

# **Endogenous Production Networks Under Supply Chain Uncertainty**

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How does uncertainty affect an economy's production network and, through that channel, macroeconomic aggregates?

Economy

## As supply lines strain, some corporations rewrite production playbook

Many firms are eyeing regional networks to replace globe-circling supply chains

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Trucks line up to enter a Port of Oakland shipping terminal in Oakland, Calif. (Noah Berger/AP)

*"As the disruptions persist, executives are embracing more lasting measures, moving production to new suppliers or different countries and relaxing their traditional fixation with low costs."*

## Approach and results

We construct a model of **endogenous network formation** under uncertainty

- Firms create links with suppliers to acquire intermediate inputs
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  - Tradeoff between buying goods whose prices are **low** vs **stable**

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- **Uncertainty lowers expected GDP**
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- **Uncertainty lowers expected GDP**
  - Mechanism operates through the endogenous response of the network
  - Firms seek stability at the cost of lower efficiency
- Shocks can have **counterintuitive effects**
  - Higher firm-level expected productivity can lead to lower expected GDP
  - Higher firm-level volatility can lead to more stable GDP

## Approach and results

We **calibrate** the model to the United States economy

- The model is able to replicate the relationship between shocks and the structure of the network well.
- Letting the network adjust to shocks has large impact on welfare
- The impact of uncertainty on the network is small on average but can be substantial during high-volatility events like the Great Recession

## Survey evidence

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- COVID-19 pandemic: 70% agreed that the pandemic pushed companies to favor **higher supply chain resiliency** instead of purchasing from the lowest-cost supplier (Foley & Lardner, 2020)

## Slightly less anecdotal evidence

Use detailed U.S. data on firm-to-firm relationship (Factset 2003–2016)

Regress a dummy for link destruction on supplier uncertainty measures

- Instruments from Alfaro, Bloom and Lin (2019)

► Details

	Dummy for last year of supply relationship		
	(1) OLS	(2) IV	(3) IV
$\Delta \text{Vol}_{t-1}$ of supp.	0.026** (0.010)	0.097*** (0.029)	0.1494** (0.064)
1st moment of IVs	No	Yes	Yes
Type of volatility	Realized	Realized	Implied
Fixed effects	Yes	Yes	Yes
Observations	35,629	35,620	26,195
F-statistic	—	39.0	23.2

All specifications include year  $\times$  customer  $\times$  supplier industry (2SIC) fixed effects. Standard errors are two-way clustered at the customer and the supplier levels. F-statistics are Kleibergen-Paap. \* , \*\* , \*\*\* indicate significance at the 10%, 5%, and 1% levels, respectively.

- Doubling volatility  $\Rightarrow$  12 p.p. increase in probability link destroyed (IV)

## Related literature

### Uncertainty

- Bloom (2009); Fernandez-Villaverde et al (2011); Bloom (2014); Bloom et al (2018); and many others ...

### Exogenous production networks

- Long and Plosser (1983); Dupor (1999); Horvath (2000); Acemoglu et al (2012); Carvalho and Gabaix (2013); and many others ...

### Endogenous production networks

- Oberfield (2018); Acemoglu and Azar (2020); Boehm and Oberfield (2020); Taschereau-Dumouchel (2021); Acemoglu and Tahbaz-Salehi (2021).

## Model

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# Model

Static model with two types of agents

1. **Representative household:** owns the firms, supplies labor and consumes
2. **Firms:** produce differentiated goods using labor and intermediate inputs
  - There are  $n$  industries/goods, indexed by  $i \in \{1, \dots, n\}$
  - Representative firm that behaves **competitively**

## Production technique

Each firm  $i$  has access to a set of production techniques  $\mathcal{A}_i$ .

A technique  $\alpha_i \in \mathcal{A}_i$  specifies

- The set of intermediate inputs to be used in production
- The proportion in which these inputs are combined
- A productivity shifter  $A_i(\alpha_i)$  for the firm

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These techniques are Cobb-Douglas production functions

- We identify  $\alpha_i = (\alpha_{i1}, \dots, \alpha_{in})$  with the input shares

$$F(\alpha_i, L_i, X_i) = e^{\varepsilon_i} \zeta(\alpha_i) A_i(\alpha_i) L_i^{1 - \sum_{j=1}^n \alpha_{ij}} \prod_{j=1}^n X_{ij}^{\alpha_{ij}},$$

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Set  $\mathcal{A}_i$  allows adjustment along the intensive and extensive margins

$$\mathcal{A}_i = \left\{ \alpha \in [0, 1]^n : \sum_{j=1}^n \alpha_j \leq \bar{\alpha}_i < 1 \right\}.$$



## Production technique

### Assumption

$A_i(\alpha_i)$  is smooth and strictly log-concave.

### Implications:

- There are ideal input shares  $\alpha_{ij}^\circ$  that maximize  $A_i$
- Deviating from these ideal shares reduces productivity

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## Example

$$\log A_i(\alpha_i) = - \sum_{j=1}^n \kappa_{ij} (\alpha_{ij} - \alpha_{ij}^\circ)^2 - \kappa_{i0} \left( \sum_{j=1}^n \alpha_{ij} - \sum_{j=1}^n \alpha_{ij}^\circ \right)^2,$$

## Source of uncertainty

Firms are subject to **productivity shocks**  $\varepsilon$

- Shocks are jointly normal  $\varepsilon = (\varepsilon_1, \dots, \varepsilon_n) \sim \mathcal{N}(\mu, \Sigma)$ 
  - $\mu$  captures **optimism/pessimism** about productivity
  - $\Sigma$  captures **uncertainty** and **correlations**

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### Key restriction

Each firm/industry  $i$  can only adopt one production technique.

## Production technique

### Example

- A car manufacturer can use steel or carbon fiber for certain parts
- All else equal the manufacturer prefers carbon fiber
- If carbon fiber is expensive ( $\mu_{\text{carbon fiber}}$  small) or its price is volatile ( $\Sigma_{\text{carbon fiber}}$  large), the manufacturer switches to steel.

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### A production network

- Together the techniques  $\alpha \in \mathcal{A}$  of the firms form a production network.
- Beliefs ( $\mu, \Sigma$ ) affect the structure of the network.
- Model allows for intensive and extensive adjustments in the network.

## Household

The representative household makes decisions after  $\varepsilon$  is realized

- Owns the firms
- Supplies one unit of labor inelastically
- Chooses *state-contingent consumption*  $(C_1, \dots, C_n)$  to maximize

$$u \left( \left( \frac{C_1}{\beta_1} \right)^{\beta_1} \times \cdots \times \left( \frac{C_n}{\beta_n} \right)^{\beta_n} \right),$$

subject to the *state-by-state* budget constraint

$$\sum_{i=1}^n P_i C_i \leq 1,$$

where  $u$  is CRRA with relative risk aversion  $\rho \geq 1$ .

▶ Details

## Household

Two key quantities from the household's problem

1. The **stochastic discount factor** of the household is

$$\Lambda = u'(Y) / \bar{P}$$

where  $Y = \prod_{i=1}^n (\beta_i^{-1} C_i)^{\beta_i}$  is **consumption** and **GDP** and  $\bar{P} = \prod_{i=1}^n P_i^{\beta_i}$ .

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2. **GDP as a function of prices**

$$y = -\beta' p,$$

where  $y = \log Y$ ,  $p = (\log(P_1), \dots, \log(P_n))$  and  $\beta = (\beta_1, \dots, \beta_n)$ .

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⇒ We only need prices to compute GDP

## Problem of the firm: Labor and intermediate inputs

For a given technique  $\alpha_i$ , the **cost minimization** problem of the firm is

$$K_i(\alpha_i, P) = \min_{L_i, X_i} \left( L_i + \sum_{j=1}^n P_j X_{ij} \right), \text{ subject to } F(\alpha_i, L_i, X_i) \geq 1$$

where  $K_i(\alpha_i, P)$  is the **unit cost** of production.

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Note:

1. **Constant returns to scale**  $\Rightarrow K_i$  does not depend on the size of the firm.
2. Given that each technique is Cobb-Douglas,

$$K_i(\alpha_i, P) = \frac{1}{e^{\varepsilon_i} A_i(\alpha_i)} \prod_{j=1}^n P_j^{\alpha_{ij}}.$$

3. Since we have perfect competition, it must be that in **equilibrium**

$$P_i = K_i(\alpha_i, P) \text{ for all } i \in \{1, \dots, n\}.$$

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$\Rightarrow$  For a given network  $\alpha$  we can compute equilibrium prices

## Problem of the firm: Production technique

Firm  $i$  chooses a technique  $\alpha_i \in \mathcal{A}_i$  to solve

$$\alpha_i^* \in \arg \max_{\alpha_i \in \mathcal{A}_i} E[\Lambda Q_i(P_i - K_i(\alpha_i, P))]$$

where  $Q_i$  is the equilibrium demand for good  $i$ .

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### Lemma

$\lambda = \log(\Lambda)$ ,  $k_i = \log(K_i)$ ,  $q_i = \log(Q_i)$  are normally distributed so that

$$\begin{aligned}\alpha_i^* \in \arg \min_{\alpha_i \in \mathcal{A}_i} & E[k_i(\alpha_i, \alpha^*)] + \frac{1}{2} V[k_i(\alpha_i, \alpha^*)] \\ & + \text{Cov}[k_i(\alpha_i, \alpha^*), \lambda(\alpha^*) + q_i(\alpha^*)].\end{aligned}$$

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Three channels:

1. Minimize expectation  $E[k_i]$  of unit cost
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2. Minimize variance  $V[k_i]$  of unit cost

$$V[k_i] = \text{cte} + \underbrace{\sum_{j=1}^n \alpha_{ij}^2 V[p_j]}_{\text{stable prices}} + \underbrace{\sum_{j \neq k} \alpha_{ij} \alpha_{ik} \text{Cov}[p_j, p_k]}_{\text{uncorrelated prices}} + \underbrace{2 \text{Cov}\left[-\varepsilon_i, \sum_{j=1}^n \alpha_{ij} p_j\right]}_{\text{uncorrelated with own TFP}}$$

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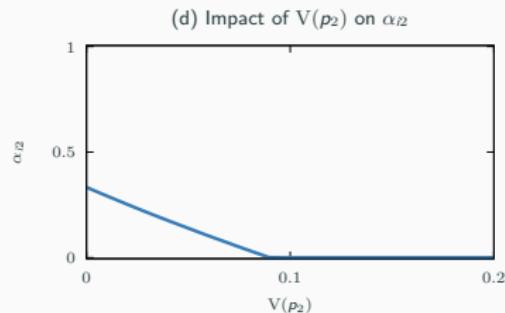
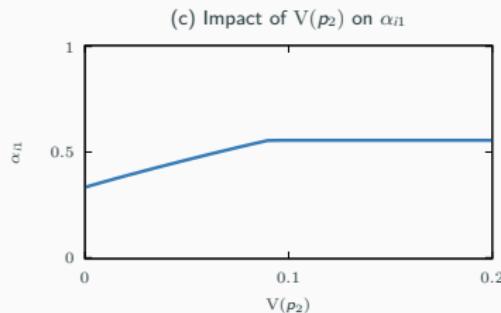
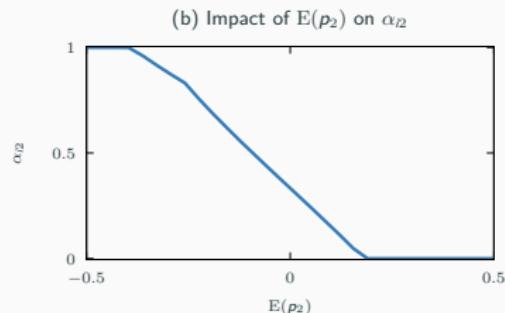
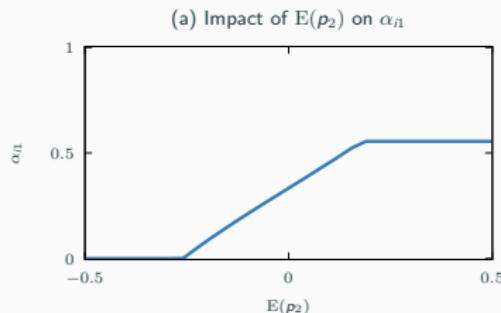
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3. Importance of aggregate conditions through  $\text{Cov}[k_i, \lambda + q_i]$ 
  - Seek low unit costs when **high demand** ( $q_i$ ) and **high marginal utility** ( $\lambda$ ).
  - Because of the SDF the firm inherits the **risk aversion of the household**.

## Back to our example

- Firm  $i$  can use steel (input 1) or carbon fiber (input 2)
- Look at impact of  $E(p_2)$  and  $V(p_2)$  on the shares  $\alpha_{i1}$  and  $\alpha_{i2}$



## Definition

An equilibrium is a choice of technique for every firm  $\alpha^*$  and a stochastic tuple  $(P^*, C^*, L^*, X^*, Q^*, \Lambda^*)$  such that:

1. (Unit cost pricing) For each  $i \in \{1, \dots, n\}$ ,  $P_i^* = K_i(\alpha_i^*, P^*)$ .
2. (Optimal technique choice) For each  $i \in \{1, \dots, n\}$ , factor demand  $L_i^*$  and  $X_i^*$ , and the technology choice  $\alpha_i^* \in \mathcal{A}_i$  solves the firm's problem.
3. (Consumer maximization) The consumption vector  $C^*$  solves the household's problem.
4. (Market clearing) For each  $i \in \{1, \dots, n\}$ ,

$$Q_i^* = C_i^* + \sum_{j=1}^n X_{ji}^*,$$

$$Q_i^* = F_i(\alpha_i^*, L_i^*, X_i^*),$$

$$\sum_{i=1}^n L_i^* = 1.$$

## **Fixed-network economy**

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## GDP in a fixed-network economy

### Lemma

Under a given network  $\alpha$ , the log of GDP  $y = \log Y$  is given by

$$y = \beta' \mathcal{L}(\alpha) (\varepsilon + a(\alpha)),$$

where  $a(\alpha) = (\log A_1(\alpha_1), \dots, \log A_n(\alpha_n))$  and  $\mathcal{L}(\alpha) = (I - \alpha)^{-1}$  is the Leontief inverse.

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The importance of a firm's shock for GDP is given by its **Domar weight**

$$\omega_i := \beta' \mathcal{L}(\alpha) 1_i = \underbrace{\frac{P_i Q_i}{PC}}_{\text{sales share in value added}}$$

Domar weights capture the **importance of a firm as a supplier**

1. Role of  $\beta$ : Goods in high demand have larger impact on GDP
2. Role of  $\mathcal{L} = I + \alpha + \alpha^2 + \dots$ : Important suppliers matter more for GDP

## Impact of shocks on GDP

Moments of  $y$ :  $E[y] = \beta' \mathcal{L}(\alpha) (\mu + a(\alpha))$  and  $V[y] = \beta' \mathcal{L}(\alpha) \Sigma \mathcal{L}(\alpha)' \beta$

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Proposition (Hulten's Theorem in expectation)

For a fixed network  $\alpha$ ,

1. The impact of  $\mu_i$  on expected GDP  $E[y]$  is given by

$$\frac{\partial E[y]}{\partial \mu_i} = \omega_i.$$

2. The impact of  $\Sigma_{ij}$  on the variance of GDP  $V[y]$  is given by

$$\frac{\partial V[y]}{\partial \Sigma_{ij}} = \begin{cases} \omega_i^2 & i = j, \\ 2\omega_i \omega_j & i \neq j. \end{cases}$$

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$$\frac{\partial E[y]}{\partial \mu_i} = \omega_i.$$

2. The impact of  $\Sigma_{ij}$  on the variance of GDP  $V[y]$  is given by

$$\frac{\partial V[y]}{\partial \Sigma_{ij}} = \begin{cases} \omega_i^2 & i = j, \\ 2\omega_i \omega_j & i \neq j. \end{cases}$$

Under a fixed network:

1. Sales shares  $\omega$  are enough to understand GDP (Hulten's Theorem).
2. Since  $\omega_i > 0$  shocks have intuitive impact.

## **Flexible-network economy**

---

### Proposition

There exists an efficient equilibrium. Furthermore, the equilibrium production network  $\alpha^*$  solves

$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} E[y(\alpha)] - \frac{1}{2} (\rho - 1) V[y(\alpha)]$$

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$$\mathcal{W} := \max_{\alpha \in \mathcal{A}} E[y(\alpha)] - \frac{1}{2} (\rho - 1) V[y(\alpha)]$$

## Implications:

1. The economy is undistorted by externalities or imperfections.
2. Complicated network formation problem  $\Rightarrow$  simple optimization problem.

## **Economic forces at work**

---

## Impact of beliefs on the network

Domar weights are constant when the network is fixed. When it is flexible...

### Proposition

The Domar weight  $\omega_i$  of firm  $i$  is increasing in  $\mu_i$  and decreasing in  $\Sigma_{ii}$ .

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This result is intuitive

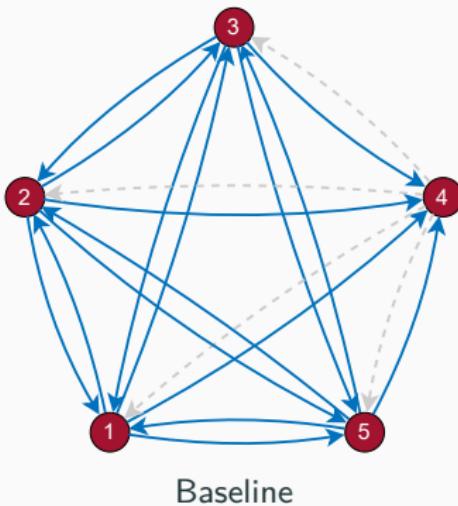
1. **Equilibrium's perspective:** Firms rely more on high- $\mu_i$  and low- $\Sigma_{ii}$  firms as suppliers.
2. **Planner's perspective:** The importance of high- $\mu_i$  and low- $\Sigma_{ii}$  firms for welfare is magnified if they are important suppliers.

▶ Impact on  $\alpha$

## Example: Impact of beliefs on the network

Simple economy

- Five firms with uncorrelated shocks.
- Firms are identical except that  $i = 4$  is less productive ( $\mu_4$  low).

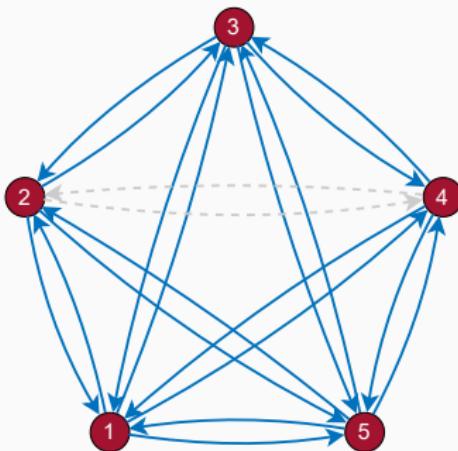


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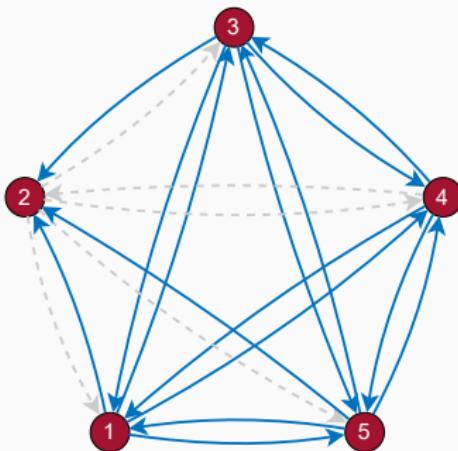
Small increase in  $\Sigma_{22}$

▶ Details

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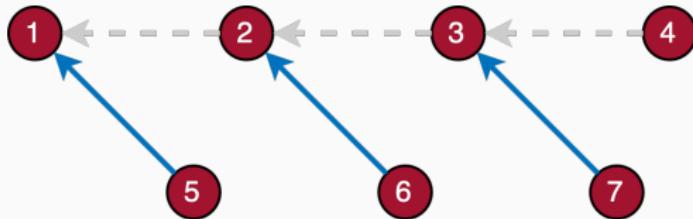
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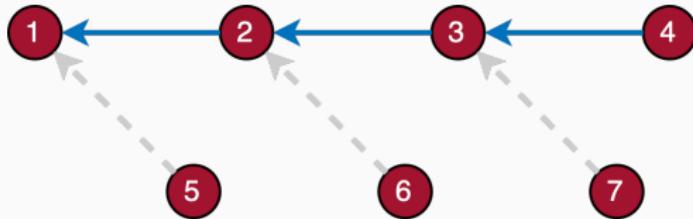
Large increase in  $\Sigma_{22}$

▶ Details

## Example: Cascading effect of uncertainty



(a) High uncertainty about  $\varepsilon_4$



(b) Low uncertainty about  $\varepsilon_4$

## Effect of uncertainty on GDP

### Proposition

Uncertainty lowers the expected value of GDP in equilibrium, such that  $E[y]$  is largest when  $\Sigma = 0_{n \times n}$ .

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This result is intuitive

1. **Equilibrium's perspective:** When there is no uncertainty firms purchase from the lowest expected price (highest expected utility) supplier. This maximizes expected GDP.
2. **Planner's perspective:** When  $\Sigma = 0_{n \times n}$  the variance of GDP is 0. Only objective is to maximize  $E[y]$ .

## Proposition

1. The impact of an increase in  $\mu_i$  on expected welfare is given by

$$\frac{d\mathcal{W}}{d\mu_i} = \frac{\partial E[y]}{\partial \mu_i} = \omega_i.$$

2. The impact of an increase in  $\Sigma_{ij}$  on expected welfare is given by

$$\frac{d\mathcal{W}}{d\Sigma_{ij}} = \begin{cases} -\frac{1}{2}(\rho - 1) \left( \frac{\partial E[y]}{\partial \mu_i} \right)^2 = -\frac{1}{2}(\rho - 1)\omega_i^2 & i = j, \\ -(\rho - 1) \frac{\partial E[y]}{\partial \mu_i} \frac{\partial E[y]}{\partial \mu_j} = -(\rho - 1)\omega_i\omega_j & i \neq j. \end{cases}$$

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The impact of shocks on welfare is intuitive

1. Higher productivity leads to higher welfare.
2. Higher correlation or uncertainty leads to lower welfare.

## Effect of shocks on GDP

Impact of shocks on

- Welfare: intuitive
- GDP when the network is fixed: intuitive
- GDP when the network is flexible: ???

# Effect of shocks on GDP

Impact of shocks on

- Welfare: intuitive
- GDP when the network is fixed: intuitive
- GDP when the network is flexible: ???

Decompose a shock to, say,  $\mu_i$  as

$$\frac{dE[y]}{d\mu_i} = \underbrace{\frac{\partial E[y]}{\partial \mu_i}}_{\text{direct impact with fixed network}} + \underbrace{\frac{\partial E[y]}{\partial \alpha} \frac{d\alpha}{d\mu_i}}_{\text{network adjustment}}$$

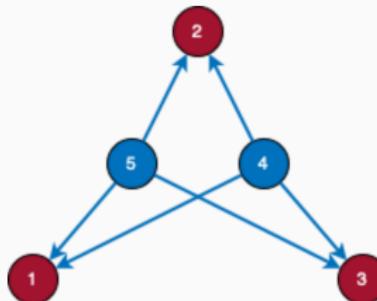
Two effects:

1. **Direct impact** keeping the network fixed = Domar weight
2. **Indirect impact** that take into account the network adjustment = ???

## Example: Surprising impact of a shock

Simple economy:

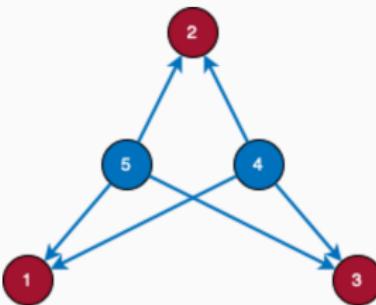
- 5 firms with uncorrelated shocks
- Firms are identical except that
  - Firm 4 is risky (high  $\Sigma_{44}$ )
  - Firm 5 is safe (low  $\Sigma_{55}$ ) but unproductive (low  $\mu_5$ )
  - $\beta_4 = \beta_5$  are very small



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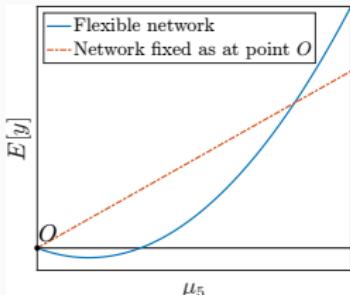
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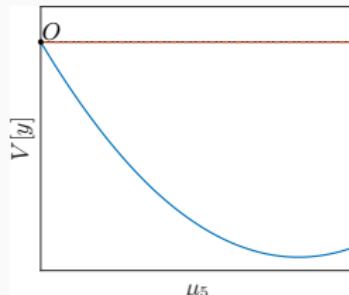
We are going to consider two shocks

1. An increase in  $\mu_5$
2. An increase in  $\Sigma_{44}$

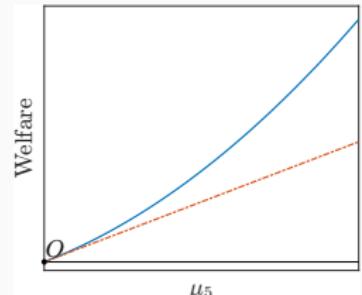
## Example: Surprising impact of a shock



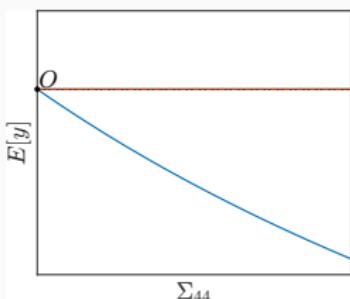
(a)  $E[y]$  as a function of  $\mu_5$



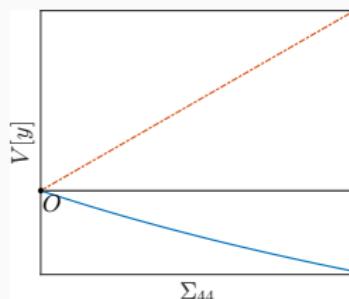
(b)  $V[y]$  as a function of  $\mu_5$



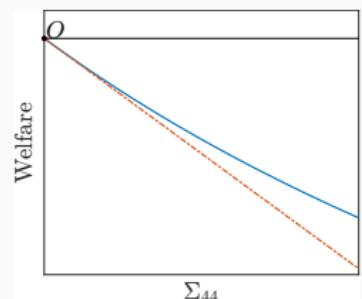
(c) Welfare as a function of  $\mu_5$



(d)  $E[y]$  as a function of  $\Sigma_{44}$



(e)  $V[y]$  as a function of  $\Sigma_{44}$



(f) Welfare as a function of  $\Sigma_{44}$

## **Quantitative exploration**

---

# Data

Data about the **United States** from vom Lehn and Winberry (2021)

- Input-output tables, sectoral total factor productivity, consumption shares
  - 37 sectors, from 1947 to 2020:
- 

Mining	Utilities	Construction
Wood products	Nonmetallic minerals	Primary metals
Fabricated metals	Machinery	Computer and electronic manuf.
Electrical equipment manufacturing	Motor vehicles manufacturing	Other transportation equipment
Furniture and related manufacturing	Misc. manufacturing	Food and beverage manufacturing
Textile manufacturing	Apparel manufacturing	Paper manufacturing
Printing products manufacturing	Petroleum and coal manufacturing	Chemical manufacturing
Plastics manufacturing	Wholesale trade	Retail trade
Transportation and warehousing	Information	Finance and insurance
Real estate and rental services	Professional and technical services	Mgmt. of companies and enterprises
Admin. and waste mgmt. services	Educational services	Health care and social assistance
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---

- Average share of 1.4% with standard deviation of 0.5% over time

# Calibration

## Preferences

- Consumption shares  $\beta$  are taken directly from the data
- Relative risk aversion  $\rho$  is **estimated**

## Production technique productivity shifters

- Function  $A_i$  as described earlier
- Set ideal shares  $\alpha_{ij}^o$  to their data average
- Costs  $\kappa_{ij}$  of deviating from  $\alpha_{ij}^o$  are **estimated**

## Process for exogenous shocks $\varepsilon_t$

- Random walk with drift  $\varepsilon_t = \gamma + \varepsilon_{t-1} + u_t^\varepsilon$ , with  $u_t^\varepsilon \sim \text{iid } \mathcal{N}(0, \Sigma_t)$ .
- Drift vec.  $\gamma$  and cov. mat.  $\Sigma_t$  are **backed out from the data given**  $(\rho, \kappa)$ .

**Loss function:** Target the full set of shares  $\alpha_{ijt}$  and the GDP growth.

## Calibrated economy

Estimated risk aversion:  $\rho = 4.27$

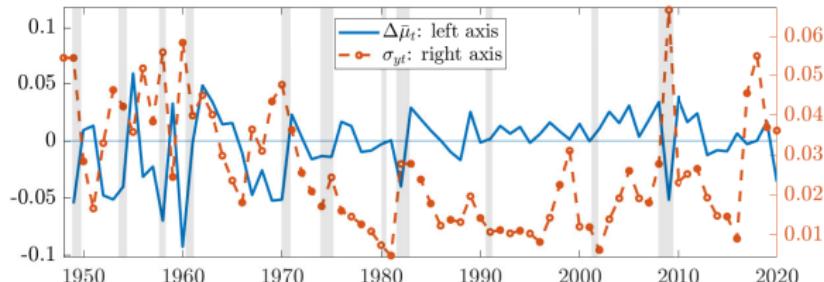
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### Evolution of beliefs

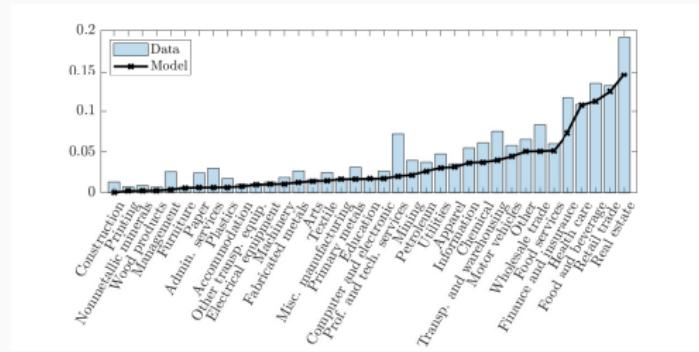


$$\Delta\bar{\mu}_t = \sum_{j=1}^n \omega_{jt} \Delta\mu_{jt} \text{ and } \sigma_{yt} = \sqrt{V[y]} = \sqrt{\omega_t' \Sigma_t \omega_t}.$$



# Calibrated economy: Domar weights

The calibrated **Domar weights** fit the data well



**Beliefs** have the expected impact on **Domar weights**

	Statistic	Data	Model
(1)	Average Domar weight $\bar{\omega}_j$	0.047	0.032
(2)	Standard deviation $\sigma(\omega_j)$	0.0050	0.0021
(3)	Coefficient of variation $\sigma(\omega_j) / \bar{\omega}_j$	0.11	0.07
(4)	$\text{Corr}(\omega_{jt}, \mu_{jt})$	0.08	0.08
(5)	$\text{Corr}(\omega_{jt}, \Sigma_{jjt})$	-0.37	-0.31

# Isolating the mechanism

Two useful counterfactuals

1. Fixed-network economy
  - to capture the full effect of network adjustments
2. Risk-neutral economy ( $\rho = 1$ )
  - to capture the impact of uncertainty

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	Fixed network	Risk neutral
Expected GDP $E[y(\alpha)]$	+2.122%	-0.008%
Std. dev. of GDP $\sqrt{V[y(\alpha)]}$	+0.131%	-0.105%
Welfare $\mathcal{W}$	+2.109%	+0.010%

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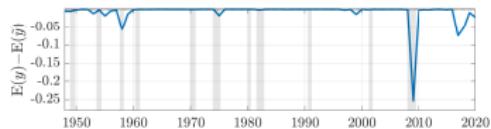
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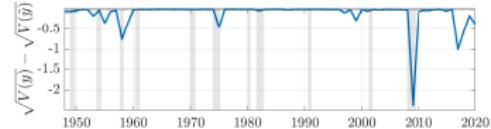
Recall: only impact of uncertainty on expected GDP is through the network

# The Great Recession

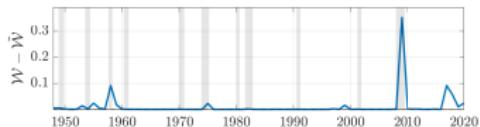
Calibrated model vs risk-neutral alternative



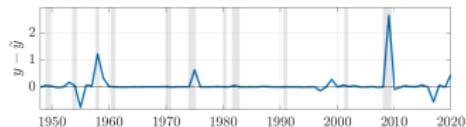
(g) Difference in expected GDP



(h) Difference in expected std. dev. of GDP



(i) Difference in expected welfare



(j) Difference in realized GDP

- During periods of **high volatility, uncertainty matters.**

## **Conclusion**

---

# Conclusion

## Main contributions

- We construct a model in which beliefs, and in particular uncertainty, affect the production network.
- During periods of high uncertainty firms purchase from safer but less productive suppliers which leads to a decline in GDP.
- Mechanism might be quantitatively important during periods of high uncertainty.

## Future research

- Use firm-level data to calibrate the model — firm-to-firm network is more sparse and links are often broken.
- Use the model to evaluate the impact of uncertainty on global supply chains.

## Details of regressions

### Volatility measures

- Supplier  $\Delta Vol_{t-1}$  is the 1-year lagged change in supplier-level volatility.
- Realized volatility is the 12-month standard deviation of daily stock returns from CRSP.
- Implied volatility is the 12-month average of daily (365-day horizon) implied volatility of at-the-money-forward call options from OptionMetrics.

### Instrument

- As in Alfaro et al. 2019 “we address endogeneity concerns on firm-level volatility by instrumenting with industry-level (3SIC) non-directional exposure to 10 aggregate sources of uncertainty shocks. These include the lagged exposure to annual changes in expected volatility of energy, currencies, and 10-year treasuries (as proxied by at-the-money forward-looking implied volatilities of oil, 7 widely traded currencies, and TYVIX) and economic policy uncertainty from Baker et al 2016.. [...] To tease out the impact of 2nd moment uncertainty shocks from 1st moment aggregate shocks we also include as controls the lagged directional industry 3SIC exposure to changes in the price of each of the 10 aggregate instruments (i.e., 1st moment return shocks). These are labeled 1st moment 1st moment of IVs.”

## Expression for $\zeta(\alpha_i)$

The function  $\zeta(\alpha_i)$  is

$$\zeta(\alpha_i) = \left[ \left( 1 - \sum_{j=1}^n \alpha_{ij} \right)^{1 - \sum_{j=1}^n \alpha_{ij}} \prod_{j=1}^n \alpha_{ij}^{\alpha_{ij}} \right]^{-1}$$

This functional form allows for a simple expression for the unit cost  $K$

## Risk aversion and $\rho$

Given the log-normal nature of uncertainty  $\rho \leq 1$  determines whether the agent is risk-averse or not. To see this, note that when  $\log C$  normally distributed, maximizing

$$E [C^{1-\rho}]$$

amounts to maximizing

$$E [\log C] - \frac{1}{2} (\rho - 1) V [\log C].$$

## Impact of $\mu$ and $\Sigma$ for $\alpha$

### Assumption (Weak complementarity)

For all  $i \in \mathcal{N}$ , the function  $a_i$  is such that  $\frac{\partial^2 a_i(\alpha_i)}{\partial \alpha_{ij} \partial \alpha_{ik}} \geq 0$  for all  $j \neq k$ .

### Lemma

Let  $\alpha^* \in \text{int}(\mathcal{A})$  be the equilibrium network and suppose that the assumption holds. There exists a  $\bar{\Sigma} > 0$  such that if  $|\Sigma_{ij}| < \bar{\Sigma}$  for all  $i, j$ , there is a neighborhood around  $\alpha^*$  in which

1. an increase in  $\mu_j$  leads to an increase in the shares  $\alpha_{kl}^*$  for all  $k, l$ ;
2. an increase in  $\Sigma_{jj}$  leads to a decline in the shares  $\alpha_{kl}^*$  for all  $k, l$ ;
3. an increase in  $\Sigma_{ij}$  leads to a decline in the shares  $\alpha_{kl}^*$  for all  $k, l$ .

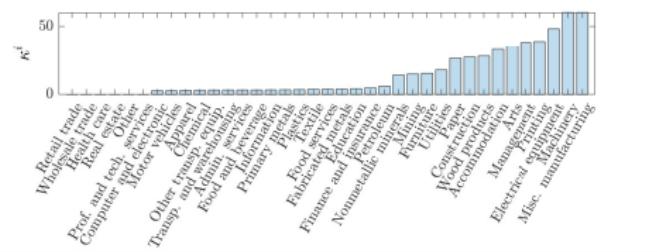
## Pentagon example: parameter value

Details of the simulation:

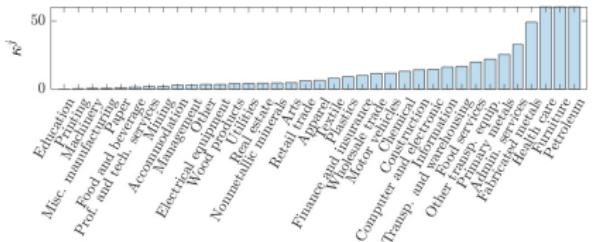
1.  $a$  function:  $\kappa$  equal to 1, except  $\kappa_{ii} = \infty$ ,  $\alpha^\circ$  are 1/10 except  $\alpha_{ii}^\circ = 0$ .
2.  $\rho = 5$ ,  $\beta = 0.2$ .  $\mu = 0.1$  except for  $\mu_4 = 0.0571$ .  $\Sigma = 0.3 \times I_{n \times n}$  in Panel (a).
3. Panel (b): same as Panel (a) except  $\text{Corr}(\varepsilon_2, \varepsilon_4) = 1$ .
4. Panel (c): same in Panel (a) except  $\Sigma_{22} = 1$ .

## Calibrated $\kappa$

We assume that  $\kappa = \kappa^i \times \kappa^j$  where  $\kappa^i$  is an  $n \times 1$  column vector and  $\kappa^j$  is an  $1 \times (n+1)$  row vector.



**Figure 1:** Vector of costs  $\kappa^i$



**Figure 2:** Vector of costs  $\kappa^j$