§1 Day 1: (Sept 9, 2025)

Life is very random and uncertain, with interesting problems to solve (e.g. whats the probability that I win the lottery). This course uses certain mathematics to study the uncertainty of probabilities. You will be able to solve problems like this:

Problem 1.1. Which is more likely: getting at least one six when rolling a fair 6 sided die 4 times, or getting one pair of sixes when rolling two six sided dice 24 times?

5% of your grade is poll-based, more info here.

Definition 1.2 (Sample Space)**.** A non-empty set containing all possible outcomes, written S.

e.g. coin-flipping: $S = \{\text{Heads}, \text{Tails}\}$, two die: $S = \{1,2,3,4,5,6\} \times \{1,2,3,4,5,6\}$

Definition 1.3 (Event)**.** Any subset $A \subseteq S$ is an event.

Prof says in some continuous sample spaces, there may exist some non-measurable subsets to which the probability measure defined later won't work on, but don't worry about it in this course. (yay!!)

Definition 1.4 (Probability)**.** For any event A, define probability $\mathrm{P}(A)$ that satisfies:

- For all $A \in \mathcal{P}(S)$, $0 \leq \mathrm{P}(A) \leq 1$
- $A = S$, $\mathrm{P}(A) = \mathrm{P}(S) = 1$
- $A = \emptyset$, corresponding to no outcome, then $\mathrm{P}(A) = \mathrm{P}(\emptyset) = 0$
- **Additivity**: if $A \cap B = \emptyset$, then $P(A \cup B) = P(A) + P(B)$

If $A_1, \ldots, A_n$ are disjoint[1] events, we have

- **Finite Additivity**: For some $n \in \mathbb{N}$,

$$\mathrm{P}\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} \mathrm{P}(A_i)$$

- **Countable Additivity**:

$$\mathrm{P}\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} \mathrm{P}(A_i)$$

[2]

When looking at the probability of getting heads from a coinflip, $\mathrm{P}(H)$ with $S = \{T, H\}$ is really shorthand for $\mathrm{P}(\{H\})$, since H may not be a subset of S. For uniformly picking any number between 0 and 1, denoted $\text{Uniform}[0,1]$, we can define a probability $\mathrm{P}([a,b]) = b - a$ whenever $0 \leq a \leq b \leq 1$. (don't know what uniform means yet)

Note that by definition of probability, $\mathrm{P}(A_i)$ is positive, so the right hand side is an absolutely convergent series (prof didn't mention this).

[1] prof said this but i think he meant pairwise disjoint?

[2] we could've actually done this with just 3, see probability axioms