EBU7240
Computer Vision

- Features: edges, corners -

Semester 1, 2021

Changjae Oh

Content

- Edges
- Corners

Content

- **Edges**
- Corners

Derivatives and Edges

- **Edges: intensity discontinuities**
  - Ramp edge
  - Step edge

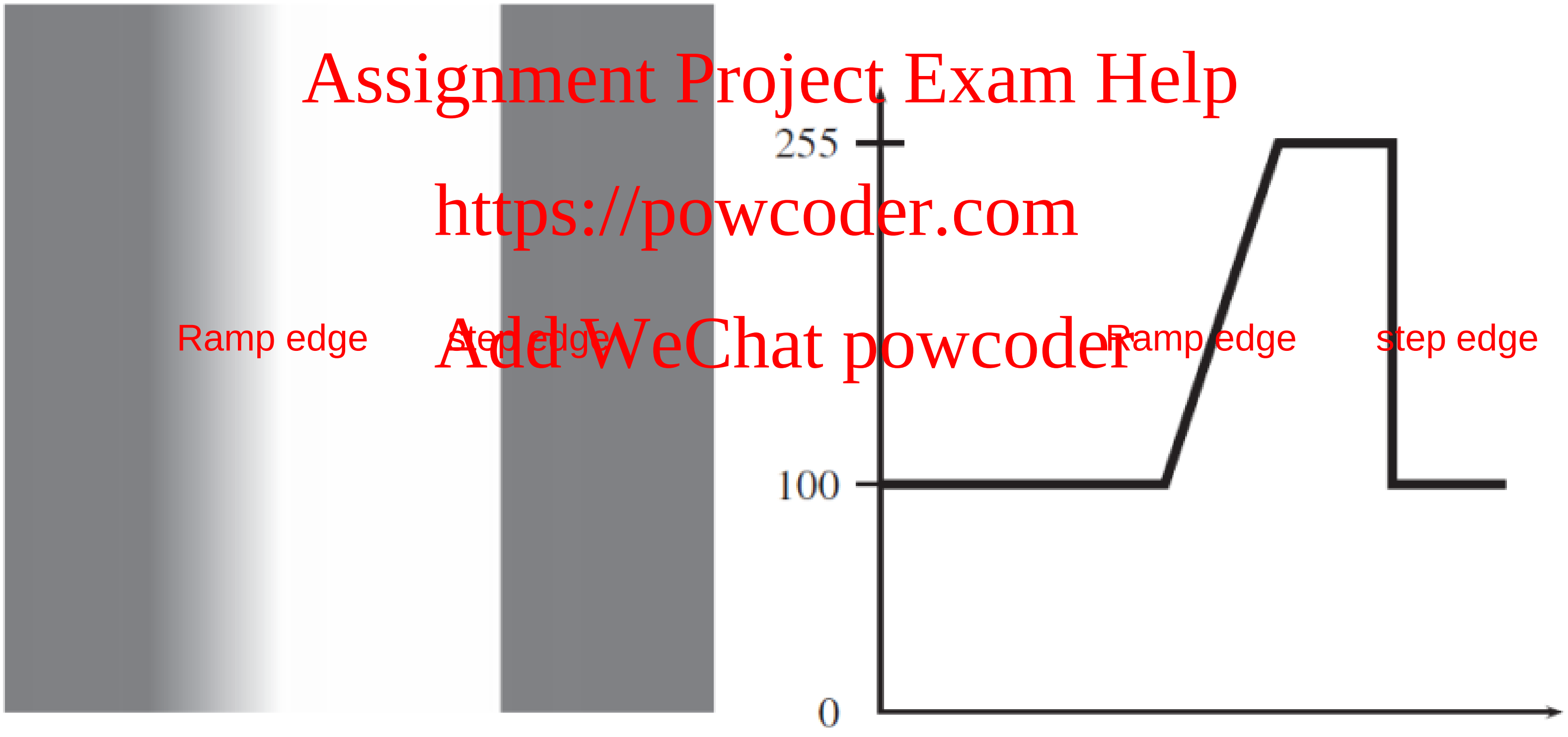


FIGURE 9.14 *Edges and their profiles.*

Derivatives and Edges

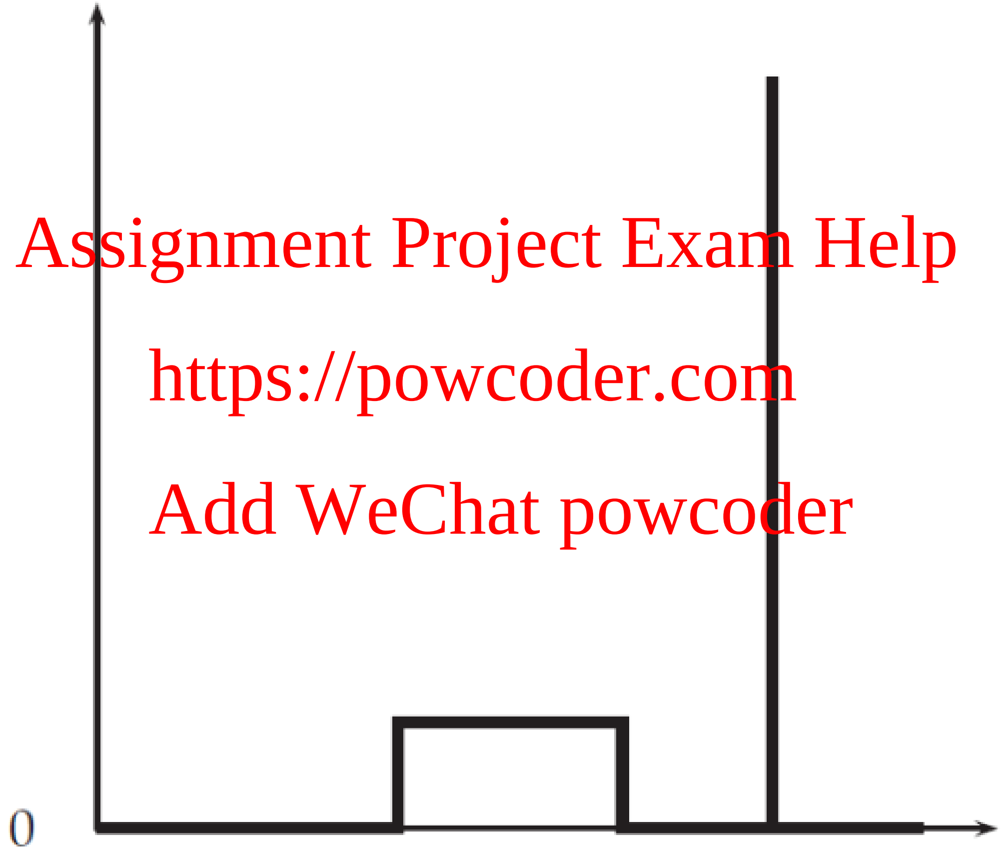
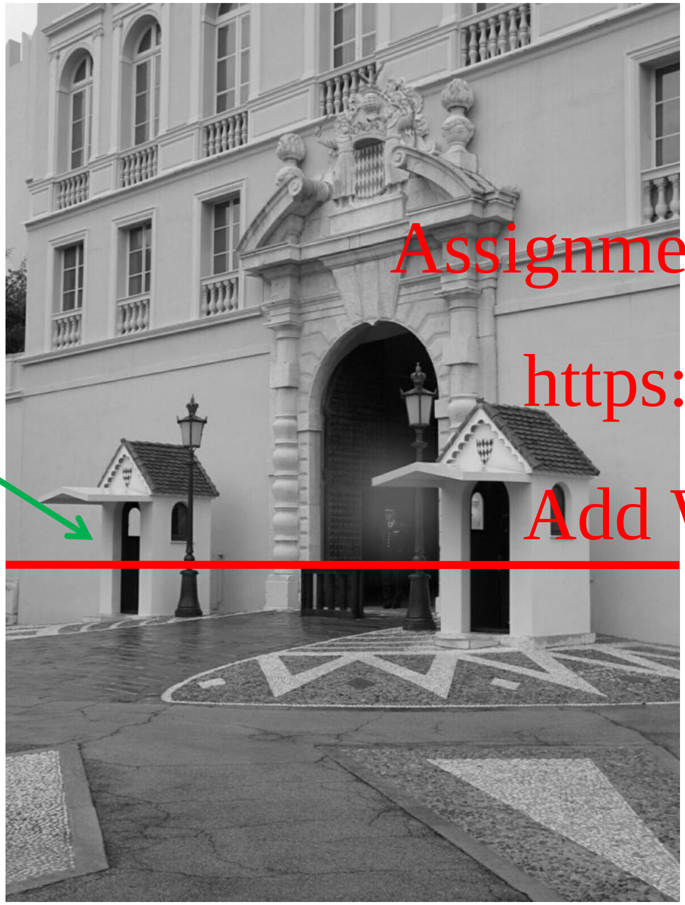
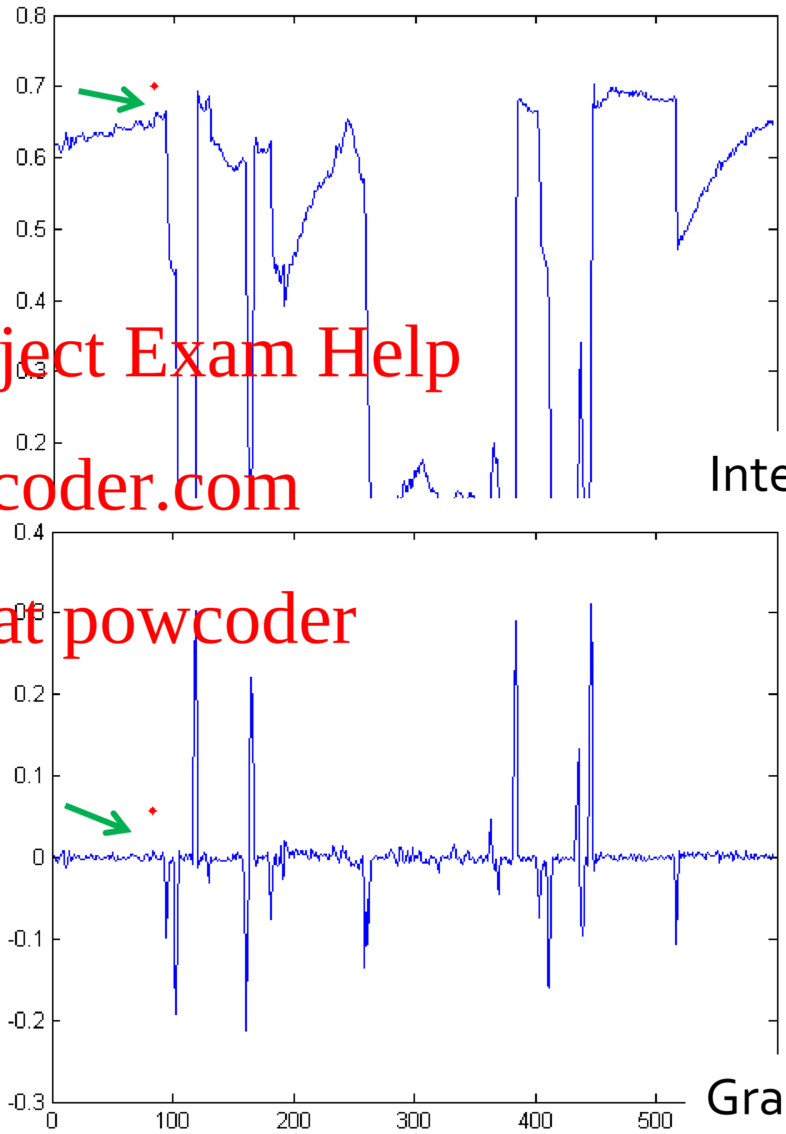


FIGURE 9.15 *The derivative of the edge profile.*

Effects of Noise in Edge Detection

Intensity profile



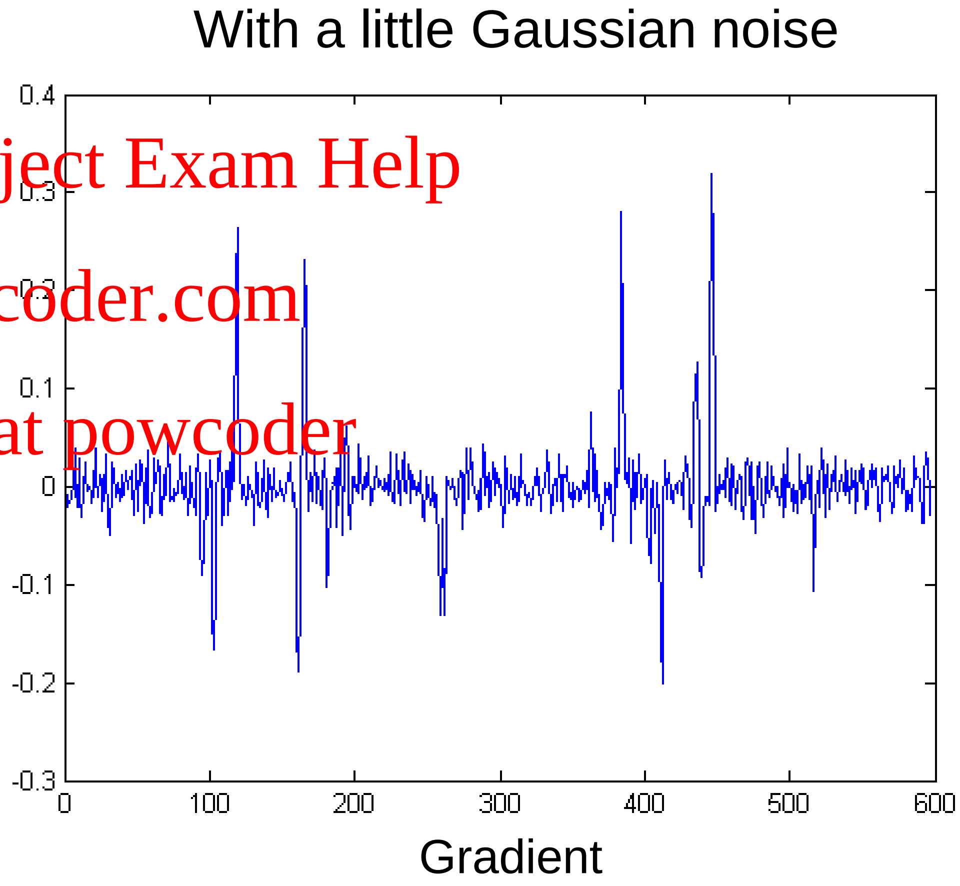


Intensity

Gradient

Source: D. Hoiem

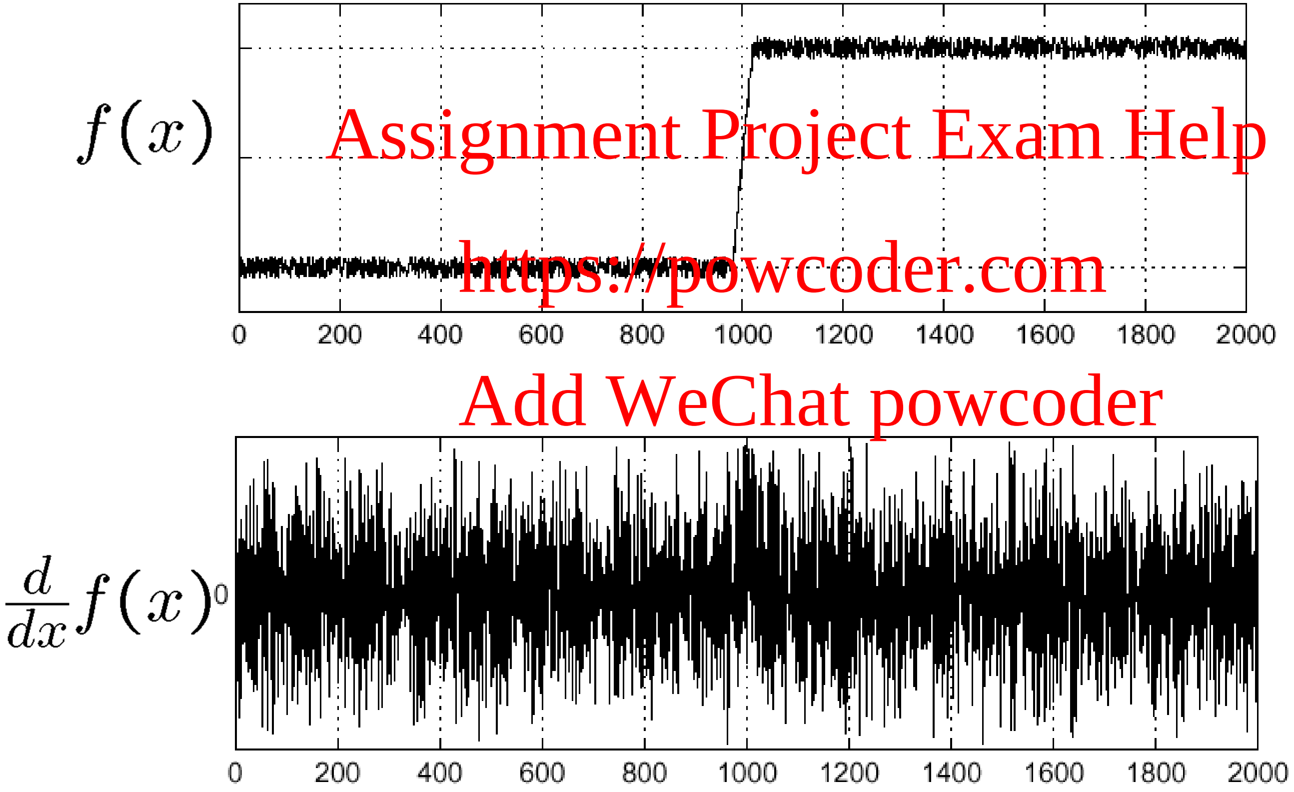
Effects of Noise in Edge Detection



Source: D. Hoiem

Effects of Noise in Edge Detection

- **Consider a single row or column of the image**
  - Plotting intensity as a function of position



Where is the edge?

Effects of Noise in Edge Detection

- **Difference filters respond strongly to noise**
  - Image noise results in pixels that look very different from their neighbors
  - Generally, the larger the noise the stronger the response

- **What can we do about it?**

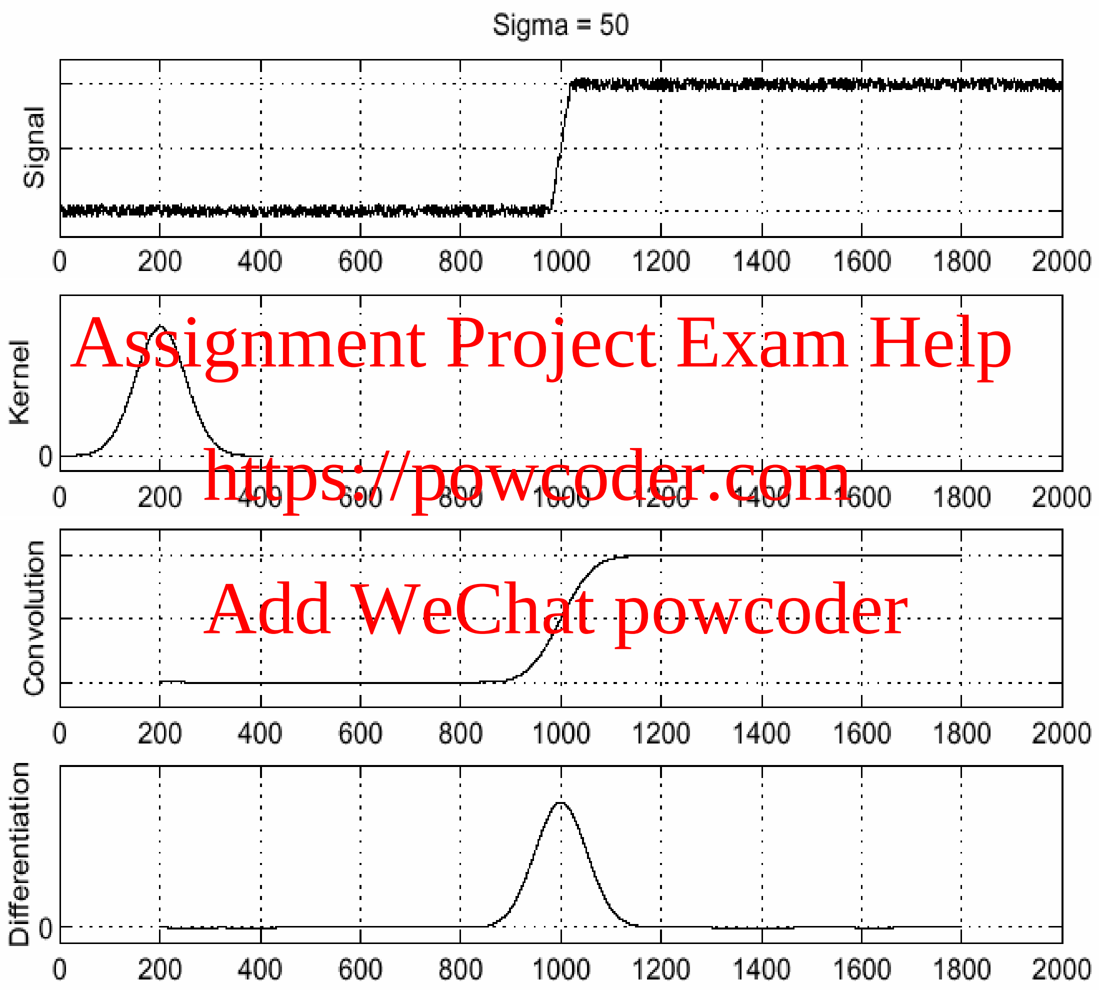
Solution: Smooth First

f

g

$f * g$

$\frac{d}{dx}(f * g)$



To find edges, look for peaks in $\frac{d}{dx}(f * g)$

Source: S. Seitz

Derivative Theorem of Convolution

- **Differentiation is convolution, and convolution is associative:**

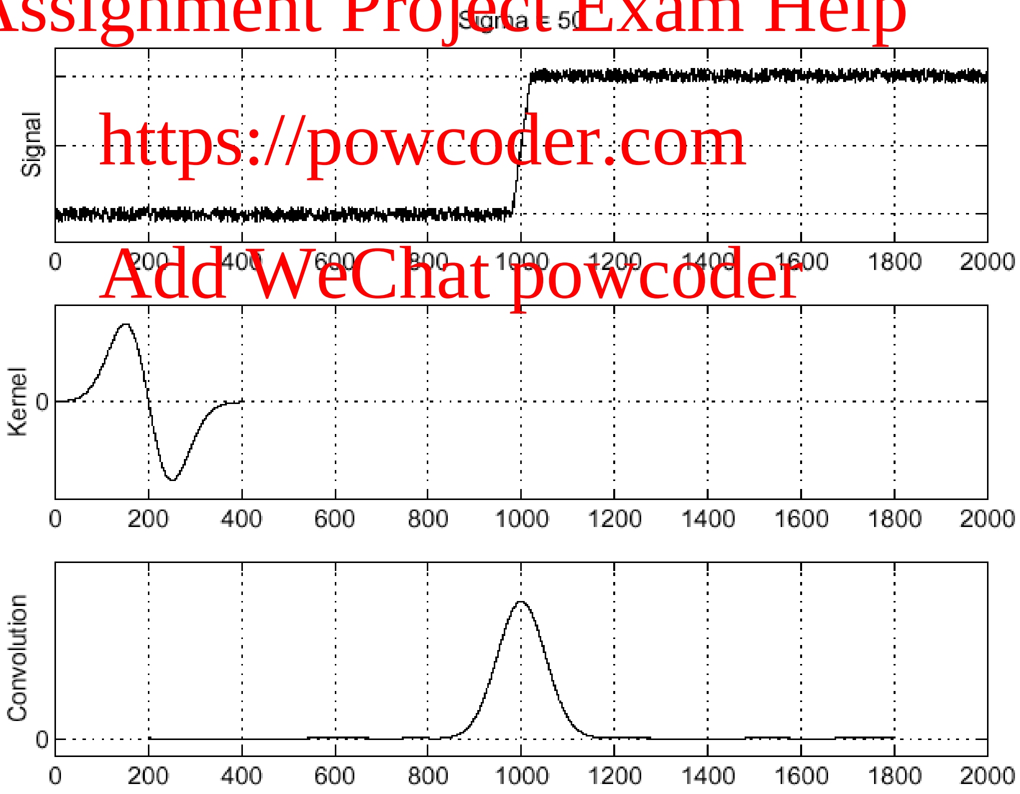
$$\frac{d}{dx}(f * g) = f * \frac{d}{dx} g$$

- **This saves us one operation:**

f

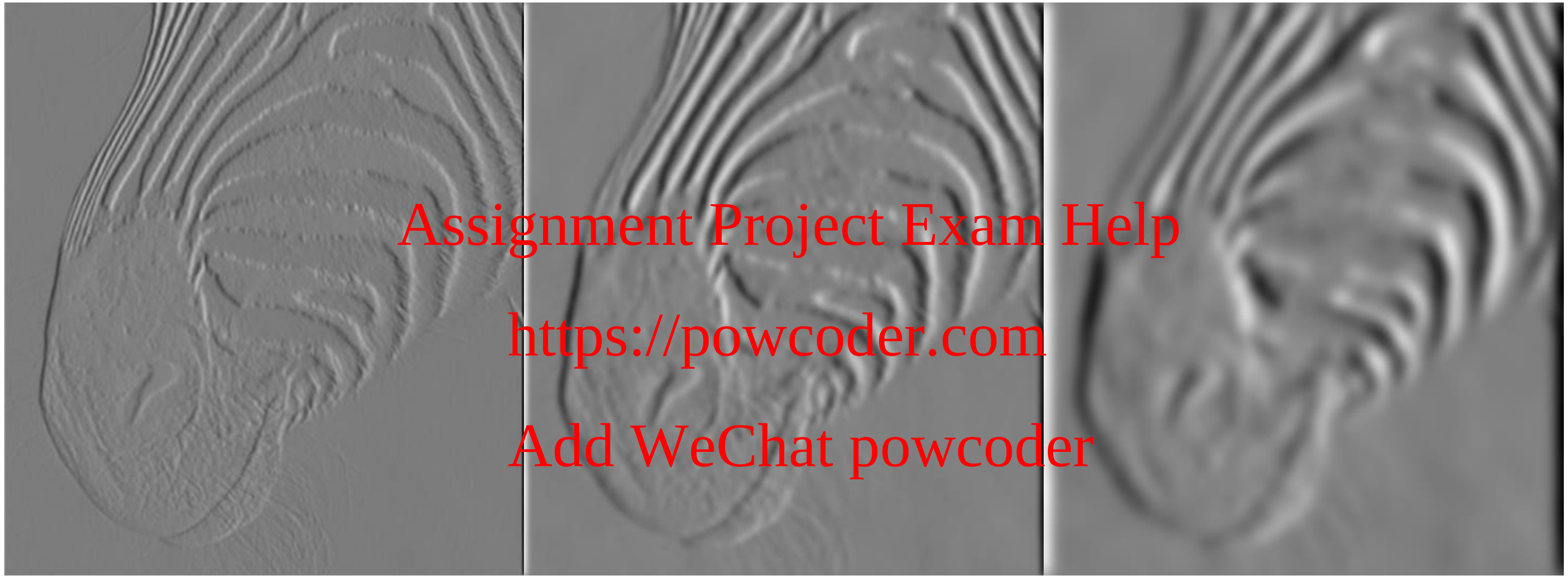
$\frac{d}{dx} g$

$f * \frac{d}{dx} g$



Source: S. Seitz

Trade-off Between Smoothing and Localization

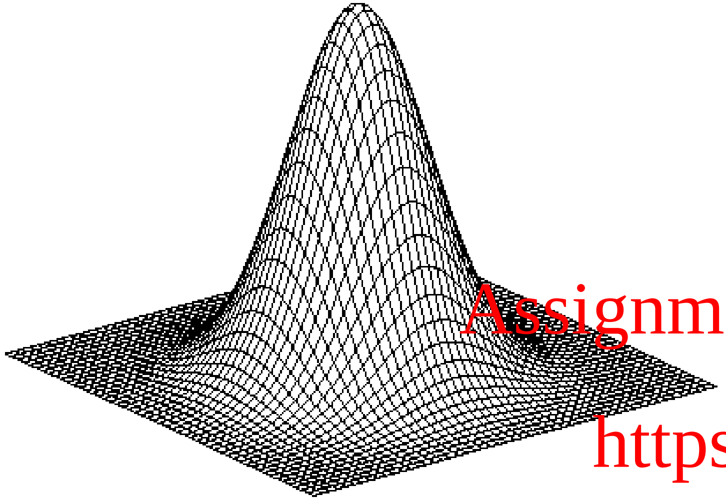


1 pixel | 3 pixels | 7 pixels

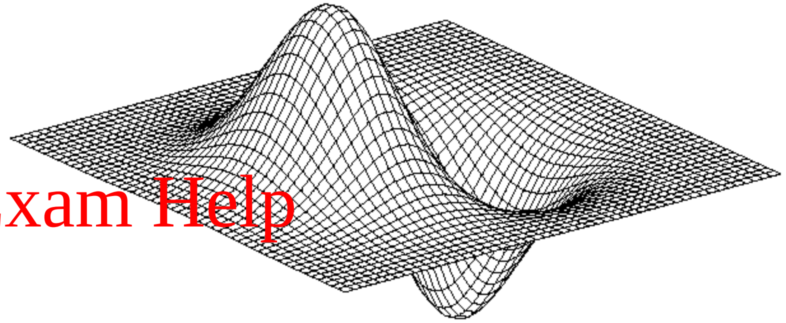
- **Smoothed derivative removes noise, but blurs edge.**

Source: D. Forsyth

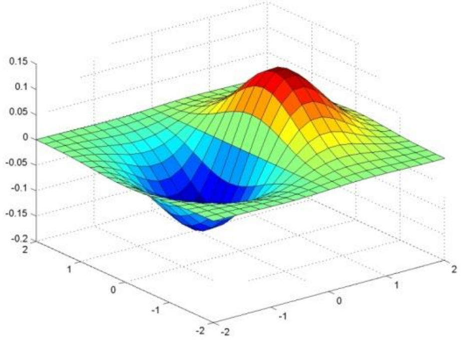
Edge Detection: Derivative of Gaussian Filter



* [1 -1] =

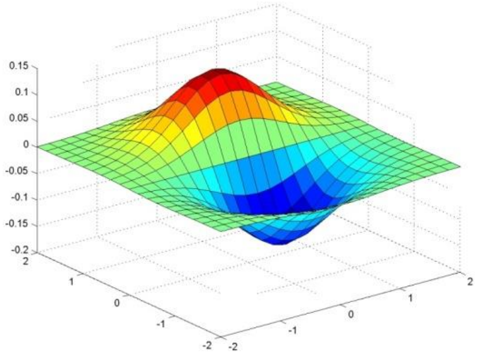


Note) Sobel operators approximate the derivative of Gaussian filter in a simple form.



≈

-1	0	1
-2	0	2
-1	0	1



≈

1	2	1
0	0	0
-1	-2	-1

Edge Detection: Sobel Filter

- **Sobel filter**

$$\nabla I = (I_x, I_y) = (S_x * I, S_y * I)$$

Sobel filter output $M(x,y) = \sqrt{{I_x}^2 + {I_y}^2}$ or $|I_x| + |I_y|$

For color image,
$M(x,y) = \big(M_R(x,y) + M_G(x,y) + M_B(x,y)\big)/3$

S_x

-1	0	1
-2	0	2
-1	0	1

S_y

-1	-2	-1
0	0	0
1	2	1

Gaussian filtering is often applied as pre-processing.

1) Apply Gaussian filter
2) Apply Sobel filter

Edge Detection: Laplacian Filter

- **Laplacian filter**

0	1	0
1	-4	1
0	1	0

or

1	1	1
1	-8	1
1	1	1

More general form

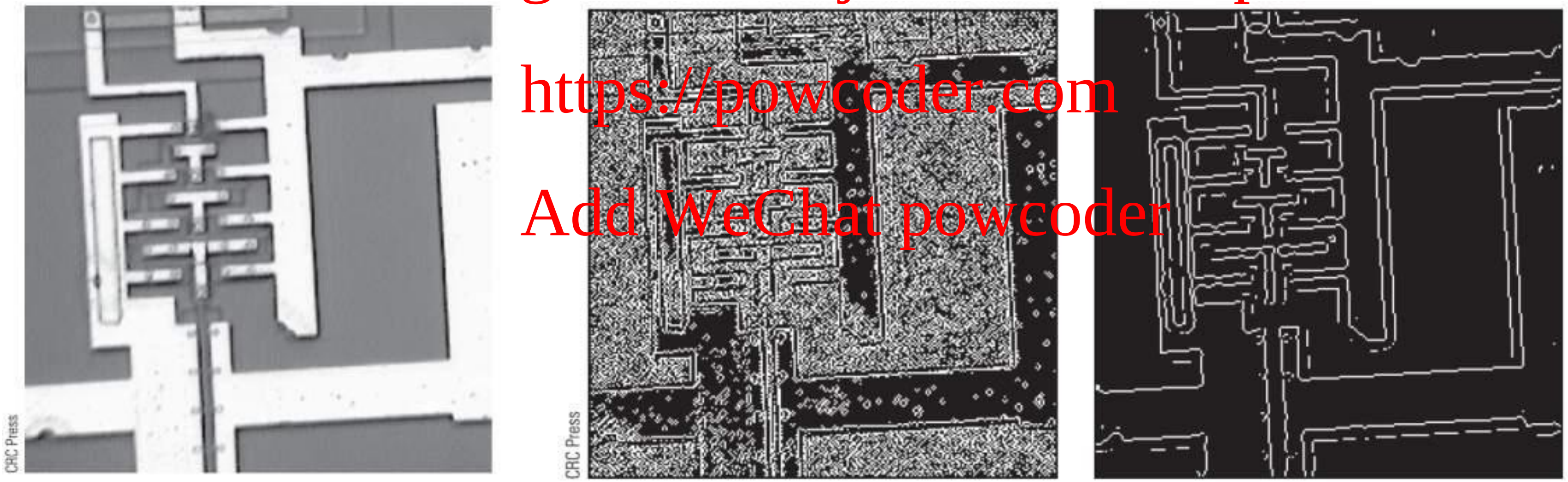
$$\begin{bmatrix} \frac{\alpha}{1+\alpha} & \frac{1-\alpha}{1+\alpha} & \frac{\alpha}{1+\alpha} \\ \frac{1-\alpha}{1+\alpha} & \frac{-4}{1+\alpha} & \frac{1-\alpha}{1+\alpha} \\ \frac{\alpha}{1+\alpha} & \frac{1-\alpha}{1+\alpha} & \frac{\alpha}{1+\alpha} \end{bmatrix}$$

$$O = |L * I|$$

For color image,
$O(x,y) = \left(O_R(x,y) + O_G(x,y) + O_B(x,y)\right)/3$

Edge Detection: Laplacian of Gaussian (LoG)

- **Marr-Hildreth Method (Laplacian of Gaussian: LoG)**
  1. Apply the Gaussian filter for removing unnecessary noise.
  2. Apply the Laplacian filter



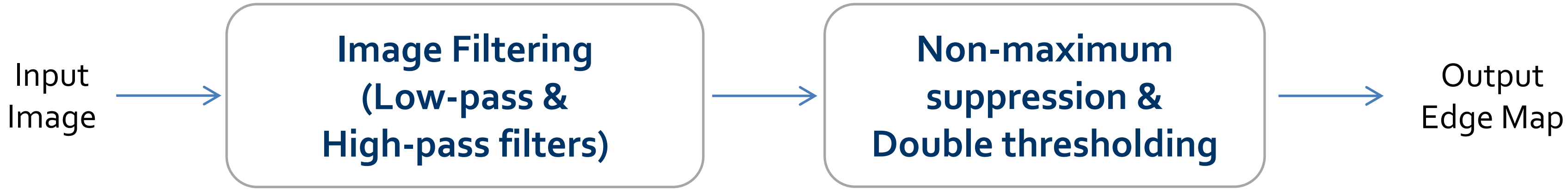
(a) Without Gaussian smoothing

(b) With Gaussian smoothing

Canny Edge Detector

- **The most popular method for edge detection**

- **Three criteria for edge detection**
  - Low error rate of detection: finding all edges
  - Localization of edges: computing precise locations of edges
  - Single response: returning a single pixel for a single edge



Canny Edge Detector

Step 1) Image Gradient using Low-pass & High-pass filters

1. Apply low-pass and high-pass filters
   ex)
   - Gaussian filter → Sobel filter ('default implementation in OpenCV')
   - 1D derivative of Gaussian filter
   - Difference of Gaussian filter

S_x

-1	0	1
-2	0	2
-1	0	1

S_y

-1	-2	-1
0	0	0
1	2	1

$$\nabla G = (G_x, G_y) = \left(\frac{\partial G}{\partial x}, \frac{\partial G}{\partial y}\right)$$

2. Compute the magnitude and angle of edge

$$M(x,y) = \sqrt{{G_x}^2 + {G_y}^2} \qquad A(x,y) = tan^{-1}\left(\frac{G_y}{G_x}\right)$$

Canny Edge Detector

Step 2) Non-maximum suppression

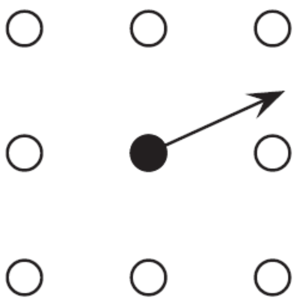
- Key idea: Survive only pixels with **a larger edge magnitude** $M(x,y)$ within a small window

$$M(x,y) = \sqrt{{G_x}^2 + {G_y}^2} \qquad A(x,y) = tan^{-1}\left(\frac{G_y}{G_x}\right)$$

- **Procedure**

1. Within a small window (e.g. 3×3 window) centered at (x,y), find neighbor pixels in direction $A(x,y)$

2. Compare the edge magnitudes $M(x,y)$ of these two neighbor pixels

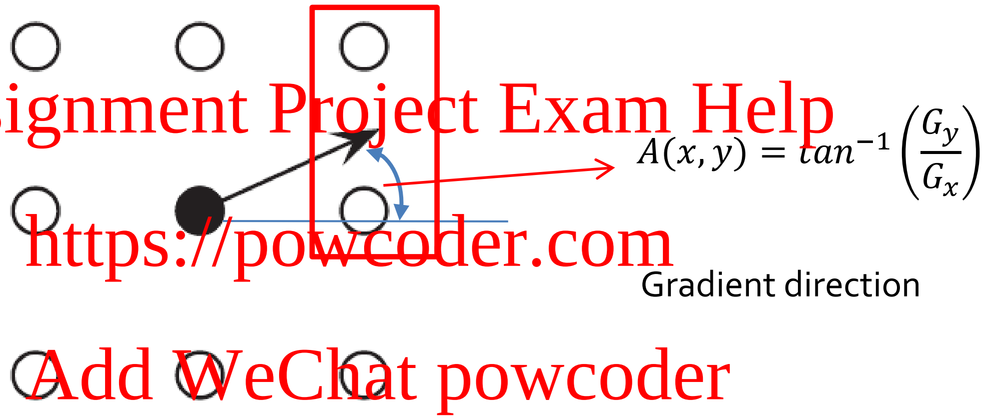


The edge direction at a pixel

FIGURE 9.26 *Nonmaximum suppression in the Canny edge detector.*

Canny Edge Detector

- **But, the image is a discrete signal.**
  - So, quantize the gradient direction



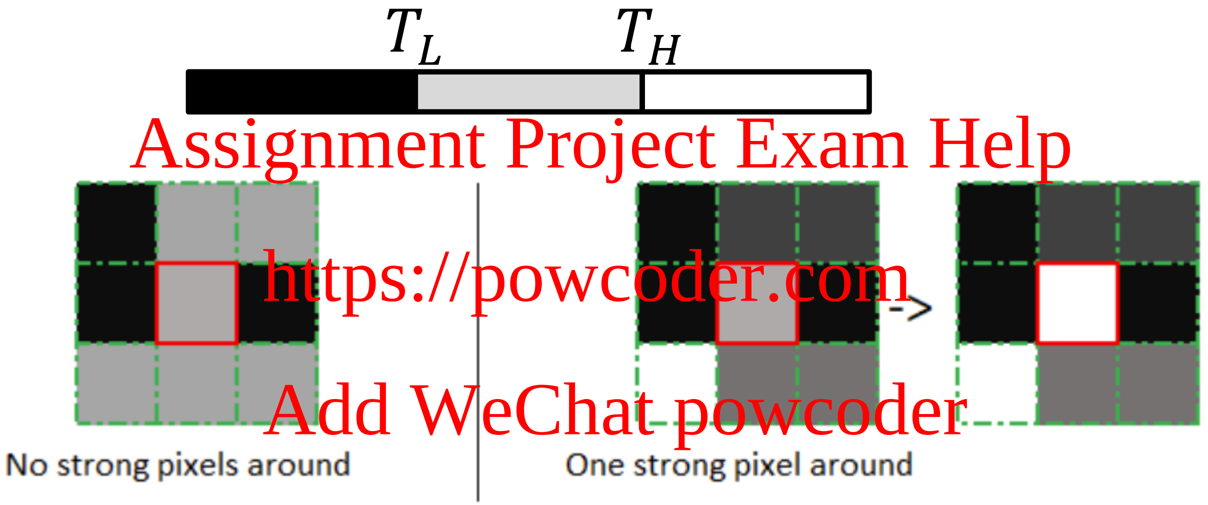
The edge direction at a pixel

FIGURE 9.26 *Nonmaximum suppression in the Canny edge detector.*

Canny Edge Detector

Step 3) Double thresholding

- Hysteresis thresholding (T_L, T_H)

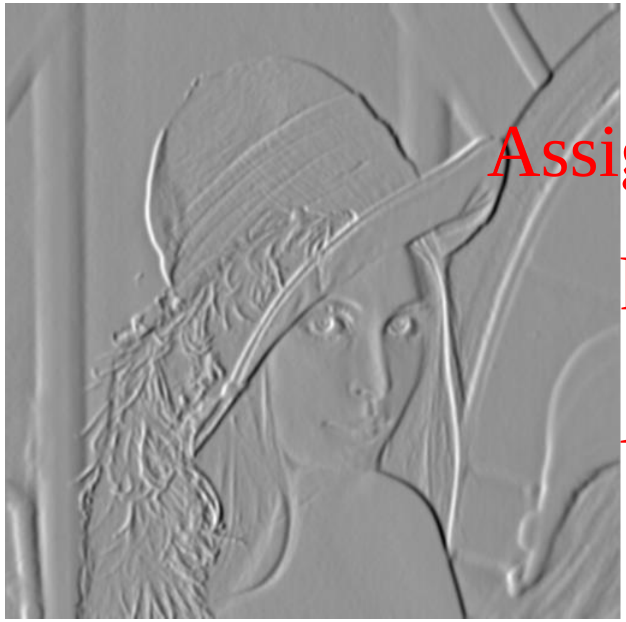


- If $M(x,y) > T_H$, then (x,y) is an edge
- If $M(x,y) < T_L$, then (x,y) is **NOT** an edge
- If $T_L \leq M(x,y) \leq T_H$,
  - If the **neighboring** pixels of (x,y) is an edge, then (x,y) is an edge.
  - Otherwise, then (x,y) is **NOT** an edge.

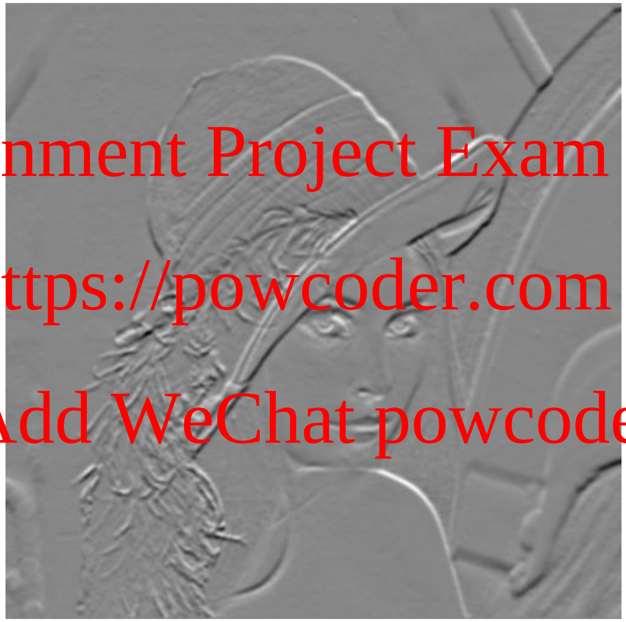
Canny Edge Detector Example

original image (Lena)

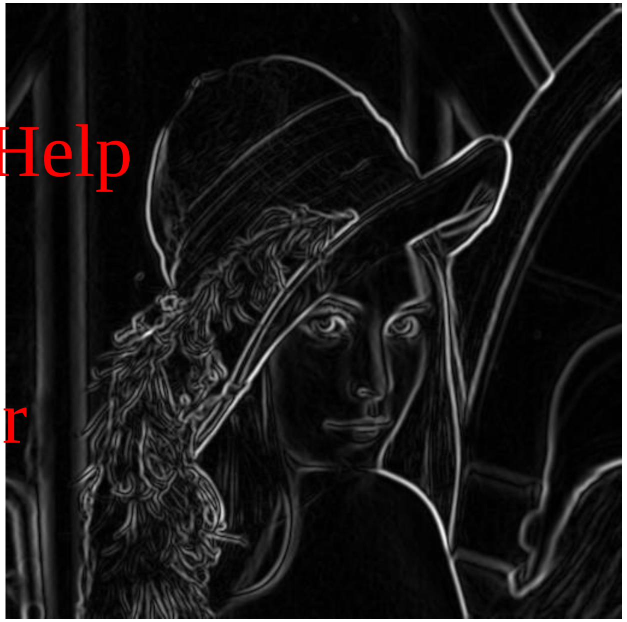
Compute Gradients



X-Derivative of Gaussian

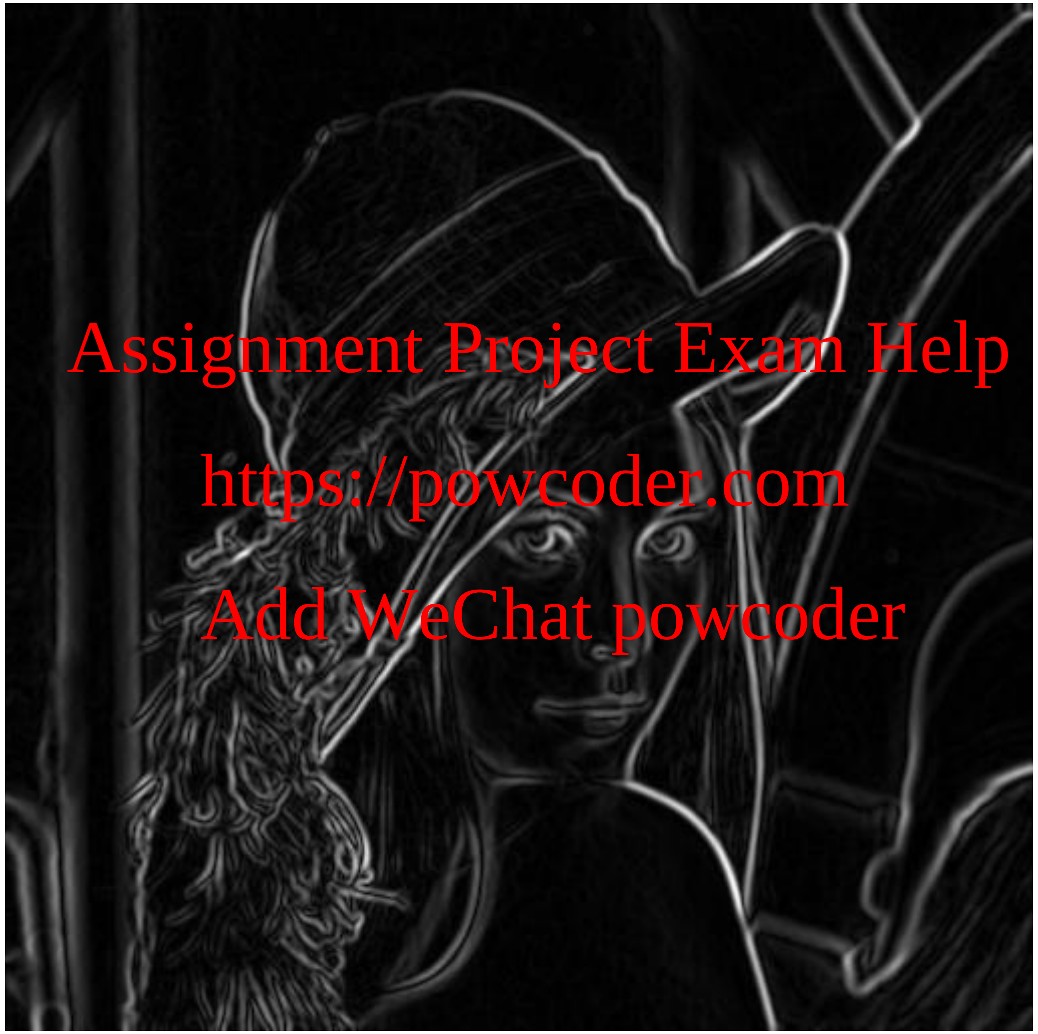


Y-Derivative of Gaussian

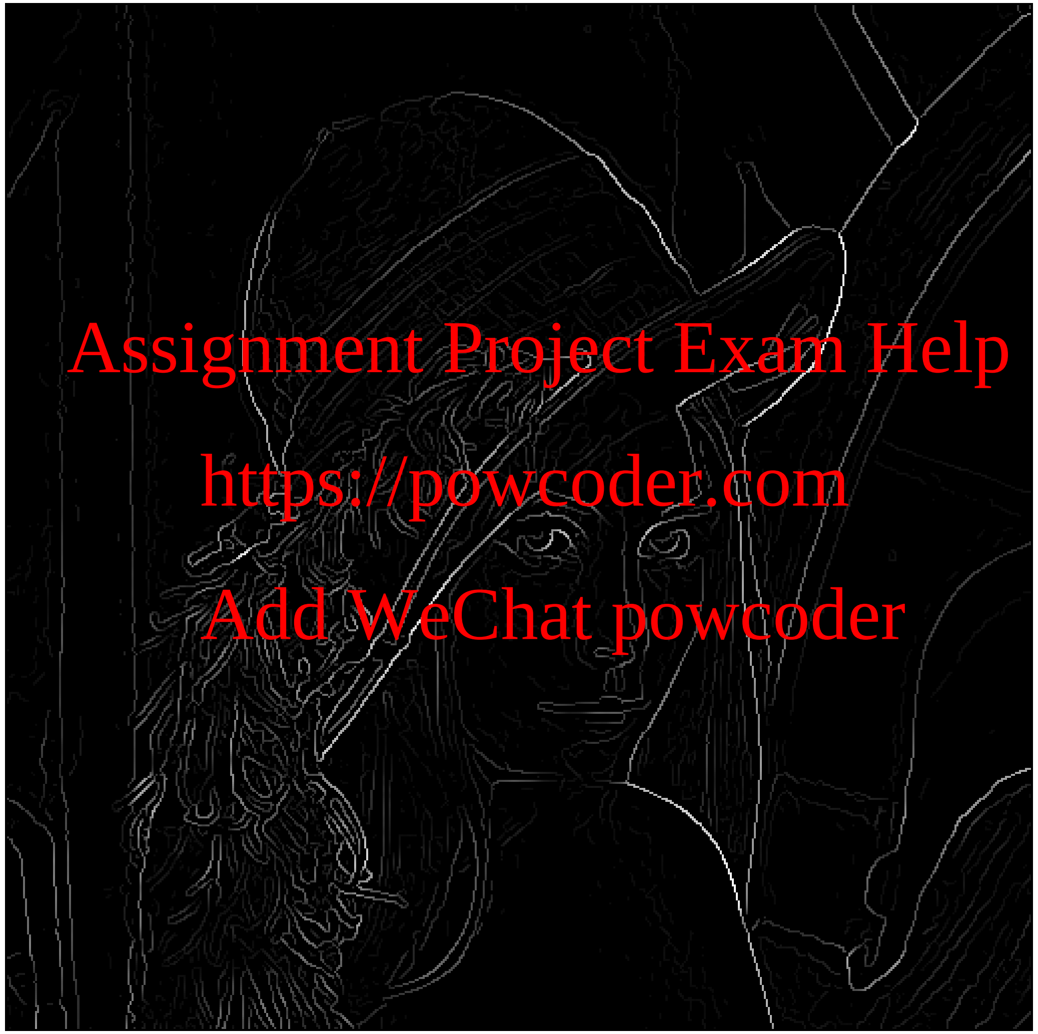


Gradient Magnitude

Before Non-max Suppression

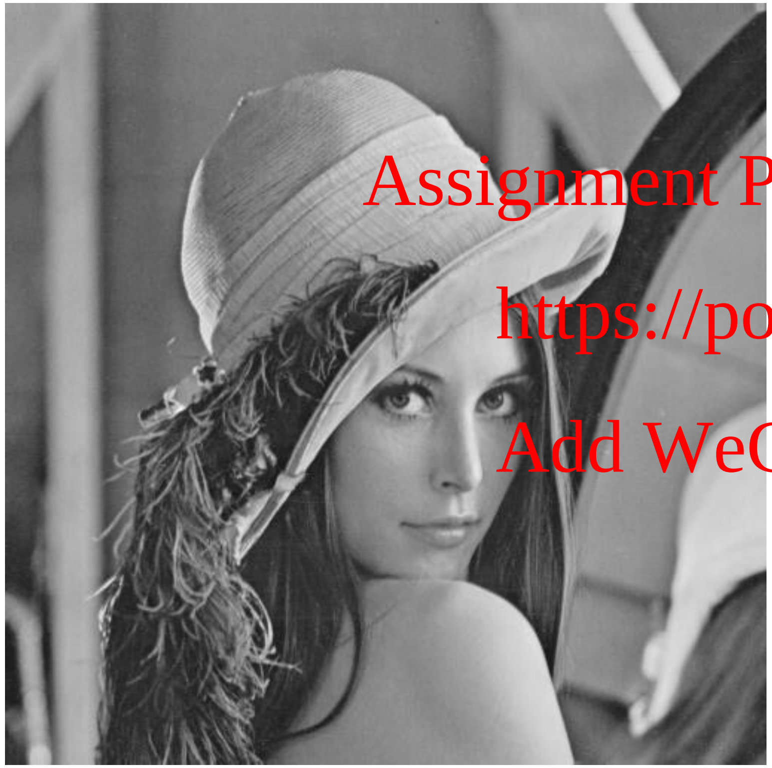


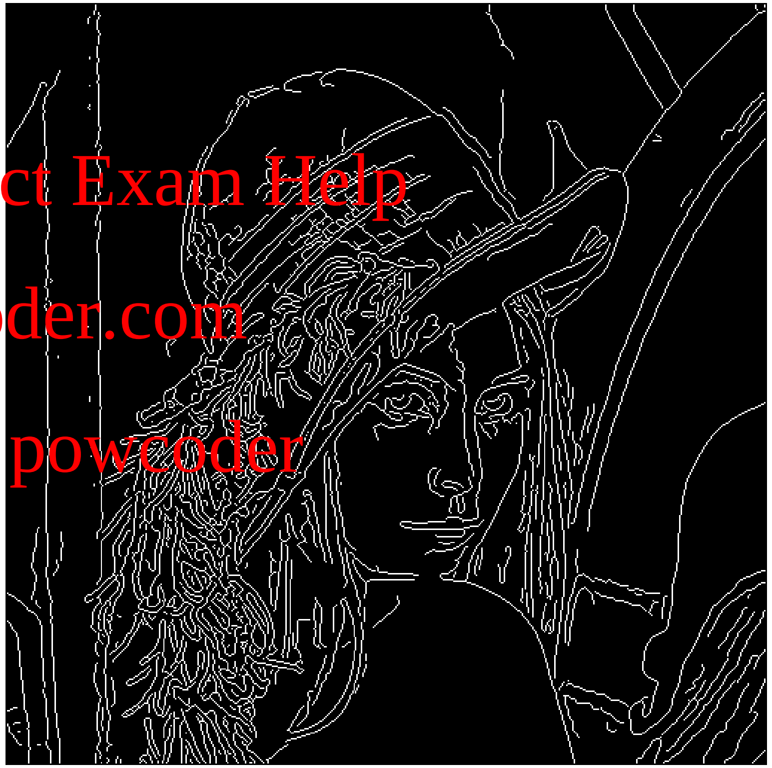
After Non-max Suppression



Final Canny Edges

- **After applying Hysteresis thresholding (Double thresholding)**

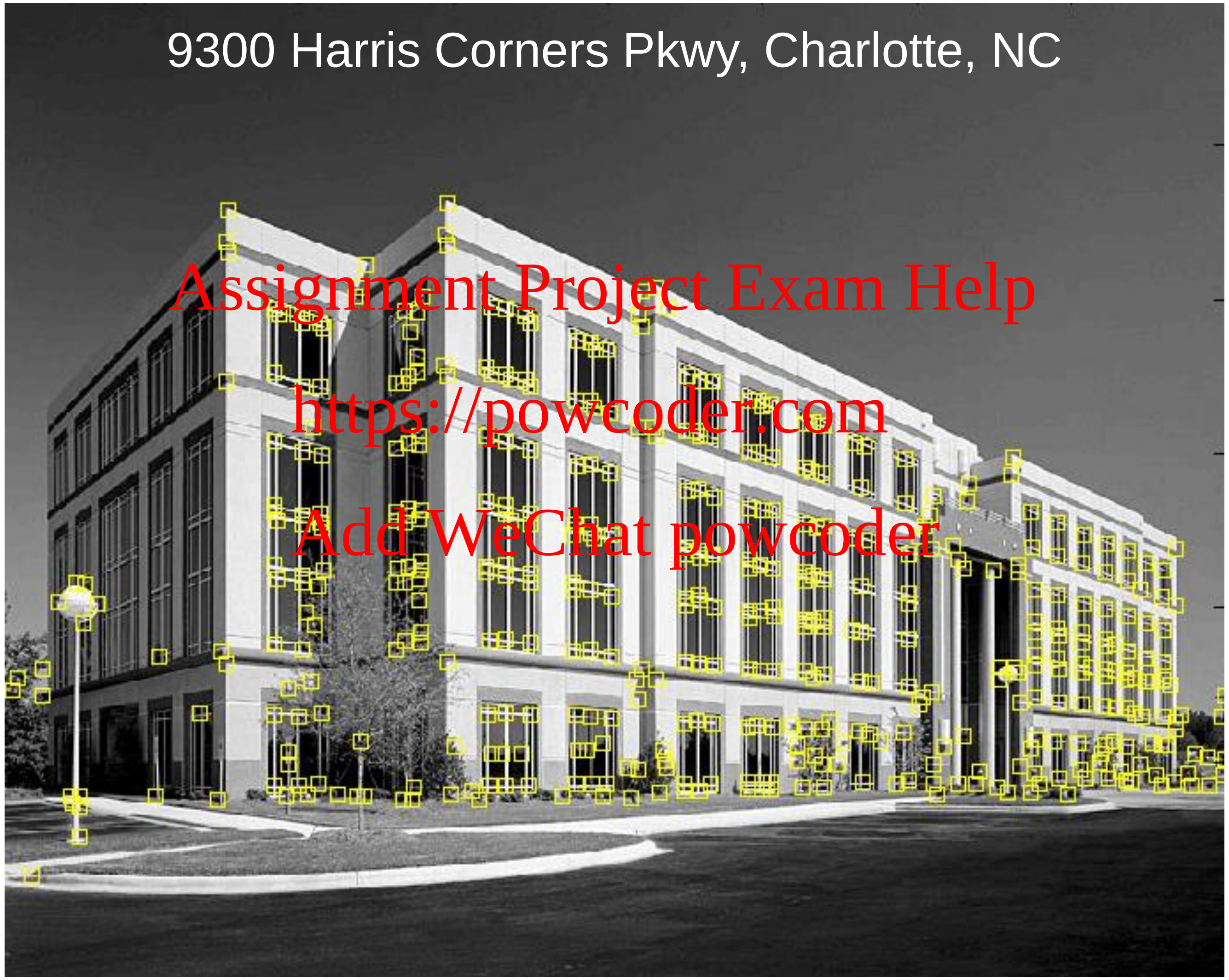




Content

- Edges
- Corners

Interest points: Corners



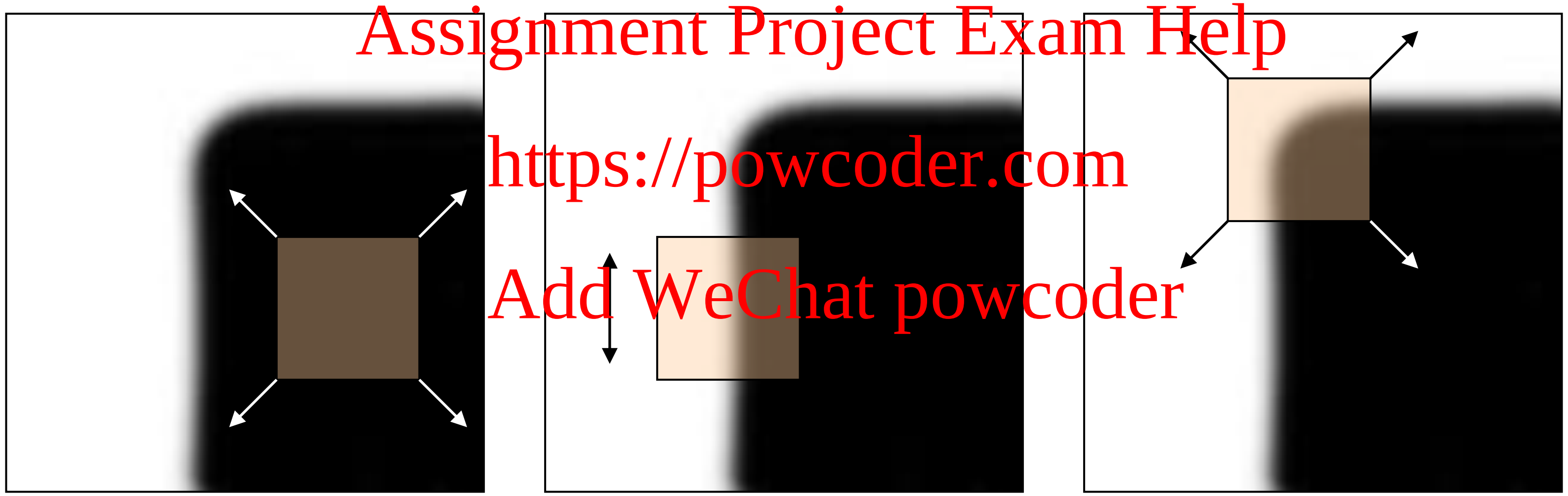
Slides from Rick Szeliski, Svetlana Lazebnik, and Kristin Grauman

Corners: Applications

- **Corners are used for:**
  - Image alignment/3D reconstruction
    - Find correspondences across different views
  - Motion tracking/robot navigation
    - Which points are good to track?
  - Object recognition/Indexing and database retrieval
    - Find patches likely to tell us something about object category

Corner Detection: Basic Idea

- We should easily recognize the point by looking through a small window
- Shifting a window in *any direction* should give *a large change* in intensity



"flat" region:
no change in all di
rections

"edge":
no change along the
edge direction

"corner":
significant change in
all directions

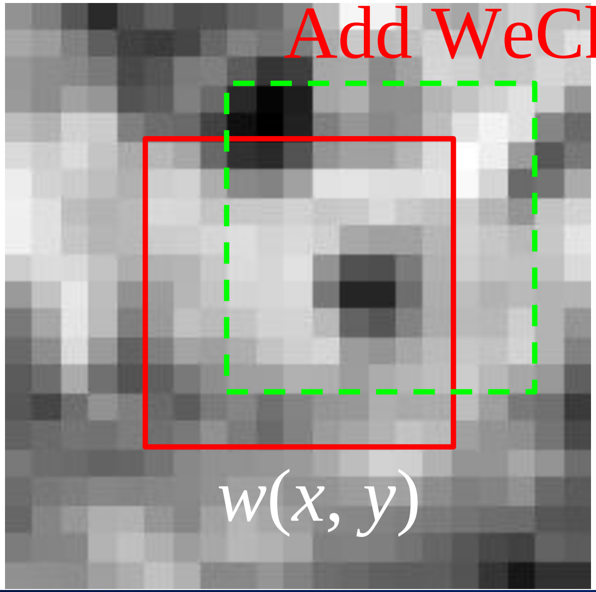
Source: A. Efros

Corner Detection: Mathematics

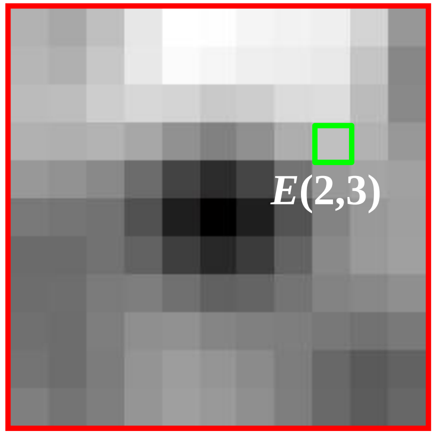
Change in appearance of window $w(x,y)$
for the shift $[u,v]$:

$$E(u,v)=\sum_{x,y} w(x,y)\left[I(x+u,y+v)-I(x,y)\right]^2$$

$I(x, y)$



$E(u, v)$



Corner Detection: Mathematics

The quadratic approximation simplifies to

$$E(u,v) \approx [u \;\; v] \; M \begin{bmatrix} u \\ v \end{bmatrix}$$

where M is a *second moment matrix* computed from image derivatives:

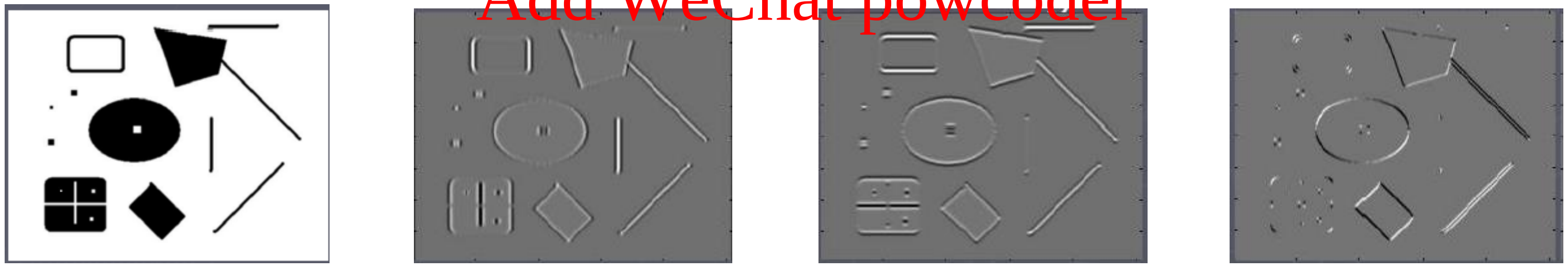
$$M = \sum_{x,y} w(x,y) \begin{bmatrix} I_x^2 & I_x I_y \\ I_x I_y & I_y^2 \end{bmatrix}$$

$$M = \begin{bmatrix} \sum I_x I_x & \sum I_x I_y \\ \sum I_x I_y & \sum I_y I_y \end{bmatrix} = \sum \begin{bmatrix} I_x \\ I_y \end{bmatrix} [I_x \; I_y] = \sum \nabla I (\nabla I)^T$$

Corners as Distinctive Interest Points

$$M = \sum w(x, y) \begin{bmatrix} I_x I_x & I_x I_y \\ I_x I_y & I_y I_y \end{bmatrix}$$

2 x 2 matrix of image derivatives
(averaged in neighborhood of a point)

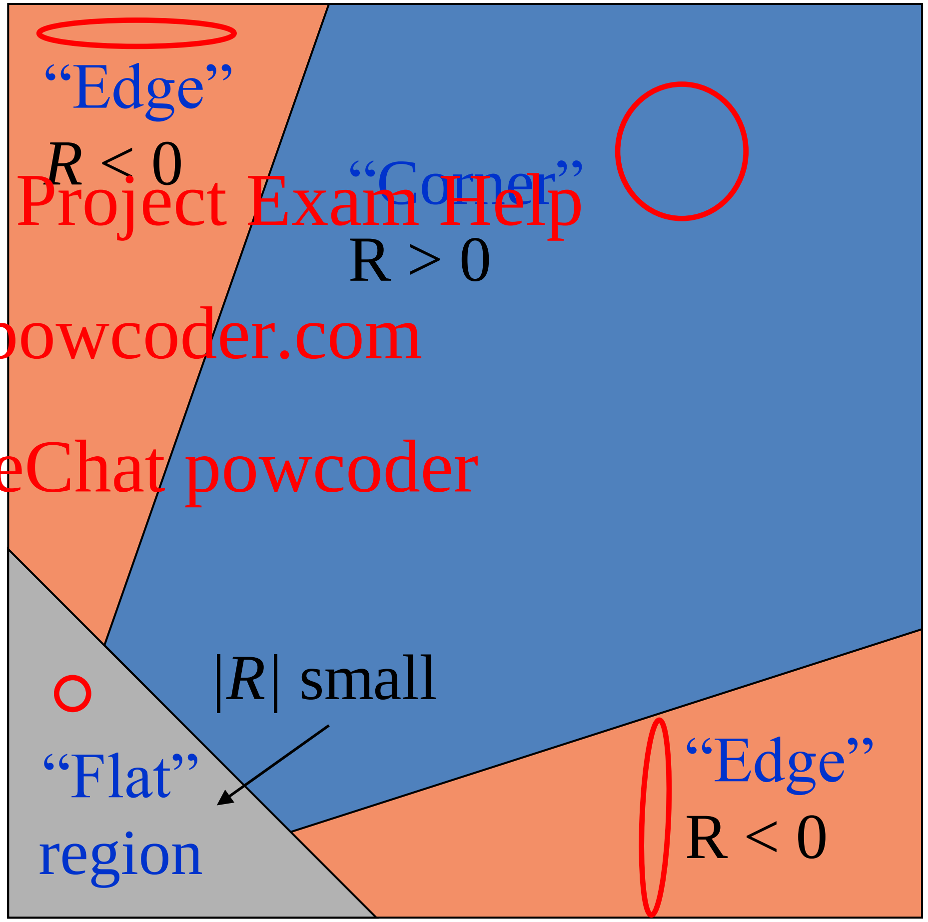


Notation: $I_x \Leftrightarrow \frac{\partial I}{\partial x}$ $\quad I_y \Leftrightarrow \frac{\partial I}{\partial y}$ $\quad I_x I_y \Leftrightarrow \frac{\partial I}{\partial x}\frac{\partial I}{\partial y}$

Corner Response Function

$$R = \det(M) - \alpha \operatorname{trace}(M)^2 = \lambda_1 \lambda_2 - \alpha(\lambda_1 + \lambda_2)^2$$

α: constant (0.04 to 0.06)



Note) The eigenvalues are not needed. Instead, we can use the determinant and trace of the second moment matrix.

Harris Corner Detector: Procedure

Step 1) Compute *M* matrix for each window to get their *cornerness* **scores**.

Step 2) Find points whose surrounding window gave large corner response ($R >$ threshold)

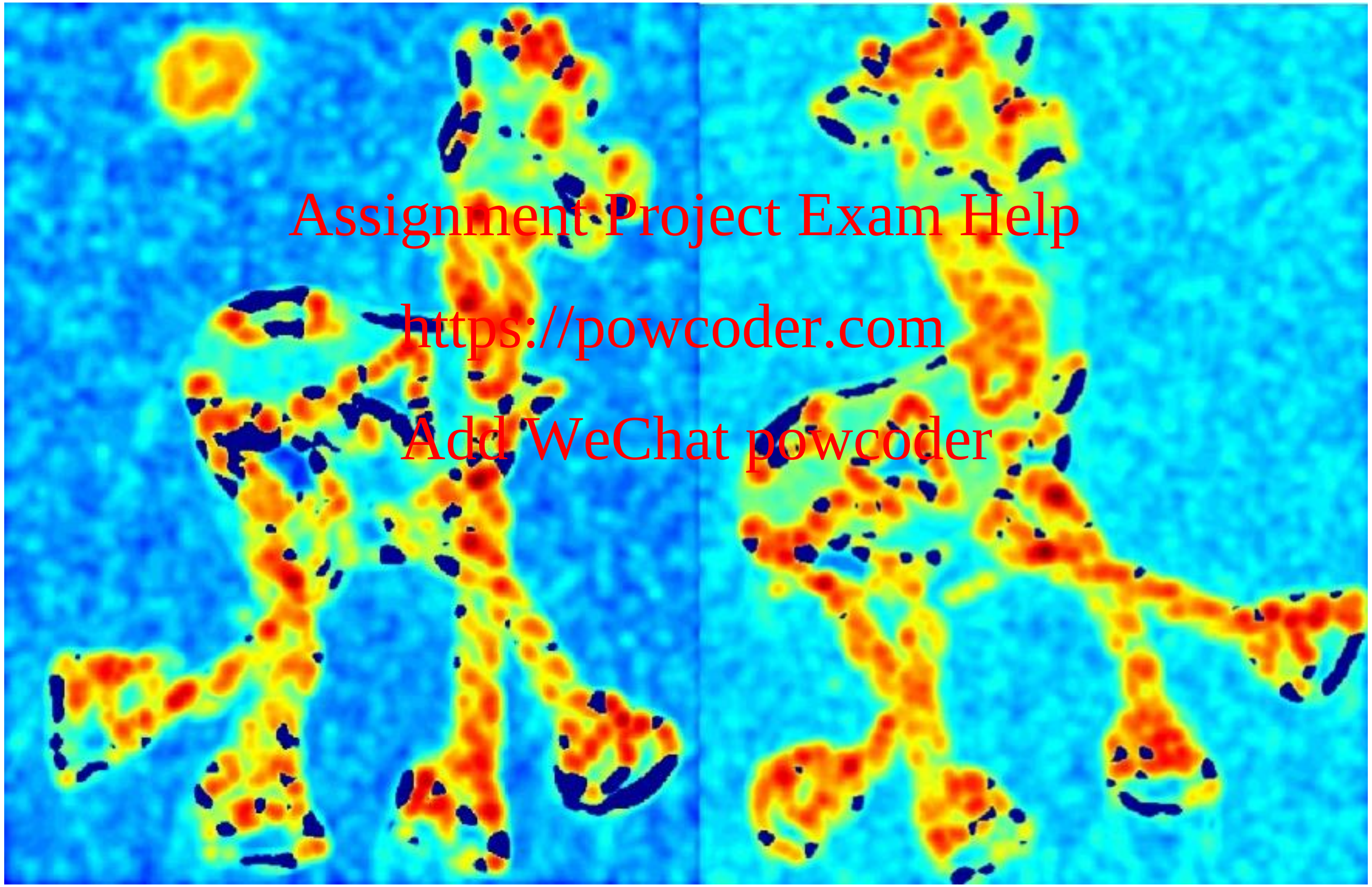
Step 3) Take the points of local maxima, i.e., perform non-maximum suppression

Harris Corner Detector: Input

Performing corner detection to each image independently

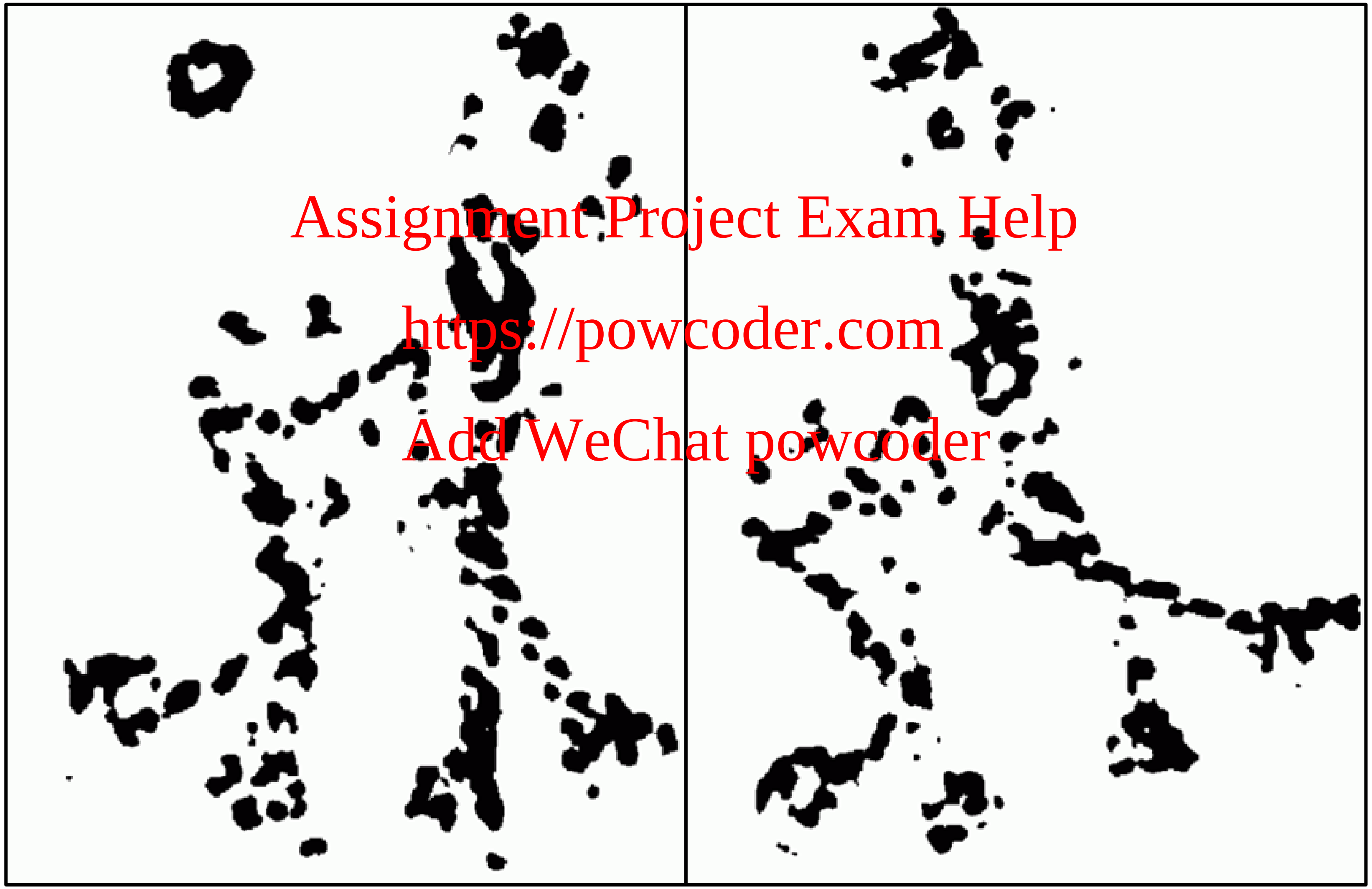
Harris Corner Detector: Step 1)

Compute corner response *R*



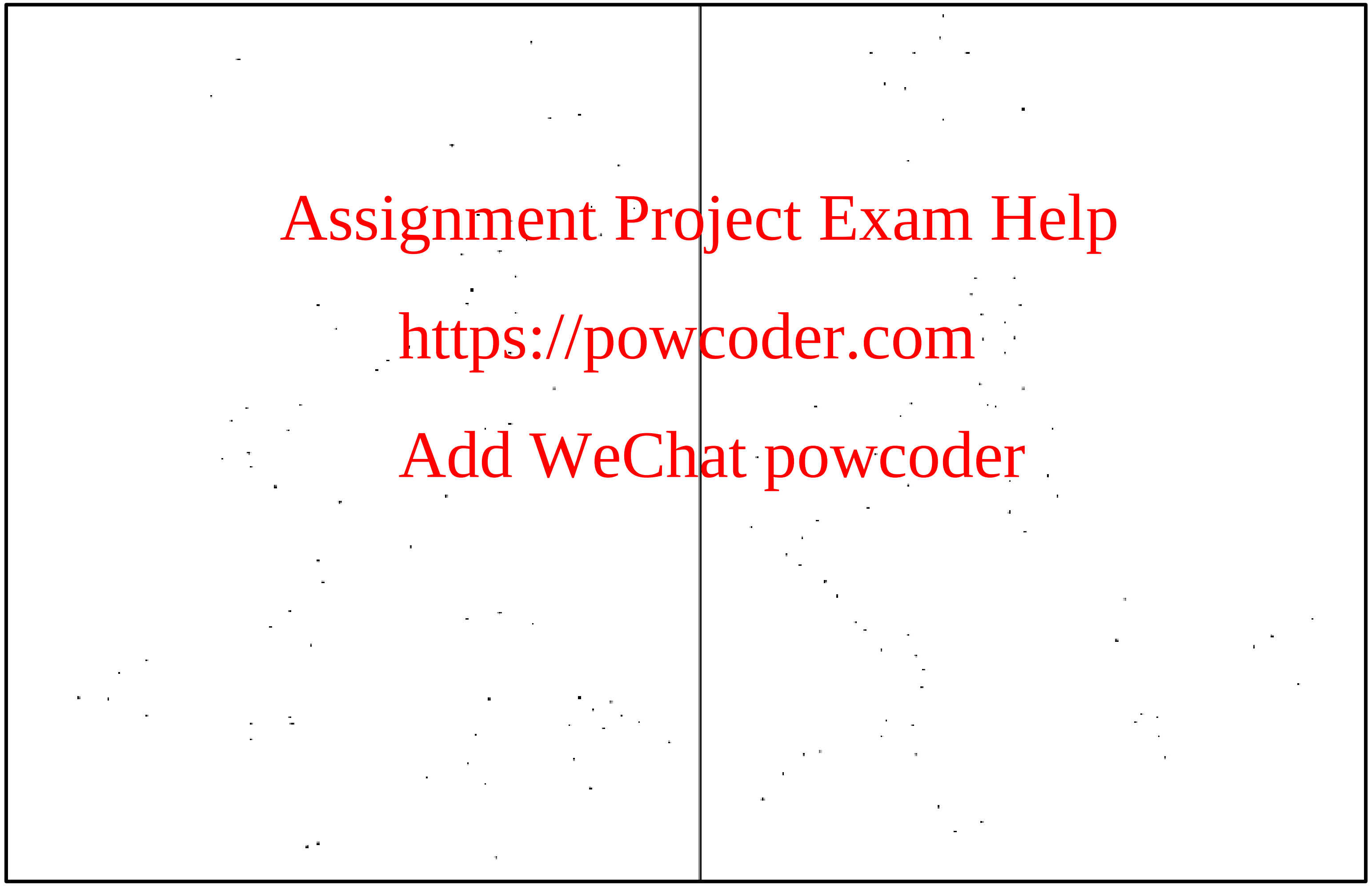
Harris Corner Detector: Step 2)

Find points with large corner response: *R*>threshold



Harris Corner Detector: Step 3)

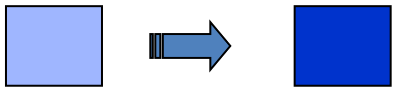
Take only the points of local maxima of R



Harris Corner Detector: Results

Advantages? Drawbacks?

Affine intensity change

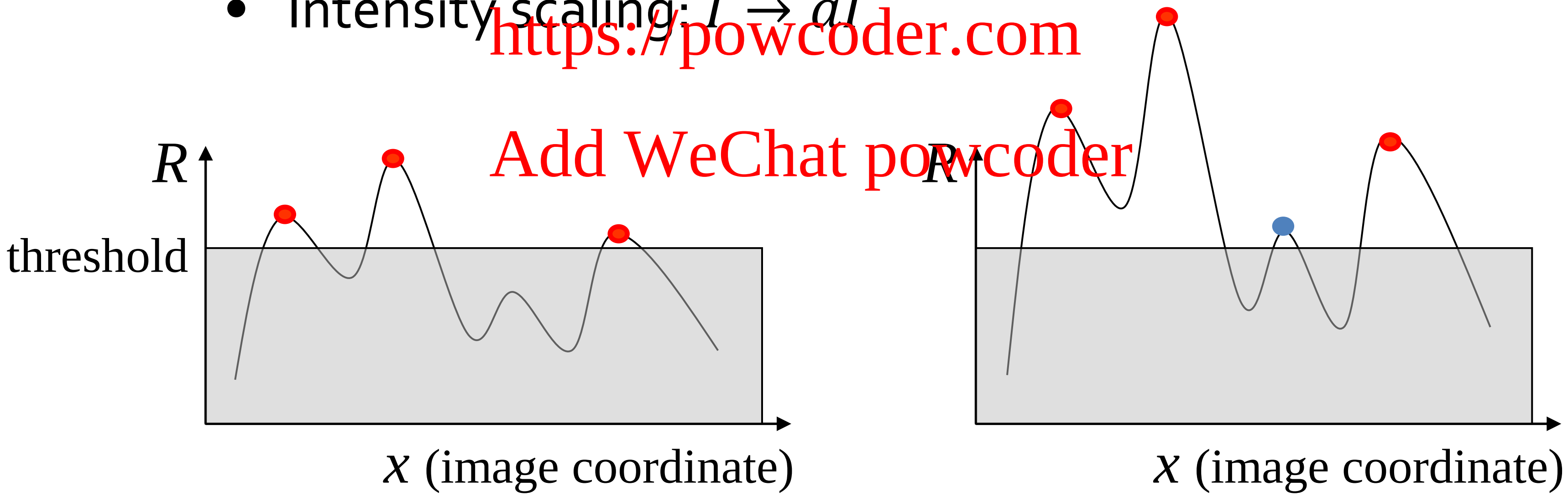


$$I \to aI + b$$

- Only derivatives are used

=> invariance to intensity shift: $I \to I + b$

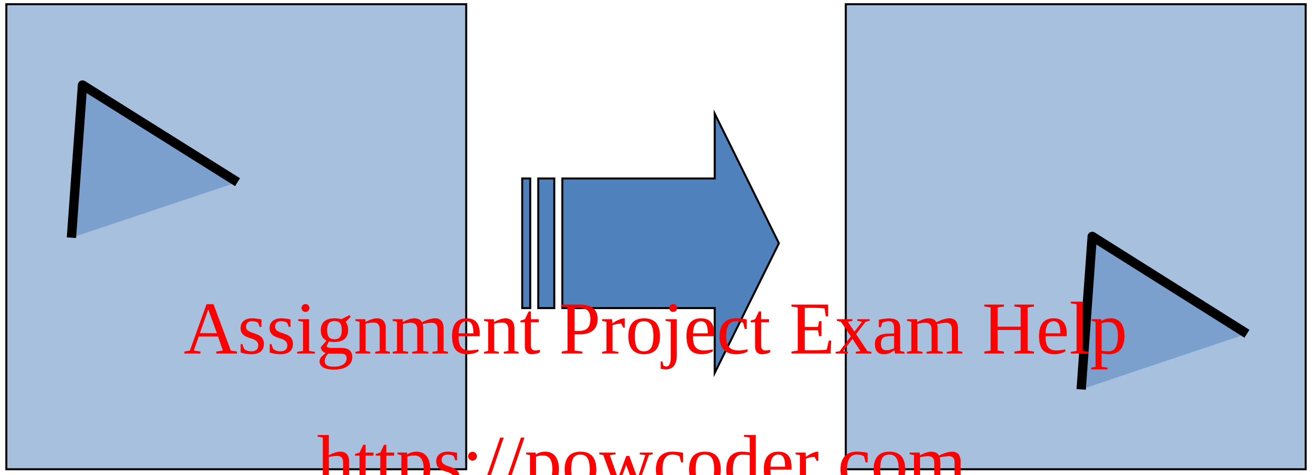
- Intensity scaling: $I \to aI$



- *Partially invariant* to affine intensity change

Slide from J. Hays

Image translation

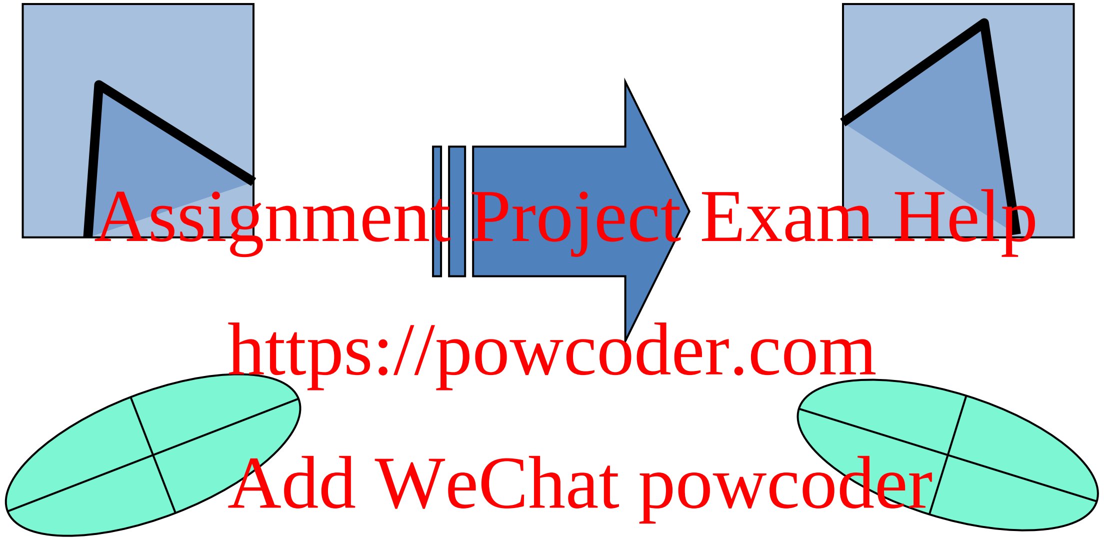


- Derivatives and window function are shift-invariant

Corner location is covariant w.r.t. translation

Slide from J. Hays

Image rotation

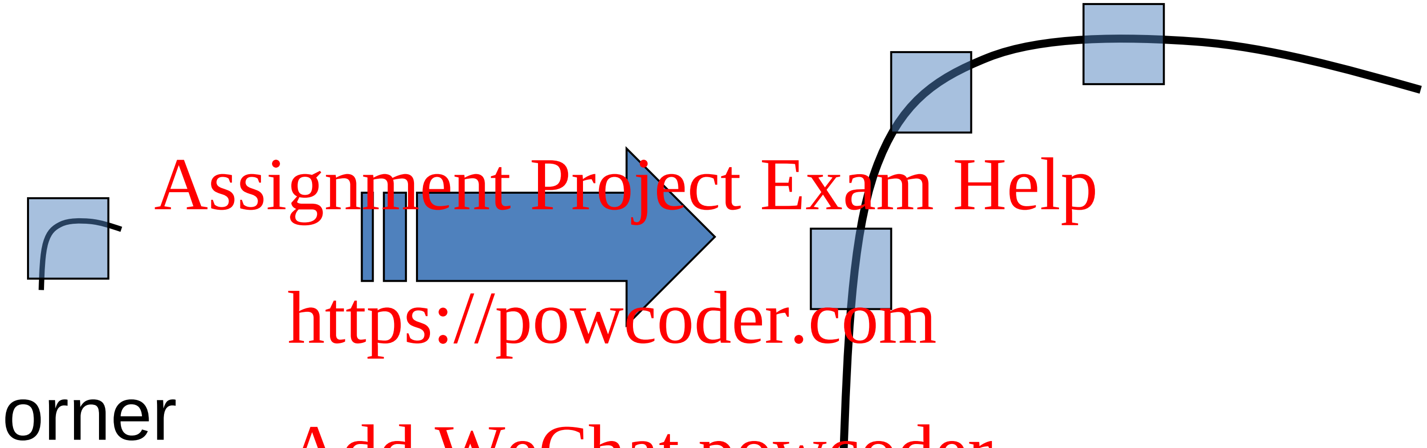


Second moment ellipse rotates but its shape (i.e. eigenvalues) remains the same

Corner location is covariant w.r.t. rotation

Slide from J. Hays

Scaling



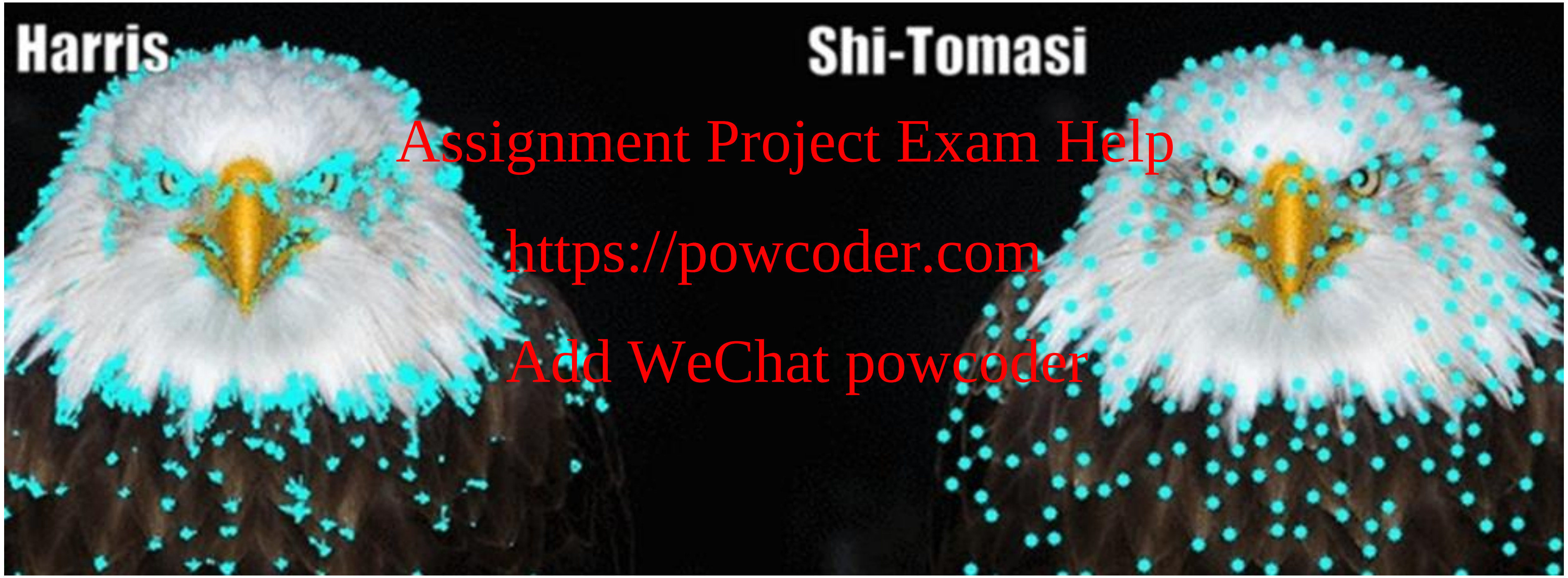
Corner

All points will be classified as edges

Corner location is not covariant to scaling!

Slide from J. Hays

Not always the best



https://medium.com/pixel-wise/detect-those-corners-aba0f034078b

Corner detection: pros & cons

- Corner detection can localize in x-y, but not scale

Slide from J. Hays

Summary

- **Edge detection**
  - Identify sudden changes (discontinuities) in an image
  - Derivative of Gaussian, Sobel, Laplacian, Laplacian of Gaussian, Canny detector

- **Canny edge detector**
  - The most popular edge detector
    1. Applying low- and high-pass filtering
    2. Non-maximum suppression
    3. Double thresholding

- **Harris corner detector**
  1. Compute the second moment matrix
  2. Non-maximum suppression

EBU7240

Computer Vision

- Features: SIFT, SURF, and matching -

Semester 1, 2021

Changjae Oh

Contents

- **Feature Descriptor**
  - Scale Invariant Feature Transform (SIFT) descriptor
  - Speeded Up Robust Features (SURF) descriptor

- **Feature Matching**
  - Nearest Neighbor (NN) Matching

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- **Feature Descriptor**
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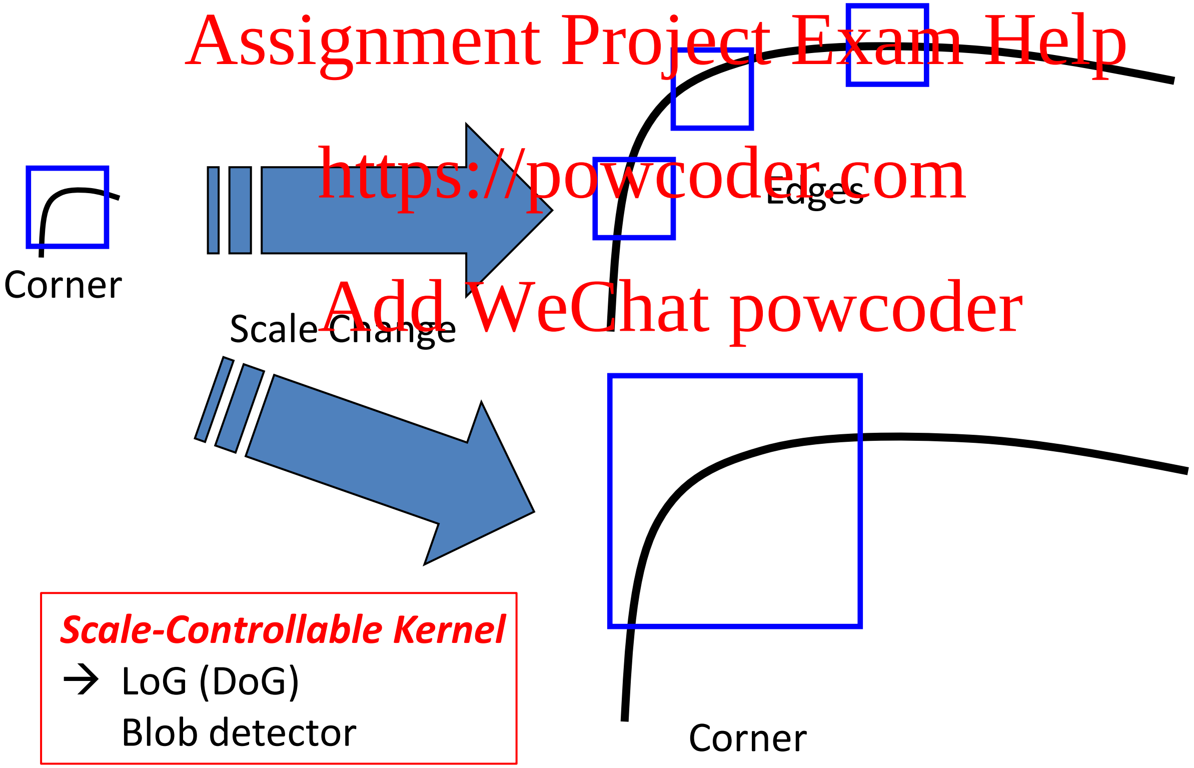
- **Feature Matching**
  - Nearest Neighbor (NN) Matching

Harris corner detector – Pros & Cons

– Robust against shift/rotation transformations and brightness changes,
– Not robust against scale changes.
– Solution: Scale invariant blob detector by SIFT

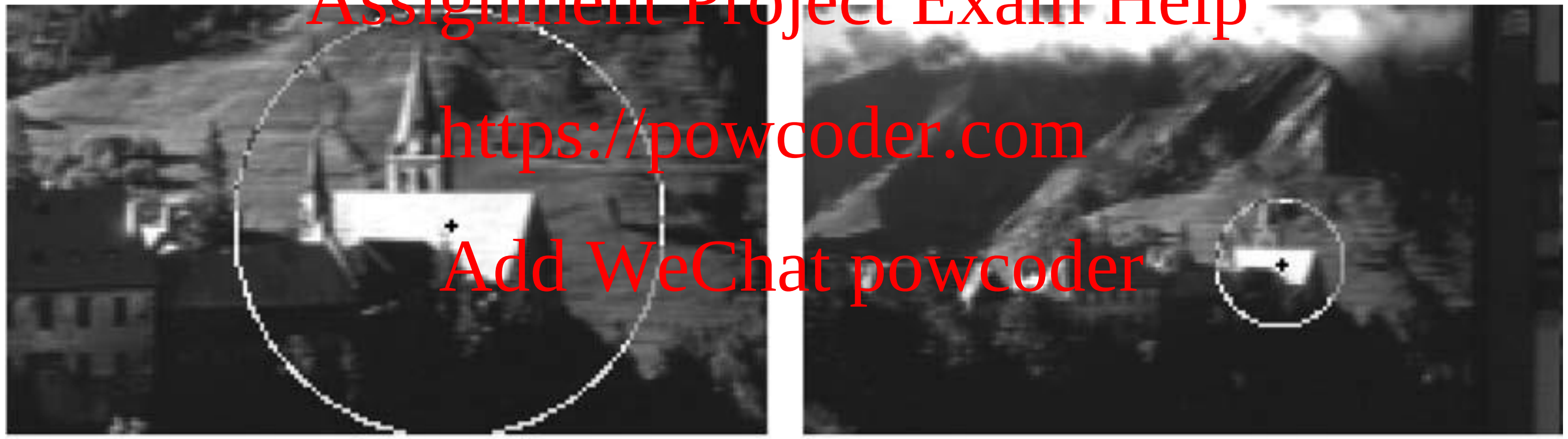


Scale-Controllable Kernel
→ LoG (DoG)
Blob detector

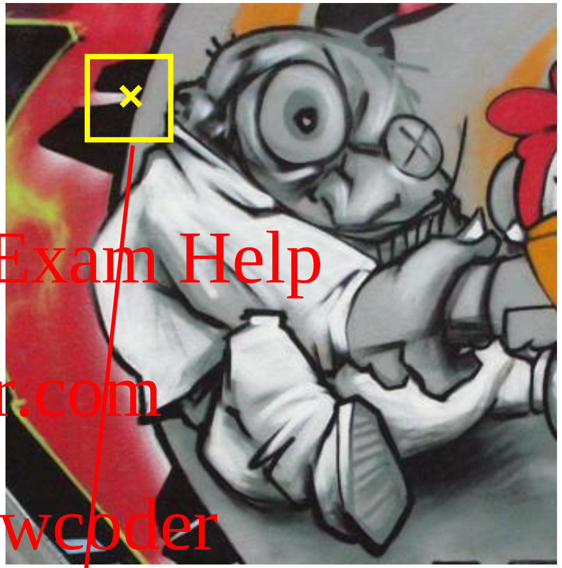
[Ref: Distinctive Image Features from Scale-Invariant Keypoints, IJCV 2004]

Keypoint detection with scale selection

- We want to extract keypoints with characteristic scales that are covariant with respect to the image transformation



Automatic scale selection



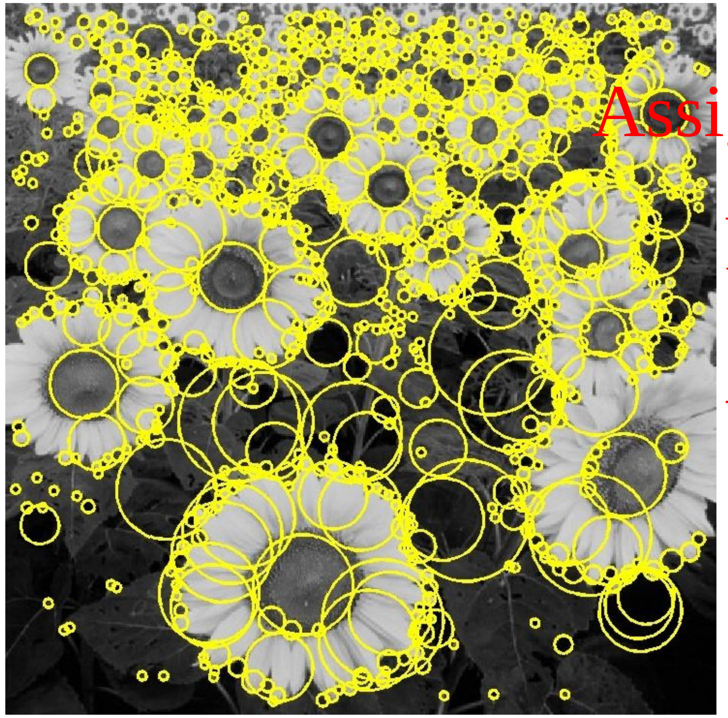
$$f(I_{i_1\ldots i_m}(x,\sigma)) = f(I_{i_1\ldots i_m}(x',\sigma'))$$

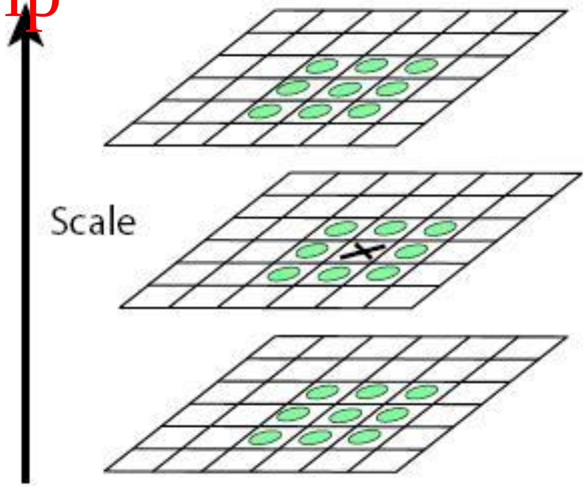
How to find corresponding patch sizes?

A: Blob detector

Basic idea

- Convolve the image with a “blob filter” at multiple scales and look for extrema of filter response in the resulting scale space

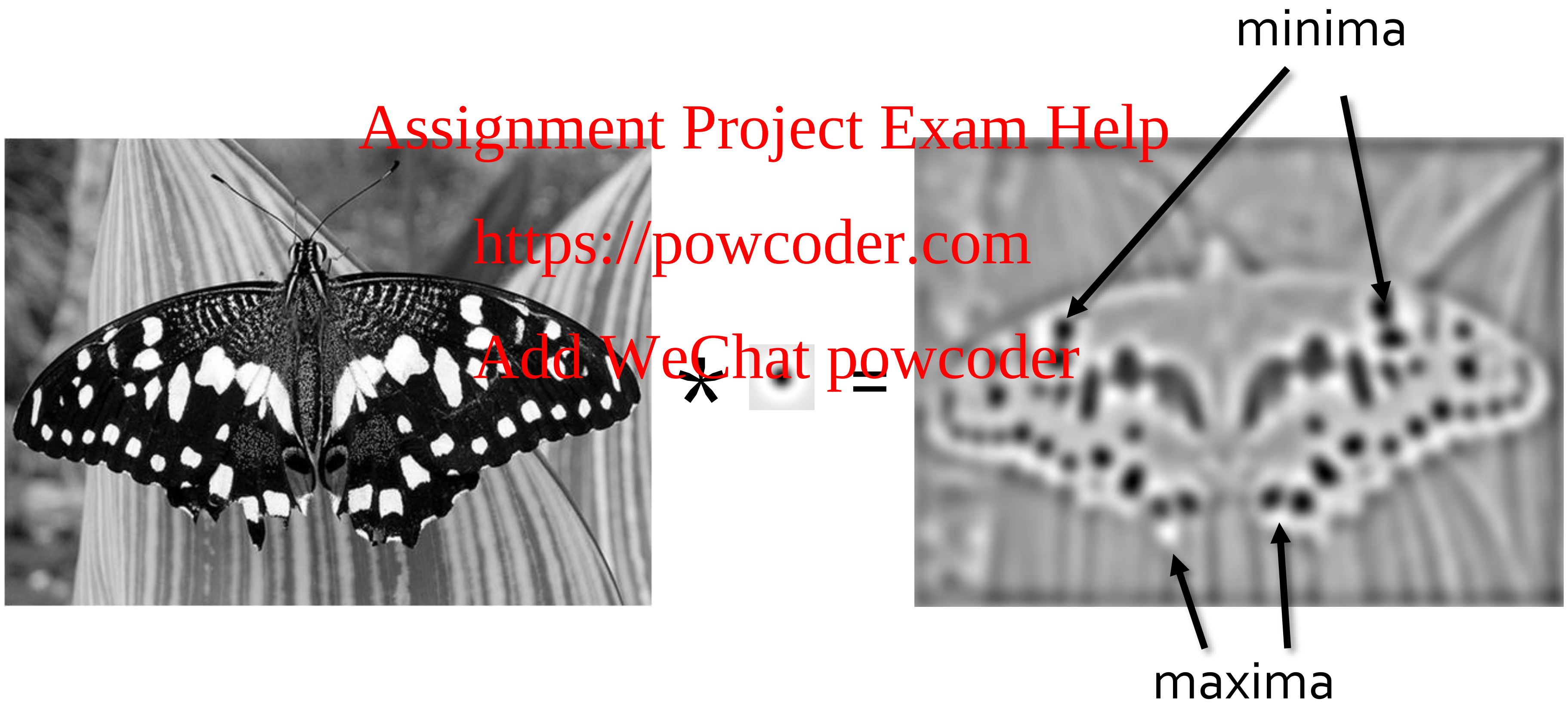




T. Lindeberg, [Feature detection with automatic scale selection](), *IJCV* 30(2), pp 77-116, 1998

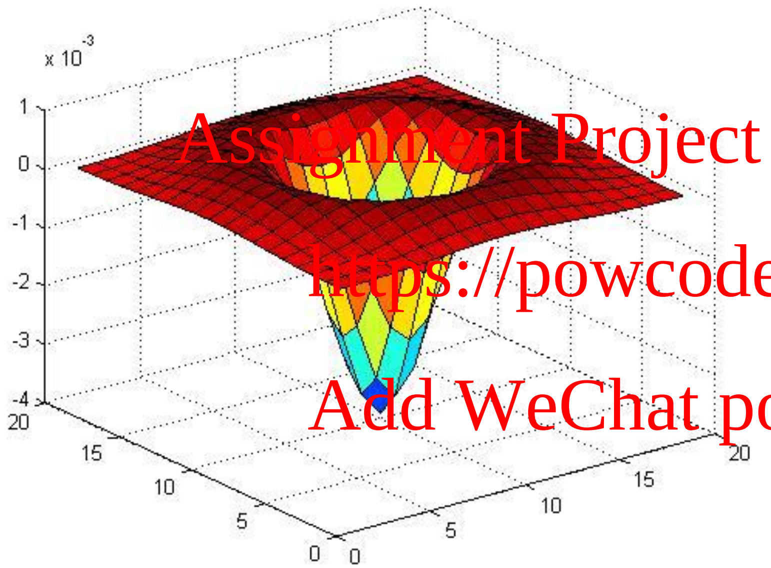
Blob detection

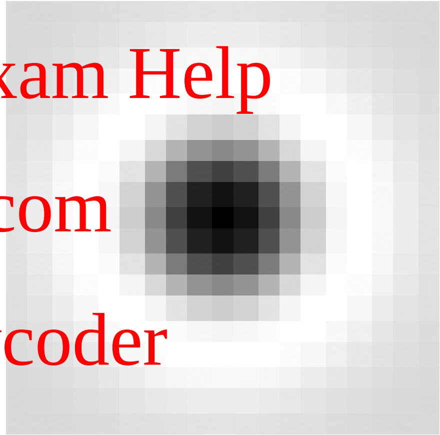
- Find *maxima* and *minima* of blob filter response in space and scale



Blob filter

- **Laplacian of Gaussian**
  - Circularly symmetric operator for blob detection in 2D



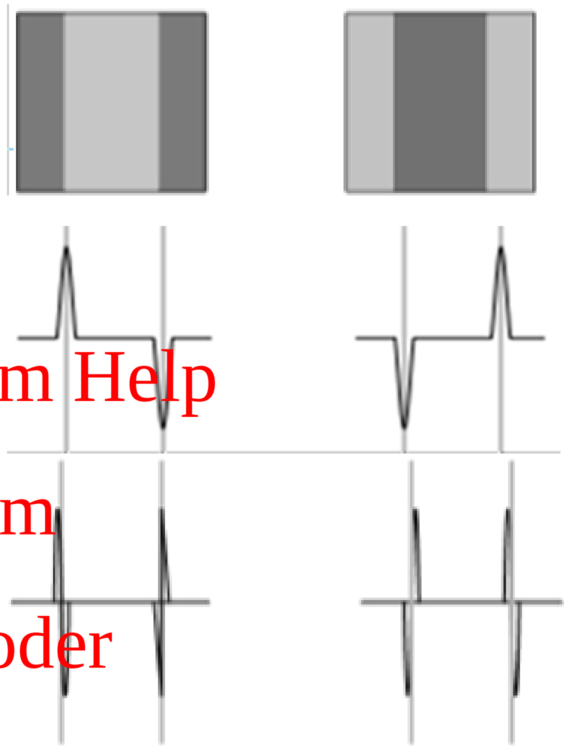


$$\nabla^2 g = \frac{\partial^2 g}{\partial x^2} + \frac{\partial^2 g}{\partial y^2}$$

Laplacian of Gaussian (LoG)

$$\nabla^2 I = \frac{\partial^2 I(x,y)}{\partial x^2} + \frac{\partial^2 I(x,y)}{\partial y^2} \quad \textit{Laplacian} \qquad I \to \boxed{G} \to \boxed{L} \to$$

$$\nabla^2 * (G * I) = (\underbrace{\nabla^2 * G}_{\text{LoG}}) * I \quad \textit{LoG} \qquad I \to \boxed{\text{LoG}} \to$$

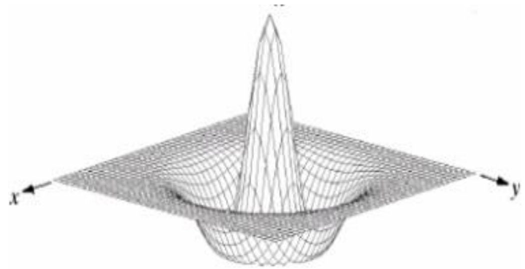


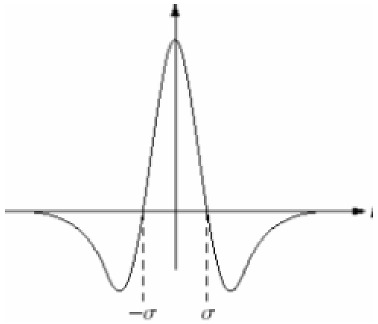
$$G(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

$$\nabla^2 G = \left(\frac{\partial^2 G}{\partial x^2} + \frac{\partial^2 G}{\partial y^2}\right)$$

$$= \left[\frac{x^2+y^2-2\sigma^2}{2\pi\sigma^6}\right] \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right)$$

LoG function





σ : LoG Parameter

0	0	-1	0	0
0	-1	-2	-1	0
-1	-2	16	-2	-1
0	-1	-2	-1	0
0	0	-1	0	0

Recall: Edge detection- Laplacian

f

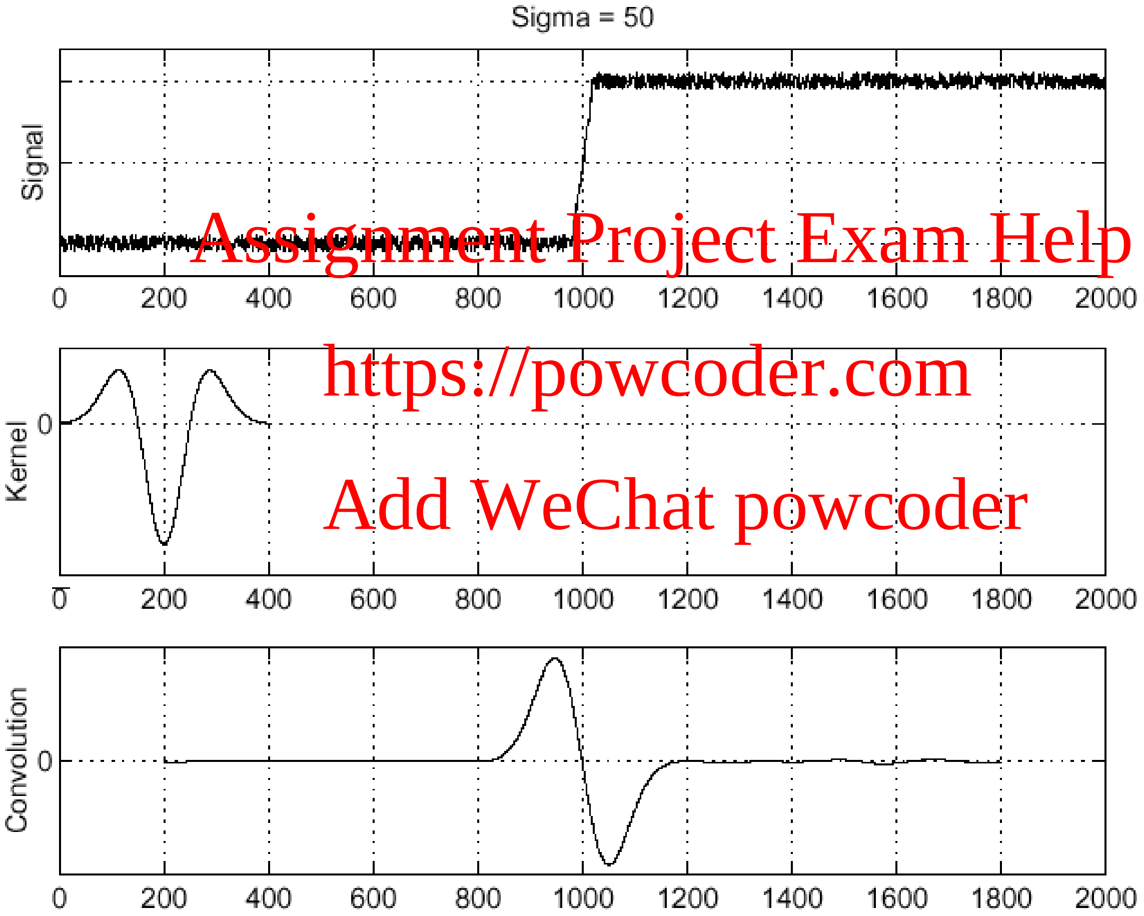
Edge

$$\frac{d^2}{dx^2} g$$

Second derivative of Gaussian (Laplacian)

$$f * \frac{d^2}{dx^2} g$$

Edge = zero crossing of second derivative

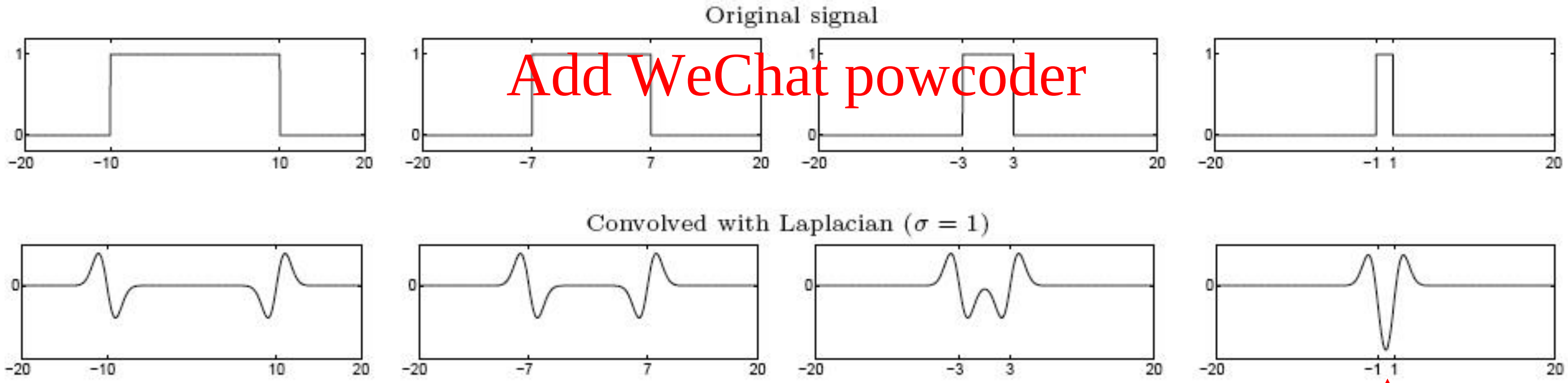


Source: S. Seitz

Laplacian of Gaussian (LoG)

- **LoG applied on Edge**
  - Ripple
- **LoG applied on Pulse (Blob):**
  - Superposition of two ripples, generating an extremal value at the center of the pulse (blob)

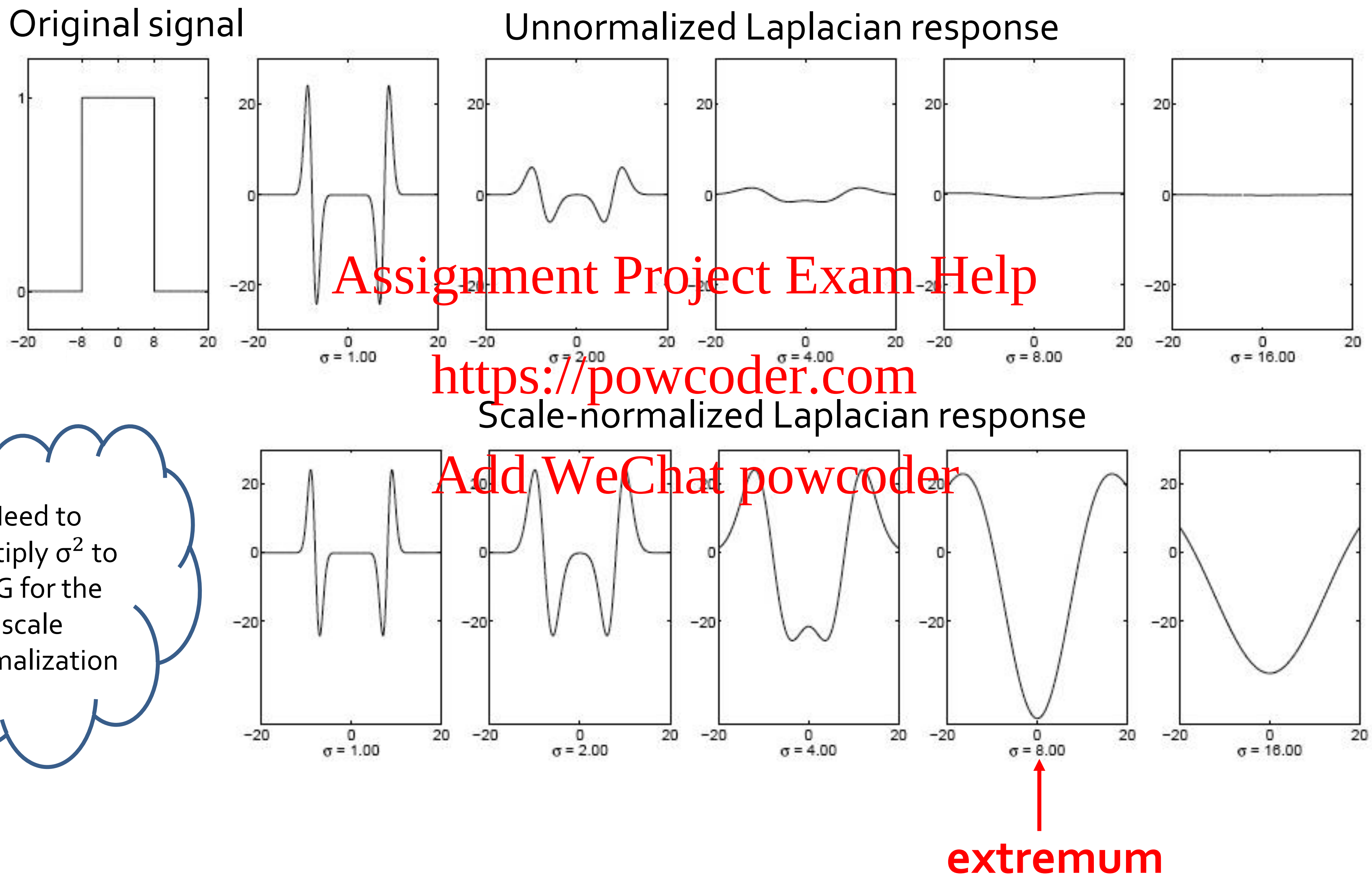
Convolving with a pulse signal, 1-D LoG generates an extremal pulse when its sigma matches the width of the pulse.



extremum

Slides from Chee Sun Won

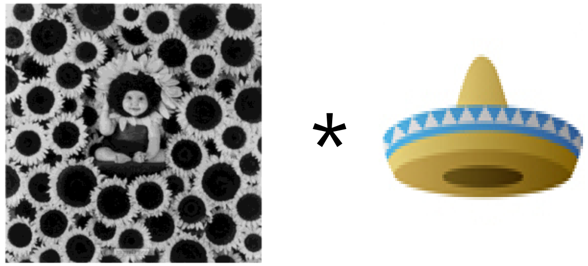
Scale-normalized LoG

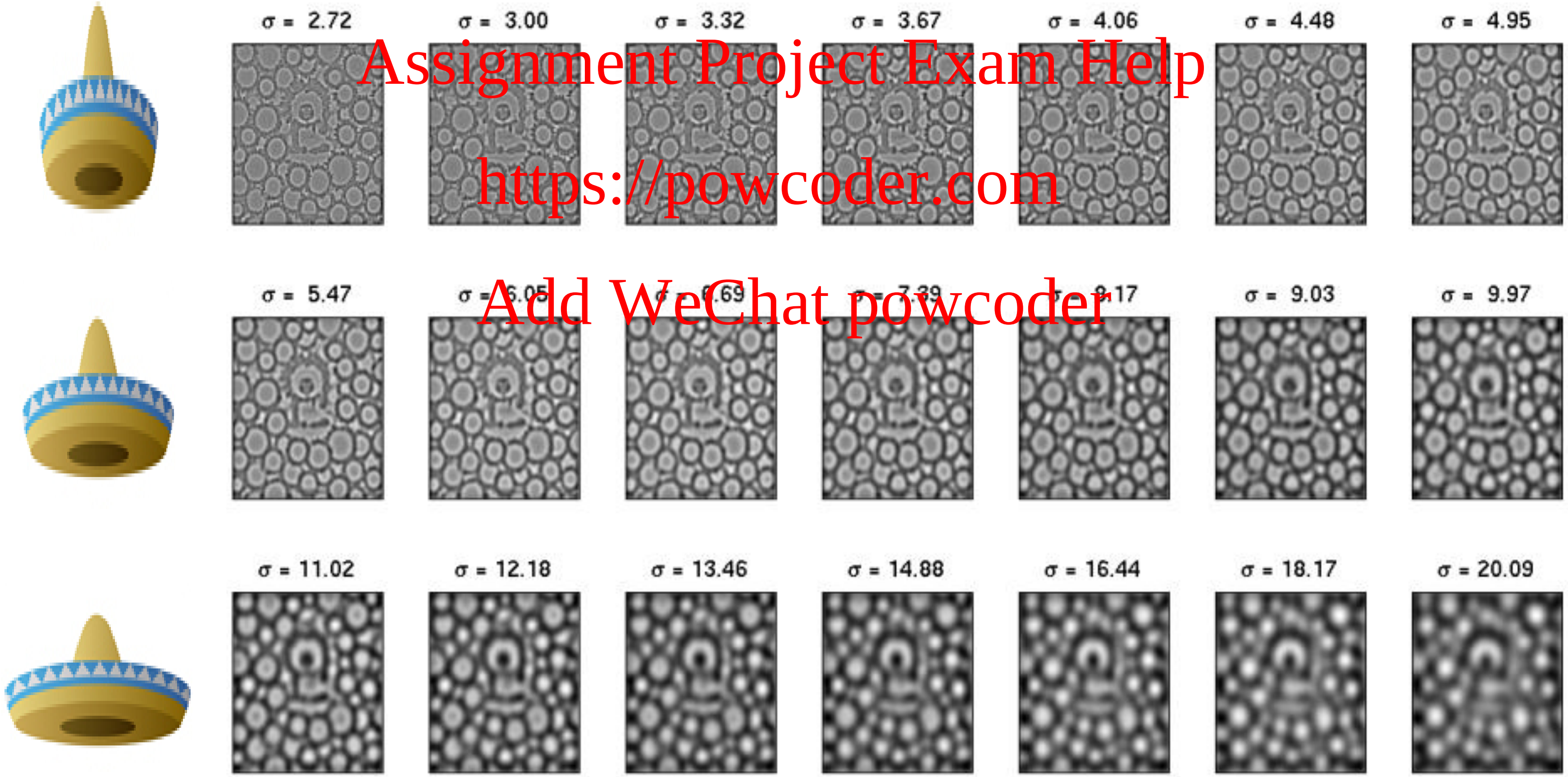


Need to multiply σ^2 to LoG for the scale normalization

Blob Detection with 2-D LoG

LoG filtered scale-space



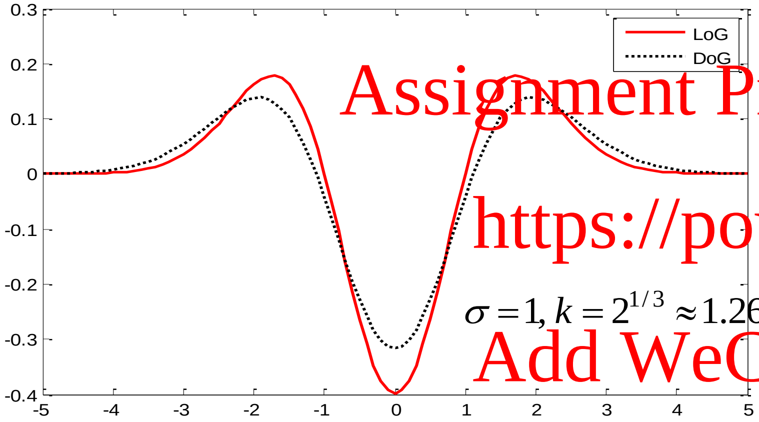


Source: http://www.cs.utah.edu/~jfishbau/advimproc/project1/

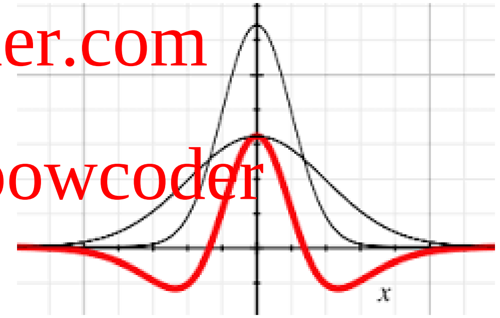
DoG (Difference of Gaussian)

Normalized LoG ≈(k-1) x Difference of Gaussian (DoG)

$$LoG_{norm} = \sigma^2 \left(G_{xx}(x, y, \sigma) + G_{yy}(x, y, \sigma) \right) \qquad DoG = G(x, y, k\sigma) - G(x, y, \sigma)$$



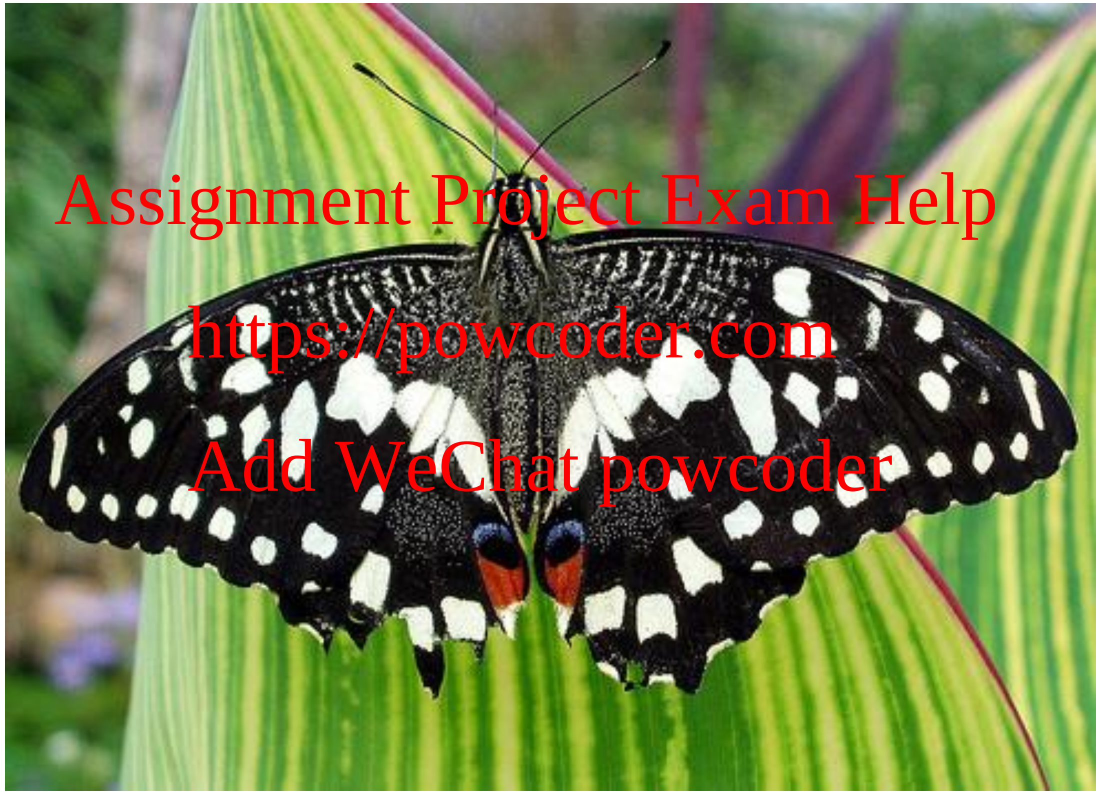
$$DoG(x, y, \sigma) \approx (k-1) LoG_{norm}(x, y, \sigma)$$



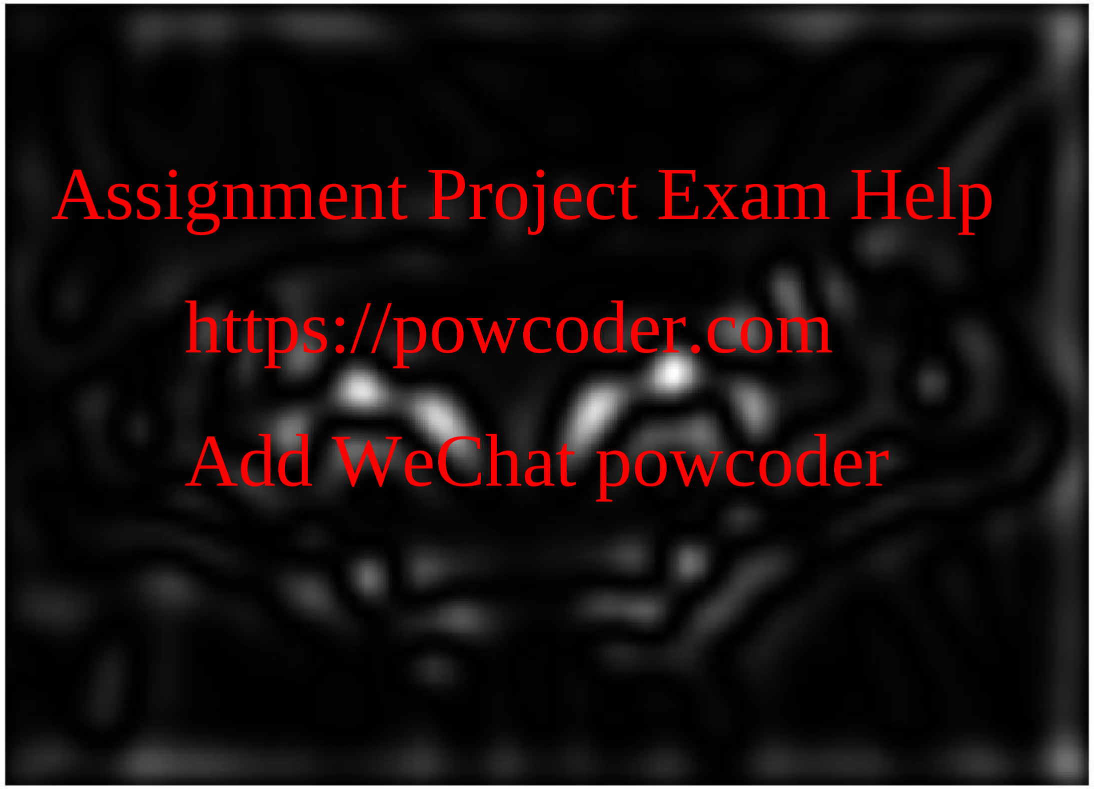
- Use DoG instead of LoG
  1) Gaussian Filtering : G(x,y,σ), G(x,y,kσ)
  2) Subtraction

$$D(x, y, \sigma) = I * G(x, y, k\sigma) - I * G(x, y, \sigma)$$
$$= I * \left(G(x, y, k\sigma) - G(x, y, \sigma) \right) = I * DoG(x, y, k\sigma, \sigma)$$

Scale-space blob detector: Example

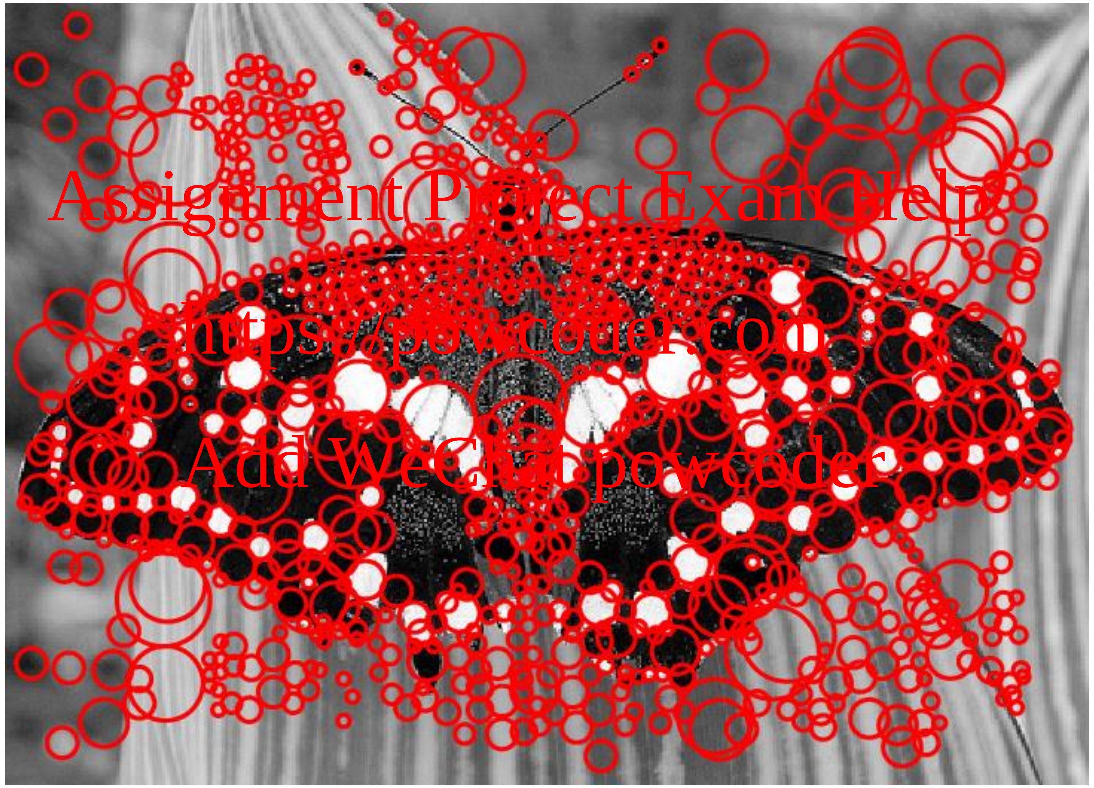


Scale-space blob detector: Example



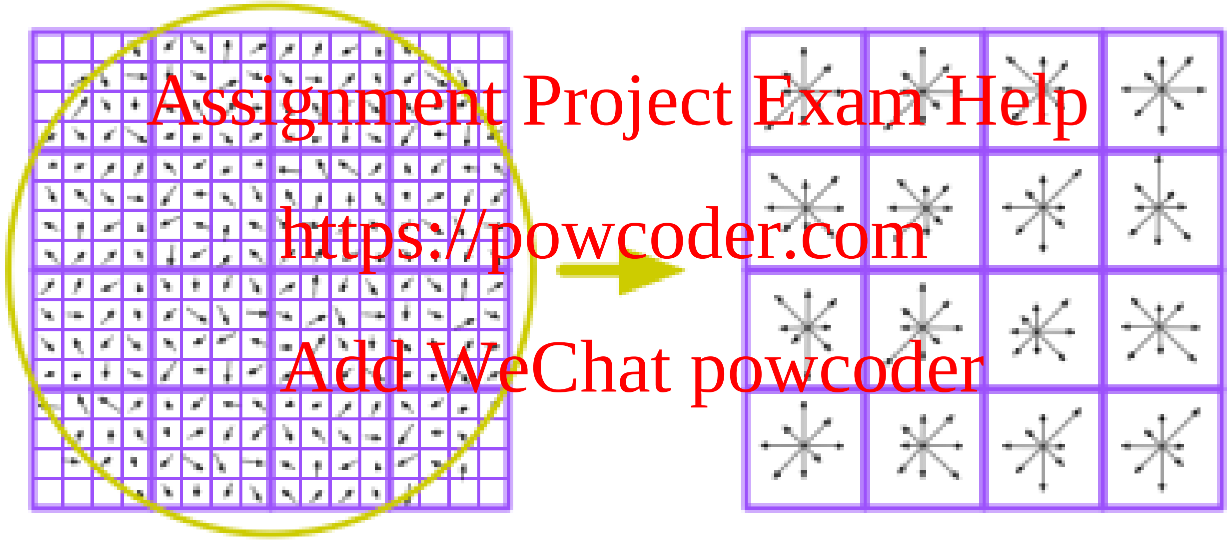
sigma = 11.9912

Scale-space blob detector: Example



SIFT descriptors

- **Inspiration: complex neurons in the primary visual cortex**



D. Lowe, [Distinctive image features from scale-invariant keypoints](), *IJCV* 60 (2), pp. 91-110, 2004

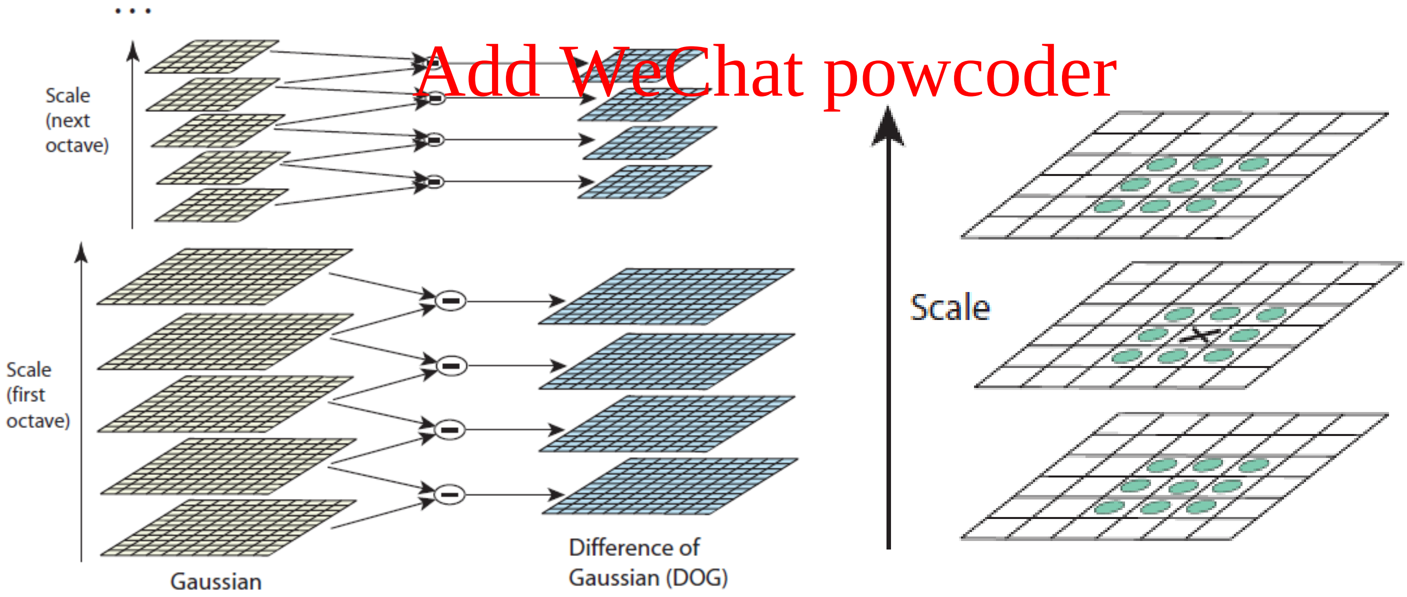
SIFT: Step 1) Scale Space Extrema Detection

Scale-Space Extrema Detection

Detect the candidates of interest points, which are extrema points in the scale-space domains.

Scale-Space Extrema detection → Key Point Localization → Orientation Assignment → Key Point Descriptor Construction

Detector ⟷ Descriptor



Source: D. Lowe

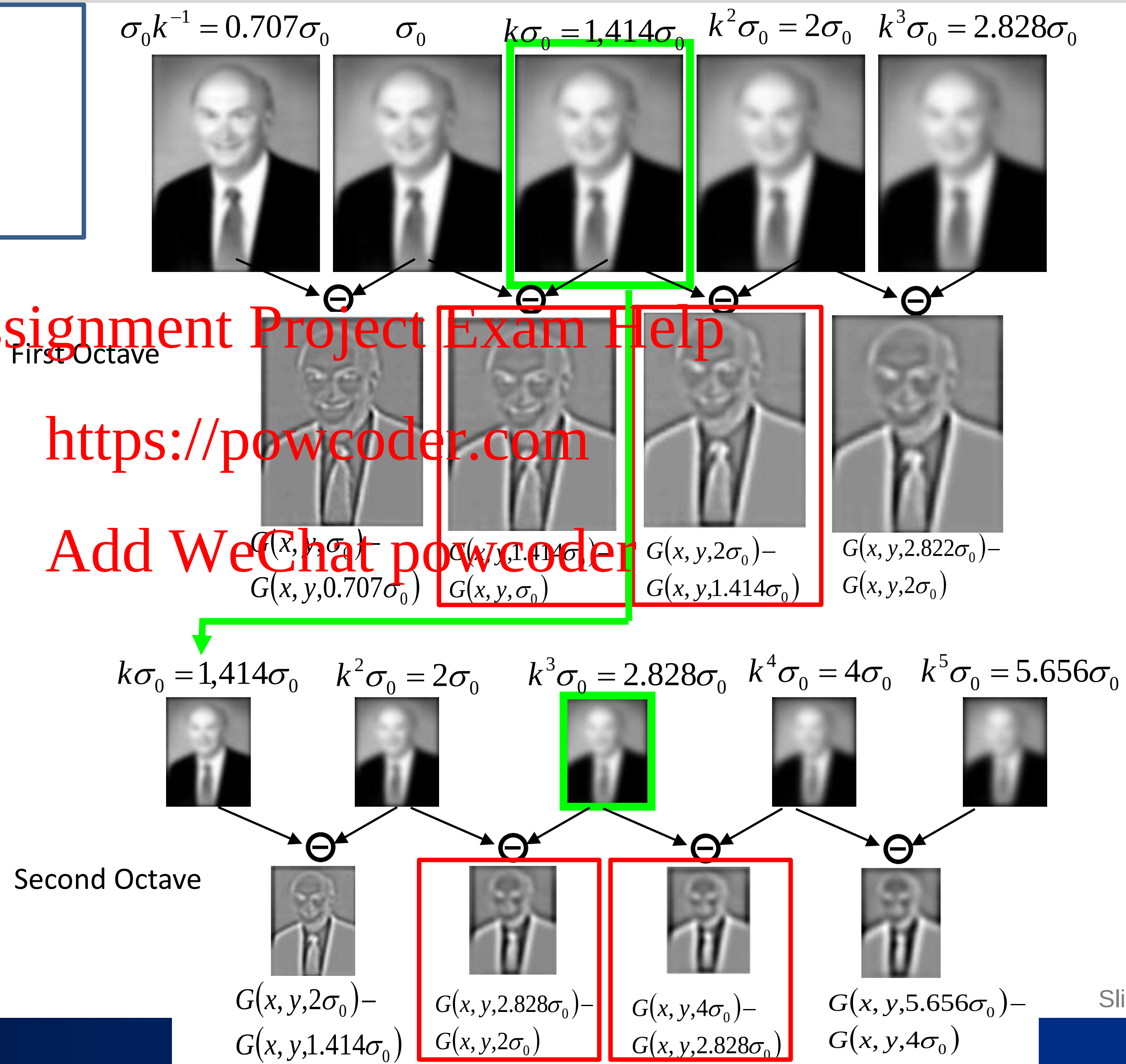
SIFT: Step 1) Scale Space Extrema Detection

Ex) 2 Levels (S=2)

2 Octave

- Need to generate S+3=2+3=5 blurred images per octave
- $k = 2^{1/S} = 2^{1/2} = 1.414$

FirstOctave : -1



SIFT: Step 1) Scale Space Extrema Detection

Scale-Space Extrema Detection

- Compare a pixel with 26 pixels in current and adjacent scales (Green circles)
- Select a pixel as an extrema if larger/smaller than neighboring 26 pixels
- Needs further localization and sampling

Source: D. Lowe

SIFT: Step 2) Key Point Localization

Scale-Space Extrema detection → **Key Point Localization** → Orientation Assignment → Key Point Descriptor Construction

(1) Sub-pixel localization and removal of extrema points with low contrast:

Use Taylor series expansion of the scale-space function D to find maximum of surface

$$D(X) = D + \frac{\partial D^T}{\partial X} X + \frac{1}{2} X^T \frac{\partial^2 D}{\partial X^2} X$$

D and its derivatives are evaluated at the sample point

The sub-pixel location of the extremum

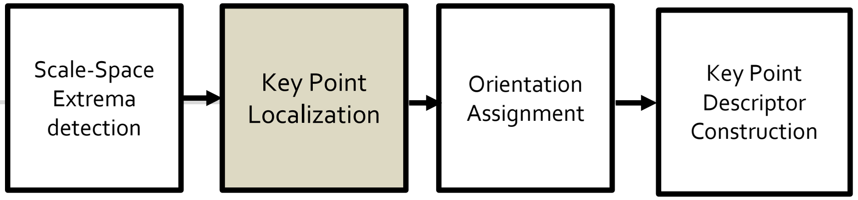
$$\hat{X} = -\left(\frac{\partial^2 D}{\partial X^2}\right)^{-1} \frac{\partial D}{\partial X}$$

The function value at the extremum, $D(\hat{X})$

$$D(\hat{X}) = D + \frac{1}{2} \frac{\partial D^T}{\partial X} \hat{X}$$

If $|D(\hat{X})| < Th$ then discard the extremum.

SIFT: Step 2) Key Point Localization



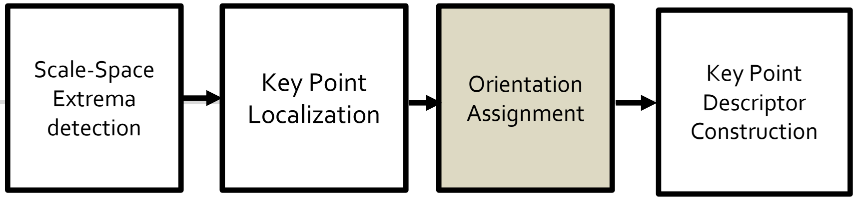
(2) Delete edge-like features by calculating the curvature

H: Hessian matrix

$$H = \begin{bmatrix} D_{xx} & D_{xy} \\ D_{xy} & D_{yy} \end{bmatrix} \qquad r = \frac{Trace(H)^2}{|H|} = \frac{(\lambda_1 + \lambda_2)^2}{\lambda_1 \lambda_2} = \frac{\left(1 + \frac{\lambda_2}{\lambda_1}\right)^2}{\frac{\lambda_2}{\lambda_1}}$$

If $r > Th$, then delete the extreme point.

SIFT: Step 3) Orientation Assignment

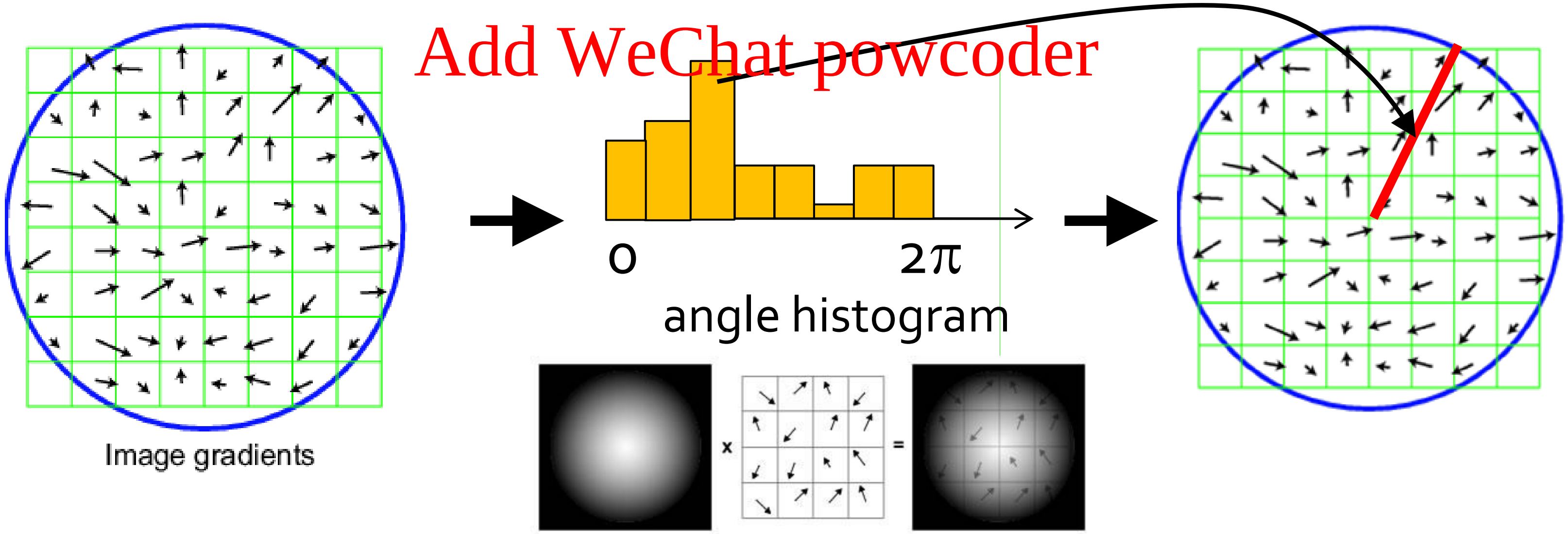


1. Take 16x16 square window
2. Compute edge orientation for each 2x2 block in 16x16 square

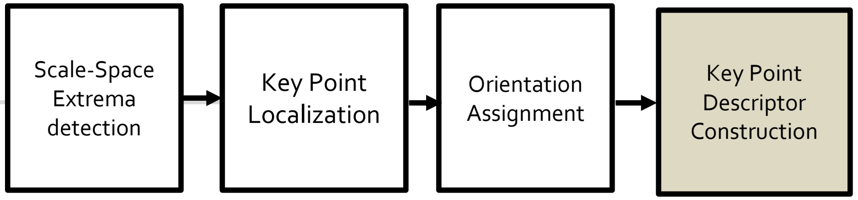
$$L(x,y) = G(x,y,\sigma) * I(x,y) \qquad m(x,y) = \sqrt{\left(L(x+1,y) - L(x-1,y)\right)^2 + \left(L(x,y+1) - L(x,y-1)\right)^2}$$

$$\theta(x,y) = \tan^{-1}\left(\left(L(x,y+1) - L(x,y-1)\right) / \left(L(x+1,y) - L(x-1,y)\right)\right)$$

3. Throw out weak edges (threshold gradient magnitude)
4. Create histogram by accumulating the Gaussian weighted edge magnitude

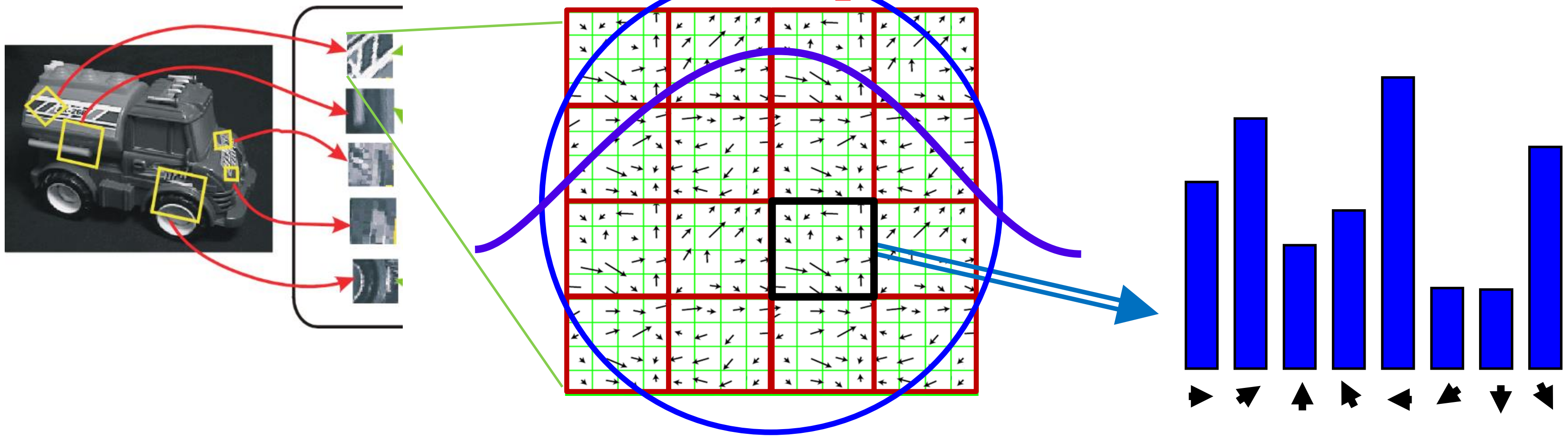


SIFT: Step 4) Descriptor Construction



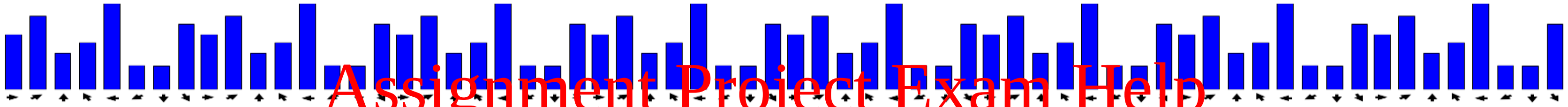
1. Normalize the window as 16x16 window using orientation/scale.
2. For each 4x4 block, compute gradient histogram over 8 directions. Gradient magnitudes are weighted by a Gaussian of variance half the window (for s mooth fall-off)

SIFT descriptor: 128-D vector



SIFT: Step 4) Descriptor Construction

3. **Concatenate 8-D vectors of 4x4 arrays and normalise the magnitude 128-D vector to 1.**

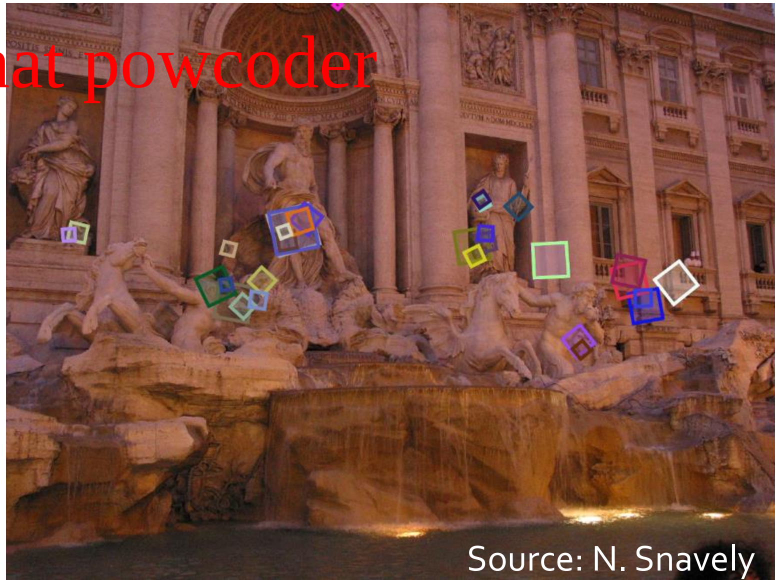


SIFT Descriptor: Binning of Spatial Coordinates and Gradient Orientations

4. **Threshold gradient magnitudes to avoid excessive influence of high gradients**
    1. after normalization, clamp gradients >0.2
    2. Renormalize the vector

Properties of SIFT

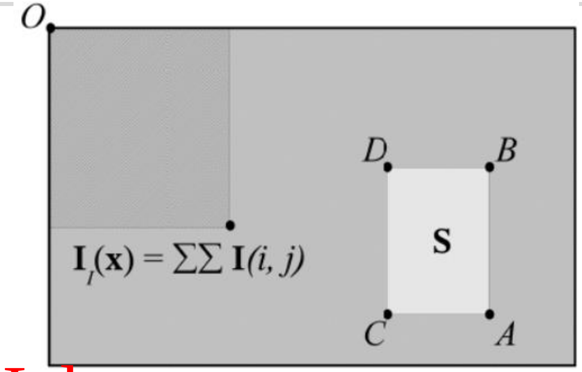
- **Extraordinarily robust detection and description technique**
  - Can handle changes in viewpoint
    - Up to about 60 degree out-of-plane rotation
  - Can handle significant changes in illumination
    - Sometimes even day vs. night
  - Fast and efficient—can run in real time



Source: N. Snavely

SURF (Speeded Up Robust Features)

- SIFT is one of the best but slow
- Using *integral images* for an efficient implementation
- Detect and describe SIFT like features
- SURF describes image 3 times faster than SIFT
- SURF is not as well as SIFT on invariancae to illumination change and viewpoint change



$$S = (A - B - C + D)$$

Integral images: accumulated sum of gray scale pixel values of images

- Keypoint detection based on Hessian matrix: blob-like features

$$H(p,\sigma) = \begin{bmatrix} L_{xx}(p,\sigma) & L_{xy}(p,\sigma) \\ L_{xy}(p,\sigma) & L_{yy}(p,\sigma) \end{bmatrix} \qquad |H(p,\sigma)| = L_{xx}(p,\sigma)L_{yy}(p,\sigma) - L_{xy}(p,\sigma)^2$$

where $L_{xx}(p,\sigma)$ is the convolution of the second order Gaussian derivative $\partial^2 G(p,\sigma)/\partial x^2$ with the image *I* in *p* and similarly for $L_{xy}(p,\sigma)$ and $L_{yy}(p,\sigma)$.

Bay, H., Tuytelaars, T., Gool, L.V., "SURF: Speeded Up Robust Features", ECCV 2006.
Bay, H. et al., “Speeded-Up Robust Features (SURF)”, Computer Vision and Image Understanding, 2008

SURF

- SURF is more than three times faster than SIFT
- SURF is inferior to SIFT for luminance and viewpoint changes.
- SURF integrates the gradient information within a subpatch, whereas SIFT depends on the orientations of the individual gradients. This makes SURF *less sensitive to noise*.

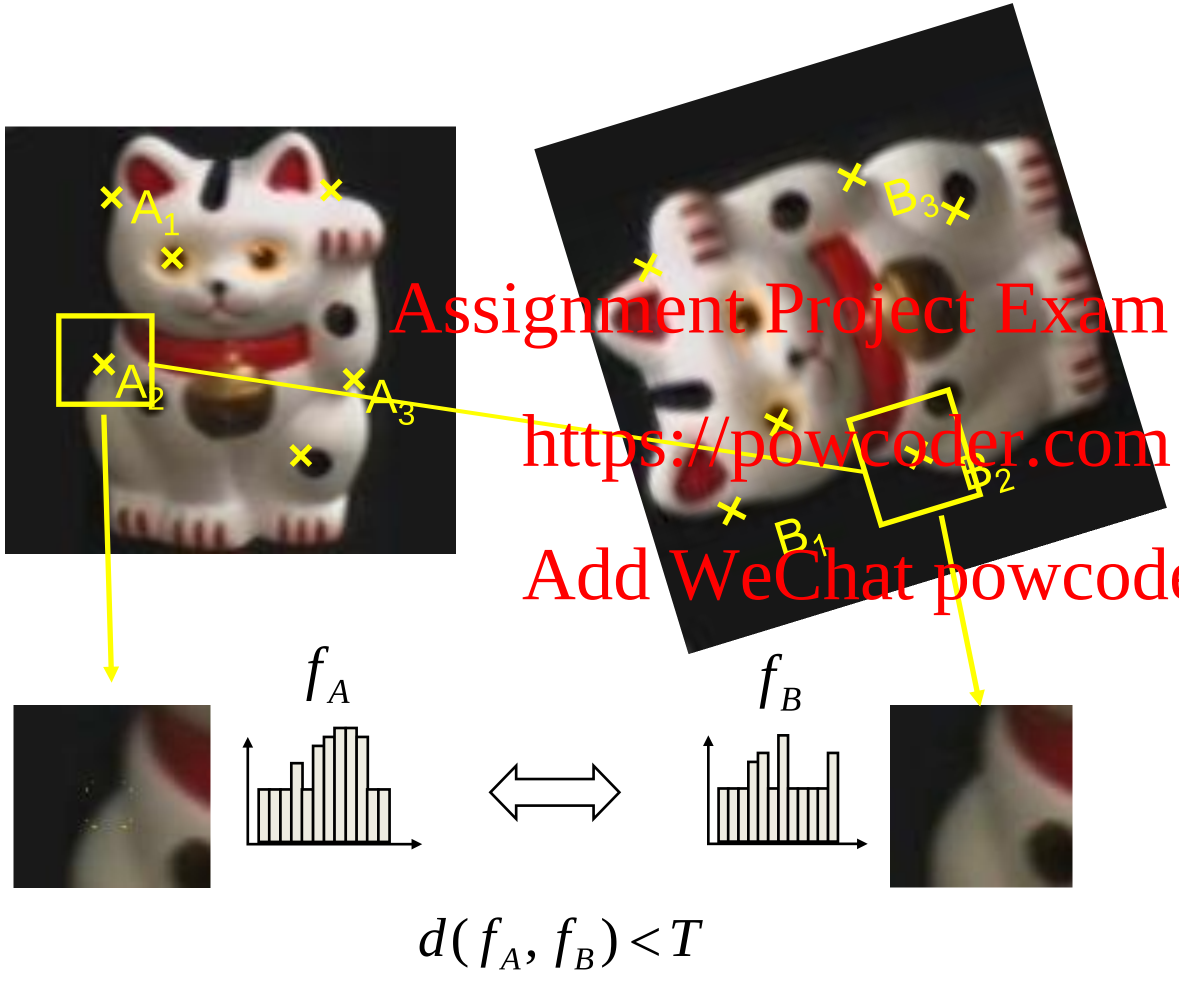
Contents

- **Feature Descriptor**
  - Scale Invariant Feature Transform (SIFT) descriptor
  - Speeded Up Robust Features (SURF) descriptor

- **Feature Matching**
  - Nearest Neighbor (NN) Matching

How do we decide which features match?

Overview of Feature Matching



1. Find a set of distinctive keypoints
2. Define a region around each keypoint
3. Extract and normalize the region content
4. Compute a local descriptor from the normalized region
5. Match local descriptors

Slide from K. Grauman, B. Leibe

Feature Matching

- **Nearest neighbor matching**
  - One feature matches to another if those features are nearest neighbors and their distance is below some threshold.

$$\{f_i | i = 1, \ldots, N\} \text{ for } I_1 \text{ and } \{g_j | j = 1, \ldots, M\} \text{ for } I_2$$

$$k = \min_j dist(f_i, g_j) \ \& \ dist(f_i, g_k) < T \rightarrow NN(f_i) = g_k$$

- **Problems**
  - Threshold T is difficult to set
  - Non-distinctive features could have lots of close matches, only one of which is correct

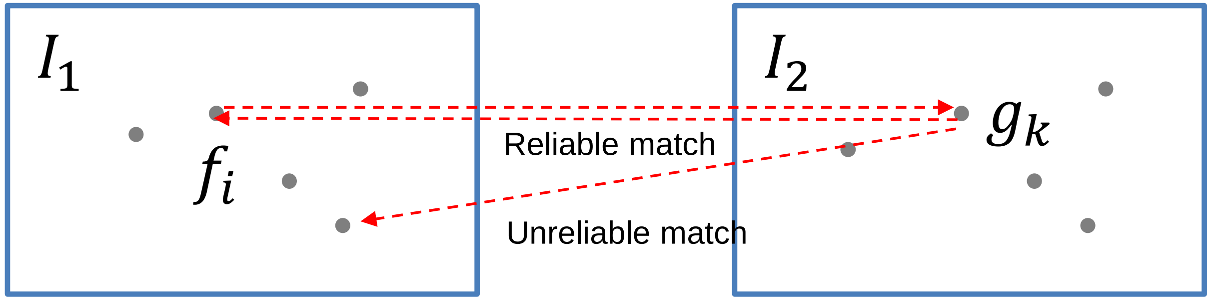
Feature Matching

- **Simple Solution: Cross-checking technique**

$$k = \min_j dist(f_i, g_j) \;\rightarrow\; NN(f_i) = g_k$$

$$l = \min_i dist(f_i, g_k) \;\rightarrow\; NN(g_k) = f_l$$

If $i = l$, the matching is assumed to be reliable.
Otherwise, the matching is unreliable.



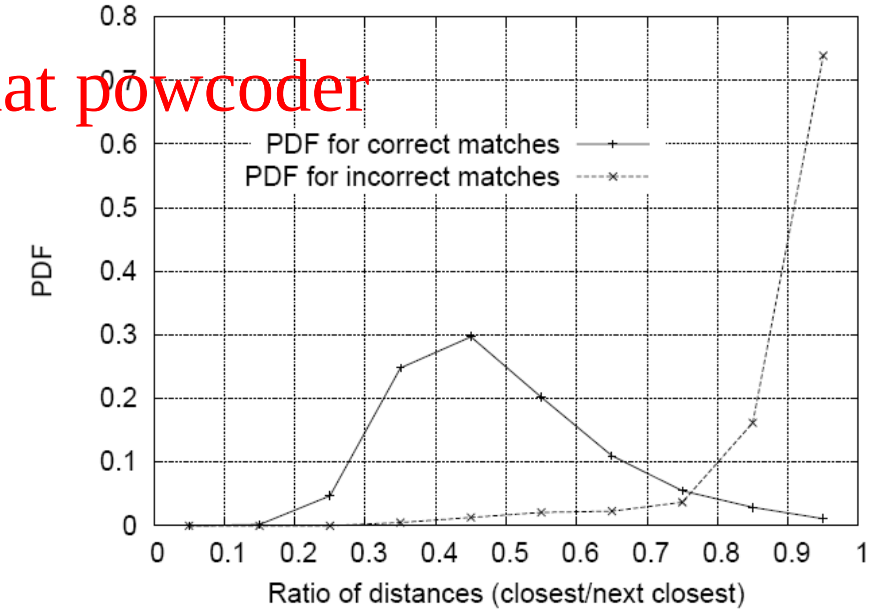
Feature Matching

- **Simple Solution**
  - Refine matched points using threshold ratio of nearest to 2nd nearest descriptor

$$k_1 = \min_j dist(f_i, g_j)$$

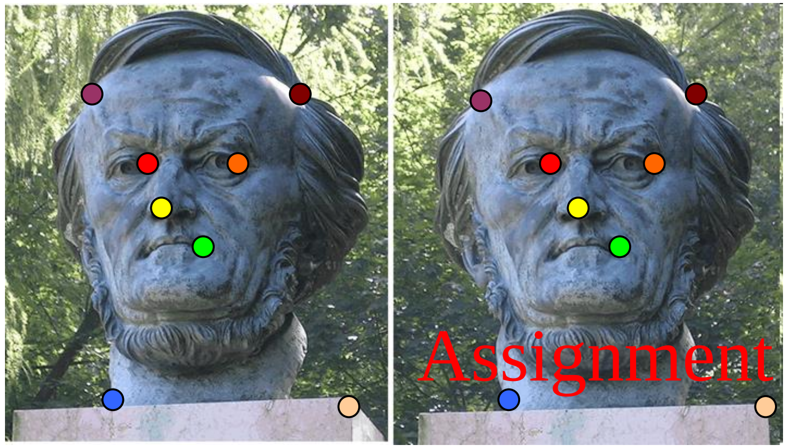
$$k_2 = \text{second}\min_j dist(f_i, g_j)$$

$$\frac{dist(f_i, g_{k_1})}{dist(f_i, g_{k_2})} < T_r \rightarrow NN(f_i) = g_{k_1}$$

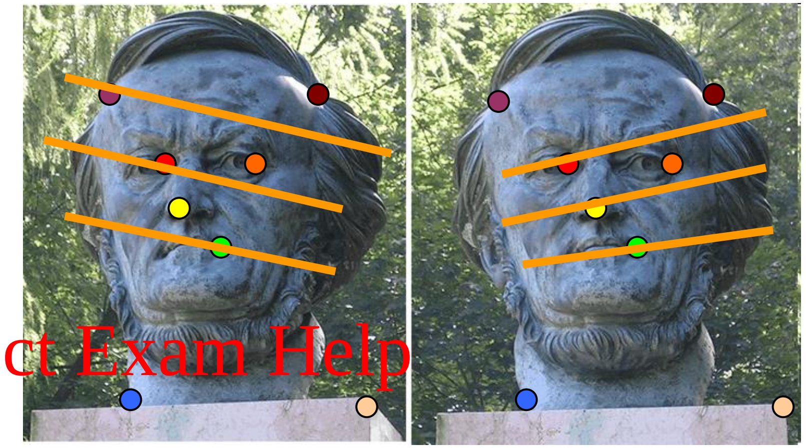


This gets rid of 90% false matches, 5% of true matches in Lowe's study

Feature Matching and Fitting



Feature matching



Fundamental
matrix estimation

Feature matching

Image stitching using
geometric transform
estimation

Next topic

- **We've learned how to process pixels and detect features, e.g. edges, corners, blobs.**
  Now what?

- **A higher-level, more compact representation of the features in the image by grouping multiple features according to a simple model**

  - Prerequisite

    - Review EBU6230 Image/Video Processing – Week3: Interest points
    - Review EBU7240 Computer Vision – Week1: feature (what you learned today)