that is released at the end of the period. By changing  $p^*$ , the planner can, for instance, increase the gap between the expected realizations of  $m$  in different states of the world. When it does so,  $m$  becomes more informative as the signal-to-noise ratio increases. Notice that when  $\beta = 0$  the first-order condition (12) collapses to the equilibrium cut-off rule (6), such that the efficient allocation coincide with the equilibrium.

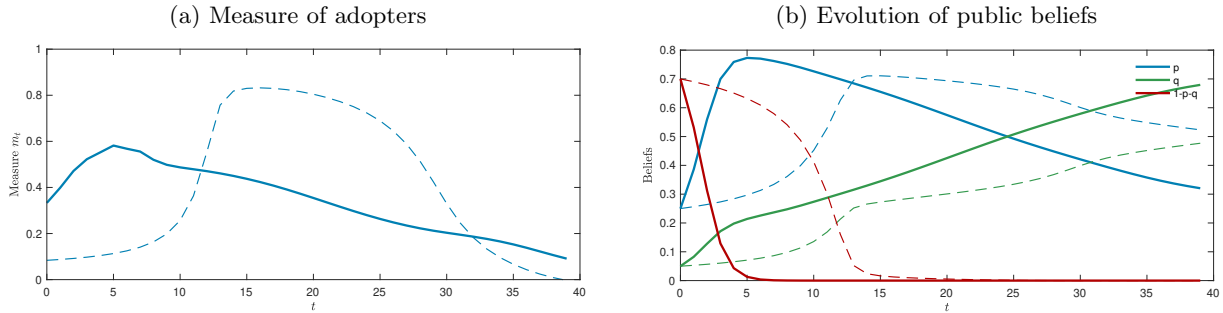
We describe in Appendix A.6 how the efficient allocation can be implemented as an equilibrium through an adoption tax  $\tau^*$ . That tax in general varies with public beliefs and pushes agents to internalize the social benefits of their adoption decisions on the release information.

### Example: Efficiency in the 3-state model

To further explore the role played by inefficiencies in the model, we now go back to our earlier example of Figure 4, and compare the evolution of the equilibrium and the efficient allocations. The results are presented in Figure 7 where, as before, the economy is in the false-positive state. We see from Panel (a) that, in the equilibrium (dashed lines), agents are initially cautious and only about 10% of them adopt the new technology. To make the public signal more informative, the planner pushes more agents to adopt (solid lines). It follows that the public beliefs move more rapidly in the efficient allocation (Panel b), and agents quickly learn that the bad-technology can



be ruled out. In later periods, starting from about  $t = 11$ , the situation changes as optimistic signals about the economy accumulate. The public beliefs are so positive that private agents would neglect their private information to adopt the technology massively. The planner instead is more cautious and the efficient allocation features less adoption than in the equilibrium. The planner leans-against-the-wind so that more private information is transmitted into public signals, and that beliefs can move more rapidly. Overall, we see that in the planner's allocation there is a brief boom followed by a steady decline that begins at  $t = 5$ . In the equilibrium, however, the economy enters an information cascade and aggregate adoption rates remain very high for an extended period. In Appendix A.6 we describe the optimal tax that would need to be implemented in this economy so that the equilibrium coincides with the efficient allocation.



Notes: Bold lines correspond to the efficient allocation and thin lines correspond to the equilibrium. The true value of the fundamental is  $(\theta, \xi) = (\theta_L, \bar{\xi})$ , the false-positive state. The simulation was performed with the same parameters as for Figure 4 and with  $\beta = 0.9$  and  $\lambda = 0.9$ .

Figure 7: Endogenous boom-bust cycles in the efficient allocation

## 4 A Business Cycle Model with Herding

We now embed the mechanism from the simple model into a business cycle framework. Our objective is threefold. First, we want to examine the robustness of the mechanism in a more realistic environment that involves more moving parts (e.g., prices and constraints). Second, we want to investigate under what conditions the hump-shaped evolution of beliefs produced by our simple learning model may lead to a contraction deep enough to go below the trend. Finally, a more realistic setup is required to explore the quantitative implications of the theory, which we do in the next section.

### 4.1 Foreword

A key lesson from the news (or noise)-driven business cycle literature is that standard models have difficulty producing realistic business cycles out of fluctuations in beliefs. In particular they are

unable to generate a general macroeconomic expansion, with positive comovements in macro aggregates, during the optimistic phase of the cycle, followed by a recession when beliefs are reversed.

To circumvent this difficulty, we propose a model that features nominal rigidities and two types of capital. Following [Lorenzoni \(2009\)](#), nominal rigidities ensure that aggregate output and hours may increase in response to a surge in demand driven by optimistic expectations. Under a sufficiently accommodative monetary policy, in line with standard estimates, the economy is demand-driven and a muted response of interest rates helps sustain the expansion in demand caused by a positive wealth effect.

We also include two types of capital, a new-technology-specific capital (e.g., IT capital) and a traditional form of capital, in the model to achieve two objectives. First, under the assumption that the new technology is intensive in IT capital, investment must increase for agents to benefit from the technological innovation, ensuring positive comovements between consumption and investment along the booming phase of the cycle. Second, the two types of capital also allow the model to generate a decline below trend during the bust through a *misallocation* channel: when people realize that they overestimated the quality of the technology, the IT capital stock loses its value and the economy must work its way back up to trend by reinvesting in traditional capital.

## 4.2 Household

Our business cycle framework builds on the core entrepreneurial economy presented in [Section 2](#). In addition to the entrepreneurs who face a technology adoption choice, the economy is composed of i) a representative household, ii) retailers, who are the only agents facing price rigidities, and iii) a monetary authority. The household lives forever, consumes, supplies labor and is the owner of all the firms and capital stocks in the economy. The preferences of the household are given by

$$E \left[ \sum \beta^t \log \left( C_t - \frac{L_t^{1+\psi}}{1+\psi} \right) \right], \quad \psi \geq 0,$$

where  $C_t$  is the consumption of the final good and  $L_t$  is labor. The household can save in a risk-free one-period nominal bond  $B_t$  in zero net supply and in two different forms of capital: a traditional type (T) in quantity  $K_t^T$  and IT capital in quantity  $K_t^{IT}$ . The household is subject to the real budget constraint

$$C_t + \sum_{i=T,IT} I_t^i + \frac{B_t}{P_t} = w_t L_t + \sum_{i=T,IT} z_t^i K_t^i + \frac{1+R_{t-1}}{1+\pi_t} \frac{B_{t-1}}{P_{t-1}} + \Pi_t,$$

where  $I_t^i$ ,  $i = T, IT$ , is the investment in each capital type,  $z_t^i$  is the corresponding real rental rate,  $w_t$  is the real wage,  $\Pi_t$  is total profits,  $R_{t-1}$  is the nominal interest rate on government debt issued at date  $t-1$ ,  $P_t$  is the nominal price level and  $1+\pi_t = P_t/P_{t-1}$  is the inflation rate. As usual, the

law of motion for each type of capital,  $i = T$  or  $IT$ , is given by

$$K_{t+1}^i = (1 - \delta) K_t^i + I_t^i,$$

where  $\delta$  is the depreciation rate.

### 4.3 Technology

There are four sectors: i) an entrepreneur sector, ii) a wholesale sector, iii) a retail sector and iv) a final good sector. The most important one, the entrepreneur sector, is the analog of the simple model from Section 2.

#### Entrepreneur sector

As in the simple model, the entrepreneurial sector is populated by a unit continuum of entrepreneurs indexed by  $j \in [0, 1]$  who face a choice between an *old* versus a *new* technology. As before, the old technology is characterized by a known constant productivity  $A^o$  and the new technology by an unknown stochastic return  $A_t^n$ . In contrast with the earlier model, however, we now assume that these productivities can be combined with capital and labor in a production function. The old production technology is Cobb-Douglas in some capital bundle  $K_{jt}^o$ , to be described shortly, and labor  $L_{jt}^o$ ,

$$Y_{jt}^o = A^o (K_{jt}^o)^\alpha (L_{jt}^o)^{1-\alpha}, \quad 0 \leq \alpha \leq 1.$$

Unexpectedly, at date 0, the new technology becomes available with production function

$$Y_{jt}^n = A_t^n (K_{jt}^n)^\alpha (L_{jt}^n)^{1-\alpha}.$$

As before, the TFP of the new technology  $A_t^n$  is initially as productive as the old one,  $A_t^n = A^o$ , before it matures with fixed probability  $\lambda > 0$ . After maturation, the true nature of the technology is revealed and  $A_t^n$  is then characterized by a constant fundamental  $\theta \in \{\theta_H, \theta_L\}$ ,  $\theta_H > A^o > \theta_L$ .

In addition to differing in TFP, the two technologies differ in the capital bundle they use as input. The capital bundle used by each technology  $i = o, n$  is given by

$$K_{jt}^i = \kappa_i \left( \omega_i (K_{it}^{IT})^{\frac{\zeta-1}{\zeta}} + (1 - \omega_i) (K_{it}^T)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}}, \quad \zeta > 0, \quad (13)$$

where  $\kappa_i = \left( \omega_i^\zeta + (1 - \omega_i)^\zeta \right)^{-\frac{1}{\zeta-1}}$  and with the assumption that the intensity in IT capital is greater for the new than for the old technology,  $0 \leq \omega_o < \omega_n \leq 1$ .<sup>17</sup> We denote by  $z_t^i$ ,  $i = o, n$ , the rental

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<sup>17</sup>The value of the parameter  $\kappa_i$  is set such that permanent changes in the measure of new technology adopters  $m_t$  have no effect on steady-state output for equal productivities.

price of each bundle.

After date  $t \geq 0$ , entrepreneurs face a technology choice problem. As before, we assume that a fraction  $0 \leq \mu \leq 1$  of entrepreneurs are “noise entrepreneurs”, and that a fraction  $\varepsilon_t$  of them adopt the new technology, where  $\varepsilon_t$  is i.i.d, distributed according to a CDF  $F^\varepsilon$  with support  $[0, 1]$ . The remaining  $1 - \mu$  entrepreneurs are rational and choose the best of the two technologies, based on public and private information:

$$i_{jt} = \operatorname{argmax}_{i_{jt} \in \{0,1\}} i_{jt} E[\Pi_t^n | \mathcal{I}_{jt}] + (1 - i_{jt}) E[\Pi_t^o | \mathcal{I}_{jt}],$$

where  $\Pi_t^i$ ,  $i = o, n$ , are the profits from using technology  $i$ ,  $\mathcal{I}_{jt}$  is the information set of entrepreneur  $j$  at time  $t$ , and  $i_{jt}$  is a dummy capturing the technology adoption decision. Finally, we assume that entrepreneurs are monopolistic producers of differentiated varieties that sell their output to the wholesale sector.

### Wholesale sector

The wholesale, retail and final good sectors play no major role in the model other than separating price rigidities from the technology choice problem of the entrepreneurs.

The wholesale sector is modeled as a representative firm which produces a wholesale good with CES technology

$$Y_t^w = \left( \int_0^1 Y_{jt}^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma \geq 0, \quad (14)$$

where  $Y_{jt}$  is the quantity of inputs it purchases from the monopolistic entrepreneurs. The wholesale sector is perfectly competitive, giving rise to the demand schedule,  $Y_{jt} = (P_{jt}/P_t^w)^{-\sigma} Y_t^w$ , where  $P_t^w = \left( \int_0^1 P_{jt}^{1-\sigma} dj \right)^{\frac{1}{1-\sigma}}$  is the price of the wholesale good and  $P_{jt}$  the price of each differentiated entrepreneur good.

### Retail sector

The retail sector is composed of a unit continuum of monopolistic producers who buy the wholesale good at  $P_t^w$  and costlessly differentiate it using a one-to-one technology. Retail sector firms are the only ones to face price rigidities. We assume that they face Calvo-style frictions: firms can only reset their price with probability  $1 - \chi$ , leading to a standard Phillips curve.

### Final good sector

The final good sector, similar to the wholesale sector, is modeled as a representative firm that operates under perfect competition and produces the final good, used for consumption and investment,

using inputs from the retail sector. It uses the CES technology,

$$Y_t = \left( \int_0^1 (Y_{jt}^r)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}},$$

where  $Y_{jt}^r$  is the quantity purchased from each retail firm  $j$  and  $\sigma$  is the same elasticity of substitution as in (14).

#### 4.4 Monetary Authority

To close the model, we need to specify the policy followed by the monetary authority. As is common in the literature, we assume that the central bank follows a Taylor rule,

$$\frac{1 + R_t}{1 + \bar{R}} = \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y}, \quad (15)$$

where  $\bar{R}$ ,  $\bar{\pi}$  and  $\bar{Y}$  correspond to the values of, respectively, the target nominal interest rate, inflation and output, which we define later.

#### 4.5 Information

The information structure for entrepreneurs mimics the one in the simple model of Section 2. As before, the true technology  $\theta$  and the common noise  $\xi$  are drawn once-and-for-all at date 0. The ex-ante probability that  $\theta = \theta_H$  is denoted by  $p_0$ . Entrepreneurs receive a private signal  $s_j$  drawn from  $F_{\theta+\xi}$  at date 0. In addition, entrepreneurs and all other agents in the economy (household, central bank, retailers, etc.) collect public information over time through the observation of an exogenous signal  $S_t$  and the measure of entrepreneurs  $m_t = \int_0^{1-\mu} i_{jt} dj + \mu \varepsilon_t$ . Importantly, because the productivity of the new technology is identical to  $A^o$  until maturation, there is no other source of information in the economy. In equilibrium, prices and aggregate quantities are solely functions of  $m_t$  and of the public information up to time  $t$ . As a result, prices and quantities provide no other information than is already contained in  $m_t$ . The updating rules for beliefs  $\Lambda_t$  and  $\Lambda_{jt}$  are identical to those in the simple model and given by equations (3), (9) and (10).

#### 4.6 Timing

Before date 0, the economy is in a deterministic, no-inflation initial steady state that corresponds to the economy before the introduction of the new technology. At date 0, the new technology fundamental  $\theta$ , the common noise component  $\xi$  and the private signals  $s_j$  are drawn once-and-for-all. For all date  $t \geq 0$ ,

1. Entrepreneurs choose whether to adopt the new technology based on the capital stocks  $(K_t^{IT}, K_t^T)$ , the relative rental rates  $z_t^i$ ,  $i = T, IT$ , and their information set  $(\Lambda_t, s_j)$  (Stage

- A),
2. The measure of technology adopters  $m_t$  is realized,
  3. The new technology matures with probability  $\lambda$ ,
  4. Simultaneously (Stage B),
    - (a) All agents observe  $\{m_t, S_t\}$  and update their information,
    - (b) The household chooses consumption, investment and labor supply,
    - (c) Production takes place,
    - (d) The monetary authority sets the policy rate,
    - (e) Markets clear.

The notation “Stage A” and “Stage B” is used in Appendix B.1 to identify when decisions are made in the full equilibrium definition.

#### 4.7 Investment Decision

The technology adoption decision is more complicated than in the simple model because of the presence of general equilibrium effects. When choosing whether to use the new technology, agents have to forecast the profits from either technology. Profits in equilibrium depend not only on productivity but also on the level of demand from wholesalers  $Y_t^w$ , prices and the real marginal costs  $mc_t^i = \frac{1}{A_{jt}} \left( \frac{z_t^i}{\alpha} \right)^\alpha \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha}$  from using each technology  $i = o, n$ :

$$\Pi_t^i = (P_{jt} - P_t mc_t^i) Y_{jt}. \quad (16)$$

Solving the model by linearizing the equations of the DSGE model, entrepreneur  $j$  ultimately chooses to invest if and only if

$$E \left[ \hat{A}_t^n - \alpha \hat{z}_t^n \mid \mathcal{I}_{jt} \right] \geq E \left[ \underbrace{\hat{A}_t^o}_{=0} - \alpha \hat{z}_t^o \mid \mathcal{I}_{jt} \right], \quad (17)$$

where the hatted variables are log-deviations from a moving steady state that we define in Section B.3 of the appendix and  $\hat{z}_t^i$ ,  $i = n, o$ , are the rental rates on the capital bundles (13). As equation (17) demonstrates, entrepreneurs not only have to forecast the technology  $A_t^n$  but also factor prices, as they are now competing for the same inputs.

This concludes the exposition of the general business cycle model. Additional details with a full definition of the equilibrium, all model equations and a discussion of the resolution method can be found in Appendix B.

## 5 Quantitative Illustration

We now turn to a quantitative illustration of our general macroeconomic model. We calibrate the model to a specific episode in US history and examine its ability to explain broad features of the data.

### 5.1 Calibration

As we argued before, our model offers a theory of infrequent endogenous booms-and-busts. For that reason, we do not expect to explain general business cycle patterns in the absence of other shocks, but rather to provide a narrative for certain episodes. We thus focus our calibration exercise on a particular episode in recent US history that best fits the description of a technology-driven boom and bust cycle: the late 1990's dot-com bubble. We map the new technology in our model to the introduction of IT technologies in the 1990s and we focus more specifically on the late part of the cycle which covers the period that preceded the stock market collapse in the NASDAQ composite index starting from a trough in 1995Q4 to the crash in 2001Q1.

Our goal with this exercise is not to show that the mechanism can precisely replicate the behavior of the economy during the dot-com period. Rather, we want to determine whether a reasonable calibration of the model is able to generate boom-bust cycles that are similar in terms of magnitude and comovements to what we see in the data. Since we focus on a single historical episode, there is limited data to pin down certain parameters with confidence. For those, we rely on the best data available and provide robustness tests in Appendix B to show that the mechanism does not hinge on specific parameter values. We are, however, careful to properly discipline moments that are key for the mechanism, such as the dispersion of private beliefs, as we explain in more details below.

| Parameter  | Value | Target   |
|------------|-------|--|
| $\alpha$   | 0.36  | Labor share  |
| $\beta$    | 0.99  | 4% annual interest rate                                    |
| $\psi$     | 2     | Frisch elasticity of labor supply (Chetty et al., 2011)    |
| $\chi$     | 0.75  | 1 year price duration                                      |
| $\sigma$   | 10    | Markups of about 11%                                       |
| $\phi_y$   | 0.125 | Clarida et al. (2000)                                      |
| $\phi_\pi$ | 1.5   | Clarida et al. (2000)                                      |
| $\zeta$    | 1.71  | Elasticity between types of capital (Boddy and Gort, 1971) |

Table 1: Standard parameters

The model is solved at a quarterly frequency. Table 1 lists a first set of standard parameters that we take from the literature. The labor intensity  $\alpha$  is set to target a standard labor share of

36%. The discount factor  $\beta$  matches an annual real interest rate of about 4%. The household's preference over consumption is logarithmic and the Frisch elasticity is set to 2, within the range of standard macro-level estimates (Chetty et al., 2011). The Calvo price-setting parameter  $\chi$  yields a standard average price duration of 1 year (Basu and Bundick, 2017). The elasticity of substitution between varieties  $\sigma$  is set to 10 to match an average markup of 11%. The Taylor rule parameters  $(\phi_y, \phi_\pi)$  are within the estimates of Clarida et al. (2000). The target inflation rate  $(\bar{\pi})$  is set to 0 and that for output  $(\bar{Y})$  and the interest rate  $(\bar{R})$  are set to their respective values in the zero-inflation steady-state before the arrival of the technology. Finally, we pick the elasticity  $\zeta$  between the different types of capital within the firm from early estimates by Boddy and Gort (1971).

Table 2 below lists the more important parameters that attempt to match features of the dot-com bubble. We set the IT-capital shares  $\omega_i, i = o, n$ , to match a share of IT capital of 3% before the introduction of the new technology in 1991 and 14% in 2007 (Strauss and Samkharadze, 2011). The probability of maturation for the new technology  $\lambda$  is set to  $1/22$  to match an average waiting time of 22 quarters, corresponding to the length of our period of interest: 1995Q4-2001Q1. We now turn to the technology parameters.  $A^o$  is normalized to 1. We use the Survey of Professional Forecaster (SPF) mean real GDP growth forecast over the current quarter to discipline  $\theta_H$  and  $\theta_L$ . Under the assumption that factors are fixed in the short run, this identifies changes in the productivity parameter  $\theta$ . The highest forecast for growth was 4.19% in 2000Q2 in annualized terms. Correcting for a mean growth trend in GDP of 2.4% over 1991-1998, this yields  $\theta_H = 1.099$ . Similarly, targeting the lowest growth forecast of 0.80% in 2001Q1, we obtain an estimate of  $\theta_L = 0.912$ .<sup>18</sup>

For the common noise, we adopt the same structure as in Section 3.1 and assume that the pair  $(\theta, \xi)$  can only take the three values

$$(\theta, \xi) \in \{(\theta_L, 0), (\theta_H, 0), (\theta_L, \bar{\xi})\} \text{ with } \theta_L < \theta_L + \bar{\xi} < \theta_H,$$

where  $(\theta_L, 0)$  is the *bad-technology* state,  $(\theta_H, 0)$  is the *good-technology* state and  $(\theta_L, \bar{\xi})$  is the *false-positive* state. As before, we let  $p_t$  and  $q_t$  denote the public beliefs about the good-technology and false-positive states. The distribution of private signals is assumed to be Gaussian, centered on  $\theta + \xi$  with standard deviation  $\sigma_s$ . To set the dispersion  $\sigma_s$ , we target the average dispersion of growth forecasts in the SPF over 1995Q4-2001Q1, which yields  $\sigma_s = 0.156$ . Finally, we must assume a distribution for the fraction of noise traders that adopt the new technology with support over  $[0, 1]$ . We choose a beta distribution with parameters (2,2).<sup>19</sup>

Five parameters remain to calibrate for which there does not exist widely accepted estimates or

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<sup>18</sup>The point estimate  $\theta_L < A^o$  is not to be interpreted literally as implying that IT technologies are unproductive *per se*, but merely that the market was not fully ripe in 2000 for an economy-wide adoption due to temporary disruption costs (habits, processes) and because other complementary investments (e.g., infrastructure, workforce quality), not modeled here, had not taken place yet. See Appendix B.4 for more details about how we calibrate  $\theta_L$  and  $\theta_H$ .

<sup>19</sup>The distribution Beta(1,1) is uniform and produces a flat learning response. As a result, we pick a Beta(2,2) distribution which is symmetric around its mode of 0.5. This assumption is relatively unimportant.



| Parameter     | Value                              | Target  |
|---------------|------------------------------------|---|
| $\omega_o$    | 0.11                               | Share of IT capital 1991 (3%)                   |
| $\omega_n$    | 0.26                               | Share of IT capital 2007 (14%)                  |
| $\lambda$     | 1/22                               | Duration of NASDAQ boom-bust 1995Q4-2001Q1      |
| $\theta_H$    | 1.099                              | SPF highest growth forecast over 1998-2001      |
| $\theta_L$    | 0.912                              | SPF lowest growth forecast over 1998-2001       |
| $s_j$         | $\mathcal{N}(\theta + \xi, 0.156)$ | SPF avg. dispersion in forecasts over 1998-2001 |
| $\varepsilon$ | Beta(2, 2)                         | Non-uniform distribution over [0, 1]            |
| $\mu$         | 15%                                | Fraction of noise traders                       |
| $p_0$         | 0.20                               | Prior on the “good technology” state            |
| $q_0$         | 0.15                               | Prior on the “false positive” state             |
| $\bar{\xi}$   | $0.95(\theta_h - \theta_l)$        | See text  |
| $\sigma^u$    | $3 \times \sigma^s$                | See text  |

Table 2: Dot-Com episode related parameters

natural targets. The first one is the fraction of noise entrepreneurs  $\mu$ , which controls the informativeness of the social learning channel. While some estimates exist in the literature regarding the informativeness of markets (see for instance [David et al., 2016](#)), these estimates do not cover social learning about new technologies. We conduct sensitivity analysis on this parameter but start with a benchmark value of  $\mu = 15\%$ . We must also specify the priors  $p_0$  and  $q_0$  that agents associate with the states of the world “good technology” and “false positive”. There is no obvious moment that we can target for these parameters given that we focus on one historical episode. We start with a benchmark parametrization that assigns relatively low values to those parameters and do some robustness analysis in the appendix. Our benchmark parametrization attempts to capture the idea that if new technologies can be invented frequently, only few of them lead to deep economic transformations like the ones considered in this paper. This suggests a small value for  $p_0$ . Similarly, the false-positive signal  $\xi$  cannot happen too frequently, otherwise agents would distrust their private signals and the boom-bust cycles would never arise. As a benchmark, we therefore pick  $p_0 = 0.2$  and  $q_0 = 0.15$  but we show in [Appendix B](#) that boom-bust episodes are robust to some variation in these values. We must also set a value for the common noise term  $\bar{\xi}$ . Again, neither the literature nor the data provides much guidance. Since one of our goals is to evaluate the potential of the model in generating boom-bust cycles, we pick a relatively large shock and set  $\bar{\xi} = 0.95(\theta_H - \theta_L)$ . Finally, we need a value for the exogenous public signal about  $\theta$ . For the mechanism to operate, we need that signal to be mostly uninformative, otherwise agents know the true value of  $\theta$  and boom-bust cycles can obviously never arise. We set  $\sigma_u = 3 \times \sigma_s$  in the benchmark simulations so that a private signal has about the same information as three exogenous public signals. [Appendix](#)

B conducts sensitivity analysis over all the parameters mentioned in this paragraph. The results are fairly robust.

## 5.2 Boom-and-Bust Cycles

Our interpretation of the dot com bubble is that the economy was in the false-positive state, and that investors' optimism about IT technologies triggered the initial boom. As more information became available, agents became pessimistic, and the bust followed.<sup>20</sup> In line with this interpretation, Figure 8 presents the impulse responses of our model economy in the false-positive state with all the other shocks are set to zero, that is  $\varepsilon_t = 0$  and  $u_t = 0$  for all  $t$ . We also assume that the technology never matures through the  $\lambda$  shock, so that the bust that we observe is purely endogenous.<sup>21</sup>

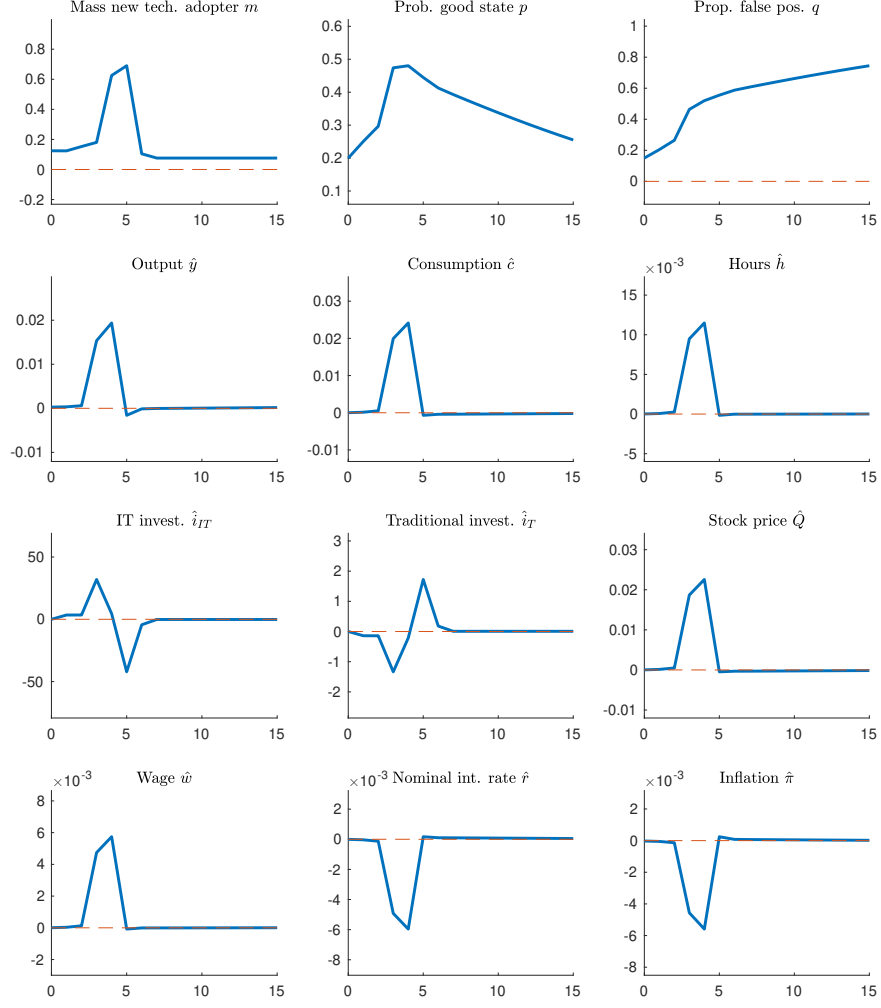
In period  $t = 0$ , a new technology is discovered and entrepreneurs receive encouraging private signals about the true value of that technology. Since the false-positive state is initially deemed unlikely, entrepreneurs begin to adopt the new technology and the economy goes through a growth phase with  $m_t$  moving upward. While this growth in  $m_t$  is initially consistent with both the “good technology” and the “false-positive” state, there comes a point at which agents start to realize that the data is more consistent with  $\theta = \theta_L$ , and the likelihood  $p_t$  starts to decline. As a result, the mass of entrepreneurs who adopt the new technology collapses around  $t = 5$ , pushing the economy into a crash.

While the behavior of the mass of adopters  $m_t$  is similar to what we have observed in the simplified model of Section 3.1, Figure 8 shows how this pattern and the evolution of beliefs translate to other macroeconomic variables. As agents become more optimistic after observing people rushing to adopt the new technology, the household anticipates higher productivity growth in the future and higher income, resulting in upward pressure on consumption due to a positive income effect. With expectations of higher productivity from the new technology, the demand for IT capital rises and the household responds by increasing IT investment. The new technology being less intensive in the other form of capital, the demand for traditional capital falls and so does traditional investment. The rise in consumption and investment in IT capital, despite being accompanied by a moderate decline in traditional investment, contribute to an overall rise in aggregate demand. Price rigidities play an important role in turning this surge in demand into a general macroeconomic boom. In a

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<sup>20</sup>Ofek (2002) looks at the earnings trajectory that would have been required to justify the stock valuation of internet companies in early 2000. He finds that, according to a standard asset pricing model, the earnings growth of internet companies would have had to exceed 40% for a decade to justify their valuations. These large growth rates did not materialize. This analysis is consistent with investors having a very optimistic view about the fundamental that turned out to be mistaken, in line with our false-positive interpretation.

<sup>21</sup>An alternative view of the data is that the bust was triggered by the technology maturing. While we cannot completely rule out that story, we note that measures of labor productivity in the technology industry have kept increasing through the bust, which pushes against a scenario in which the IT technology would be revealed to be of low productivity. Also, the literature does not seem to agree, even roughly, on any given event (a potential  $\lambda$  shock) that could have triggered the bust.



Notes: The impulse responses are reported in log-deviations from the initial non-stochastic steady state. All shocks after the realization of  $(\theta, \xi)$  are set to 0 ( $\varepsilon_t = 0$  and the technology never matures exogenously).

Figure 8: Impulse response in the false-positive state of the world

real business cycle model, the rise in aggregate demand should be offset by a sharp rise in the real interest rate. With sticky prices, the interest rate response is muted if the monetary authority is sufficiently accommodative. As a result, aggregate demand remains high. Firms, satisfying demand, respond by raising output and employment. Because of a higher labor demand, wages increase, but inflation remains low because firms anticipate greater productivity and lower marginal costs in the future. As evidenced by the variable  $\hat{Q}$ , which captures the value of the firms, the economy also experiences a stock market boom along the expansion.

These dynamic effects are reversed when the crash occurs and agents realize that the new technology is actually of low quality. While agents abandon the new technology, a recession occurs, with GDP falling below trend, because agents wake up after having invested too much in IT capital and not enough in traditional capital. This misallocation of resources, combined with a negative

income effect, is the essential ingredient that puts downward pressure on aggregate demand and pushes the economy below the trend in the recovery.

A few comments are in order at this point. First, while the model is able to generate a recession with a significant peak-to-trough gap (about 2%), it remains smaller than the one in the data (about 3%). This result seems, however, a feature of belief-driven cycles that our model shares with most of the news/noise-driven business cycle literature. Second, while the model is able to generate a recession with output falling below trend, that effect is somewhat weak in our simulation. Given that the capital stocks can be adjusted rapidly, the misallocation channel responsible for the dip does not lead to a large drop in output. The introduction of debt and bankruptcy in the model would provide another channel through which the crash could result in a deeper recession. Third, we can also compute the frequency at which boom-and-bust cycles arise in our model. While the existing consensus is that such cycles are rare in models with rational agents,<sup>22</sup> our benchmark calibration suggests that boom-bust cycles may arise in our calibrated model at the fairly high frequency of  $q_0 = 15\%$  after the introduction of a new technology. We view this number as quite encouraging for the ability of rational herding models in explaining the data.

Overall, the impulse responses of Figure 8 suggest that it is possible to generate a realistic macroeconomic boom-bust cycle that is entirely driven by the internal forces of the model—without the need of an exogenous shock to trigger the bust after the initial introduction of a new technology.<sup>23</sup>

## 6 Conclusion

This paper explores whether rational herding can generate endogenous business cycle fluctuations. We propose a novel theory of herding which captures many essential features of more traditional models (Banerjee, 1992; Bikhchandani et al., 1992; Chamley, 2004), while being tractable enough to be embedded into a general equilibrium business cycle framework. We show that the model is able to endogenously generate a boom-and-bust pattern without the need for a particular sequence of shocks. Our model has predictions on the frequency, the timing and the conditions under which such cycles emerge or burst. It can thus be used to analyze the role of stabilization policy, including investment-specific taxes or monetary policy.

We have restricted our attention to technology-driven boom-and-bust cycles, but the implications of the theory go beyond this context and we believe our herding model can be used in other environments to analyze herding behavior following any sort of innovation, be it financial innovations or innovations to the demand for certain types of goods (new products, housing, etc).

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<sup>22</sup>Chamley (2004) suggests that a boom-bust cycles arises with a probability of  $10^{-6}$  in the traditional model of herding of Avery and Zemsky (1998).

<sup>23</sup>In Appendix B.5, we investigate the role of lean-against-the-wind monetary policy in this economy and find that it only has a limited impact on boom-bust cycles.

Several extensions are worth investigating. First, our current macroeconomic model ignores the role of debt. An interesting extension would be to study how the rising pattern of optimism during the growth stage of the cycle could relax financial constraints and lead to an expansion in credit, triggering a wave of bankruptcies at the time of the crash. Another natural extension would be to consider a financial market application of our herding model and examine, in particular, the role of speculation. We leave these ideas to future research.

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## A Appendix of Section 2

### A.1 Equations for the three-state model

This section provides the specific model equations that characterize beliefs in the three-state model. Equation (3) that builds private beliefs from the public ones becomes

$$\begin{aligned} p_{jt} &= p_j(p_t, q_t, s_j) = \frac{p_t f_{\theta_H}^s(s_j)}{p_t f_{\theta_H}^s(s_j) + q_t f_{\theta_L + \bar{\xi}}^s(s_j) + (1 - p_t - q_t) f_{\theta_L}^s(s_j)}, \\ q_{jt} &= q_j(p_t, q_t, s_j) = \frac{q_t f_{\theta_L + \bar{\xi}}^s(s_j)}{p_t f_{\theta_H}^s(s_j) + q_t f_{\theta_L + \bar{\xi}}^s(s_j) + (1 - p_t - q_t) f_{\theta_L}^s(s_j)}. \end{aligned} \quad (18)$$

Equation (9) that defines the interim beliefs after observing  $S_t$  is simply

$$\begin{aligned} p_{t|R_t} &= \frac{p_t f^u(S_t - \theta_H)}{p_t f^u(S_t - \theta_H) + (1 - p_t) f^u(S_t - \theta_L)}, \\ q_{t|R_t} &= \frac{q_t f^u(R_t - \theta_L)}{p_t f^u(S_t - \theta_H) + (1 - p_t) f^u(S_t - \theta_L)}. \end{aligned}$$

Finally, in the three state model, the optimal adoption strategy characterized by Equation (10) that defines the law of motion of beliefs after observing  $m_t$  becomes

$$\begin{aligned} p_{t+1} &= \frac{p_{t|R_t} f^\varepsilon((m_t - (1 - \mu) \bar{F}_{\theta_H}^s(s^*(p_t, q_t))) / \mu)}{denom_t}, \\ q_{t+1} &= \frac{q_{t|R_t} f^\varepsilon((m_t - (1 - \mu) \bar{F}_{\theta_L + \bar{\xi}}^s(s^*(p_t, q_t))) / \mu)}{denom_t}, \end{aligned}$$

where

$$\begin{aligned} denom_t &= p_{t|R_t} f^\varepsilon((m_t - (1 - \mu) \bar{F}_{\theta_H}^s(s^*(p_t, q_t))) / \mu) + q_{t|R_t} f^\varepsilon((m_t - (1 - \mu) \bar{F}_{\theta_L + \bar{\xi}}^s(s^*(p_t, q_t))) / \mu) \\ &\quad + (1 - p_{t|R_t} - q_{t|R_t}) f^\varepsilon((m_t - (1 - \mu) \bar{F}_{\theta_L}^s(s^*(p_t, q_t))) / \mu). \end{aligned}$$

### A.2 Propositions

**Proposition 1.** *There exists a unique equilibrium.*

*Proof.* The threshold  $p^*$  is uniquely determined by (6). The result is established recursively. Fix the fundamental  $(\theta, \xi)$  and the realization of the shocks  $\{u_0, \varepsilon_0, u_1, \varepsilon_1, \dots\}$ . At any date  $t$ , given public beliefs  $\Lambda_t$ , (3) and (5) yield a unique distribution of private beliefs  $\{\Lambda_{jt}\}_{j \in [0,1]}$  and  $\{p_{jt}\}_{j \in [0,1]}$ . Given these, under the tie-breaking rule that indifferent agents adopt the technology, there is a unique  $m_t^\varepsilon$  derived from (8) and, therefore a unique  $m_t$  from (7). As a result, updating beliefs through (9) and (10) yields unique  $\Lambda_{t|R}$  and  $\Lambda_{t+1}$ . We have shown that the updating of public beliefs yields a unique  $\Lambda_{t+1}$  from  $\Lambda_t$  and the realization of shocks  $\{u_t, \varepsilon_t\}$ . Starting from public

beliefs  $\Lambda_0$ , there is therefore a unique equilibrium path  $\{\Lambda_0, \Lambda_1, \dots\}$  for any history of shocks, and all other quantities can be uniquely determined from it.  $\square$

**Lemma 1.** *In the three-state model, for  $\theta_L < \theta_L + \bar{\xi} < \theta_H$  and  $\{F_x^s\}$  satisfying the MLRP condition, the optimal adoption strategy is characterized by a cutoff rule in the private signal  $s^*(p_t, q_t)$ , decreasing in  $p_t$ . That is, an agent adopts the technology if and only if  $s_j \geq s^*(p_t, q_t)$ . The measure of rational adopters is given by*

$$m^e(p_t, q_t, \theta, \xi) = (1 - \mu) \bar{F}_{\theta+\xi}^s(s^*(p_t, q_t)).$$

*Proof.* The proof is straightforward. Under the above conditions, rewrite the individual probability of the good-technology state as

$$p_j(p_t, q_t, s_j) = \frac{p_t}{p_t + q_t \frac{f_{\theta_L + \bar{\xi}}^s(s_j)}{f_{\theta_H}^s(s_j)} + (1 - p_t - q_t) \frac{f_{\theta_L}^s(s_j)}{f_{\theta_H}^s(s_j)}}.$$

Under the assumption of MLRP and  $\theta_L < \theta_L + \bar{\xi} < \theta_H$ ,  $p_j$  is clearly increasing in  $s_j$ . Hence, for all  $(p_t, q_t)$ , there exists a cutoff  $s^*(p_t, q_t) \in \mathbb{R} \cup \{-\infty, \infty\}$  such that  $s_j \geq s^*(p_t, q_t) \Leftrightarrow p_j(p_t, q_t, s_j) \geq p^*$ . Also, because  $p_j$  is increasing in  $p_t$ , the implicit function theorem ensures that  $s^*(p_t, q_t)$  is decreasing in  $p_t$ . The measure of rational adopters is thus

$$m^e(p_t, q_t, \theta, \xi) = (1 - \mu) \int \mathbb{I}(p_j(p_t, q_t, s_j) \geq p^*) f_{\theta+\xi}^s(s_j) ds_j = (1 - \mu) \bar{F}_{\theta+\xi}^s(s^*(p_t, q_t)).$$

$\square$

**Proposition 2.** *In the Gaussian case, i.e.,  $F^\xi \sim \mathcal{N}(0, \sigma_\xi^2)$ ,  $F^s | \theta, \xi \sim \mathcal{N}(\theta + \xi, \sigma_s^2)$ ,  $F^\varepsilon \sim \mathcal{N}(0, \sigma_\varepsilon^2)$ ,  $F^u \sim \mathcal{N}(0, \sigma_u^2)$ , for  $\theta$  and  $\xi$  independent and signal  $S_t$  sufficiently uninformative ( $\sigma_u$  low), there exists a large enough  $\underline{\xi}$  such that all shocks  $\xi \geq \underline{\xi}$  generate a boom-and-bust cycle in the impulse response of beliefs  $p_t$  to a false-positive shock  $(\theta_L, \xi)$ .*

*Proof.* Our strategy is to show that there exists a sufficiently large  $\underline{\xi}$ , such that for all shock  $\xi \geq \underline{\xi}$  the public beliefs about the good state in date 1,  $p_1$ , increases after observing  $m_0$ . Since beliefs must converge to the truth in the long-run ( $p_t \rightarrow 0$ ), due to the strictly positive flow of information, and the law of large numbers, this guarantees the existence of a boom-and-bust cycle in beliefs. We start under the assumption that  $S_t$  is totally uninformative,  $\sigma_u = \infty$ .

First, we establish that the optimal strategy in the Gaussian case follows a cutoff strategy in  $s^*$ . The probability that individual  $j$  puts on the good state is given by

$$p_j(p_0, s_j) = \frac{\int \Lambda_0(\theta_H, \xi) f_{\theta_H+\xi}^s(s_j) d\xi}{\int \Lambda_0(\theta_H, \xi) f_{\theta_H+\xi}^s(s_j) d\xi + \int \Lambda_0(\theta_L, \xi) f_{\theta_L+\xi}^s(s_j) d\xi}.$$

Since  $\xi$  is independent from  $\theta$ ,  $\Lambda_0(\theta_H, \xi) = p_0 f^\xi(\xi)$  and  $\Lambda_0(\theta_L, \xi) = (1 - p_0) f^\xi(\xi)$ . Notice, then, that  $\int f^\xi(\xi) f_{\theta+\xi}^s(s_j) d\xi$  is the pdf of  $s_j$  given  $\theta$ , which is a normal,  $s_j|\theta \sim \mathcal{N}(\theta, \sigma_\xi^2 + \sigma_s^2)$ . Denote  $\phi$  the pdf of a unit normal, we have:

$$p_j(p_0, s_j) = \frac{1}{1 + \frac{(1-p_0) \int f^\xi(\xi) f_{\theta_L+\xi}^s(s_j) d\xi}{p_0 \int f^\xi(\xi) f_{\theta_H+\xi}^s(s_j) d\xi}} = \frac{1}{1 + \frac{1-p_0}{p_0} \phi\left(\frac{s_j - \theta_L}{\sqrt{\sigma_\xi^2 + \sigma_s^2}}\right) / \phi\left(\frac{s_j - \theta_H}{\sqrt{\sigma_\xi^2 + \sigma_s^2}}\right)}.$$

Since the Gaussian family satisfies the MLRP property,  $p_j$  is increasing in  $s_j$ . Hence, the optimal adoption strategy at date 0 takes a cutoff form  $\hat{s}_0$ .

Under the assumption that  $S_t$  is uninformative, the public belief about the good state at the beginning of period 1,  $p_1$ , is given by

$$\begin{aligned} p_1 &= \int \Lambda_1(\theta_H, \xi) d\xi \\ &= \frac{\int \Lambda_0(\theta_H, \xi) f^\varepsilon((m_0 - m^e(\Lambda_0, \theta_H, \xi))/\mu) d\xi}{\int \Lambda_0(\theta_H, \xi) f^\varepsilon((m_0 - m^e(\Lambda_0, \theta_H, \xi))/\mu) d\xi + \int \Lambda_0(\theta_L, \xi) f^\varepsilon((m_0 - m^e(\Lambda_0, \theta_L, \xi))/\mu) d\xi}. \end{aligned}$$

Using the independence property between  $\theta$  and  $\xi$  and the cutoff property, the above formula can be rewritten as

$$p_1 = \frac{1}{1 + \frac{1-p_0}{p_0} \frac{\int f^\xi(\xi) f^\varepsilon((m_0 - (1-\mu)\bar{F}_{\theta_L+\xi}^s(s_0^*))/\mu) d\xi}{\int f^\xi(\xi) f^\varepsilon((m_0 - (1-\mu)\bar{F}_{\theta_H+\xi}^s(s_0^*))/\mu) d\xi}}.$$

Denoting  $\xi_0$  the true shock, the impulse response in  $m_t$  yields  $m_0 = (1 - \mu) \bar{F}_{\theta_L+\xi_0}^s(s_0^*)$ , which goes to  $1 - \mu$  as  $\xi_0 \rightarrow \infty$ . Because the MLRP property implies first-order stochastic dominance, we have  $\bar{F}_{\theta_L+\xi}^s(s_0^*) < \bar{F}_{\theta_H+\xi}^s(s_0^*)$ . Since  $f^\varepsilon(\varepsilon)$  is decreasing for  $\varepsilon \geq 0$ , we have

$$f^\varepsilon((m_0 - (1 - \mu) \bar{F}_{\theta_L+\xi}^s(s_0^*))/\mu) < f^\varepsilon((m_0 - (1 - \mu) \bar{F}_{\theta_H+\xi}^s(s_0^*))/\mu)$$

for all  $\xi \leq \theta_L - \theta_H + \xi_0$ . Decompose the difference between the denominator and numerator can be written

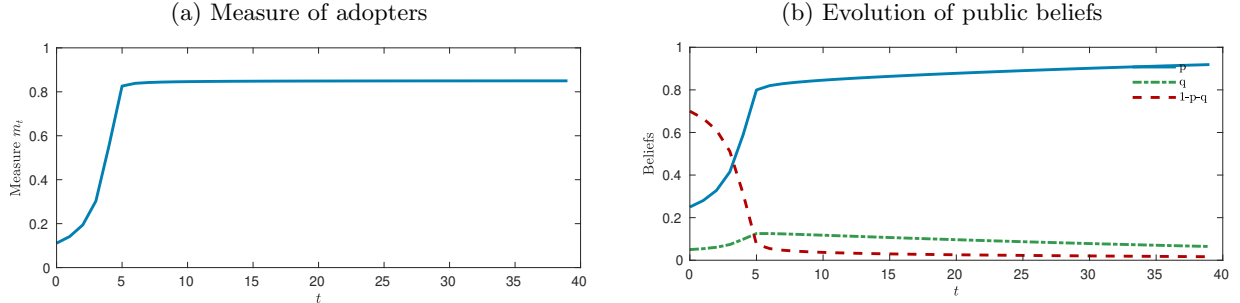
$$\begin{aligned} &\int f^\xi(\xi) f^\varepsilon((m_0 - (1 - \mu) \bar{F}_{\theta_H+\xi}^s(s_0^*))/\mu) d\xi - \int f^\xi(\xi) f^\varepsilon((m_0 - (1 - \mu) \bar{F}_{\theta_L+\xi}^s(s_0^*))/\mu) d\xi \\ &\xrightarrow{\xi_0 \rightarrow \infty} \int_{-\infty}^{\infty} f^\xi(\xi) [f^\varepsilon((1 - \mu - (1 - \mu) \bar{F}_{\theta_H+\xi}^s(s_0^*))/\mu) - f^\varepsilon((1 - \mu - (1 - \mu) \bar{F}_{\theta_L+\xi}^s(s_0^*))/\mu)] d\xi > 0 \end{aligned}$$

The difference converges to a strictly positive term. Thus, there exists  $\underline{\xi}$  such that for all  $\xi_0 > \underline{\xi}$

$$\begin{aligned} &\int f^\xi(\xi) f^\varepsilon(((1 - \mu) \bar{F}_{\theta_L+\xi_0}^s(s_0^*) - (1 - \mu) \bar{F}_{\theta_L+\xi}^s(s_0^*))/\mu) d\xi \\ &< \int f^\xi(\xi) f^\varepsilon(((1 - \mu) \bar{F}_{\theta_L+\xi_0}^s(s_0^*) - (1 - \mu) \bar{F}_{\theta_H+\xi}^s(s_0^*))/\mu) d\xi \end{aligned}$$

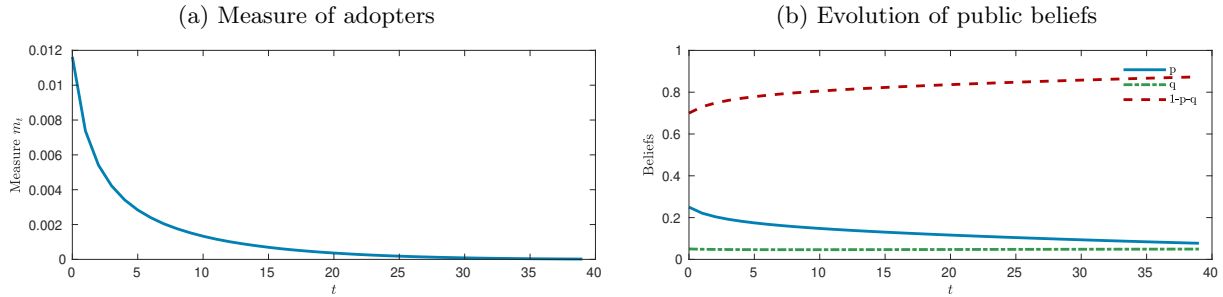
and  $p_1 > p_0$ . The shock is large enough for agents to attribute it mostly to the good state, initiating the growth stage of the cycle. By continuity of the belief updating equations in  $\sigma_u$ . There must also exist a sufficiently large  $\sigma_u$  ( $S_t$  sufficiently uninformative) for which  $p_1 > p_0$  after  $\xi_0 \geq \underline{\xi}$ .  $\square$

### A.3 Additional figures



Notes: The simulation was performed with parameters:  $A^o = 1$ ,  $\theta_H = 1.25$ ,  $\theta_L = 0.6$ ,  $\bar{\xi} = 0.55$ ,  $\mu = 0.15$ . The priors are set to  $p_0 = 0.25$  and  $q_0 = 0.05$ . The distributions of all signals are Gaussian with standard deviations  $\sigma_s = 0.6$ ,  $\sigma_\varepsilon = 1$ ,  $\sigma_u = 3$ .

Figure 9: Impulse response in the case of a true positive



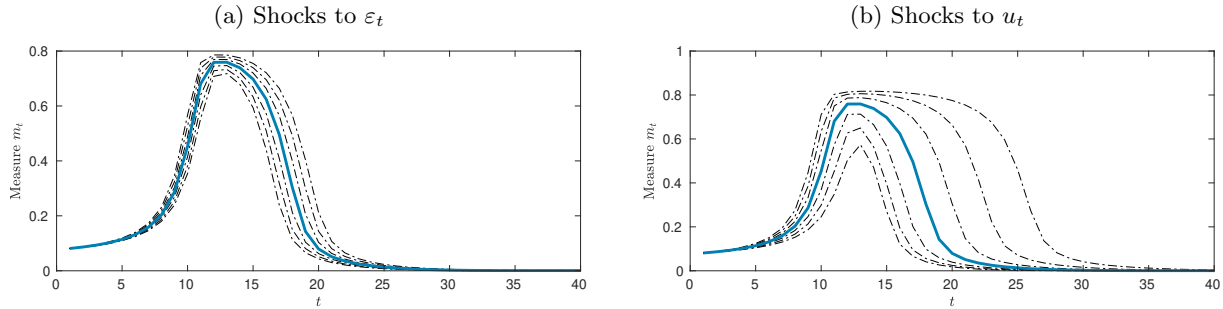
Notes: The simulation was performed with parameters:  $A^o = 1$ ,  $\theta_H = 1.25$ ,  $\theta_L = 0.6$ ,  $\bar{\xi} = 0.55$ ,  $\mu = 0.15$ . The priors are set to  $p_0 = 0.25$  and  $q_0 = 0.05$ . The distributions of all signals are Gaussian with standard deviations  $\sigma_s = 0.6$ ,  $\sigma_\varepsilon = 1$ ,  $\sigma_u = 3$ .

Figure 10: Impulse response in the case of a true negative

### A.4 The role of random shocks

To highlight the dynamics of the model, the simulations presented in Figures 3 and 4 assumed that there were no shocks to the public signal  $S_t$  ( $u_t = 0$ ) and no shock coming from the noise entrepreneurs ( $\varepsilon_t = 0$ ). But these random shocks, by influencing the signals that agents observe, can also play an important role in driving aggregate adoption. Because single shocks do not have much effect, we illustrate this by conducting a series of simulations using the same economy as in Figure 3 but in which we fix the shocks to some number  $\bar{x}$ . As a result, we can see how the

economy evolves when agents continuously receive optimistic signals and, inversely, when they get a constant flow of pessimistic news. We plot the results in Figure 11. In the left panel we set  $u_t = 0$  and  $\varepsilon_t = \bar{x}$  and vary  $\bar{x}$  from  $-0.005$  to  $0.005$ . In the right panel we set  $\varepsilon_t = 0$  and  $u_t = \bar{x}$  and vary  $\bar{x}$  from  $-0.1$  to  $0.1$ . The blue lines represent the simulation with  $\bar{x} = 0$ , and the dashed lines represent simulations with  $\bar{x} \neq 0$ . As we can see from the figure, there is quite a bit of dispersion across simulations and shocks to  $u_t$  and  $\varepsilon_t$  can push the economy through very different dynamics. For the more optimistic signals  $(u_t, \varepsilon_t)$ , the economy enters an information cascade and adoption remains high for a sustained period. In contrast, for the more pessimistic signals, adoption slowly declines and we never observe a boom-bust cycle. Notice also that shocks to  $\mu_t$  and  $\varepsilon_t$ , as they both influence the overall level of optimism in the economy, have similar effects on the dynamic of adoption.<sup>24</sup>



Notes: Simulations performed with parameters:  $A^o = 1$ ,  $\theta_H = 1.25$ ,  $\theta_L = 0.6$ ,  $\bar{\xi} = 0.53$ ,  $\mu = 0.15$ . The priors are set to  $p_0 = 0.25$  and  $q_0 = 0.05$ . The distributions of all signals are Gaussian with standard deviations  $\sigma_s = 0.6$ ,  $\sigma_\varepsilon = 1$ ,  $\sigma_u = 3$ . Left panel: the signals are  $u_t = 0$  and  $\varepsilon_t = \bar{x}$  and we vary  $\bar{x}$  from  $-0.005$  (lowest line) to  $0.005$  (highest line) in seven increments. Right panel: the signals are  $\varepsilon_t = 0$  and  $u_t = \bar{x}$  and we vary  $\bar{x}$  from  $-0.1$  (lowest line) to  $0.1$  (highest line) in seven increments.

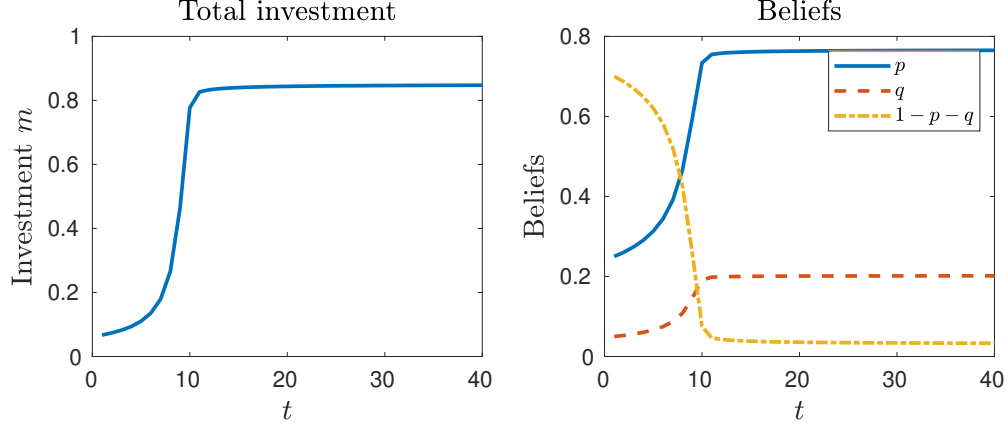
Figure 11: The role of  $u_t$  and  $\varepsilon_t$  in driving aggregate adoption

## A.5 Permanent information cascades

In this appendix, we show that a slight modification of the environment considered in Figure 4 in the main text can lead to a permanent information case in which no information is provided by the observation of the mass of adopters. We depart from that specification by imposing a lower bound  $\underline{s} = 0$  on the distribution of private signal and by making the exogenous public signal uninformative ( $\sigma_u = \infty$ ). Figure 12 shows the behavior of the economy in the false-positive state under this new parametrization. As we can see, the economy quickly converges to a permanent plateau at  $m = 1$ . The public beliefs are so optimistic at that point that even the most pessimistic agent ( $s_j = 0$ ) adopts the technology. Since all agents adopt regardless of the true state of the world, observing the mass of adopters provides no information and the economy remains in a permanently elevated

<sup>24</sup>Some of the curves in Figure 11 intersect each other. For instance, in the case with  $\bar{x} = 0$  agents learn fairly slowly about the true state of the world. In contrast, in the simulation with a slightly higher  $\bar{x}$  the behavior of aggregate adoption leads to a high flow of information and agents quickly learn that the true fundamental is bad.

state of adoption.



Notes: The simulation was performed with parameters:  $A^o = 1$ ,  $\theta_H = 1.25$ ,  $\theta_L = 0.6$ ,  $\bar{\xi} = 0.55$ ,  $\mu = 0.15$ . The priors are set to  $p_0 = 0.25$  and  $q_0 = 0.05$ . The distributions of all signals are Gaussian with standard deviations  $\sigma_s = 0.6$ ,  $\sigma_\varepsilon = 1$ ,  $\sigma_u = \infty$ . The private signal distribution is bounded below by zero.

Figure 12: Impulse response in the false-positive state with permanent information cascades

## A.6 Optimal taxation

To gain further insight into the nature of the model's inefficiencies, it is useful to look at a particular implementation of the efficient allocation using a tax (or subsidy)  $\tau^*$  that entrepreneurs must pay to adopt the new technology. The next proposition characterizes  $\tau^*$ .

**Proposition 3.** *The efficient allocation can be implemented as an equilibrium by an adoption tax*

$$\tau^* = \left( E_{\theta, \xi} \left[ f_{\theta+\xi}^p(p^*) \mid \mathcal{I} \right] \right)^{-1} \beta \frac{\partial E_{\theta, \xi} [V(\mathcal{I}') \mid \mathcal{I}]}{\partial p^*}, \quad (19)$$

and a lump-sum transfer to all entrepreneurs.

*Proof.* We consider a tax  $\tau$  that agents must pay to adopt the new technology. Under that tax, (6) shows that the marginal adopter  $p^*$  is such that  $\lambda(p^*\theta_H + (1-p^*)\theta_L) + (1-\lambda)A^o - \tau = A^o$ . Combining with (12) and reorganizing yields (19).  $\square$

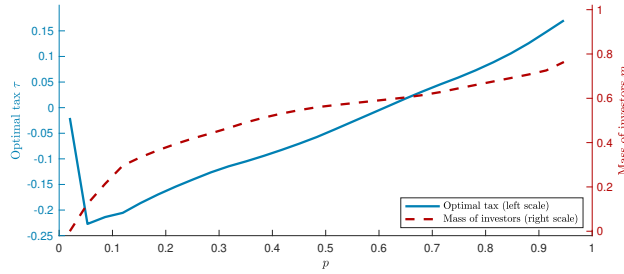
The optimal tax  $\tau^*$  balances the distortion in adoption it creates (first term in the product) with the potential benefit on information acquisition (second term).

We plot in Figure 13 how this optimal tax varies with the public beliefs  $p$  in the example of Figure 7.<sup>25</sup> We see that the tax tends to be negative for low values of  $p$  and positive for larger values. As we described in Section 3.1, when agents are pessimistic (low  $p$ ), few of them adopt the technology and the endogenous public signal does not reveal much information. The planner therefore sets

<sup>25</sup>To draw this plot, we fix  $q$  to some arbitrary value  $q = 0.01$  and plot  $m$  in the good-technology state.

$\tau^* < 0$  to encourage entry and make the observed mass of adopters a more precise signal. The opposite happens when many entrepreneurs adopt (high  $p$ ). In this case, the planner sets  $\tau^* > 0$  to discourage adoption, once again to make the endogenous public signal more informative. The tax for intermediate values of  $p$  reflects these information concerns.<sup>26</sup>

In this particular Gaussian case, the tax incentivizes agents to behave against the crowd in what amounts to a *leaning-against-the-wind* pattern: the tax is negative when no agent wants to adopt the technology, and positive when agents adopt massively.<sup>27</sup> We will see in our quantitative model that these same forces have important consequences for the conduct of monetary policy.



Notes: Mass of adopters and optimal tax as a function of  $p$  with the same parameters as Figure 7 and with  $\beta = 0.9$  and  $\lambda = 0.9$ . These curves are generated by fixing  $q = 0.01$ .  $m$  is computed as  $(1 - \mu) \bar{F}_{\theta_H}(\hat{s}(p, q))$ .

Figure 13: Optimal tax as a function of  $p$

<sup>26</sup>The planner begins to phase out the tax as  $p$  approaches 0. In this case, the public beliefs are so extreme that there is very little uncertainty about the true state of the world. Since there is not much more to learn, the planner sets the tax close to zero to minimize the distortion in adoption.

<sup>27</sup>In the more general case, the optimal policy that maximizes information collection may look more complicated. It remains true, however, that the planner always has an incentive to lean against the market for extreme public beliefs ( $p_t$  high or low) as an information cascade occurs and  $\bar{F}_{\theta+\xi}^s(s^*)$  goes to 0 or 1.

## B Online Appendix of Section 5

This appendix contains additional details about the full quantitative model, our resolution method, the calibration as well as exercises to test the robustness of the simulations of Figure 8.

### B.1 Equilibrium definition

In what follows, to lighten notation, we denote by  $\Omega_t^A$  the aggregate state space at the beginning of a period upon making the technology adoption decision (stage A of the timing) and  $\Omega_t^B$  the aggregate state space at the end of the period when consumption, production and market clearing take place (stage B). Before maturation of the technology, the state space that describes the information set of the household and any participant that is not an entrepreneur is  $\Omega_t^A = (K_t^{IT}, K_t^T, \Lambda_t)$ . The individual state space that describes the information set of entrepreneurs at stage A is  $(\Omega_t^A, s_j)$ . Finally, the aggregate state space that describes the information set at stage B is  $\Omega_t^B = (K_t^{IT}, K_t^T, \Lambda_t, m_t, S_t)$ . After maturation, all the uncertainty related to the technology is resolved, so the aggregate state spaces can be described by  $\Omega_t^A = (K_t^{IT}, K_t^T, \theta)$  and  $\Omega_t^B = (K_t^{IT}, K_t^T, \theta, m_t)$ .

We are now ready to define an equilibrium for the full quantitative model. All the model's equations that the definition refers to have been compiled in subsection B.2.

**Definition 2.** An equilibrium is a set of a) entrepreneur  $j$ 's technology adoption decision  $i_{jt}(\Omega_t^A, s_j)$ , b) a measure of new technology adopters  $m_t(\Omega_t^A, \theta, \xi, \varepsilon_t)$ , c) household decisions  $\{C_t, L_t, I_t^{IT}, I_t^T, K_{t+1}^{IT}, K_{t+1}^T, B_t\}$  which are functions of  $\Omega_t^B$ , d) firm decisions  $\{Y_t, Y_t^w, Y_t^n, Y_t^o, Y_{jt}^r, L_t^n, L_t^o, K_t^n, K_t^o, K_t^{n,IT}, K_t^{n,T}, K_t^{o,IT}, K_t^{o,T}, \Pi_t^{IT}, \Pi_t^T\}$  which are functions of  $\Omega_t^B$ , e) prices  $\{P_t, P_t^w, P_t^n, P_t^o, w_t, R_t, \pi_t, z_t^n, z_t^o, z_t^{IT}, z_t^T, \mu_t\}$  which are functions of  $\Omega_t^B$  for all  $t$  such that

1. The entrepreneurs' technology adoption decision and the measure of new technology adopters solve (42)-(43);
2. The household decisions satisfy the household's optimality conditions and constraints (26)-(32);
3. The firm's decisions satisfy the its optimality conditions (33)-(41) and the production functions (20)-(25);
4. Prices satisfy (49)-(49), the Phillips curve (56), the (15) and the market clearing constraints (44)-(48).

### B.2 Model equations

We provide below all the 37 nonlinear equations that characterize the full business cycle model (36 unknowns).<sup>28</sup>

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<sup>28</sup>One equation is redundant because of Walras' Law, but we keep it in the equilibrium definition for the sake of completeness.



## 1. Production functions and capital bundles

$$Y_t^n = A_t^n (K_t^n)^\alpha (L_t^n)^{1-\alpha} \quad (\text{individual production new technology}) \quad (20)$$

$$Y_t^o = A^o (K_t^o)^\alpha (L_t^o)^{1-\alpha} \quad (\text{individual production old technology}) \quad (21)$$

$$Y_t^w = \left( \int_0^1 m_t (Y_t^n)^{\frac{\sigma-1}{\sigma}} + (1-m_t) (Y_t^o)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (\text{aggregate production wholesale sector}) \quad (22)$$

$$Y_t = \left( \int_0^1 (Y_{jt}^r)^{\frac{\sigma-1}{\sigma}} dj \right)^{\frac{\sigma}{\sigma-1}} \quad (\text{aggregate production final good}) \quad (23)$$

$$K_t^n = \kappa_n \left( (1-\omega_n) \left( K_t^{n,T} \right)^{\frac{\zeta-1}{\zeta}} + \omega_n \left( K_t^{n,IT} \right)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \quad (\text{new tech. capital bundle}) \quad (24)$$

$$K_t^o = \kappa_o \left( (1-\omega_o) \left( K_t^{o,T} \right)^{\frac{\zeta-1}{\zeta}} + \omega_o \left( K_t^{o,IT} \right)^{\frac{\zeta-1}{\zeta}} \right)^{\frac{\zeta}{\zeta-1}} \quad (\text{old tech. capital bundle}) \quad (25)$$

## 2. Household optimality and constraints

$$C_t^{-1} = \beta E_t \left[ C_{t+1}^{-1} \frac{1+R_t}{1+\pi_{t+1}} \right] \quad (\text{Euler eq. nominal bonds}) \quad (26)$$

$$C_t^{-1} = \beta E_t [C_{t+1}^{-1} (z_{t+1}^{IT} + 1 - \delta)] \quad (\text{Euler eq. IT capital}) \quad (27)$$

$$C_t^{-1} = \beta E_t [C_{t+1}^{-1} (z_{t+1}^T + 1 - \delta)] \quad (\text{Euler eq. T capital}) \quad (28)$$

$$L_t^\psi = w_t \quad (\text{labor supply}) \quad (29)$$

$$K_{t+1}^{IT} = (1-\delta) K_t^{IT} + I_t^{IT} \quad (\text{law of motion IT capital}) \quad (30)$$

$$K_{t+1}^T = (1-\delta) K_t^T + I_t^T \quad (\text{law of motion T capital}) \quad (31)$$

$$C_t + \sum_{i=T,IT} I_t^i + \frac{B_t}{P_t} = w_t L_t + \sum_{i=T,IT} z_t^i K_t^i + \frac{1+R_{t-1}}{1+\pi_t} \frac{B_{t-1}}{P_{t-1}} + m_t \Pi_t^{IT} + (1-m_t) \Pi_t^T \quad (\text{budget constraint}) \quad (32)$$

### 3. Firm optimality

$$\left( A_t^n (K_t^n)^\alpha (L_t^n)^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} / L_t^n = \left( A_t^o (K_t^o)^\alpha (L_t^o)^{1-\alpha} \right)^{\frac{\sigma-1}{\sigma}} / L_t^o \quad (\text{wage equalization}) \quad (33)$$

$$\frac{\alpha}{1-\alpha} = \frac{z_t^n K_t^n}{w_t L_t^n} \quad (\text{cost minimization new tech.}) \quad (34)$$

$$\frac{\alpha}{1-\alpha} = \frac{z_t^o K_t^o}{w_t L_t^o} \quad (\text{cost minimization old tech.}) \quad (35)$$

$$K_t^{n,IT} = \kappa_n^{\zeta-1} \omega_n^\zeta \left( \frac{z_t^{IT}}{z_t^n} \right)^{-\zeta} K_t^n \quad (\text{new tech. demand for IT capital}) \quad (36)$$

$$K_t^{n,T} = \kappa_n^{\zeta-1} (1 - \omega_n)^\zeta \left( \frac{z_t^T}{z_t^n} \right)^{-\zeta} K_t^n \quad (\text{new tech. demand for T capital}) \quad (37)$$

$$K_t^{o,IT} = \kappa_o^{\zeta-1} \omega_o^\zeta \left( \frac{z_t^{IT}}{z_t^o} \right)^{-\zeta} K_t^o \quad (\text{old tech. demand for IT capital}) \quad (38)$$

$$K_t^{o,T} = \kappa_o^{\zeta-1} (1 - \omega_o)^\zeta \left( \frac{z_t^T}{z_t^o} \right)^{-\zeta} K_t^o \quad (\text{old tech. demand for T capital}) \quad (39)$$

$$\Pi_t^{IT} = \frac{1}{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} P_t^{1-\sigma} (P_t^w)^\sigma Y_t^w \left( \frac{1}{A_t^n} \left( \frac{z_t^n}{\alpha} \right)^\alpha \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha} \right)^{1-\sigma} \quad (\text{new tech. profits}) \quad (40)$$

$$\Pi_t^T = \frac{1}{\sigma-1} \left( \frac{\sigma}{\sigma-1} \right)^{-\sigma} P_t^{1-\sigma} (P_t^w)^\sigma Y_t^w \left( \frac{1}{A_t^o} \left( \frac{z_t^o}{\alpha} \right)^\alpha \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha} \right)^{1-\sigma} \quad (\text{old tech. profits}) \quad (41)$$

### 4. Technology adoption decisions

$$i_{jt} = \underset{i \in [0,1]}{\operatorname{argmax}} i E [\Pi_t^n | \Omega_t^A, s_j] + (1-i) E [\Pi_t^o | \Omega_t^A, s_j] \quad (\text{optimal technology choice}) \quad (42)$$

$$m_t = \int_0^{1-\mu} i_{jt} dj + \mu \varepsilon_t \quad (\text{measure new tech. adopters}) \quad (43)$$

### 5. Market clearing constraints

$$Y_t = C_t + I_t^{IT} + I_t^T \quad (\text{final good}) \quad (44)$$

$$Y_t^w = \int Y_{jt}^r dj \quad (\text{wholesale good}) \quad (45)$$

$$L_t = m_t L_t^n + (1 - m_t) L_t^o \quad (\text{labor}) \quad (46)$$

$$K_t^{IT} = m_t K_t^{n,IT} + (1 - m_t) K_t^{o,IT} \quad (\text{IT capital}) \quad (47)$$

$$K_t^T = m_t K_t^{n,T} + (1 - m_t) K_t^{o,T} \quad (\text{T capital}) \quad (48)$$

## 6. Prices

$$z_t^n = \frac{1}{\kappa_n} \left( (1 - \omega_n)^\zeta (z_t^T)^{1-\zeta} + \omega_n^\zeta (z_t^{IT})^{1-\zeta} \right)^{\frac{1}{1-\zeta}} \quad (\text{price new tech. capital bundle}) \quad (49)$$

$$z_t^o = \frac{1}{\kappa_o} \left( (1 - \omega_o)^\zeta (z_t^T)^{1-\zeta} + \omega_o^\zeta (z_t^{IT})^{1-\zeta} \right)^{\frac{1}{1-\zeta}} \quad (\text{price old tech. capital bundle}) \quad (50)$$

$$P_{nt}/P_t = \frac{\sigma}{\sigma-1} \frac{1}{A_{nt}} \left( \frac{z_t^n}{\alpha} \right)^\alpha \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha} \quad (\text{price new tech.}) \quad (51)$$

$$P_{ot}/P_t = \frac{\sigma}{\sigma-1} \frac{1}{A_o} \left( \frac{z_t^o}{\alpha} \right)^\alpha \left( \frac{w_t}{1-\alpha} \right)^{1-\alpha} \quad (\text{price old tech.}) \quad (52)$$

$$P_t^w = \left( m_t (P_t^n)^{1-\sigma} + (1 - m_t) (P_t^o)^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \quad (\text{price wholesale}) \quad (53)$$

$$\mu_t = P_t/P_t^w \quad (\text{markup retail sector}) \quad (54)$$

## 7. New Keynesian block

$$\frac{1 + R_t}{1 + \bar{R}} = \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y} \quad (\text{Taylor rule}) \quad (55)$$

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} - \frac{(1 - \chi)(1 - \beta\chi)}{\chi} \hat{\mu}_t \quad (\text{log-linearized Phillips curve}) \quad (56)$$

where the latter is log-linearized around some steady-state that we explain in the following section.

### B.3 Resolution method

Our resolution approach relies on linearizing the equations of the economic model while keeping the full nonlinear equations that govern the evolution of beliefs. Besides providing greater tractability, this approach allows us to focus on the nonlinearities implied by our learning model while putting aside those of the DSGE model, which are usually weak.

#### Exploiting the martingale property of beliefs

Granted the benefits of the linearization approach, one difficulty remains in the need to keep track of the potentially infinite-dimensional public belief  $\Lambda_t(\theta, \xi)$ . We use a simplification proposed by [Kozłowski et al. \(2019\)](#) which exploits the fact that, due of the Law of Iterated Expectations, beliefs follow a *martingale*, that is,  $E_t [\Lambda_{t+1}(\tilde{\theta}, \tilde{\xi}) | \mathcal{I}_t] = \Lambda_t(\tilde{\theta}, \tilde{\xi})$ . The martingale property implies that any equilibrium condition of the form  $E_t [f(x_t, x_{t+1}, \Lambda_t, \Lambda_{t+1})] = 0$ , where  $f$  is a nonlinear function and  $x_t$  a vector of model variables can be approximated to a first-order as

$$E[f(x_t, x_{t+1}, \Lambda_t, \Lambda_{t+1}) | \mathcal{I}_t] \simeq E[f(x_t, x_{t+1}, \Lambda_t, \Lambda_t) | \mathcal{I}_t].$$

This implies that the model can be solved in each period as if current beliefs were constant going forward. As a consequence, we solve the model every period using a standard linear solver, compute the technology adoption decision and the evolution of beliefs in a nonlinear way, then repeat in the next period under the new beliefs.

### Linearization around moving steady states

The interaction between technology adoption and beliefs is of a highly nonlinear nature and the economy can settle on very different steady states in the long run. We have found that linearizing the economy around a fixed non-stochastic steady state produces poor results and that linearizing instead around a moving steady state that takes into account these nonlinearities yields more accurate predictions.

Our preferred approach thus involves linearizing the model every period around the long-run non-stochastic steady state in which all random variables are set equal to their means according to current beliefs.<sup>29</sup> In particular, the productivity of the new technology is equal to the current average public expectation at time  $t$  with  $\overline{A}^n(\Lambda_t) = E[A_{t+s}^n | \mathcal{I}_t] = (1 - \lambda) A^o + \lambda (p_t \theta_H + (1 - p_t) \theta_L)$ , where  $p_t = \int \Lambda_t(\theta_H, \xi) d\xi$ . We refer to this steady state as  $S.S(\Lambda_t)$ .

In a non-stochastic steady state, the rental rates on the two types of capital are equalized and the threshold  $s^{*,S.S(\Lambda_t)}$  which characterizes the marginal technology adopter solves

$$E[\theta | \Lambda_t, s^{*,S.S(\Lambda_t)}] = A^o.$$

The corresponding expected measure of new technology adopters is given by

$$m^{S.S(\Lambda_t)} = E\left[(1 - \mu) \overline{F}_{\theta+\xi}^s\left(s^{*,S.S(\Lambda_t)}\right) + \mu \varepsilon_t \mid \Lambda_t\right].$$

Every period, the model is solved forward around the long-run steady state associated to current beliefs but the variables are then translated back to a reference steady state which stays the same throughout the simulation in order to construct the impulse responses.<sup>30</sup>

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<sup>29</sup>This includes not only the productivity but also the signal noise and the mass of noise entrepreneurs.

<sup>30</sup>Note that the output gap in the Taylor rule (15), expressed in terms of the reference steady state, must be translated to the current steady state in every period.

## Technology adoption decision with GE effects

In each period, given the current capital stocks and public beliefs  $\Omega_t^A = (K_t^{IT}, K_t^T, \Lambda_t)$ , we solve nonlinearly for the marginal technology adopter threshold  $s_t^* = s^*(\Omega_t, \Lambda_t)$ <sup>31</sup> which solves

$$E \left[ \hat{A}_t^n - \alpha \hat{z}_t^n \mid \Lambda_t, s_t^* \right] = E \left[ \underbrace{\hat{A}_t^o}_{=0} - \alpha \hat{z}_t^o \mid \Lambda_t, s_t^* \right],$$

where the hatted variables indicate log-deviations from the current steady state around which the economy is linearized.

Solving for the threshold thus requires solving for the relative rental rates of capital. For a given stock of capital  $(K_t^{IT}, K_t^T)$ ,  $m_t$  and productivity realization  $A_t^n$ , the relative rental rate of capital  $\hat{z}_t^n - \hat{z}_t^o = \widehat{z_t^n / z_t^o}$  can be computed by solving a static system of equations. We first combine the firms' first-order conditions over capital and labor to express the relative demands for capital

$$\frac{k_{nt}}{k_{ot}} = \left( \frac{A_t^n}{A_o} \right)^{\sigma-1} \left( \frac{z_t^n}{z_t^o} \right)^{-\sigma(1-(1-\alpha)\frac{\sigma-1}{\sigma})}. \quad (57)$$

Using the IT and traditional capital market clearing constraints

$$K_t^{IT} = m_t \kappa_n^{\zeta-1} \omega_n^\zeta \left( \frac{z_t^{IT}}{z_t^n} \right)^{-\zeta} k_t^n + (1 - m_t) \kappa_o^{\zeta-1} \omega_o^\zeta \left( \frac{z_t^{IT}}{z_t^o} \right)^{-\zeta} k_t^o \quad (58)$$

$$K_t^T = m_t \kappa_n^{\zeta-1} (1 - \omega_n)^\zeta \left( \frac{z_t^T}{z_t^n} \right)^{-\zeta} k_t^n + (1 - m_t) \kappa_o^{\zeta-1} (1 - \omega_o)^\zeta \left( \frac{z_t^T}{z_t^o} \right)^{-\zeta} k_t^o, \quad (59)$$

and definitions of the capital bundle prices

$$z_t^n = \frac{1}{\kappa_n} \left( (1 - \omega_n)^\zeta (z_t^T)^{1-\zeta} + \omega_n^\zeta (z_t^{IT})^{1-\zeta} \right)^{\frac{1}{1-\zeta}} \quad (60)$$

$$z_t^o = \frac{1}{\kappa_o} \left( (1 - \omega_o)^\zeta (z_t^T)^{1-\zeta} + \omega_o^\zeta (z_t^{IT})^{1-\zeta} \right)^{\frac{1}{1-\zeta}}, \quad (61)$$

we obtain a system of 5 equations (57)-(61) in 5 unknowns  $(k_t^n, k_t^o, z_t^{IT}, z_t^T, z_t^n / z_t^o)$  which can be solved using a nonlinear solver.

Our procedure to solve for  $m_t$  in a given period with state variables  $(K_t^{IT}, K_t^T, \Lambda_t)$  involves a simple bisection algorithm which we describe as follows:

1. Guess  $s_t^*$ ;
2. Using certainty equivalence, set  $A_t^n \equiv E[A_t^n \mid \Lambda_t, s_t^*]$  and  $m_t \equiv E_t[m_t \mid \Lambda_t, s_t^*]$  and solve for

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<sup>31</sup>Due to the presence of general equilibrium effects, even under the assumption of MLRP for  $\{F_x^s\}$ , the technology adoption decision is not guaranteed to take the form of a cutoff rule. We check numerically that the necessary monotonicity condition holds to ensure the existence of a cutoff  $s_t^*$ .

- $\hat{z}_t^n - \hat{z}_t^o$  using the system (57)-(61).<sup>32</sup>;
3. Check if  $\left| E \left[ \hat{A}_t^n - \alpha (\hat{z}_t^n - \hat{z}_t^o) \mid \Lambda_t, s_t^* \right] \right| < \varepsilon$  with  $\varepsilon$  small. If yes, stop, otherwise return to (1) with a lower  $s_t^*$  if the expectation strictly positive, or a higher  $s_t^*$  if negative.

We denote the resulting technology adoption threshold by  $s^* (K_t^{IT}, K_t^T, \Lambda_t)$ .

### Dynamics of $m_t$

The previous paragraph explains how  $s_t^*$  can be computed in a given period  $t$  for a certain state space  $(K_t^{IT}, K_t^T, \Lambda_t)$ . One difficulty remains in the fact that agents in the economic model need to form expectations about future  $m_{t+s}$ ,  $s > 0$ , and that one equation in our linearized system should govern the dynamics of  $m_t$ . We do so by numerically linearizing the expectation of  $m_{t+s}$  as a function of  $(K_{t+s}^{IT}, K_{t+s}^T, \Lambda_t)$ .

To fix notation, denote  $m^{exp}$  the public expectation of  $m$  that corresponds to the technology adoption threshold  $s^* (K^{IT}, K^T, \Lambda_t)$ :

$$m^{exp} (K^{IT}, K^T, \Lambda_t) = E \left[ (1 - \mu) \bar{F}_{\theta+\xi}^s (s^* (K^{IT}, K^T, \Lambda_t)) + \mu \varepsilon_t \mid \Lambda_t \right].$$

In the long-run steady state around which the economy is linearized, the expected measure of new technology adopters  $m^{S.S(\Lambda_t)}$  coincides with  $m^{exp}$  when the stocks of capital have reached their steady states  $(K^{IT, S.S(\Lambda_t)}, K^{T, S.S(\Lambda_t)})$ :

$$m^{S.S(\Lambda_t)} = m^{exp} (K^{IT, S.S(\Lambda_t)}, K^{T, S.S(\Lambda_t)}, \Lambda_t).$$

The linearized equation that governs the expectation of  $m$  is given by

$$\begin{aligned} E[m_{t+s} \mid \Lambda_t] &= m^{exp} (K_{t+s}^{IT}, K_{t+s}^T, \Lambda_t) = m^{S.S(\Lambda_t)} + \frac{\partial m^{exp}}{\partial K^{IT}} (K^{IT, S.S(\Lambda_t)}, K^{T, S.S(\Lambda_t)}, \Lambda_t) (K_{t+s}^{IT} - K^{IT, S.S(\Lambda_t)}) \\ &\quad + \frac{\partial m^{exp}}{\partial K^T} (K^{IT, S.S(\Lambda_t)}, K^{T, S.S(\Lambda_t)}, \Lambda_t) (K_{t+s}^T - K^{T, S.S(\Lambda_t)}). \end{aligned} \quad (62)$$

We use numerical differentiation to evaluate the two derivatives  $\frac{\partial m^{exp}}{\partial K^{IT}}$  and  $\frac{\partial m^{exp}}{\partial K^T}$  and feed equation (62) to our linear rational expectation model solver.

### Resolution method step by step

We start by selecting a reference steady state (indicated with the upperscript *ref*) with respect to which all the impulse responses are expressed. We choose the long-run nonstochastic steady state associated with the initial public beliefs  $\Lambda_0$  at the time 0 with the corresponding technology adoption  $m^{S.S(\Lambda_0)}$ .

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<sup>32</sup>Since the economy is eventually linearized, the use of certainty equivalence has no bearing on the results, which remain correct to the first order.

In what follows, hatted variables  $\hat{x}^{ref}$  or  $\hat{x}^{S.S(\Lambda_t)}$  denote (log) deviations from the particular steady state it refers to (*ref* or  $S.S(\Lambda_t)$ ). The various steps of the resolution method can be described as follows:

1. At a given date  $t$  with state variables  $(\hat{K}_t^{IT,ref}, \hat{K}_t^{T,ref}, \Lambda_t)$ , compute the long-run steady state associated with current beliefs  $S.S(\Lambda_t)$ :  $(Y^{S.S(\Lambda_t)}, m^{S.S(\Lambda_t)}, K^{IT,S.S(\Lambda_t)}, K^{T,S.S(\Lambda_t)}, \dots)$ ;
2. Translate the state variables  $(\hat{K}_t^{IT,ref}, \hat{K}_t^{T,ref})$  to  $(\hat{K}_t^{IT,S.S(\Lambda_t)}, \hat{K}_t^{T,ref(\Lambda_t)})$ ;
3. Compute the threshold  $s^*(K_t^{IT}, K_t^T, \Lambda_t)$  and the realization of  $m_t = (1 - \mu) \bar{F}_{\theta+\xi}^s(s^*(K_t^{IT}, K_t^T, \Lambda_t)) + \mu \varepsilon_t$ ; express it in deviation from the current steady state  $\hat{m}_t^{S.S(\Lambda_t)} = m_t - m^{S.S(\Lambda_t)}$ ;
4. Use numerical differentiation to evaluate the law of  $m^{exp}(K_{t+s}^{IT}, K_{t+s}^T, \Lambda_t)$  and express it with respect to the current steady state associated to  $\Lambda_t$ ;
5. Using a linear rational expectation solver, solve for the economy linearized around the current steady state  $S.S(\Lambda_t)$ , keeping beliefs constant;
6. Evaluate the impulse response associated with the chosen set of shocks from  $t$  onward;
7. Collect the time- $t$  variables  $(\hat{Y}_t^{S.S(\Lambda_t)}, \hat{K}_t^{IT,S.S(\Lambda_t)}, \hat{K}_t^{T,S.S(\Lambda_t)}, \dots)$  and translate them back to the reference steady state  $(\hat{Y}_t^{ref}, \hat{K}_t^{IT,ref}, \hat{K}_t^{T,ref}, \dots)$ . Store their values as the time- $t$  realization of the desired impulse response;
8. Update beliefs using the particular realization of the signals and  $m_t$ ;
9. Set  $t := t + 1$  and go back to (1).

#### B.4 Additional details about the calibration

We use data from the “nowcast” of real GDP in the Survey of Professional Forecasters as they capture beliefs about net changes in productivity, net of changes in factors shares, because factors don’t have time to adjust over this short time horizon. The average real GDP growth between 1991 and 1998 (before the boom-bust cycle started) is 2.4%, and we assume that we are on a balanced growth path with  $A_{ot}$ ,  $\theta_{Ht}$  and  $\theta_{Lt}$  growing at rate  $\gamma = 1 + 0.024/4 = 1.006$  per quarter.

To compute  $\theta_H$ , we use the highest expectation of real GDP growth over our period, which is 4.19% in 2000Q2 in annualized term. We interpret this most optimistic scenario as a situation in which everybody is convinced that the state of the new technology is good and so  $m = 1$  and  $p = 1$ . In this case, we can write

$$\frac{\gamma(\lambda\theta_H + (1 - \lambda)A_o)}{A_o} = 1 + 0.0419/4$$

where recall  $\lambda = 1/22$  is the likelihood that the new technology becomes operational. Since  $A_o$  is normalized to one we find that  $\theta_H = 1.099$ .

We perform a similar computation to find  $\theta_L$  but in this case relying on the most pessimistic

( $m = 1$  and  $p = 1$ ) forecast (0.8% annualized in 2001Q1). We find

$$\frac{\gamma (\lambda \theta_L + (1 - \lambda) A_o)}{A_o} = 1 + 0.008/4$$

which yields  $\theta_L = 0.912$ .

## B.5 Monetary Policy Implications

In this appendix we discuss the role of monetary policy during in the quantitative model. It has long been the subject of policy debates whether monetary authorities should narrowly focus on output and inflation stabilization or, instead, intervene in response to bubble-like phenomena in investment or in the stock market.

As we explained in the context of the simplified learning model, our theory justifies the use of interventions that *lean against the wind* because of the presence of an informational externality. While evaluating optimal monetary policy in our setup is beyond the scope of this paper, we ask whether a monetary policy rule that leans against the wind would be desirable in our economy.<sup>33</sup> The answer is mostly negative and we illustrate our argument with a simple example.

We augment our Taylor rule with an explicit mandate to increase interest rates when investment in the new technology  $I_t^{IT}$  rises:<sup>34</sup>

$$\frac{1 + r_t}{1 + \bar{r}} = \left( \frac{1 + \pi_t}{1 + \bar{\pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y} \left( \frac{I_t^{IT}}{\bar{I}^{IT}} \right)^{\phi_i}, \quad \phi_i \geq 0. \quad (63)$$

We compute ex-ante welfare using Monte-Carlo simulations with all the model shocks and compute the coefficient  $\phi_i$  that maximizes welfare. Note that the model is solved around the efficient steady state corrected for monopolistic distortions. Any welfare gain thus arises mainly because of the information externality and the suboptimality of the initial monetary policy. Figure 14 reports the impulse responses to a false-positive shock with the benchmark case in blue and the welfare-maximizing leaning-against-the-wind Taylor rule in red.

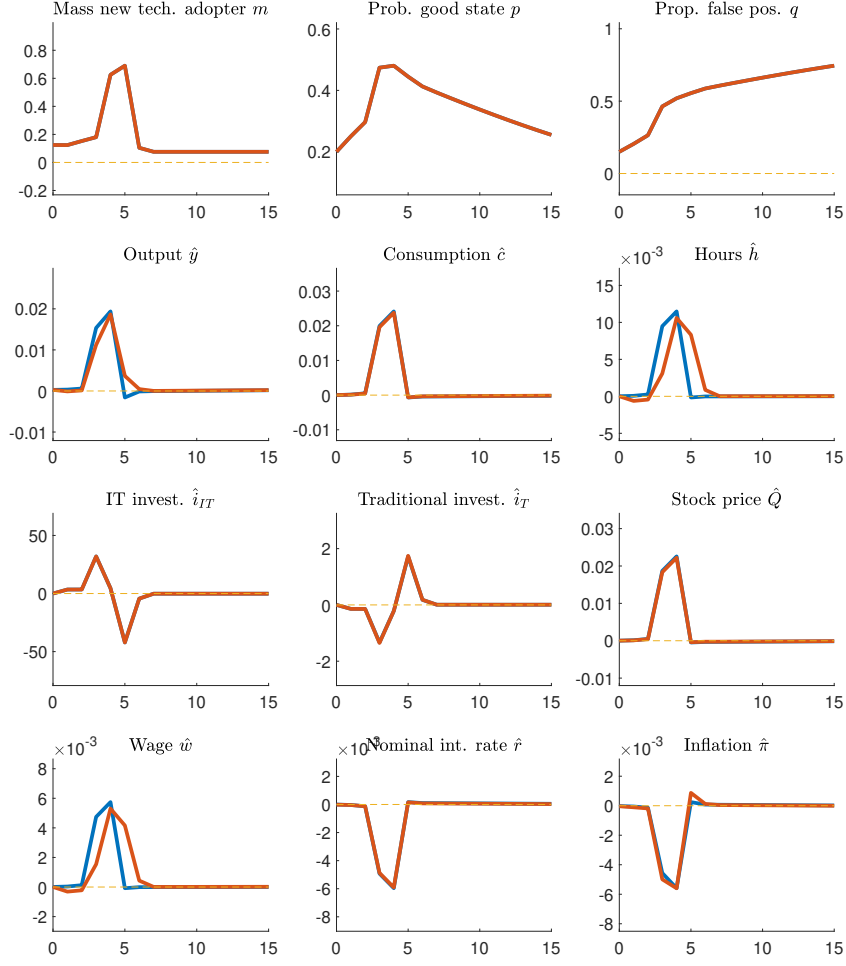
As the figure illustrates, the differences manifest themselves mostly in output, hours and wages, and to a lesser extent in the inflation rate. The other series are nearly identical. The augmented policy (63) with  $\phi_i > 0$  manages to reduce the volatility of output in response to the shock: the

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<sup>33</sup>Computing the optimal monetary policy or the optimal investment tax would require to solve the model using an entirely different, less tractable method. Since the nonlinearities in learning are essential for the result on leaning-against-the-wind policies, the resolution method cannot rely on linearization and the state space reduction result described in subsection ?? no longer holds.

<sup>34</sup>We also experimented with a Taylor ruled of the form  $\frac{1+r_t}{1+\bar{r}} = \left( \frac{1+\pi_t}{1+\bar{\pi}} \right)^{\phi_\pi} \left( \frac{Y_t}{\bar{Y}} \right)^{\phi_y} \left( \frac{m_t}{\bar{m}} \right)^{\phi_m}$ ,  $\phi_m \geq 0$ , but found no parameter  $\phi_m > 0$  that increased welfare, due to the fact that  $m_t$  lags the cycle by one period as it tracks the dynamics of IT capital.





Notes: The impulse responses are reported in log-deviations from the initial steady state. The blue curve represents the response of the economy in our benchmark calibration. The red curve corresponds to the Taylor rule described in (63) with the coefficient  $\phi_i = 0, 3.84 \cdot 10^{-5}$  that maximize ex-ante welfare.

Figure 14: Impact of leaning-against-the-wind monetary policy

economy expands slightly less during the growth stage of the cycle and the recession that would normally follow is averted. Hours are similarly less volatile. The optimal coefficient  $\phi_i = 3.84 \times 10^{-5}$  is fairly small because this policy comes at the cost of slowing booms driven by good technologies, which the monetary authority is reluctant to do because the probability of a false-positive episode remains low. Accordingly, the welfare gain is small: +0.002% in consumption equivalent relative to the benchmark calibration. Our results thus provide little support for this type of interventions.

Beyond these considerations, another reason why a monetary policy that leans against the wind is not more effective is that it is an inappropriate tool to address the information externality highlighted in subsection (3.3). We know from that section that the information externality shows up as a distortion in  $s^*$  and  $m_t$ , which govern the amount of information released by the social learning channel. In the current context, we see in the first panel of Figure 14 that monetary policy

has virtually no effect on  $m_t$ . The reason is that the technology adoption decision (17) is entirely driven by beliefs about  $\theta$  and the relative rental rates of capital  $z_t^i$ ,  $i = o, n$ . Since the monetary authority cannot affect beliefs directly (they have no additional information to provide to investors), the only channel by which it could operate is through the rental rates differential  $\hat{z}_t^n - \hat{z}_t^o$ . We find this channel quantitatively weak: by raising interest rates, the household's incentives to accumulate both types of capitals are affected, but in similar ways: the cost of the two capital bundles change by an almost identical amount and technology adoption is left nearly unchanged.<sup>35</sup> We thus find little scope for the introduction of leaning-against-the-wind motives in monetary policy in a context like ours and conclude that other policy instruments with a direct impact on technology adoption, like the investment taxes and subsidies explored in subsection 3.3, are more promising.

### B.6 Sensitivity to mass of noise entrepreneurs $\mu$

Figure 15 shows the benchmark simulation of Section 5 with a smaller ( $\mu = 0.1$ ) and larger ( $\mu = 0.2$ ) mass of noise entrepreneurs. We see that when there are fewer noise entrepreneurs, the public signals that agents receive are more precise which leads to a stronger boom that happens sooner. In contrast, if the public signals are more noisy ( $\mu = 0.2$ ), agents learn little by observing  $m$  and the economy does not go through a boom-bust cycle. This last simulation emphasizes the importance of social learning for the mechanism.

### B.7 Sensitivity to size of common noise signal $\bar{\xi}$

Figure 16 shows the same simulation as Section 5 but with various sizes for the common noise shock  $\bar{\xi}$ . Unsurprisingly, the size of the boom is correlated with  $\bar{\xi}$ . When the common noise essentially mimics  $\theta_H$  the increase in output reaches 3 percent. In contrast, when  $\bar{\xi} = 0.9 \times (\theta_H - \theta_L)$  aggregate output goes up by about 1.5 percent during the boom.

### B.8 Sensitivity to prior $p_0$ associated with the good state: $\theta = \theta_H$ and $\xi = 0$

Figure 17 shows the simulations of Section 5 under various values for the prior  $p_0$  associated with the good state. We see that the boom is much larger when entrepreneurs expect the good state to be more likely ( $p_0 = 25$ ). In that case, the economy goes in the information cascades zone as in Figure 4. In contrast, when agents expect that the good state is unlikely ( $p_0 = 0.15$ ) their pessimism is enough to prevent the economy from going through a boom-bust cycle.

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<sup>35</sup>Monetary policy could be more powerful if IT investors were differentially exposed to interest rates through borrowing, for example.

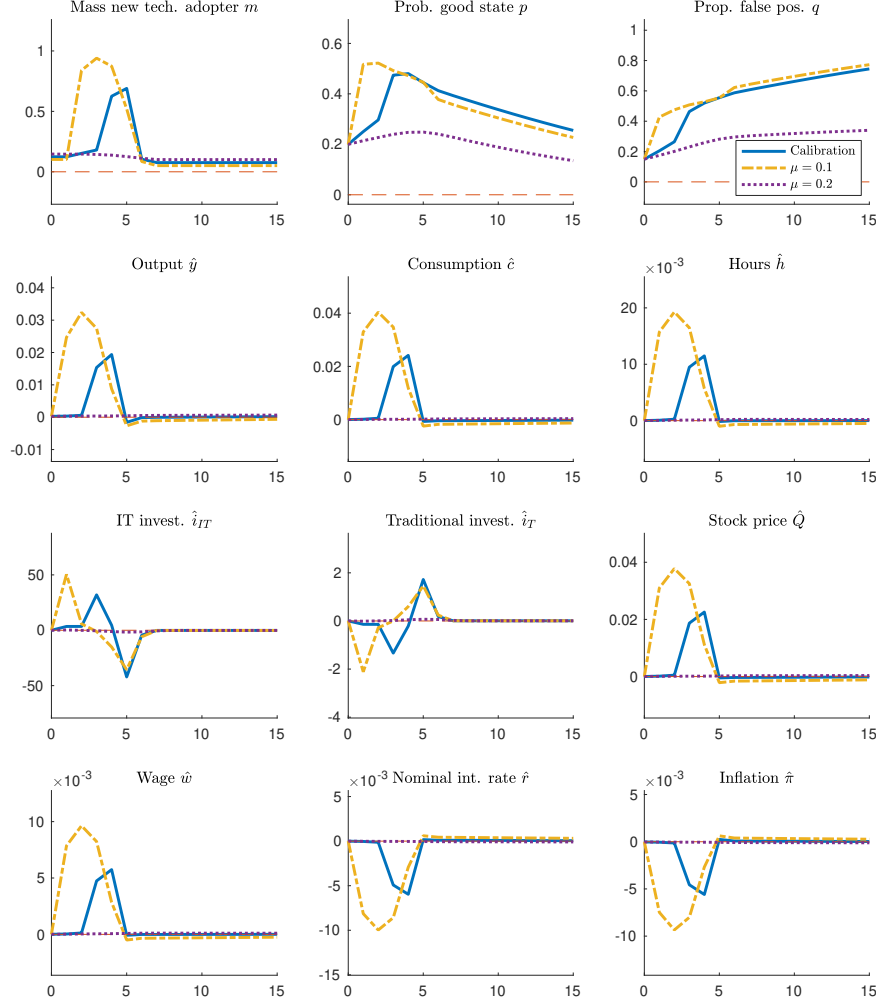


Figure 15: Simulations under different mass of noise traders  $\mu$

### B.9 Sensitivity to prior $q_0$ associated with the false positive state: $\theta = \theta_L$ and $\xi = \bar{\xi}$

Figure 18 shows what happens to the benchmark simulations of Section 5 when the prior on the false positive state changes. When that state is less likely ( $q_0 = 0.1$ ) the boom-bust cycle dynamics is stronger and the economy enters the information cascades state, as in Figure 4. If instead, the false positive state is more likely ( $q_0 = 0.2$ ), agents are particularly skeptical of good signal and the economy does go through a boom.

### B.10 Sensitivity to standard deviation of the exogenous public signal $\sigma_u$

Figure 19 shows the benchmark simulation under different values for  $\sigma_u$ . When the exogenous public signal is less information ( $\sigma_u = 4 \times \sigma_s$ ), agents have a harder time figuring out that the economy is in the false positive state and the boom is much larger, with output reaching a high

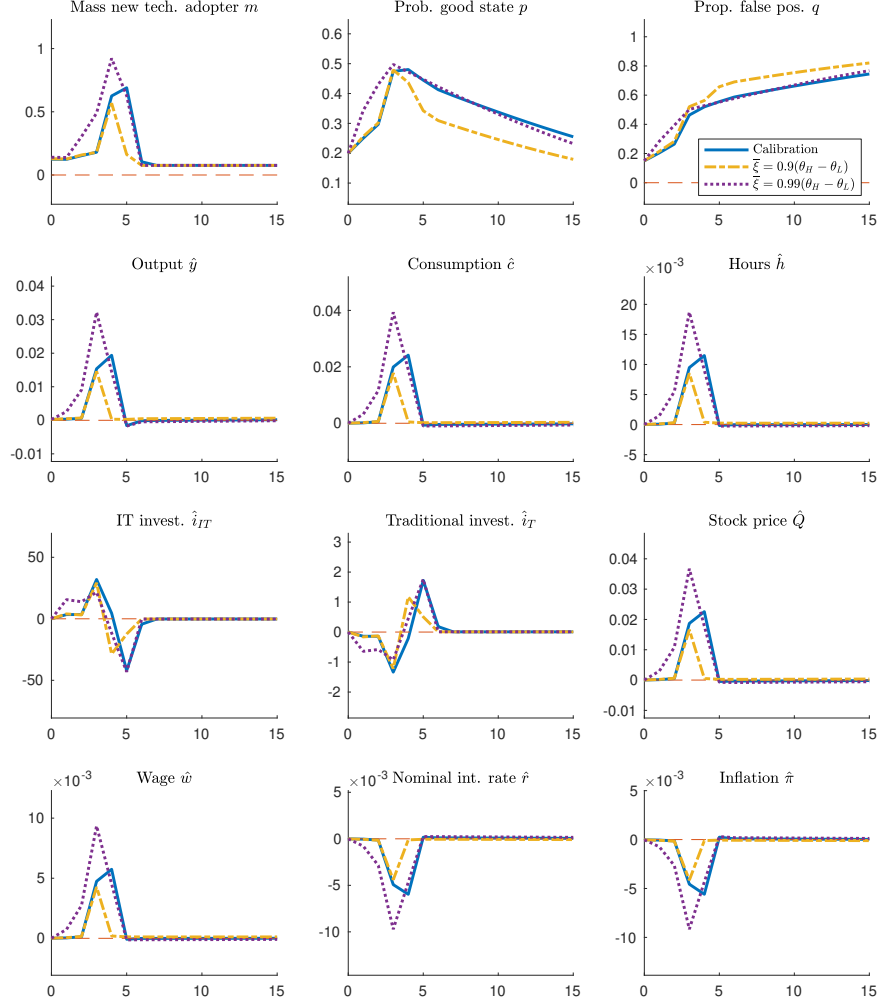


Figure 16: Simulations under different size of the common noise signal  $\bar{\xi}$

of about 3 percent above trend. In contrast, when  $\sigma_u$  is small, the exogenous public signal is very informative and agents learn rapidly that the technology is bad ( $\theta = \theta_L$ ). In this case, there is no room for our social learning mechanism to create a boom-bust cycle.

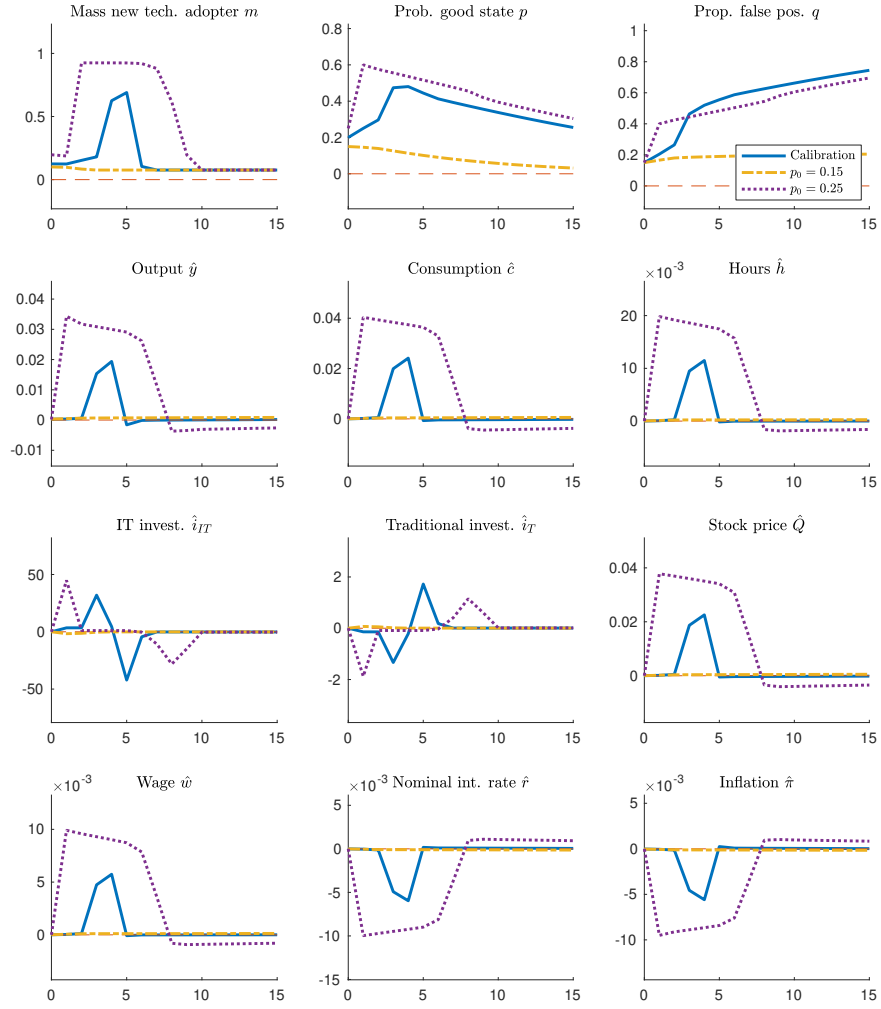


Figure 17: Simulations under different priors  $p_0$

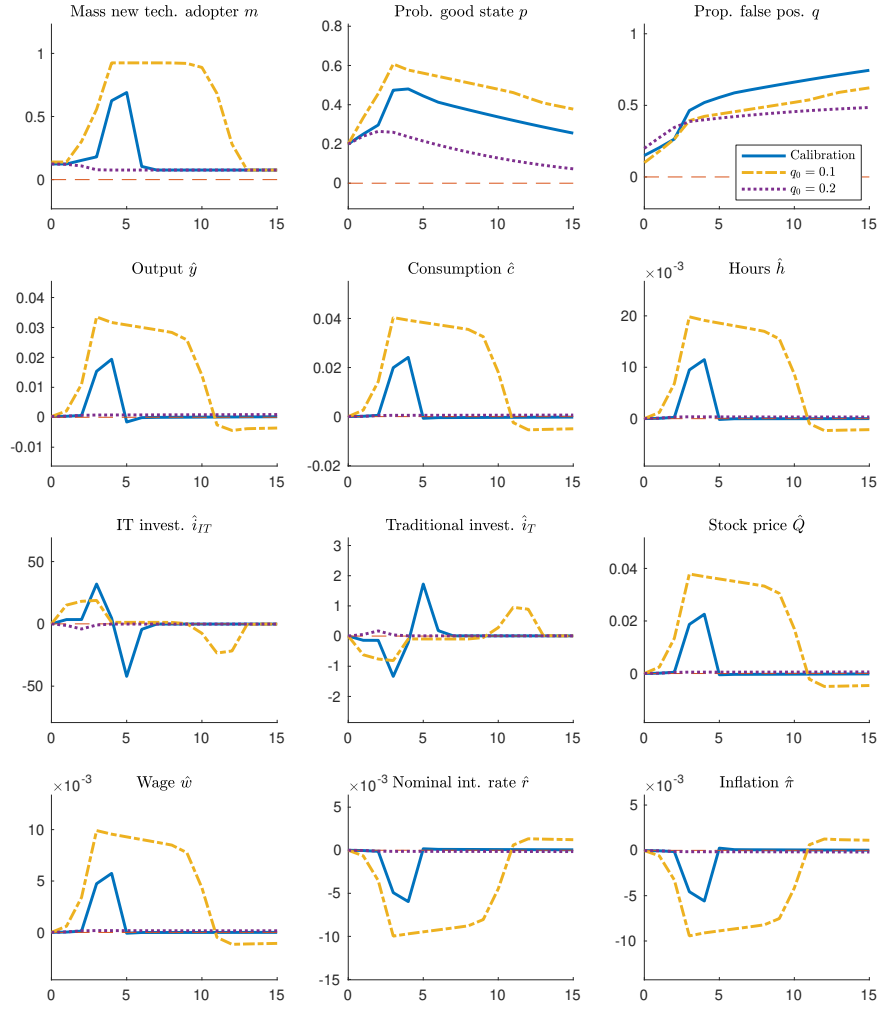


Figure 18: Simulation under different priors  $q_0$

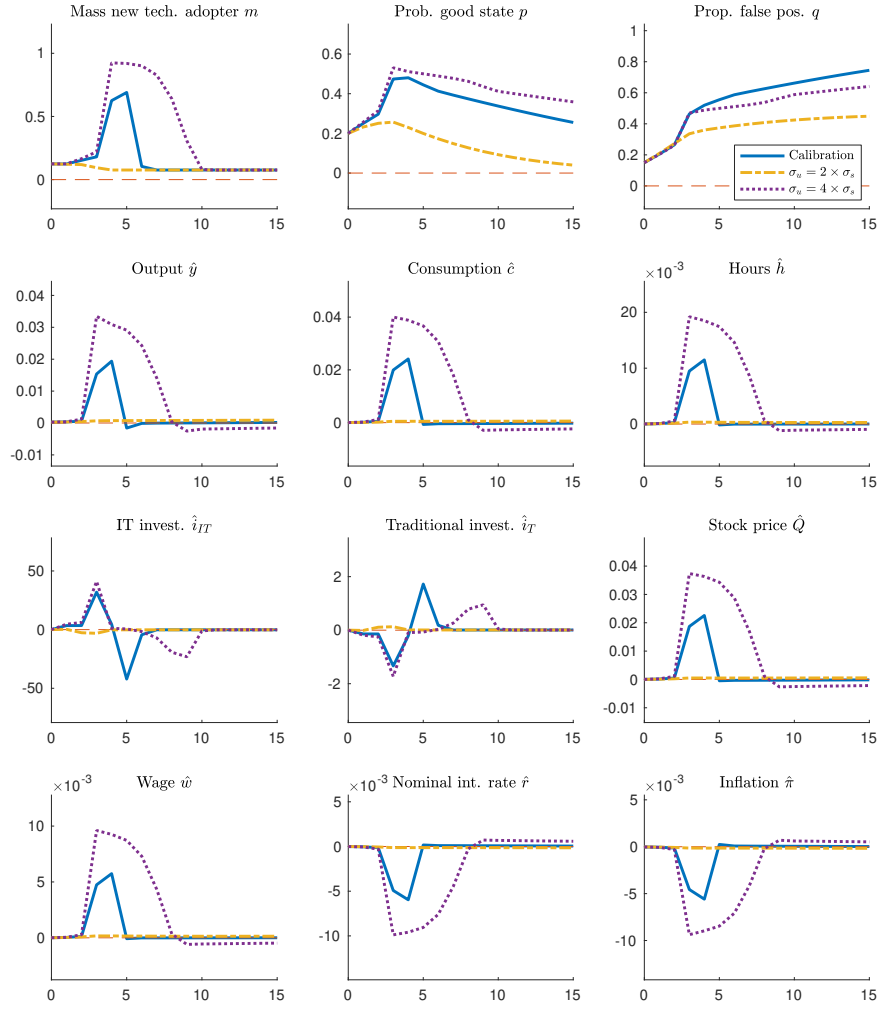


Figure 19: Simulation under different informativeness for the exogenous public signal  $\sigma_u$