$$\dot{\rho}_c + 3H\rho_c = -\sqrt{\frac{2}{3}}\beta_c(\phi)\frac{\rho_c\dot{\phi}}{M_{\text{Pl}}}$$