Linear Algebra

LECTURE :- #26

COMPLEX NUMBER matrices

Inner products

DISCRETE FOURIER

FAST TRANSFORM (FFT)

Notes: Matrix with $n \times n$; ~~take~~ n^2 multiplications.
~~two~~ multiple two matrices.
Using FFT; it takes $\boxed{\frac{1}{2} n \log_2 n}$ ✓

Length of complex Matrix:: $\rightarrow |z_1|^2 + \cdots |z_m|^2$

length = $\boxed{\bar{z}^T z}$ ✓ one symbol for this is $\boxed{z^H}$ ✓ → Hermite

ex:: $\begin{bmatrix} 1 & -i \end{bmatrix} \begin{bmatrix} 1 \\ i \end{bmatrix} = 1+1 = 2$ //

length ⇒ $\boxed{z^H z}$ (z Hermitian z)

Inner Product: for real matrix: $y^T x$

for complex matrix ⇒ $\bar{y}^T x = \boxed{y^H x}$

Symmetric: means: $\boxed{\cancel{A^T = A}}$ no good if A is complex.
→ for real

→ for complex → $\boxed{\bar{A}^T = A}$

& these matrices are called Hermitian Matrices,
(has real eigen values
& ⊥r eigen vectors

What do you mean by orthonormal vector?

$q_1, q_2, \ldots q_n$

inner product: $q_i q_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases} \rightarrow Q^T Q = I$

for complex: $\bar{q_i}^T q_j = \begin{cases} 0 & i \neq j \\ 1 & i = j \end{cases} \quad \boxed{Q^H Q = I}$

fourier Matrix $(n \times n)$

$$F_n = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & \omega & \omega^2 & \cdots & \omega^{n-1} \\ 1 & \omega^2 & \omega^4 & \cdots & \omega^{2(n-1)} \\ \vdots & \vdots & \vdots & & \\ 1 & \omega^{n-1} & \omega^{2(n-1)} & \cdots & \omega^{?} \end{bmatrix} \rightarrow (n-1)^2$$

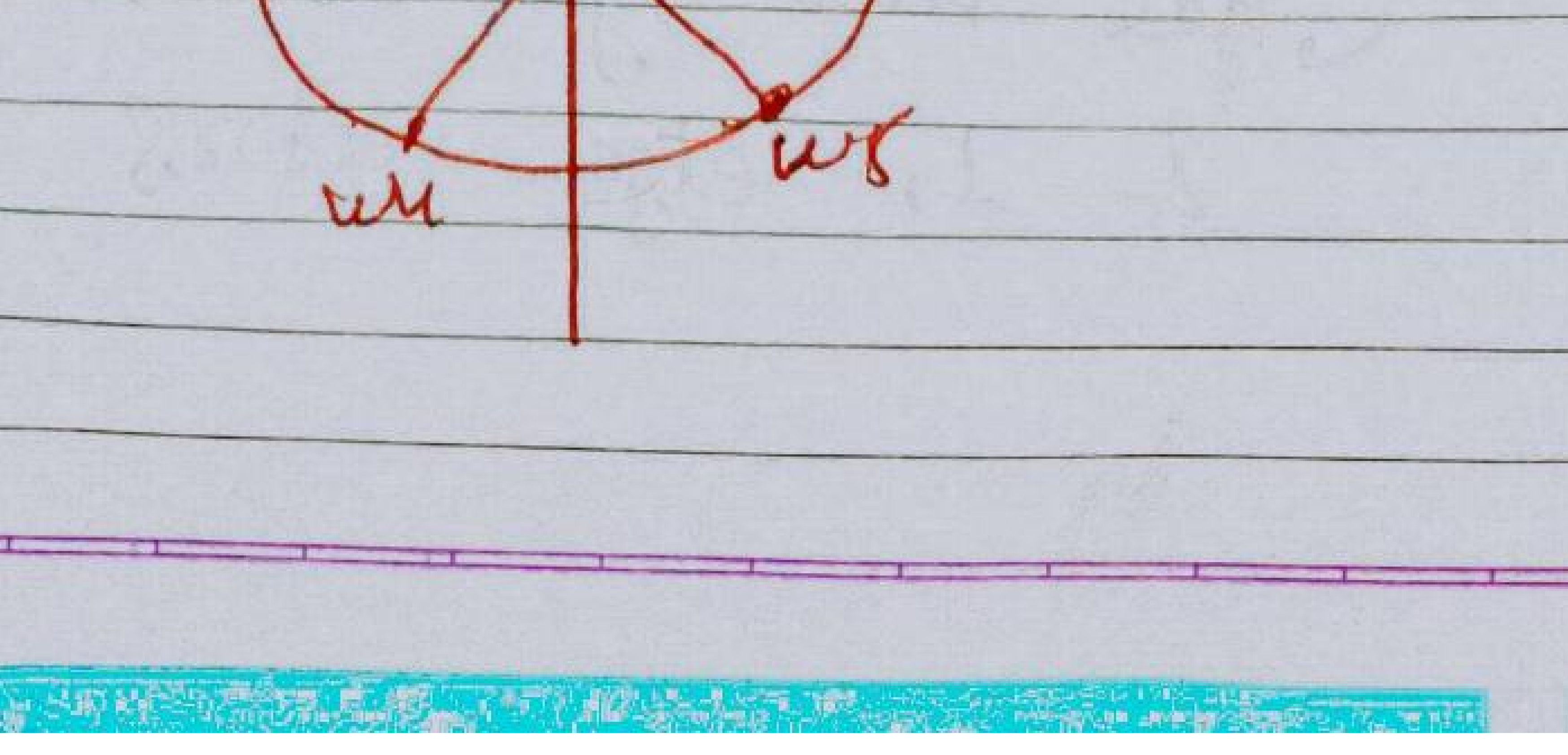
$$\boxed{(F_n)_{ij} = \omega^{ij}}$$

$i, j = 0, \ldots n-1$

Note: W is special number, whose ω^n is 1:

$$\boxed{\omega = e^{i 2\pi/n}}$$

$$= \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}$$



For n=4;

$\omega^4 = 1$

$\omega = e^{2\pi i/4} = i$

powers are $i, i^2 = -1, i^3 = -i, i^4 = 1$;

$$F_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{bmatrix},$$

four point fourier tranform of a vector with four components,

How to check that columns are orthogonal ~~vectors~~?

$\rightarrow$ inner product of column $i \neq j$ is zero

$$\boxed{\bar{q}_i^T q_j = 0} \rightarrow \begin{matrix} 0 & i \neq j \\ 1 & i = j \end{matrix}$$

they are quite orthonormal. F_4.

all colum has length 2

so divide by 2. $\boxed{F_4 = \frac{1}{2} F_4}$

Now columns are orthonormal colum

means $\boxed{F_4^H F_4 = I}$

$$F_4 = \frac{1}{2}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & i & i^2 & i^3 \\ 1 & i^2 & i^4 & i^6 \\ 1 & i^3 & i^6 & i^9 \end{bmatrix}$$

Connection b/w F(32) & F(64) ?

$$(w_{64})^2 = w_{32} \quad \because \boxed{w_n = e^{i2\pi/n}}$$

Conn:

Permutation m (P)

$$\begin{bmatrix} F_{64} \end{bmatrix} = \begin{bmatrix} I & D \\ I & -D \end{bmatrix} \begin{bmatrix} F_{32} & 0 \\ 0 & F_{32} \end{bmatrix} \begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & & 1 & \\ 1 & 1 & 1 & & & 1 \end{bmatrix}$$

64^2 (multiplication)

$$\frac{2(32)^2 + \text{fix}}{(32)}$$

odd even permutation

$$D = \begin{bmatrix} 1 & & & \\ & w & & \\ & & w^2 & \\ & & & \ddots \; w^{31} \end{bmatrix}$$

Break down (F16) (recursion)

$$\begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} \begin{matrix} I & D \\ I & -D \end{matrix} & 0 \\ 0 & \begin{matrix} I & D \\ I & -D \end{matrix} \end{bmatrix} \begin{bmatrix} F_{16} & & & \\ & F_{16} & & \\ & & F_{16} & \\ & & & F_{16} \end{bmatrix} \begin{bmatrix} P & \\ & P \end{bmatrix} \; P_6$$

$$2\left[2[16]^2 + 16\right] + 32$$

Left $\begin{pmatrix} I & D \\ I & -D \end{pmatrix}$

Right Permutations matrices

(6×32) ✓ final.

$\boxed{\frac{1}{2} n \log_2 n}$ Steps

Page No.
Date: / /

$n = 1024 = 2^{10}$

$n^2 > 1000\,000 \rightarrow 1024 \times 1024$ times

$\frac{1}{2} n \log_2 n = (1024)\frac{10}{2} = \boxed{5 \times 1024}$ times

Applications

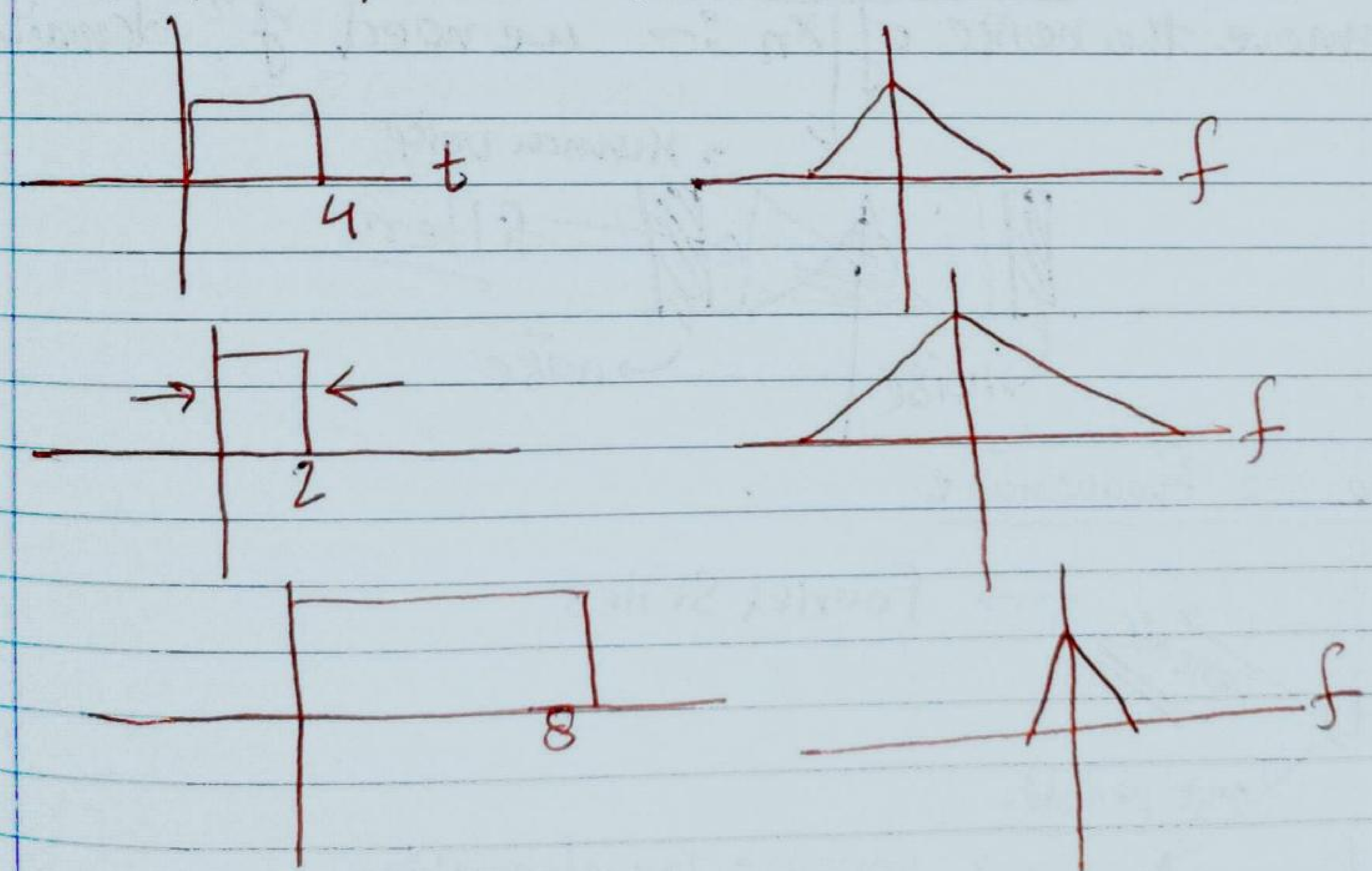
(#) frequency means how fast a signal is changing

Audio))) Hi, this is imam, please call me back!

D

DD

D/2



time Domain

frequency Domain