Lecture 30

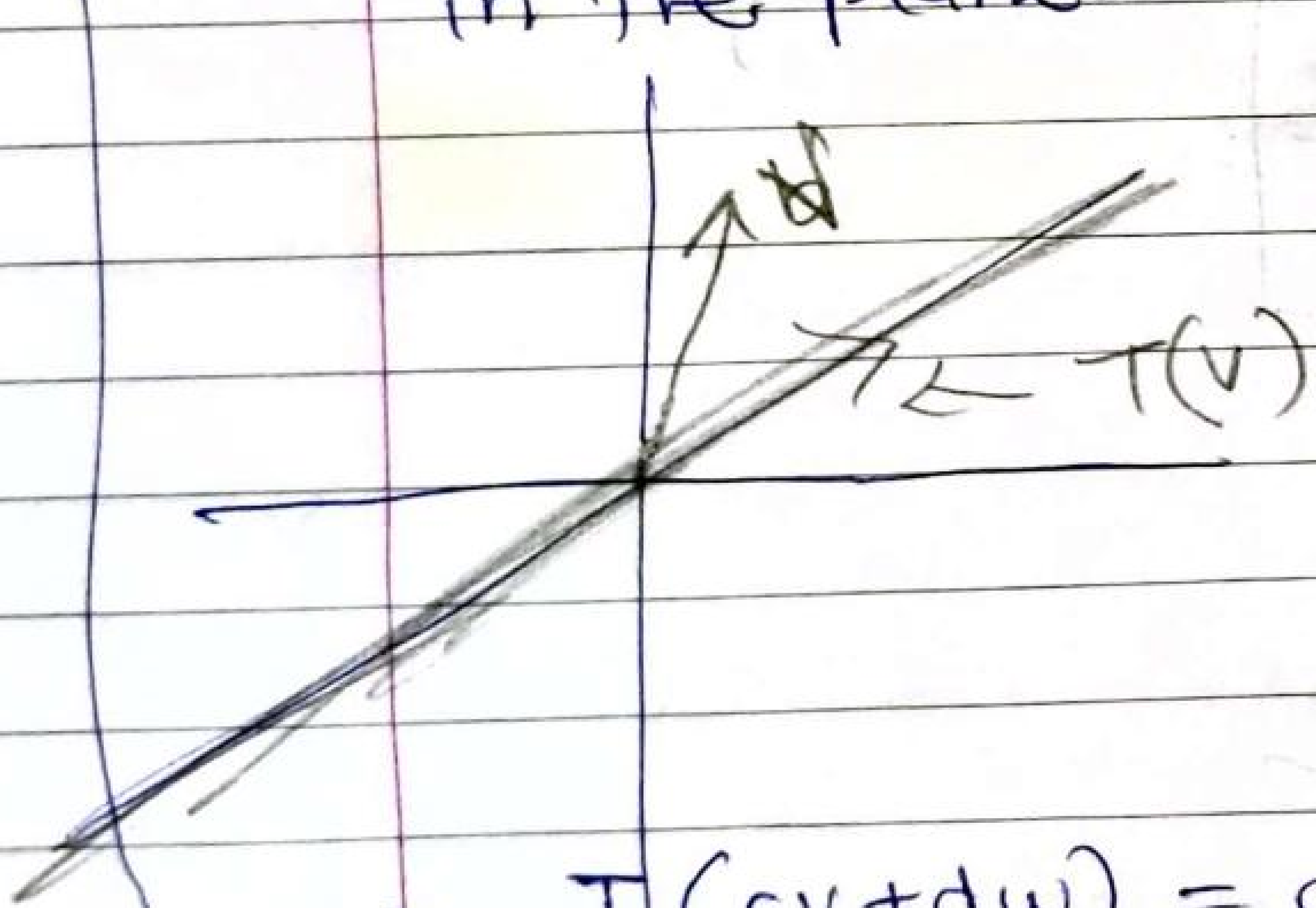
LINEAR TRANSFORMATIONS / (T)

$$\begin{cases} T(v+w) = T(v) + T(w) \\ T(cv) = cT(v) \end{cases}$$

example 1. Projection

$T : R^2 \longrightarrow R^2$

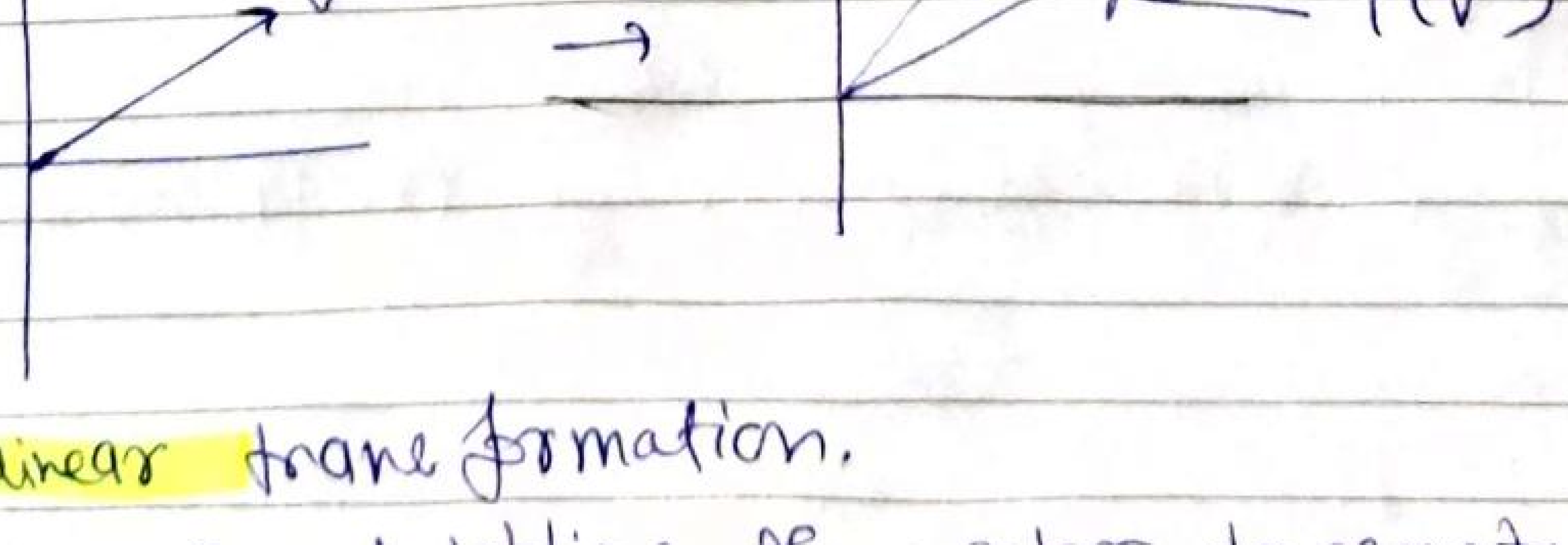
all of R^2, every vector in plane, into a vector in the plane.



(#) Projection is a linear transformation

$T(cv+dw) = cT(v) + dT(w)$

Example (2) Shift the whole plane by v_0



⇒ Not a linear transformation.

because doubling the vector doesnot double the resultant vector

In linear algebra; ~~any vector~~ zero vector must get transformed to zero.

$T(0) = 0$

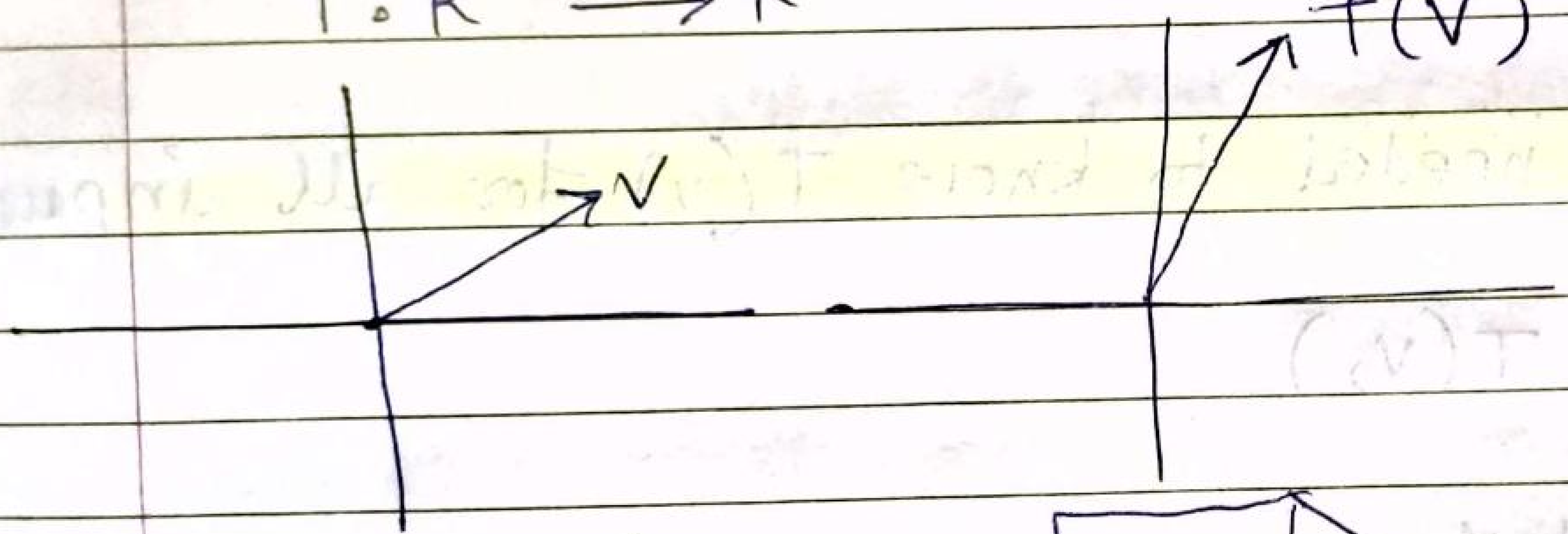
Not Linear transformation examples

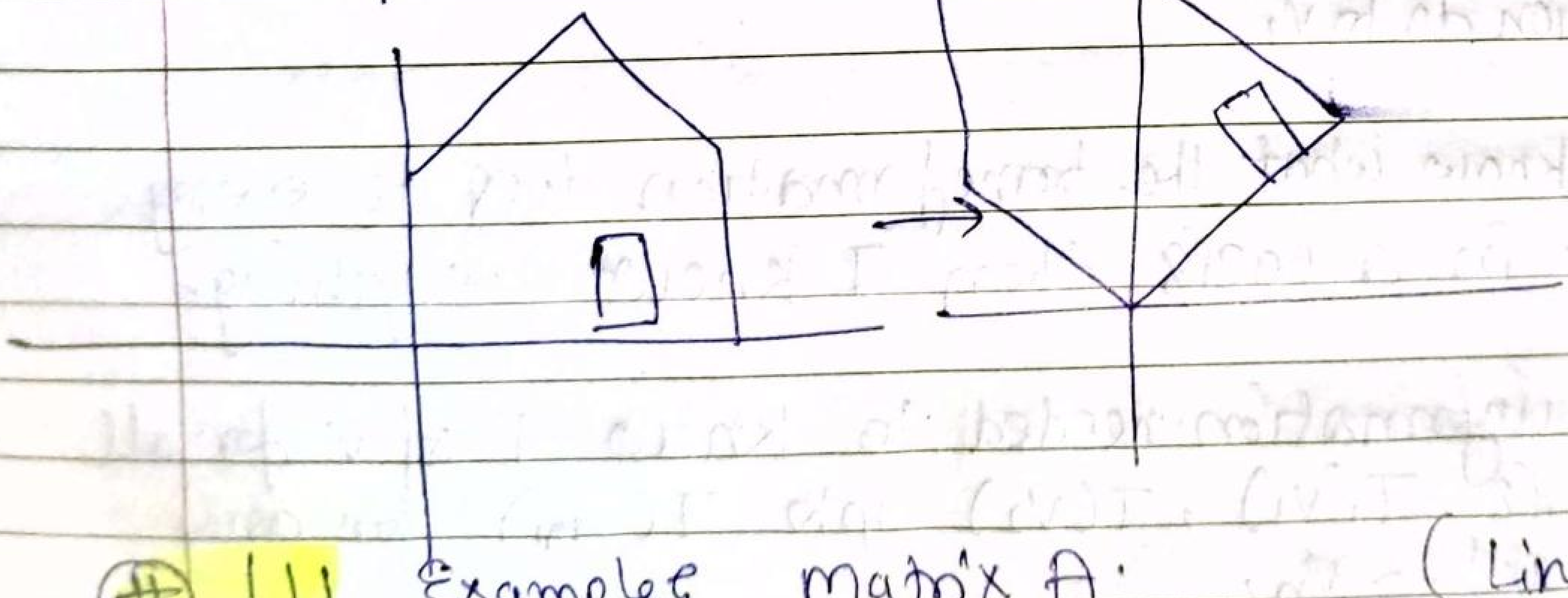
$T(V) = \|V\|$; $\boxed{T: R^3 \longrightarrow R^1}$ ✓

because $T(-V)$ is ~~no~~ not ~~a~~ minus V; ~~its length~~ it just length.

Examples Rotation by 45°

$T: R^2 \longrightarrow R^2$





!!! Examples matrix A. (Linear transformation)

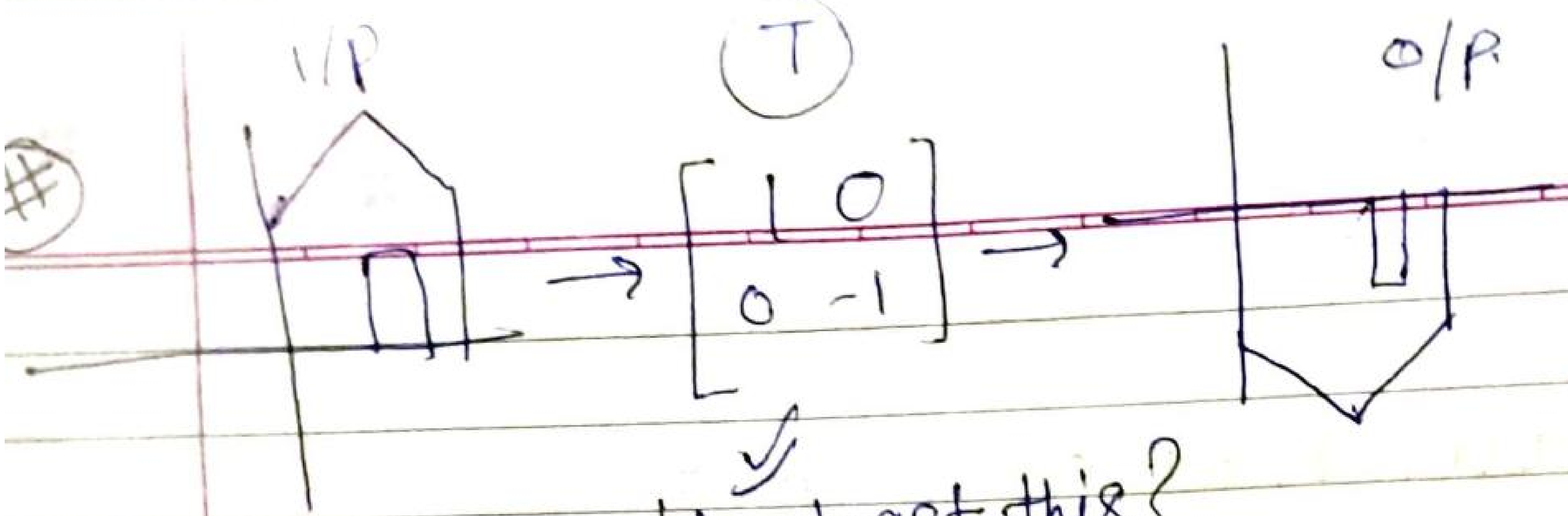
transformation will be $\boxed{T(V) = AV}$ ✓

<u>Check:</u>

$A(v+w) = Av + Aw$

$A(cv) = cAv$

whole plane is transformed by matrix multiplication.



How to get this?

Starts $R^3 \longrightarrow R^2$

3D space　　2D space.

example 6

$$\boxed{T(v) = Av}$$

~~op in R2~~ 2 by 3 matrix to get O/P

O/P in R^2　　I/P in R^3

Information needed to know $T(v)$ for all inputs.

$T(v_1)$, $T(v_2)$

↳ what's the transformation do to v_1

⇒ If I know what the transformation does to every vector in a basis then I know everything.

⇒ So the information needed to know T of v for all inputs is $T(v_1)$, $T(v_2)$ upto $T(v_n)$ for any input basis $v_1 \dots v_n$.

⇒ Because every v is some combination of these basis vectors, $c_1v_1 + \dots + c_nv_n$;

↳ then I know what T does to V

(By this linearity,

$$\boxed{T(v) = c_1 T(v_1) + \cdots + c_n T(v_n)}$$

Co-ordinates come from a basis.

Coordinates of $v = c_1 v_1 + \cdots + c_n v_n$

$$v = \begin{bmatrix} 3 \\ 2 \\ 4 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + 2\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + 4\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

standard basis

I might have chosen diff basis like eigen vector

Construct matrix A that represents linear transform T.

$$T : R^n \rightarrow R^m$$

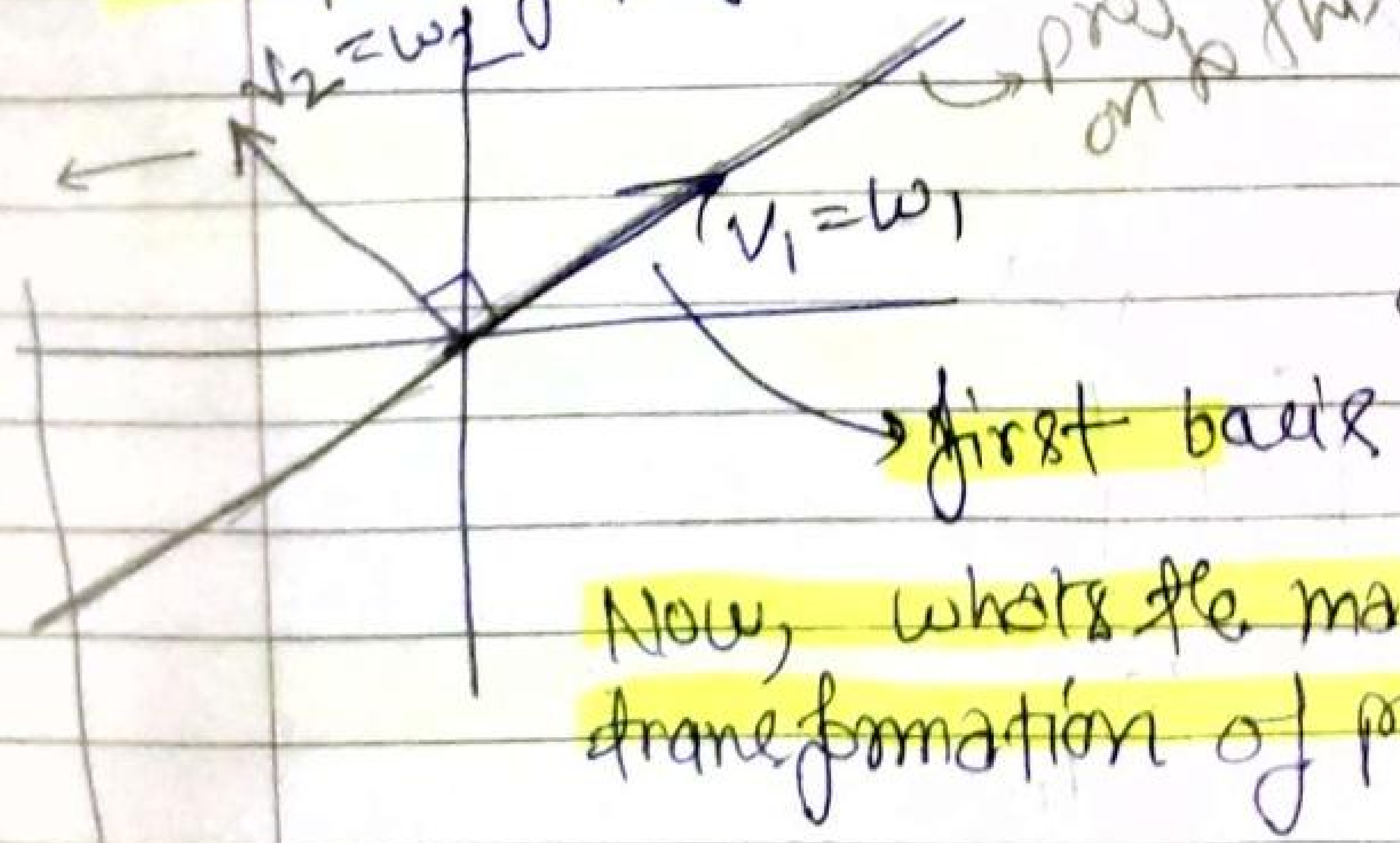
→ choose basis $v_1 \cdots v_n$ in i/p R^n;

→ choose basis $w_1 \cdots w_m$ in o/p R^m;

What I want:

matrix A

example of projection



So transformation takes every vector into plane & project on to this line.

→ first basis vector on the line

Now, what's the matrix? How do I describe this transformation of projection with respect to v_1 basis?

→ second basis vector ⊥r to line v_2), o/p basis & i/p basis are same

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Rules

Take any vector v its some combination of first basis vector & second basis vector

$$v = c_1 v_1 + c_2 v_2$$

Now, what is T of v?

$T(x) \neq$

suppose the i/p is v_1; what's the o/p? v_1.

the projection leaves v_1 alone & kills second basis to zero.

so projection do to combination.

$$\boxed{T(v) = c_1 v_1}$$

Now, I want to find matrix, that takes an input c1, c2 (the coordinates) & give me the o/ps c1, 0

i/p co-ordinates $(c_1, c_2) \rightarrow (c_1, 0)$
o/p coords

the matrix that will do that, is

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$
A

$$\underset{A}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}} \underset{\text{i/p coords}}{\begin{bmatrix} c_1 \\ c_2 \end{bmatrix}} = \underset{\text{o/p coords}}{\begin{bmatrix} c_1 \\ 0 \end{bmatrix}}$$

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⇒ In this example; i/p basis was same to o/p basis.

i/p basis & o/p basis were both along the line & ⊥r to the line

↳ They'r actually the eigenvectors of the projection

& As a result the matrix came out diagonal.

→ Infact, it came out to be lambda; Λ

↳ This is like good basis.

⇒ eigenvector basis. → leads to diagonal value. Λ

I can do whole thing in standard basis

So say we're projecting onto $45°$ line
& we use not the eigenvector basis, but the standard basis.

$$v_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix} = w_1, \quad v_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix} = w_2 \}$$

matrix ?

$$P = \frac{aa^T}{a^Ta} = \begin{bmatrix} 1/2 & 1/2 \\ 1/2 & 1/2 \end{bmatrix}$$ ← that's the matrix but not a diagonal matrix.

Rule to find matrix A: Given i/p bases $v_1 - v_n$, $w_1 - w_n$

• 1^{st} column of A : Apply $T(v_1) = a_{11}w_1 + a_{21}w_2 + \dots a_{m1}w_m$

• 2^{nd} col of A ⇒ $T(v_2) = a_{12}w_1 + \dots + a_{m2}w_m$.

A(i/p coordinate) = o/p coordinatio

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example i/p is $c_1 + c_2x + c_3x^2$ basis $1, x, x^2$

$T = \frac{d}{dx}$ o/p :- $c_2 + 2c_3x$ basis $1, x$

linear

$$A\begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} c_2 \\ 2c_3 \end{bmatrix}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$