MAT244 Notes

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§1 Day 1: Intro (Jan 05, 2026)

Our goal is to describe an unknown function using its derivatives, and hopefully obtain an explicit form.

Definition 1.1 (Proportional)**.** A is proportional to B if $A = cB$ where c is constant.

Problem 1.2

Model the following:

1. Researchers are studying a population of rabbits. Each year they measure the population, and they find that the population increases by 11% each year.
2. Based on field measurements, some reasearchers studying the population of rabbits in an area propose a model where at each moment, the population is increase at approximately 10.5% of its current size.
3. An object in freefall will accelerate towards the ground at $9.8m/s^2$ due to gravity, and will also be slowed down at a rate proportional to its velocity due to air resistance.

1. Let $P(t)$ denote the population at time t, with P_0 being the initial population. Some reasonable responses include:
   - $\frac{dP}{dt} = 0.11P$
   - $P(t) = P_0(1.11)^t$
   - $P(t) = 1.11P(t-1)$
   - $P(t) = 1.11P_0t$
   - $P(t) = 0$

   Some only grow linearly, which may not model the population well. Some need to include the base case P_0. The question is intentionally vague, so the professor thought these were fine.

2. Responses could be
   - $\frac{dP}{dt} = \ln(1.105)P_0 1.105^t$
   - $\frac{dP}{dt} = 0.105P$

   The second is response is what we're looking for (it is autonomous).

3. Have $h(t)$ be the height (in meters) of the object at t seconds, where positive is 'up'. Slowing down is a colloquialism for some acceleration in another direction.

$$\begin{aligned} a(t) &= a_{\text{gravity}}(t) + a_{\text{airres}}(t) \\ &= -9.8 - c \cdot v(t) \\ h'' &= -9.8 - ch' \end{aligned}$$

Definition 1.3 (ODE)**.** An ordinary differential equation is an equation involving a single variable function and the independent variable (input) of that function, and also derivatives up to a finite order of that function.

Definition 1.4 (PDE)**.** Partial differential equations involve derivatives w.r.t. multiple variables.

An example is the wave equation, $\frac{\partial^2 u}{\partial t^2} = k\frac{\partial^2 u}{\partial x^2}$. They are not testable, and show up in MAT351/APM346.

In this course[1], a general differential equation of order n is written as

$$y^{(n)} = f(t, y, y', \cdots, y^{(n-1)})$$

A function $\phi(t)$ is a solution when $\phi(t) \in C^n$ (is at least n times continuously differentiable), is defined on an open interval, and satisfies the equation

$$\phi^{(n)}(t) = f(t, \phi(t), \phi'(t), \cdots, \phi^{(n-1)}(t))$$

for all t in its domain. We usually have infinitely many solutions, but we can specify them using initial values.

General solutions contain one or more parameters.

Problem 1.5

Find the general solution to the DE $y' = -0.15y$. What if we add the constraint that $y(1) = -3$?

[1] so that solutions always exist

§2 Day 2: Linear DEs (Jan 07, 2026)

Somtimes, there is no better way to describe a function than as a solution of a DE. For example, e^{-x^2} has no elementary antiderivative.

Definition 2.1 (First Order Linear DE)**.** An equation of the form $y' + p(t)y = g(t)$ where p and g are some functions of t. It homogenous if $g(t) = 0$ for all t.

Problem 2.2

Find the general solutions to each of $y' = ay$, and $y' = ay + b$.

Take $y = ce^{at}$. See that for arbitrary c, $\frac{d}{dt}ce^{at} = ace^{at} = ay$

Take $y = ce^{at} - \frac{b}{a}$.

When solving for A in $A'' = -9.8 - 0.17A'$, notice that A is not present. Make the substitution $V = A'$, giving $V' = -9.8 - 0.17V$, which is a first order linear DE.

Problem 2.3

Find the general solutions to each of $y' = g(t)$, and $y' = f(t)y$.

The general solution of $y = \int g(t)\,dt$ is acceptable.

The general solution is $c\,\mathrm{Exp}(\int f(t)\,dt)$

Theorem 2.4. The general solution of $y' = p(t)y + g(t)$ is

$$y = e^{P(t)}\left(c + \int e^{-P(t)}g(t)\,dt\right)$$

where $P(t) = \int p(t)\,dt$, and c is an arbitrary constant.

TODO: Integrating factor technique