

STA257 Notes

MAX XU

'25 Fall

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§1 Day 1: (Sept 9, 2025)

Life is very random and uncertain, with interesting problems to solve (e.g. what's the probability that I win the lottery). This course uses certain mathematics to study the uncertainty of probabilities. You will be able to solve problems like this:

Problem 1.1. Which is more likely: getting at least one six when rolling a fair 6 sided die 4 times, or getting one pair of sixes when rolling two six sided dice 24 times?

5% of your grade is poll-based, [more info here](#).

Definition 1.2 (Sample Space). A non-empty set containing all possible outcomes, written S .

e.g. coin-flipping: $S = \{\text{Heads}, \text{Tails}\}$, two die: $S = \{1, 2, 3, 4, 5, 6\} \times \{1, 2, 3, 4, 5, 6\}$

Definition 1.3 (Event). Any subset $A \subseteq S$ is an event.

Prof says in some continuous sample spaces, there may exist some non-measurable subsets to which the probability measure defined later won't work on, but don't worry about it in this course. (yay!!)

Definition 1.4 (Probability). For any event A , define probability $P(A)$ that satisfies:

- For all $A \in \mathcal{P}(S)$, $0 \leq P(A) \leq 1$
- $A = S$, $P(A) = P(S) = 1$
- $A = \emptyset$, corresponding to no outcome, then $P(A) = P(\emptyset) = 0$
- **Additivity:** if $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$

If A_1, \dots, A_n are disjoint¹ events, we have

- **Finite Additivity:** For some $n \in \mathbb{N}$,

$$P\left(\bigcup_{i=1}^n A_i\right) = \sum_{i=1}^n P(A_i)$$

- **Countable Additivity:**

$$P\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} P(A_i)$$

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When looking at the probability of getting heads from a coinflip, $P(H)$ with $S = \{T, H\}$ is really shorthand for $P(\{H\})$, since H may not be a subset of S . For uniformly picking any number between 0 and 1, denoted $\text{Uniform}[0, 1]$, we can define a probability $P([a, b]) = b - a$ whenever $0 \leq a \leq b \leq 1$. (don't know what uniform means yet)

Note that by definition of probability, $P(A_i)$ is positive, so the right hand side is an absolutely convergent series (prof didn't mention this).

¹prof said this but i think he meant pairwise disjoint?

²we could've actually done this with just 3, see [probability axioms](#)