

Cascades and Fluctuations in an Economy with an Endogenous Production Network

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Introduction

- Production in modern economies involves a complex network of producers supplying and demanding goods from each other
- The shape of this network
 - ▶ is an important determinant of how micro shocks aggregate into macro fluctuations
 - ▶ is also constantly changing in response to micro shocks
 - For instance, after a severe shock a producer might shut down which might lead its neighbors to shut down as well, etc...
 - Cascade of shutdowns that spreads through the network

This paper proposes a

Theory of network formation and aggregate fluctuations

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Overview of the Model

Simple framework

- Set of n firms that use inputs from connected firms
- Fixed cost to operate
 - ▶ Firms operate or not depending on economic conditions
 - Links between firms are active or not
 - Endogenously shape the network

Modeling choice motivated by the data

- In the U.S., about 70% of link destructions occur with exit of supplier or customer

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Key economic force: Complementarities in operation decisions of nearby firms

Efficient organization of production

- Create tightly connected clusters centered around productive firms
- Small changes can trigger large reorganization of the network

Cascades of firm shutdowns

- Well-connected firms are hard to topple but create big cascades
- Elasticities of substitution matter for size and propagation of cascades

Aggregate fluctuations

- Recessions feature fewer well-connected firms and less clustering
- Allowing the network to adjust yields substantially smaller fluctuations

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Key difficulty in solving the planner's problem

- The Karush-Kuhn-Tucker conditions do not apply
 - ▶ Reason 1: Discrete choice about network formation
 - ▶ Reason 2: Complementarities in decisions of nearby firms
- Novel approach that relies on a *reshaping* of the problem
 - ▶ Theory: works if network is highly connected
 - ▶ Practice: works very well even for sparse networks

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- Endogenous network formation
 - ▶ Atalay et al (2011), Oberfield (2013), Carvalho and Voigtländer (2014), Acemoglu and Azar (2017)
- Network of sectors and fluctuations
 - ▶ Long and Plosser (1983), Horvath (1998), Dupor (1999), Acemoglu et al (2012), Baqaee (2016), Acemoglu et al (2016), Lim (2017), Baqaee and Farhi (2017)
- Non-convex adjustments in networks
 - ▶ Bak, Chen, Woodford and Scheinkman (1993), Elliott, Golub and Jackson (2014)
- Propagation of shocks through networks
 - ▶ Barrot and Sauvagnat (2016), Carvalho et al (2017)

I. Model

Model

- There are n units of production (firm) indexed by $j \in \{1, \dots, n\}$
 - ▶ Each unit produces a differentiated good
 - ▶ Differentiated goods can be used to
 - produce a final good
 - produce other differentiated goods
- Representative household
 - ▶ Consumes the final good
 - ▶ Supplies L units of labor inelastically

$$Y \equiv \left(\sum_{j=1}^n \left(y_j^0 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

Model

- Firm j produces good j

$$y_j = \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \left(\sum_{i=1}^n x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

- Firm j can only use good i as input if there is a *connection* from firm i to j
 - ▶ $\Omega_{ij} = 1$ if connection and $\Omega_{ij} = 0$ otherwise
 - ▶ A connection can be *active* or *inactive*
 - ▶ Matrix Ω is *exogenous*
- A firm can only produce if it pays a fixed cost f in units of labor
 - ▶ $\theta_j = 1$ if j is operating and $\theta_j = 0$ otherwise
 - ▶ Vector θ is *endogenous*

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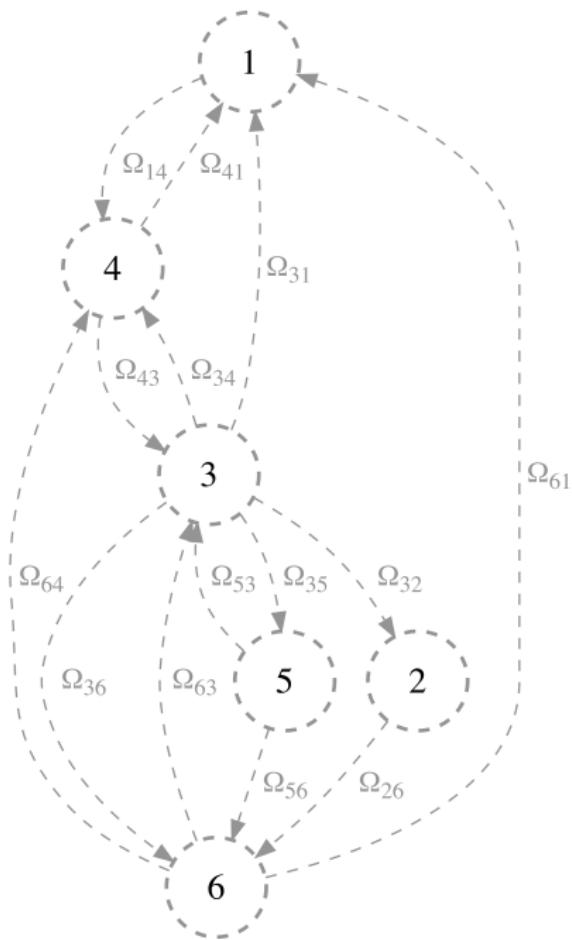
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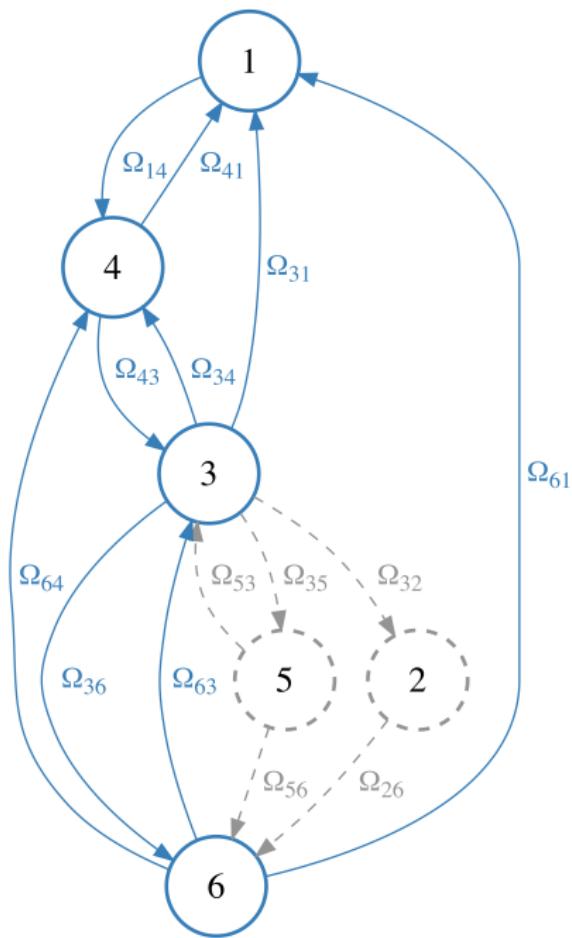
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Social Planner

Problem \mathcal{P}_{SP} of a social planner

$$\max_{\substack{y^0, x, l \\ \theta \in \{0,1\}^n}} \left(\sum_{j=1}^n \left(y_j^0 \right)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}$$

subject to

1. a resource constraint for each good j

$$y_j^0 + \sum_{k=1}^n x_{jk} \leq \frac{A}{\alpha^\alpha (1-\alpha)^{1-\alpha}} z_j \theta_j \left(\sum_{i=1}^n \Omega_{ij} x_{ij}^{\frac{\epsilon-1}{\epsilon}} \right)^{\alpha \frac{\epsilon}{\epsilon-1}} l_j^{1-\alpha}$$

2. a resource constraint on labor

$$\sum_{j=1}^n l_j + f \sum_{j=1}^n \theta_j \leq L$$

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1. a resource constraint for each good j (**Lagrange multiplier:** λ_j)

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2. a resource constraint on labor (**Lagrange multiplier:** w)

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II. Social Planner with Exogenous θ

Define $q_j = w/\lambda_j$

- From the FOCs, output is $(1 - \alpha) y_j = q_j l_j$
- q_j is the *labor productivity* of firm j

Proposition 1

In the efficient allocation,

$$q_j = z_j \theta_j A \left(\sum_{i=1}^n \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}} \quad (1)$$

Furthermore, there is a unique vector q that satisfies (1).

Social Planner with Exogenous θ

Knowing q we can solve for all other quantities easily.

Lemma 1

Aggregate output is

$$Y = Q \left(L - f \sum_{j=1}^n \theta_j \right)$$

where $Q \equiv \left(\sum_{j=1}^n q_j^{\sigma-1} \right)^{\frac{1}{\sigma-1}}$ is aggregate labor productivity.

III. Social Planner with Endogenous θ

Social Planner with Endogenous θ

Planner's problem is now

$$\max_{\theta \in \{0,1\}^n} Q \left(L - f \sum_{j=1}^n \theta_j \right)$$

with

$$q_j = z_j \theta_j A \left(\sum_{i=1}^n \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}}$$

Trade-off: making firm j produce ($\theta_j = 1$)

- increases labor productivity of the network (Q)
- reduces the amount of labor into production $\left(L - f \sum_{j=1}^n \theta_j \right)$

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“Very hard problem” (MINLP — NP Hard)

- The set $\theta \in \{0, 1\}^n$ is not convex
- Objective function is not concave

Naive approach

- For any vector $\theta \in \{0, 1\}^n$ iterate on q and evaluate the objective function
- 2^n vectors θ to try ($\approx 10^6$ configurations for 20 firms)
- Impossible for n large

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Alternative approach

New solution approach: Find an alternative problem such that

- P1 The alternative problem is easy to solve
- P2 A solution to the alternative problem also solves \mathcal{P}_{SP}

Social Planner with Endogenous θ

Consider the relaxed and reshaped problem \mathcal{P}_{RR}

$$\max_{\theta \in \{0,1\}^n} Q \left(L - f \sum_{j=1}^n \theta_j \right)$$

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Parameters $a > 0$ and $b \geq 0$ are *reshaping constants*

- Reshape the objective function away from optimum (i.e. when $0 < \theta_j < 1$)
 - ▶ For a : if $\theta_j \in \{0,1\}$ then $\theta_j^a = \theta_j$
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P1 The alternative problem \mathcal{P}_{RR} is easy to solve

Proposition 2

If $\Omega_{jj} = c_i d_j$ for some vectors c and d then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to \mathcal{P}_{RR} .

Proposition 3

Let $\sigma = \epsilon$ and suppose that $f > 0$ and $\bar{z} - z > 0$ are not too big. If Ω is sufficiently connected, then the Karush-Kuhn-Tucker conditions are necessary and sufficient to characterize a solution to \mathcal{P}_{RR} .

These propositions

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- Later: Test the approach on thousands of economies

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P2 A solution to the alternative problem also solves \mathcal{P}_{SP}

Proposition 4

If θ^* solves \mathcal{P}_{RR} and that $\theta_j^* \in \{0, 1\}$ for all j , then θ^* also solves \mathcal{P}_{SP} .

Solution θ^* to \mathcal{P}_{RR} is such that $\theta_j^* \in \{0, 1\}$ for all j (P2) if

- there are many firms and they are sufficiently connected
- the reshaping parameters take the values

$$a = \frac{1}{\sigma - 1} \quad \text{and} \quad b = 1 - \frac{\epsilon - 1}{\sigma - 1} \quad (*)$$

In practice, we verify numerically that this is satisfied

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Reshaping

Intuition:

- First-order condition on θ_j :

$$\text{Marginal Benefit}(\theta_j, F(\theta)) - \text{Marginal Cost}(\theta_j, G(\theta)) = \bar{\mu}_j - \underline{\mu}_j$$

- Under \star the marginal benefit of θ_j only depends on θ_j through aggregates
- For large connected network F and G are independent of θ_j



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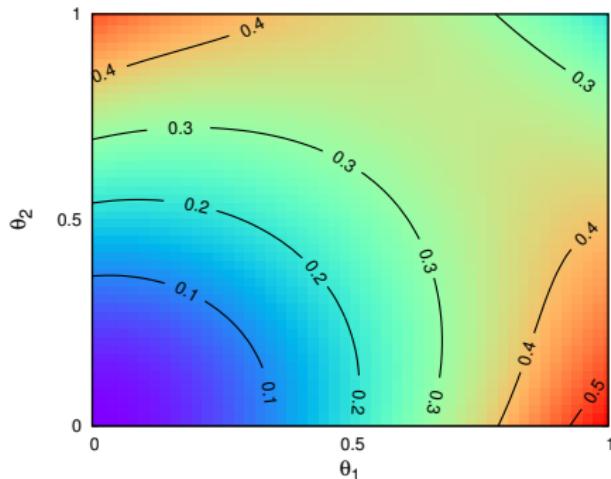
- Under \star the marginal benefit of θ_j only depends on θ_j through aggregates
- For large connected network F and G are independent of θ_j

▶ FOCs

Example with $n = 2$

Relaxed problem **without** reshaping

$$V(\theta) = Q(\theta) \left(L - f \sum_{j=1}^n \theta_j \right) \text{ with } q_j = z_j \theta_j A \left(\sum_{i=1}^n \Omega_{ij} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}}$$



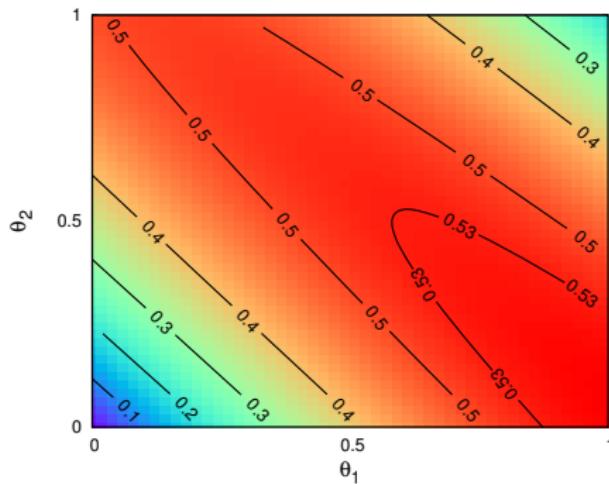
Problem: V is not concave

- ⇒ First-order conditions are not sufficient
- ⇒ Numerical algorithm can get stuck in local maxima

Example with $n = 2$

Relaxed problem **with** reshaping

$$V(\theta) = Q(\theta) \left(L - f \sum_{j=1}^n \theta_j \right) \text{ with } q_j = z_j \theta_j^{\frac{1}{\sigma-1}} A \left(\sum_{i=1}^n \Omega_{ij} \theta_i^{1 - \frac{\epsilon-1}{\sigma-1}} q_i^{\epsilon-1} \right)^{\frac{\alpha}{\epsilon-1}}$$



Problem: V is now (quasi) concave

- ⇒ First-order conditions are necessary and sufficient
- ⇒ Numerical algorithm converges to global maximum

Testing the approach on small networks

For small networks we can solve \mathcal{P}_{SP} directly by trying all possible vectors θ

- Comparing approaches for a million different economies:

	Number of firms n			
	8	10	12	14
A. With reshaping				
Firms with correct θ_j	99.9%	99.9%	99.9%	99.8%
Error in output Y	0.00039%	0.00081%	0.00174%	0.00171%
B. Without reshaping				
Firms with correct θ_j	84.3%	83.2%	82.3%	81.3%
Error in output Y	0.84%	0.89%	0.93%	0.98%

Notes: Parameters $f \in \{0.05/n, 0.1/n, 0.15/n\}$, $\sigma_z \in \{0.34, 0.39, 0.44\}$, $\alpha \in \{0.45, 0.5, 0.55\}$, $\sigma \in \{4, 6, 8\}$ and $\epsilon \in \{4, 6, 8\}$. For each combination of parameters 1000 different economies are created. For each economy, productivity is drawn from $\log(z_k) \sim \text{iid } \mathcal{N}(0, \sigma_z)$ and Ω is drawn randomly such that each link Ω_{ij} exists with some probability such that a firm has on average five possible incoming connections. A network is kept in the sample only if the first-order conditions give a solution in which θ hits the bounds.

The errors come from

- firms that are particularly isolated
- two θ configurations with almost same output

Testing the approach on large networks

For large networks we cannot solve \mathcal{P}_{SP} directly by trying all possible vectors θ

- After all the 1-deviations θ are exhausted:

	With reshaping	Without reshaping
Firms with correct θ	99.98%	69.3%
Error in output Y	0.00007%	0.696%

Notes: Simulations of 200 different networks Ω and productivity vectors z that satisfy the properties of the calibrated economy.

- Very few “obvious errors” in the allocation found by the approach

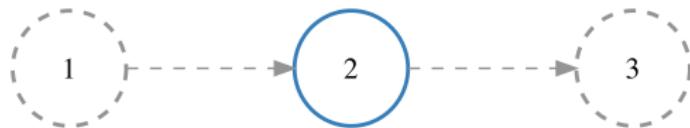
IV. Economic Forces at Work

Complementarities



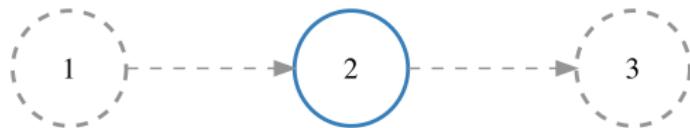
- Impact of operating 2 on the incentives to operate 1 and 3
 - ▶ Operating 3 leads to a larger q_3 because 2 is operating
 - ▶ Operating 1 increases q_2 because 2 is operating
- Complementarity between operating decisions of nearby firms
 - Cascades of firm shutdowns may arise

Complementarities



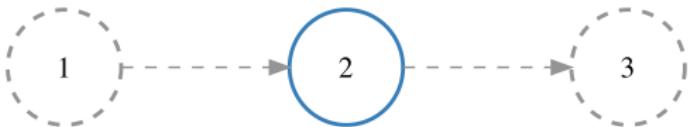
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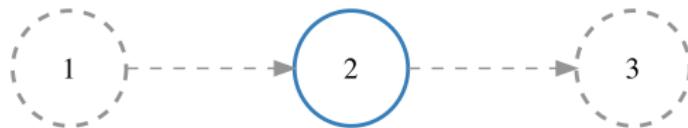
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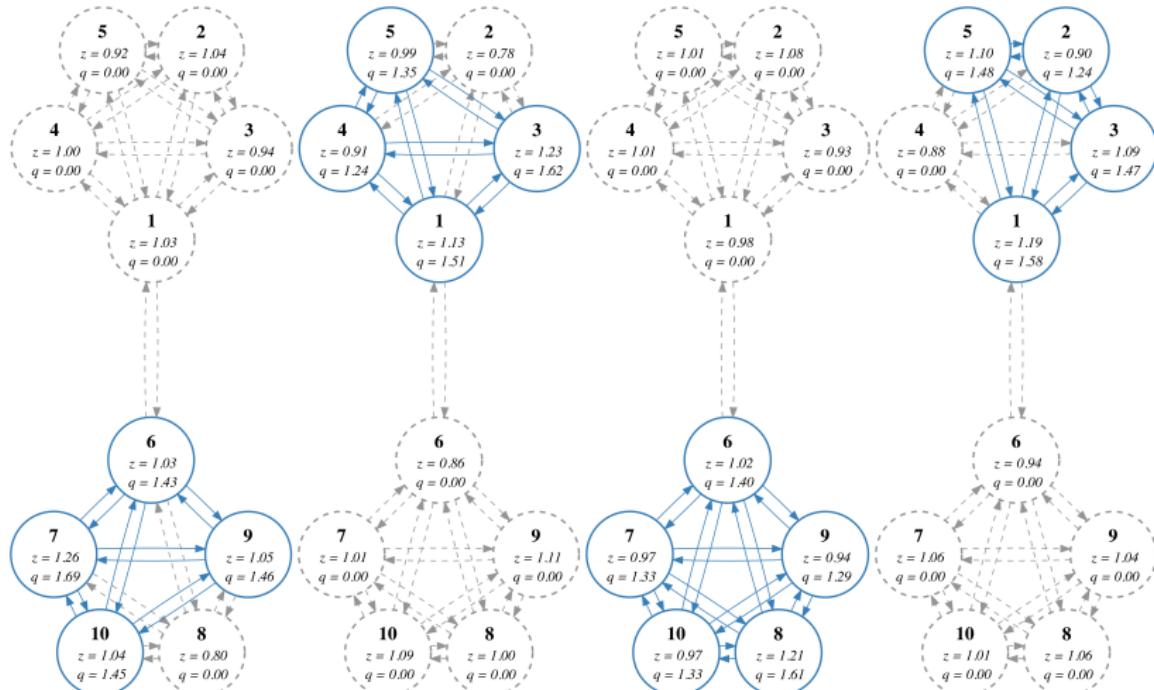
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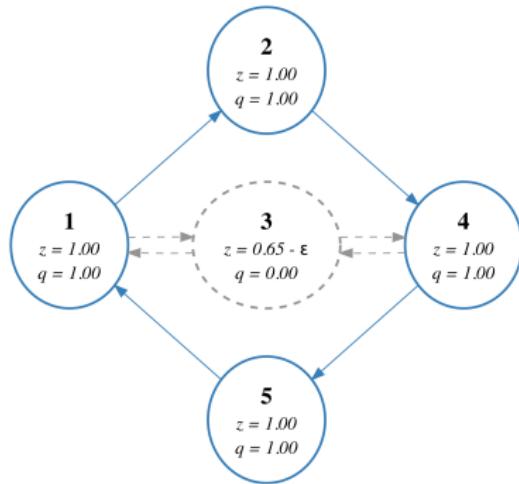
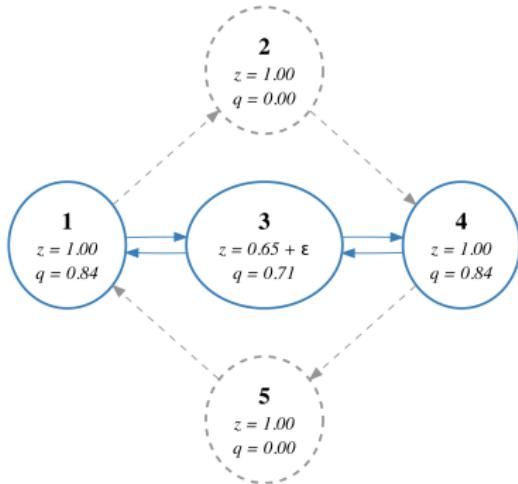
Complementarities lead to clustering



Characterization

Non-convex nature of the economy:

- A small shock can lead to a large reorganization...



V. Quantitative Exploration

Network data

Two datasets that cover the U.S. economy

- Compustat
 - ▶ Public firms must self-report important customers (>10% of sales)
 - ▶ Cohen and Frazzini (2008) and Atalay et al (2011) use fuzzy-text matching algorithms to build the network
- Factset Revere
 - ▶ Includes public and private firms, and less important relationships
 - ▶ Analysts gather data from 10-K, 10-Q, annual reports, investor presentations, websites, press releases, etc

	Year	Firms/year	Links/year
Compustat			
Atalay et al (2001)	1976 - 2009	1,300	1,500
Cohen and Frazzini (2006)	1980 - 2004	950	1,100
Factset	2003 - 2016	13,000	46,000

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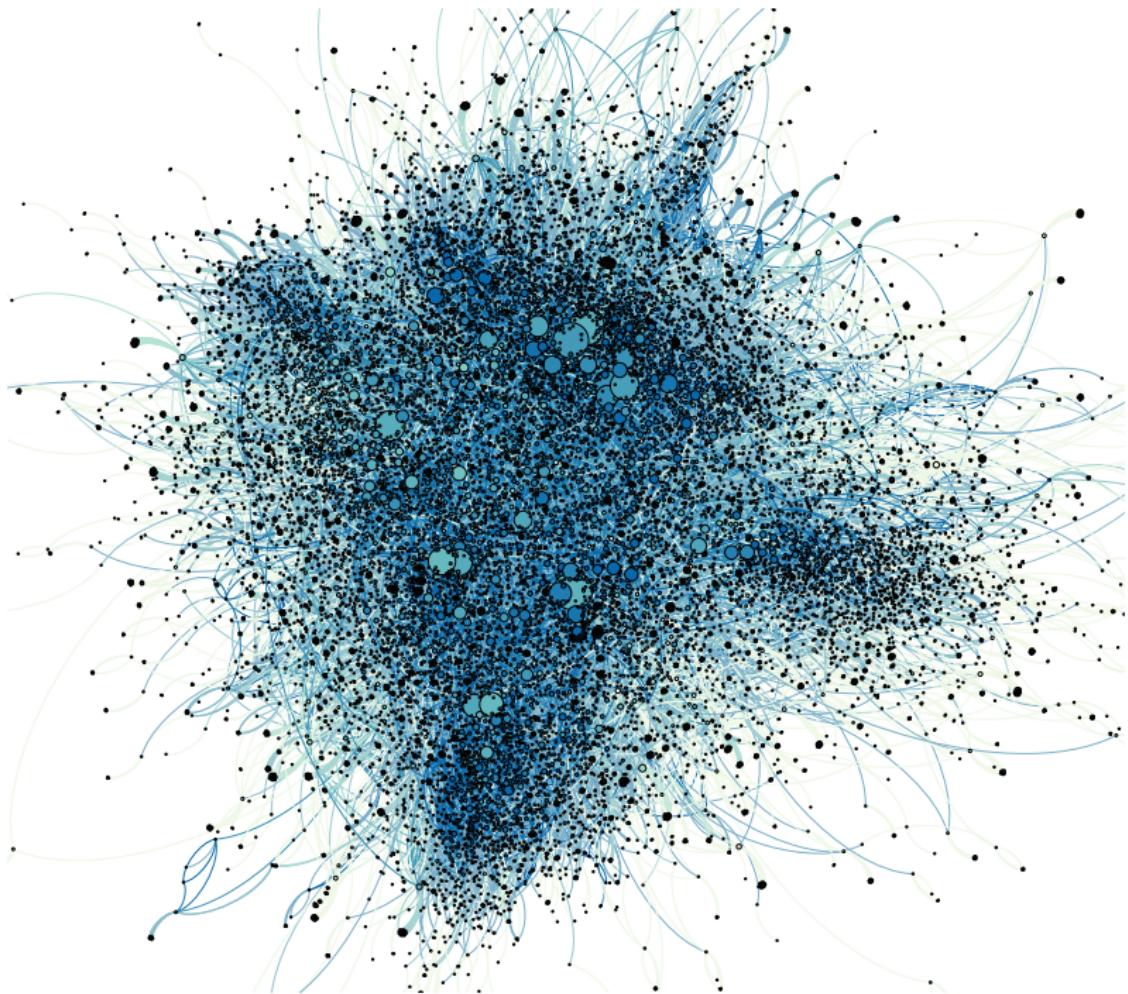
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Parameters

Parameters from the literature

- $\alpha = 0.5$ to fit the share of intermediate (Jorgenson et al 1987, Jones 2011)
- $\sigma = \epsilon = 5$ average of estimates (Broda et al 2006)
 - ▶ Robustness with smaller ϵ in the paper
- Firm productivity follows AR1
 - ▶ $\log(z_{it}) \sim \mathcal{N}(0, 0.39^2)$ from Bartelsman et al (2013)
 - ▶ $\rho_z = 0.81$ from Foster et al (2008)
- $f \times n = 5\%$ to fit employment in management occupations
- Set $n = 1000$ for high precision while limiting computations

Unobserved network Ω :

- Pick to match the *observed* in-degree distribution in Factset
- Generate thousands of such Ω 's and report averages

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Shape of the network

	Model	Data		
		Factset	AHRS	CF
Power law exponents				
In-degree distribution	1.07	0.95	1.13	1.32
Out-degree distribution	1.02	0.81	2.24	2.22
Global clustering coefficient	0.51	0.64	0.013	0.014

Notes: Global clustering coefficients are multiplied by the square roots of the number of nodes for better comparison.

Shape of the network

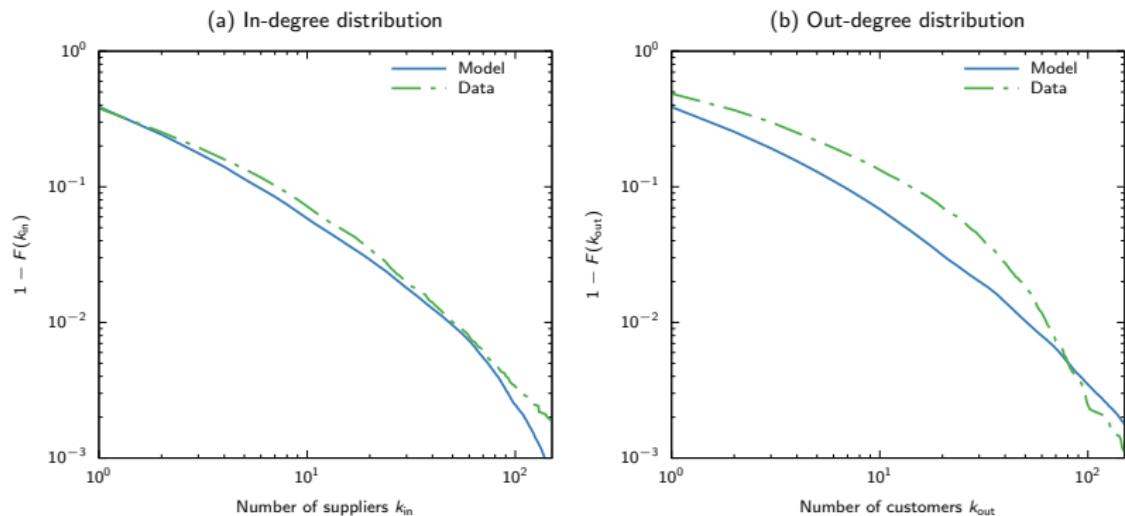


Figure: Model and Factset data for 2016

Shape of the network

What does an optimally designed network looks like?

- Compare **optimal networks** to completely **random networks**
- Differences highlights how efficient allocation shapes the network

	Model	
	Optimal network	Random network
Power law exponents		
In-degree distribution	1.07	1.22
Out-degree distribution	1.02	1.21
Global clustering coefficient	0.51	0.30

Notes: Clustering coeff. multiplied by the square roots of number of nodes for better comparison.

Efficient network features

- More highly connected firms
- More clustering of firms

► Def. clust. coeff.

Firm-level distributions

In the efficient allocation:

- **Selection:** Low productivity firms do not operate
- **Magnification:** High productivity firms benefit from clustering

	Optimal network economy		Random network economy	
	Productivity q	Empl. /	Productivity q	Empl. /
Mean	2.40	0.0018	1.75	0.0018
Std. dev.	0.30	1.26	0.46	1.94
Skewness	0.53	1.04	-0.05	0.05

Because of the optimal organization of the network

- Distributions are positively skewed

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Cascades of shutdowns

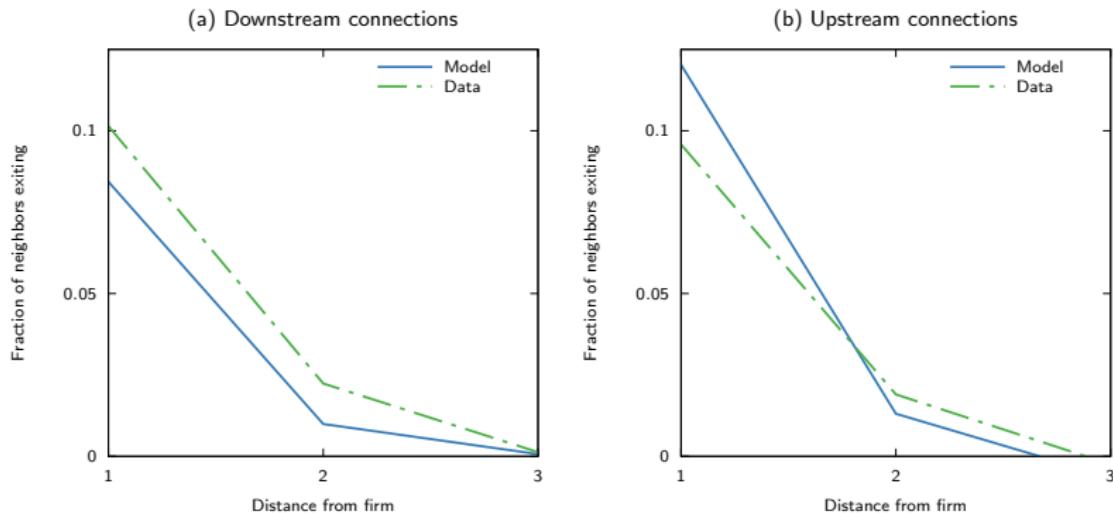
For each firm in each year:

- Look at all neighbors upstream and downstream
- Regress the fraction of these neighbors that exits on whether the original firm exits and some controls

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Resilience of firms

Size of cascades and probability of exit by degree of firm

	Size of cascades		Probability of exit	
	Model	Data	Model	Data
Average firm	0.4	0.9	16.3%	12.2%
High degree firm	5.9	7.9	2.3%	3.4%

Implications:

- Highly-connected firms are hard to topple but upon shutting down they create large cascades

* Robustness

Resilience of firms

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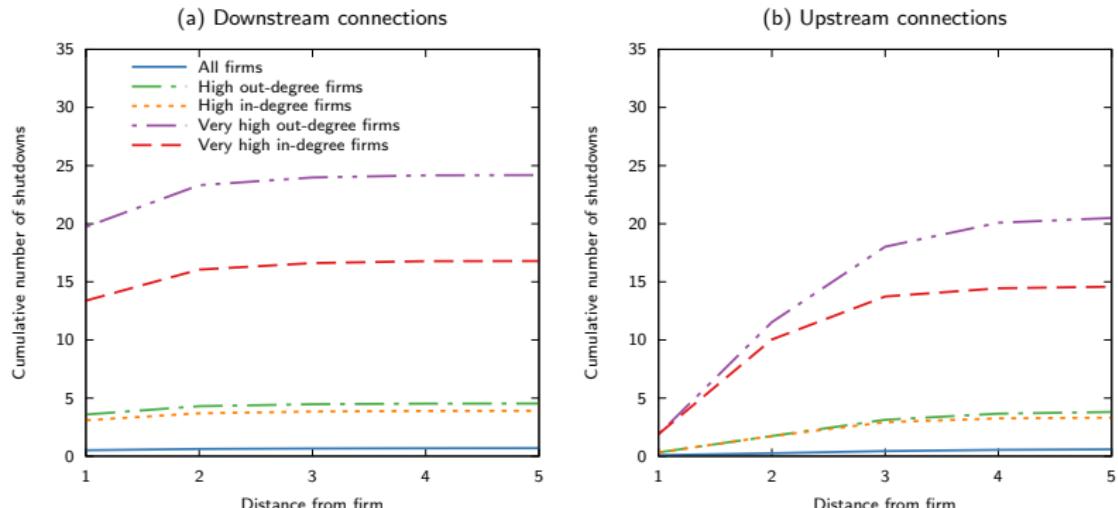
Implications:

- Highly-connected firms are hard to topple but upon shutting down they create large cascades

▶ Robustness

Resilience of firms

Causal impact of a firm exits on its neighbors



Implications:

- Cascades mostly propagate downstream
- Firms with higher degree create larger cascades

▶ Robustness

Aggregate fluctuations

The shape of the network changes with the business cycle

	Model	Data		
		Factset	AHRS	CF
Power law exponents				
In-degree distribution	-0.57	-0.85	-0.35	-0.12
Out-degree distribution	-0.67	-0.94	-0.30	-0.11
Global clustering coefficient	0.46	0.68	0.17	0.20

Implications:

- Recessions are periods with fewer highly-connected firms and in which clustering activity around most productive firms is costly

Aggregate fluctuations

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Aggregate fluctuations

Size of fluctuations

$$Y = Q \left(L - f \sum_j \theta_j \right)$$

	Output Y	Labor Prod. Q	Prod. labor $L - f \sum_j \theta_j$
Optimal network	0.10	0.10	0.009
Fixed network	0.13	0.13	0

Implications:

- Fluctuations are more than 30% smaller in optimal network economy
- The difference comes from changes in the production network

• Definition

Aggregate fluctuations

Size of fluctuations

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- Fluctuations are more than 30% smaller in optimal network economy
- The difference comes from changes in the production network

▶ Intuition

Conclusion

Summary

- Theory of endogenous network formation and aggregate fluctuations
- The optimal network features complementarities between operating decisions of firms that lead to
 - ▶ clustering of activity
 - ▶ large impact of small changes
 - ▶ cascades of shutdowns/restarts
- Compared to U.S. data the model is able to replicate
 - ▶ intensity and occurrence of cascades of shutdowns
 - ▶ correlation between shape of network and business cycles
- The endogenous reorganization of the network limits the size of fluctuation
- Methodological contribution: approach to easily solve certain non-convex optimization problems

Details of reshaping

Simpler to consider

$$\mathcal{P}'_{RD}: \max_{\theta \in [0,1]^n, q} \left(\sum_{j=1}^n q_j^{\sigma-1} \right)^{\frac{1}{\sigma-1}} \left(L - f \sum_{j=1}^n \theta_j \right)$$
$$q_j \leq A z_j \theta_j^a AB_j^\alpha \quad (\text{LM: } \beta_j)$$

where $B_j = \left(\sum_{i=1}^n \Omega_{ij} \theta_i^b q_i^{\epsilon-1} \right)^{\frac{1}{\epsilon-1}}$.

First order condition with respect to θ_k :

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial Q}{\partial q_k} \left(L - f \sum_{j=1}^n \theta_j \right) - fQ + \sum_{j=1}^n \beta_j \left(\frac{\partial q_k}{\partial \theta_k} \frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} \right) \frac{\partial q_j}{\partial B_j} = \bar{\mu}_k - \underline{\mu}_k$$

The terms are

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial Q}{\partial q_k} = z_k a \theta_k^{a-1} AB_k^\alpha \times (z_k \theta_k^a AB_k^\alpha)^{\sigma-2} Q^{2-\sigma}$$

$$\frac{\partial q_k}{\partial \theta_k} \frac{\partial B_j}{\partial q_k} + \frac{\partial B_j}{\partial \theta_k} = B_j \theta_k^{b-1} \Omega_{kj} \left(\frac{z_k \theta_k^a AB_k^\alpha}{B_j} \right)^{\epsilon-1} \left(a + \frac{b}{\epsilon-1} \right)$$

[◀ Return](#)

Clustering coefficient

- Triplet: three connected nodes (might be overlapping)
- Triangles: three fully connected nodes (3 triplets)

$$\text{Clustering coefficient} = \frac{3 \times \text{number of triangles}}{\text{number of triplets}}$$

[◀ Return](#)

Cascades of shutdowns

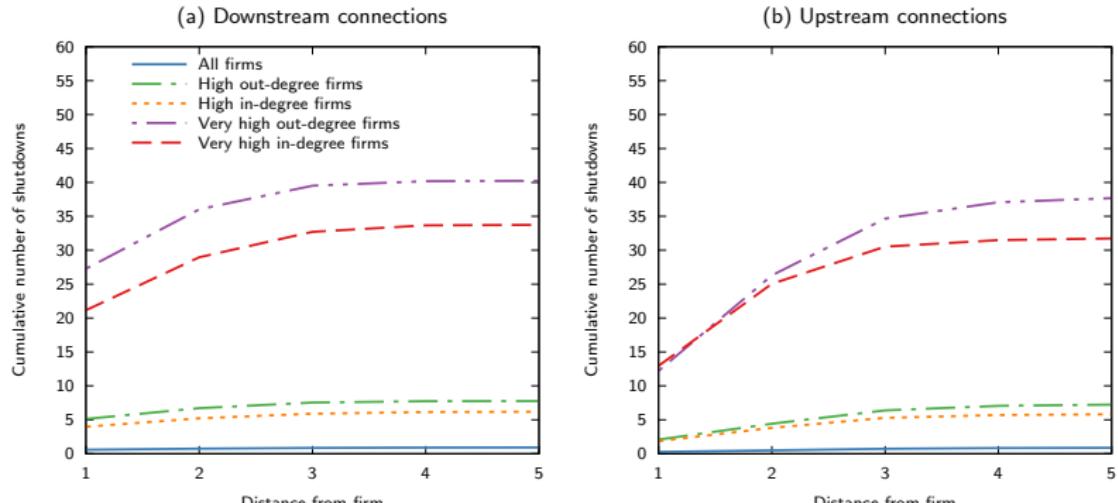


Figure: $\epsilon = 3$

◀ return

	Probability of exit			
	Benchmark	$\alpha = 0.75$	$\sigma = 7$	$\varepsilon = 4$
Average firm	16.3%	17.0%	25.2%	15.0%
High degree firm	2.3%	1.2%	2.8%	1.0%

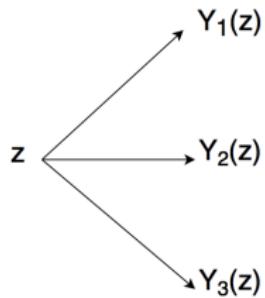
◀ return

Intuition

A given network θ^k is a function that maps $z \rightarrow Y_k(z)$

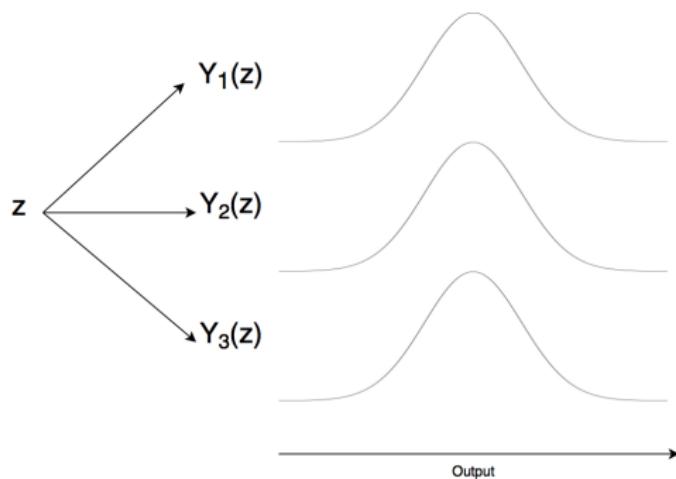
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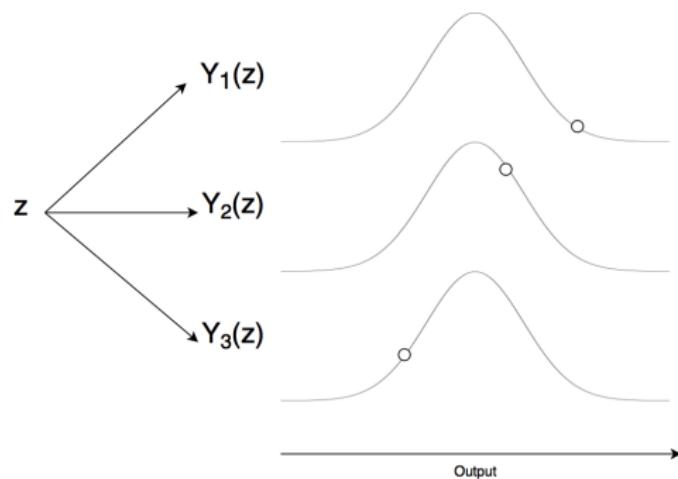
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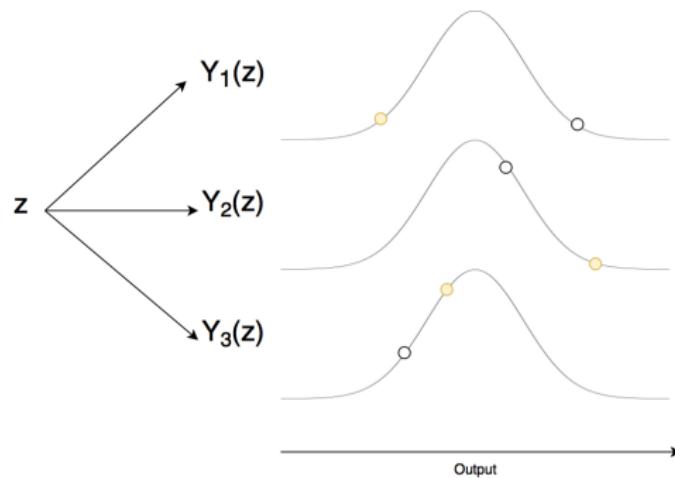
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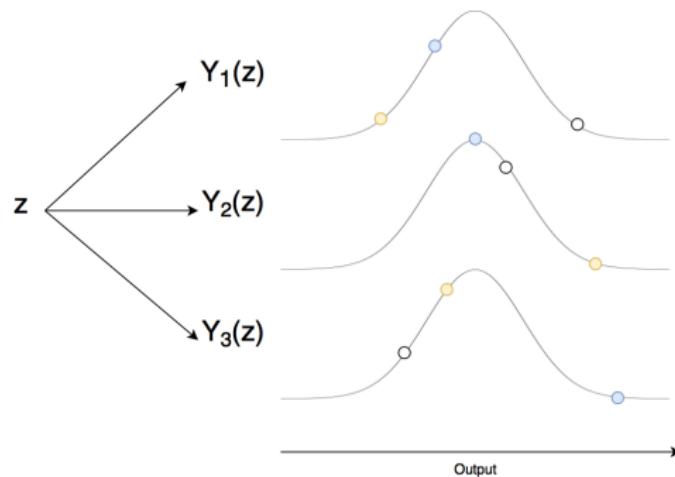
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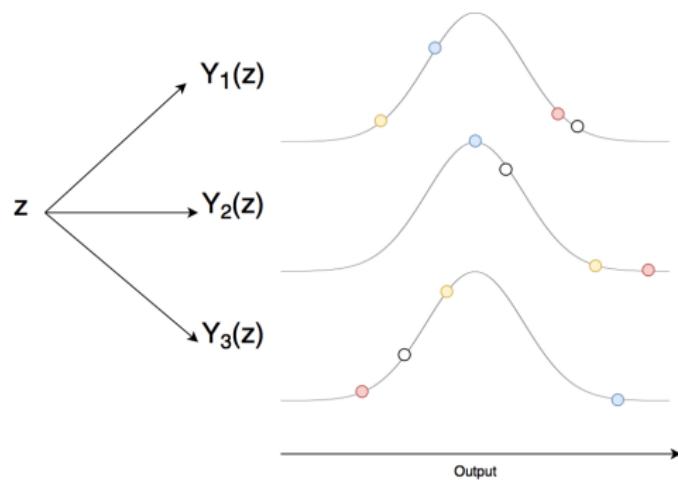
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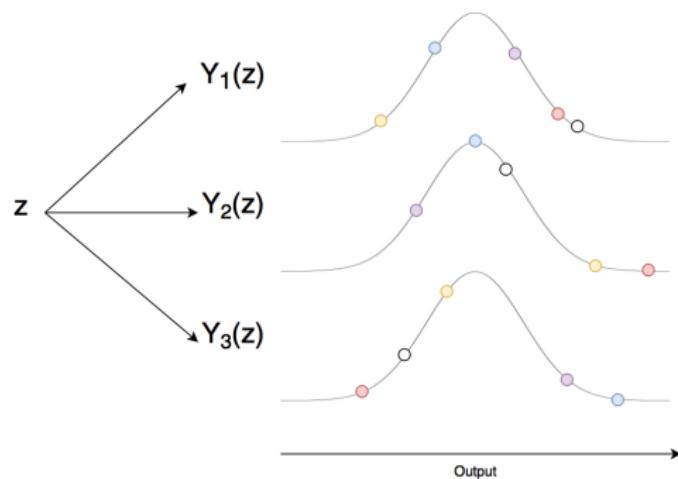
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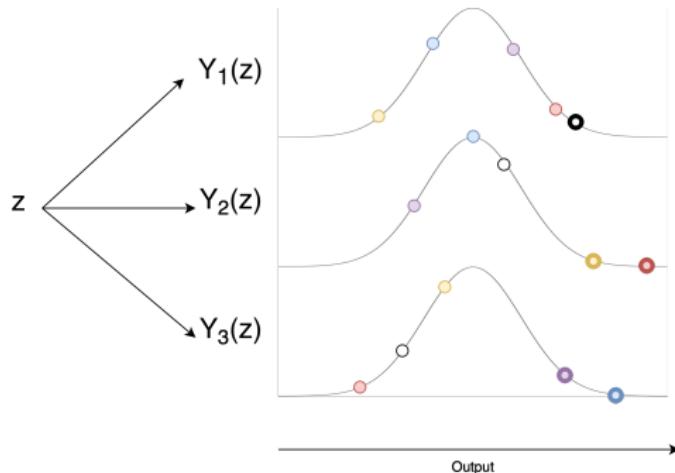
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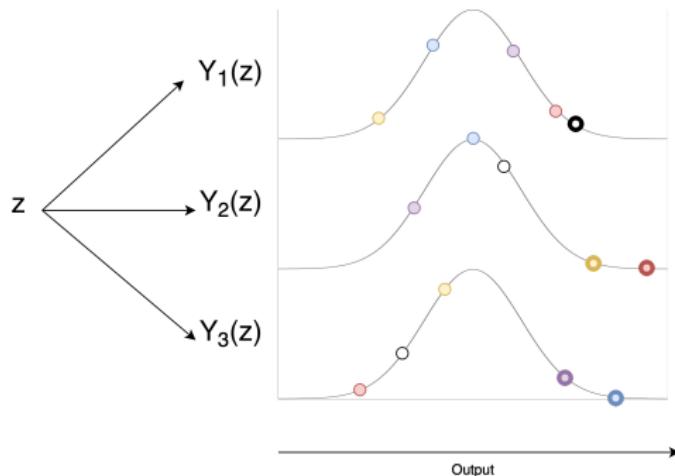
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From extreme value theory

$$\text{Var}(Y) = \text{Var} \left(\max_{k \in \{1, \dots, 2^n\}} Y_k \right)$$

declines rapidly with n

◀ return