$\boxed{\frac{1}{2} n \log_2 n}$ Steps

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$n = 1024 = 2^{10}$

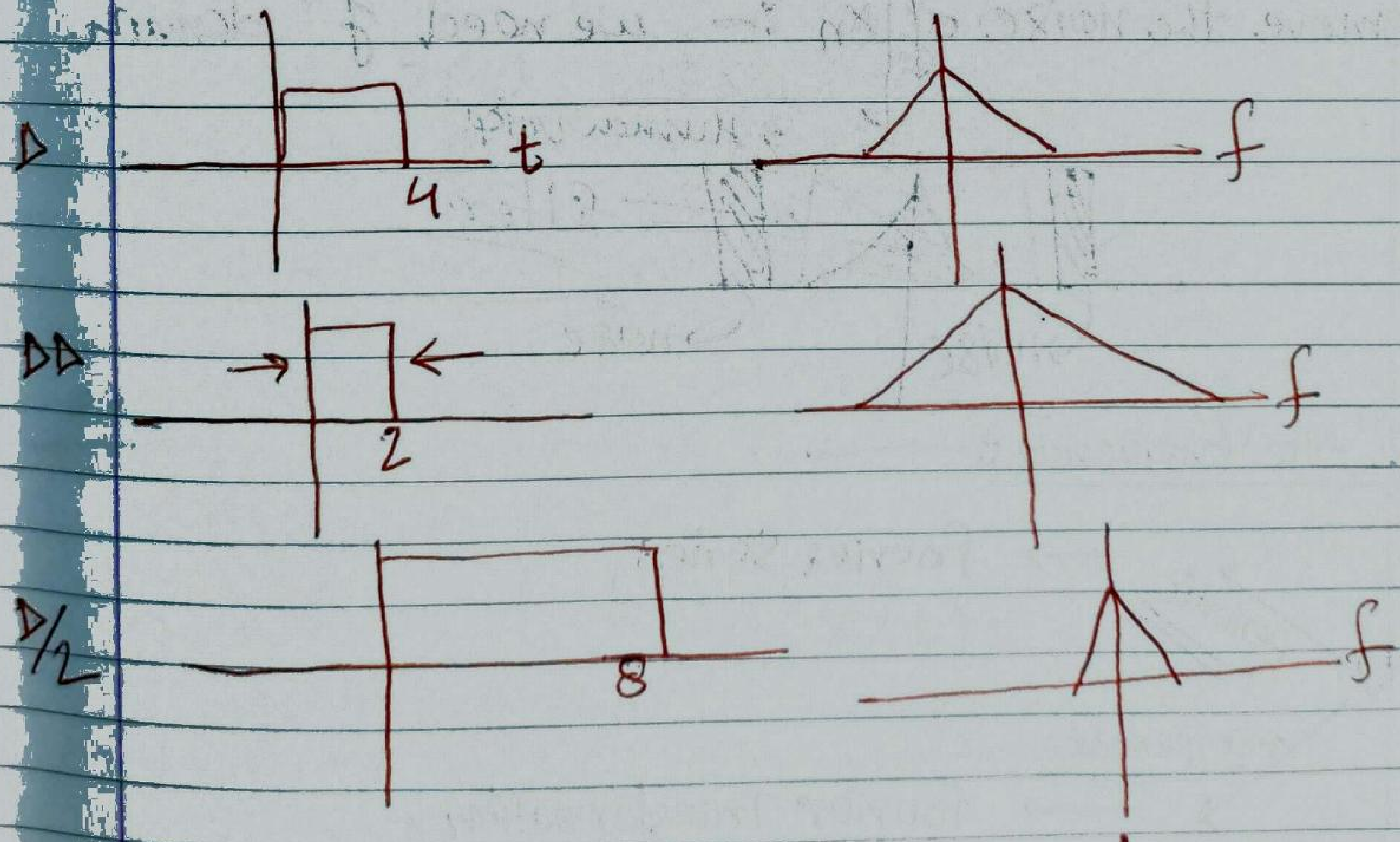
$n^2 > 1000\,000 \rightarrow 1024 \times 1024$ times

$\frac{1}{2} n \log_2 n = (1024)\frac{10}{2} = \boxed{5 \times 1024}$ times

Applications

(f) frequency means how fast a signal is changing

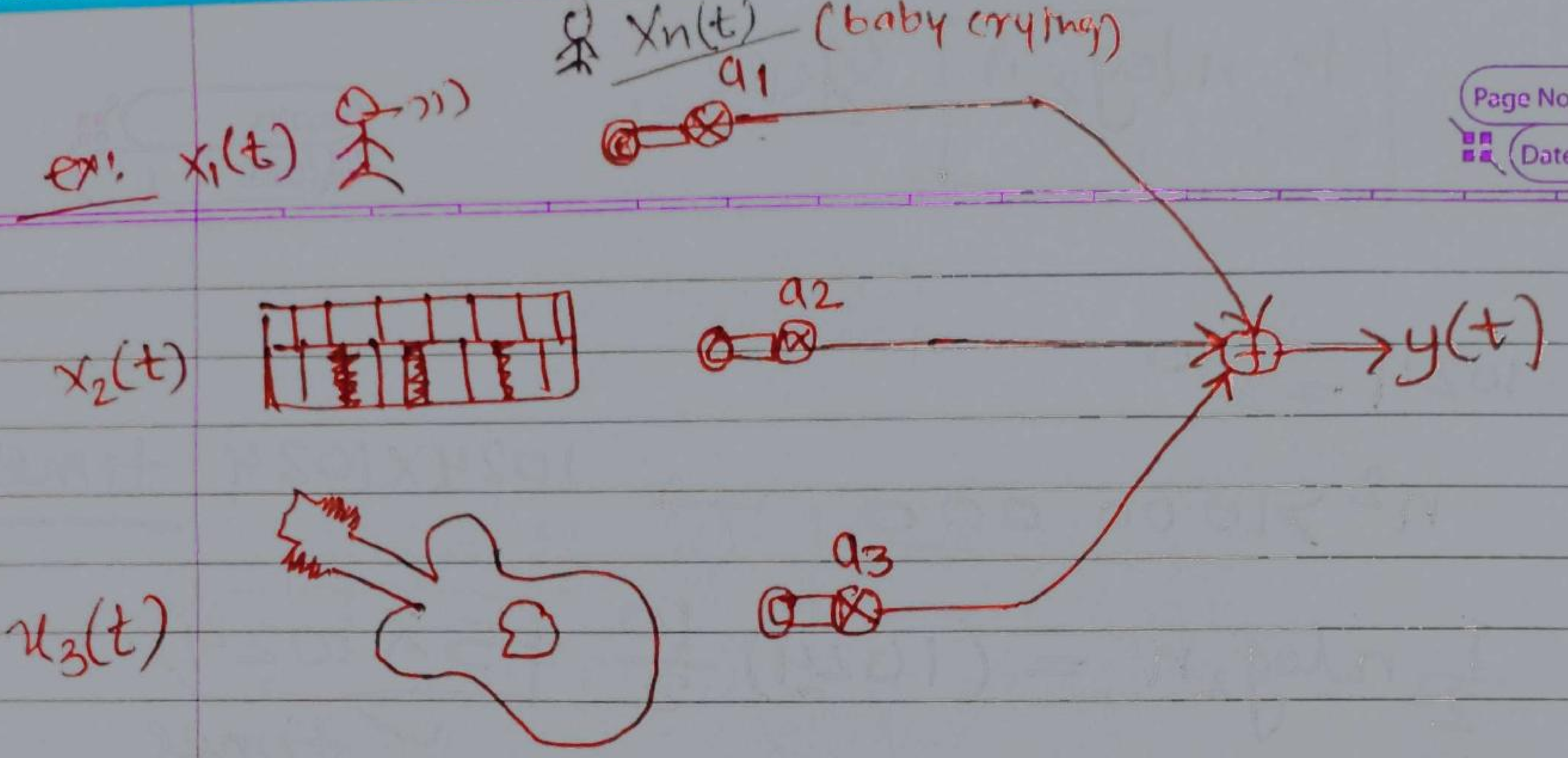
Audio)) Hi, this is imam, please call me back!



time Domain

frequency Domain

ex:



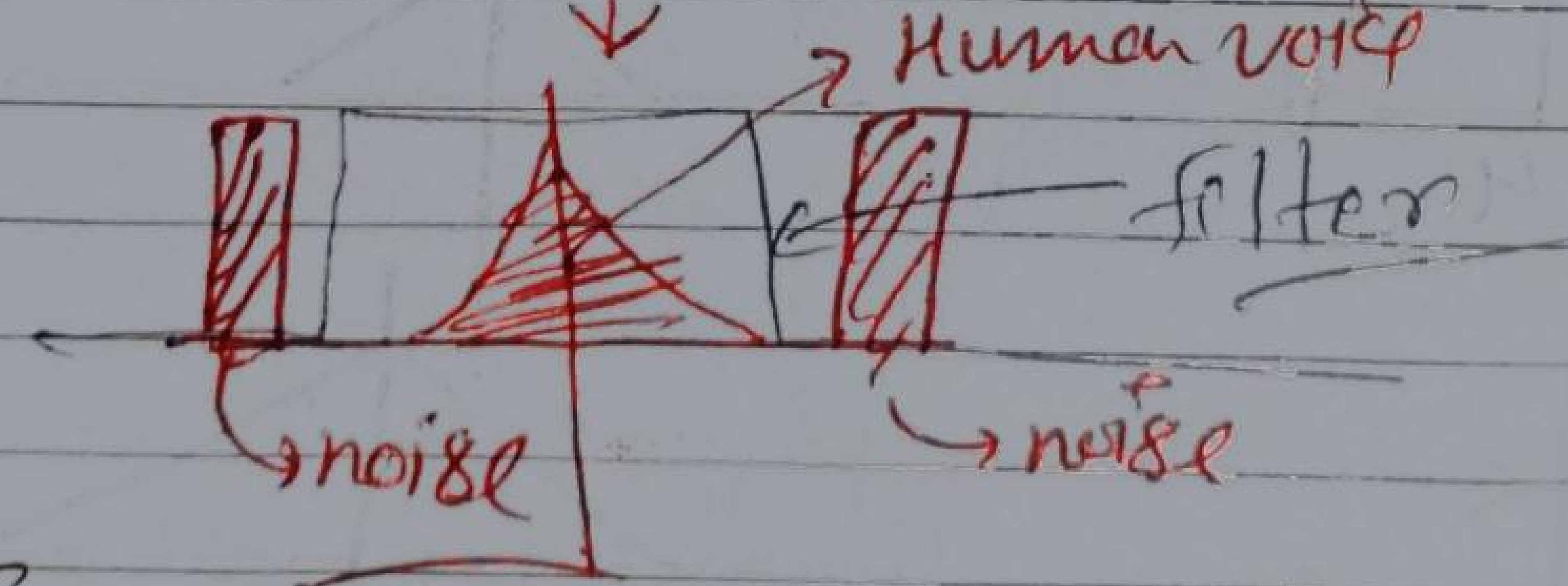
$$y(t) = a_1x_1(t) + a_2x_2(t) + a_3x_3(t)$$

a_1	a_2	a_3
1	1	1
0	1	3
1	1	3

with noise

$$y(t) = \boxed{a_1(x_1(t) + x_n(t))} + a_2x_2(t) + a_3x_3(t)$$

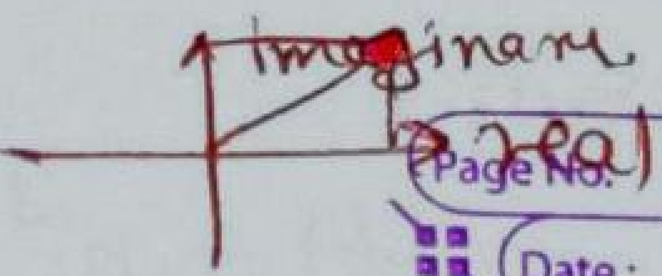
to remove the noise of x_n :- we need f^n domain



Time to Frequency:

$x(t)$ periodic → Fourier Series

$x(t)$ not periodic → fourier Transformation, Complex Numbers → Cartesian



→ Polar

↓

$$\boxed{z = \underline{r}\, e^{j\theta}}$$

Cartesian

$z = x + yi$

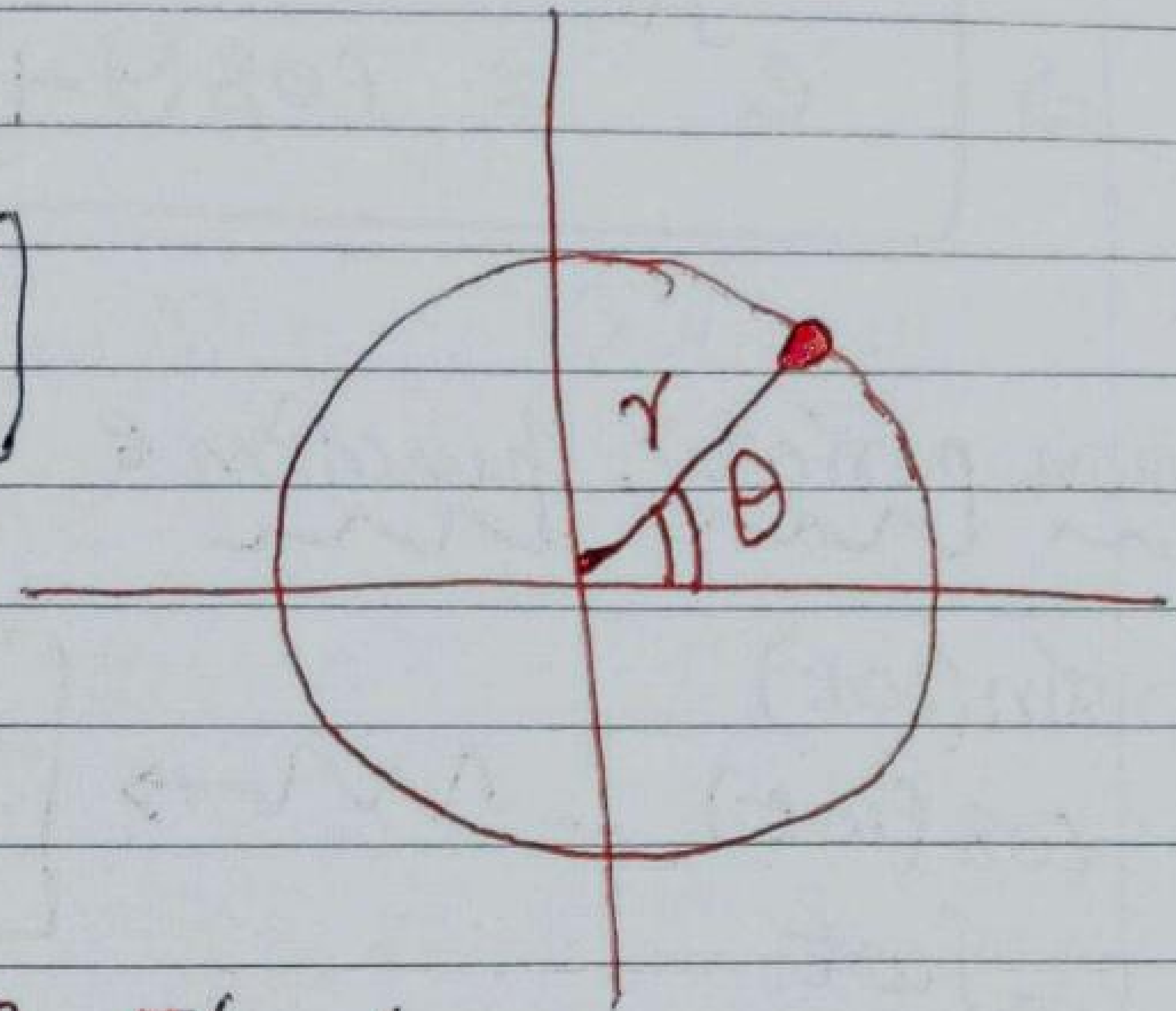
$z = 3j$

$z = -2$

$z = 4 - 3j$

$r \rightarrow$ radius

$\theta \rightarrow$ angel

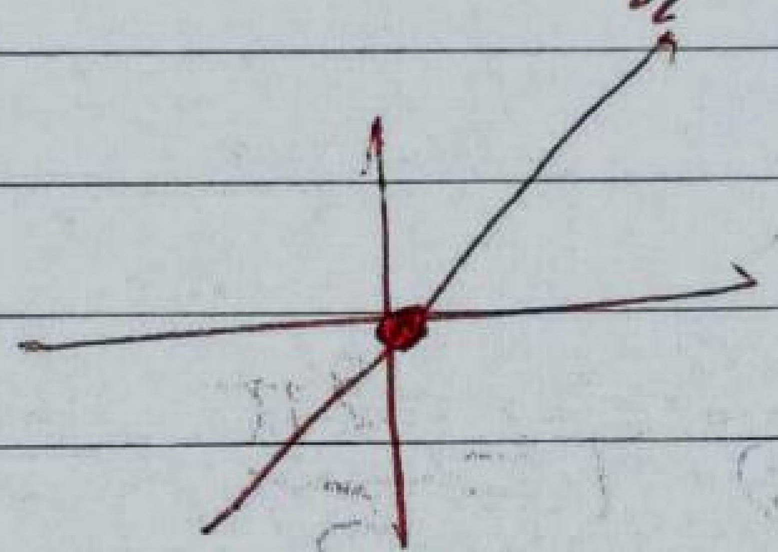


$$z = 2e^{j\pi/3}$$

= radius is 2; $\theta = \pi/3$ //

Signal is odd or even

ex: $x(t) = 2t$



symm around origin

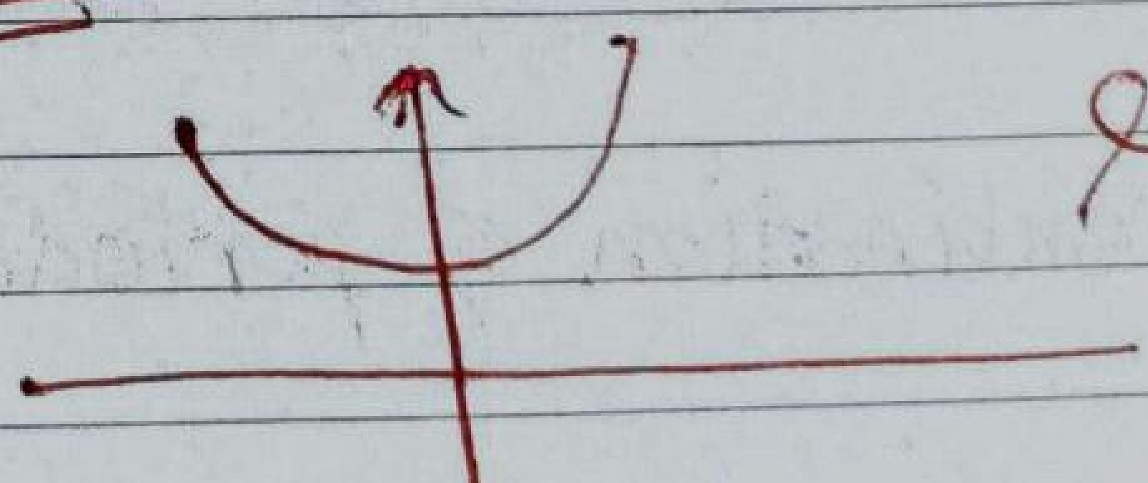
Calculate

$x(-t) = 2(-t) = -2t$

$x(t) = 2(t)$ mean

$-x(-t) = 2(t)$ $x(t) = -x(-t)$

↳ odd



symm around y-axis

ex: $x(t) = t^2 + 1$

$x(-t) = (-t)^2 + 1$

$x(t) = t^2 + 1$ ⇒ $x(t) = x(-t)$

$x(-t) = t^2 + 1$ Even

By Euler:

$$x(t) = e^{j\pi t}$$

is $\Rightarrow$ $\boxed{e^{j\theta} = \cos\theta + j\sin\theta}$

common periodic functions:

- # $\sin(\omega t)$
- # $\cos(\omega t)$ $\rightsquigarrow$ $\boxed{T = \frac{2\pi}{\omega}}$ ↳ period.
- # $e^{j\omega t}$

ex:

ⓐ $x(t) = \cos 5t$

so period is $\Rightarrow T = \frac{2\pi}{5}$

ⓑ $x(t) = \sin(\pi t + \frac{\pi}{4})$

$\omega = \pi$ shift

$T = \frac{2\pi}{\pi} = 2$

combination of periodic signals:-

$$\underbrace{x_3(t)}_{T_3} = \underbrace{x_1(t)}_{T_1} + \underbrace{x_2(t)}_{T_2}$$

$x_3(t)$ is periodic, if $\frac{T_1}{T_2}$ is rational.

if (*) is true:-

$T_3 \rightsquigarrow \text{Lcm}(T_1, T_2)$

↳ least common multiple

exe: $\sin(\pi t) + \sin(t)$ is periodic or not;

$T_1 = \frac{2\pi}{\pi} = 2$; $\quad T_2 = \frac{2\pi}{1} = 2\pi$

$\frac{T_1}{T_2} = \frac{2}{2\pi} = \cancel{\frac{1}{\pi}}$ Not periodic.

exe: $\cos(6\pi t) + \sin(30\pi t)$

$T_1 = \frac{2\pi}{6\pi} = \frac{1}{3}$; $\quad T_2 = \frac{2\pi}{30\pi} = \frac{1}{15}$

$\frac{T_1}{T_2} = \frac{1/3}{1/15} = 5$;

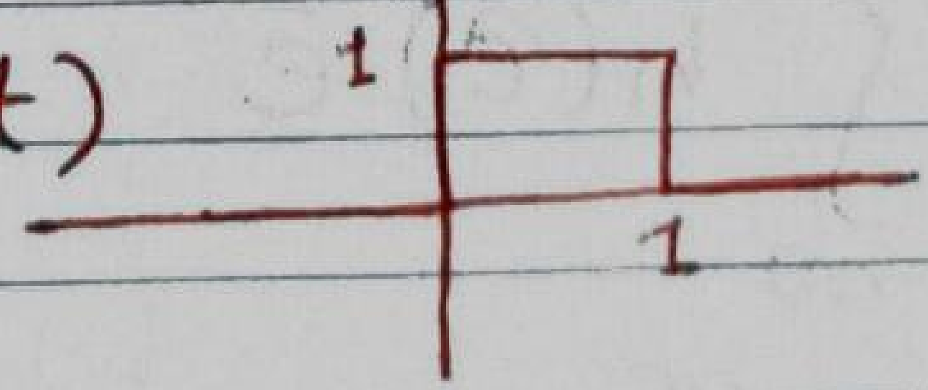
Elementary signals

LTI : Linear & Time Invariant

system is time invariant if time shift in input ~~is~~ results in the same time shift in the o/p.

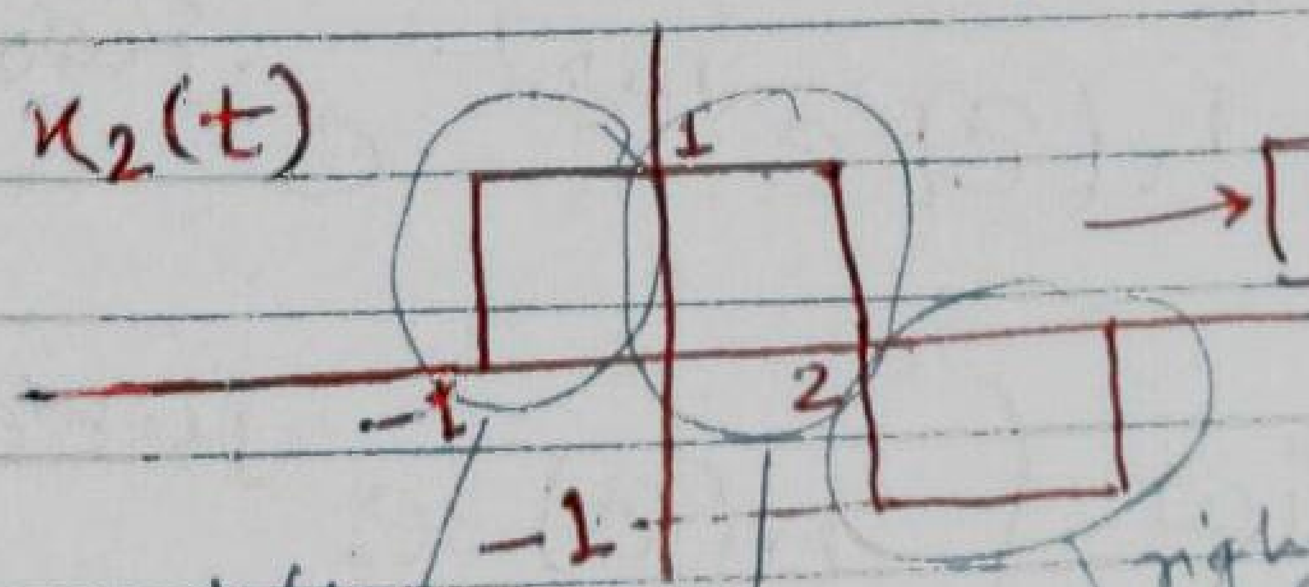
example:

$x_1(t)$



$\rightarrow \boxed{LTI} \rightarrow y_1(t)$

$\Rightarrow x_2(t)$



$\rightarrow \boxed{LTI} \rightarrow y_2(t)$ in terms of $y_1(t)$?

left shift by 1 — right shift without amplitude

$x_2(t) = x_1(t+1) + x_1(t) - x_1(t-1)$

so: $x_1(t) \rightarrow \boxed{LTI} \rightarrow y_1(t)$ $\quad x_1(t+1) \rightarrow \boxed{LTI} \rightarrow y_1(t+1)$ $\quad x_1(t-1) \rightarrow \boxed{LTI} \rightarrow y_1(t-1)$

$\rightarrow (x_2(t))$ $\qquad \rightarrow y_2(t)$

Frequency domain :- Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$ FS synthesis equations

weight $c_k = \frac{1}{T}\int_T x(t) e^{-jk\omega_0 t} dt$ FS analysis equations

→ fourier series coeff

t :- fundamental period of $x(t)$

$\omega_0 \Rightarrow$ fundamental frequency $\omega_0 = 2\pi/T$

$$x(t) = e^{j\omega_0 t} \longrightarrow \boxed{\underset{h(t)}{LTI}} \rightarrow y(t) = x(t) * h(t)$$

$$y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau \quad t \to t-\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) e^{j\omega_0 (t-\tau)} d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) e^{j\omega_0 t} e^{-j\omega_0 \tau} d\tau$$

$$= e^{j\omega} \underbrace{\int_{-\infty}^{\infty} h(\tau) \cdot e^{-j\omega_0 \tau} d\tau}_{H(\omega_0)}$$

$$= e^{j\omega_0 t} \times H(\omega_0)$$

how LTI responded to diff frequencies $\leftarrow$ f^n responce

So rewriting!

$$x(t) = \sum_{K=-\infty}^{\infty} c_K e^{jK\omega_0 t}$$

$$e^{j\omega_0 t} \longrightarrow \boxed{LTI} \longrightarrow e^{j\omega_0 t} \cdot \underbrace{H(\omega_0)}_{\text{f}^n \text{ response at } \omega_0}$$

Conclusions

① LTI system doesn't change the frequency.

properties of LTI

$$\underbrace{\sum_{K=-\infty}^{\infty} c_K e^{jK\omega_0 t}}_{x(t)} \longrightarrow \boxed{LTI} \longrightarrow \underbrace{\sum_{K=-\infty}^{\infty} c_K e^{jK\omega_0 t} \underbrace{H(K\omega_0)}_{\text{f}^n \text{ response at } \omega_0}}_{y(t)}$$

freq^n

Fourier Series

$$x(t) = \sum_{K=-\infty}^{\infty} c_K e^{jK\omega_0 t}$$

$$c_K = \frac{1}{T} \int_T x(t) e^{-jK\omega_0 t} dt$$

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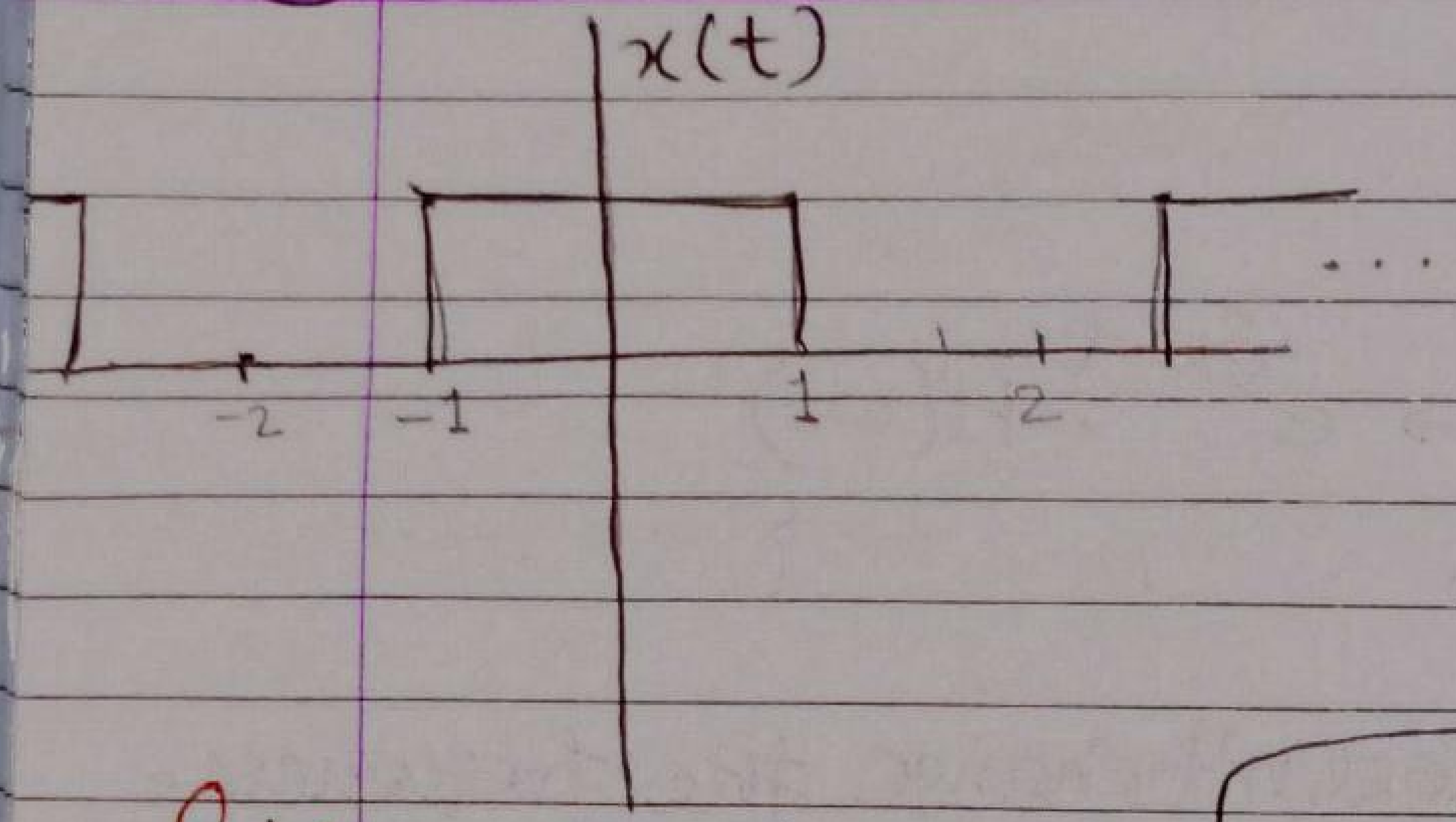
example:

① find c_k for these periodic signal:-



$c_k = ?$

Sol: $T = 4 \rightsquigarrow \boxed{\omega_0 = \frac{2\pi}{T}} = \frac{2\pi}{4} = \frac{\pi}{2}$

(weight) $c_k = \frac{1}{T}\int_T x(t)\, e^{-jk\omega_0 t}\, dt = \frac{1}{4}\int_{-2}^{2} \underline{x(t)}\, e^{-jk\omega_0 t}\, dt$

Remark

$\int e^{at}\, dt = \frac{1}{a} e^{at}$

$= \frac{1}{4}\int_{-1}^{1} e^{-jk\omega_0 t}\, dt$

($\because x(t)$ is 1 in this interval)
& $x(t)$ is 0 $-2 \to -1$, $1 \to 2$)

Remarks

$\boxed{\sin a = \frac{e^{ja} - e^{-ja}}{2j}}$

$c_k = \frac{1}{4}\frac{1}{-jk\omega_0} e^{-jk\omega_0 t}\Big|_{-1}^{1}$

$= -\frac{1}{4jk\omega_0}\left(e^{-jk\omega_0} - e^{jk\omega_0}\right)$

$= \frac{1}{4jk\omega_0}\left(e^{jk\omega_0} - e^{-jk\omega_0}\right)$

$= \frac{1}{2k\omega_0}\left(\frac{e^{jk\omega_0} - e^{-jk\omega_0}}{2j}\right)$

$= 1/2k\omega_0 \; \sin(k\omega_0)$

$$C_K = \frac{\sin(K\omega_0)}{2K\omega_0} = \frac{\sin\left(K\frac{\pi}{2}\right)}{2\cdot K\cdot\frac{\pi}{2}} =$$

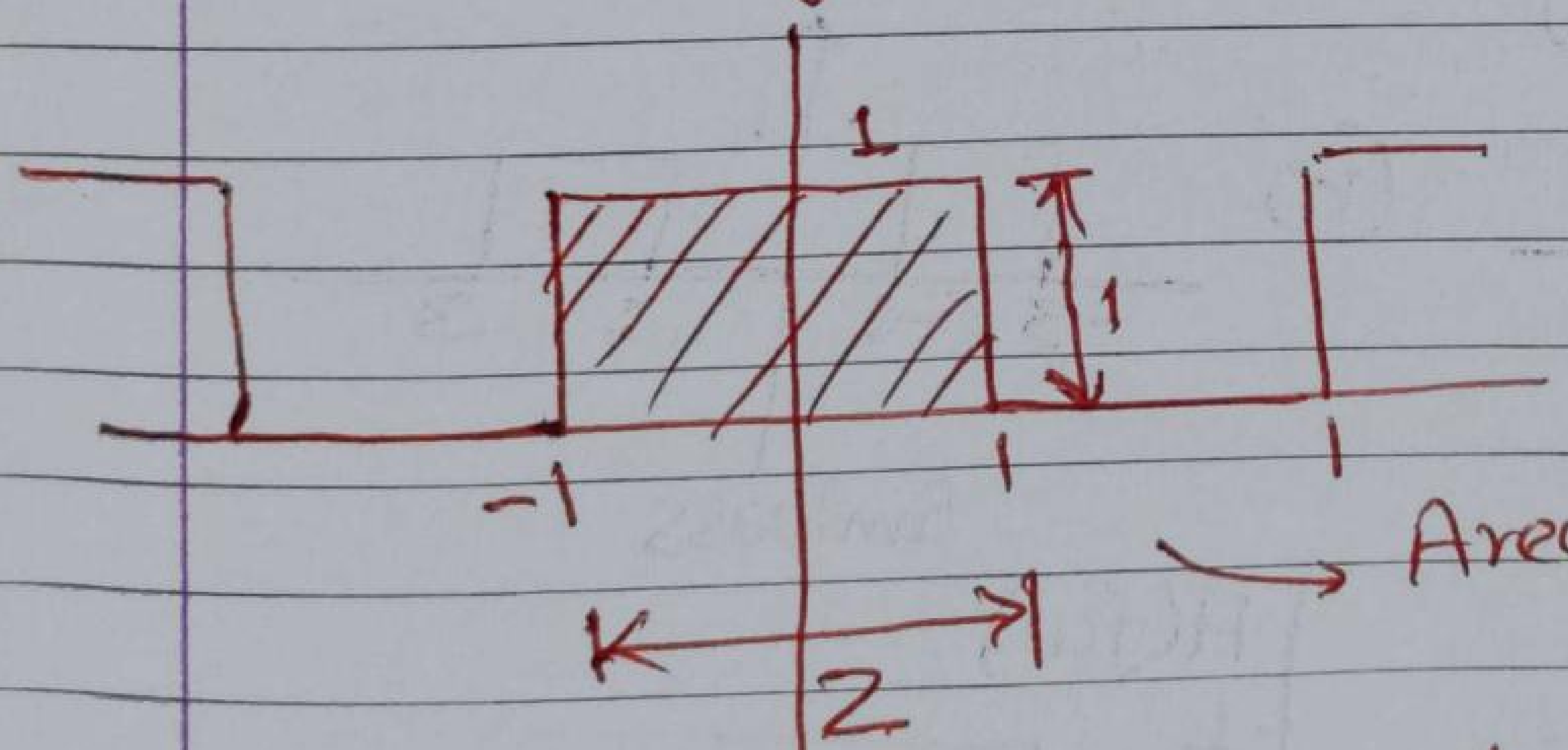
$$\boxed{C_K = \frac{\sin\left(K\cdot\frac{\pi}{2}\right)}{K\cdot\pi}} \quad K \neq 0$$

Valid only when $K \neq 0$.

$$C_K = \frac{1}{T}\int_T x(t)\, e^{-jK\omega_0 t}\, dt \rightsquigarrow \boxed{C_0 = \frac{1}{T}\int_T x(t)\,dt}$$

$\int_T$ denotes area under the curve over one period

Let's see the signal again



$\rightarrow$ Area = 2.

so: $C_0 = \frac{1}{4}\cdot 2 = 1/2$.

final answer:

$$C_K = \begin{cases} \dfrac{\sin\left(K\cdot\frac{\pi}{2}\right)}{K\cdot\pi} & K \neq 0 \\[2ex] 1/2 & K = 0 \end{cases}$$

Notes

1. if $x(t)$ is even, c_k is even $\rightsquigarrow c_k = c_{-k}$
2. if $x(t)$ is odd, c_k is odd $\rightsquigarrow c_k = -c_{-k}$
3. if $x(t)$ is real, $\rightsquigarrow c_k = c_{-k}^{*}$
4. if $x(t)$ is even and real $\rightsquigarrow c_k$ is real
5. if $x(t)$ is odd & real $\rightsquigarrow c_k$ is imaginary.

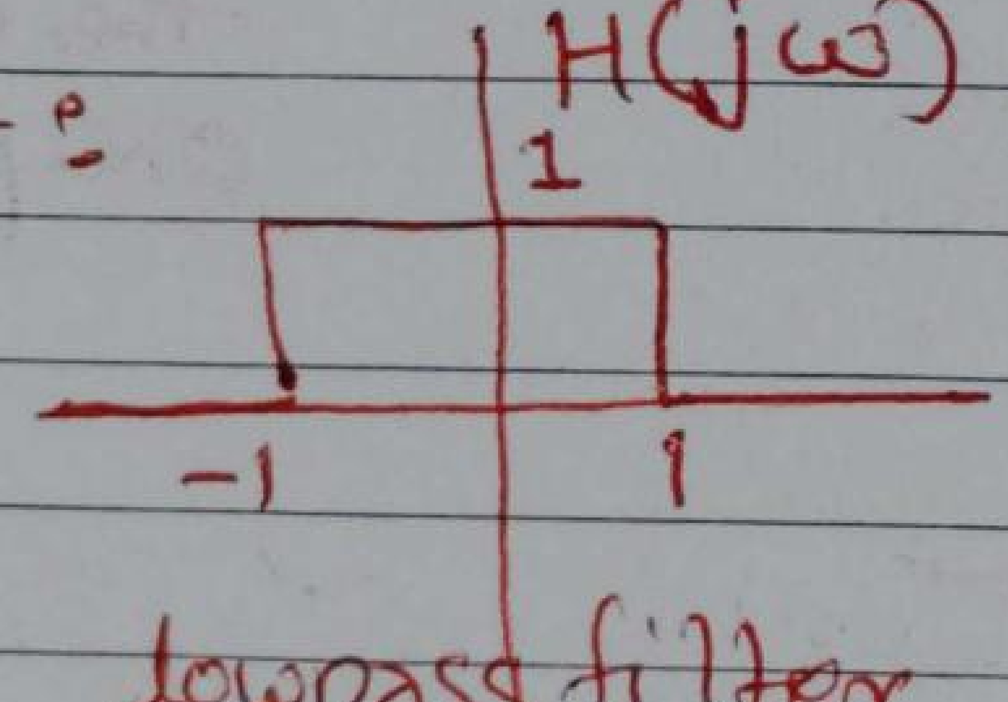
Filtering (Periodic Signals):-

Example: Signal $x(t) = 1 + 2\cos(2t) + 2\cos(4t)$ is applied to LTI system; what is the output? $y(t)$?

if:

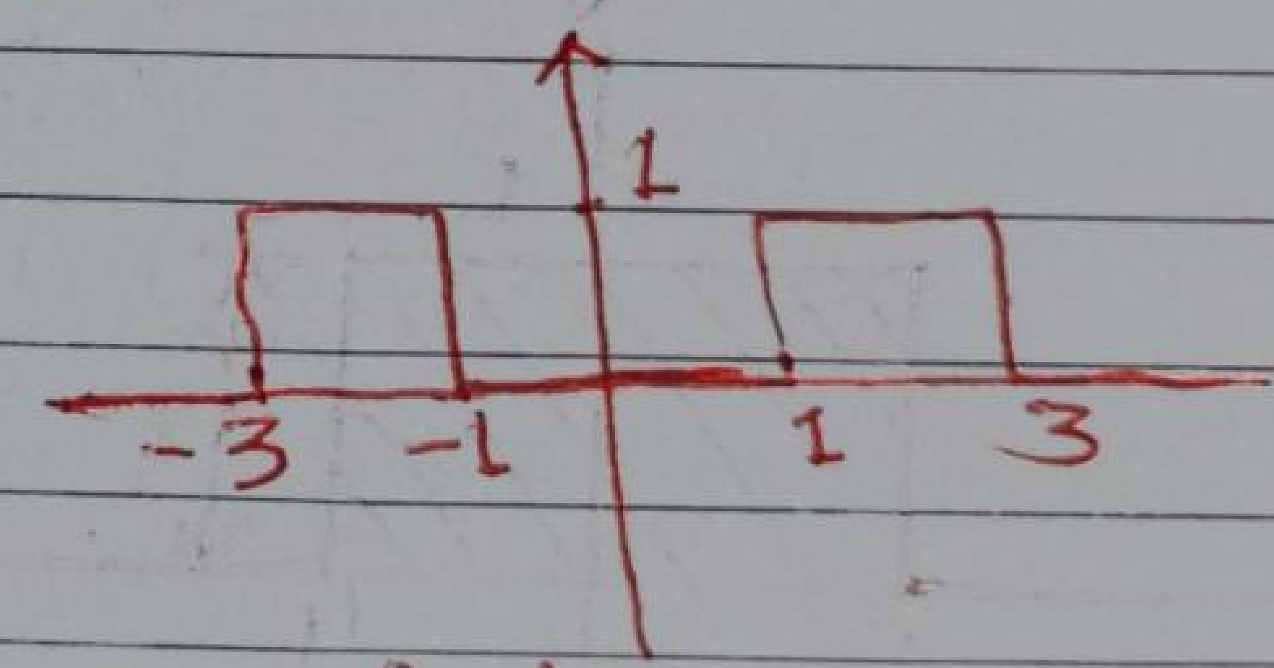
frequency response is

(a)



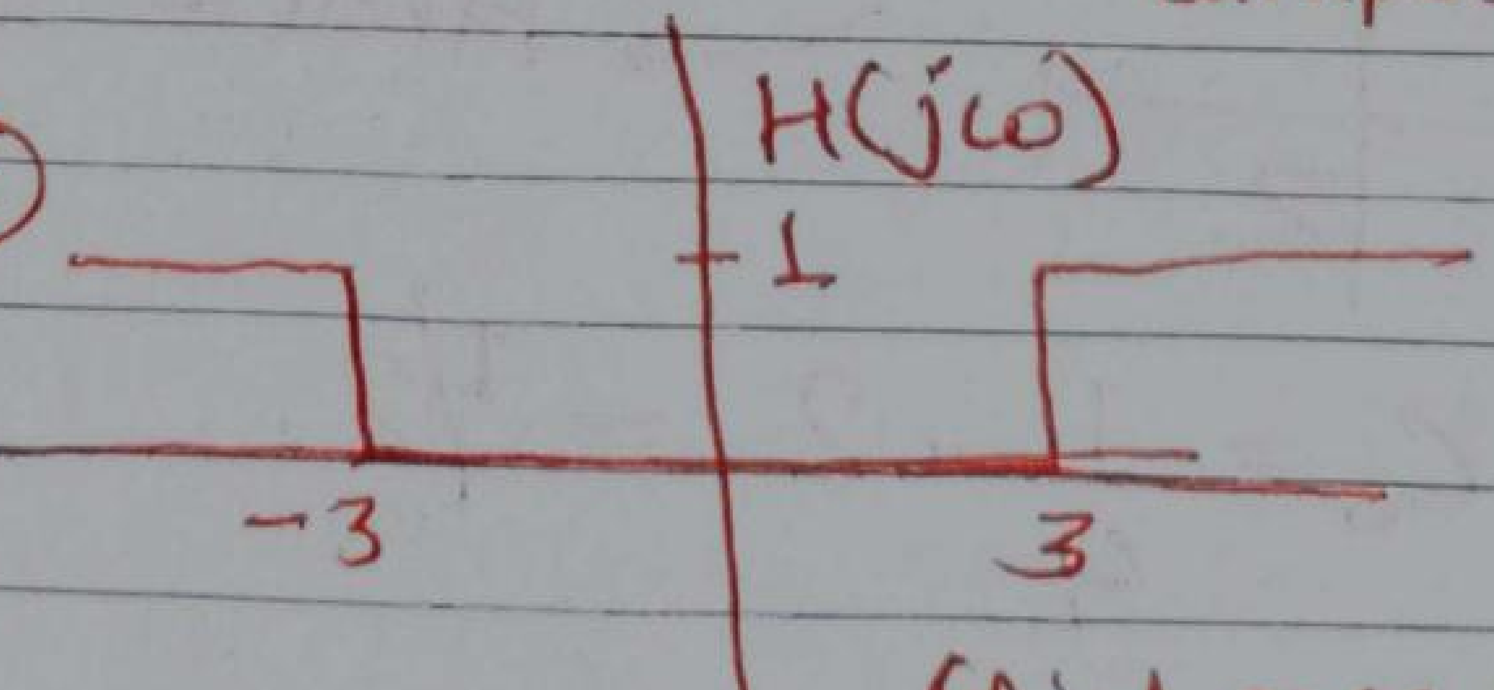
lowpass filter

(b)



Bandpass

(c)



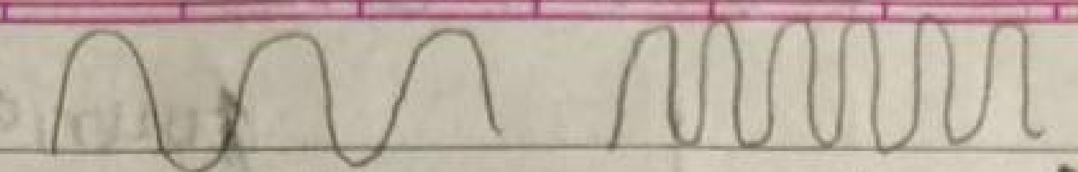
Highpass filter

Note: Step 1: Represent input into linear combination of complex exponential's.

Remark:

$$\cos(a) = \frac{e^{ja} + e^{-ja}}{2}$$

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$



mean   low   mid   High

so: $x(t) = 1 + 2\cos(2t) + 2\cos(4t)$

$$x(t) = \underset{\downarrow\, e^{j0t}}{1} + \cancel{2}\left(\frac{e^{j2t} + e^{-j2t}}{\cancel{2}}\right) + \cancel{2}\left(\frac{e^{j4t} + e^{-j4t}}{\cancel{2}}\right)$$

$$\underline{x(t)} = e^{j0t} + e^{j2t} + e^{-j2t} + e^{j4t} + e^{-j4t} \rightarrow \boxed{LTI} \rightarrow y(t)$$

As we know:

$$e^{j\omega_0 t} \longrightarrow \boxed{LTI} \longrightarrow e^{j\omega_0 t} H(j\omega_0)$$

$\boxed{LTI}$

o/p is same

$$y(t) = \cancel{e^{j0t}} + \cancel{e^{j2t}}$$

$$\underline{y(t)} = e^{j0t} H(j0) + e^{j2t} H(j2) + e^{-j2t} H(-j2) + e^{j4t} H(j4) + e^{-j4t} H(-j4)$$

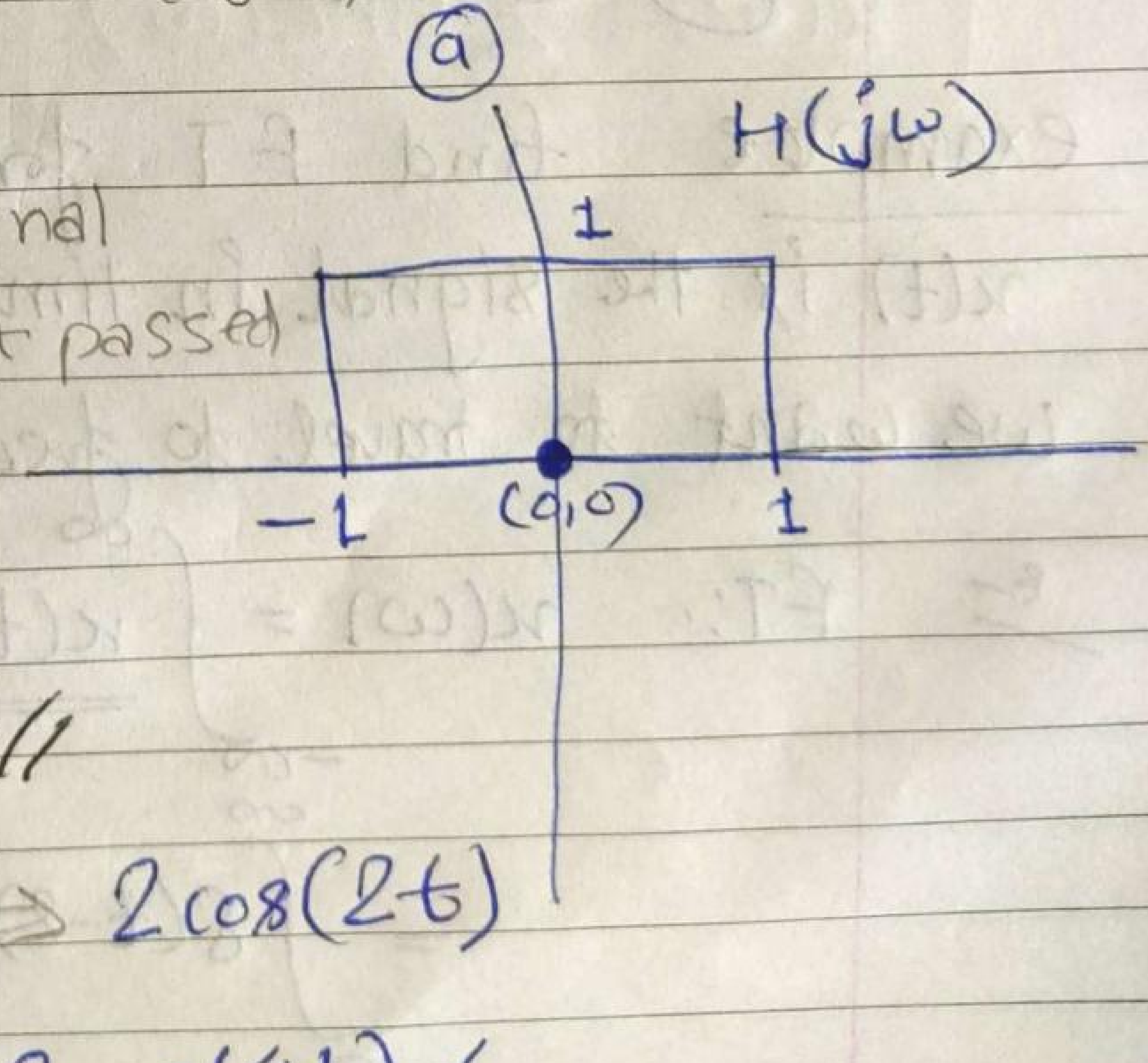
Frequency Response:

$H(0) \Rightarrow (0,0) \rightarrow$ is 1

so LTI passes this signal

similarly { for $H(2) = 0$; → signal is not passed

for $H(-2), H(4), H(-4) = 0$; signal is not passed;



(a) output: $y(t) = e^{j0t} = 1$ //

similarly:-

(b) $y(t) = 2\left(\frac{e^{j2t} + e^{-j2t}}{2}\right) \Rightarrow 2\cos(2t)$

(c) $y(t) = 2\left(\frac{e^{j4t} + e^{-j4t}}{2}\right) = 2\cos(4t)$ /

Fourier Transform

non periodic

$$\left\{\begin{aligned} x(t) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega \quad \to \boxed{IFT} \\ X(\omega) &= \int_{-\infty}^{\infty} x(t)\,e^{-j\omega t}\,dt \quad \to \boxed{FT} \end{aligned}\right.$$

Fourier Series

$$x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

$$c_k = \frac{1}{T}\int_{T} x(t)\,e^{-jk\omega_0 t}\,dt$$

periodic

example OR + OG + OB

~~$X(\omega) \Rightarrow$ weighting system for diff colors~~

$\omega \rightarrow$ color (diff- fundamental color)

Application of FT:-
1. filtering the frequen
2. Object detection by Radar.
3. Telecommunicat (Modulation & Demodulation).
4. MRI
5. Quntum mechanic.

examples find FT for $x(t) = \delta(t-2)$

$x(t)$ is the signal in time domain by using FT we want to travel to frequency domain.

FT:- $X(\omega) = \int_{-\infty}^{\infty} \underline{x(t)}\,e^{-j\omega t}\,dt$

$= \int_{-\infty}^{\infty} \delta(t-2)\,e^{-j\omega t}\,dt$

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Note: $\delta(t-2)$ is non-zero at $t=2$.

Sifting property:

$\rightsquigarrow \quad x(\omega) = e^{-j\omega 2}$ (replace t by 2)

$$\boxed{x(t) = \delta(t-2)} \xrightarrow{FT} \boxed{x(\omega) = e^{-j\omega 2}}$$

Time → frequency

example: Find IFT for $X(\omega) = 2\pi\delta(\omega-5)$ travel this signal is in frequency domain & we want to travel to time domain using Inverse FT.

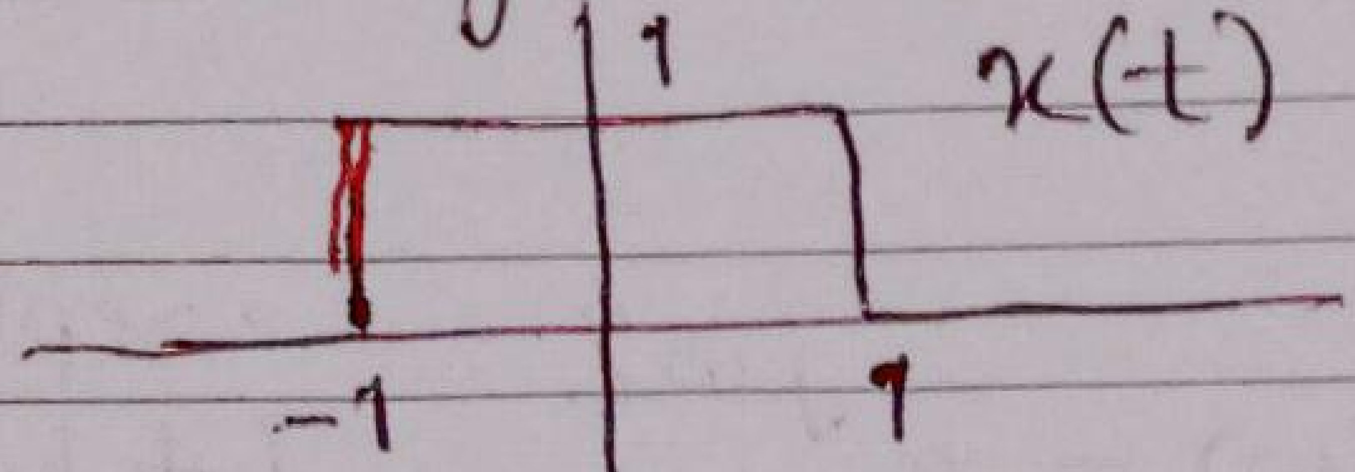
Solution:
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)\, e^{j\omega t}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\,\delta(\omega-5)\, e^{j\omega t}\, d\omega$$

$$= \int_{-\infty}^{\infty} \underbrace{\delta(\omega-5)\, e^{j\omega t}}_{\text{non-zero at } \omega=5}\, d\omega$$

Based on sifting property:

$\rightsquigarrow \quad \boxed{x(t) = e^{j5t}}$ (replace ω by 5) example6 find FT for this window



sol: $x(\omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\omega t}\, dt$

nonzero at $-1 \to 1$ so

$$= \int_{-1}^{1} e^{-j\omega t}\, dt \quad (x(t) = 1 \; (-1,+1))$$

$$x(\omega) = \frac{1}{-j\omega}\left[e^{-j\omega} - e^{j\omega}\right] = \frac{1}{j\omega}\left[e^{j\omega} - e^{-j\omega}\right]$$

$$x(\omega) = \frac{1}{\omega}\left[\frac{e^{j\omega} - e^{-j\omega}}{2j}\right] = \underbrace{\boxed{\frac{2\sin\omega}{\omega}}}_{\text{sinc}(\omega)}$$

$$\boxed{x(\omega) = 2\,\text{sinc}(\omega)}$$

Fourier Transform tables:

Table of pairs:

$$\delta(t) \longleftrightarrow 1$$

$$1 \longleftrightarrow 2\pi\delta(\omega)$$

$$\text{rect}\left(\frac{t}{T}\right) \longleftrightarrow T\,\text{sinc}\left(\frac{\omega T}{2}\right)$$

$$\frac{B}{\pi}\,\text{sinc}(Bt) \longleftrightarrow \text{rect}\left(\frac{\omega}{2B}\right)$$

$$e^{-at}u(t) \longleftrightarrow \frac{1}{j\omega + a}$$

$$e^{j\omega_0 t} \longleftrightarrow 2\pi\delta(\omega-\omega_0)$$

$$\cos\omega_0 t \longleftrightarrow \pi\left(\delta(\omega-\omega_0) + \delta(\omega+\omega_0)\right)$$

$$\sin\omega_0 t \longleftrightarrow \frac{\pi}{j}\left[\delta(\omega-\omega_0) - \delta(\omega+\omega_0)\right]$$

Time domain | frequency domain

Note: Inverse relation b/w Time & frequency.