Uncertainty Traps

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April 6, 2015
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Introduction

- Some recessions are particularly persistent
  - Slow recoveries of 1990-91, 2001
  - Recession of 2007-09: output, investment and employment still below trend ▸ Details
- Persistence is a challenge for standard models of business cycles
  - Measures of standard shocks typically recover quickly
    - Productivity/TFP, financial shocks, volatility...
  - Need strong propagation channel to transform short-lived shocks into long-lasting recessions

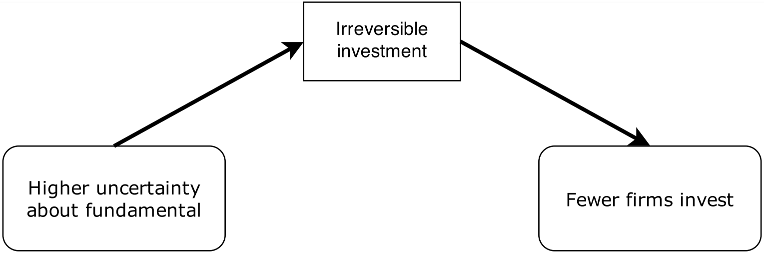
This Paper

- We develop a business cycles theory of endogenous uncertainty:
  - Large evidence of heightened uncertainty in 2007-2015 (Bloom et al.,2012; Ludvigson et al.,2013)
- This paper:
  - Provides a theory that explains why uncertainty is countercyclical
  - Proposes a propagation channel that can generate persistent recessions

Mechanism

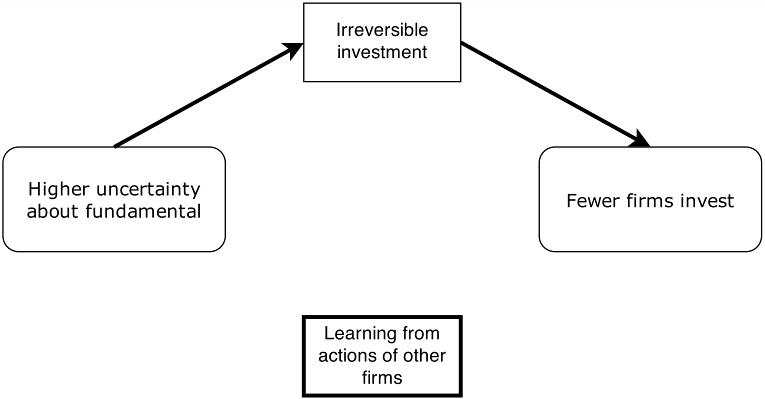
Irreversible
investment

Mechanism



Irreversible investment
Higher uncertainty about fundamental
Fewer firms invest

Mechanism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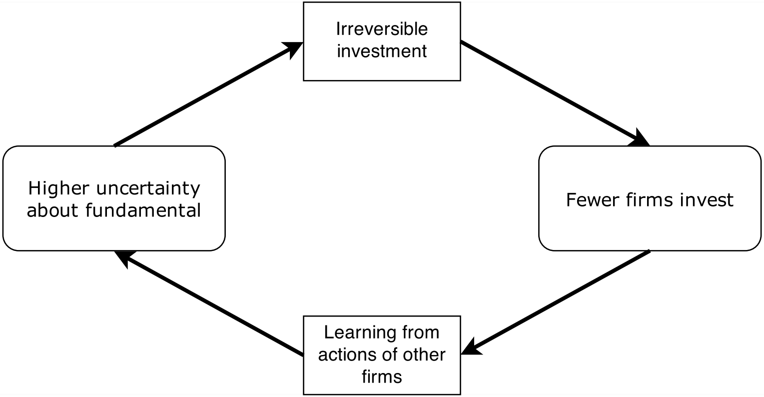
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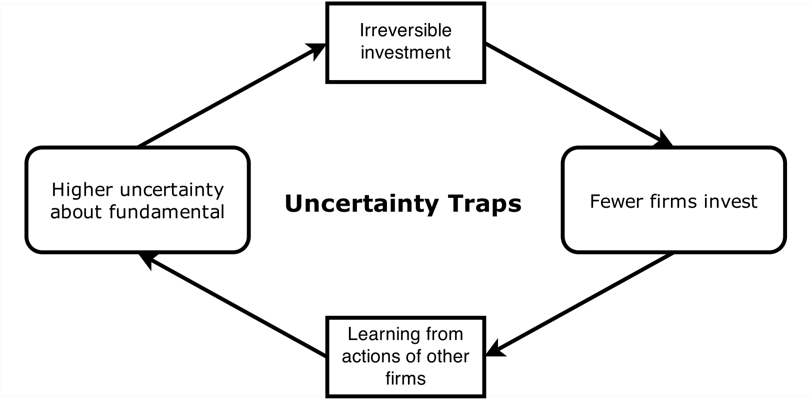


- **Uncertainty traps:**
  Self-reinforcing episodes of high uncertainty and low economic activity

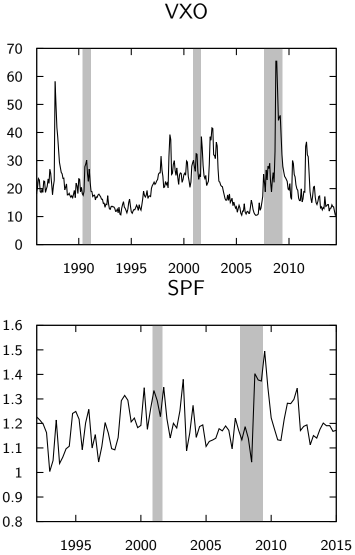
Roadmap

1. Discuss uncertainty concepts and measures
2. Start with a stylized model
   - Isolate how key forces interact to create uncertainty traps
   - Establish conditions for their existence, welfare implications
3. Extend the model to more standard RBC environment
   - Compare an economy with and without endogenous uncertainty
   - The mechanism can generate substantial persistence

Uncertainty

- Most of the uncertainty-driven business cycle literature focuses on time-varying volatility
  - Bloom (2009), Bachmann and Bayer (2009), Gilchrist et al. (2014),...
- We adopt the broader concept of *Bayesian uncertainty*:
  - Subjective uncertainty as perceived by agents is what matters for investment decisions
  - Uncertainty = variance of beliefs about variables of interest
  - Time-varying risk implies Bayesian uncertainty, not vice versa
- Empirical counterparts:
  - Ex-ante forecast errors, surveys of expectation, financial data, etc.
  - Different from measures of cross-sectional dispersion and disagreement

Measures of Aggregate Uncertainty



Theoretical Model

- Infinite horizon model in discrete time
- $\overline{N}$ atomistic firms indexed by $n \in \left\{1, \ldots, \bar{N}\right\}$ producing a homogeneous good
- Firms have CARA preferences over wealth

$$u(x) = \frac{1}{a}\left(1 - e^{-ax}\right)$$

Investment and Adjustment Costs

- Each firm n has a *unique* investment opportunity and must decide to either do the project today or wait for the next period
  - Firms face a random fixed investment cost $f \sim$ cdf F, iid, with variance σ^f
  - $N \in \{1, \cdots, \overline{N}\}$ is the endogenous number of firms that invest.
  - Firms that invest are immediately replaced by firms with new investment opportunities
- The project produces output

$$x_n = \theta + \varepsilon_n^x$$

  - Aggregate productivity (the **fundamental**) θ follows an AR(1)

$$\theta' = \rho_\theta \theta + \varepsilon^\theta$$

and $\varepsilon^\theta \sim \text{iid}\, \mathcal{N}\left(0, \left(1 - \rho_\theta^2\right) \gamma_\theta^{-1}\right)$, $\varepsilon_n^x \sim \text{iid}\, \mathcal{N}\left(0, \gamma_x^{-1}\right)$.

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Information

Firms do not observe θ directly, but receive noisy signals:

1. Public signal that captures the information released by media, agencies, etc.

$$Y = \theta + \varepsilon^y, \text{ with } \varepsilon^y \sim \mathcal{N}\left(0, \gamma_y^{-1}\right)$$

2. Output of **all** investing firms
   - Each individual signal

$$x_n = \theta + \varepsilon_n^x, \text{ with } \varepsilon_n^x \sim \text{iid}\,\mathcal{N}\left(0, \gamma_x^{-1}\right)$$

   can be summarized by the aggregate signal:

$$X \equiv \frac{1}{N}\sum_{n \in I} x_n = \theta + \frac{1}{N}\sum_{n \in I} \varepsilon_n^x \sim \mathcal{N}\left(\theta, (N\gamma_x)^{-1}\right)$$

- Remarks:
   - No bounded rationality: firms use all information from observables
   - No asymmetry of information

Timing

Each firm starts the period with common beliefs

1. Firms draw investment cost f and decide to invest or not
2. Production takes place, public signals X and Y are observed
3. Agents update their beliefs and θ' is realized

Beliefs and Uncertainty

- Before observing signals, firms share the same beliefs about θ

$$\theta | \mathcal{I} \sim \mathcal{N}\left(\mu, \gamma^{-1}\right)$$

- Our notion of uncertainty is captured by the variance of beliefs $1/\gamma$
  - *High uncertainty* means *low* γ
- Remark: no heterogeneity in beliefs, no disagreement

Law of Motion for Beliefs

- After observing signals X and Y, the *posterior about θ* is

$$\theta \quad | \quad \mathcal{I}, X, Y \sim \mathcal{N}\left(\mu_{post}, \gamma_{post}^{-1}\right)$$

with

$$\mu_{post} = \frac{\gamma\mu + \gamma_y Y + N\gamma_x X}{\gamma + \gamma_y + N\gamma_x}$$
$$\gamma_{post} = \gamma + \gamma_y + N\gamma_x$$

- Next period's *beliefs about $\theta' = \rho_\theta \theta + \varepsilon^\theta$* is

$$\mu' = \rho_\theta \mu_{post}$$
$$\frac{1}{\gamma'} = \frac{\rho_\theta^2}{\gamma_{post}} + \frac{1-\rho_\theta^2}{\gamma_\theta} \equiv \Gamma^{-1}\left(N, \gamma\right)$$

Firm Problem

- Firms choose whether to invest or not

$$V(\mu, \gamma, f) = \max \left\{ \underbrace{V^W(\mu, \gamma)}_{\text{wait}}, \underbrace{V^I(\mu, \gamma) - f}_{\text{invest}} \right\}$$

- Decision is characterized by a threshold $f_c(\mu, \gamma)$ such that

$$\text{firm invests} \Leftrightarrow f \leq f_c\left(\mu, \gamma\right)$$

Firm Problem

- Value of waiting:

$$V^W(\mu,\gamma) = \beta \mathbb{E}\left[\int V(\mu',\gamma',f')\, dF(f') \mid \mu,\gamma\right]$$

with $\mu' = \rho_\theta \frac{\gamma\mu+\gamma_y Y + N\gamma_x X}{\gamma+\gamma_y+N\gamma_x}$ and $\gamma' = \Gamma(N,\gamma)$

- Value of investing:

$$V^I(\mu,\gamma) = \mathbb{E}\left[u(x)\,|\mu,\gamma\right]$$

Aggregate Consistency

- The aggregate number of investing firms N is

$$N = \sum_n \mathbb{1}\left(f_n \leq f_c\left(\mu, \gamma\right)\right)$$

- Firms have the same ex-ante probability to invest

$$p\left(\mu, \gamma\right) = F\left(f_c\left(\mu, \gamma\right)\right)$$

- The number of investing firms follows a binomial distribution

$$N\left(\mu, \gamma\right) \sim Bin\left[\bar{N}, p\left(\mu, \gamma\right)\right]$$

Definition

An equilibrium consists of the threshold $f_c(\mu, \gamma)$, value functions $V(\mu, \gamma, f)$, $V^W(\mu, \gamma)$ and $V^I(\mu, \gamma)$, and a number of investing firms $N(\mu, \gamma, \{f_n\})$ such that

1. The value functions and policy functions solve the Bellman equation;
2. The number of investing firms N satisfies the consistency condition;
3. Beliefs (μ, γ) follow their laws of motion.

- Mean beliefs μ follow

$$\mu' = \rho_\theta \frac{\gamma\mu + \gamma_y Y + N\gamma_x X}{\gamma + \gamma_y + N\gamma_x}$$

Lemma

For a given N, mean beliefs μ follow an AR(1) with time-varying volatility s,

$$\mu'|\mu, \gamma = \rho_\theta \mu + s\,(N, \gamma)\,\varepsilon,$$

with $\frac{\partial s}{\partial N} > 0$ and $\frac{\partial s}{\partial \gamma} < 0$ and $\varepsilon \sim \mathcal{N}\,(0, 1)$.

- Precision of beliefs γ follows

$$\gamma' = \Gamma(N, \gamma) = \left(\frac{\rho_\theta^2}{\gamma + \gamma_y + N\gamma_x} + \frac{1 - \rho_\theta^2}{\gamma_\theta} \right)^{-1}$$

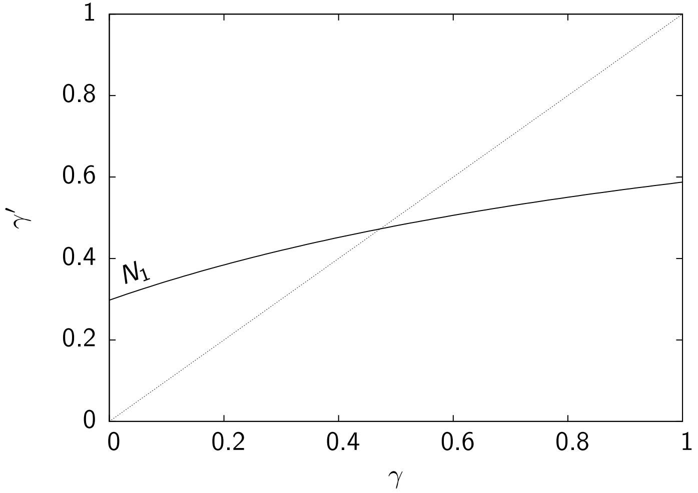
Lemma

1) Belief precision γ' increases with N and γ,
2) For a given N, $\Gamma(N, \gamma)$ admits a unique stable fixed point in γ.

Characterizing the Evolution of Beliefs

- Precision of beliefs γ follows

$$\gamma' = \Gamma(N, \gamma)$$



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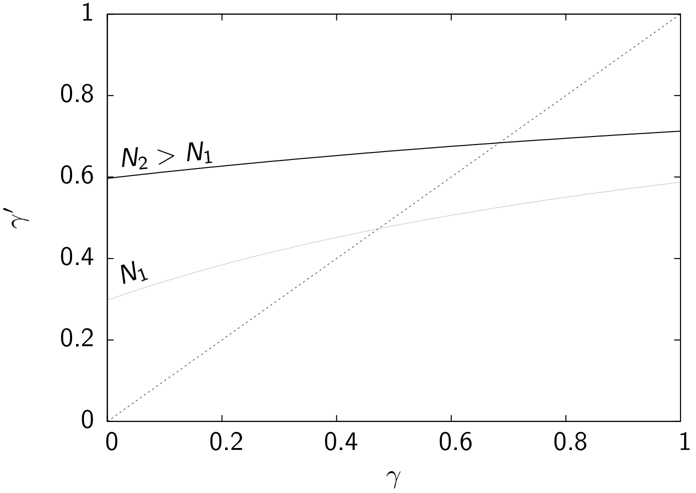


Equilibrium Characterization

Proposition

For ρ_θ and γ_θ large enough and for γ_x small,

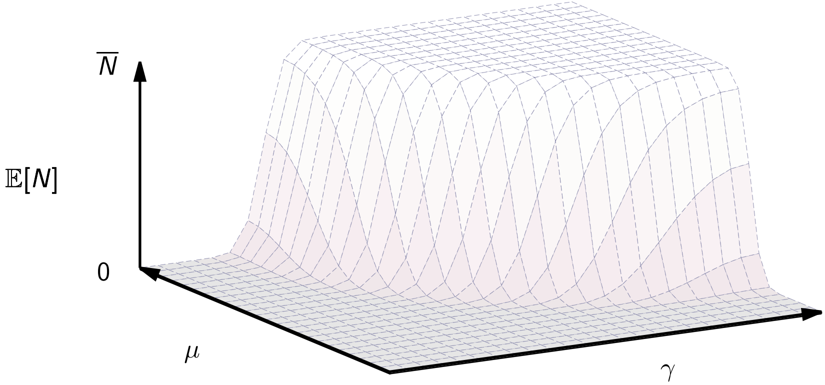
1) The equilibrium exists and is unique;
2) The investment decision of firms is characterized by the cutoff $f_c(\mu, \gamma)$ such that:

$$\textit{firm with cost } f \textit{ invests} \Leftrightarrow f \leq f_c(\mu, \gamma)$$

3) f_c is a strictly increasing function of μ and γ.

▸ Conditions

Aggregate Investment Pattern



Uncertainty Traps

- We now examine the existence of "uncertainty traps"
  - Self-reinforcing episodes of high uncertainty/low economic activity
- Take the limit as $\bar{N} \to \infty$, ▸ Details

$$\frac{N}{\bar{N}} = F\left(f_c\left(\mu, \gamma\right)\right)$$

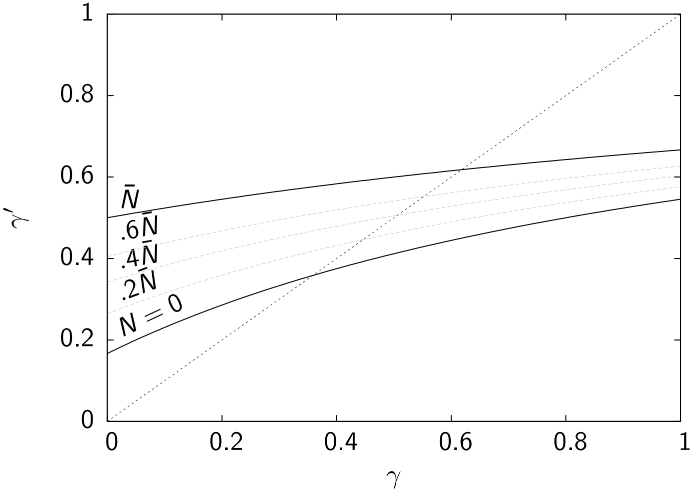
- The whole economy is described by the two-dimensional system:

$$\begin{cases} \mu' & = \rho_\theta \mu + s\left(N\left(\mu, \gamma\right), \gamma\right) \varepsilon \\ \gamma' & = \Gamma\left(N\left(\mu, \gamma\right), \gamma\right) \end{cases}$$

Equilibrium Dynamics of Belief Precision

- Precision of beliefs γ follow

$$\gamma' = \Gamma(N, \gamma)$$



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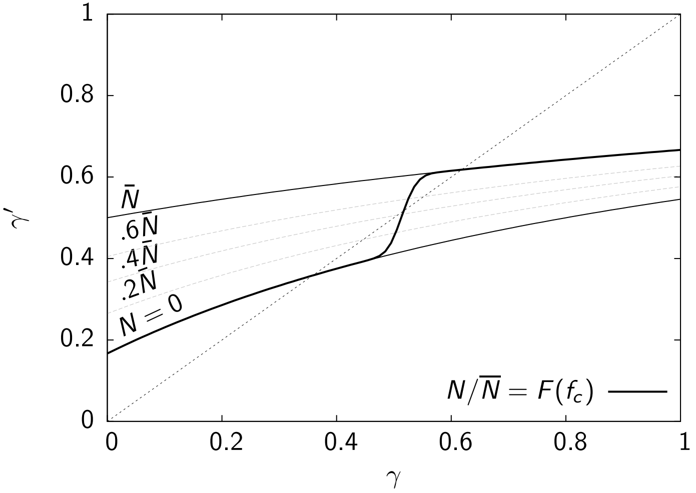

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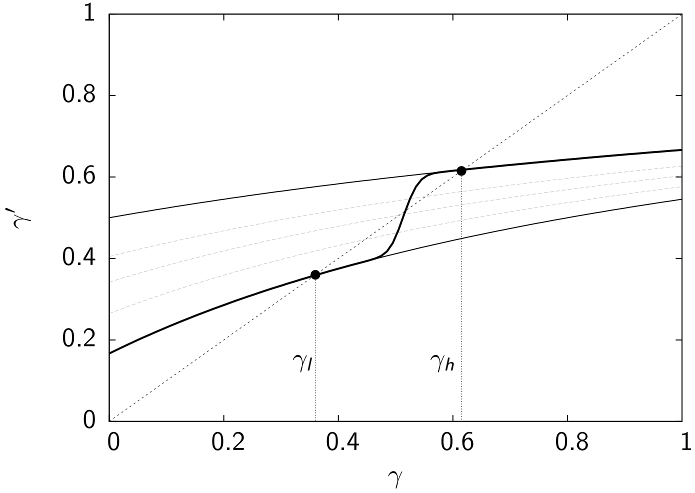

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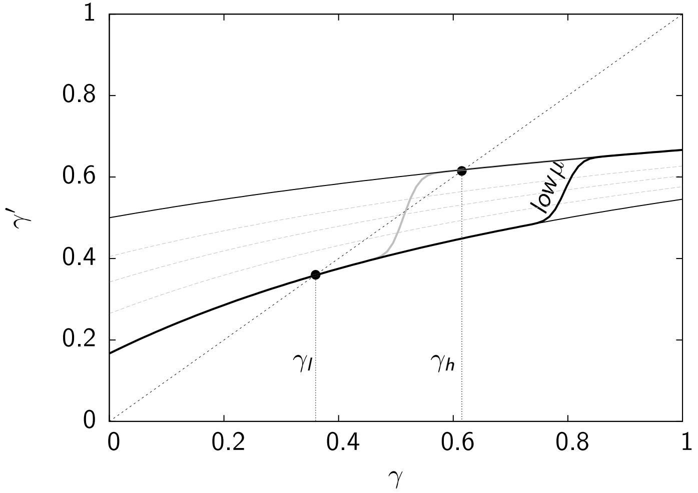

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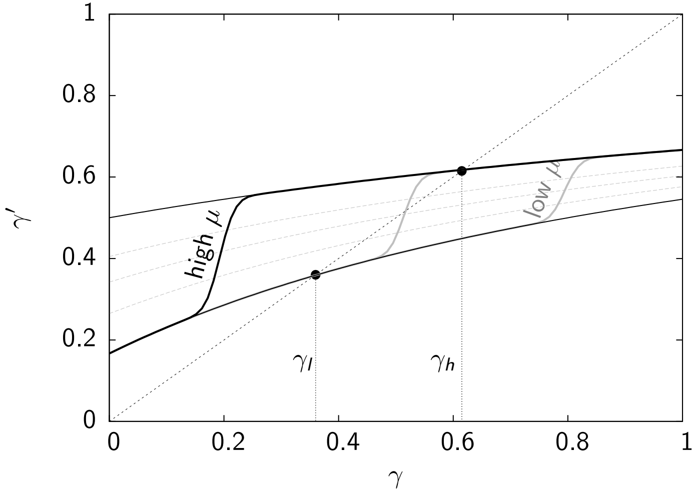
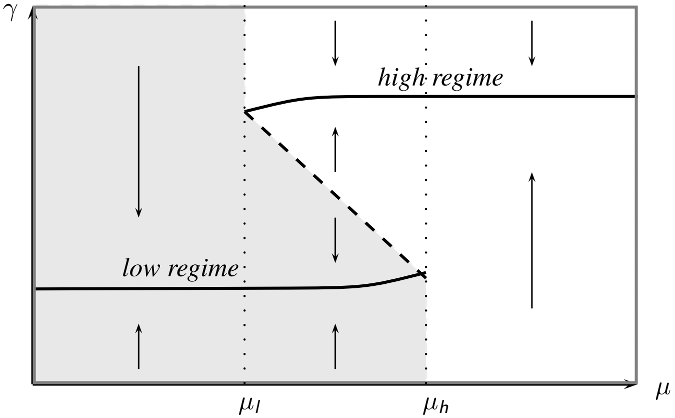


Phase Diagram



Existence of Uncertainty Traps

Definition

Given mean beliefs μ, there is an uncertainty trap if there are at least two locally stable fixed points in the dynamics of beliefs precision $\gamma' = \Gamma\left(N\left(\mu,\gamma\right),\gamma\right)$.

- Does not mean that there are multiple equilibria
  - The equilibrium is unique,
  - The past history of shocks determines which regime prevails

Proposition

For γ_x and σ^f low enough, there exists a non-empty interval $[\mu_l, \mu_h]$ such that, for all $\mu_0 \in (\mu_l, \mu_h)$, the economy features an uncertainty trap with at least two stable steady states $\gamma_l(\mu_0) < \gamma_h(\mu_0)$. Equilibrium γ_l (γ_h) is characterized by high (low) uncertainty and low (high) investment.

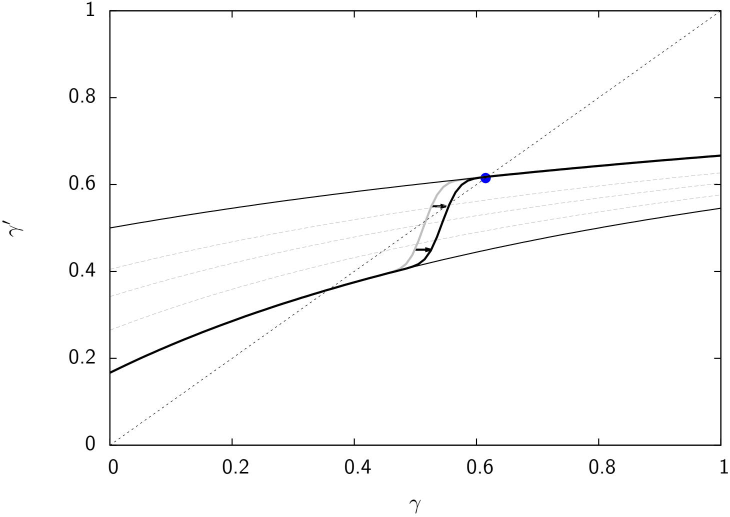
- The dispersion of fixed costs σ^f must be low enough to guarantee a strong enough feedback from information on investment

Uncertainty Traps: Falling in the Trap

- We now examine the effect of a negative shock to μ
  - Economy starts in the high regime
  - Hit the economy at $t = 5$ and last for 5 periods
  - We consider small, medium and large shocks
- Under what conditions does the economy fall into an uncertainty trap?

Uncertainty Traps: Falling in the Trap

Impact of a **small** negative shock to μ



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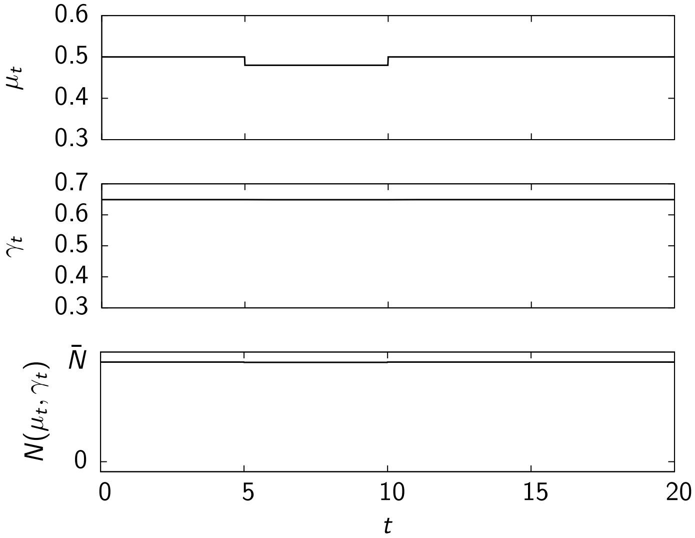
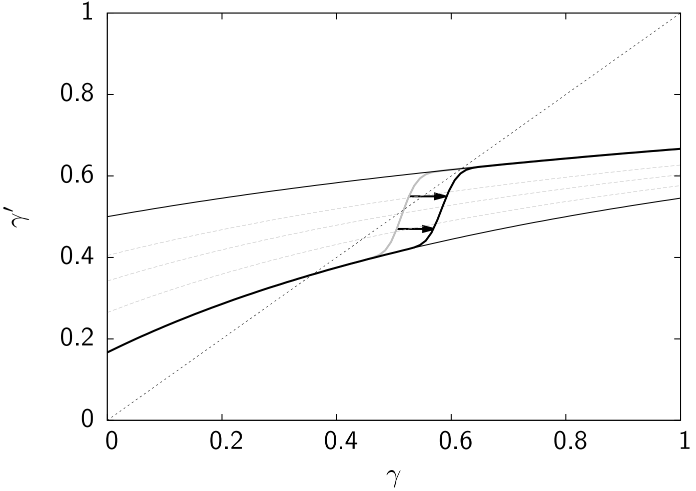


Uncertainty Traps: Falling in the Trap

Impact of a **medium**-sized negative shock to μ



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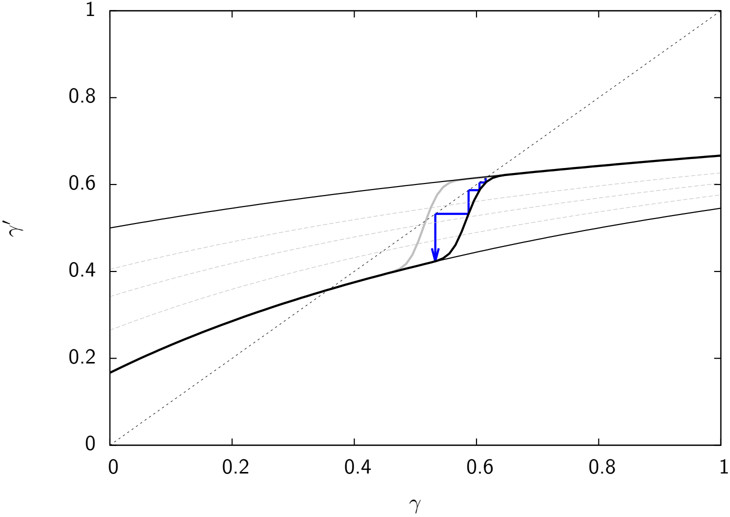

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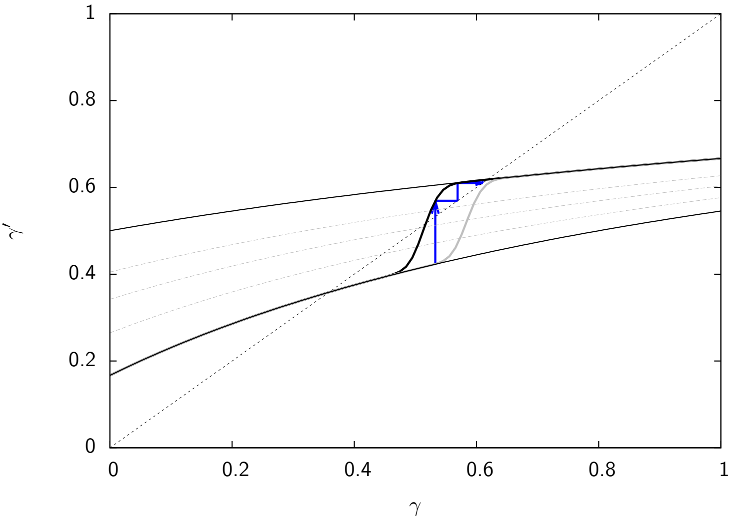

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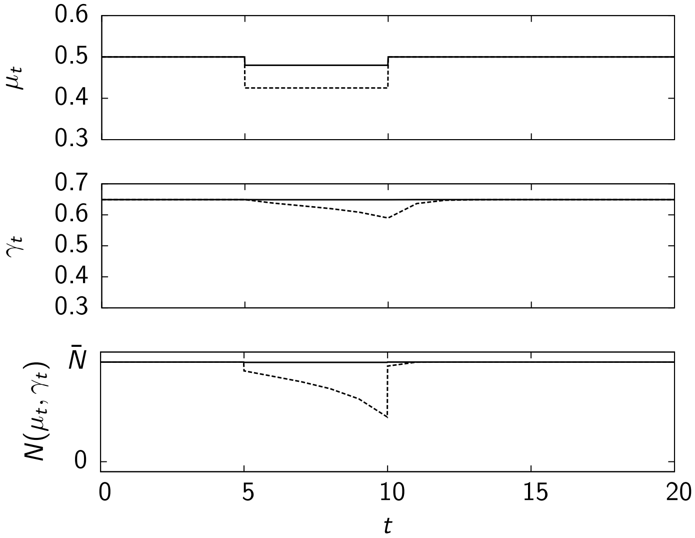
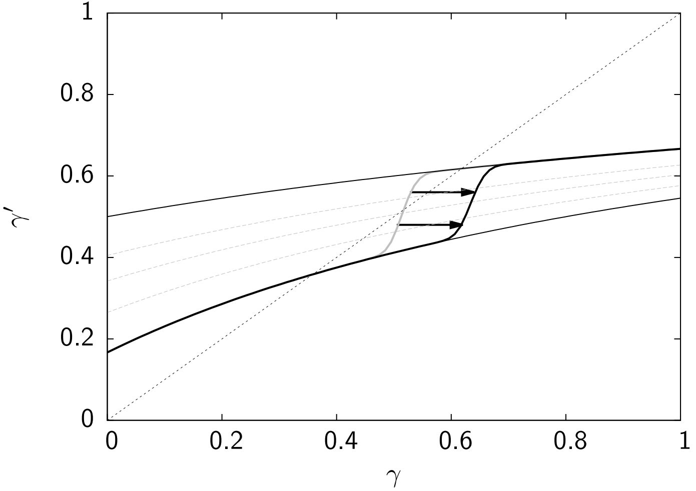


Impact of a **large** negative shock to μ



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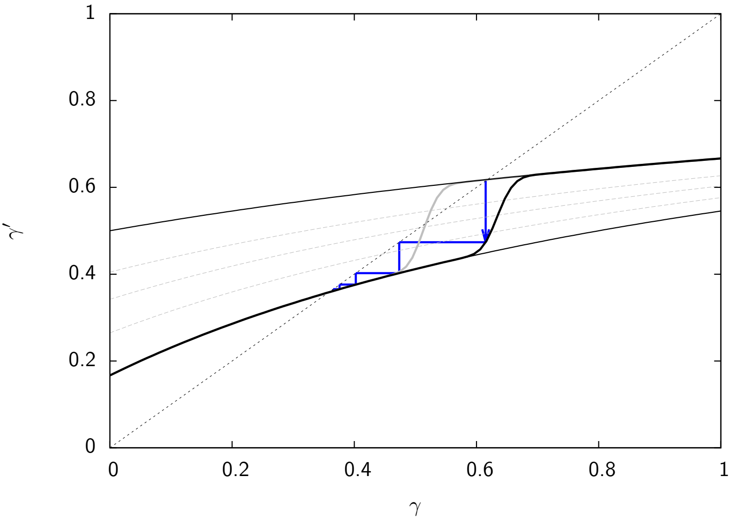

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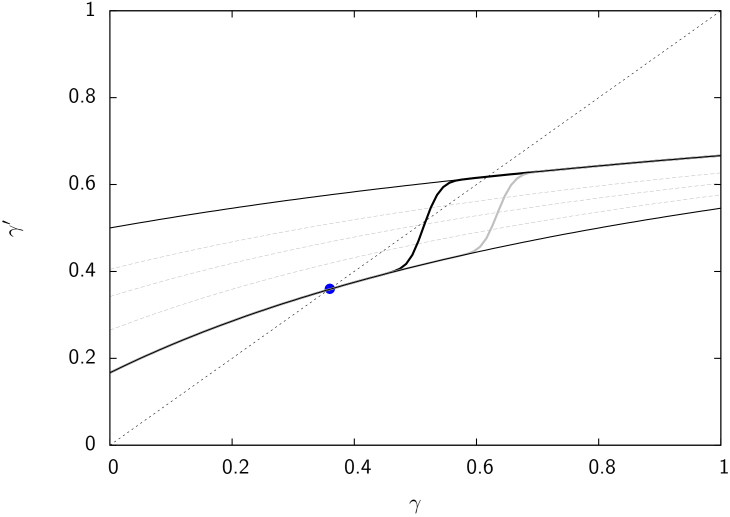

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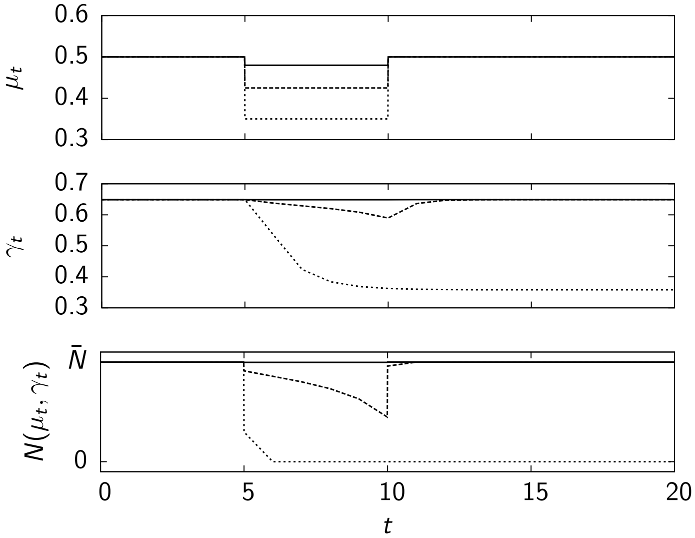
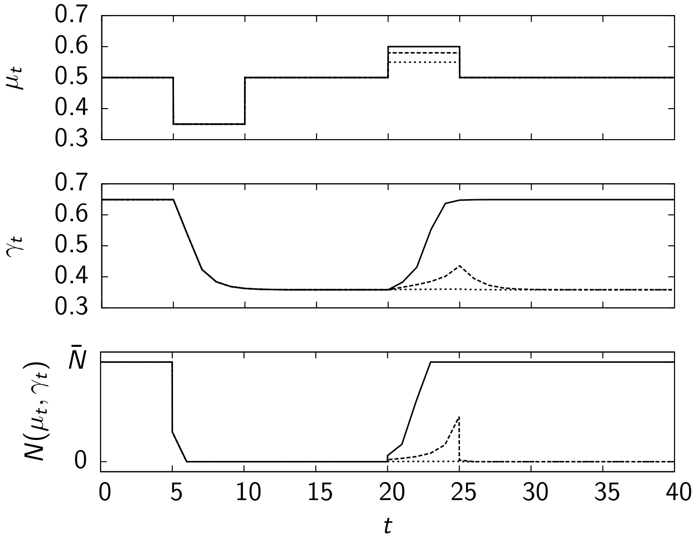


Uncertainty Traps: Escaping the Trap

- We now start after a full shift of the economy towards the low regime
- How can the economy escape the trap?

Uncertainty Traps: Escaping the Trap



- The economy displays strong non-linearities:
  - for small fluctuations, uncertainty does not matter much,
  - only large or prolonged declines in productivity (or signals) lead to self-reinforcing uncertainty events: uncertainty traps
- In such events, the economy may remain in a depressed state even after mean beliefs about the fundamental recover (μ)
  - Slow recoveries, high persistence in aggregate variables
- The economy can remain in such a trap until a large positive shock hits the economy

Welfare Implications

- The economy is inefficient because of an informational externality
  - Firms do not internalize the effect of their investments on public information

Proposition

The following results hold:
1) The competitive equilibrium is inefficient. The socially efficient allocation can be implemented with positive investment subsidies $\tau(\mu, \gamma)$*;*
2) In turn, uncertainty traps may still exist in the efficient allocation.

Extended Model

- We now introduce standard features of business cycle models
  - Evaluate the robustness of the mechanism
  - Explore numerically the potential magnitude of effects
- These features include:
  - Neoclassical production function with capital and labor
  - Long-lived firms that accumulate capital over time
  - Firms receive multiple investment opportunities stochastically
  - Competitive equilibrium and general equilibrium effects

Extended Model

- Representative household with inelastic labor supply and preferences

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^t U(C_t)$$

- Continuum of firms $j \in [0,1]$ with Cobb-Douglas technology

$$(A+Y)\,k_j^{\alpha} l_j^{1-\alpha}$$

with $Y = \theta + \varepsilon^Y$ and $\theta' = \rho_\theta \theta + \varepsilon^\theta$

- Capital accumulates over time

$$k_j' = (1-\delta+i_j)\,k_j$$

Investment Technology

- Two different types of investment:

1. *Regular* investments
    - routine maintenance and small repairs of current capital
    - convex variable cost $c(i)$, $i \in [\underline{i}, \overline{i}]$
    - does not require an investment opportunity
2. *Large* investments
    - large purchases of plants or equipment
    - requires an investment opportunity
    - fixed cost $f \sim F$ with mean $\overline{f}$ and standard deviation σ^f
    - convex variable cost $c(i)$, i unconstrained

Investment Technology

- Investment opportunities
  - Arrive at rate $\overline{q}$
  - Q is the total capital stock of firms with an opportunity
  - n is the fraction of firms with an opportunity that undertake large investment
- Large investments reveal information
  - Public signals $x_j = \theta + \varepsilon_j^x, \varepsilon_j^x \sim \mathcal{N}\left(0, (\gamma_x k_j)^{-1}\right)$
  - Individual signals aggregate into $X = \theta + \varepsilon^X, \varepsilon^X \sim \mathcal{N}\left(0, (nQ\gamma_x)^{-1}\right)$
- Aggregation (Hayashi, 1982)
  - We model the investment costs as CRS in capital: $c(i) \cdot k_j$ and $f \cdot k_j$
  - The economy admits aggregation with state variables (μ, γ, K, Q)

▸ Timing

▸ Information

Parametrization

- Standard parameters:

Parameter	Value
Time period	Month
Total factor productivity	$A = 1$
Discount factor	$\beta = (0.95)^{1/12}$
Share of capital in production	$\alpha = 0.4$
Persistence of fundamental	$\rho_\theta = (0.876)^{1/12}$
Ergodic standard deviation of fundamental	$\sigma_\theta = \gamma_\theta^{-\frac{1}{2}} = 0.03$

Parametrization: Investment

- We use quarterly firm-level data from Compustat
  - Interpret large investments as *investment spikes* in "Property, Plant and Equipment"
  - Define a spike as $i > 10\%$

Moment	Value
Average investment conditioning on spike	0.18
Average investment conditioning on no spike	0.023
Fraction of firms with spike in a quarter	0.028
Average total investment rate	0.027
Median - Average time between spikes	7-14 quarters
Parameter	**Value**
Variable cost of investment $c(i) = i + \phi i^2$	$\phi = 3.3$
Upper bound on constrained investment	$\bar{i} = 0.023$
Cost of investment	$\bar{f} = 0.1$
Depreciation rate	$\delta = (0.027)^{\frac{1}{3}}$
Probability of investment opportunity	$\bar{q} = (1/10)^{\frac{1}{3}}$

Parametrization: Information

- The information parameters γ_x and γ_y are difficult to identify
  - We perform sensitivity analysis

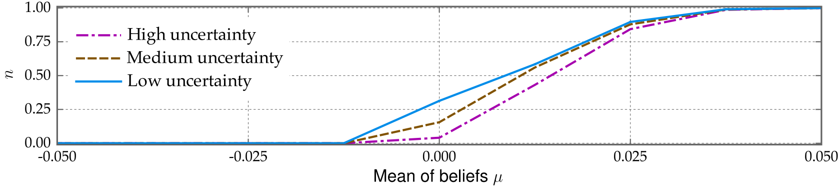
Parameter	Value
Precision of public signal γ_y	$\gamma_y = \underline{100}, 1000, 5000$[1]
Precision of individual signal γ_x	$nQ\gamma_x = \underline{10}, 5, 1 \times \gamma_y$

- Our baseline case:
  - Maximum stdev of beliefs about θ_{t+12} is 1.47% (one-year ahead)
- Survey of Professional Forecasters (SPF): Probability Forecasts
  - Maximum stdev in one-year ahead forecasts about real output is 1.53% (2009Q3)

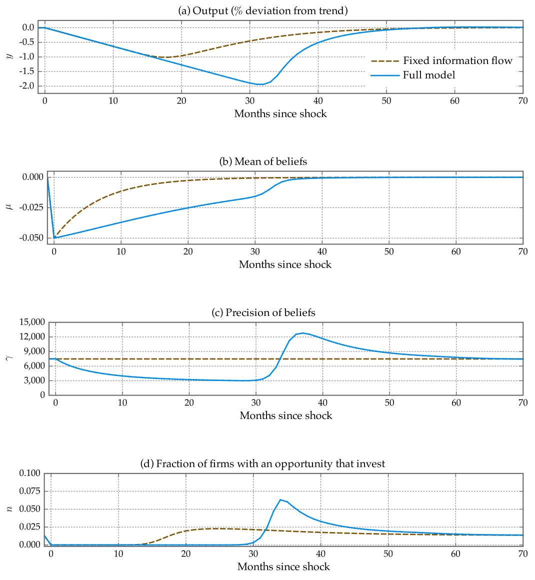
[1] stdev 10%, 3.2%, 1.4%

Numerical Illustration

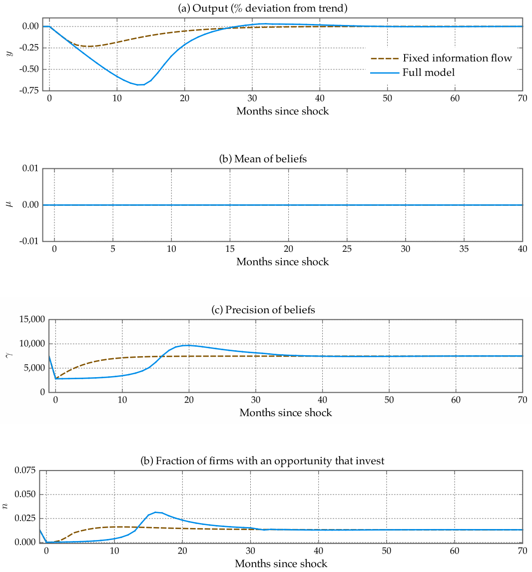
- We start with the risk neutral case and little heterogeneity in fixed costs $\left(\sigma^f = 0.001\overline{f}\right)$
  - Isolate the option value effects and extends the baseline case
  - Relax each assumption one by one later
- Option value effects have strong impact on extensive margin of large investments:



Numerical Illustration: Negative 5% shock to μ



Numerical Illustration: Negative 50% shock to γ



Numerical Illustration

- Results:
  - Endogenous uncertainty channel adds amplitude and duration to recessions in comparison to model with fixed information flow
  - Fundamental uncertainty does not necessarily imply uncertainty about endogenous variables Output uncertainty
- Evidence for model predictions:
  - Uncertainty is higher in recessions and deeper recessions feature higher uncertainty Table
  - More uncertain/deeper recessions last longer Mean recovery path
  - Large investments fall in recessions and are slow to recover the higher is uncertainty Spike share

We now relax some of our assumptions and vary parameter values:

- Risk averse preferences: logarithmic utility ▸ Risk Aversion
- Precision of individual signals ▸ γ_x
- Precision of public signals ▸ γ_y
- Heterogeneity in fixed costs ▸ σ_f

Conclusion

- We propose a model in which uncertainty fluctuates endogenously
- The complementarity between economic activity and information leads to uncertainty traps
- Uncertainty traps are robust to more general environment
  - More work needs to be done to identify size of informational frictions
- Interesting extensions:
  - Monopolistic competition: people not only care about the fundamental but also about the beliefs of others (higher-order beliefs)
  - Financial frictions: amplification through risk premium

Proposition

If $\beta e^{a(1-\rho_\theta)\overline{\mu}-\frac{a^2}{2}\frac{1-\rho_\theta^2}{\overline{\gamma}}+\frac{a^2}{2}\frac{1-\rho_\theta^2}{\gamma_\theta}} \leqslant 1$ and F is continuous, twice-differentiable with bounded first and second derivatives, for γ_x small,

1) The equilibrium exists and is unique;
2) The investment decision of firms is characterized by the cutoff $f_c(\mu,\gamma)$ such that firms invest iff $f \leq f_c(\mu,\gamma)$;
3) f_c is a strictly increasing function of μ and γ.

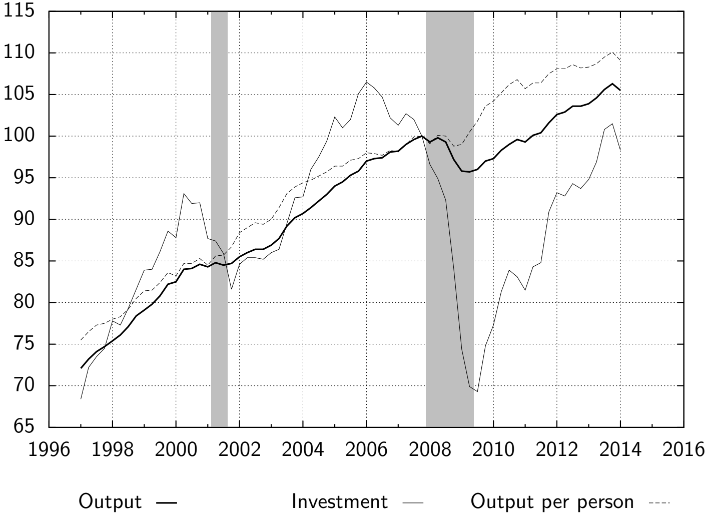
Return

Limit $N \to \infty$

- If γ_x was constant as we take the limit, a law of large number would apply and θ would be known
- To prevent agents from learning too much, we assume $\gamma_x\left(\bar{N}\right) = \gamma_x/\bar{N}$. Therefore the precision of the aggregate signal X stays constant at
$$N\gamma_x(\bar{N}) = n\gamma_x$$
where
$$n = \frac{N}{\bar{N}}$$
is the fraction of firms investing.
- Under this assumption, the updating rules for information are the same as with finite N

Return

2007-2009 Recession



Return

Timing

1. At the beginning, all firms share the same prior distribution on θ

$$\theta | \mathcal{I} \sim \mathcal{N}\left(\mu, \gamma^{-1}\right)$$

2. Firms with an investment opportunity draw $f_j \sim F$ and decide whether or not to invest
3. All firms choose investment rate i, labor l and production takes place
4. Signals X and Y are observed
5. Firms without an opportunity receive one with probability $\overline{q}$
6. Agents update their beliefs

Return

Information

- The structure of information is the same as before
  - Aggregate output reveals public signal $Y = \theta + \varepsilon^Y$ with precision γ_y
  - Social learning channel $X = \theta + \varepsilon^X$ with precision $nQ\gamma_x$
- Belief dynamics

$$\mu' = \rho_\theta \frac{\gamma\mu + \gamma_y Y + nQ\gamma_x X}{\gamma + \gamma_y + nQ\gamma_x}$$

$$\gamma' = \left(\frac{\rho_\theta^2}{\gamma + \gamma_y + nQ\gamma_x} + \frac{1-\rho_\theta^2}{\gamma_\theta} \right)^{-1}$$

◂ Return

Endogenous vs. Exogenous Variables

- Fundamental uncertainty does not imply uncertainty about endogenous variables
  - With high uncertainty, economy is more prone to fall in recession and less volatile

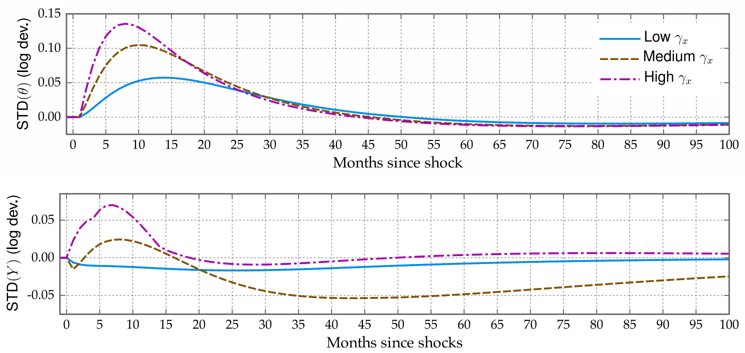


Figure : Standard deviation about θ_t vs. one-year ahead output y_{t+12}

Return

Uncertainty and Business Cycles

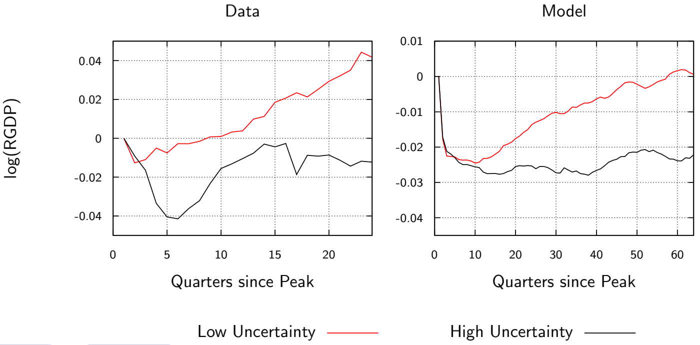
- Average uncertainty in recessions relative to expansion

Uncertainty Measure	Jurado et al. (2015)	VXO	Michigan Survey	SPF
Years available	1960-2014	1990-2014	1960-2014	1992-2014
Recessions	1.12	1.62	1.96	1.06
Small recessions	1.01	1.34	1.06	1.05
Large recessions	1.20	1.85	1.11	1.06

Return

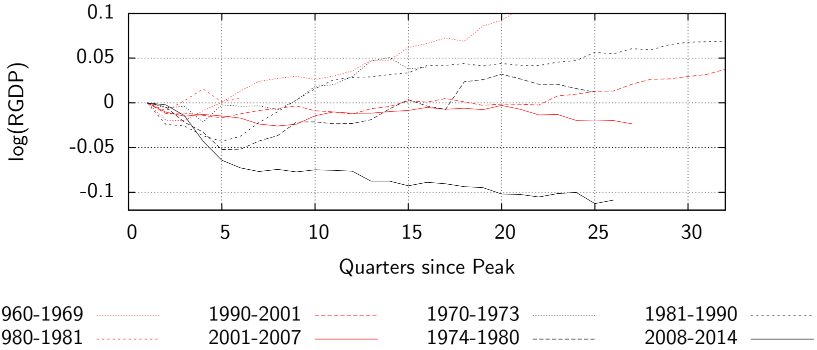
Mean Recovery Paths

- Classify recessions 1960-2015 in two bins according to average uncertainty and compute mean recovery path
- Simulate 100,000 periods in the model and compute the same



Return Data series

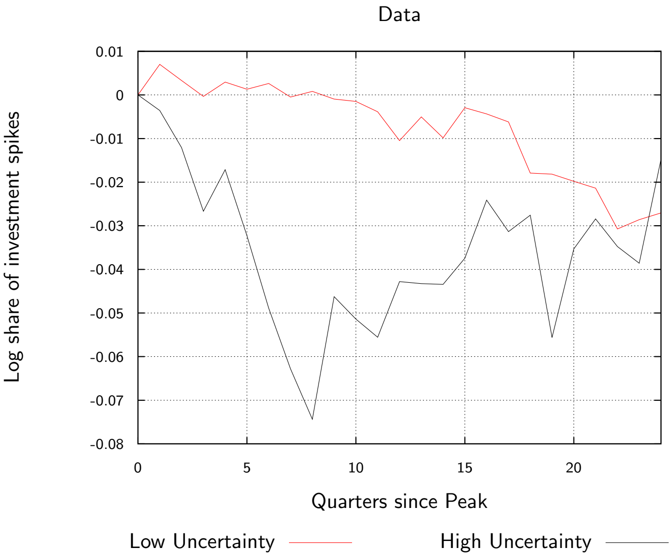
US Recessions 1960-2015



Recessions with low uncertainty in red, high uncertainty in black

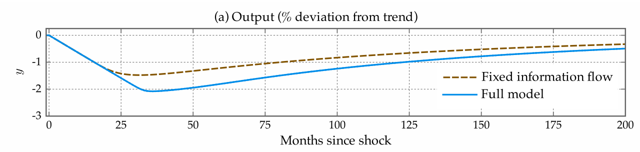
Mean Recovery Paths

Share of Investment Spikes



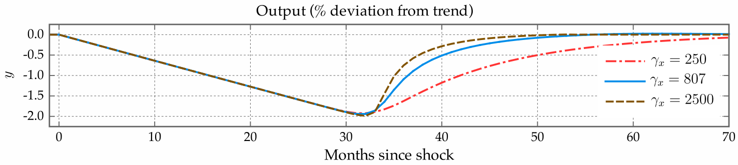
Return

Sensitivity: Risk Aversion



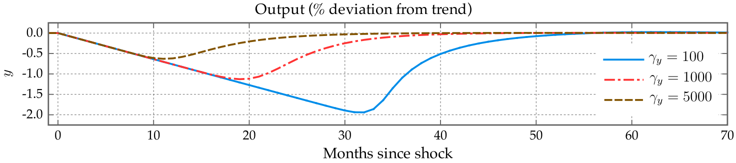
Sensitivity

Sensitivity to γ_x



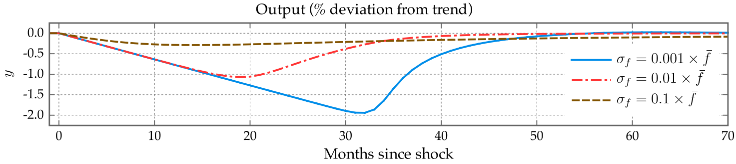
▸ Sensitivity

Sensitivity to γ_y



Sensitivity

Sensitivity to σ^f



◂ Sensitivity