Doctoral Thesis Defense

Spine-driven deformation with Local Volume Preservation

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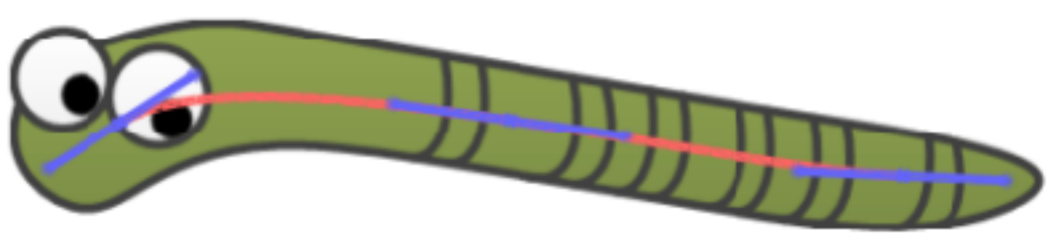
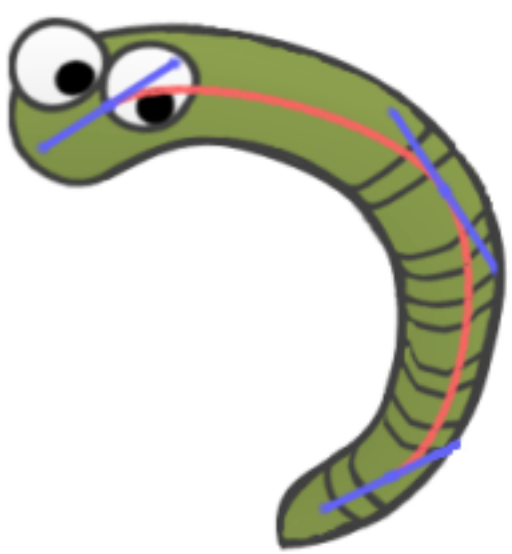
Goal

1. Propose and implement a framework for spine-driven deformation

2. Derive the mathematical model for local volume (or area in 2D) preservation



before



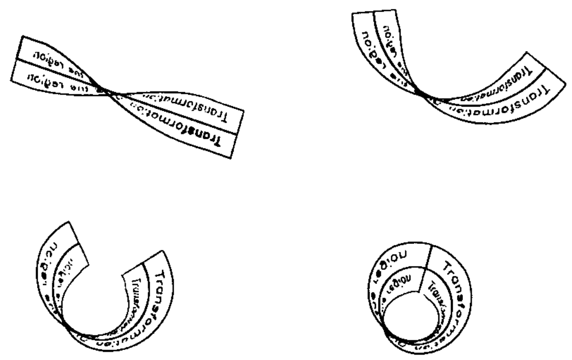
after

Outline

Deformation

Primitive

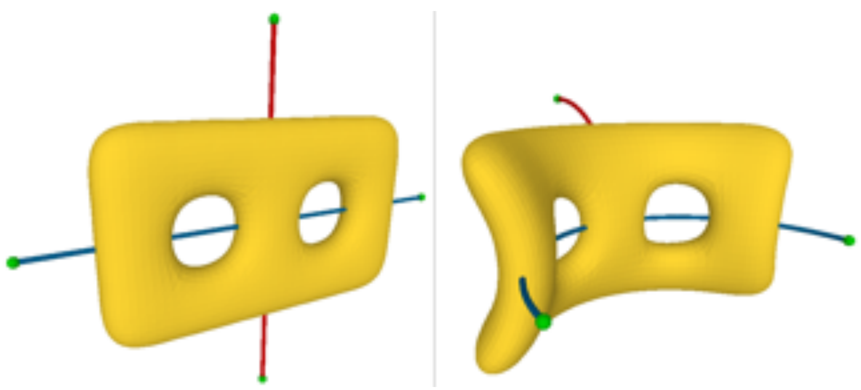
translation, rotation, shearing, scaling
bending



Barr 1984

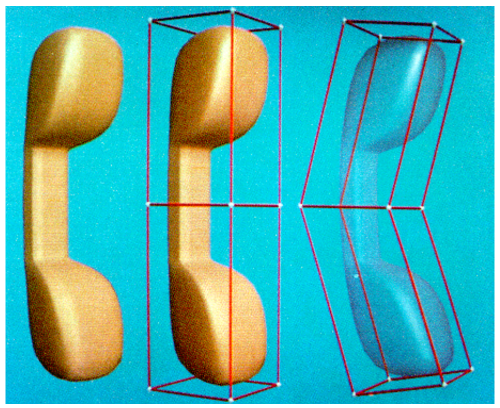
Skeleton-driven

deformation is controlled by skeletons



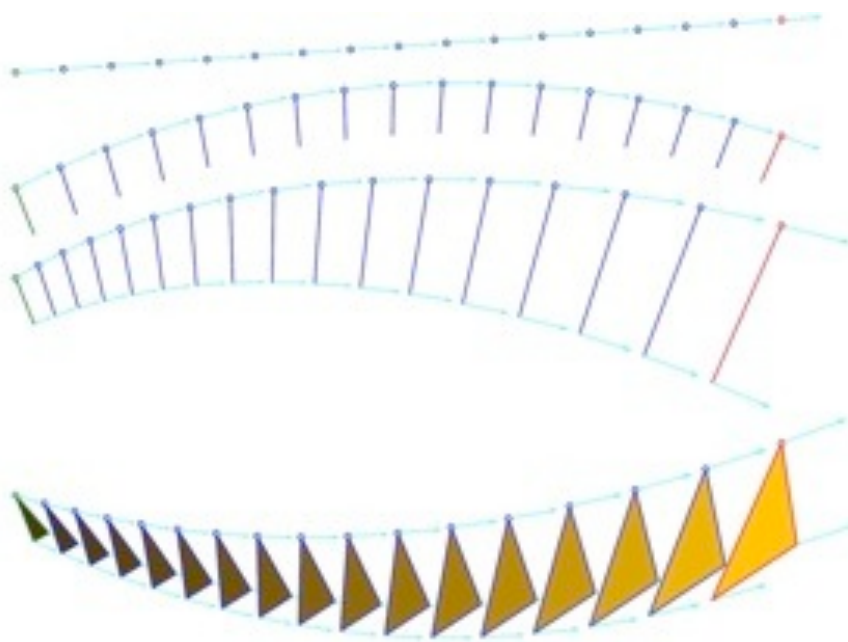
Freeform (FFD)

any mapping from R3 to R3, e.g. trivariate or polynomial mapping



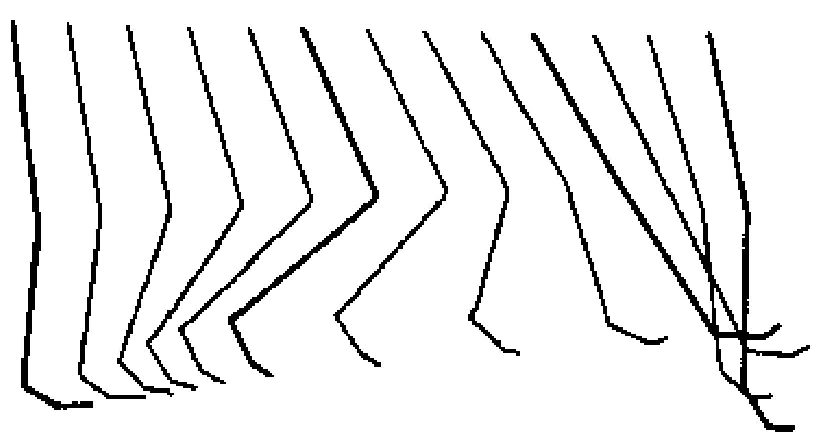
Sederberg and Parry 1986

Application to Animation



Rossignac and Vinacua 2011

Burtnyk and Wein 1976



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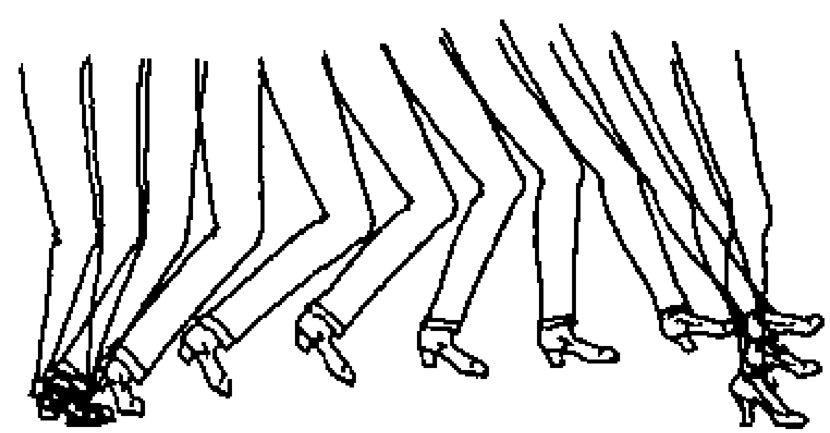
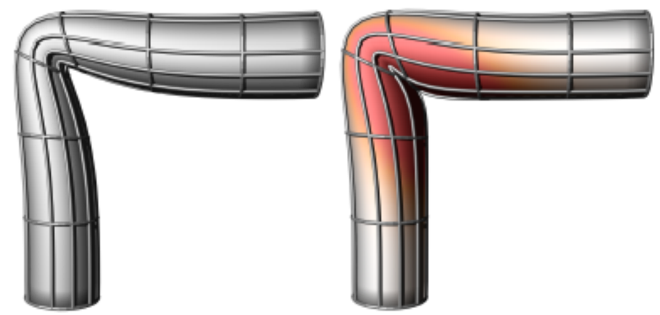


Skeleton-driven Deformation

Polygonal Skeleton

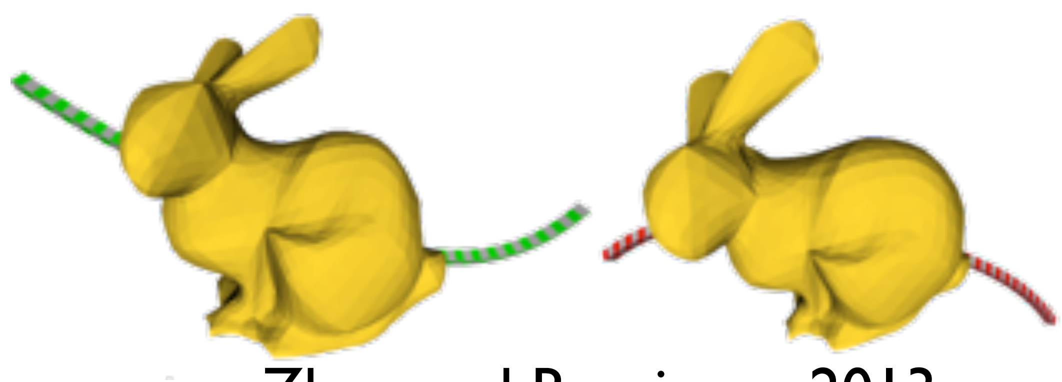
with bifurcations
Special care taken at junctions



Rohmer, Hahmann, Cani, 2009

Smooth Spine Curve

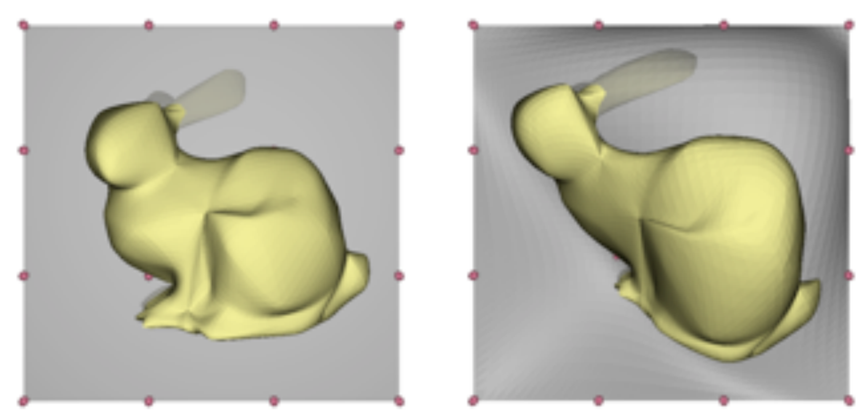
extension of polygonal curve
without bifurcations



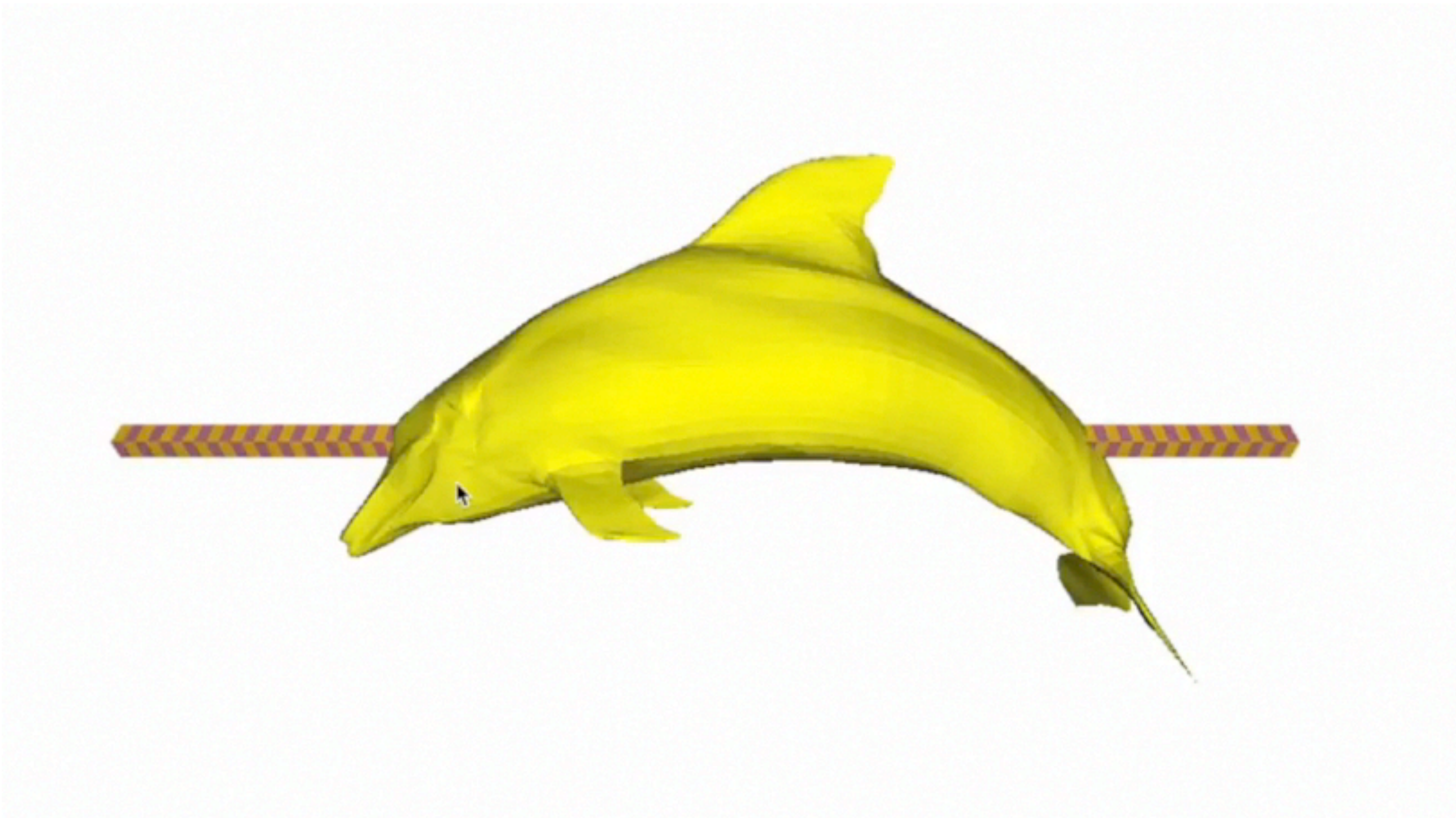
Zhuo and Rossignac, 2013

Smooth Spine Surface

extension of spine curve



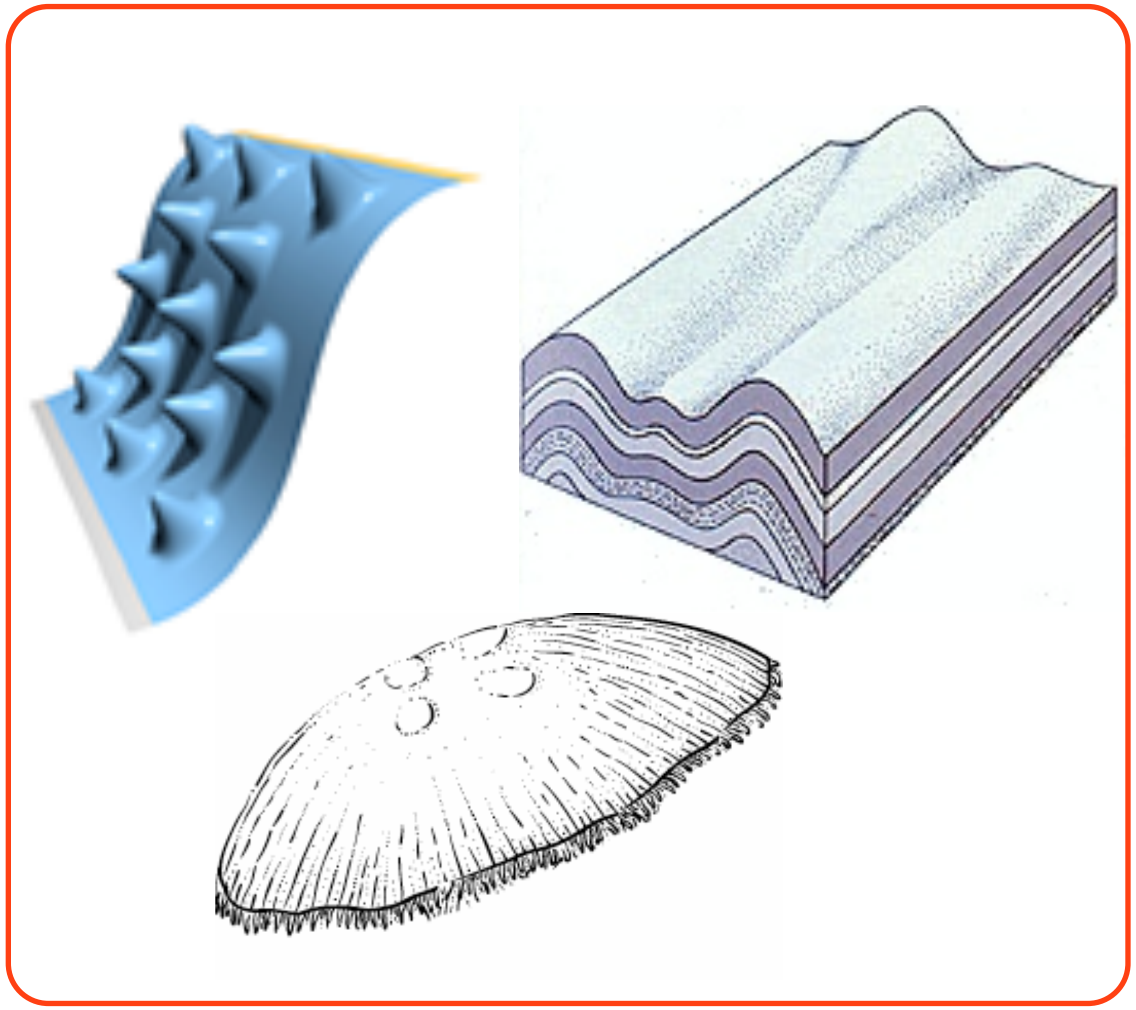
Spine-driven Deformation



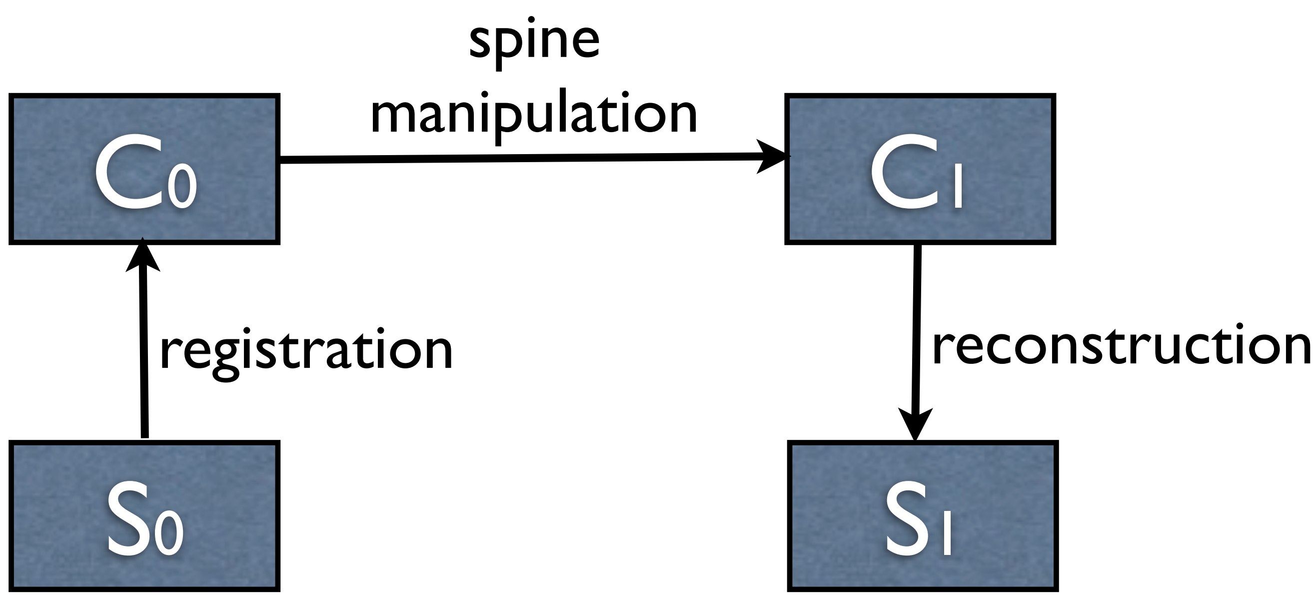
Applications

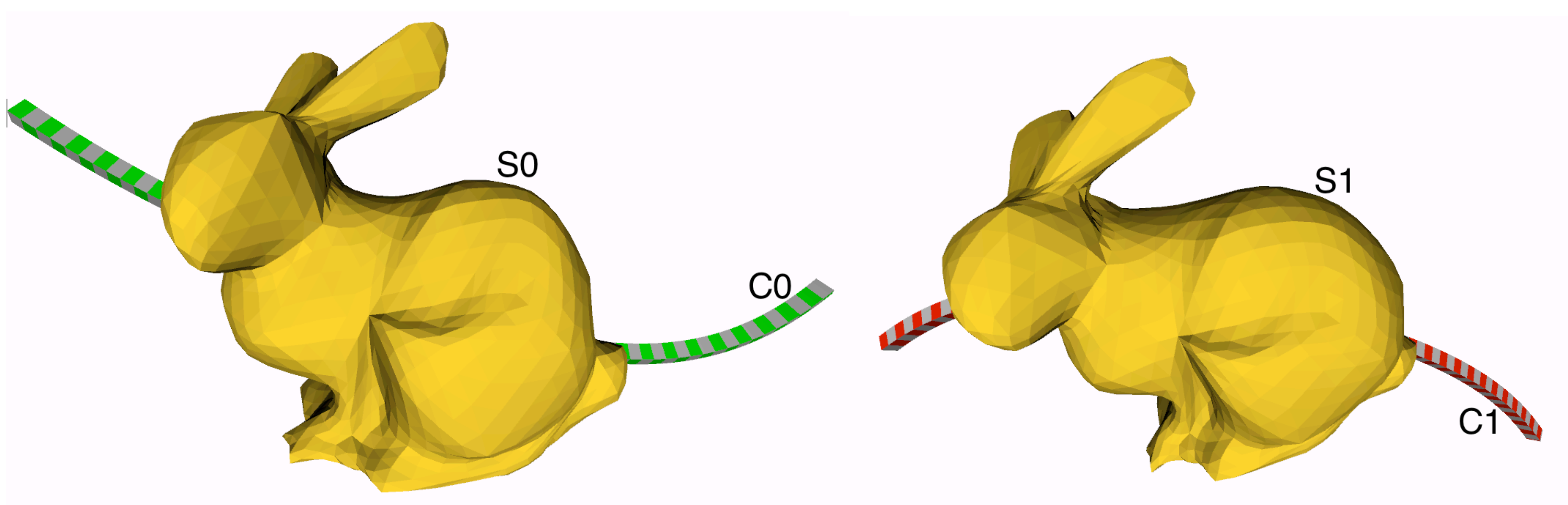
driven by spine curve

driven by spine surface



Overview

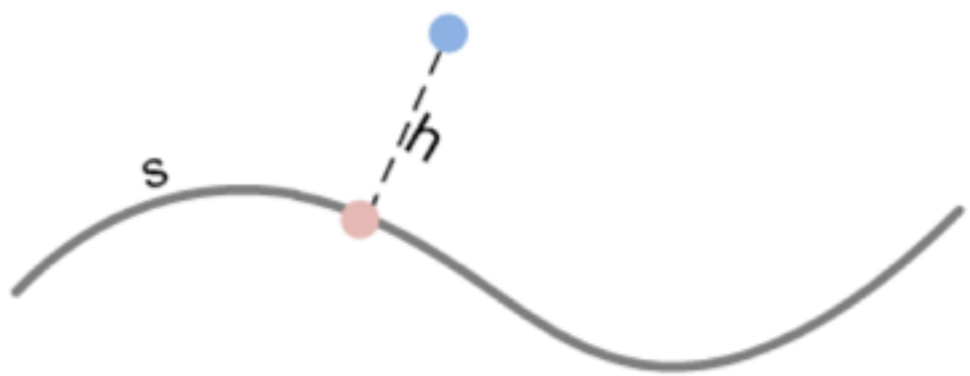


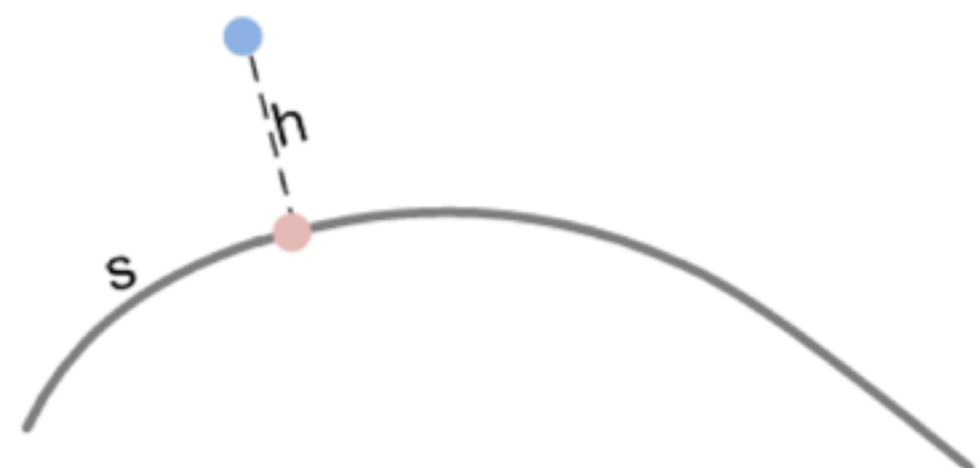


Overview of the steps in the deformation driven by a spine curve

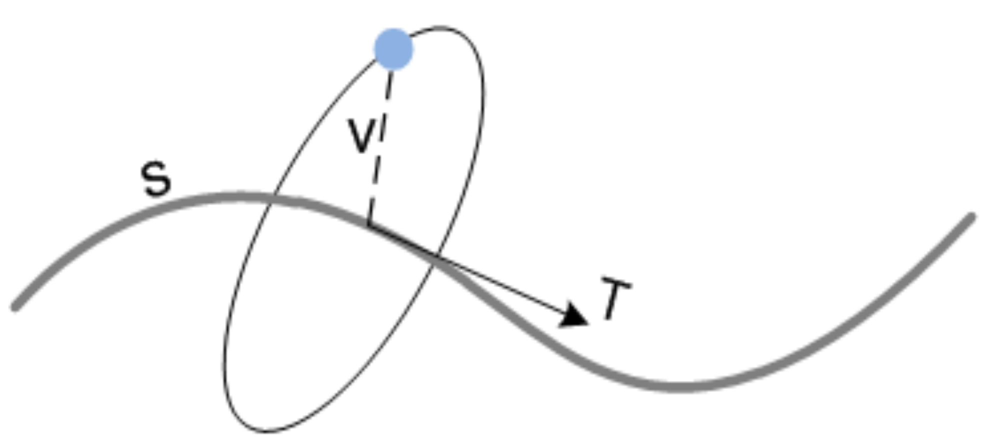
Assumptions

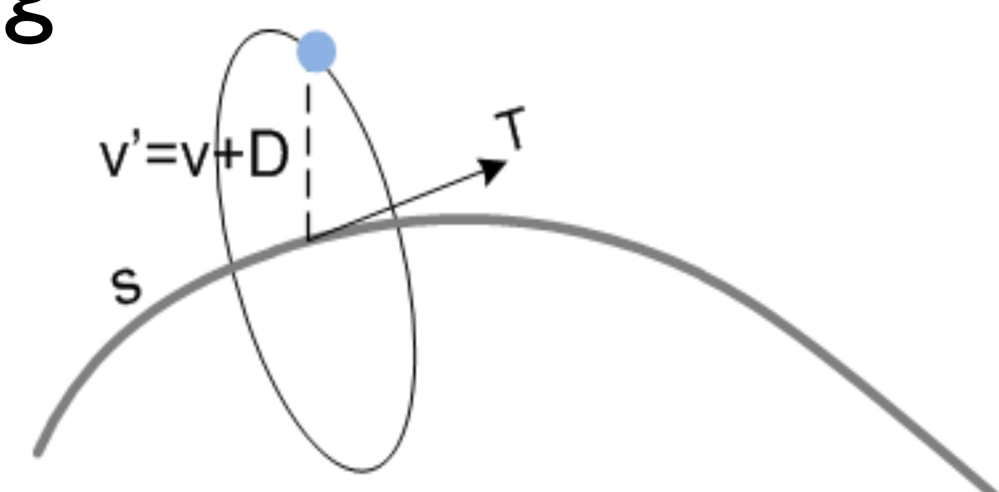
2D: Normal offset preserving





3D: Cross section preserving



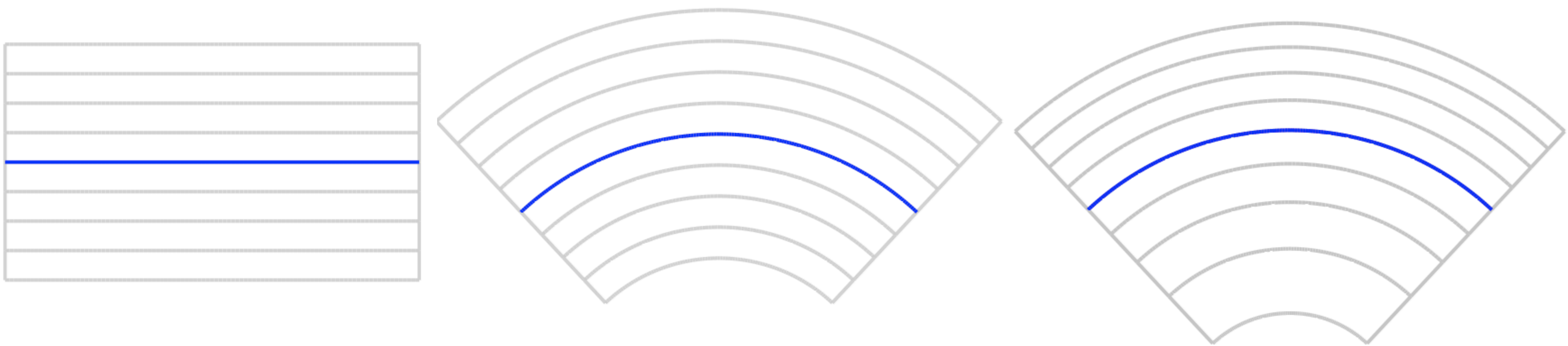


- Preserve the parameter s of the closest projection on the spine
- Simplest model to support the behavior of physical phenomena

We will discuss the problem first in 2D on how to preserve the area and later in 3D on how to preserve the volume.

Research Challenge

How to preserve the volume or area of a deformed shape locally?

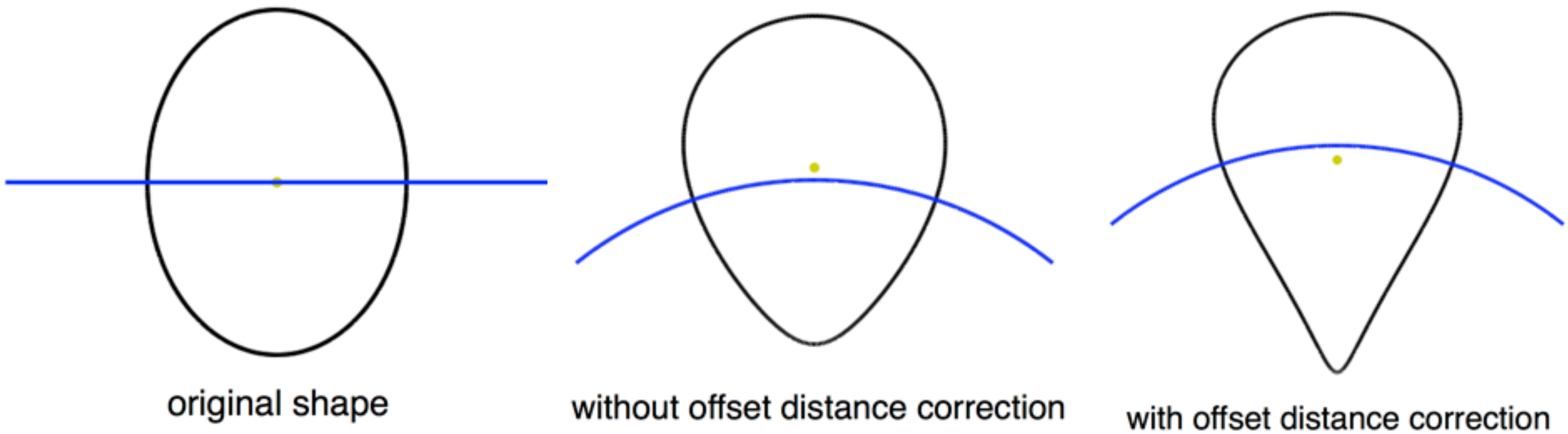


Original Naive Correct

Observation - thickness of each layer should change in order to preserve its area

Locally Area Preserving

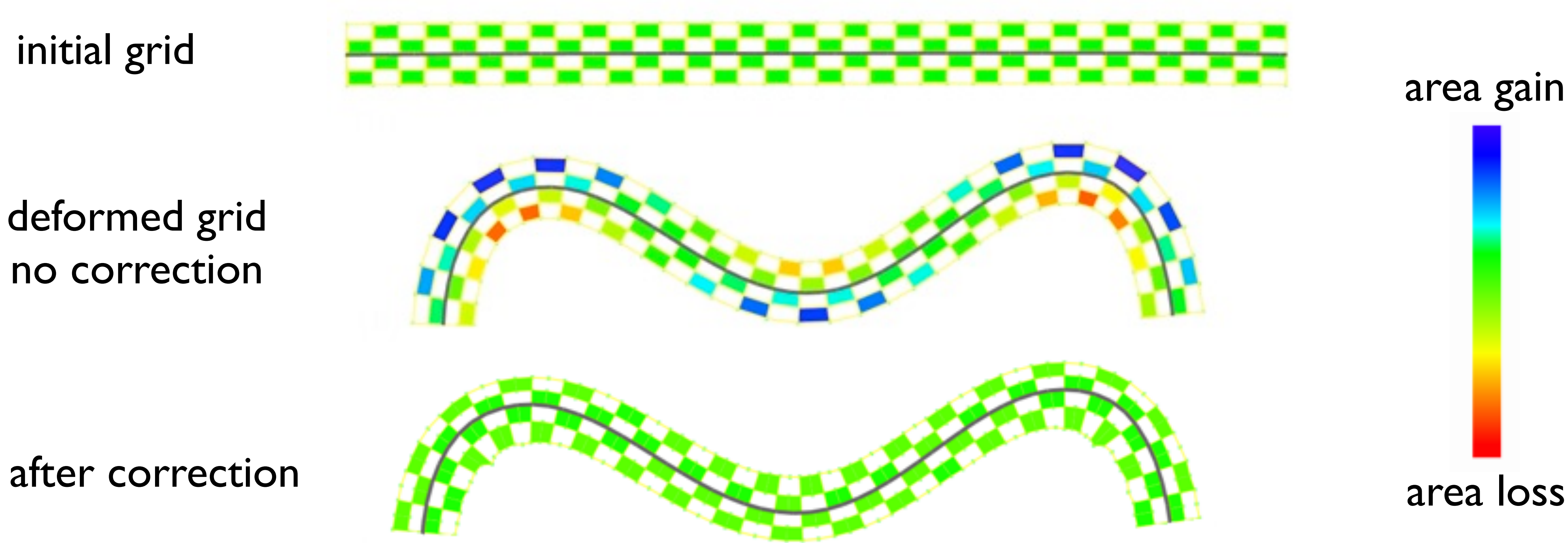
Insight to the solution: correcting the offset distance from based on the local curvature



In this way, the area loss or gain is compensated

Problem in 2D

Preserve the local area of any region or cell in space by adjusting the initial offset distance based on the curvature



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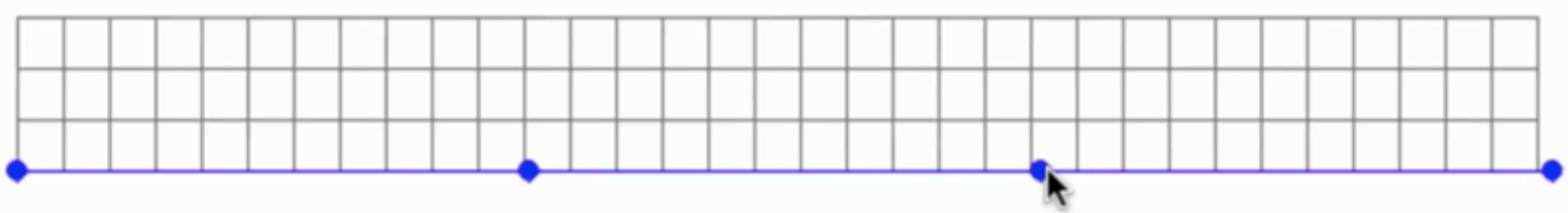
Prior Solution in 2D

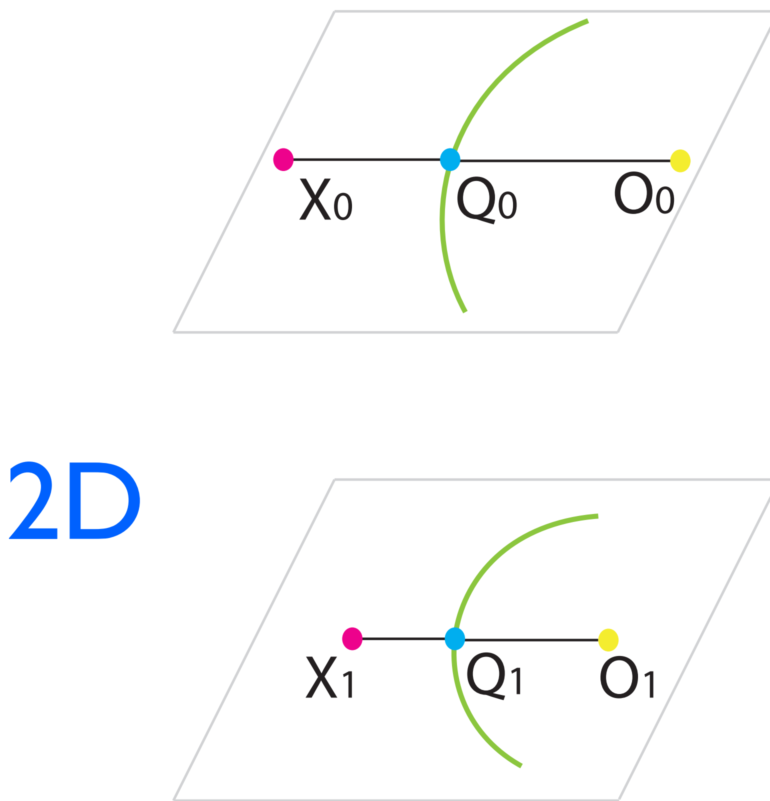
$h_1 - \frac{1}{2} k_1 h_1^2 = (h_0 - \frac{1}{2} k_0 h_0^2)\sigma$

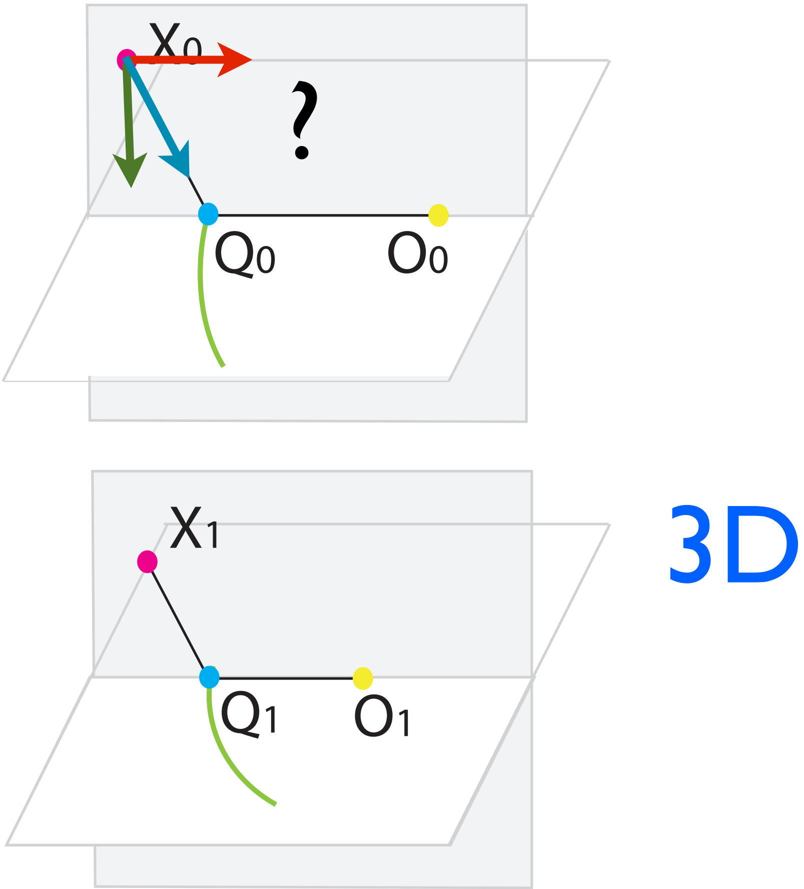
discovered by Chirikjian in 1995 and Moon in 2008



Problem in 3D

Preserve the local volume of any chunk in space by adjusting the direction and distance of the offset within the cross section





Challenge - additional degree of freedom in 3D

Our Solutions in 3D

Derivations

1. Express a point P in space as
$P(s, x, y) = C(s) + xN(s) + yB(s)$ or
$P(s, r, \theta) = C(s) + r\cos\theta N(s) + r\sin\theta B(s)$
2. Set the Jacobian determinant $det(\frac{\partial P_1}{\partial P_0})$ to 1
3. Solve the corresponding PDE

Three fundamental cases in a possible family of solutions

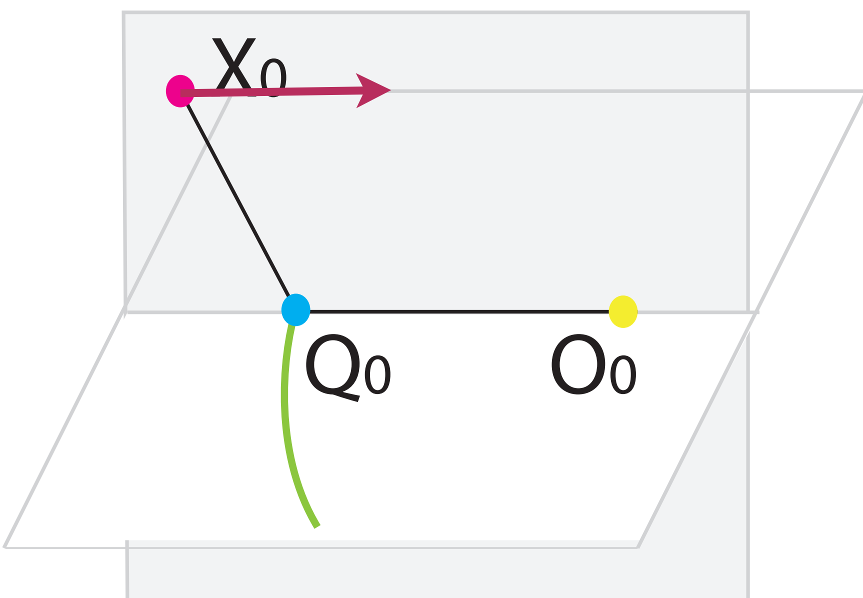
Normal $x_1 - k_1 x_1^2 = (x_0 - k_0 x_0^2)\sigma$

Radial $r_1^2 - \frac{2}{3}k_1 \cos\theta_1 r_1^3 = (r_0^2 - \frac{2}{3}k_0 \cos\theta_0 r_0^3)\sigma$

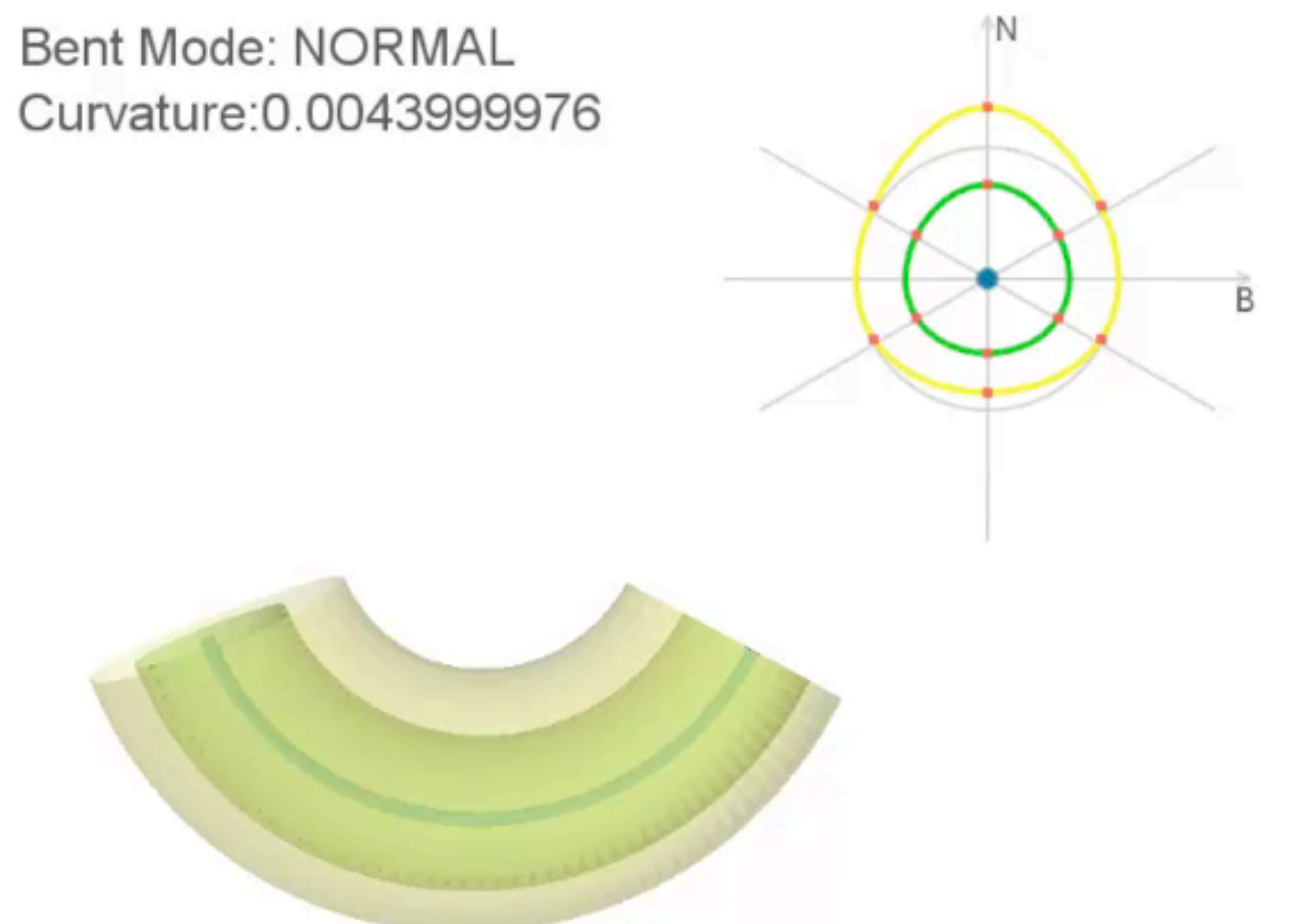
Binormal $(1 - k_1 x_1)y_1 = (1 - k_0 x_0)y_0\sigma$

Normal Solution

$$x_1 - k_1 x_1^2 = (x_0 - k_0 x_0^2)\sigma$$

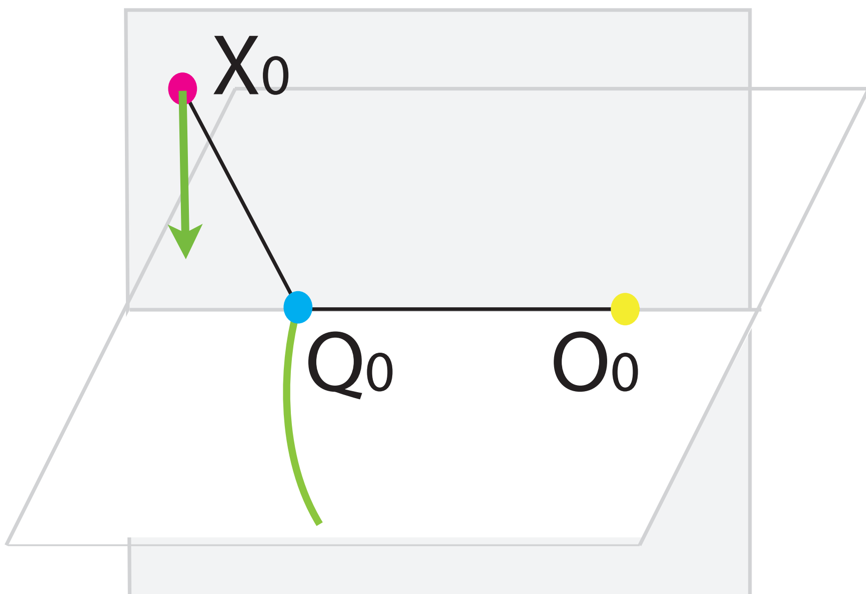


translation parallel to the osculating plane

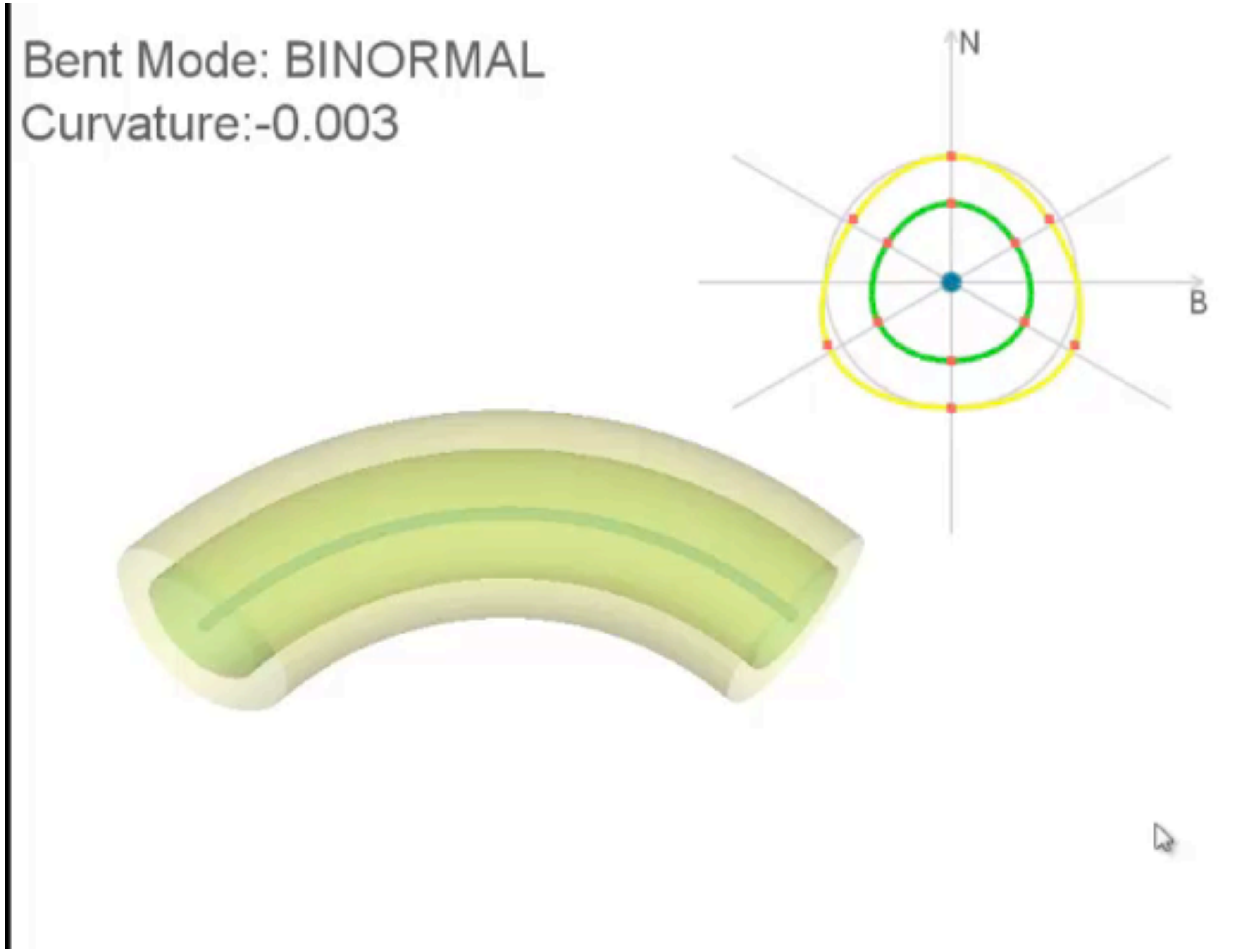


Binormal Solution

$$(1 - k_1 x_1) y_1 = (1 - k_0 x_0) y_0 \sigma$$

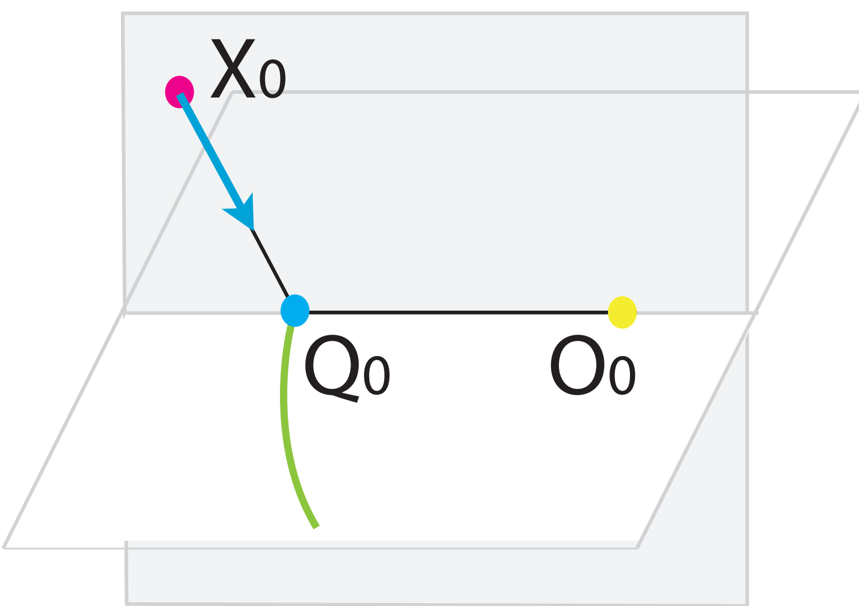


translation orthogonal to the osculating plane

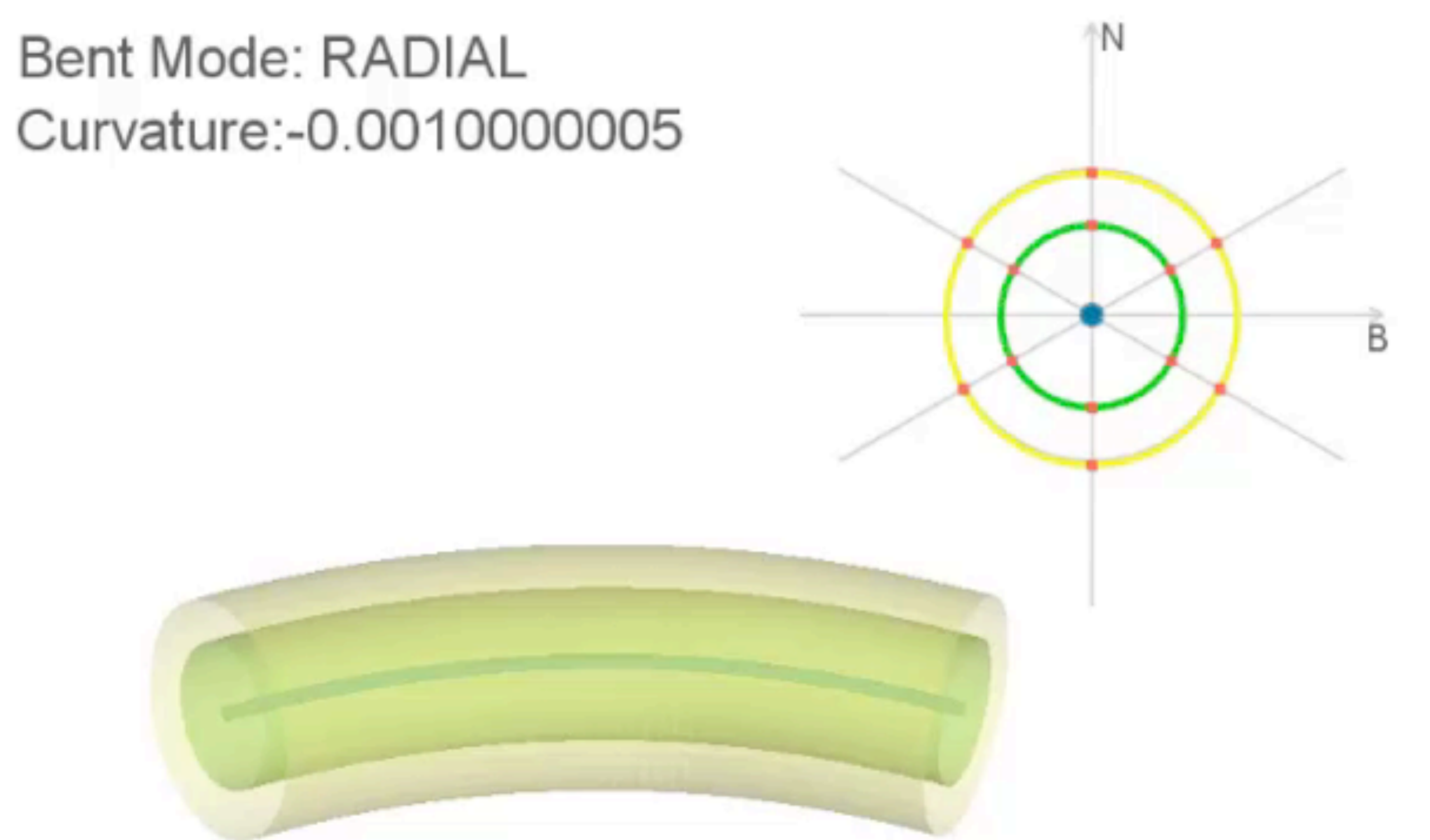


Radial Solution

$$r_1^2 - \frac{2}{3}k_1 \cos\theta_1 r_1^3 = (r_0^2 - \frac{2}{3}k_0 \cos\theta_0 r_0^3)\sigma$$



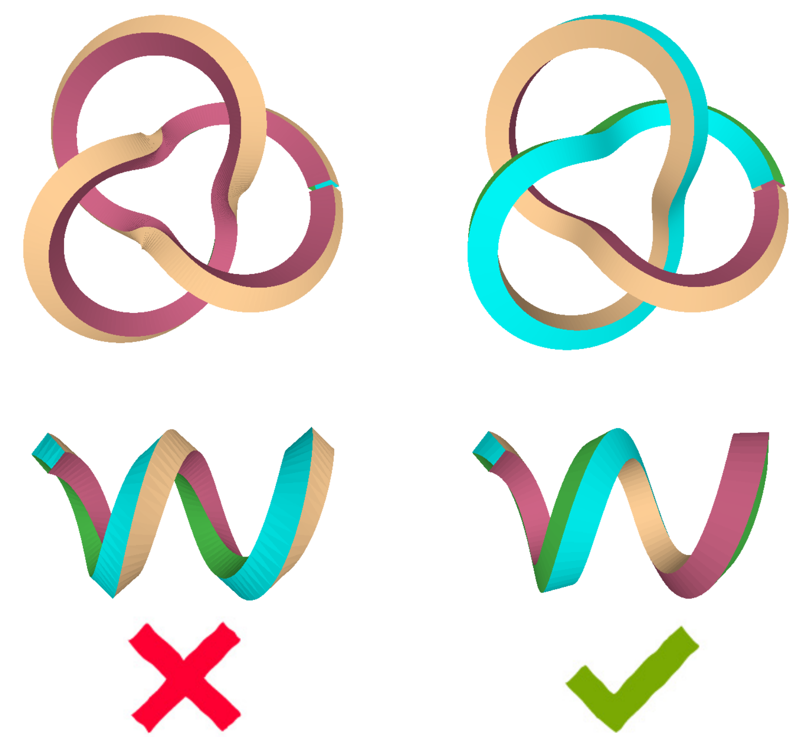
translate towards or away from the closest projection



Implementation

What happens without local volume preservation?

$$
\begin{tikzcd}
C_0 \arrow[r, "\text{spine manipulation}"] & C_1 \arrow[d, "\text{reconstruction}"] \\
S_0 \arrow[u, "\text{registration}"] & S_1
\end{tikzcd}
$$



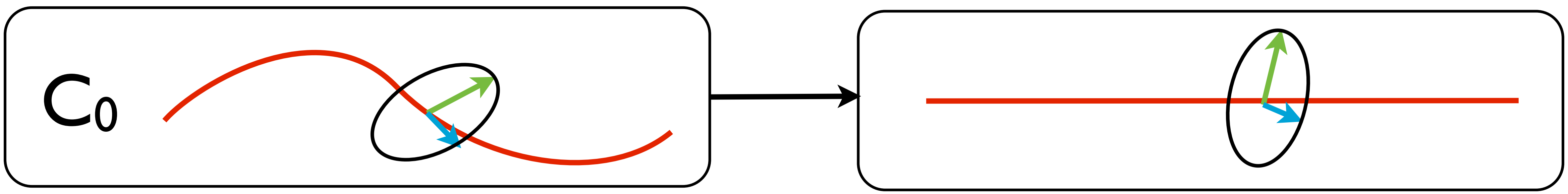
Registration with respect to Frenet frame produces unacceptable result.
We use twist compensated frame, implement using parallel transform.

Reference: Hanson and Ma 1995; Wang, Jüttler, Zheng and Liu 2008

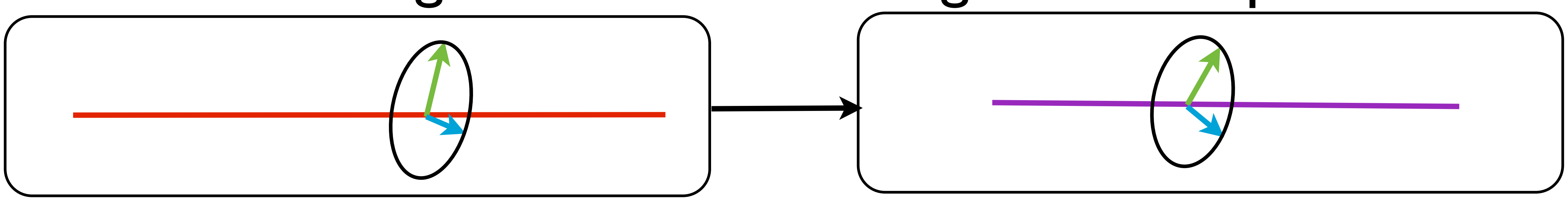
Our Implementation in 3D

Unbending-transfer-bending

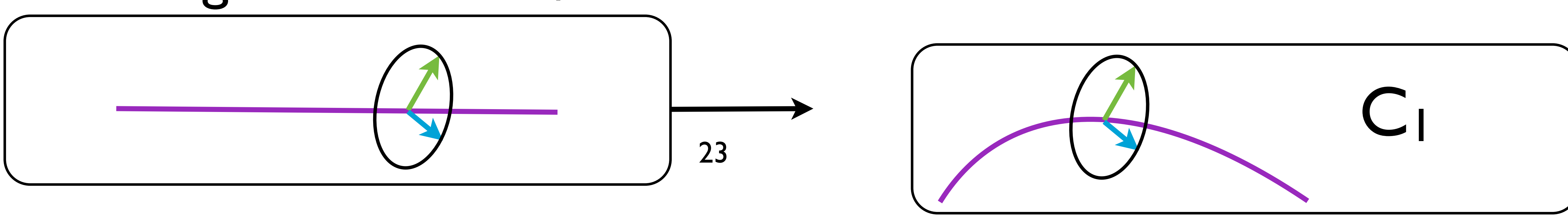
1. Unbending - straighten C_0



2. Transfer - change basis for reflecting twist compensation



3. Bending: bend into C_1



Publications

Wei Zhuo and Jarek Rossignac, Curvature-based Distance Offset, Implementation and Applications, SMI 2012

Wei Zhuo and Jarek Rossignac, Spine-driven Bending with Local Volume Preservation, EG 2013

Extensions: stretchable spine curve, spine surface, and problem with bifurcations

Stretchable Spine Curve

Let C(s) represent a 3D stretchable curve where s may not be the arc-length parameter. Let l be the arc-length. The derivative relationship between s and l is $dl = |C'(s)|ds.$

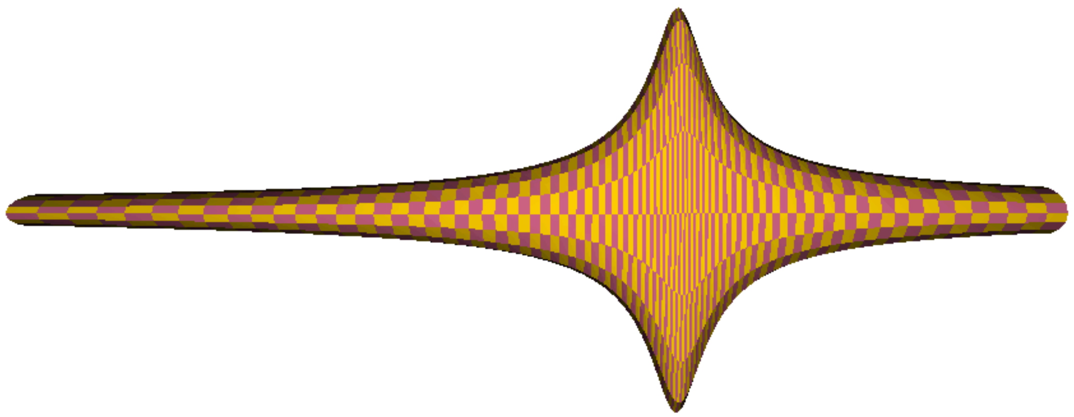
Difference from the non-stretchable curve:

Let $\sigma=|C'_0(s)|/|C'_1(s)|$. Scale the normal or the binormal offset distance by σ. Or scale the radial offset distance by $\sqrt{\sigma}$.

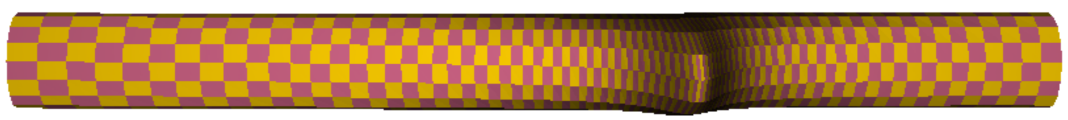
Examples

original cylinder and control point positions

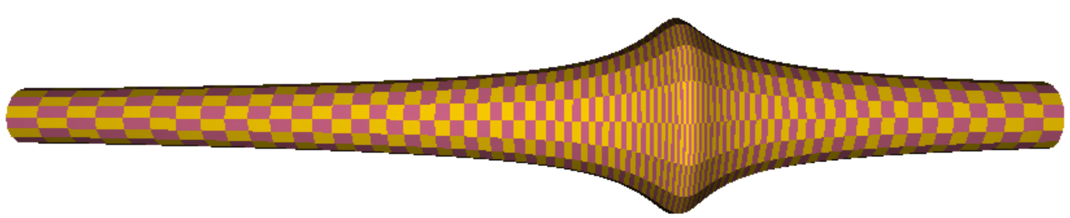
control point positions after manipulation



Normal

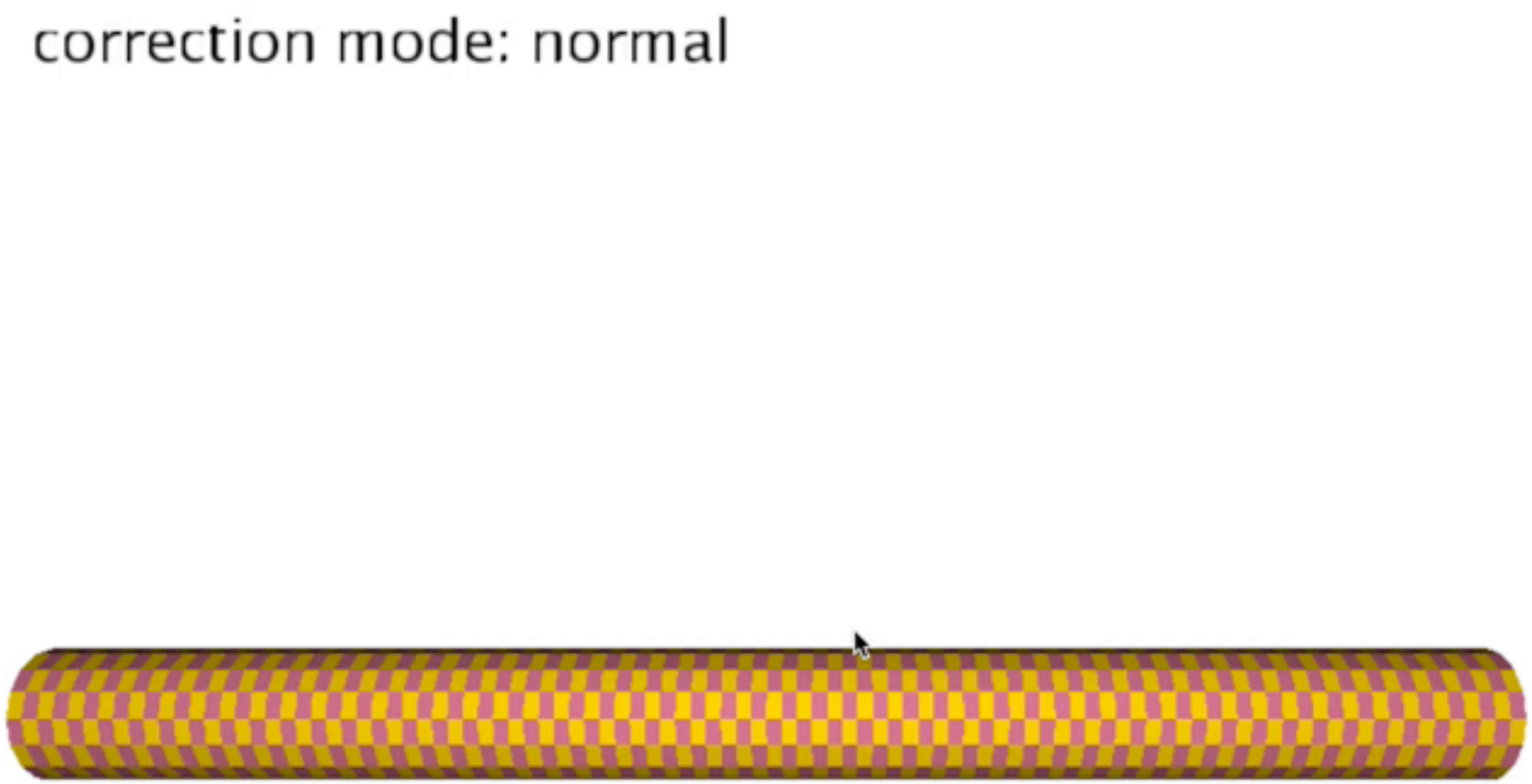


Binormal



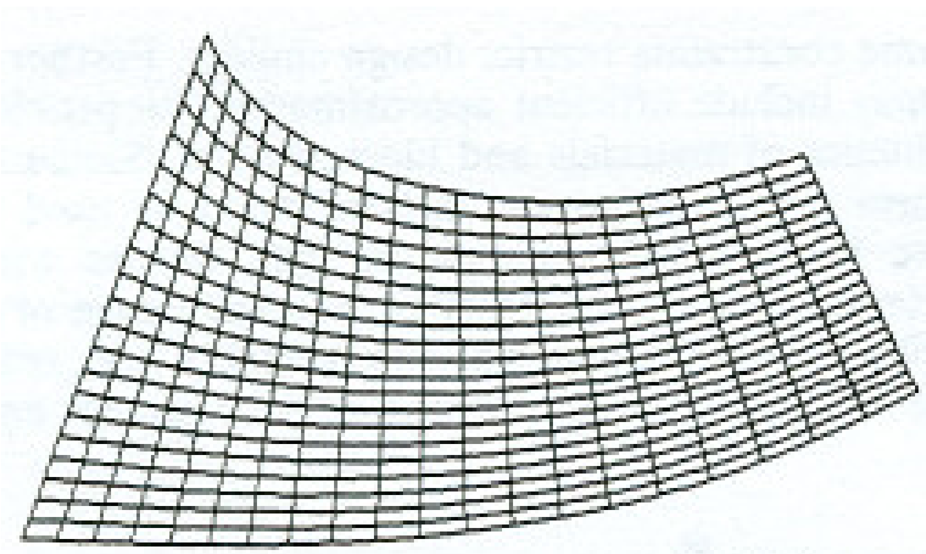
Radial

Cylinder



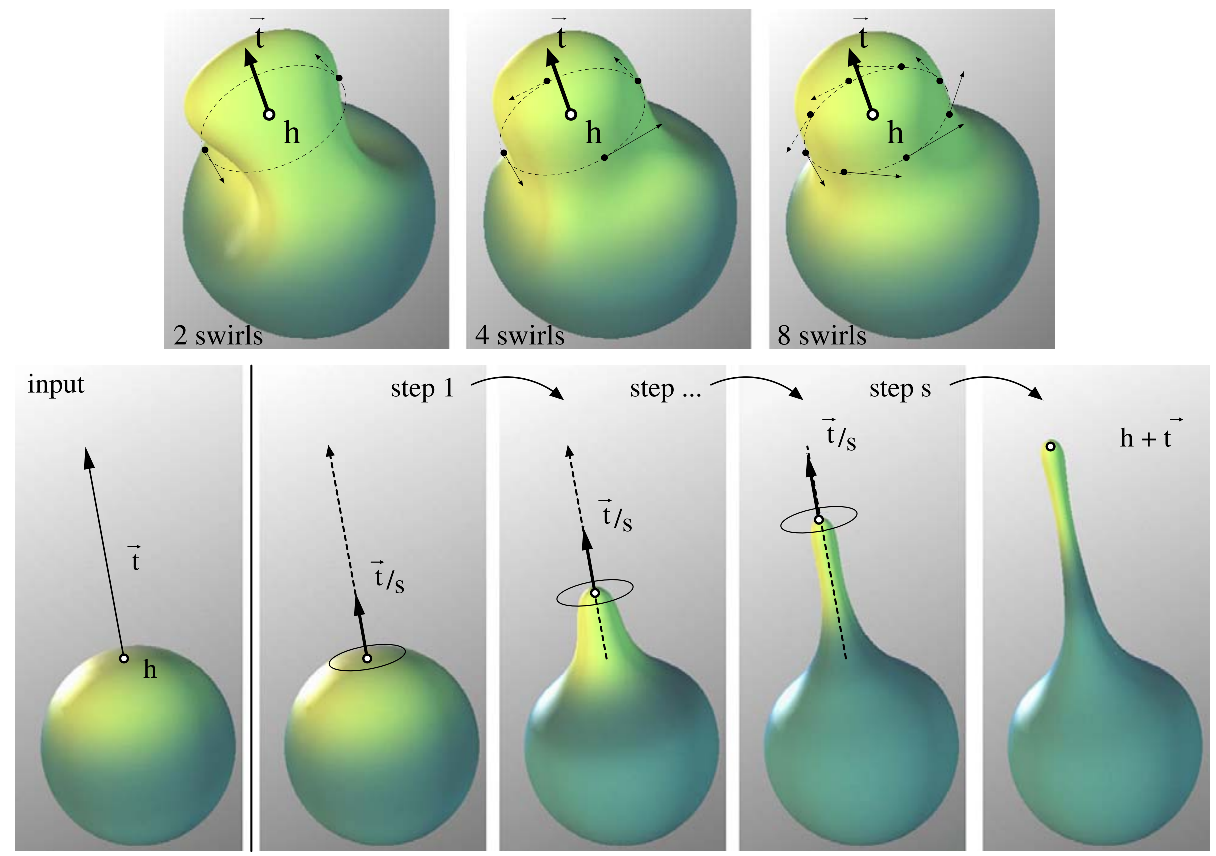
Bunny

Prior Art



stretch and bending
solutions in 2D

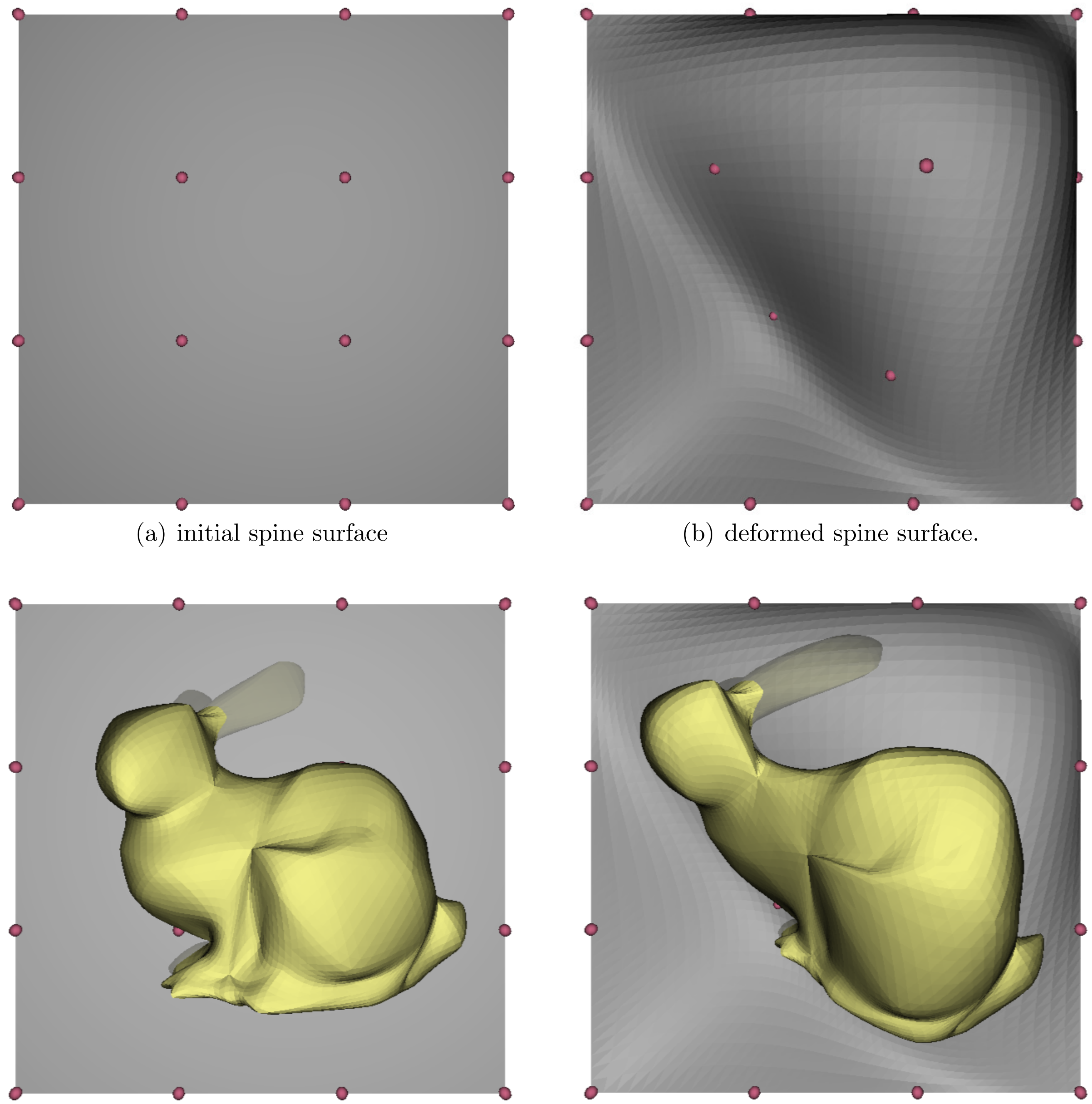
Gregory S. Chirikjian 1995



Alexis Angelidis, Marie-Paule Cani, Geoff Wyvill, Scott King 2005

Stretchable Spine Surface

The deformation is driven by stretching and bending a spine surface



(a) initial spine surface

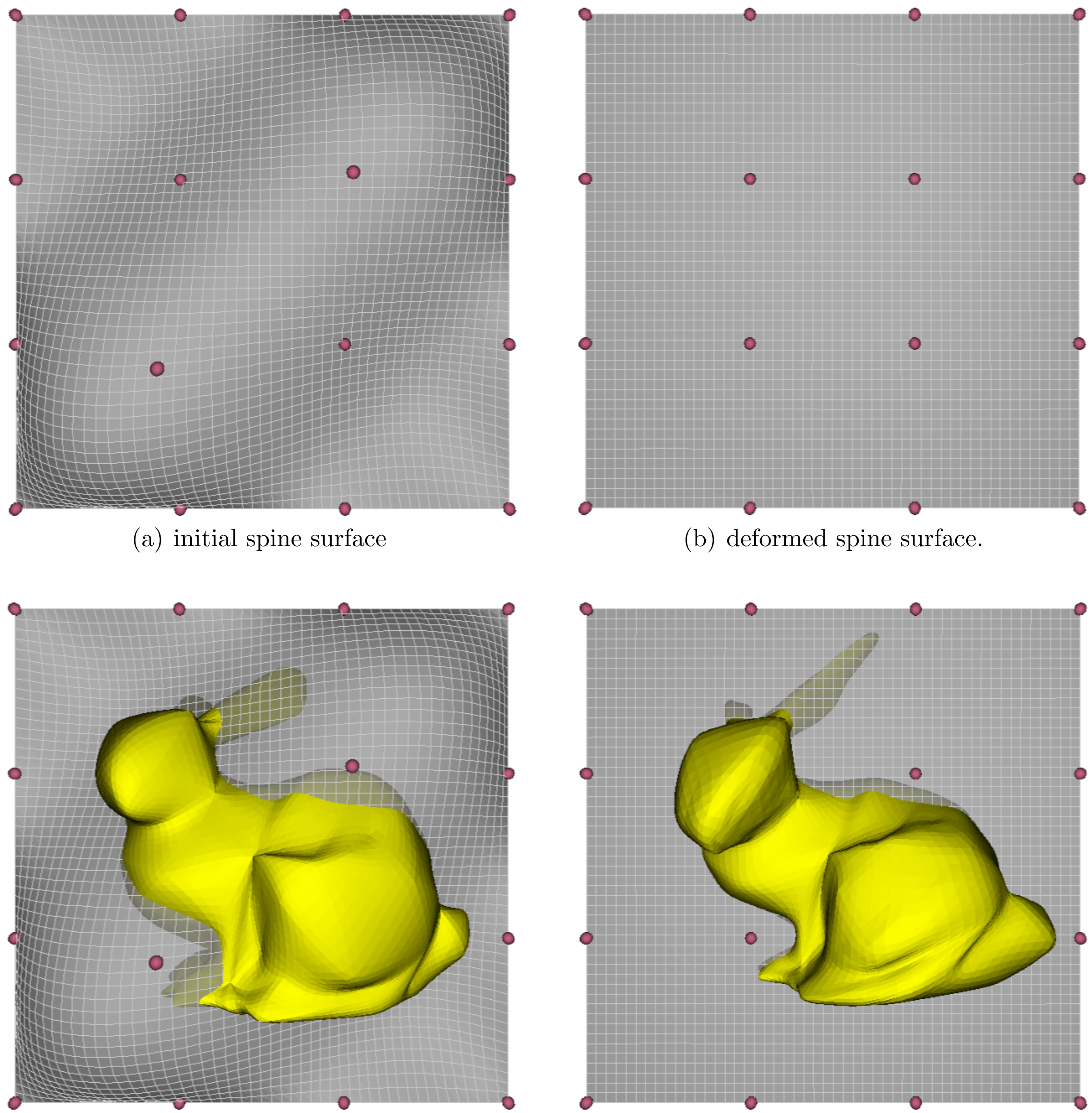
(b) deformed spine surface.

(c) original bunny registered to the initial spine surface

(d) deformed bunny driven by the spine surface

Stretchable Spine Surface

The initial surface does not need to be flat



(a) initial spine surface (b) deformed spine surface.

(c) original bunny registered to the initial spine surface (d) deformed bunny driven by the spine surface

Formula for spine surface

Let P denote a offset point from surface S

$$P(u,v,h) = S(u,v) + hN(u,v).$$

The mean curvature is the divergence of the normal field at S(u, v), and the Gaussian curvature is the cross product of the Hessian of the normal

$$det(\frac{\partial P}{\partial(h,u,v)}) = (1 - 2hm + h^2 g)|S_u \times S_v \cdot N|$$

Let $det(\frac{\partial P_1}{\partial P_0}) = 1$, we have the following solution

$$\boxed{(h_1 - h_1^2 m_1 + \frac{h_1^3}{3} g_1)\boxed{|S_{1u} \times S_{1v} \cdot N_1|} = (h_0 - h_0^2 m_0 + \frac{h_0^3}{3} g_0)\boxed{|S_{0u} \times S_{0v} \cdot N_0|}}$$

compensate for stretching

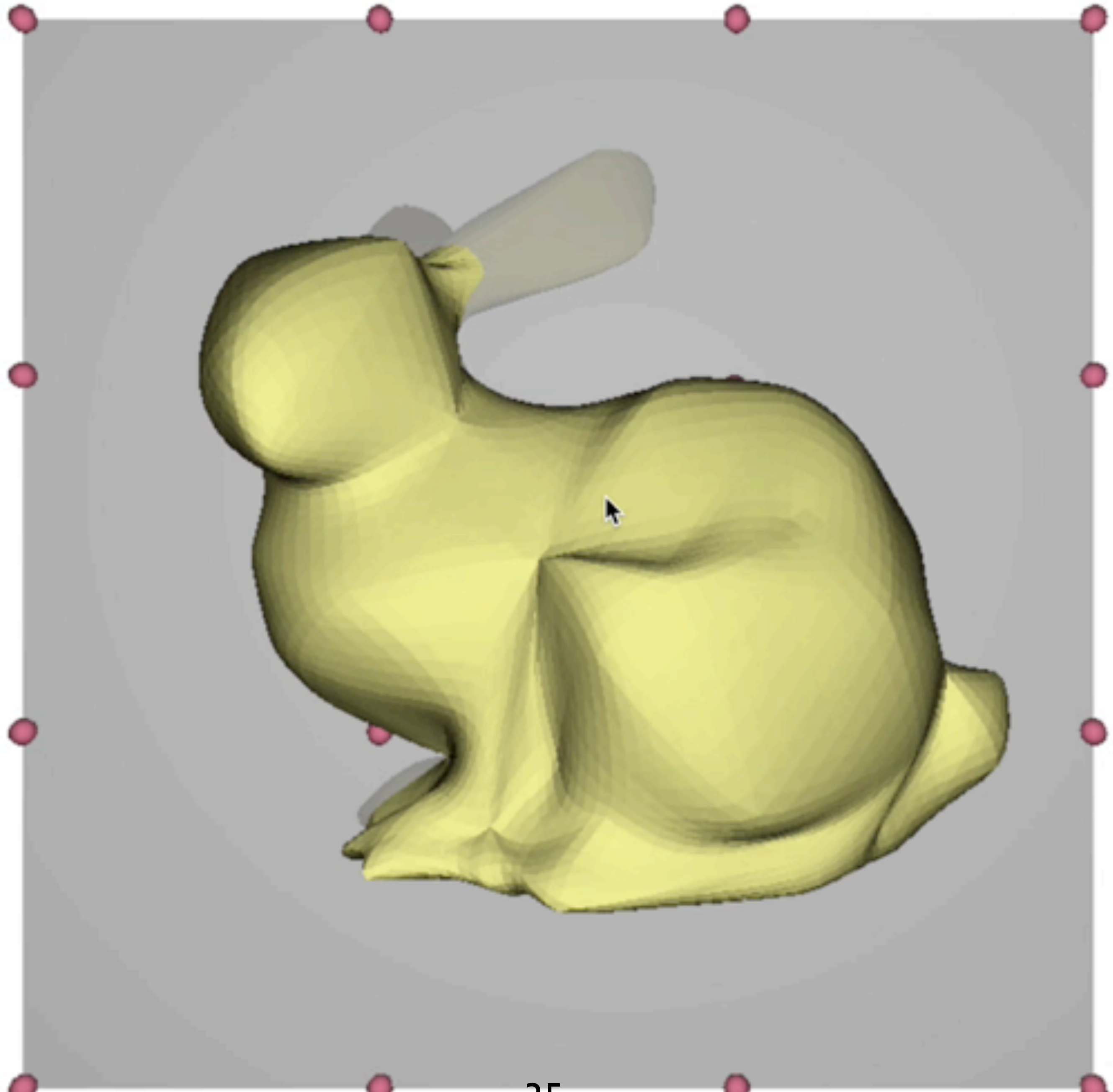
Implementation

The offset distance is always along the normal. The normal for registration and for updating the offset distance are the same: we do not need to have the transfer step to change the basis: The formula can be used to compute the solution directly.

$$h_1 - h_1^2 m_1 + \frac{h_1^3}{3} g_1 = (h_0 - h_0^2 m_0 + \frac{h_0^3}{3} g_0)/\sigma.$$

$\sigma = \frac{|S_{1u} \times S_{1v} \cdot N_1|}{|S_{0u} \times S_{0v} \cdot N_0|}.$ is the local stretch parameter

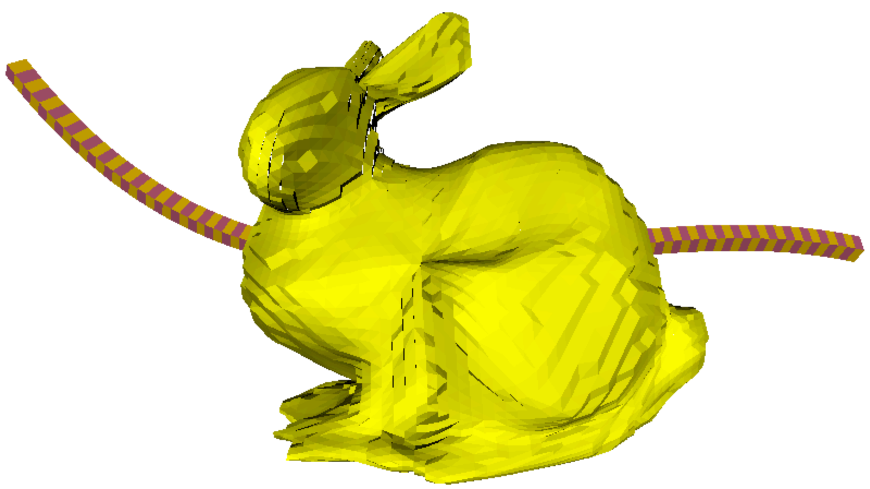
Result

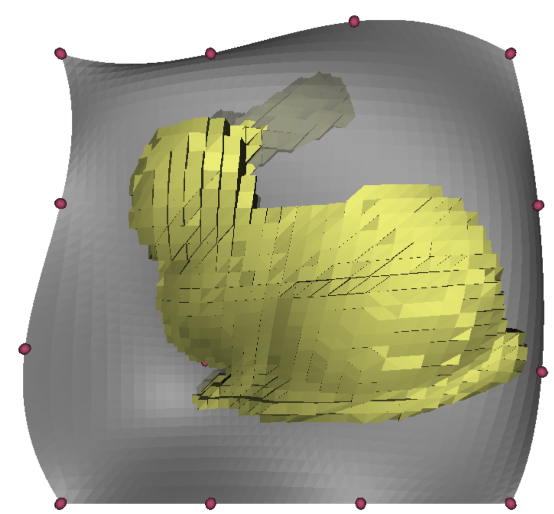


The spine is approximated by polygonal curve/ triangulated surface, snapping the closest projection to the closest vertex yields unacceptable results, we need to compute the closest projection on the polygonal curve/ triangulated surface.

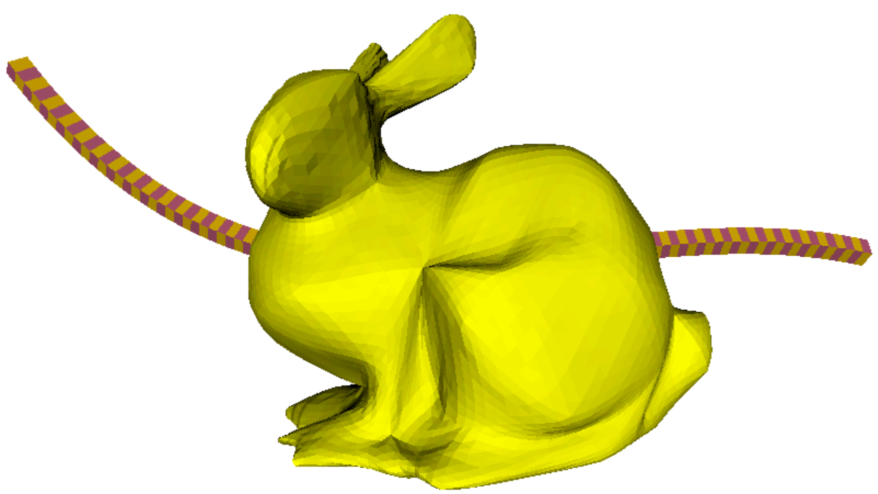
Comparison

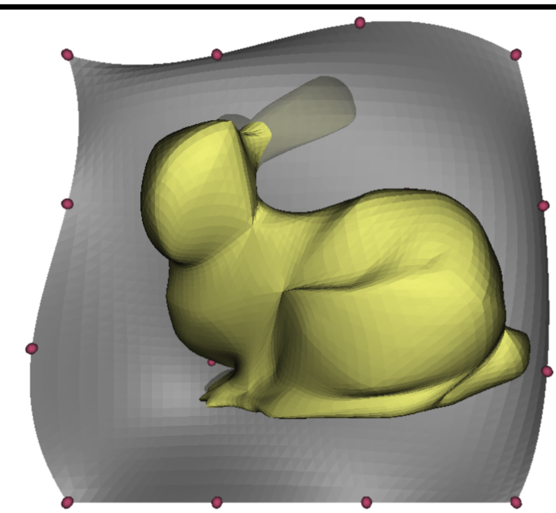
projection point approximated by the closest vertex





by the closest projection point on polyline or polygonal surface

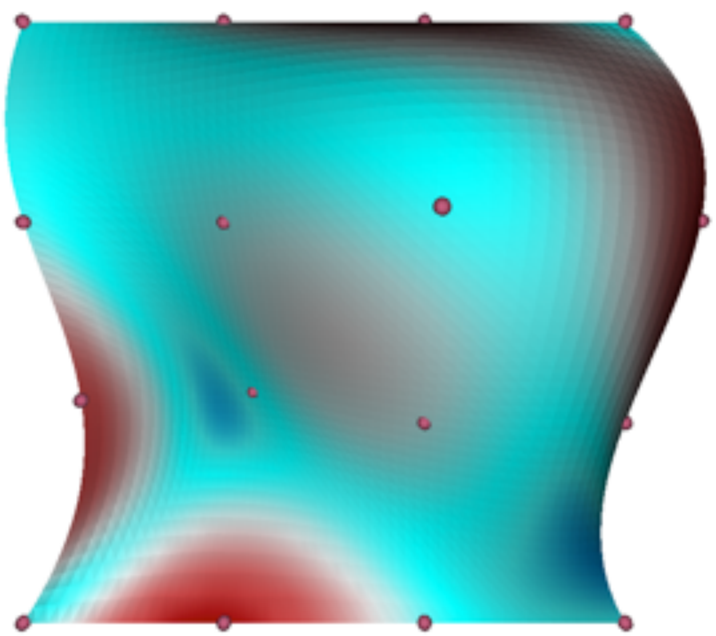




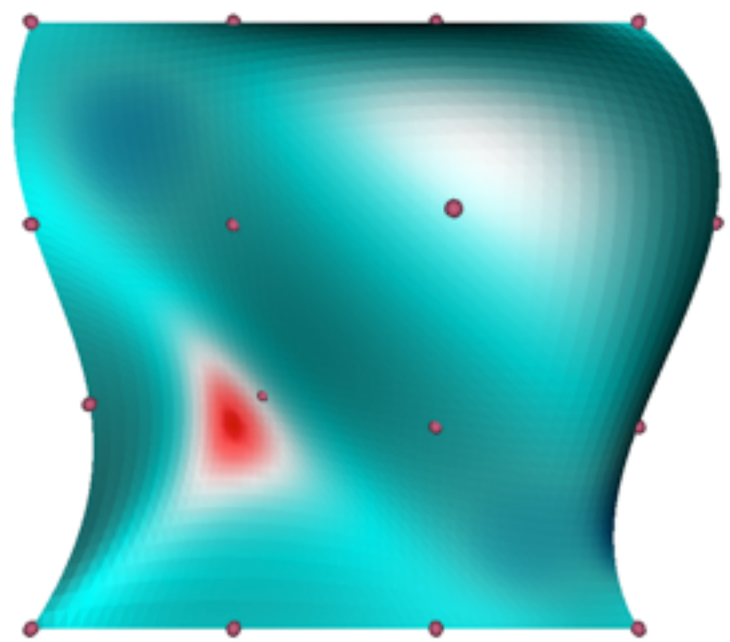
But how do we estimate the local parameters (curvature, stretch, ...) at these closest projection points?

Local Parameter Evaluation

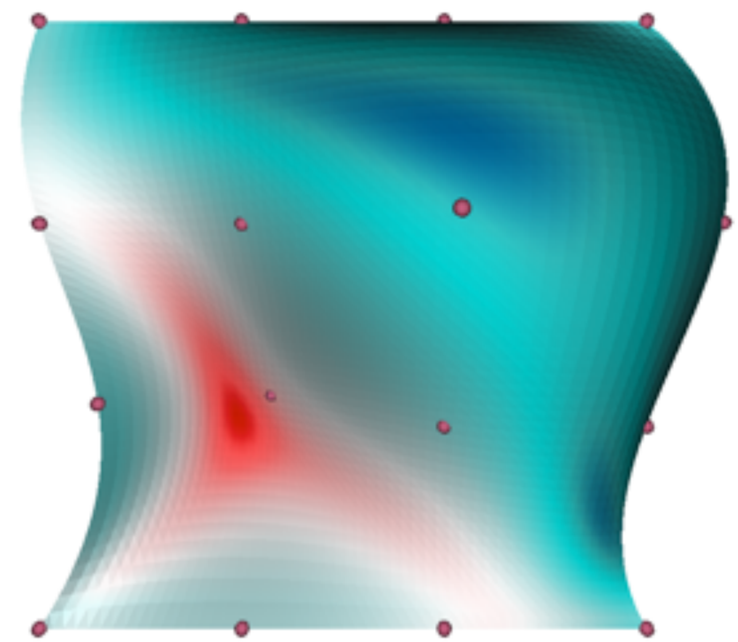
- Evaluate the local parameters at lattice grid points
- The curvature at the closest projection is a linear interpolation of the curvature at vertices of the triangle that contains the closest projection



local stretch

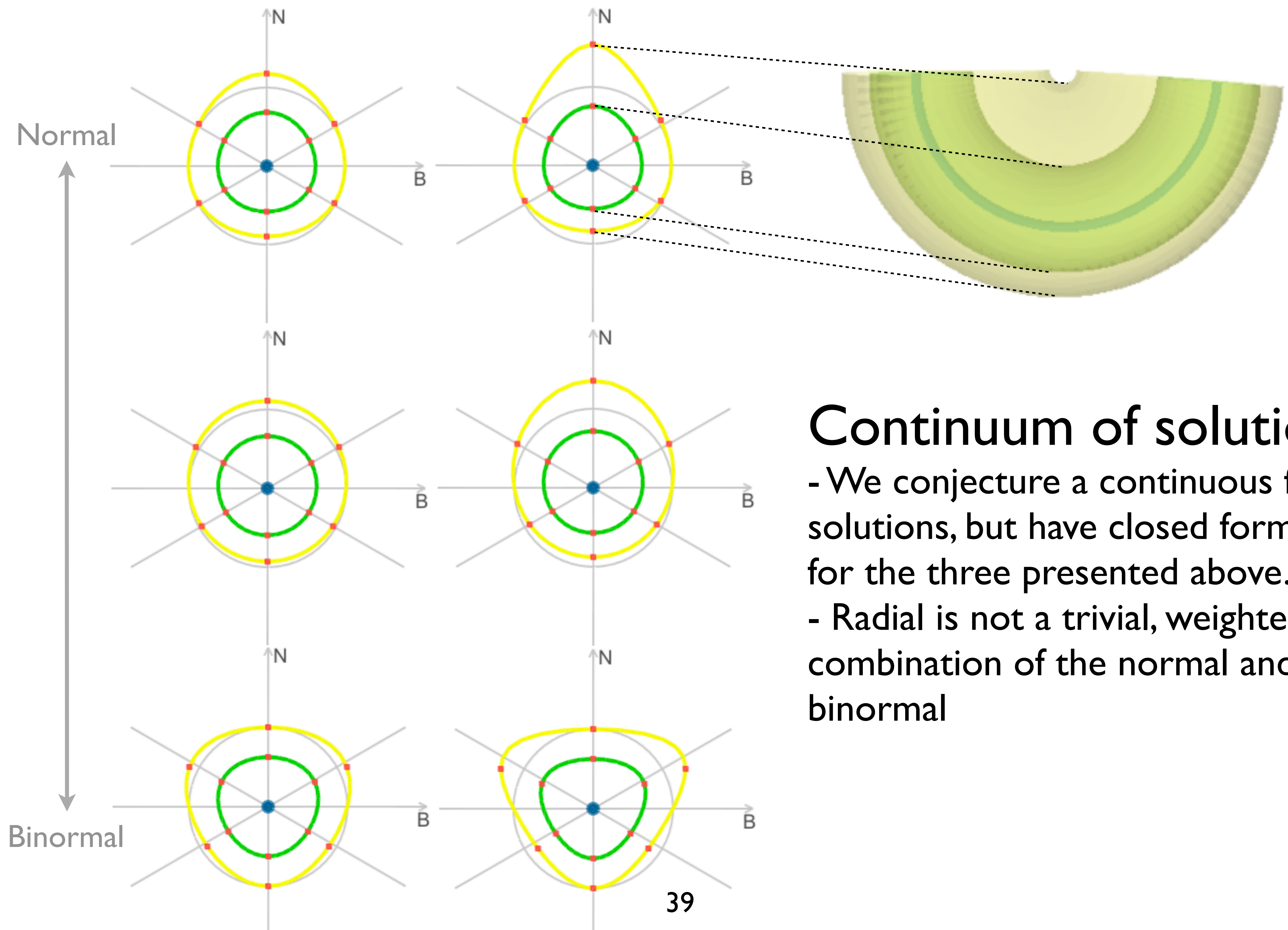


gaussian curvature



mean curvature

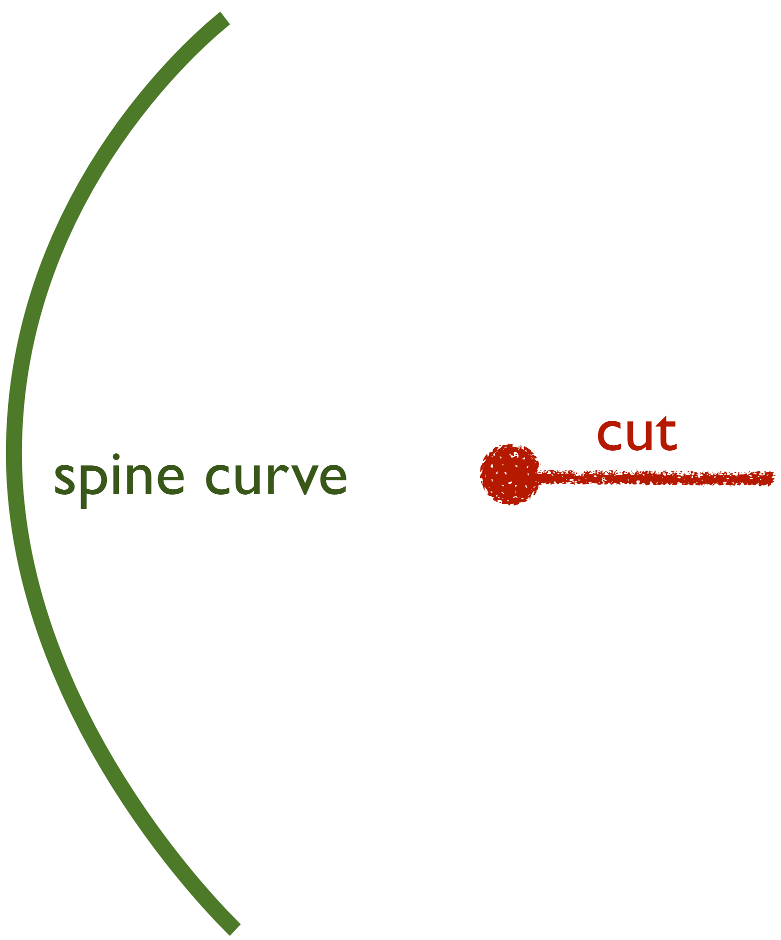
Deformation Behaviors



Continuum of solutions?

- We conjecture a continuous family of solutions, but have closed form only for the three presented above.
- Radial is not a trivial, weighted combination of the normal and binormal

Validity

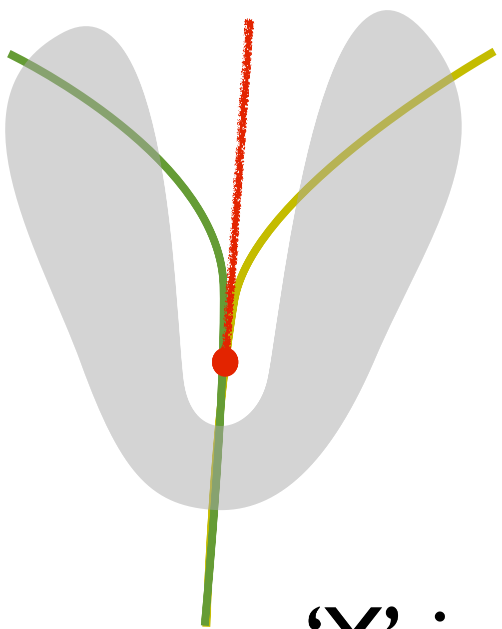


Define *cut* as the set of points that each of them has more than one closest projection.

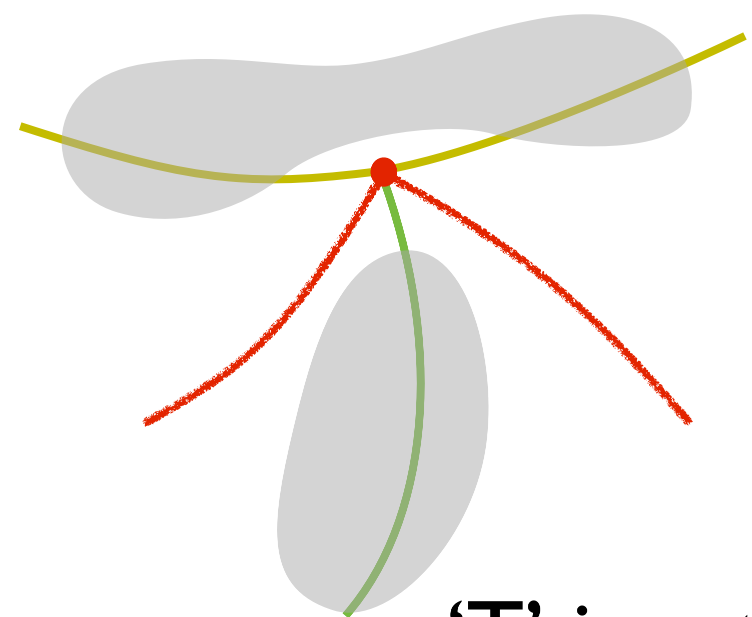
The initial shape should not intersect the cut of C_0 or the pre-image of the cut of C_1.

Otherwise, our solution does not work.

Spine Bifurcations



'Y' junction



'T' junction

With bifurcations, the cut touches the spine at junctions. The solutions work as long as the solid is in the valid region, i.e. does not overlap with the cut.

Contributions

- Derived closed-form solutions of the offset distances based on the local curvature and stretch of the spine.
- Devised a technique that decompose the deformation into three steps: unbending, transfer and bending.
- Extended the solutions to stretchable spine curve, surface; studied issues related to accuracy, sampling and existence conditions