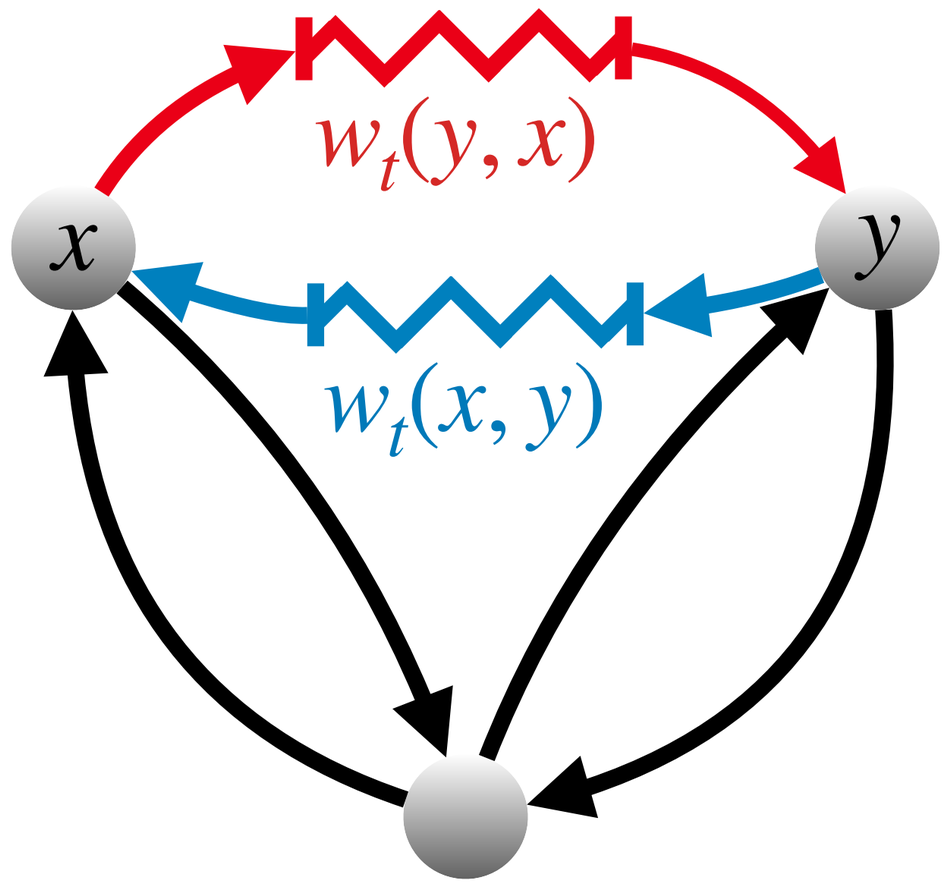
Kinetic Optimal Problem

$$\min_{u_t} \int_0^1 \mathbb{E}_{X_t \sim p_t} \| u_t(X_t) \|^2 dt$$

Continuous

Discrete

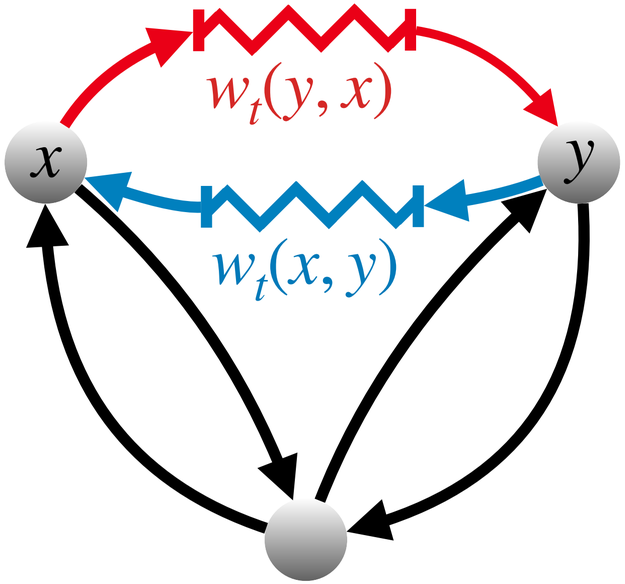
$$\min_{u_t} \int_0^1 \mathbb{E}_{X_t \sim p_t} \left(\sum_{y \neq x} w_t(y, X_t) u_t(y, X_t)^2 \right) dt$$



Constraint 1: u_t generates $X_t \sim p_t$

Constraint 2: u_t is non-negative

Kinetic Optimal Problem



Continuous

$$\min_{u_t} \int_0^1 \mathbb{E}_{X_t \sim p_t} \|u_t(X_t)\|^2 dt$$

Discrete

$$\min_{u_t} \int_0^1 \mathbb{E}_{X_t \sim p_t} \left(\sum_{y \neq x} w_t(y, X_t) u_t(y, X_t)^2 \right) dt$$

Constraint 1: u_t generates $X_t \sim p_t$

Constraint 2: u_t is non-negative

Kinetic Optimal Solution

Theorem: For a symmetric weighting the kinetic optimal velocity is

$$u_t(y, x) = \frac{1}{p_t(x)} \left[p_t(x)\dot{p}_t(y) - \dot{p}_t(x)p_t(y) \right]_+$$