运行说明

代码思路

$$f(x)=\frac{x^3}{3}-x$$
$$f'(x)=x^2-1$$
$Newton$迭代法：$x_{k+1}=x_k-\frac{f(x_k)}{f'(x_k)},k=0,1,2,\dots$

收敛阶为2。

弦截法：$x_{k+1}=x_k-\frac{f(x_k)(x_k-x_{k-1})}{f(x_k)-f(x_{k-1})},k=0,1,2,\dots$

收敛阶为$\frac{1+\sqrt{5}}{2}$

实现细节

对两种求根方法分别设置函数：对Newton迭代法传入x0，待求函数式及其导数，收敛判断标准epsilon，对弦截法传入x0, x1, 待求函数式，收敛判断标准epsilon。在函数中依据理论公式计算每一次迭代所得根，并储存于数组中。数组中的数值用于收敛阶的分析。具体见代码注释

分析收敛阶：

记收敛阶为α, 在迭代所用函数内计算每一步迭代之后的$\frac{|e_k|}{|e_{k-1}|^\alpha}$

若随着k的增大，表达式值有收敛于常数的趋势，则可验证收敛阶正确性

运行结果

```
Enter full name of method:
newton
Enter x0 (q to quit): 0.1
x0 = 1.000000000000000e-01, k = 3, x = 0.000000000000000e+00
Analyse order:
k = 1 , order = 6.505221574305985e-01
k = 2 , order = 5.769015970140415e-01
k = 3 , order = 5.773502693253441e-01
Enter x0 (q to quit): 0.2
x0 = 2.000000000000000e-01, k = 4, x = 0.000000000000000e+00
Analyse order:
k = 1 , order = 7.402954192947491e-01
k = 2 , order = 5.736642711733040e-01
k = 3 , order = 5.773503453998806e-01
k = 4 , order = 5.773502691896258e-01
Enter x0 (q to quit): 0.9
x0 = 9.000000000000000e-01, k = 7, x = -1.732050807568877e+00
Analyse order:
k = 1 , order = 6.196580364155928e+00
k = 2 , order = 2.034906009228296e-01
k = 3 , order = 2.505361375912547e-01
k = 4 , order = 2.807428063535430e-01
k = 5 , order = 2.883427033381300e-01
k = 6 , order = 2.886745599258799e-01
k = 7 , order = 2.886751345930968e-01
Enter x0 (q to quit): 9.0
x0 = 9.000000000000000e+00, k = 10, x = 1.732050807568877e+00
Analyse order:
k = 1 , order = 8.221687836487031e-02
k = 2 , order = 1.288753578078540e-01
k = 3 , order = 2.053141342924570e-01
k = 4 , order = 3.310932028720108e-01
k = 5 , order = 5.240104039907028e-01
k = 6 , order = 7.418945128981742e-01
k = 7 , order = 8.528362334139225e-01
k = 8 , order = 8.658942653614247e-01
k = 9 , order = 1.853602114439544e+00
k = 10 , order = 4.503599627370496e+15
Enter x0 (q to quit): q

D:\计算方法\1\Project1\x64\Debug\Project1.exe (进程 19920)已退出，代码为 0。
按任意键关闭此窗口. . .
```

对于Newton迭代法，所得order值除最后一组外均区域收敛。且一二组所得根数值相同，order收敛值也相近，符合收敛阶等于2的结论。最后一组未收敛可能是由于初值与根差别较大，导致达到精度时还未收敛。

```
Enter full name of method:
secant
Enter x0 (q to quit): -0.1
Enter x1 (q to quit): 0.1
x0 = -1.000000000000000e-0111296x1 = 1.000000000000000e-01, k = 2, x = 0.000000000000000e+00
Analyse order:
k = 1 , order = 6.127620222179342e-01
k = 2 , order = 7.840618286660553e-01
Enter x0 (q to quit): -0.2
Enter x1 (q to quit): 0.2
x0 = -2.000000000000000e-0111296x1 = 2.000000000000000e-01, k = 2, x = 0.000000000000000e+00
Analyse order:
k = 1 , order = 5.278195160495076e-01
k = 2 , order = 8.685250406004802e-01
Enter x0 (q to quit): -2.0
Enter x1 (q to quit): 0.9
x0 = -2.000000000000000e+0011296x1 = 9.000000000000000e-01, k = 12, x = 1.732050807753635e+00
Analyse order:
k = 1 , order = 9.879143065877728e-02
k = 2 , order = 2.642710845322459e+02
k = 3 , order = 1.622614998602000e-04
k = 4 , order = 1.120314941258988e+00
k = 5 , order = 5.780769971189714e+00
k = 6 , order = 1.177814004413585e-02
k = 7 , order = 7.116569988014343e+00
k = 8 , order = 2.783714087401807e-01
k = 9 , order = 2.333312904345379e+00
k = 10 , order = 5.203552820324061e-01
k = 11 , order = 1.290550957082472e+00
k = 12 , order = 7.398135786119525e-01
Enter x0 (q to quit): 0.9
Enter x1 (q to quit): 9.0
x0 = 9.000000000000000e-0111296x1 = 9.000000000000000e+00, k = 14, x = 1.732050807453568e+00
Analyse order:
k = 1 , order = 9.786156238509617e+00
k = 2 , order = 3.268545727571222e-02
k = 3 , order = 1.107610470250098e+00
k = 4 , order = 8.835786678217934e+00
k = 5 , order = 3.426525911077309e-02
k = 6 , order = 9.784717936782918e-01
k = 7 , order = 3.230422759493619e+00
k = 8 , order = 3.068372790008557e-01
k = 9 , order = 1.216150119306951e+00
k = 10 , order = 1.042249057264974e+00
k = 11 , order = 9.000287736028982e-01
k = 12 , order = 9.025521904465439e-01
k = 13 , order = 9.259411769433482e-01
k = 14 , order = 9.083092601959522e-01
```

对于Newton迭代法，收敛速度较快，一二组不易判断。第三组order趋于收敛，符合收敛阶等于（1+sqrt(5)）/2的结论。最后一组未收敛可能是由于初值与根差别较大，导致达到精度时还未收敛。