Bending with Local Volume Preservation

A Thesis Proposal Presented to The Academic Faculty

by Wei (Tina) Zhuo

Outline

- Overview of Bending and Related Deformations
- Problem Formulation
- Prior Art and Published Results
- Plans for Remaining Contributions

Overview of Deformations

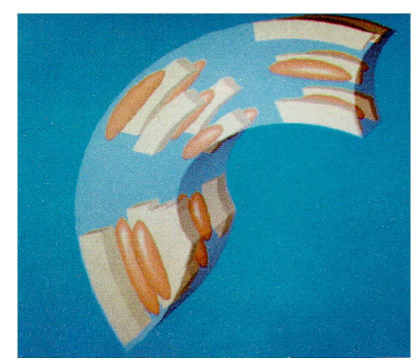
Jarek Rossignac and Alvar Vinacua 2011,
Steady Affine Motions and Morphs

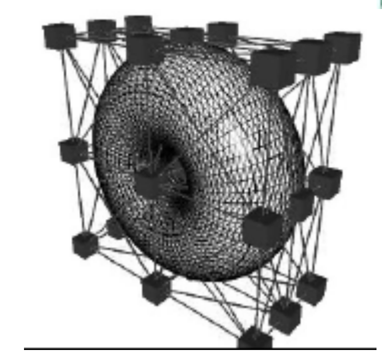
Primitive Deformation

-Combinations of translation, rotation shearing, scaling

Free-form

-Polynomial mapping from R3 to R3





Thomas Sederberg and Scott Parry 1986,
Free-form deformation of solid geometric models

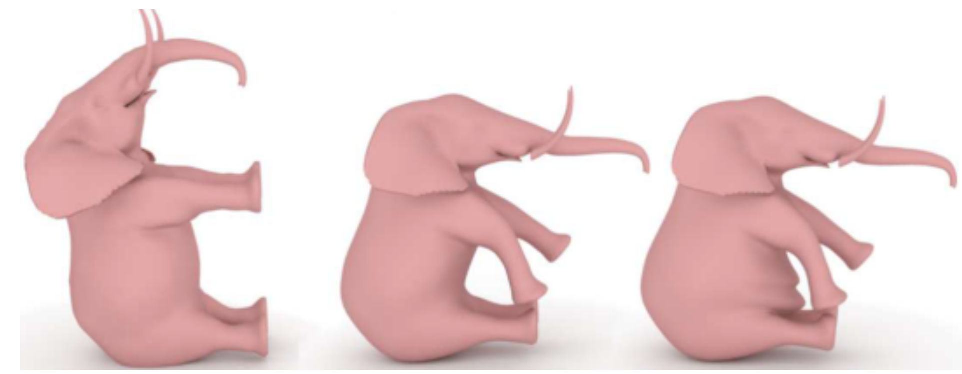
Skeletal Deformation

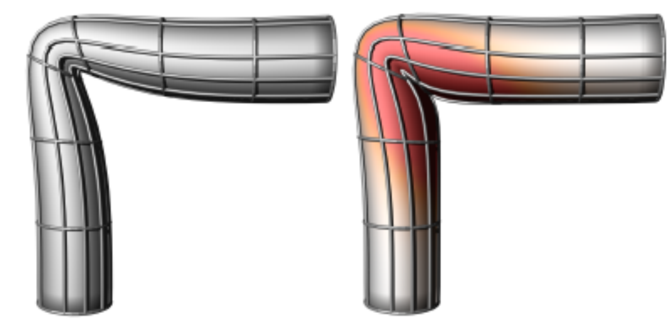
-The deformation is controlled by motions of the skeletons

Skeletal Deformation

Polygonal Skeleton

-Using exponential distance factor or different correction maps (ripples)
-Iterative radial displacement until total volume is recovered

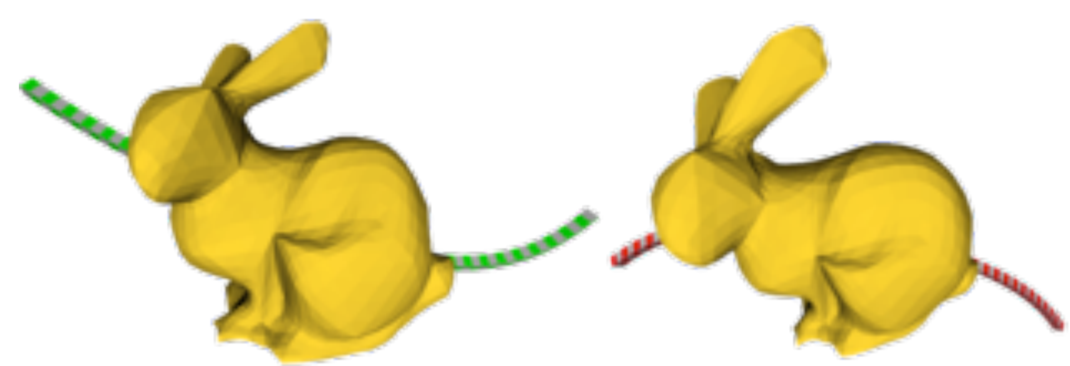




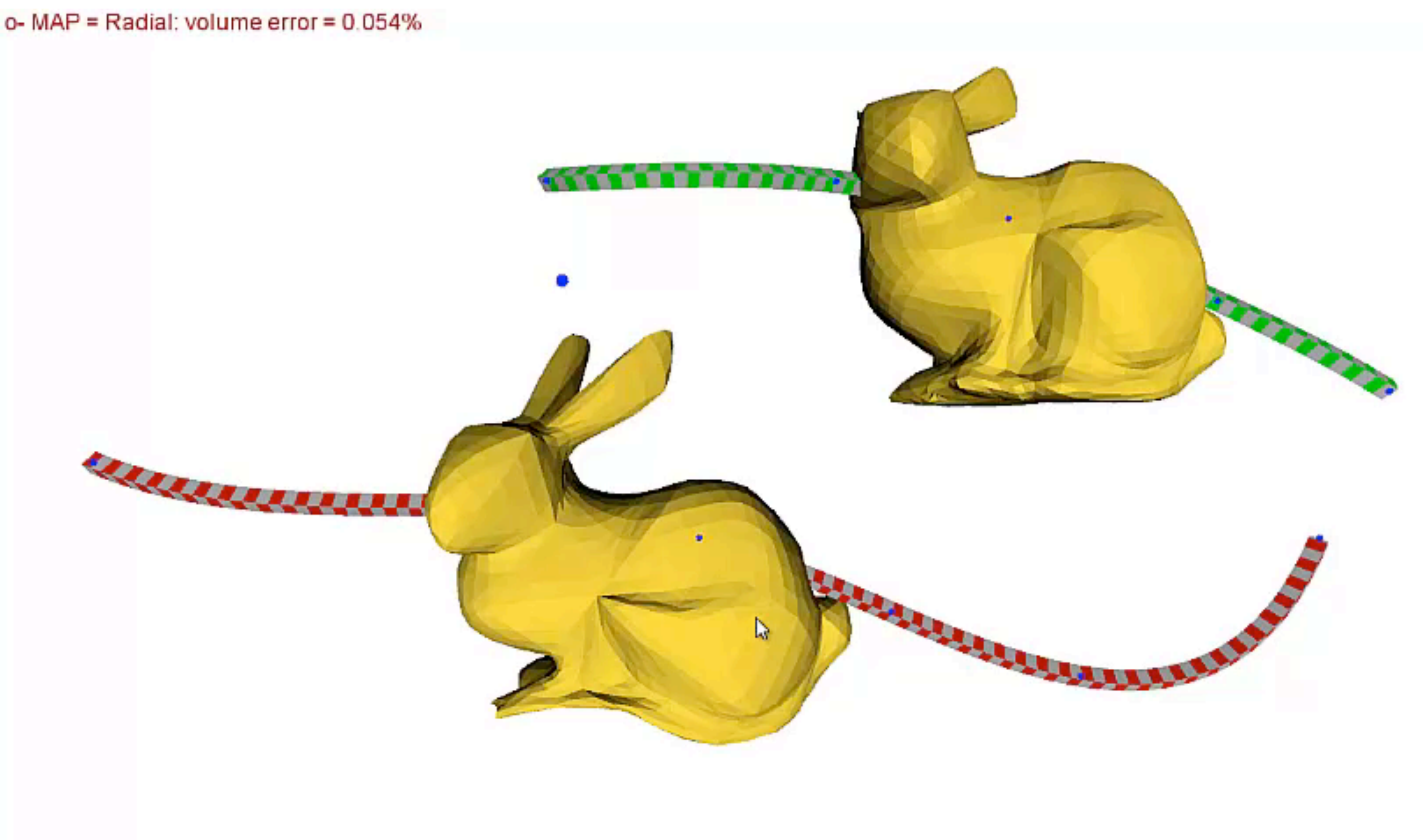
Rohmer, Hahmann & Cani, 2008, 2009

Smooth Spine Curves

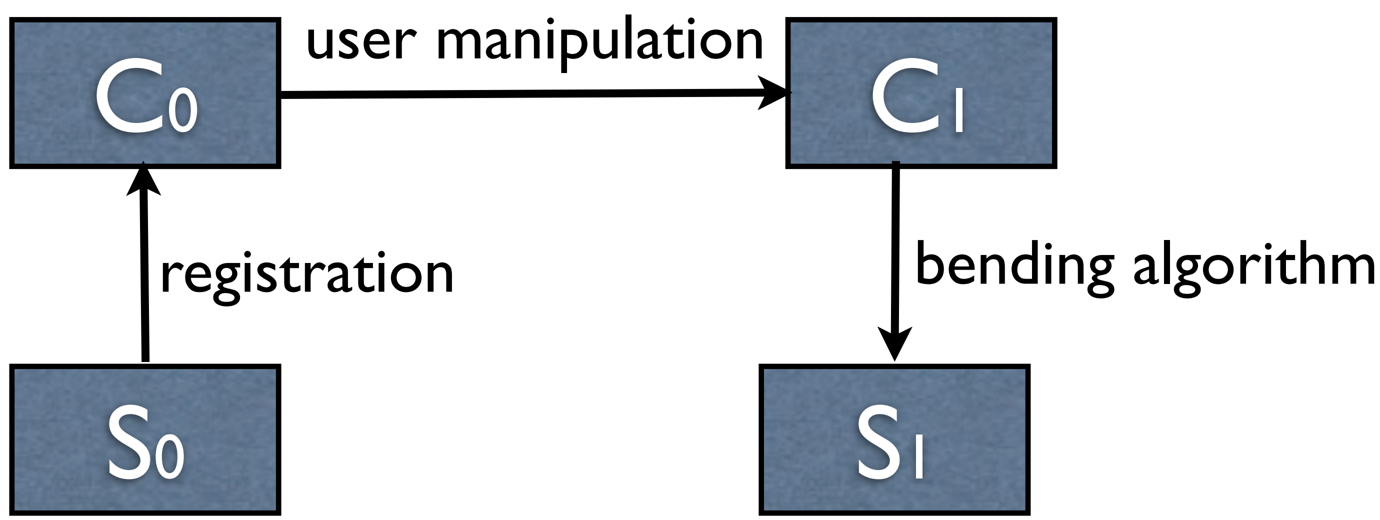
-Smooth spine curve (no branch)
-Preserve cross-section
-Preserve local volume exactly
Closed form (no iteration)

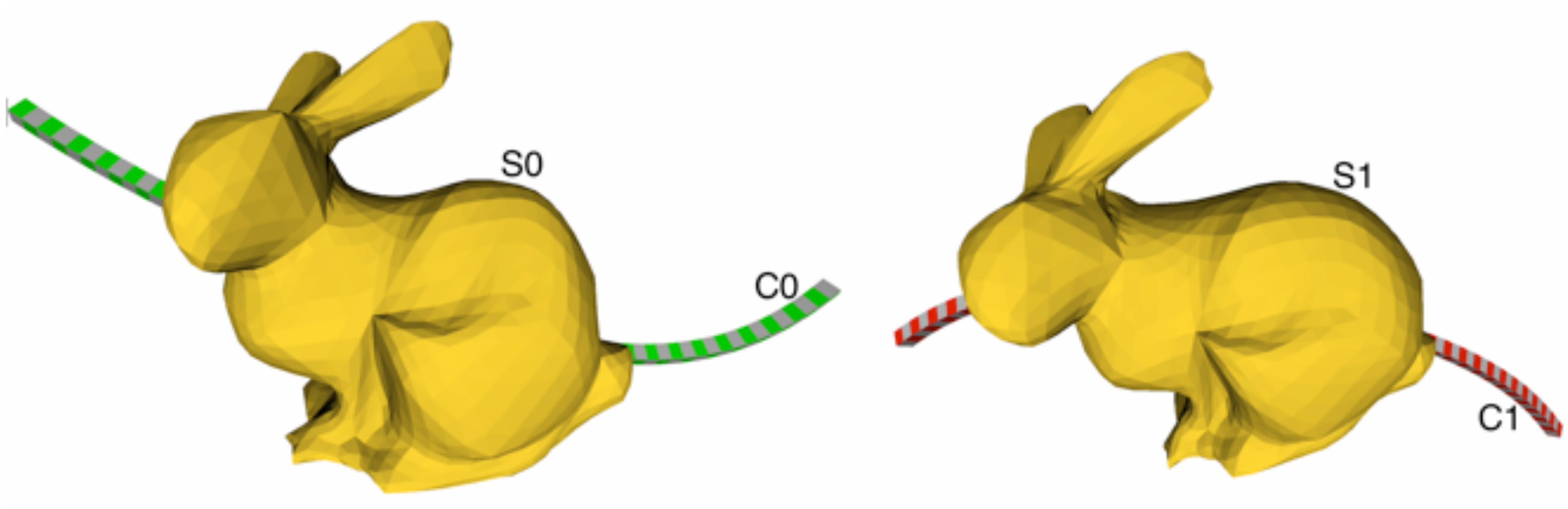


Zhuo and Rossignac, 2013


o- MAP = Radial: volume error = 0.054%

Overview of Bending





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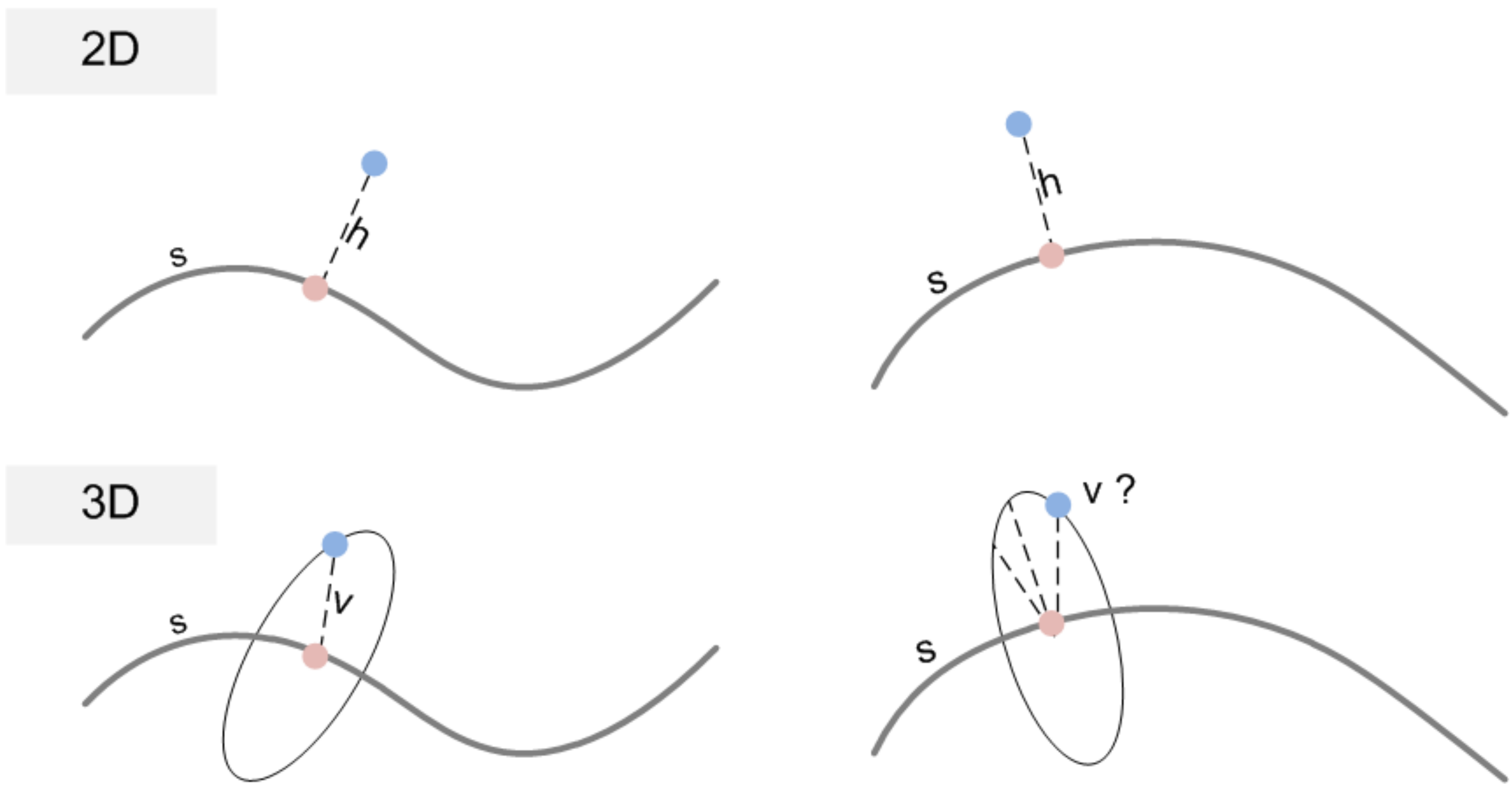
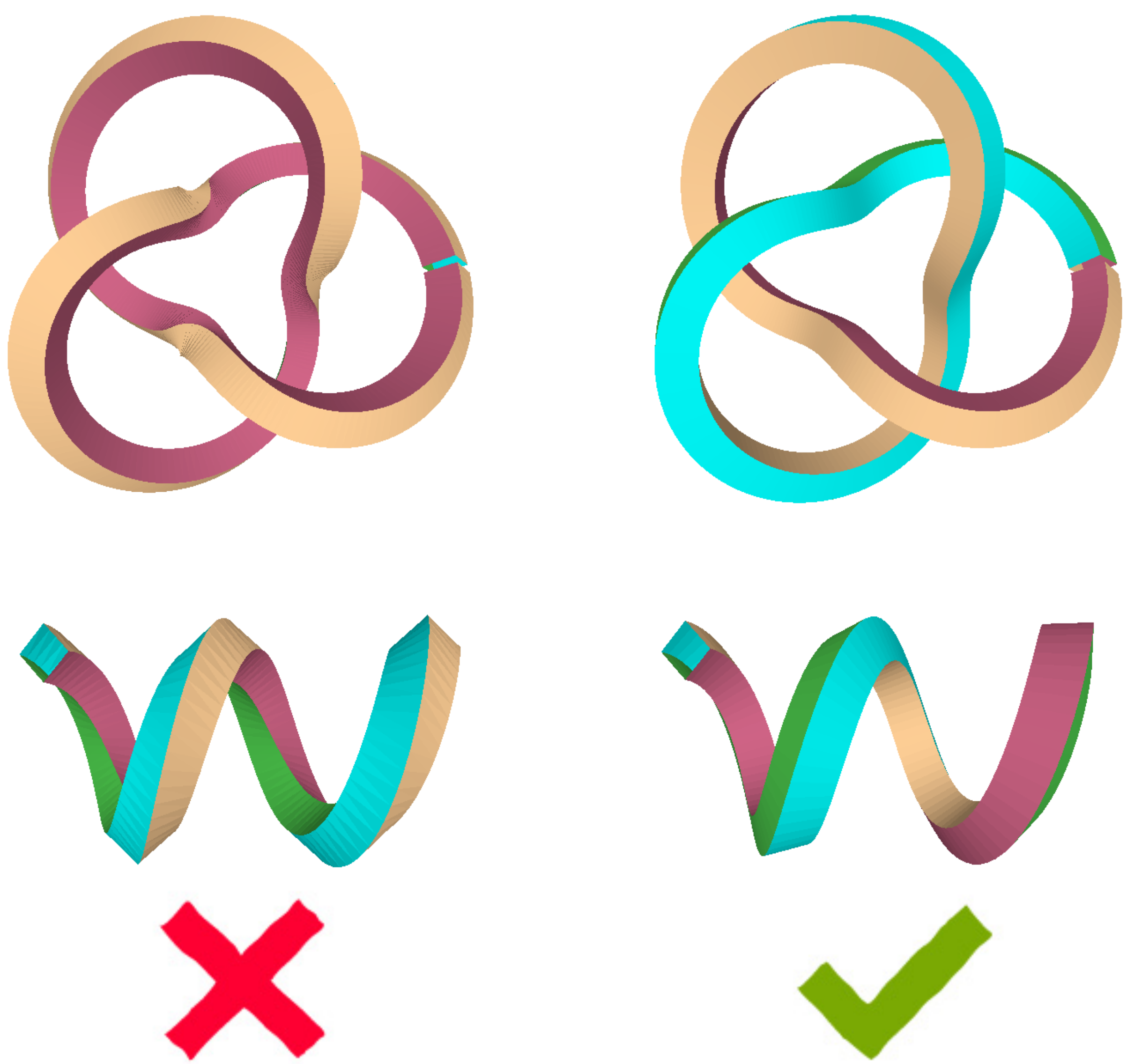
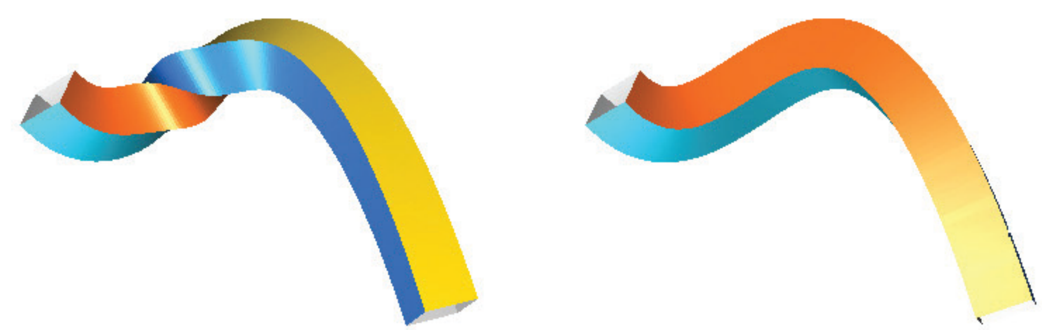
Registration



which normal to use?

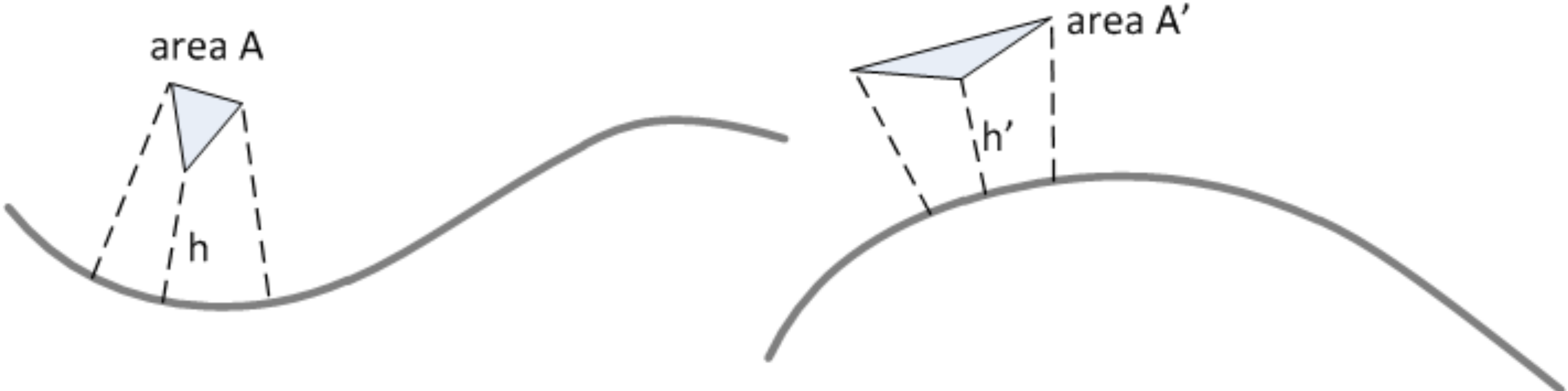
Twist-compensated Frame





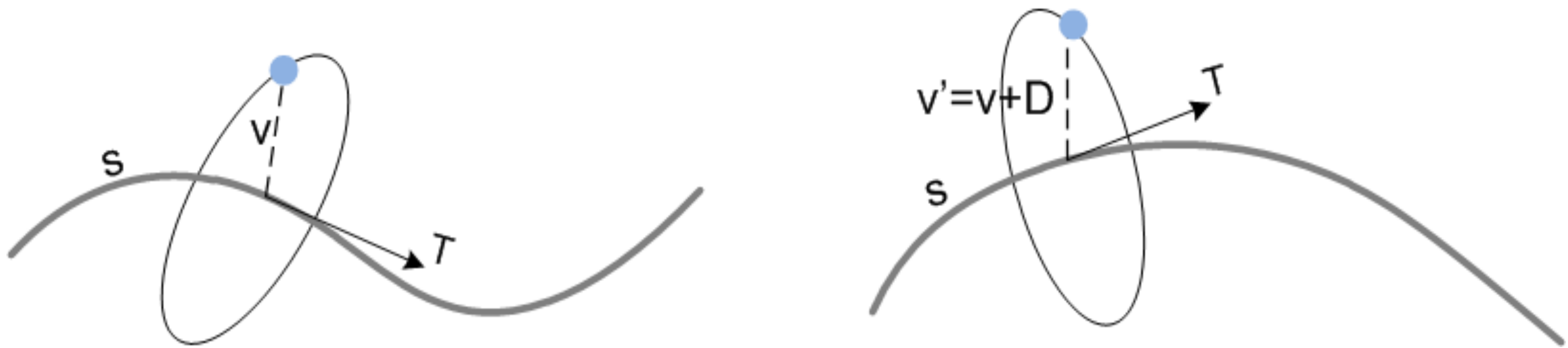
Wang, Juttler, Zheng & Liu 2008

Our Problem in 2D



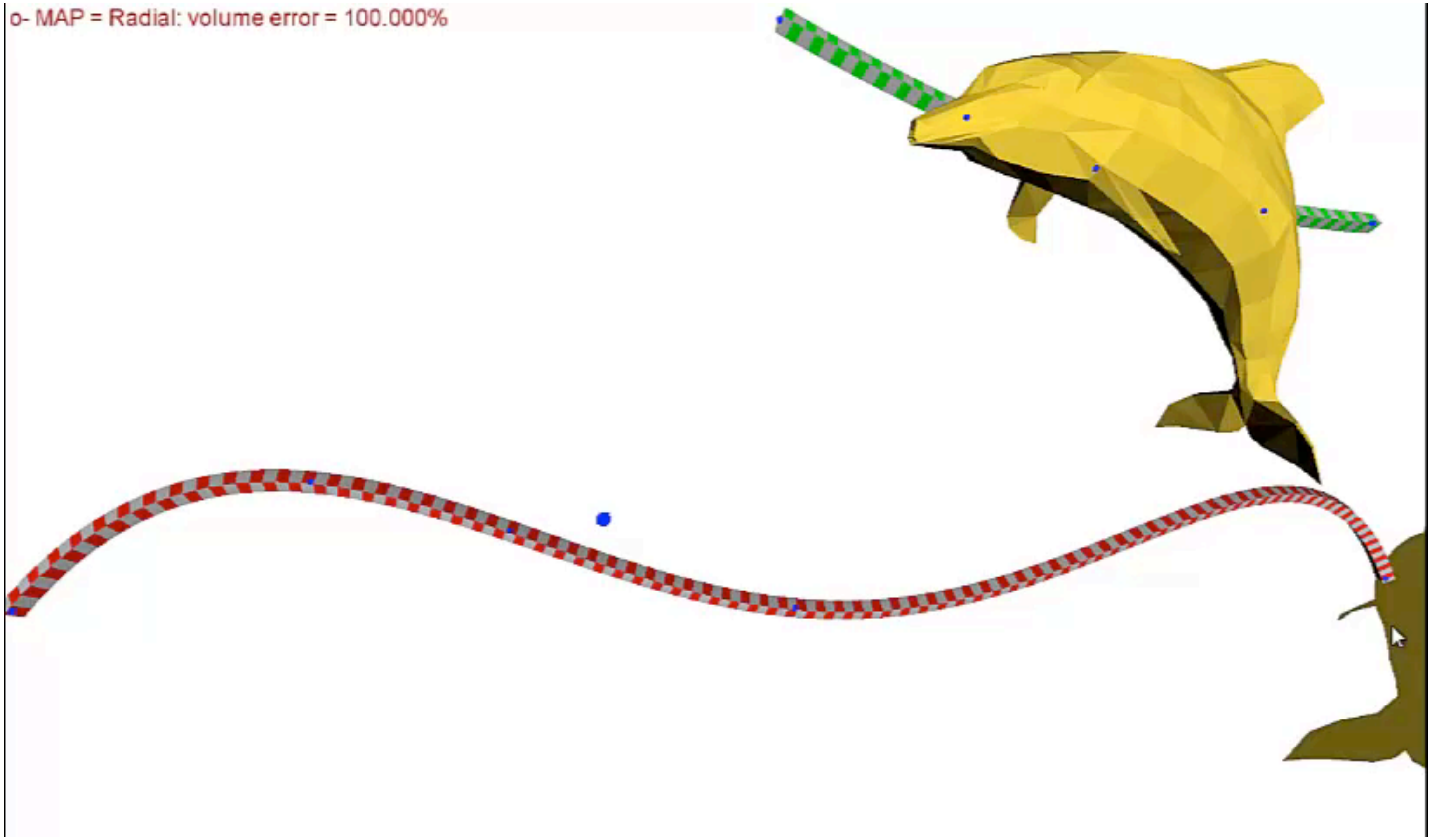
Preserve the local area of any “chunk” in space.
Want to adjust h (based on the curvature) so that A=A’.

Our Problem in 3D



Preserve the local volume of any "chunk" in space.
Want to adjust v (v'=v+D), where D is a vector orthogonal to T.
constraint: we preserve cross-section planarity

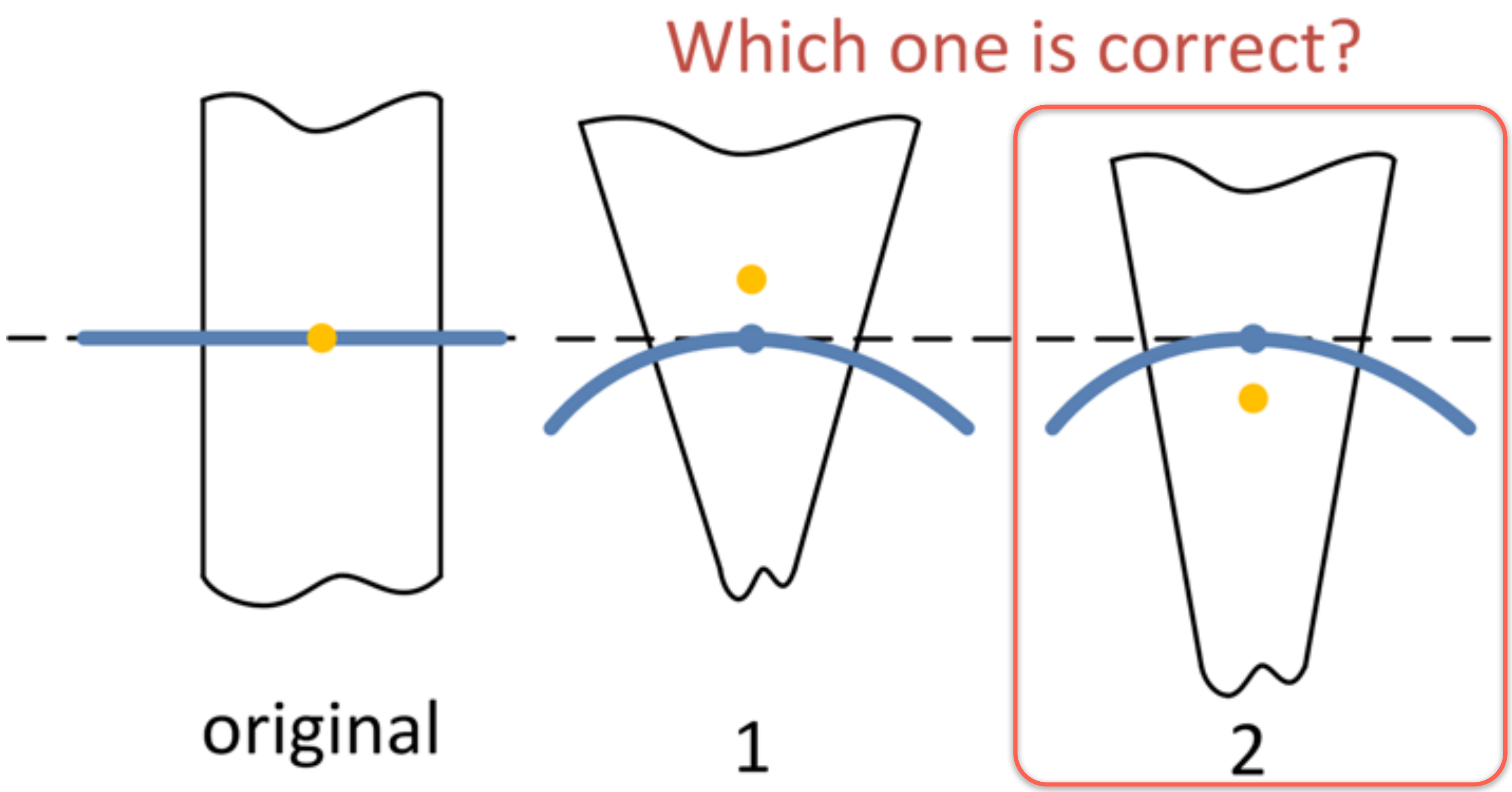
Global Slide and Twist



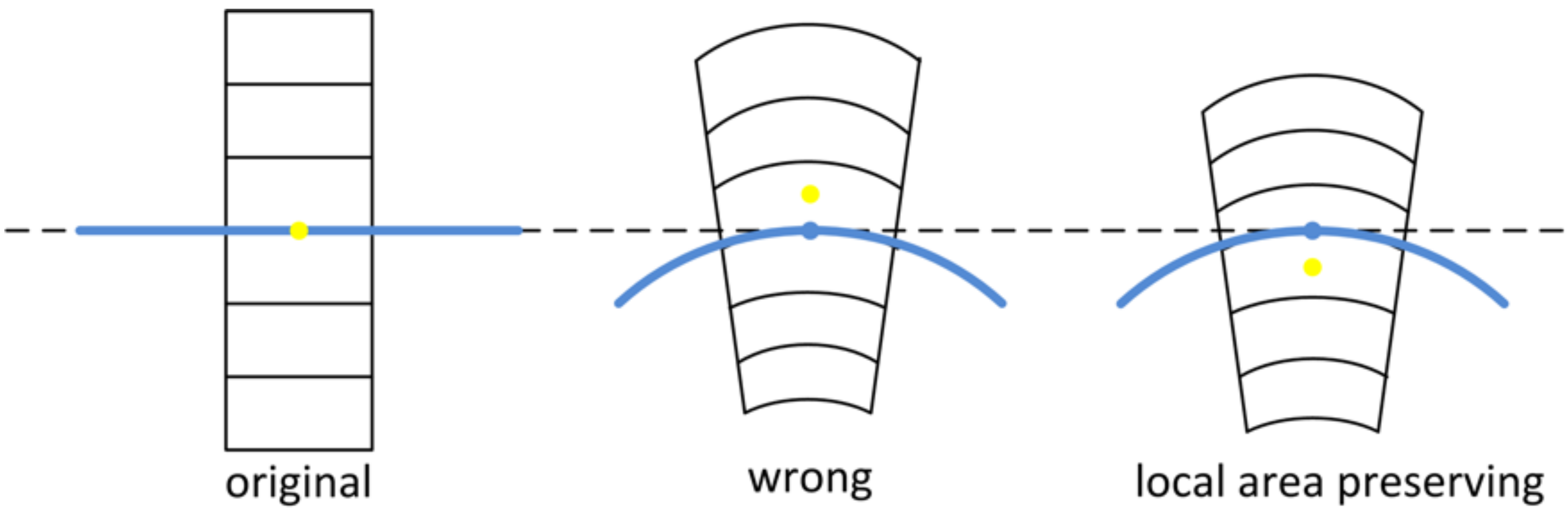
Local Volume Preservation

- Physically plausible simulations that involve biological creatures or deformable shapes made of incompressible materials require that the volume be preserved.
- During an physically based animation, the momentum and kinetic energy both depend on the mass and hence of the volume. Hence, changes of volume during an animation will result in surprising changes of velocity.

Local Volume Preservation in Bending



Local Volume Preservation in Bending



Problem Formulation

The designer starts with a initial shape S_0 and specifies an initial spine C_0. Then the designer deforms C_0 to C_1. The following is a list of requirements for the mapping M, which maps S_0 to S_1

Topology-independent M should operate on any shape topology.

Homeomorphism M should be a homeomorphism between S_0 and S_1

Cross-section Preserving M should preserve the parameters on the spine

Locally Volume Preserving M should preserve the volume of any subset of S

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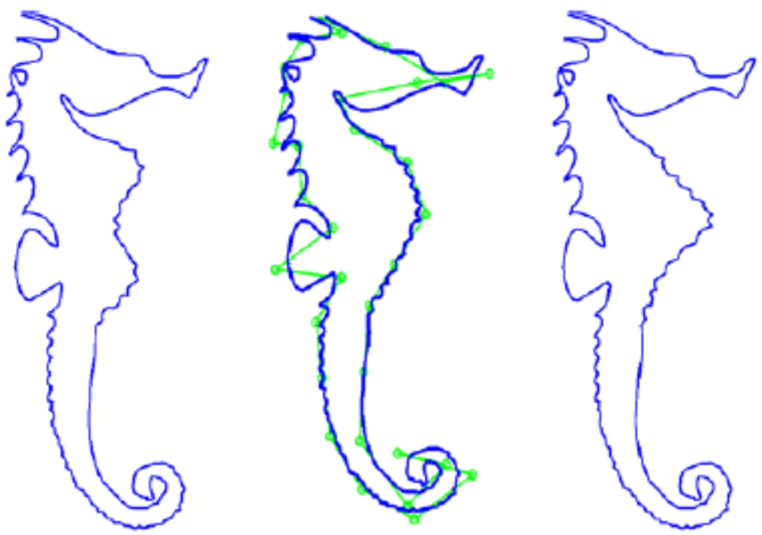
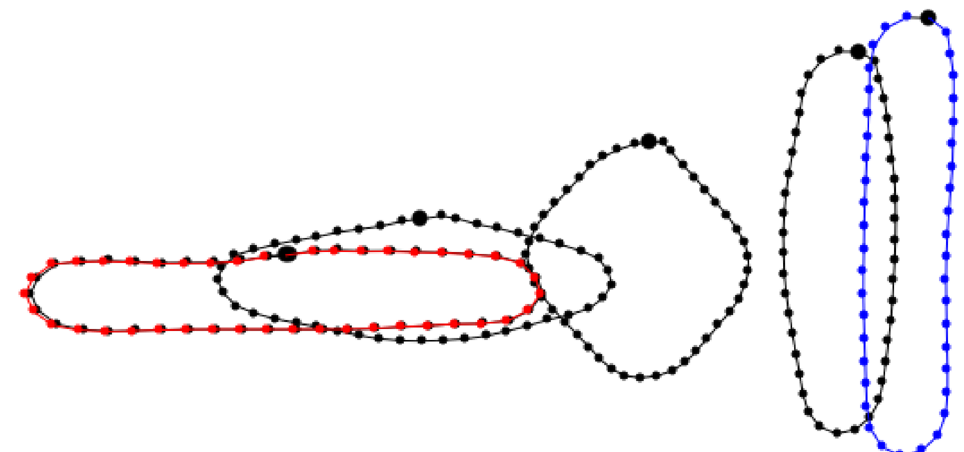
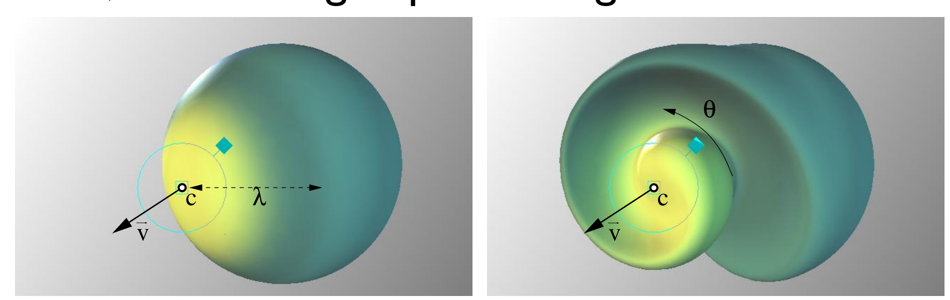
Literature Survey



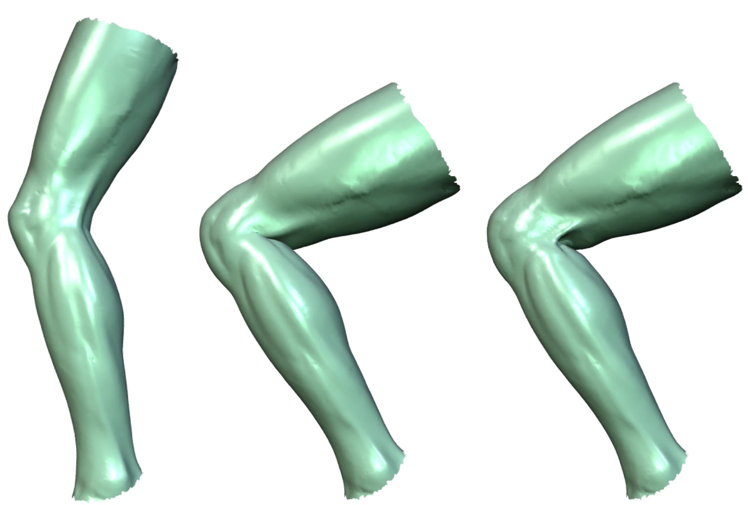
Stefanie Hahmann, Basile Sauvage and Georges-Pierre Bonneau 2005, Area preserving deformation of multiresolution curves



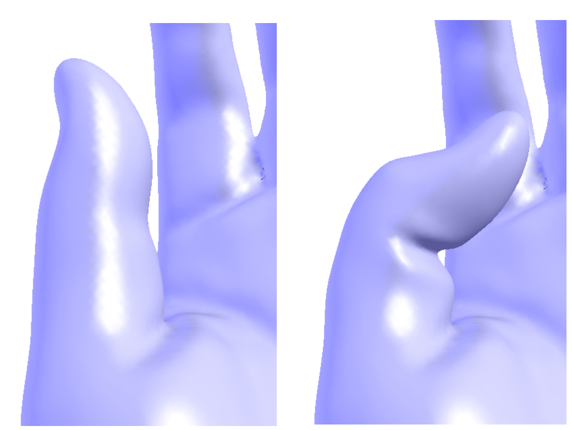
Karthik Raveendran, Nils Thuerey, Chris Wojtan, Greg Turk, Controlling Liquids Using Meshes



Alexis Angelidis, Marie-Paule Cani, Geoff Wyvill, Scott King 2006, Swirling-sweepers: Constant-volume modeling

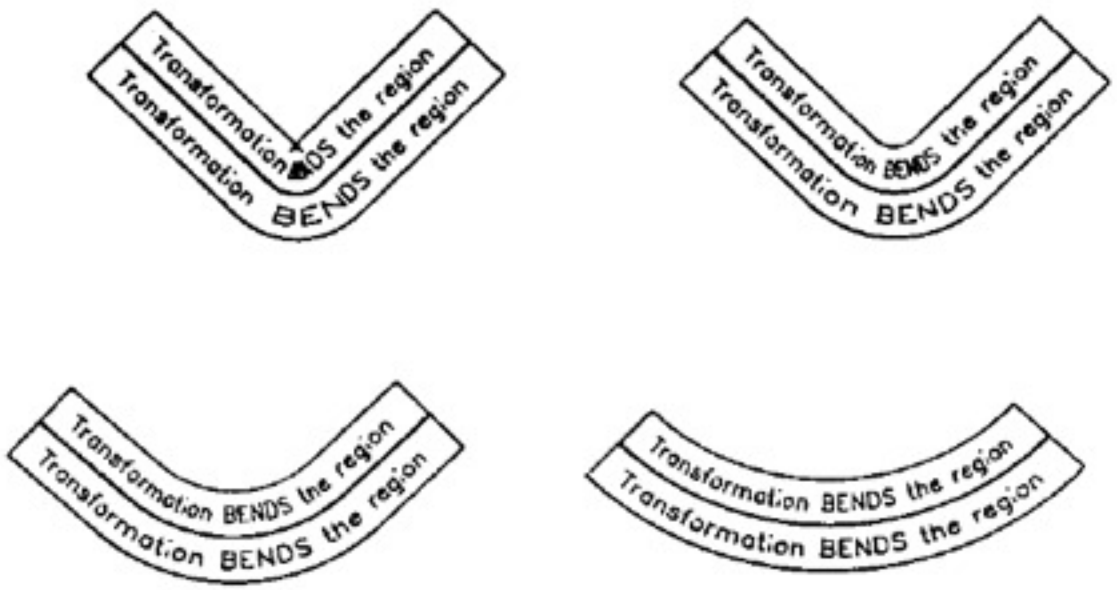


Mario Botsch and Leif Kobbelt, 2003, Multiresolution Surface Representation Based on Displacement Volumes

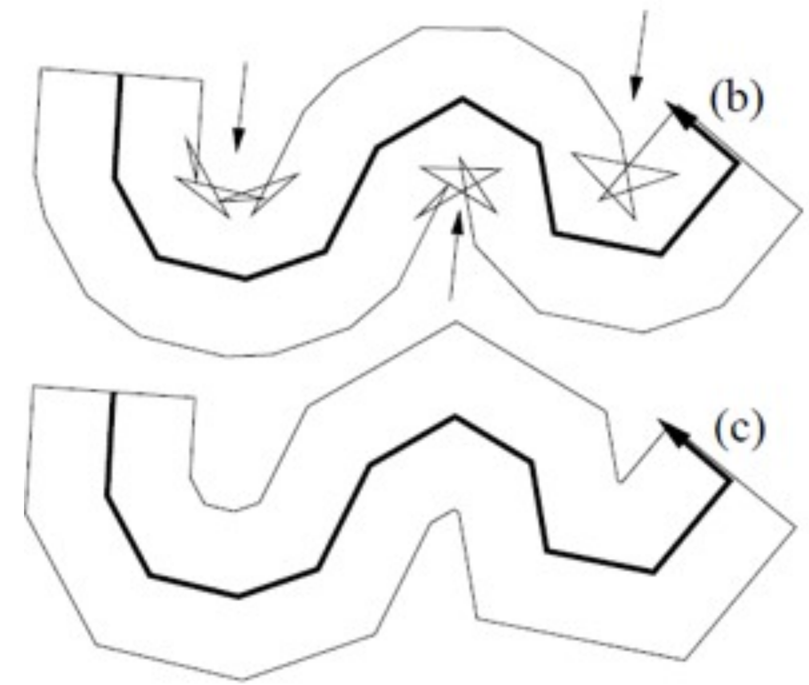


Wolfram von Funck, Holger Theisel, Hans-Peter Seidel 2006, Vector Field Based Shape Deformations

Prior Art



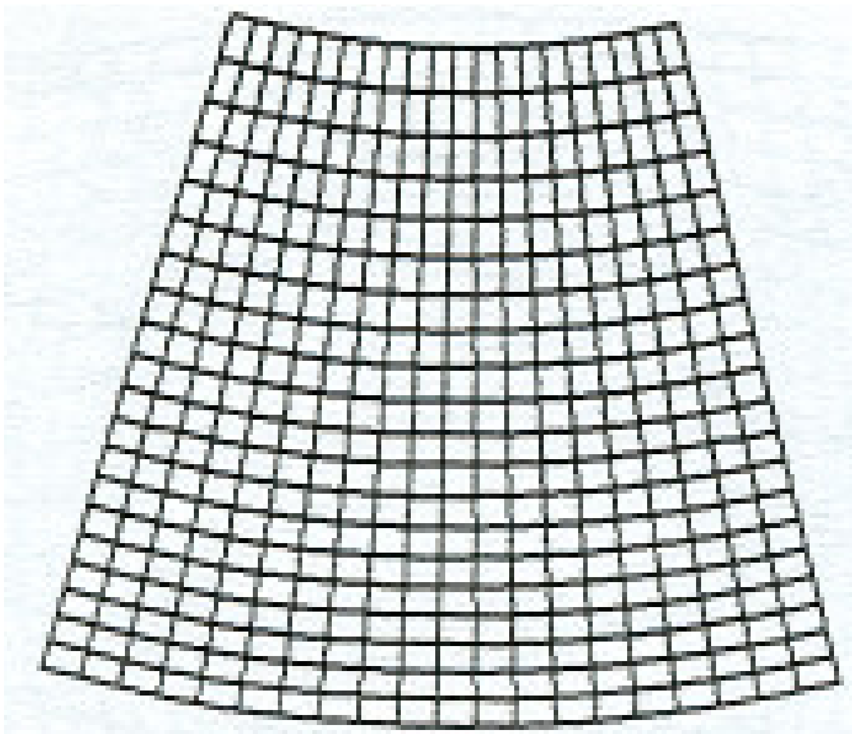
Barr 1984, Global and local deformation of solid primitives



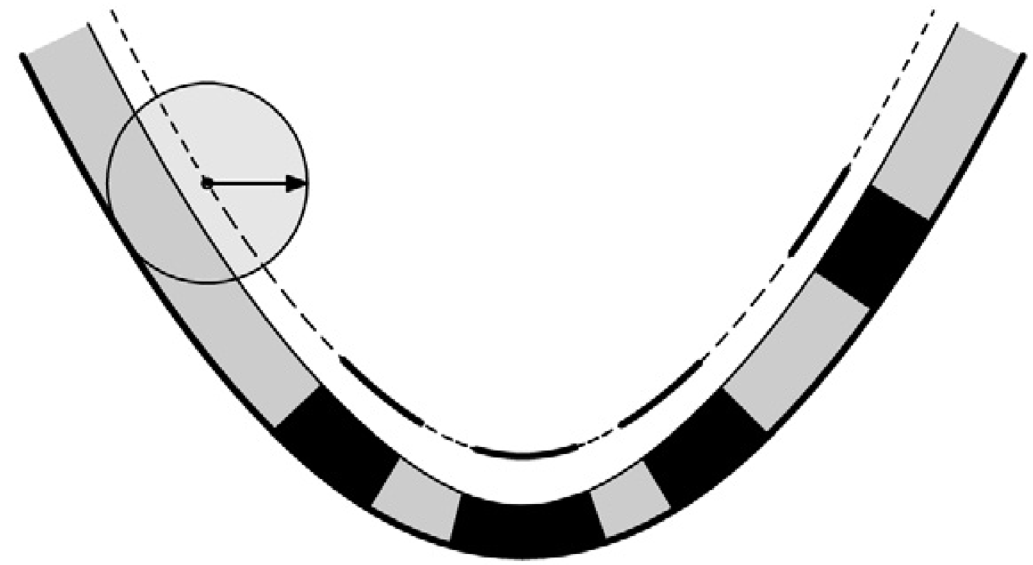
Hsu, Lee and Wiseman 1994, Skeletal strokes

The two approaches mentioned above do not preserve area or volume.

Prior Art



G.S. Chirikjian, Closed-form Primitives for Generating Locally Volume Preserving Deformations, 1995



HP Moon, Equivolumetric Offsets for 2D Machining with Constant Material Removal Rate, 1995

$$h - \frac{kh^2}{2} = r$$

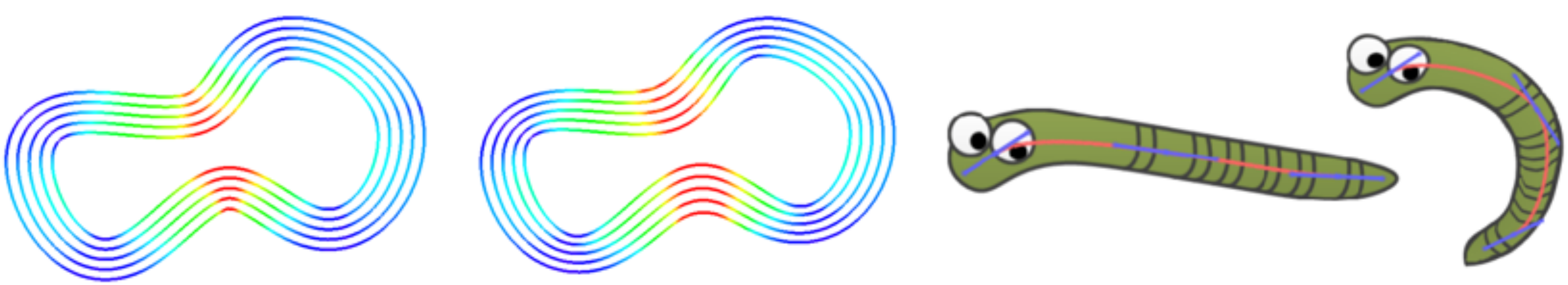
The locally volume-preserving formula in 2D has been derived in the context of bending and machining. (r is the original offset distance or removed area per unit length; h is the updated offset distance or cutting depth)

Published Results: 2012

Wei Zhuo and Jarek Rossignac, Curvature-based Distance Offset, Implementation and Applications, SMI 2012

-- Solve three related problems: (1) exact volume compensation with constant distance offset (2) successive offsets of even area distribution (3) bending with local area preservation

-- Developed techniques: Selective smoothing, Spine-aligned grid.

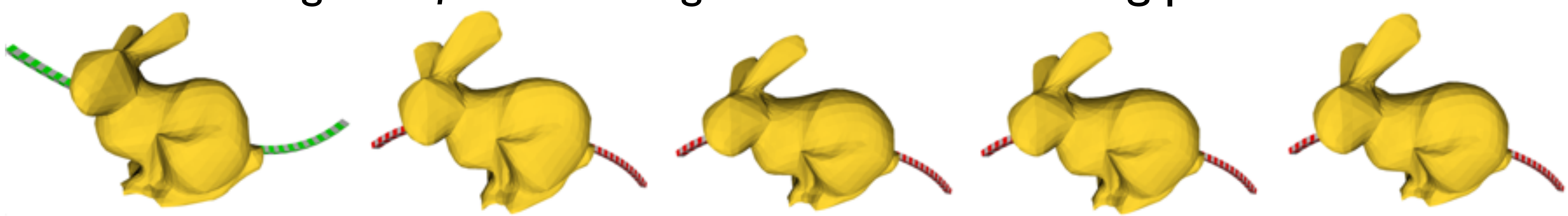


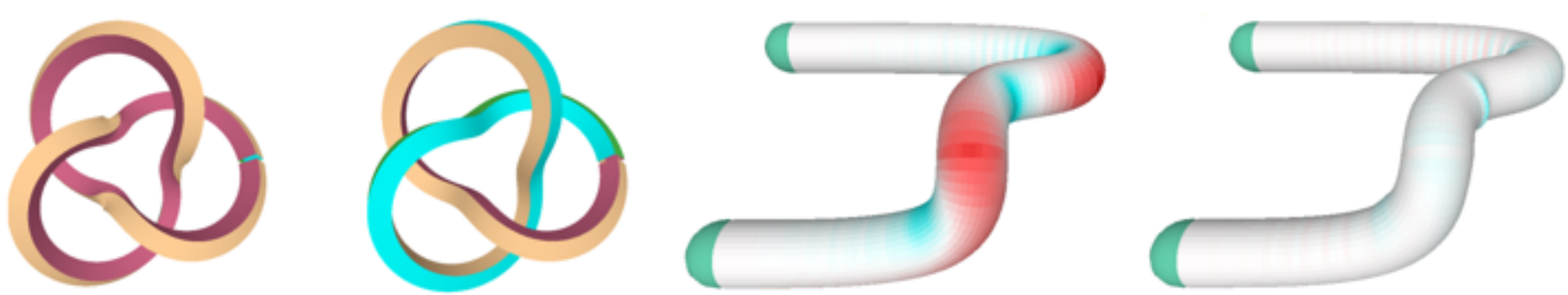
Published Results: 2013

Wei Zhuo and Jarek Rossignac, Spine-driven Bending with Local Volume Preservation, EG 2013

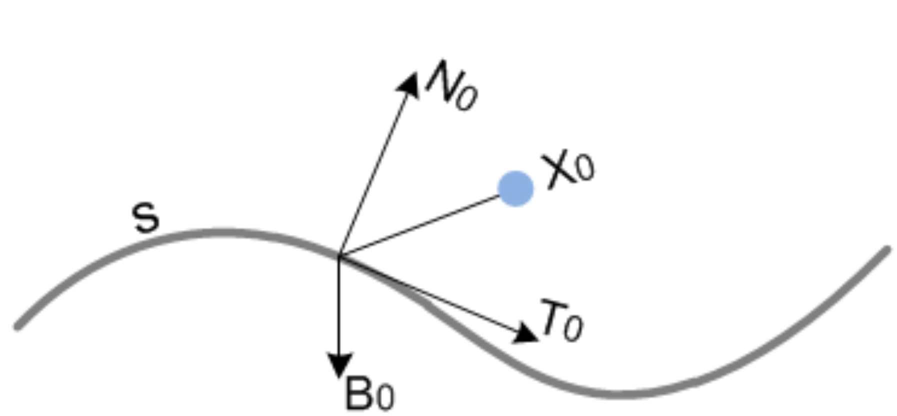
-- Extend 2D bending to 3D and derive a family of solutions with local volume preservation: Radial, Normal and Binormal, depending on the direction in which the point move to compensate the local volume

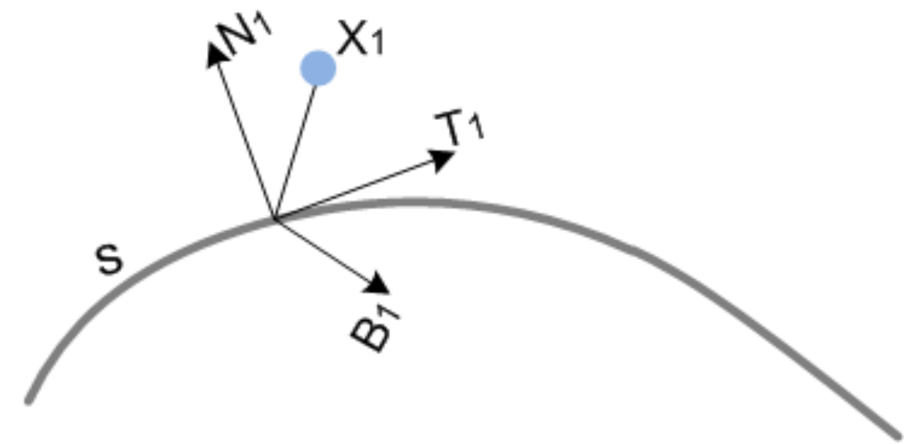
-- Use *bending-transfer-unbending* to solve the twisting problem





Bending-Transfer-Unbending

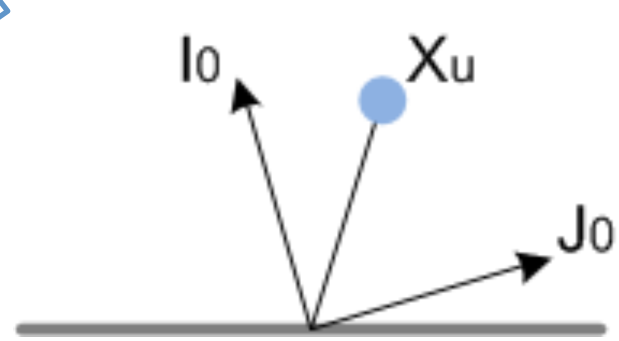


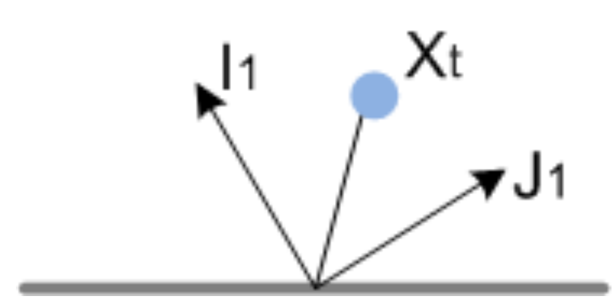


The local parameters used for unbending and re-bending are measured in the Frenet frame

The registration uses the twist-compensated frame

We need to combine them





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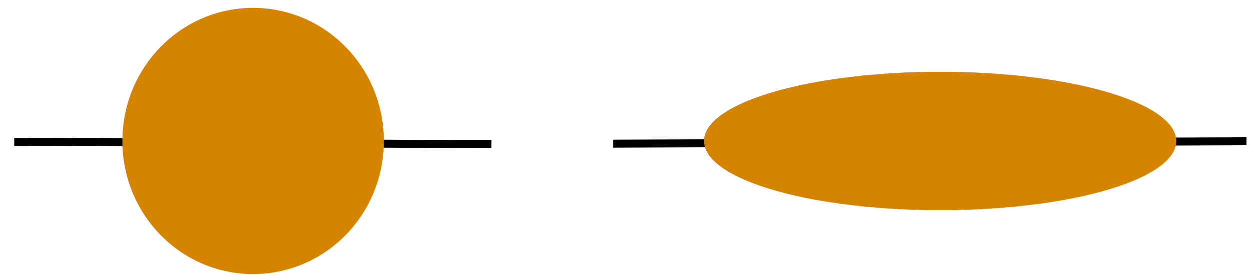
2D Stretchable Spine



$$l(s) = \int_{s_0}^{s} |C'(s)| ds \longrightarrow dl = |C'(s)| ds$$

$$P(s,h) = C(s) + hN(s) \longrightarrow$$

$$det(\frac{\partial P_1}{\partial P_0}) = (1 - k_1 x_1)|C_1'(s)| dh_1 / (1 - k_0 x_0)|C_0'(s)| dh_0$$

$$\boxed{(h_1 - \frac{1}{2} k_1 h_1^2)|C_1'(s)| = (h_0 - \frac{1}{2} k_0 h_0^2)|C_0'(s)|}$$

The difference from non-stretchable spine is that we need to have a discrete approximation of the local stretch, |C'(s)|, of the spine curve. The advantage is that we do not need to compute arc-length parameterization for C

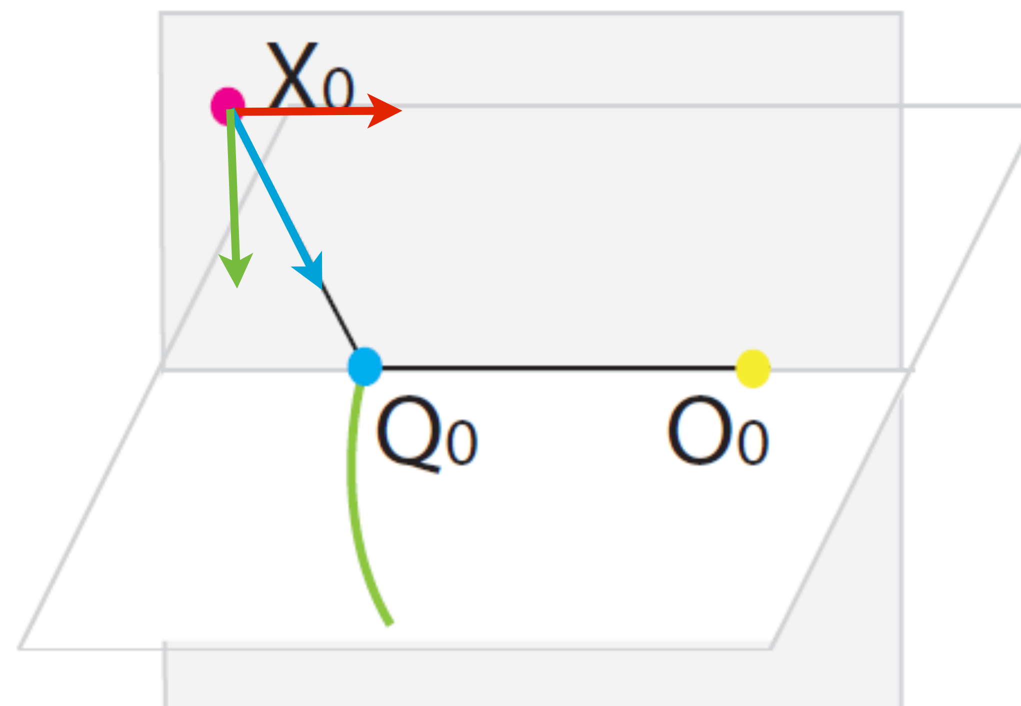
3D Stretchable Spine

$$l(s) = \int_{s_0}^{s} |C'(s)| ds \longrightarrow dl = |C'(s)| ds$$

Same derivative relationship holds in 3D

depending on the direction in which the point moves to compensate the local volume, there are three solutions:

normal, bi-normal and radial



3D Stretchable Spine

$P = C(s) + xN(s) + yB(s)$ **or** $P = C(s) + r\cos\theta N(s) + r\sin\theta B(s)$

let $det(\frac{\partial P_1}{\partial P_0}) = 1$ we have

Normal Solution: $(x_1 - k_1 x_1^2/2)|C_1'(s)| = (x_0 - k_0 x_0^2/2)|C_0'(s)|$

Binormal Solution: $(1 - k_1 x_1)|C_1'(s)|y_1 = (1 - k_0 x_0)|C_0'(s)|y_0$

Radial Solution: $-\frac{2}{3}k_1\cos\theta_1 r_1^3 + r_1^2 = \frac{|C_1'(t)|}{|C_0'(t)|}(-\frac{2}{3}k_0\cos\theta_0 r_0^3 + r_0^2)$

Stretchable Surface

$$P(u,v,h) = S(u,v) + hN(u,v) \Rightarrow \frac{\partial P}{\partial(u,v,h)} = \begin{bmatrix} S_u(u,v) + hN_u(u,v) \\ S_v(u,v) + hN_v(u,v) \\ N(u,v) \end{bmatrix} \Rightarrow$$

We make use of the proposition: Km=Div(N), Kg = det(J(N))

$$\Rightarrow det(\frac{\partial P_1}{\partial P_0}) = 1 \Rightarrow$$

$$\boxed{h_1 - h_1^2 H_1 + \frac{h_1^3}{3} G_1 |S_{1u} \times S_{1v} \cdot N_1| = h_0 - h_0^2 H_0 + \frac{h_0^3}{3} G_0 |S_{1u} \times S_{1v} \cdot N_1|}$$

h changes from h_0 to h_1 to compensate the local volume change

Sampling and Approximation

--More Accurate Projection

--Normal and Curvature Interpolation

--Experiment with different Level of Subdivisions of the spine

Month	Nov, 2013					Dec, 2013				Jan, 2014		
Week	1	2	3	4	5	6	7	8	9	10	11	12
3.1												
3.1.1												
3.1.2												
3.1.3												
3.2												
3.2.1												
3.2.2												
3.3												
write up paper												

Schedule after the proposal