一、
1. A 2. B 3. C 4. D 5. B 6. C
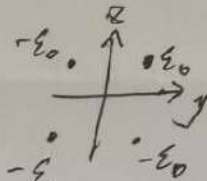7. D 8. A

二、

1. (1) $\sum_{l=0}^{\infty}\left(A_l r^l + \dfrac{B_l}{r^{l+1}}\right)P_l(\cos\theta)$

(2) 记住 $P_0(x)=1$, $P_1(x)=x$, $P_2(x)=\dfrac{1}{2}(3x^2-1)$ 即可

2. (1) $\nabla^2 G(\vec{r},\vec{r_0}) = -\delta(\vec{r}-\vec{r_0})$

$G(\vec{r},\vec{r_0})\big|_{y=0} = 0$

$G(\vec{r},\vec{r_0})\big|_{z=0} = 0$

(2) $\Rightarrow G = ?$

三、

1. $z^{2n} = a^{2n} e^{i(2k+1)\pi}$, 解得 $z_k = a e^{i\frac{2k+1}{2n}\pi}$, $k=0,1,\cdots,2n-1$,

其中 $k=0,1,\cdots,n-1$ 在上半平面

$\therefore I = 2\pi i \sum_{k=0}^{n-1} \mathrm{Res}\, f(z_k) = 2\pi i \sum_{k=0}^{n-1} \dfrac{z_k^{2n-2}}{2n\, z_k^{2n-1}} = 2\pi i \sum_{k=0}^{n-1} \dfrac{1}{2na} e^{-i\frac{2k+1}{2n}\pi}$

用等比数列求和公式

2. $\begin{cases} \dfrac{\partial u}{\partial t} - a^2 \dfrac{\partial^2 u}{\partial x^2} = 0 & (0<x<l, t>0) \\ \dfrac{\partial u}{\partial x}\big|_{x=0}=0, \ \dfrac{\partial u}{\partial x}\big|_{x=l}=q & (t\geq 0) \\ u\big|_{t=0} = \varphi(x) & (0\leq x\leq l) \end{cases}$

$u(x,t) = v(x,t) + \dfrac{q}{2l}x^2$ 先齐次化

后用本征函数展开法