Endogenous Production Networks Under Supply Chain Uncertainty

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How does uncertainty affect an economy's production network and, through that channel, macroeconomic aggregates?

Approach and results

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- Sizable role for uncertainty during high-volatility events like the Great Recession

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Reduced-form evidence for the model mechanisms

- Links with riskier suppliers are more likely to be destroyed
- Riskier firms have lower Domar weights

Related literature

Uncertainty

- Bloom (2009); Fernandez-Villaverde et al (2011); Bloom (2014); Bloom et al (2018); and many others ...

Exogenous production networks

- Long and Plosser (1983); Dupor (1999); Horvath (2000); Acemoglu et al (2012); Carvalho and Gabaix (2013); and many others ...

Endogenous production networks

- Oberfield (2018); Acemoglu and Azar (2020); Boehm and Oberfield (2020); Taschereau-Dumouchel (2021); Acemoglu and Tahbaz-Salehi (2021); and many others ...

Model

Model

Static model with two types of agents

1. Representative household: owns the firms, supplies labor and consumes
2. Firms: produce differentiated goods using labor and intermediate inputs
   - There are n industries/goods, indexed by $i \in \{1, \ldots, n\}$
   - Representative firm that behaves competitively

Production technique

Each firm i has access to a set of production techniques $\mathcal{A}_i$.

A technique $\alpha_i \in \mathcal{A}_i$ specifies

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These techniques are Cobb-Douglas production functions

▸ ζ

- We identify $\alpha_i = (\alpha_{i1}, \ldots, \alpha_{in})$ with the input shares

$$F(\alpha_i, L_i, X_i) = e^{\varepsilon_i}\zeta(\alpha_i) A_i(\alpha_i) L_i^{1-\sum_{j=1}^{n}\alpha_{ij}} \prod_{j=1}^{n} X_{ij}^{\alpha_{ij}},$$

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Allow adjustment along intensive and extensive margins: $\mathcal{A}_i = \left\{ \alpha_i \in [0,1]^n : \sum_{j=1}^n \alpha_{ij} \leq \overline{\alpha}_i < 1 \right\}$.

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Example: A car manufacturer can use only steel or only carbon fiber, or a combination of both.

Production technique

Assumption

$A_i(\alpha_i)$ is smooth and strictly log-concave.

Implication: There are ideal input shares α_{ij}° that maximize A_i

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Example

$$\log A_i(\alpha_i) = -\sum_{j=1}^{n} \kappa_{ij} \left(\alpha_{ij} - \alpha_{ij}^{\circ}\right)^2 - \kappa_{i0} \left(\sum_{j=1}^{n} \alpha_{ij} - \sum_{j=1}^{n} \alpha_{ij}^{\circ}\right)^2,$$

Source of uncertainty and timing

Firms are subject to productivity shocks $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_n) \sim \mathcal{N}(\mu, \Sigma)$

- Vector μ captures optimism/pessimism about productivity
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Timing

1. Before ε is realized: Production techniques are chosen
   - Beliefs (μ, Σ) affect technique choice $\rightarrow$ production network $\alpha \in \mathcal{A}$ is endogenous
2. After ε is realized: All other decisions are taken
   - Only impact of uncertainty on decisions is through technique choice

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Key restriction

Each firm/industry i can only adopt one production technique.

Microfound.

Household

The representative household makes decisions after ε is realized

- Owns the firms
- Supplies one unit of labor inelastically
- Chooses *state-contingent* consumption $(C_1, \ldots, C_n)$ to maximize

$$u\left(\left(\frac{C_1}{\beta_1}\right)^{\beta_1} \times \cdots \times \left(\frac{C_n}{\beta_n}\right)^{\beta_n}\right),$$

subject to the *state-by-state* budget constraint

$$\sum_{i=1}^{n} P_i C_i \leq 1,$$

where u is CRRA with relative risk aversion $\rho \geq 1$.

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Details

- We refer to aggregate consumption $Y = \prod_{i=1}^{n} \left(\beta_i^{-1} C_i\right)^{\beta_i}$ as GDP.

Problem of the firm

Firms solve a two-stage problem

1. Before ε is drawn: Choose production technique α_i
    - ex ante decision under uncertainty
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Problem of the firm: Labor and intermediate inputs

For a given technique α_i, the cost minimization problem of the firm is

$$K_i\left(\alpha_i, P\right) = \min_{L_i, X_i} \left(L_i + \sum_{j=1}^{n} P_j X_{ij} \right), \text{ subject to } F\left(\alpha_i, L_i, X_i\right) \geq 1$$

where $K_i\left(\alpha_i, P\right)$ is the unit cost of production.

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1. Constant returns to scale $\rightarrow$ K_i does not depend on firm size
2. Given that each technique is Cobb-Douglas,

$$K_i(\alpha_i, P) = \frac{1}{e^{\varepsilon_i} A_i(\alpha_i)} \prod_{j=1}^{n} P_j^{\alpha_{ij}}.$$

3. Since we have perfect competition, it must be that in equilibrium

$$P_i = K_i(\alpha_i, P) \text{ for all } i \in \{1, \ldots, n\}.$$

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- For a given network $\alpha \in \mathcal{A}$ we can compute equilibrium prices $P(\alpha)$
- From prices we can compute GDP

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Problem of the firm: Production technique

Firm i chooses a technique $\alpha_i \in \mathcal{A}_i$ to maximize profits

$$\alpha_i^* \in \arg\max_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i \left(P_i - K_i\left(\alpha_i, P\right)\right)\right]$$

where Q_i is the equilibrium demand for good i and Λ is the SDF.

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Lemma

In equilibrium, $\lambda\left(\alpha^*\right)$, $k_i\left(\alpha_i, \alpha^*\right)$ and $q_i\left(\alpha^*\right)$ are normally distributed, and the technique choice of the representative firm in sector i solves

$$\alpha_i^* \in \arg\min_{\alpha_i \in \mathcal{A}_i} \mathrm{E}\left[k_i\left(\alpha_i, \alpha^*\right)\right] + \mathrm{Cov}\left[\lambda\left(\alpha^*\right), k_i\left(\alpha_i, \alpha^*\right)\right]. \tag{1}$$

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The firm prefers techniques with low

1. expected unit cost
2. unit cost when marg. utility is high $\rightarrow$ firm "inherits" the household's risk aversion through λ

Problem of the firm: Production technique

We can expand the two terms to minimize

$$\mathrm{E}\left[k_i\left(\alpha_i, \alpha^*\right)\right] = -a_i\left(\alpha_i\right) + \sum_{j=1}^{n} \alpha_{ij}\,\mathrm{E}\left[p_j\right]$$

Firm prefers techniques with high TFP and low average input prices.

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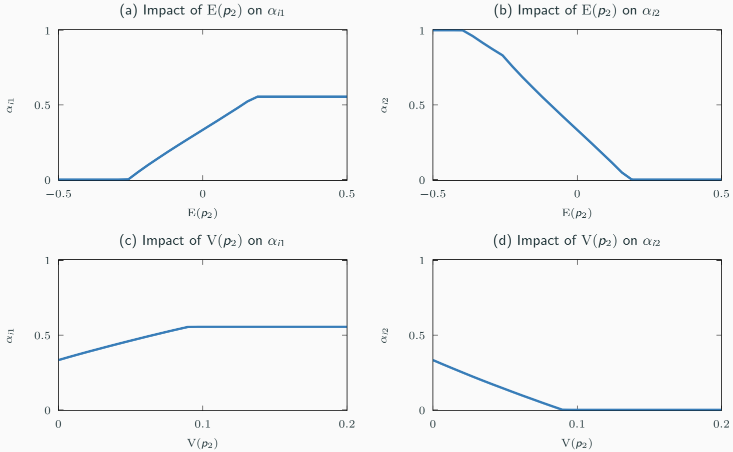
In general $\mathrm{Corr}\left[\lambda, k_i\right] > 0 \rightarrow$ Minimize variance of k_i

$$\mathrm{V}\left[k_i\right] = \text{cte} + \underbrace{\sum_{j=1}^{n} \alpha_{ij}^2\, \mathrm{V}\left[p_j\right]}_{\text{stable prices}} + \underbrace{\sum_{j \neq k} \alpha_{ij}\alpha_{ik}\, \mathrm{Cov}\left[p_j, p_k\right]}_{\text{uncorrelated prices}} + \underbrace{2\, \mathrm{Cov}\left[-\varepsilon_i, \sum_{j=1}^{n} \alpha_{ij} p_j\right]}_{\text{uncorrelated with own } \varepsilon_i}$$

Back to our example

- Car manufacturer i can use steel (input 1) or carbon fiber (input 2)
- Look at impact of $\mathrm{E}\, p_2$ and $\mathrm{V}\, p_2$ on the shares α_{i1} and α_{i2}



Definition

An equilibrium is a technique for every firm α^* and a stochastic tuple $(P^*, C^*, L^*, X^*, Q^*, \Lambda^*)$ such that

1. (Unit cost pricing) For each $i \in \{1, \ldots, n\}$, $P_i^* = K_i(\alpha_i^*, P^*)$.
2. (Optimal technique choice) For each $i \in \{1, \ldots, n\}$, factor demand L_i^* and X_i^*, and the technology choice $\alpha_i^* \in \mathcal{A}_i$ solves the firm's problem.
3. (Consumer maximization) The consumption vector C^* solves the household's problem.
4. (Market clearing) For each $i \in \{1, \ldots, n\}$,

$$Q_i^* = C_i^* + \sum_{j=1}^{n} X_{ji}^*,$$

$$Q_i^* = F_i(\alpha_i^*, L_i^*, X_i^*),$$

$$\sum_{i=1}^{n} L_i^* = 1.$$

Fixed-network economy

GDP in a fixed-network economy

Define a firm's Domar weight ω_i as its sales share

$$\omega_i(\alpha) := \frac{P_i Q_i}{PC}$$

GDP in a fixed-network economy

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$$\omega_i(\alpha) := \frac{P_i Q_i}{PC} = \beta' \mathcal{L}(\alpha) 1_i$$

Domar weights depend on

1. Demand from the household through β
2. Demand from intermediate good producers through $\mathcal{L}(\alpha) = (I - \alpha)^{-1} = I + \alpha + \alpha^2 + \ldots$

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→ Domar weights are constant for a fixed network

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Lemma (Hulten's Theorem)

Under a given network α, the log of GDP $y = \log Y$ is given by

$$y = \omega(\alpha)' (\varepsilon + a(\alpha)).$$

Impact of beliefs on GDP

Proposition (Hulten's Theorem in expectation)

For a fixed network α,

1. The impact of μ_i on expected log GDP is given by
$$\frac{\partial \mathrm{E}[y]}{\partial \mu_i} = \omega_i.$$
2. The impact of Σ_{ij} on the variance of log GDP is given by
$$\frac{\partial \mathrm{V}[y]}{\partial \Sigma_{ij}} = \begin{cases} \omega_i^2 & i = j, \\ 2\omega_i\omega_j & i \neq j. \end{cases}$$
3. μ does not affect $\mathrm{V}[y]$ and Σ does not affect $\mathrm{E}[y]$.

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For a fixed network

1. Domar weights ω are enough to understand log GDP
2. Since $\omega_i > 0$ shocks have intuitive impact.

Flexible-network economy

Equilibrium and efficiency

The economy is fully competitive and undistorted by frictions or externalities.

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Proposition

1. There exists an efficient equilibrium
2. That equilibrium production network solves

$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} \mathrm{E}\left[y(\alpha)\right] - \frac{1}{2}(\rho - 1)\,\mathrm{V}\left[y(\alpha)\right]$$

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Implications

1. The planner prefers networks that balance high $\mathrm{E}\left[y(\alpha)\right]$ with low $\mathrm{V}\left[y(\alpha)\right]$
2. Complicated network formation problem $\rightarrow$ simpler optimization problem.

Economic forces at work

Impact of beliefs on the network

Domar weights are constant when the network is fixed. But when it is flexible...

Proposition

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Intuition

1. Equilibrium: Firms rely more on high-μ_i and low-Σ_{ii} firms as suppliers.
2. Planner: Planner wants high-μ_i and low-Σ_{ii} firms to be more important for GDP.

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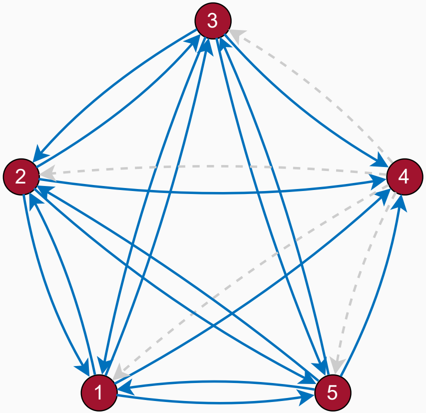
Flexible network $\rightarrow$ beneficial changes are amplified while adverse changes are mitigated.

Data Impact on α

Example: Impact of beliefs on the network

Simple example of possible substitution patterns



Baseline

Details

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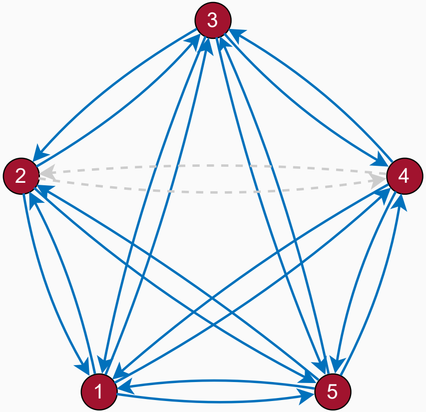

Small increase in Σ_{22} $\rightarrow$ Firms also purchase from 4 to diversify

Details

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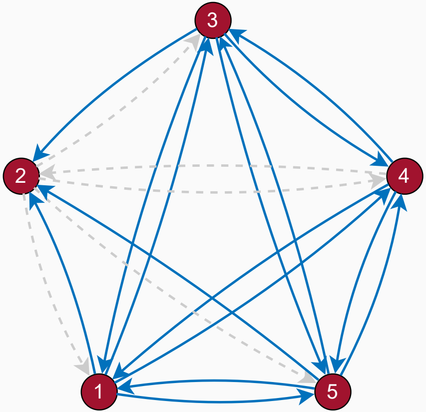
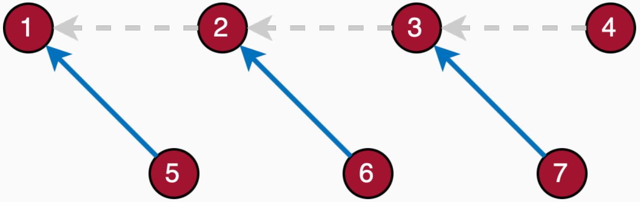


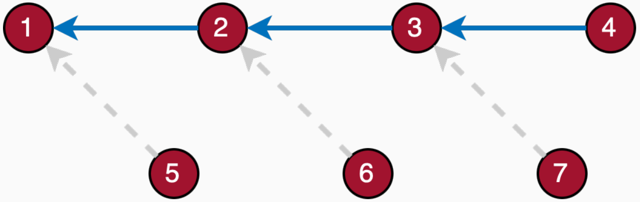
Large increase in Σ_{22} $\rightarrow$ Firms drop 2 as a supplier

Details

Example: Cascading effect of uncertainty



(a) High uncertainty about ε_4



(b) Low uncertainty about ε_4

Effect of uncertainty on GDP

Proposition

Uncertainty lowers expected GDP in equilibrium, in the sense that $\mathrm{E}[y]$ is largest when $\Sigma = 0_{n\times n}$.

Effect of uncertainty on GDP

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Intuition

1. Equilibrium: With uncertainty, firms seek stability at the cost of efficiency.

Effect of uncertainty on GDP

Proposition

Uncertainty lowers expected GDP in equilibrium, in the sense that $\mathrm{E}[y]$ is largest when $\Sigma = 0_{n\times n}$.

Intuition

1. Equilibrium: With uncertainty, firms seek stability at the cost of efficiency.
2. Planner: Only objective is to maximize $\mathrm{E}[y]$.

$$\mathcal{W} := \max_{\alpha\in\mathcal{A}} \mathrm{E}[y(\alpha)] - \xcancel{\frac{1}{2}(\rho-1)\mathrm{V}[y(\alpha)]}$$

Effect of beliefs on welfare

Proposition

1. The impact of μ_i on welfare is given by

$$\frac{d\mathcal{W}}{d\mu_i} = \omega_i.$$

2. The impact of Σ_{ij} on welfare is given by

$$\frac{d\mathcal{W}}{d\Sigma_{ij}} = \begin{cases} -\frac{1}{2}\left(\rho - 1\right)\omega_i^2 & i = j, \\ -\left(\rho - 1\right)\omega_i \omega_j & i \neq j. \end{cases}$$

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The impact of beliefs on welfare is intuitive

1. Higher expected productivity increases welfare
2. Higher correlation or uncertainty lowers welfare

Effect of beliefs on GDP

Impact of shocks on

- Welfare: intuitive
- GDP when the network is fixed: intuitive
- GDP when the network is flexible: ???

Effect of beliefs on GDP

Impact of shocks on

- Welfare: intuitive
- GDP when the network is fixed: intuitive
- GDP when the network is flexible: ???

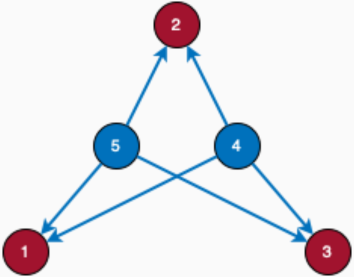
Decompose a shock to, say, μ_i as

$$\frac{d\,\mathrm{E}[y]}{d\mu_i} = \underbrace{\frac{\partial\,\mathrm{E}[y]}{\partial\mu_i}}_{\text{direct impact with fixed network}} + \underbrace{\frac{\partial\,\mathrm{E}[y]}{\partial\alpha}\frac{d\alpha}{d\mu_i}}_{\text{network adjustment}}$$

Two effects

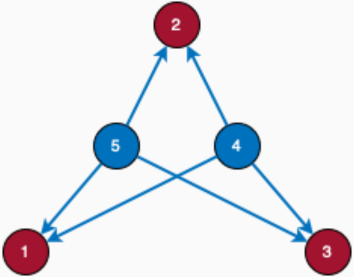
1. Direct impact keeping the network fixed = Domar weight
2. Indirect impact that take into account the network adjustment = ???

Example: Counterintuitive impact of a change in (μ, Σ)



- Firm 4 is risky (high Σ_{44}) but productive (high μ_4)
- Firm 5 is safe (low Σ_{55}) but unproductive (low μ_5)

Example: Counterintuitive impact of a change in (μ, Σ)



- Firm 4 is risky (high Σ_{44}) but productive (high μ_4)
- Firm 5 is safe (low Σ_{55}) but unproductive (low μ_5)

Consider two shocks

1. Increase μ_5
   - Move away from high-μ firm 4 toward low-μ firm 5 $\Rightarrow$ $\mathrm{E}[y]$ falls
2. Increase Σ_{44}
   - Move away from high-Σ firm 4 toward low-Σ firm 5 $\Rightarrow$ $\mathrm{V}[y]$ falls

▸ Details

Quantitative exploration

Calibration

Data

- Annual United States data from 1947 to 2020 about 37 sectors

Calibration

- Consumption shares β and ideal shares α° taken from the data
- Risk-aversion ρ and cost of deviating κ are **estimated**
- ε_t is random walk with drift and time-varying uncertainty and is **estimated**

▸ Data details ▸ Estimation details

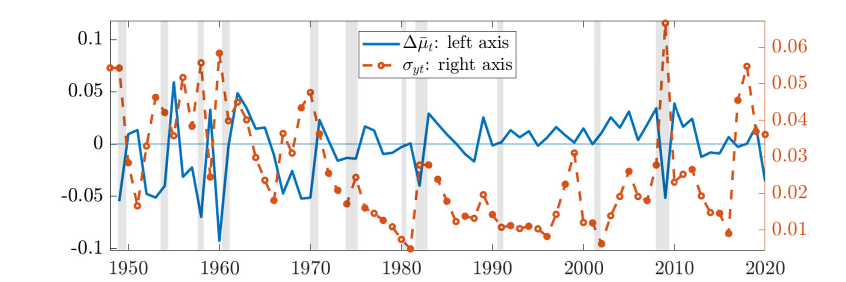
Calibrated economy

Estimated risk aversion: $\rho = 4.27$

Calibrated economy

Estimated risk aversion: $\rho = 4.27$

Estimated evolution of beliefs

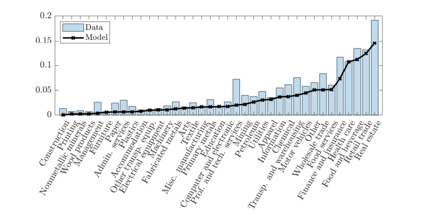


$$\Delta\bar{\mu}_t = \sum_{j=1}^{n} \omega_{jt}\Delta\mu_{jt} \text{ and } \sigma_{yt} = \sqrt{\mathbb{V}[y]} = \sqrt{\omega_t'\Sigma_t\omega_t}.$$

▶ κ's

Calibrated economy: Domar weights

The calibrated Domar weights fit the data reasonably well



Beliefs have the expected impact on Domar weights

	Statistic	Data	Model
(1)	Average Domar weight $\bar{\omega}_j$	0.047	0.032
(2)	Standard deviation $\sigma(\omega_j)$	0.0050	0.0021
(3)	Coefficient of variation $\sigma(\omega_j)/\bar{\omega}_j$	0.11	0.07
(4)	Corr (ω_{jt}, μ_{jt})	0.08	0.08
(5)	Corr $(\omega_{jt}, \Sigma_{jjt})$	-0.37	-0.31

Isolating the mechanism

Two useful counterfactuals

1. Fixed-network economy
   - No change in network $\rightarrow$ capture the full effect of network adjustments
2. "Risk-neutral" economy ($\rho = 1$)
   - Uncertainty has no impact on network $\rightarrow$ capture the impact of uncertainty
   - Recall: only impact of uncertainty on expected GDP is through the network

Isolating the mechanism

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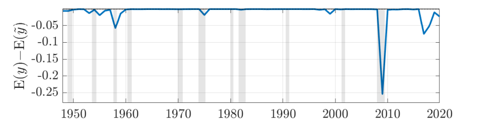
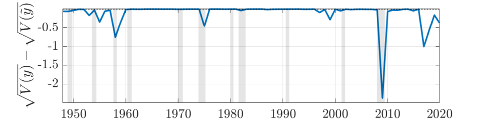
	Baseline model compared to...	
	Fixed network	Risk neutral
Expected GDP $\mathrm{E}[y(\alpha)]$	$+2.122\%$	-0.008%
Std. dev. of GDP $\sqrt{\mathrm{V}[y(\alpha)]}$	$+0.131\%$	-0.105%
Welfare $\mathcal{W}$	$+2.109\%$	$+0.010\%$

The Great Recession

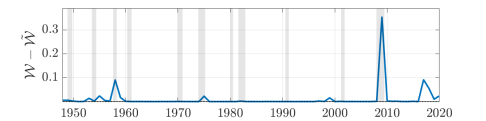
Calibrated model vs risk-neutral alternative



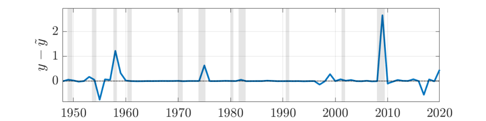
(a) Difference in expected GDP



(b) Difference in expected std. dev. of GDP



(c) Difference in expected welfare



(d) Difference in realized GDP

- During periods of high volatility, uncertainty matters.

Reduced-form evidence for the model mechanisms

Links with riskier suppliers are more likely to be destroyed

Use detailed U.S. data on firm-to-firm relationship (Factset 2003–2016)

Regress a dummy for link destruction on supplier uncertainty measures

- Instruments from Alfaro, Bloom and Lin (2019)

Details

	Dummy for last year of supply relationship		
	(1) OLS	(2) IV	(3) IV
ΔVol_{t-1} of supp.	0.026**	0.097***	0.1494**
	(0.010)	(0.029)	(0.064)
1st moment of IVs	No	Yes	Yes
Type of volatility	Realized	Realized	Implied
Fixed effects	Yes	Yes	Yes
Observations	35,629	35,620	26,195
F-statistic	—	39.0	23.2

All specifications include year × customer × supplier industry (2SIC) fixed effects. Standard errors are two-way clustered at the customer and the supplier levels. *F*-statistics are Kleibergen-Paap. *,** ,*** indicate significance at the 10%, 5%, and 1% levels, respectively.

- Doubling volatility → 12 p.p. increase in probability link destroyed (IV)

Domar weights and uncertainty in the data

Firms with higher uncertainty have lower Domar weights, in line with the model

- Specifications, uncertainty measures and instruments from Alfaro, Bloom and Lin (2019)

	Change in Domar weight		
	(1) OLS	(2) IV	(3) IV
$\Delta\text{Volatility}_{i,t-1}$	-0.043^{***}	-0.250^{***}	-0.672^{***}
	(0.004)	(0.076)	(0.185)
1st moment of IVs	No	Yes	Yes
Type of volatility	Realized	Realized	Implied
Fixed effects	Yes	Yes	Yes
Observations	111,587	26,962	16,862
F-statistic	—	17.0	9.8

All specifications include year and firm fixed effects. Standard errors are clustered at the industry (3SIC) level. F-statistics are Kleibergen-Paap. $^{*}, ^{**}, ^{***}$ indicate significance at the 10%, 5%, and 1% levels, respectively.

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Conclusion

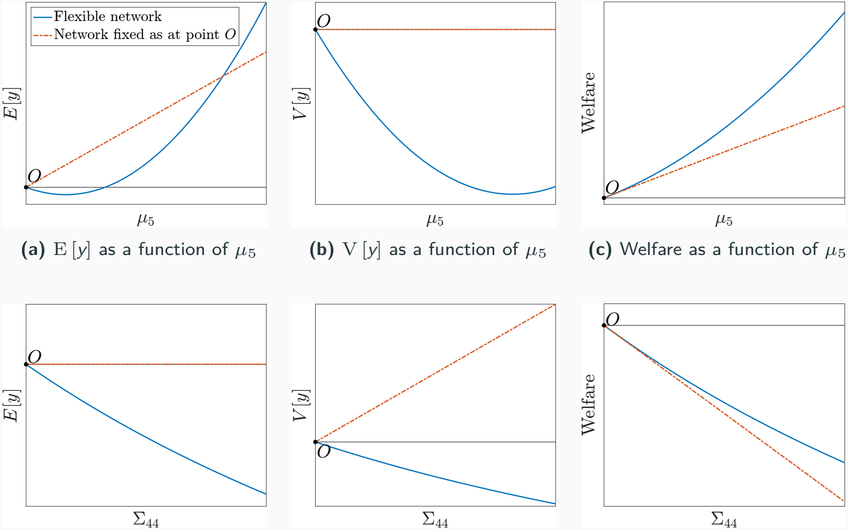
Conclusion

Main contributions

- We construct a model in which beliefs, and in particular uncertainty, affect the production network.
- During periods of high uncertainty firms purchase from safer but less productive suppliers which leads to a decline in GDP.
- Mechanism might be quantitatively important during periods of high uncertainty.

Future research

- Use firm-level data to calibrate the model — firm-to-firm network is more sparse and links are often broken.
- Use the model to evaluate the impact of uncertainty on global supply chains.



(a) $\mathrm{E}[y]$ as a function of μ_5 **(b)** $\mathrm{V}[y]$ as a function of μ_5 **(c)** Welfare as a function of μ_5

(d) $\mathrm{E}[y]$ as a function of Σ_{44} **(e)** $\mathrm{V}[y]$ as a function of Σ_{44} **(f)** Welfare as a function of Σ_{44}

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More about the data

United States data from vom Lehn and Winberry (2021)

- Input-output tables, sectoral total factor productivity, consumption shares

Mining	Utilities	Construction
Wood products	Nonmetallic minerals	Primary metals
Fabricated metals	Machinery	Computer and electronic manuf.
Electrical equipment manufacturing	Motor vehicles manufacturing	Other transportation equipment
Furniture and related manufacturing	Misc. manufacturing	Food and beverage manufacturing
Textile manufacturing	Apparel manufacturing	Paper manufacturing
Printing products manufacturing	Petroleum and coal manufacturing	Chemical manufacturing
Plastics manufacturing	Wholesale trade	Retail trade
Transportation and warehousing	Information	Finance and insurance
Real estate and rental services	Professional and technical services	Mgmt. of companies and enterprises
Admin. and waste mgmt. services	Educational services	Health care and social assistance
Arts and entertainment services	Accommodation	Food services
Other services		

- Average share of 1.4% with standard deviation of 0.5% over time

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More about the estimation

Preferences

- Consumption shares β are taken directly from the data
- Relative risk aversion ρ is **estimated**

Production technique productivity shifters

- Function A_i as described earlier
- Set ideal shares α_{ij}° to their data average
- Costs κ_{ij} of deviating from α_{ij}° are **estimated**

Process for exogenous shocks ε_t

- Random walk with drift $\varepsilon_t = \gamma + \varepsilon_{t-1} + u_t^{\varepsilon}$, with $u_t^{\varepsilon} \sim$ iid $\mathcal{N}(0, \Sigma_t)$.
- Drift vec. γ and cov. mat. Σ_t are **backed out from the data given** (ρ, κ)**.**

Loss function: Target the full set of shares α_{ijt} and the GDP growth.

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More about the calibration

- Random walk with drift $\varepsilon_t = \gamma + \varepsilon_{t-1} + u_t$, with $u_t \sim$ iid $\mathcal{N}(0, \Sigma_t)$.
  - We estimate the vector γ by averaging $\Delta\varepsilon_t = \varepsilon_t - \varepsilon_{t-1}$ over time
  - We estimate Σ_t as

$$\hat{\Sigma}_{ijt} = \sum_{s=1}^{t-1} \lambda^{t-s-1} u_{is} u_{js}$$

where $\hat{\lambda} = 0.47$ is set to the sectoral average of the corresponding parameters of a GARCH(1,1) model estimated on each sector's productivity innovation u_{it}

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Expression for $\zeta(\alpha_i)$

The function $\zeta\left(\alpha_i\right)$ is

$$\zeta\left(\alpha_i\right) = \left[\left(1 - \sum_{j=1}^{n} \alpha_{ij}\right)^{1-\sum_{j=1}^{n} \alpha_{ij}} \prod_{j=1}^{n} \alpha_{ij}^{\alpha_{ij}}\right]^{-1}$$

This functional form allows for a simple expression for the unit cost K

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Microfoundation for "one technique" restriction and cost minimization

- Each industry $i \in \{1, \ldots, n\}$ has a continuum of firms $l \in [0,1]$.
- Buyers use *shoppers* to purchase goods
  - Shoppers face an *information problem* and cannot differentiate between producers within an industry
  - Uniform allocation: each producer gets mass $Q_i dl$ of shoppers
  - Shoppers from firm m in industry j faces average price $\tilde{P}_i^{jm} = \int_0^1 \tilde{P}_{il}^{jm} dl$ for good i.
- When a shopper m from j meets a producer l from $i \rightarrow$ Nash bargaining

$$\tilde{P}_{il}^{jm} - K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right) = \gamma\left(B_i^{jm} - K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right)\right)$$

- Technique choice problem

$$\max_{\alpha_i^l \in \mathcal{A}_i} \mathrm{E}\left[\Lambda \sum_{j=0}^{n} Q_{ji} dl \int_0^1 \gamma\left(B_i^{jm} - K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right)\right) dm\right] \longrightarrow \min_{\alpha_i^l \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right)\right]$$

Microfoundation for "one technique" restriction and cost minimization

- Take limit $\gamma \to 0$
  - Nash bargaining implies $\tilde{P}_{il}^{jm} = K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right) \to \tilde{P}_{il}^{jm}$ does not depend on j, $m \to \tilde{P}_i^{jm} \equiv P_i$.
  - $K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right) \to K_i\left(\alpha_i^l, P\right)$
  - Cost minimization problem

$$\min_{\alpha_i^l \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i K_i\left(\alpha_i^l, \left\{\tilde{P}_k^{il}\right\}_k\right)\right] \longrightarrow \min_{\alpha_i^l \in \mathcal{A}_i} \mathrm{E}\left[\Lambda Q_i K_i\left(\alpha_i^l, P\right)\right]$$

  - We have the same pricing equation as in benchmark model with all firms in i choosing same technique

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Risk aversion and ρ

Given the log-normal nature of uncertainty $\rho \lesseqgtr 1$ determines whether the agent is risk-averse or not. To see this, note that when $\log C$ normally distributed, maximizing

$$\mathrm{E}\left[C^{1-\rho}\right]$$

amounts to maximizing

$$\mathrm{E}\left[\log C\right] - \frac{1}{2}\left(\rho - 1\right)\mathrm{V}\left[\log C\right].$$

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Impact of μ and Σ for α

Assumption (Weak complementarity)

For all $i \in \mathcal{N}$, the function a_i is such that $\frac{\partial^2 a_i(\alpha_i)}{\partial \alpha_{ij} \partial \alpha_{ik}} \geq 0$ for all $j \neq k$.

Lemma

Let $\alpha^* \in \text{int}(\mathcal{A})$ be the equilibrium network and suppose that the assumption holds. There exists a $\overline{\Sigma} > 0$ such that if $|\Sigma_{ij}| < \overline{\Sigma}$ for all i, j, there is a neighborhood around α^* in which

1. an increase in μ_j leads to an increase in the shares α^*_{kl} for all k, l;
2. an increase in Σ_{jj} leads to a decline in the shares α^*_{kl} for all k, l;
3. an increase in Σ_{ij} leads to a decline in the shares α^*_{kl} for all k, l.

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Pentagon example: parameter value

Details of the simulation:

1. *a* function: κ equal to 1, except $\kappa_{ii} = \infty$, α° are 1/10 except $\alpha^{\circ}_{ii} = 0$.
2. $\rho = 5$, $\beta = 0.2$. $\mu = 0.1$ except for $\mu_4 = 0.0571$. $\Sigma = 0.3 \times I_{n \times n}$ in Panel (a).
3. Panel (b): same as Panel (a) except $\mathrm{Corr}\left(\varepsilon_2, \varepsilon_4\right) = 1$.
4. Panel (c): same in Panel (a) except $\Sigma_{22} = 1$.

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Calibrated κ

We assume that $\kappa = \kappa^i \times \kappa^j$ where κ^i is an $n \times 1$ column vector and κ^j is an $1 \times (n+1)$ row vector.

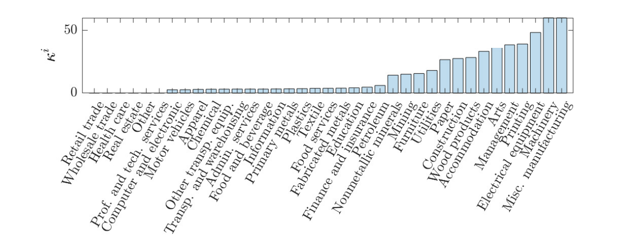
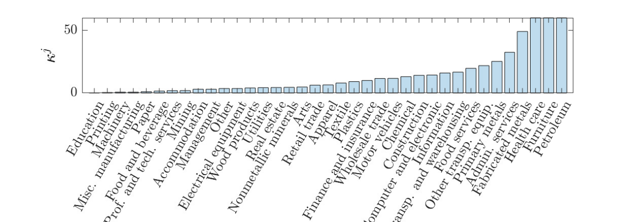


Figure 1: Vector of costs κ^i



Details of regressions

Volatility measures

- Supplier ΔVol_{t-1} is the 1-year lagged change in supplier-level volatility.
- Realized volatility is the 12-month standard deviation of daily stock returns from CRSP.
- Implied volatility is the 12-month average of daily (365-day horizon) implied volatility of at-the-money-forward call options from OptionMetrics.

Instrument

- As in Alfaro et al. 2019 "we address endogeneity concerns on firm-level volatility by instrumenting with industry-level (3SIC) non-directional exposure to 10 aggregate sources of uncertainty shocks. These include the lagged exposure to annual changes in expected volatility of energy, currencies, and 10-year treasuries (as proxied by at-the-money forward-looking implied volatilities of oil, 7 widely traded currencies, and TYVIX) and economic policy uncertainty from Baker et al 2016.. [...] To tease out the impact of 2nd moment uncertainty shocks from 1st moment aggregate shocks we also include as controls the lagged directional industry 3SIC exposure to changes in the price of each of the 10 aggregate instruments (i.e., 1st moment return shocks). These are labeled 1st moment 1st moment of IVs."

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