

运行说明

代码思路

最小二乘法

$$\begin{aligned}\varphi(x) &= a\sin(x) + b\cos(x) \\ L &= \sum_i (\varphi(x_i) - y_i)^2 = \sum_i (a\sin(x_i) + b\cos(x_i) - y_i)^2 \\ \frac{\partial L}{\partial a} &= 2(a\sum_i \sin(x_i)^2 + b\sum_i \sin(x_i)\cos(x_i) - \sum_i y_i \sin(x_i)) \\ \frac{\partial L}{\partial b} &= 2(a\sum_i \sin(x_i)\cos(x_i) + b\sum_i \cos(x_i)^2 - \sum_i y_i \cos(x_i)) \\ L \text{最小时对应} \frac{\partial L}{\partial a} &= 0, \frac{\partial L}{\partial b} = 0. \\ \begin{cases} a\sum_i \sin(x_i)^2 + b\sum_i \sin(x_i)\cos(x_i) &= \sum_i y_i \sin(x_i) \\ a\sum_i \sin(x_i)\cos(x_i) + b\sum_i \cos(x_i)^2 &= \sum_i y_i \cos(x_i) \end{cases} \\ \text{记} A &= \sum_i \sin(x_i)^2, B = \sum_i \sin(x_i)\cos(x_i), C = \sum_i \cos(x_i)^2, \\ D &= \sum_i y_i \sin(x_i), E = \sum_i y_i \cos(x_i). \\ \text{则} \\ \begin{cases} a &= \frac{CD-BE}{AC-BB} \\ b &= \frac{AE-BD}{AC-BB} \end{cases} \\ \text{均方误差} mse &= L/i. \end{aligned}$$

实现细节

输入插值节点个数，使用动态数组储存x、y，输入插值节点x、y，计算A、B、C、D、E解得a、b。代入L计算均方误差。细节见代码注释。

运行结果

```
Enter number of nodes:
10
Enter list of x:
0.25
0.5
0.75
1.0
1.25
1.5
1.75
2.0
2.25
2.50
Enter list of y:
1.284
1.648
2.117
2.718
3.427
2.798
3.534
4.456
5.465
5.894
a = 4.407264412474432e+00
b = -1.675711420892508e+00
均方误差 = 1.258139370828297e+00
```