EBU7240

Computer Vision

- Restoration: spatial filtering -

Semester 1, 2021

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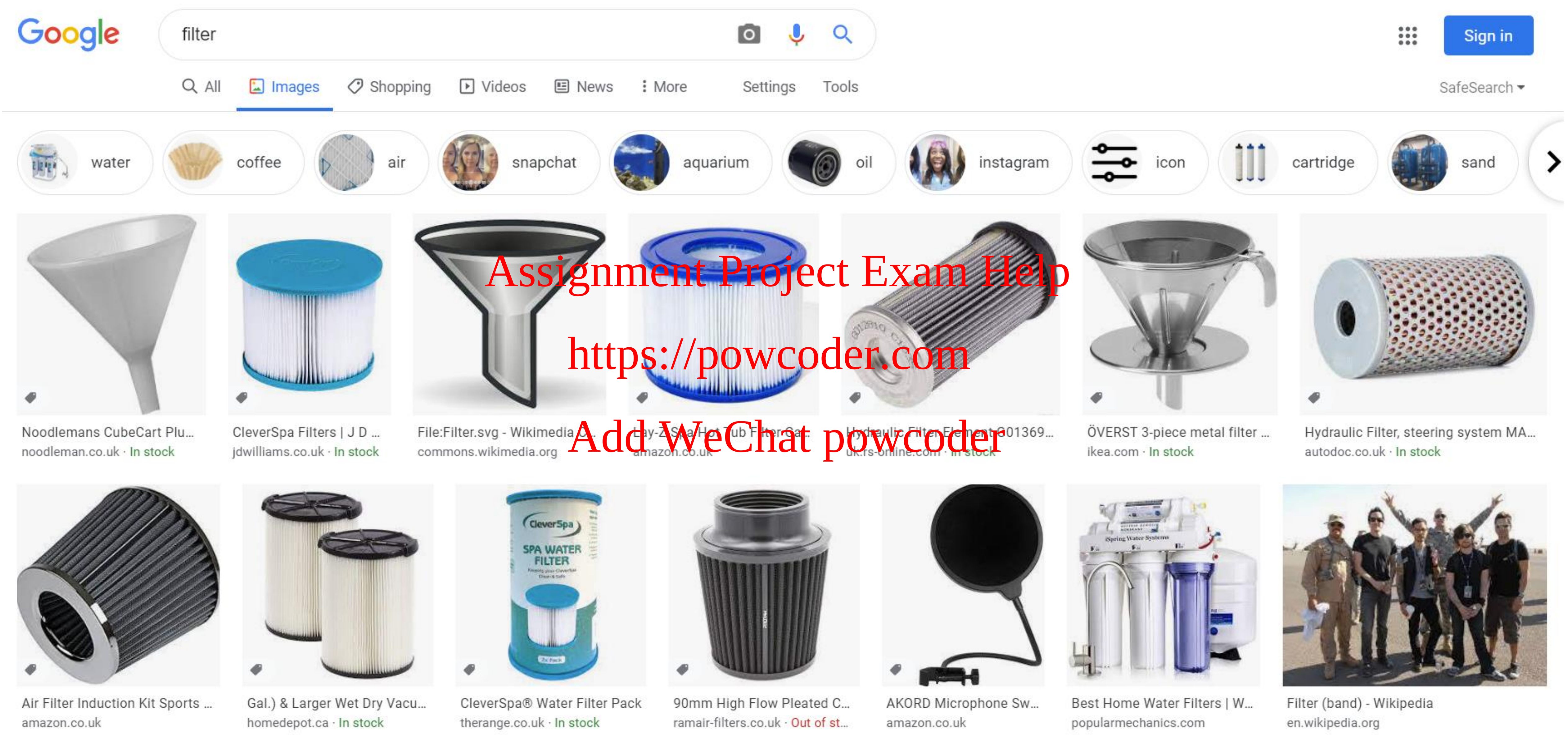
Google
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CleverSpa Filters | J D ...
jdwilliams.co.uk · In stock
File:Filter.svg - Wikimedia C...
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ikea.com · In stock
Hydraulic Filter, steering system MA...
autodoc.co.uk · In stock
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homedepot.ca · In stock
CleverSpa® Water Filter Pack
therange.co.uk · In stock
90mm High Flow Pleated C...
ramair-filters.co.uk · Out of st...
AKORD Microphone Sw...
amazon.co.uk
Best Home Water Filters | W...
popularmechanics.com
Filter (band) - Wikipedia
en.wikipedia.org

Content

- **Neighborhood in an image**
- **Convolution: review**
  – From 'Signal Processing' Lecture
- **Spatial Filtering**
  – Low-pass (or high-pass) filter
  – Gaussian (or Laplacian) filter
  – Mean, median, or mode filter
  – Advanced filters: Bilateral filter, Non-local means filter

Neighborhood in an image

- Spatial filtering is performed for each pixel (i, j) using neighborhoods of (i, j).
  - $I(i, j)$: input image $O(i, j)$: output image
  - $w(s, t)$: filtering kernel (a.k.a. mask)

$$O(i, j) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s, t) I(i + s, j + t)$$

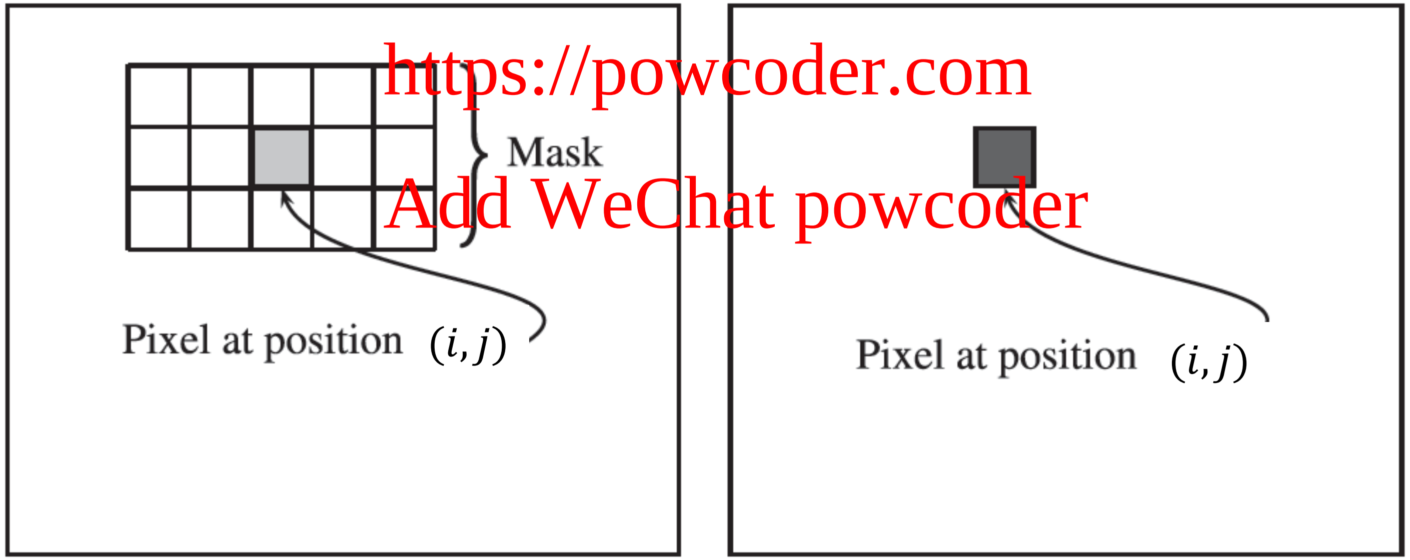
- The filtering operation is defined according to what kind of $w(s, t)$ is used.

- The filter serves as an essential building block for many applications.
  - Blurring, sharpening, image restoration, and so on

Spatial Filtering: Filter Kernel (Mask)

- **Filter kernel** should be defined accordingly, depending on applications.

$$O(i,j) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) I(i+s, j+t)$$

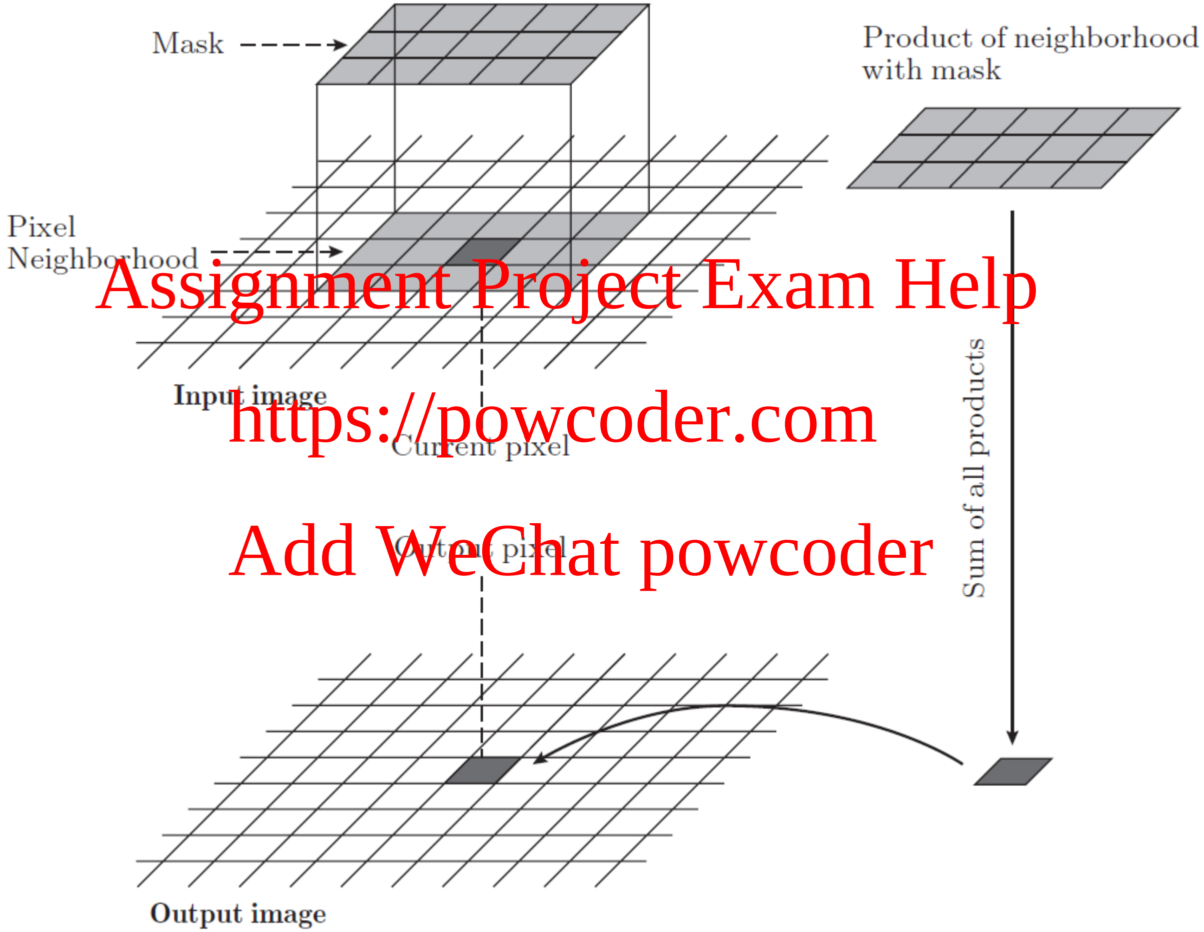


(a) (b)

Filtering = SUM (Mask .× Neighborhood)

$$O(i,j) = \sum \left(\begin{bmatrix} w(-1,-2) & w(-1,-1) & w(-1,0) & w(-1,1) & w(-1,2) \\ w(0,-2) & w(0,-1) & w(0,0) & w(0,1) & w(0,2) \\ w(1,-2) & w(1,-1) & w(1,0) & w(1,1) & w(1,2) \end{bmatrix} .\times \begin{bmatrix} I(i-1,j-2) & I(i-1,j-1) & I(i-1,j) & I(i-1,j+1) & I(i-1,j+2) \\ I(i,j-2) & I(i,j-1) & I(i,j) & I(i,j+1) & I(i,j+2) \\ I(i+1,j-2) & I(i+1,j-1) & I(i+1,j) & I(i+1,j+1) & I(i+1,j+2) \end{bmatrix} \right)$$

Filtering = SUM (Mask .× Neighborhood)



Convolution: From the perspective of signal processing

- **Filtering = Convolution!**
  - When the filter kernel is symmetric

- Let's start with 1-D convolution (from signal processing)
  - Convolution integral

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

For continuous signal

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

For discrete signal

$$y[n] = \cdots + x[-1]\mathrm{h}[n+1] + x[0]h[n] + x[1]h[n-1] + \cdots$$

Convolution: From the perspective of signal processing

- **From the perspective of signal processing**
  - An output signal $y[n]$ is obtained by passing an input signal $x[n]$ to a discrete system with a response function $h[n]$.

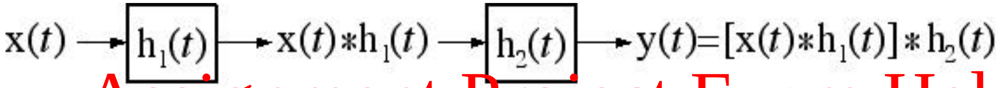
$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

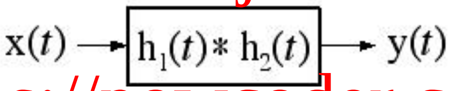
$$x[n] \longrightarrow \boxed{h[n]} \longrightarrow y[n]$$

Note **convolution** in spatial domain = **multiplication** in frequency domain

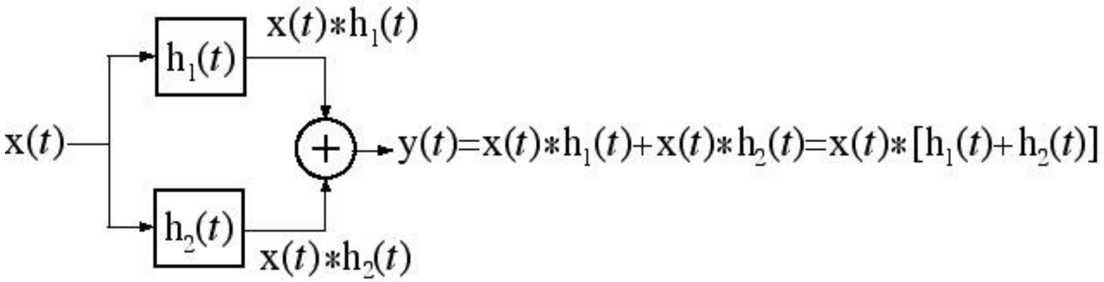
Convolution Properties

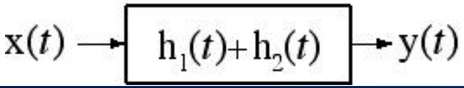
- **Associative property**
  - Impulse response of a cascade connection
    = Convolution of the individual impulse responses





- **Distributive property**
  - Impulse response of a parallel connection of LTI systems
    = Sum of the individual impulse responses.



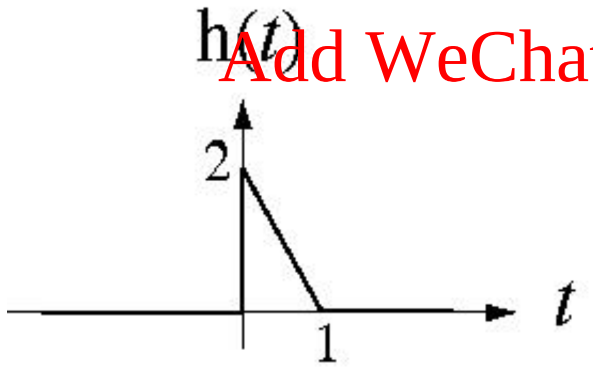


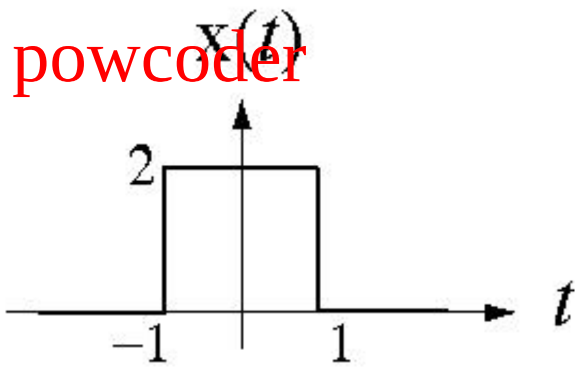
Example

- **The convolution on the continuous domain**

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

For illustration, let an input signal *x(t)* and the impulse response *h(t)* be the two functions below.





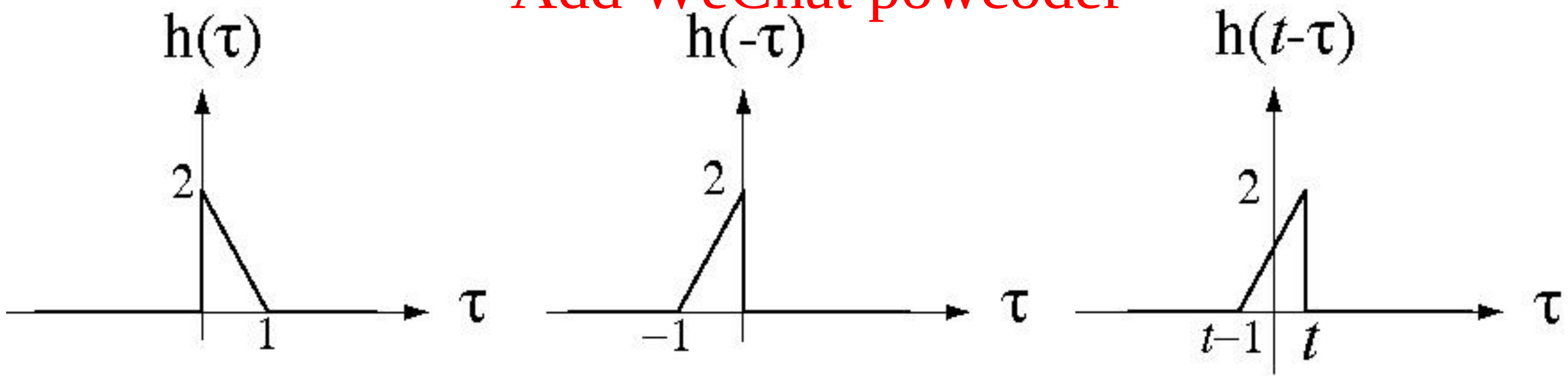
Example

- **The convolution on the continuous domain**

$$y(t) = x(t) * h(t) = \int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau$$

  – The functional transformation from $h(t)$ to $h(t-\tau)$

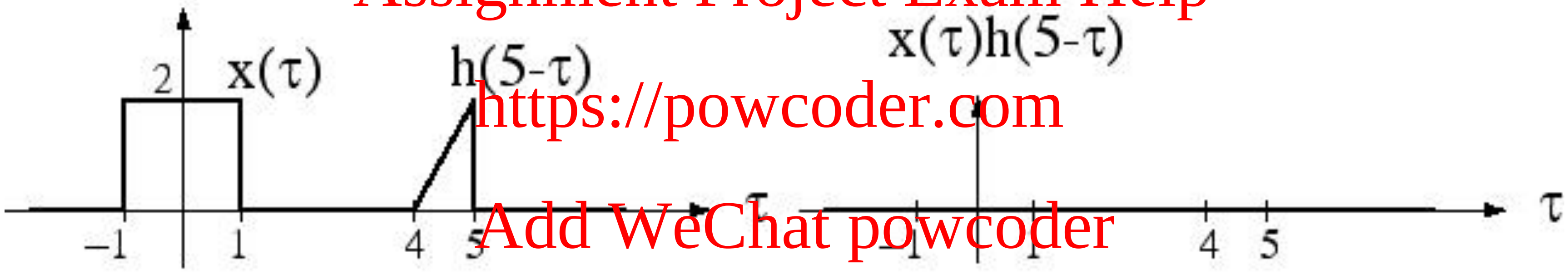
$$\mathrm{h}(\tau) \xrightarrow{\tau \to -\tau} \mathrm{h}(-\tau) \xrightarrow{\tau \to \tau - t} \mathrm{h}(-(\tau - t)) = \mathrm{h}(t-\tau)$$



Example

- **The convolution on the continuous domain**

The convolution value at t:
The area under the product of $x(t)$ and $h(t-\tau)$.
For example, let $t=5$.



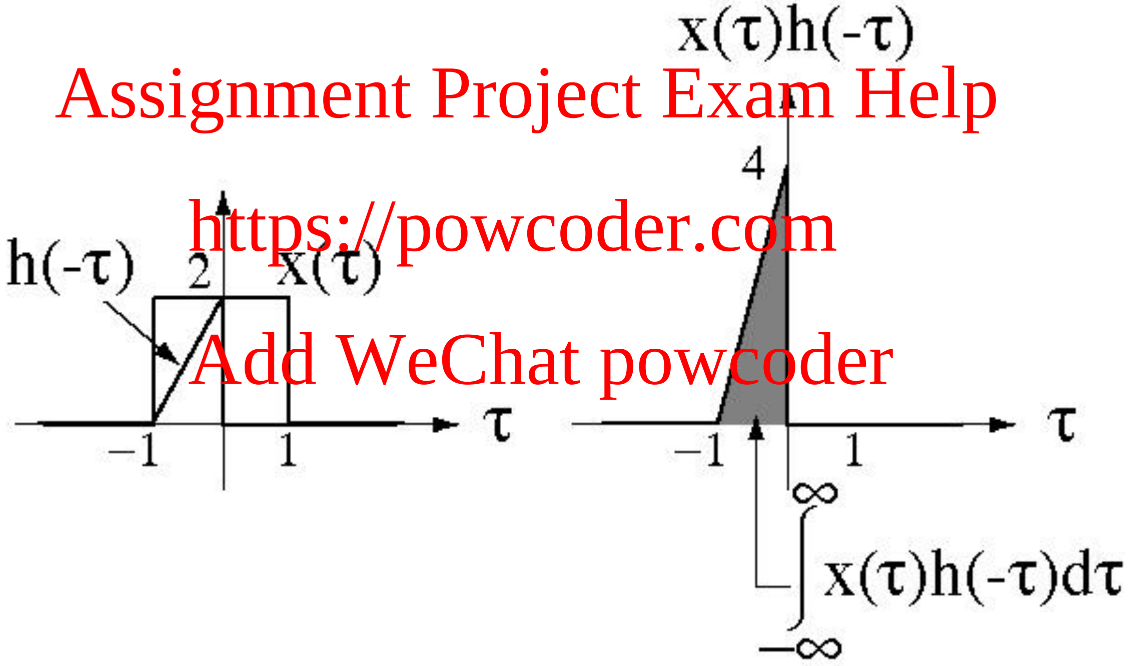
For $t=5$, the area under the product is zero.

$$y(5)=x(5)*h(5)$$
$$=\int_{-\infty}^{\infty} x(\tau)h(5-\tau)d\tau=0$$

Example

- **The convolution on the continuous domain**

When $t = 0$,

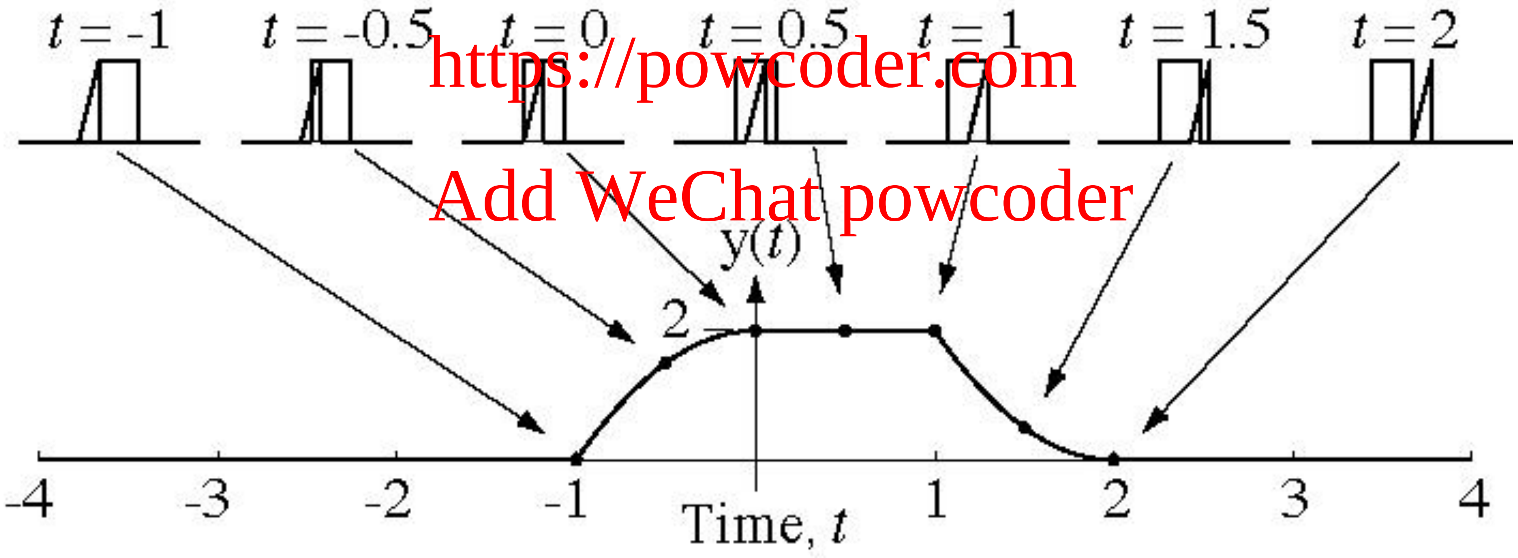


$$y(0) = x(0) * h(0) = \int_{-\infty}^{\infty} x(\tau)h(-\tau)d\tau = 2$$

Example

- **The convolution on the continuous domain**

The process of convolving to find $y(t)$ is illustrated below.

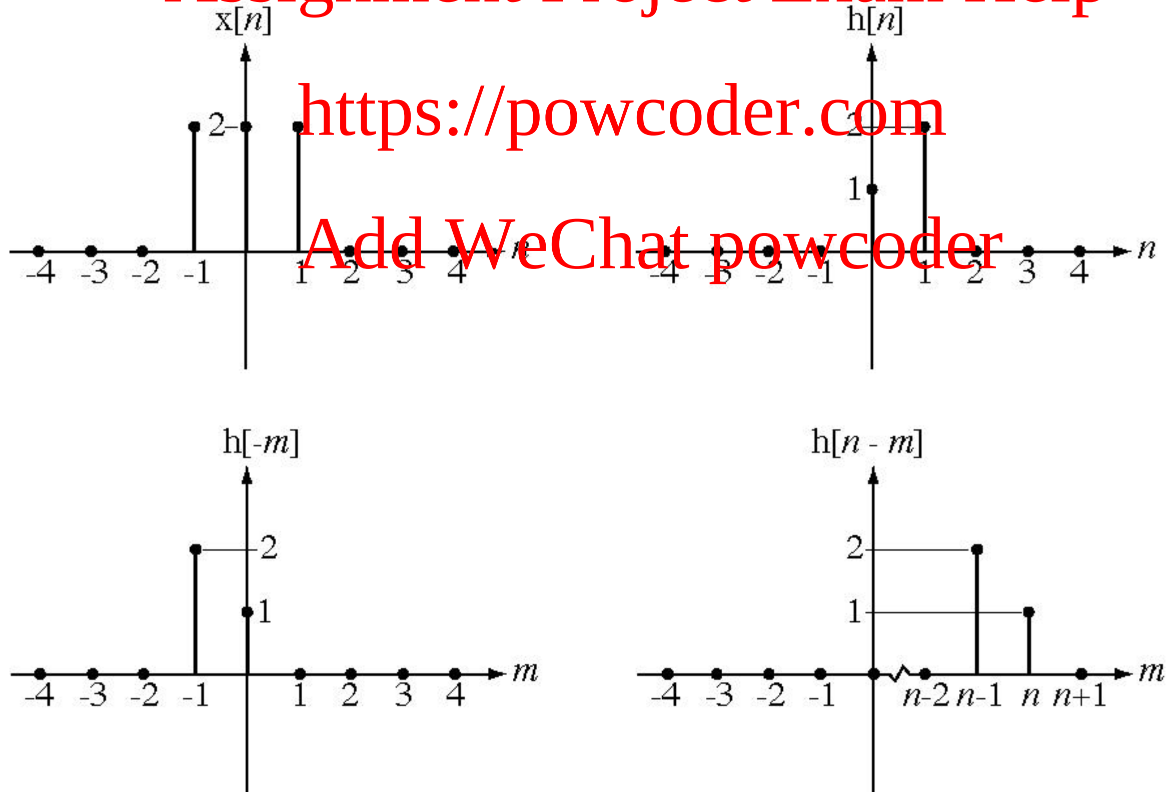


Example

- **The convolution on the discrete domain**

$$y[n] = x[n] * h[n] = \sum_{m=-\infty}^{\infty} x[m]h[n-m]$$

What is $y[n]$?



Note) x: M points, h: N points
→ y: $M + N - 1$ points

Convolution vs. Filtering

1D convolution

$$y[i] = x[i] * h[i]$$
$$= \sum_{s=-\infty}^{\infty} h[s]\,x[i-s]$$

Note) This formulation is different from previous slides, but the result is identical. Namely, $x * h = h * x$.

1D filtering

$$O[i] = \sum_{s=-a}^{a} w[s]\,I[i+s]$$

Remember 2D filtering

$$O(i,j) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t)\,I(i+s, j+t)$$

Suppose $h[i]$ exists for $-a \le i \le a$ and is symmetric, i.e., $h[-i] = h[i]$.
Then, 1D convolution = 1D filtering!
This also applies to N-D convolution and N-D filtering ($N \ge 2$).

2D convolution - example

Original

*

0	0	0
0	1	0
0	0	0

= ?

Source from D. Lowe

2D convolution - example

Original

*

•0	•0	•0
•0	•1	•0
•0	•0	•0

=

Filtered
(no change)

2D convolution - example

Original

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} = \ ?$$

Source from D. Lowe

2D convolution - example

Original

$$* \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} =$$

Shifted right
By 1 pixel

Source from D. Lowe

2D convolution - example

Original

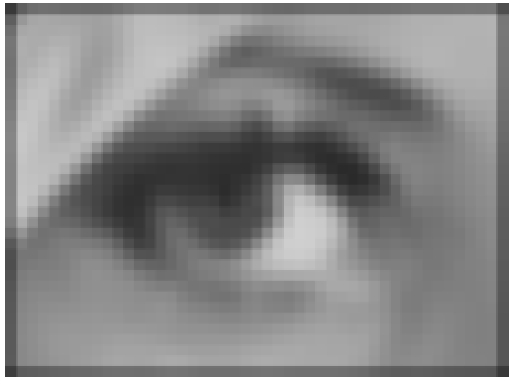
$$* \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

Source from D. Lowe

2D convolution - example

Original

$$* \ \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} =$$



Blur (with a box filter)

Source from D. Lowe

2D convolution - example

Original

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = ?$$

(Note that filter sums to 1)

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}}_{\text{“details of the image”}}$$

Source from D. Lowe

2D convolution - example

- Removing Blurred region means:

−

=

- Adding it back means:

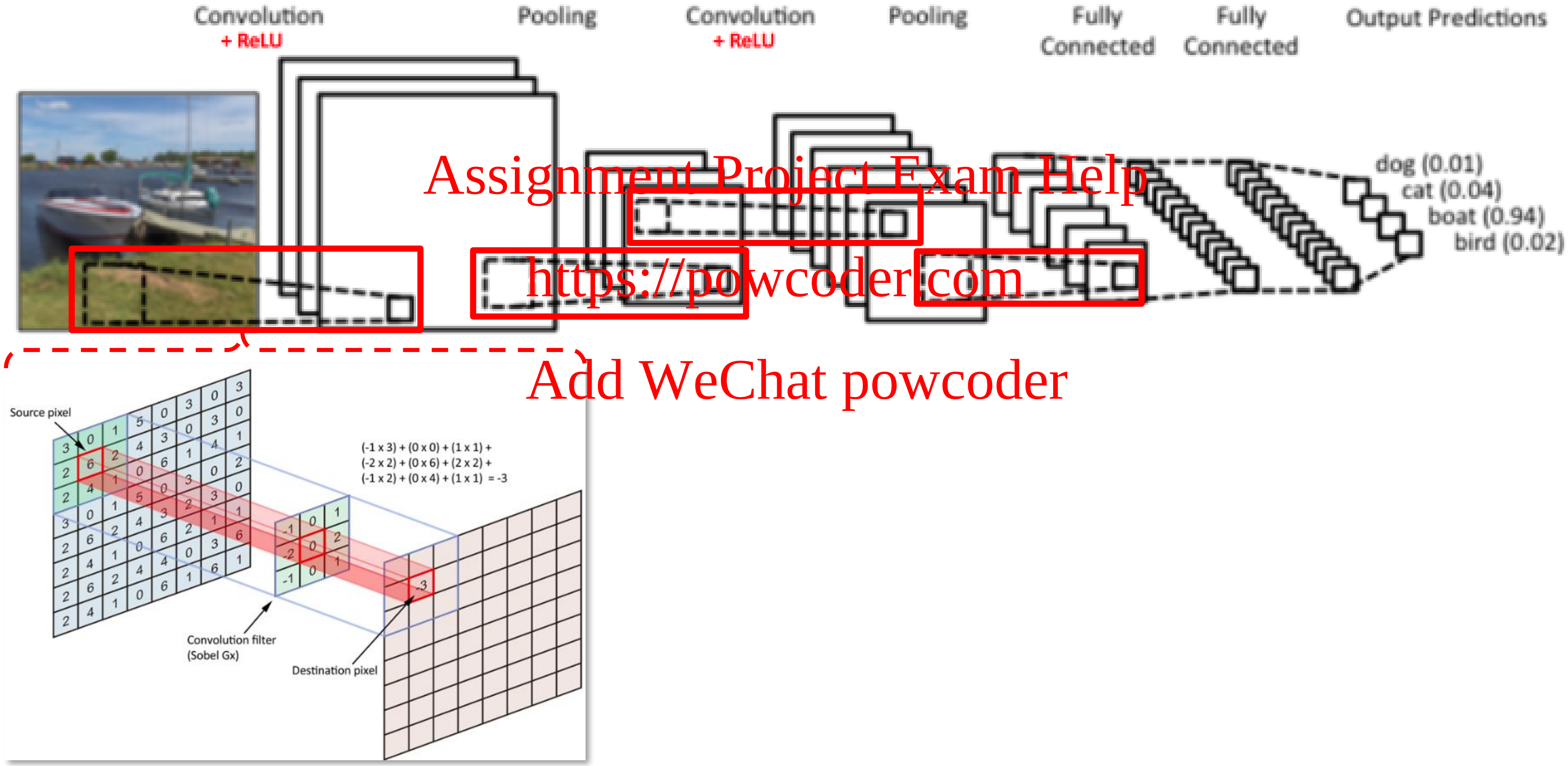
+ a

=

Source from L. Fei-Fei

2D convolution - example

- Deep learning: Convolutional Neural Network



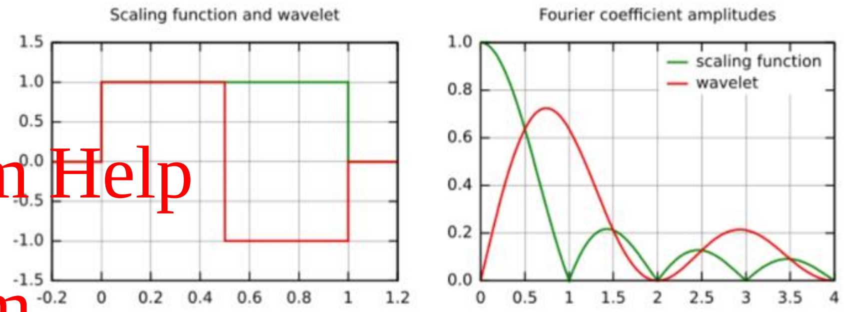
Credit: https://ujjwalkarn.me/2016/08/11/intuitive-explanation-convnets/

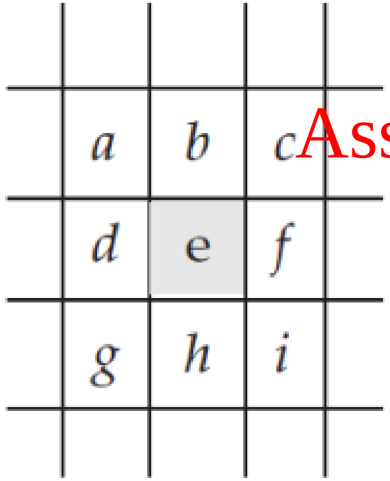
2D Image Filtering: Uniform Mean Filter

- **Uniform mean filter**
  - The simplest **low-pass** filter

EBU6018- remember?

Haar functions in frequency domain





$$\rightarrow \frac{1}{9}(a+b+c+d+e+$$

Filter kernel

$$\begin{bmatrix} \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \\ \frac{1}{9} & \frac{1}{9} & \frac{1}{9} \end{bmatrix} = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

2D Image Filtering: Uniform Mean Filter

- **How to process pixels at image boundary?**
  - 1) Mirroring at an image boundary

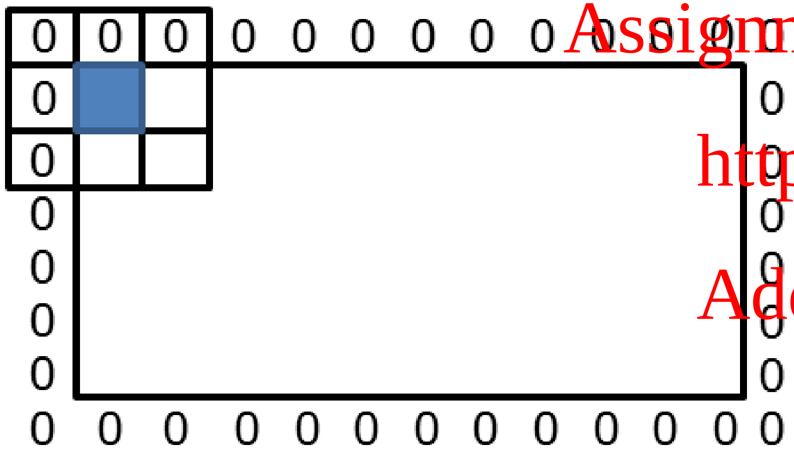
2	2	3	4	5	4	3	2	1	2	3	4	4
2	2	3	4	5	4	3	2	1	2	3	4	4
3	3										3	3
4	4										4	4
5	5										5	5
6	6										6	6
7	7	6	5	4	3	2	1	2	3	4	5	5
7	7	6	5	4	3	2	1	2	3	4	5	5

$$O(i,j) = \sum_{s=-1}^{1}\sum_{t=-1}^{1} w(s,t)\, I'(i+1+s, j+1+t)$$

I': mirror image

2D Image Filtering: Uniform Mean Filter

- **How to process pixels at image boundary?**
  - 2) Zero padding at an image boundary
  - Common way in convolutional neural networks

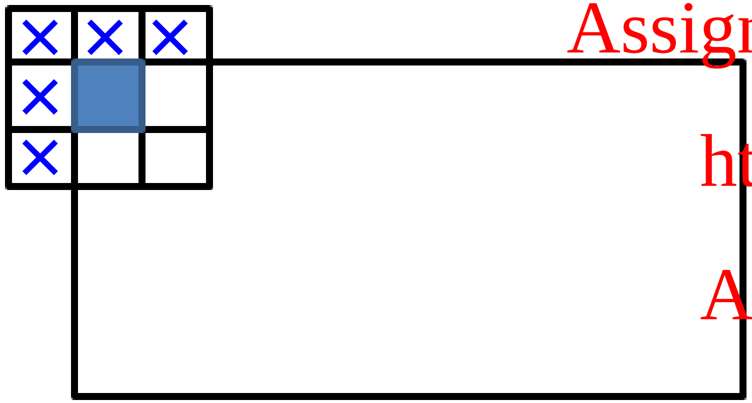


$$O(i,j) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} w(s,t) I'(i+1+s, j+1+t)$$

I': zero-padded image

2D Image Filtering: Uniform Mean Filter

- **How to process pixels at image boundary?**
  - 3) Adjusting filter kernel



$$O(i,j) = \frac{\sum_{s=-1}^{1}\sum_{t=-1}^{1} w'(s,t) I(i+s, j+t)}{\sum_{s=-1}^{1}\sum_{t=-1}^{1} w'(s,t)}$$

$$w'(s,t) = (0 \le i+s \le H-1 \;\&\; 0 \le j+t \le W-1 \; ? \; w(s,t) : 0)$$

2D Image Filtering: Uniform Mean Filter

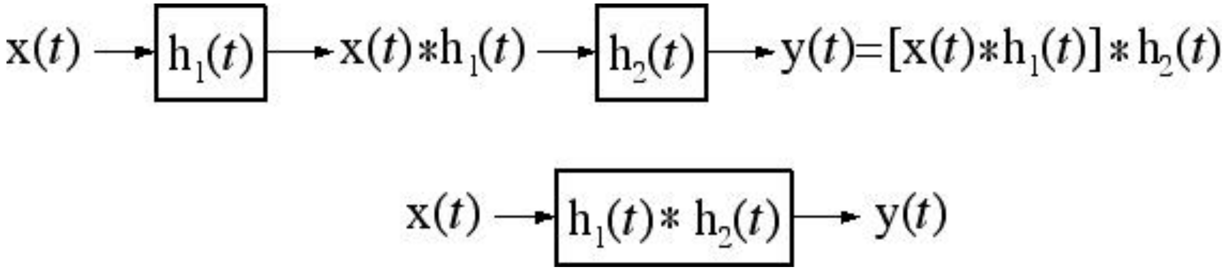
(a) original image, (b) 3×3 filter with zero padding: $\frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, (c) 9×9 filter with zero padding: $\frac{1}{81}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$,(d) 25×25 filter with zero padding: $\frac{1}{625}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$, (e) 25×25 filter with mirroring: $\frac{1}{625}\begin{bmatrix} 1 & \cdots & 1 \\ \vdots & \ddots & \vdots \\ 1 & \cdots & 1 \end{bmatrix}$

2D Image Filtering: Uniform Mean Filter

- **Uniform mean filtering is separable.**
  - Separable filtering is VERY important in terms of runtime.

$$w(s,t) = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = w_s(s,0) * w_s(0,t)$$

Cascade Connection



2D Image Filtering: Gaussian Filter

- **Gaussian distribution**
  - One of the most commonly used parametric models
  - $\boldsymbol{Fourier\ transform}\,(Gaussian\ func.) = \ Gaussian\ func.$
  - Rotationally symmetric
  - Separable filter
  - $Gaussian\ *\ Gaussian\ =\ Gaussian$

1-D

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

2-D

$$f(x,y) = \frac{1}{2\pi\sigma_x\sigma_y}\exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x{}^2}-\frac{(y-\mu_y)^2}{2\sigma_y{}^2}\right)$$

N-D

$$f(\boldsymbol{m}) = \frac{1}{|2\pi|^{N/2}|\Sigma|^{1/2}}\exp(-(\boldsymbol{m}-\boldsymbol{\mu})^T\Sigma^{-1}(\boldsymbol{m}-\boldsymbol{\mu}))$$

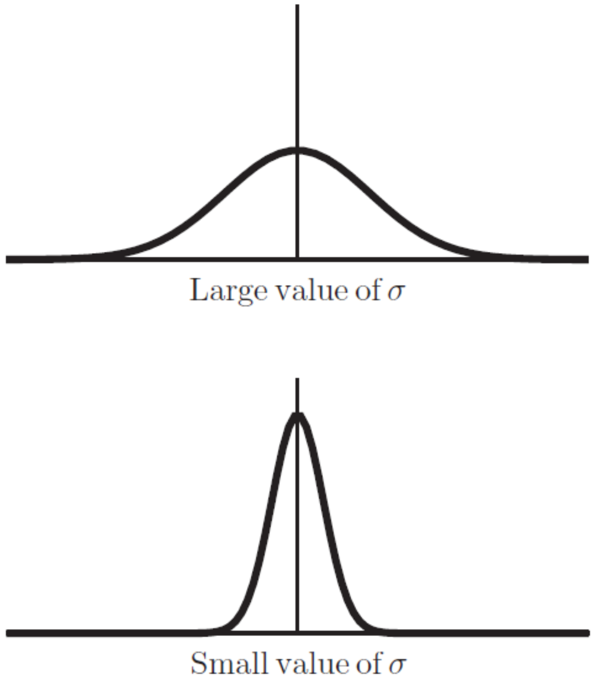
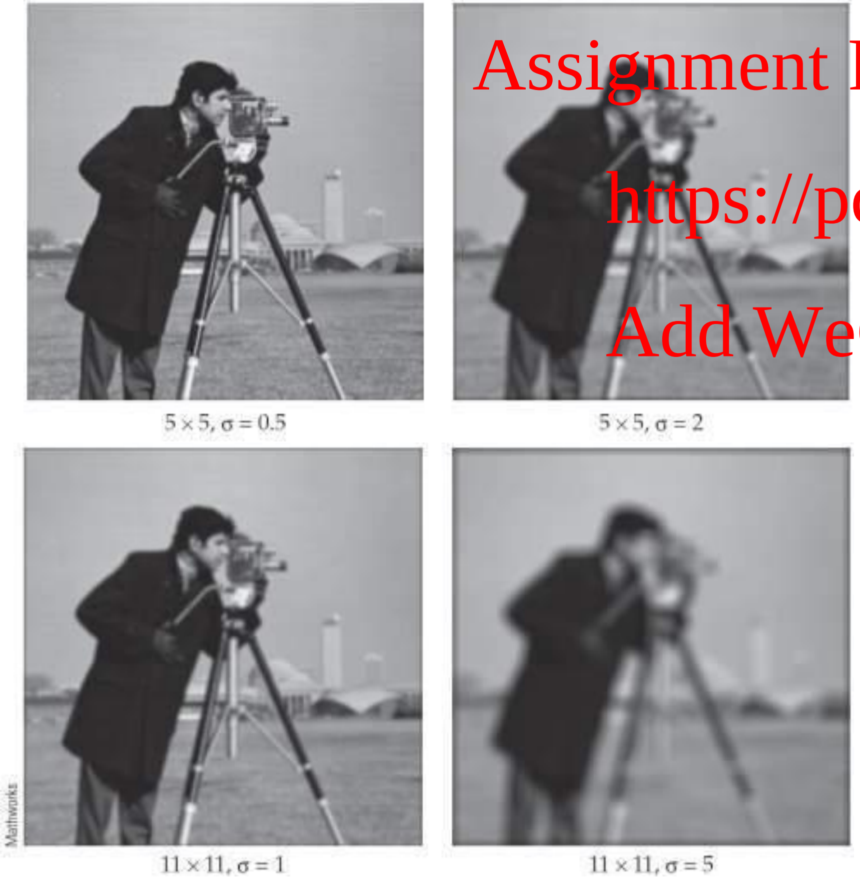


FIGURE 5.7 *One-dimensional Gaussians.*

Q: What if σ approaches infinite?

2D Image Filtering: Gaussian Filter

- **Gaussian filter's advantage**
  – It considers spatial distances within neighborhoods
  – Blurred results looks more natural compared to mean filter.



<u>Practical Implementation</u>

$$O(i,j) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) I(i+s, j+t)$$

Zero mean Gaussian filter

$$w(s,t) = \frac{1}{\sum_{m=-a}^{a} \sum_{n=-b}^{b} \exp\left(-\frac{m^2}{2{\sigma_s}^2} - \frac{n^2}{2{\sigma_t}^2}\right)} \exp\left(-\frac{s^2}{2{\sigma_s}^2} - \frac{t^2}{2{\sigma_t}^2}\right)$$

In results, $\mu_s = \mu_t = 0$ and $\sigma_s = \sigma_t = \sigma$

2D Image Filtering: Gaussian Filter

- **Gaussian filter's advantage**
  - Separable

$$\frac{1}{2\pi\sigma_s\sigma_t}\exp\left(-\frac{s^2}{2{\sigma_s}^2}-\frac{t^2}{2{\sigma_t}^2}\right)=\frac{1}{\sqrt{2\pi}\sigma_s}\exp\left(-\frac{s^2}{2{\sigma_s}^2}\right)\frac{1}{\sqrt{2\pi}\sigma_t}\exp\left(-\frac{t^2}{2{\sigma_t}^2}\right)$$

<u>**Practical implementation**</u>

$$O(i,j)=\sum_{s=-a}^{a}\sum_{t=-b}^{b}w(s,t)I(i+s,j+t)$$

$$w(s,t)=\frac{1}{\sum_{m=-a}^{a}\sum_{n=-b}^{b}\exp\left(-\frac{m^2}{2{\sigma_s}^2}-\frac{n^2}{2{\sigma_t}^2}\right)}\exp\left(-\frac{s^2}{2{\sigma_s}^2}-\frac{t^2}{2{\sigma_t}^2}\right)$$

$$\Updownarrow$$

$$O(i,j)=\sum_{s=-a}^{a}w_s(s)\sum_{t=-b}^{b}w_t(t)\,I(i+s,j+t)$$

$$w_s(s)=\frac{1}{\sum_{m=-a}^{a}\exp\left(-\frac{m^2}{2{\sigma_s}^2}\right)}\exp\left(-\frac{s^2}{2{\sigma_s}^2}\right)$$

$$w_t(t)=\frac{1}{\sum_{n=-b}^{b}\exp\left(-\frac{n^2}{2{\sigma_t}^2}\right)}\exp\left(-\frac{t^2}{2{\sigma_t}^2}\right)$$

Frequency: Low- and High-pass Filters

- **Frequency in an image**
  - How often do intensity values vary in neighbourhoods?
- **Low-pass filter:** uniform average filter, Gaussian filter
- **High-pass filter:** Sobel filter, Laplacian filter

(this estimates intensity change.)

Example of Low-pass filter: $\frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$

Example of High-pass filter: $\begin{bmatrix}0 & 1 & 0\\ 1 & -4 & 1\\ 0 & 1 & 0\end{bmatrix}$

2D Image Filtering: Sobel Filter

- Using the first order gradient

$$\nabla I = (I_x, I_y) = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y}\right)$$

Practical implementation

$$\nabla I = (I_x, I_y) = (|S_x * I|, |S_y * I|)$$

For color image,
$M(x,y) = (M_R(x,y) + M_G(x,y) + M_B(x,y))/3$

Sobel filter output $M(x,y) = \sqrt{{I_x}^2 + {I_y}^2}$ or $|I_x| + |I_y|$

S_x

-1	-2	-1
0	0	0
1	2	1

S_y

-1	0	1
-2	0	2
-1	0	1

2D Image Filtering: Laplacian Filter

- **Using the second order gradient**

Laplacian Filtering output $G(x,y) = \left| \frac{\partial^2 I}{\partial x^2} + \frac{\partial^2 I}{\partial y^2} \right|$

- 1st derivative of an image $I(x,y)$

$$\nabla I(x,y) = \left(\frac{\partial I}{\partial x}, \frac{\partial I}{\partial y} \right)$$

$$I_x(x,y) = \frac{\partial I}{\partial x} = I(x+1,y) - I(x,y) \qquad I_y(x,y) = \frac{\partial I}{\partial y} = I(x,y+1) - I(x,y)$$

- 2nd derivative of an image $I(x,y)$

$$\nabla^2 I(x,y) = \left(\frac{\partial^2 I}{\partial x^2}, \frac{\partial^2 I}{\partial y^2} \right)$$

$$\frac{\partial^2 I}{\partial x^2} = I_x(x,y) - I_x(x-1,y) = I(x+1,y) + I(x-1,y) - 2I(x,y)$$

$$\frac{\partial^2 I}{\partial y^2} = I_y(x,y) - I_y(x,y-1) = I(x,y+1) + I(x,y-1) - 2I(x,y)$$

2D Image Filtering: Laplacian Filter

- **Using the second order gradient**

Laplacian filter L

0	1	0
1	-4	1
0	1	0

or

1	1	1
1	-8	1
1	1	1

More general form

$\frac{\alpha}{1+\alpha}$	$\frac{1-\alpha}{1+\alpha}$	$\frac{\alpha}{1+\alpha}$
$\frac{1-\alpha}{1+\alpha}$	$\frac{-4}{1+\alpha}$	$\frac{1-\alpha}{1+\alpha}$
$\frac{\alpha}{1+\alpha}$	$\frac{1-\alpha}{1+\alpha}$	$\frac{\alpha}{1+\alpha}$

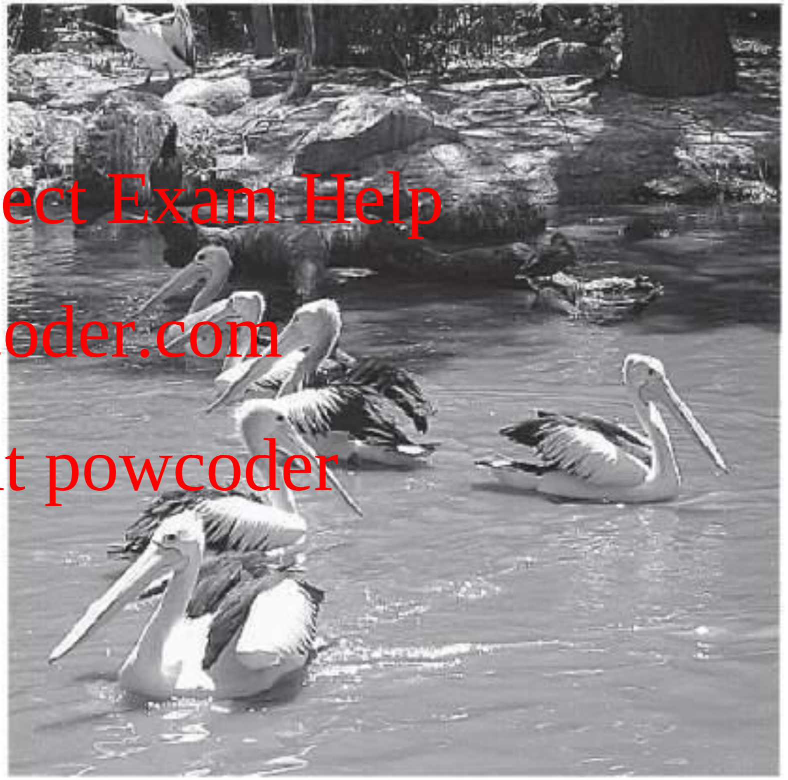
$$O = |L * I|$$

For color image,
$O(x,y) = \left(O_R(x,y) + O_G(x,y) + O_B(x,y)\right)/3$

Unsharp Masking

- Makes an image look sharper by boosting high-frequency components.

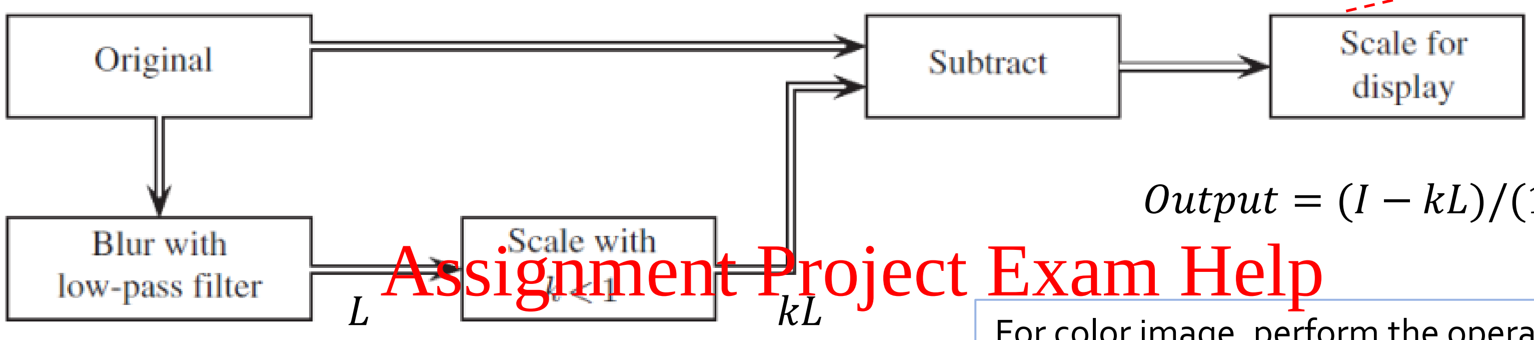
Original image



Unsharp mask result

Unsharp Masking

Scaling matters!



$$Output = (I - kL)/(1 - k)$$

For color image, perform the operation for RGB channels independently.

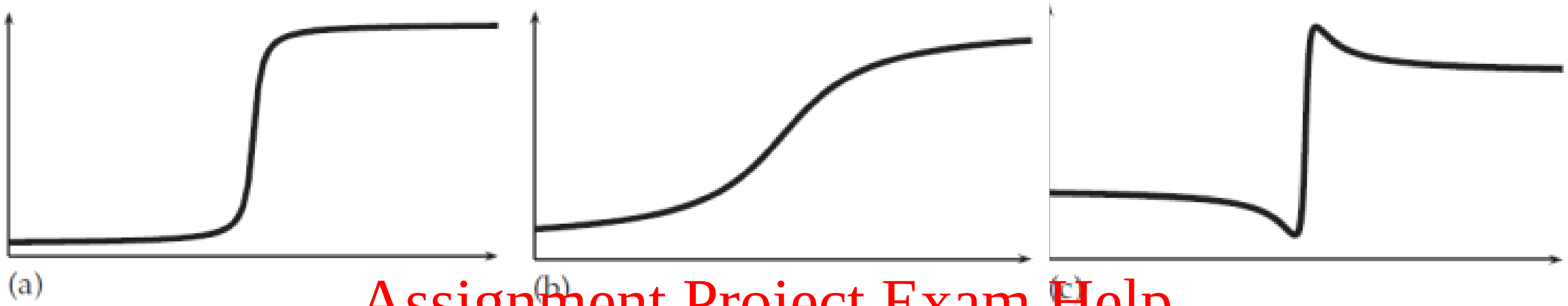
Matlab Code

```
x = double(imread('cameraman.tif'))/255;

f = fspecial('average');
xf = filter2(f,x);
figure, imshow(xf)

% k: parameter determining the amount of reducing low-frequency component
% 0<=k<=1
% As k increases, the output image looks sharper. But, it is recommended using k below 0.5;
k = 0.5;
fi = zeros(3);      fi(2,2)=1;
f2 = (fi - k*f)/(1-k);
xf2 = filter2(f2,x);
figure, imshow(xf2)
```

Unsharp Masking



$Output = (I - kL)/(1 - k)$

$$w = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - k \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

Scaling

$$w = \frac{1}{1-k} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} - \frac{k}{1-k} \begin{bmatrix} 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \\ 1/9 & 1/9 & 1/9 \end{bmatrix}$$

2D Image Filtering: Nonlinear Filter

- **Linear vs. Nonlinear filters?**
  - Definition of linearity, and its advantages

- **Some instances of nonlinear filter**
  - Max, Min, Median filter


Original


Max filter


Min filter


Median filter

2D Image Filtering: Nonlinear Filter

- **Median Filter**
  1. Sort pixels within the window centered at reference pixel
  2. Select the median value as an output.

Q: Mean vs. Median?

Original

Max filter

Min filter

Median filter

Summary

- **Spatial Filtering**
  - Depends on how to define a filter kernel.
  - Linear vs. Nonlinear filter: Linear filter is **separable**!
  - Mean, median, maximum, minimum, low-pass, high-pass, Gaussian, Laplacian filter
  - Edge Sharpening: combination of low-pass and high-pass filters.

- **Convolution**
  - Same as filter
  - Commutativity, Cascade property, Parallel property

EBU7240

Computer Vision

- Restoration: noise and noise removal -

Semester 1, 2021

Changjae Oh

Contents

- **Image Degradation Model**
  - Image Noise
- **Noise removal**
  - Salt-and-Pepper Noise Removal
  - Gaussian Noise Removal
  - Periodic Noise Removal

Image Degradation Model

- **Image restoration**
  - Aims to reduce the image degradation
- **Types of image degradation**
  - Noise, our-of-focus blur, motion blur

Image Degradation Model

- When an input degraded image $g(x,y)$ is given, our goal is to estimate $f(x,y)$.

$$g(x,y) = h(x,y) * f(x,y) + n(x,y)$$

- $n(x,y)$: Additive noise
- $h(x,y)$: Blur kernel, which is the same as filtering mask.

- In frequency domain (using 2D DFT)?

$$G(u,v) = H(u,v)F(u,v) + N(u,v)$$

$$\Rightarrow F(u,v) = \big(G(u,v) - N(u,v)\big)/H(u,v)$$

(x,y): 2D image coordinate
(u,v): 2D frequency coord.

Is that it? It is not such a simple problem.
1) $N(u,v)$ is not known.
2) What if $H(u,v) = 0$?

Image Noise

- **Noise**
  - Any kind of degradation in an image caused by external disturbance
  - To make the problem simple, we pre-assume the noise models.
    The most appropriate restoration may vary depending on the noise types.

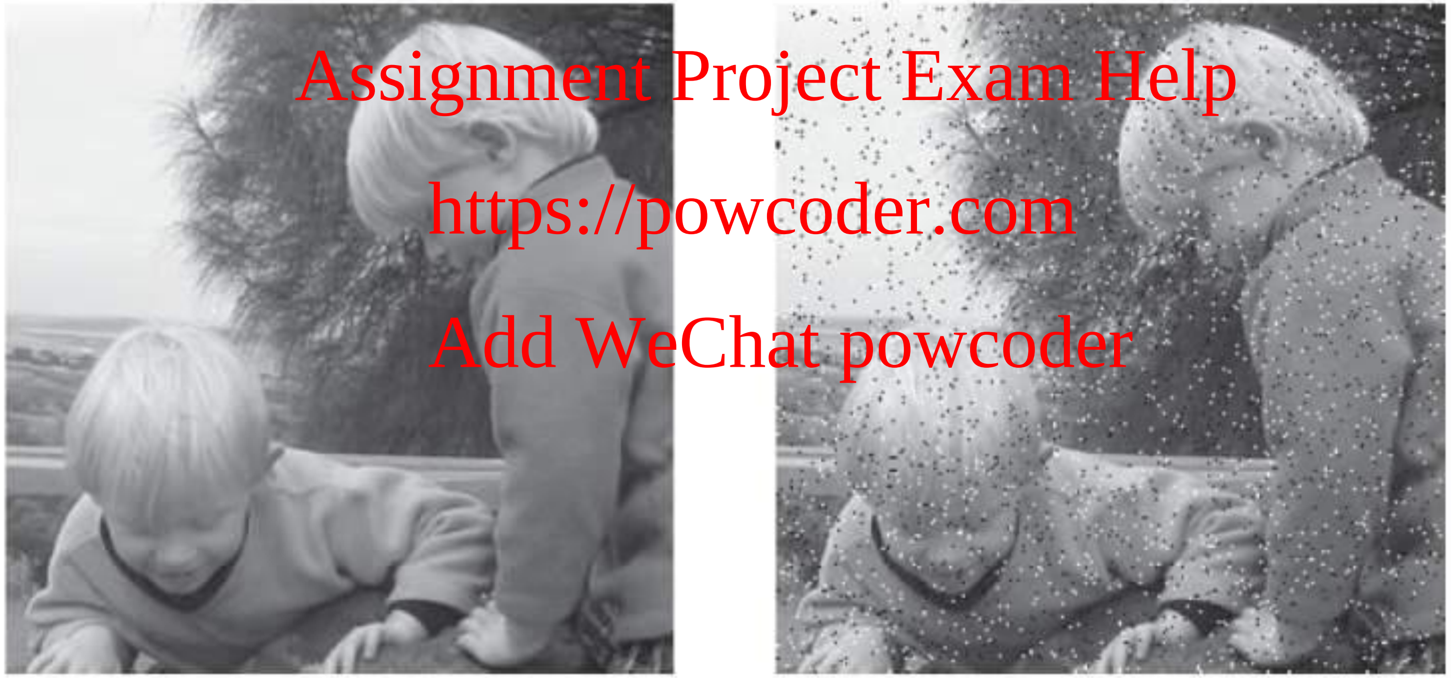
- **Noise types**
  - Periodic noise, Salt and Pepper noise, Gaussian noise, Speckle noise

- **In the beginning, let's think of the case using $h(x,y)$=1**

$$g(x, y) = f(x, y) + n(x, y)$$

Image Noise: Salt and Pepper Noise

- Sharp and sudden disturbances
- Image is randomly scattered as white (salt) or black (pepper) pixels.



(a) (b)

FIGURE 8.1 *Noise on an image. (a) Original image. (b) With added salt and pepper noise.*

Image Noise: Gaussian Noise

- **Additive White Gaussian Noise (AWGN)**
  - **Additive** noise: $I_G(x, y) = I(x, y) + N(x, y)$
  - **White** noise: randomly fluctuated and normally distributed
  - **Most approaches assume this type of noise.**

  - Usually, zero mean AWGN is assumed ($\mu = 0$).
    The probability density function (PDF) P of a Gaussian random variable z is

$$P(z = N) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{(z-\mu)^2}{2\sigma^2}\right)$$

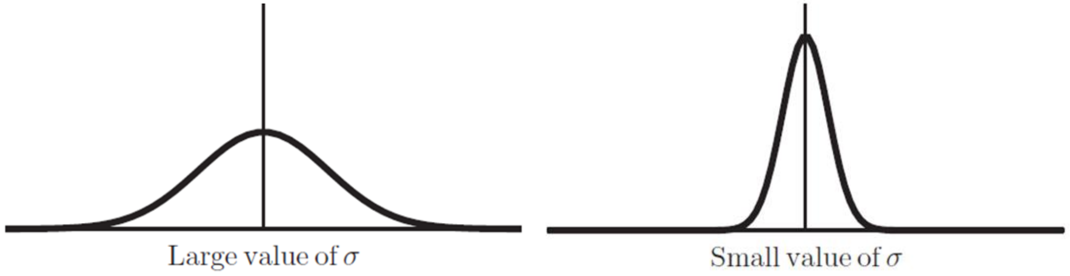
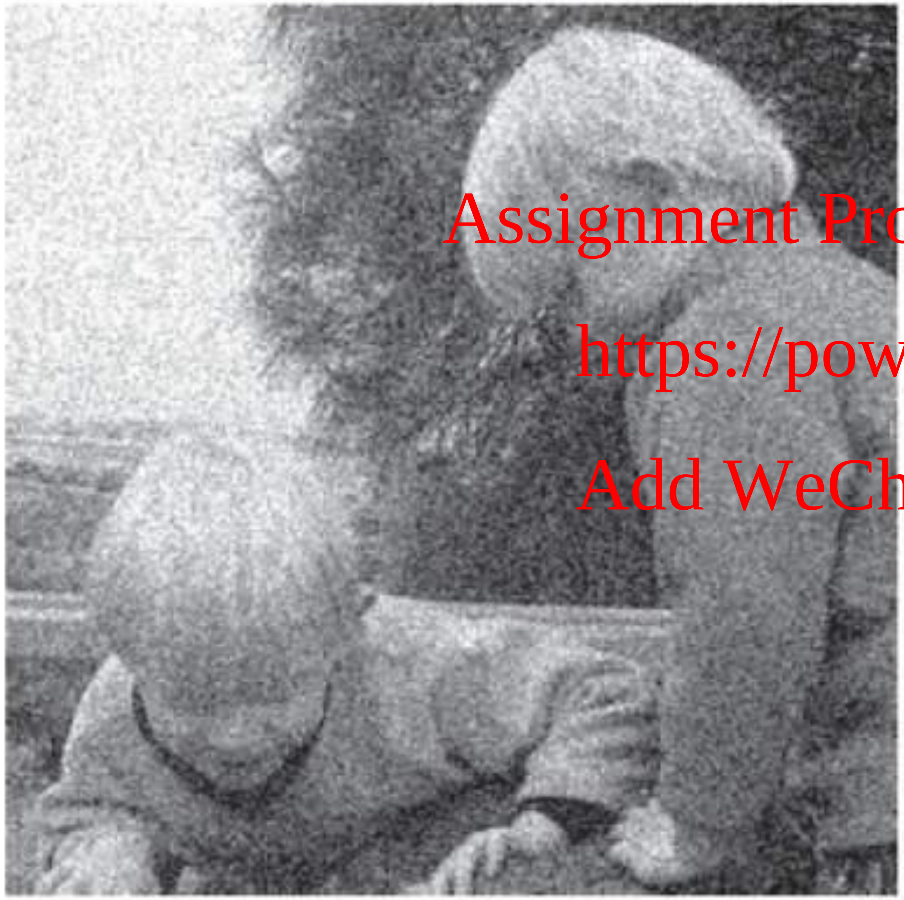


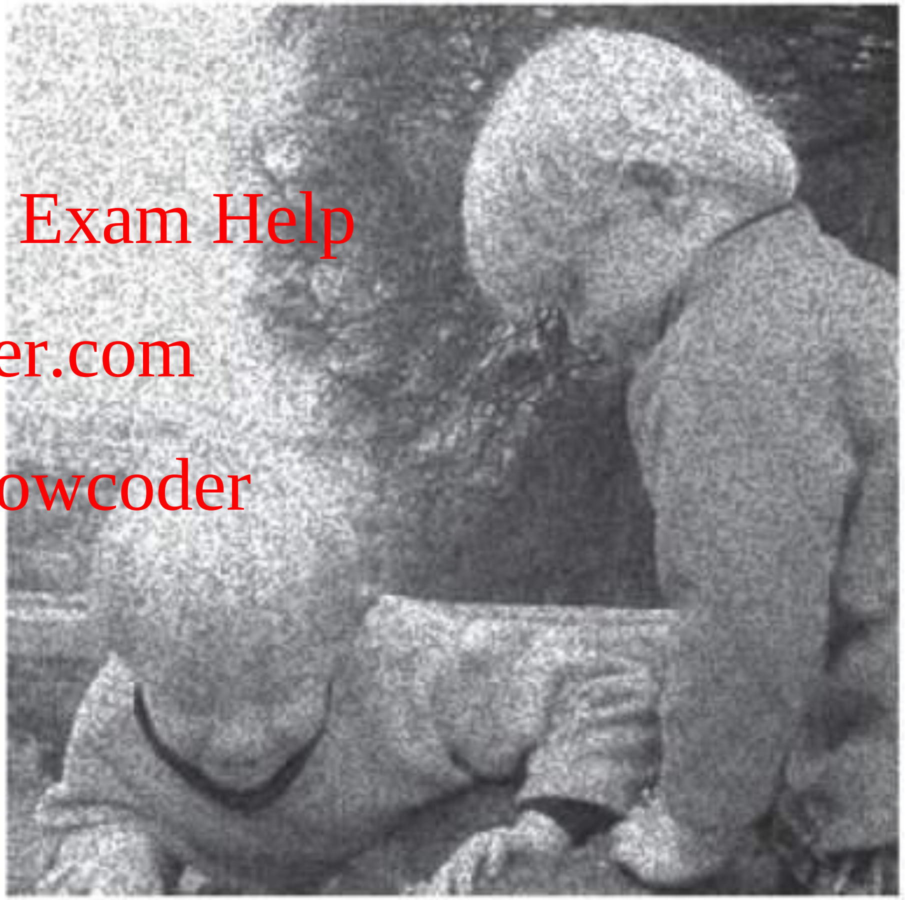
Image Noise: Speckle Noise

- **Multiplicative Noise**
  - $I_1(x,y) = I(x,y) + I(x,y)N(x,y)$
  - $N(x,y)$: zero mean uniform distributed function with σ
  - This noise is usually in the active radar, synthetic aperture radar (SAR), medical ultrasound and optical coherence tomography images.
  - The reduction of Speckle noise is MUCH more difficult due to the multiplication

Image Noise: Gaussian Noise vs. Speckle Noise



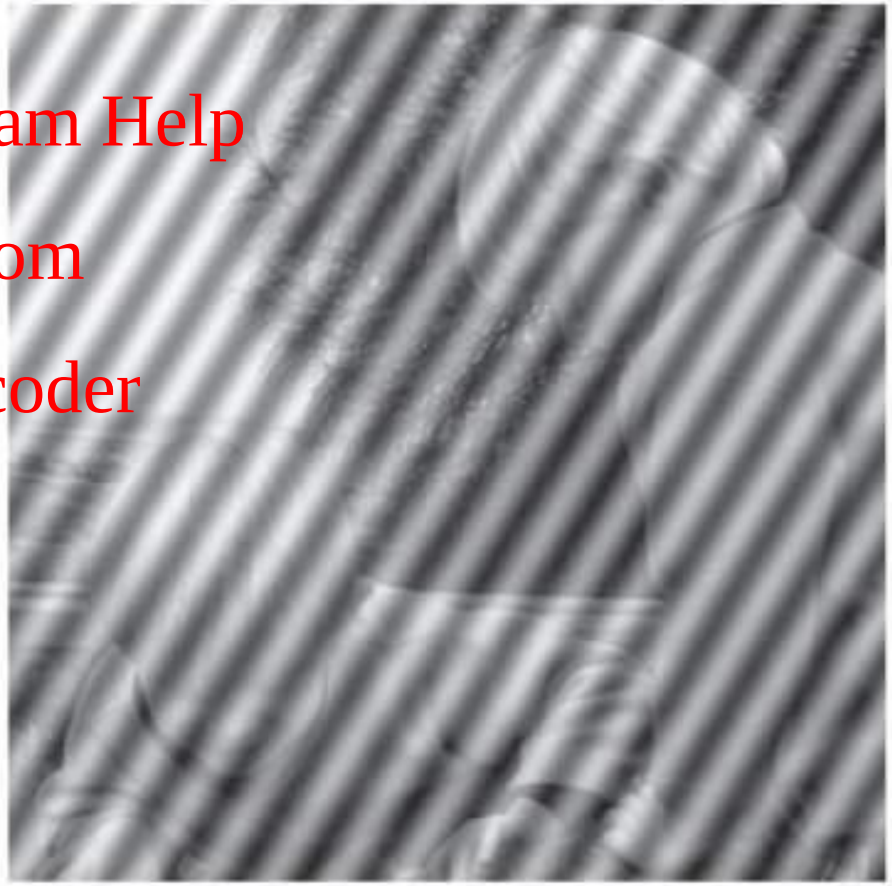
Gaussian Noise



Speckle Noise

Image Noise: Periodic Noise

- **Periodic fluctuation**
  - Electrical or electromechanical interference during image acquisition
  - Spatially dependent noise
  - It can be modeled as sinusoid waves



Contents

- **Image Degradation Model**
  - Image Noise
- **Noise removal**
  - Salt-and-Pepper Noise Removal
  - Gaussian Noise Removal
  - Periodic Noise Removal

Salt and Pepper Noise Removal

- **Low-Pass Filtering**
  - For instance, uniform averaging filter or Gaussian averaging filter
  - Not so effective

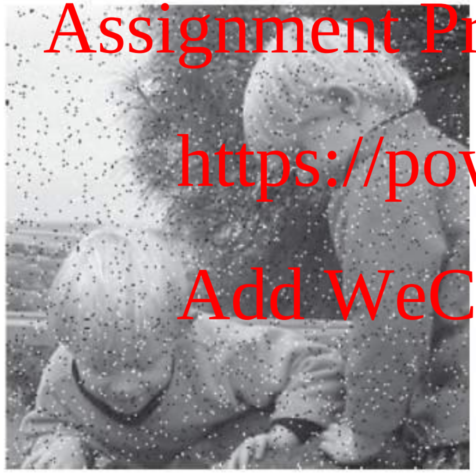
- **Median Filtering**
  - Median filter works well in the salt-and-pepper noise removal.
  - Rank-order Filtering is a general formulation of median filter.

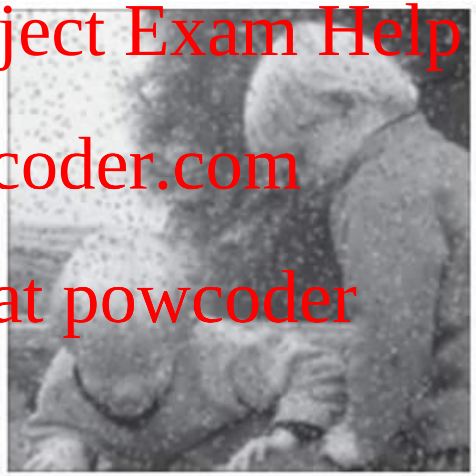
- **Outlier Rejection Method**

Salt and Pepper Noise Removal

- **Low-Pass Filtering**
  - NOT effective in removing the salt and pepper noise.


Original Image


10% salt and pepper noise
(salt: 5%, pepper: 5%)


3x3 average filter


7x7 average filter

Salt and Pepper Noise Removal: Median Filtering

- **Median filtering procedure**
  1. Sort pixels within the window centered at reference pixel
  2. Select the median value as an output.

$$\begin{array}{|c|c|c|}\hline 50 & 65 & 52 \\ \hline 63 & 255 & 58 \\ \hline 61 & 60 & 57 \\ \hline\end{array} \longrightarrow 50 \quad 52 \quad 57 \quad 58 \quad \boxed{60} \quad 61 \quad 63 \quad 65 \quad 255 \longrightarrow 60$$

  - Good performance in the salt-and-pepper noise removal

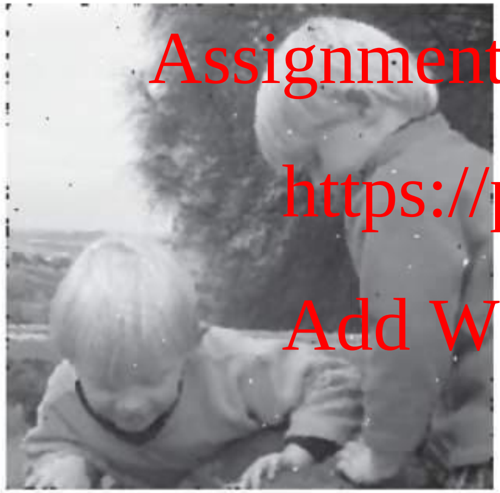
    Why?

3 × 3 median filter

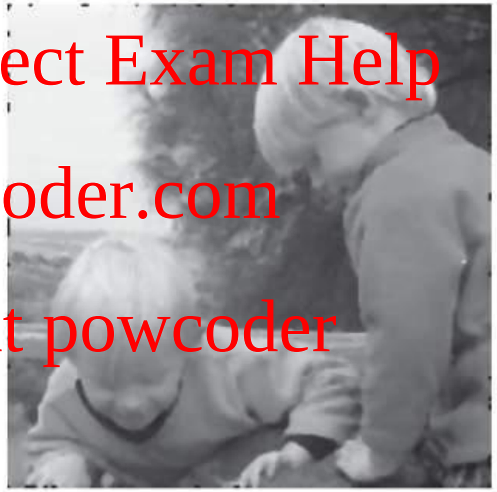
Salt and Pepper Noise Removal: Median Filtering

- More results using median filter

20% salt and pepper noise (salt: 10%, pepper: 10%)



Result using 3×3 median filter



Result applying 3×3 median filter twice

Result applying 5×5 median filter

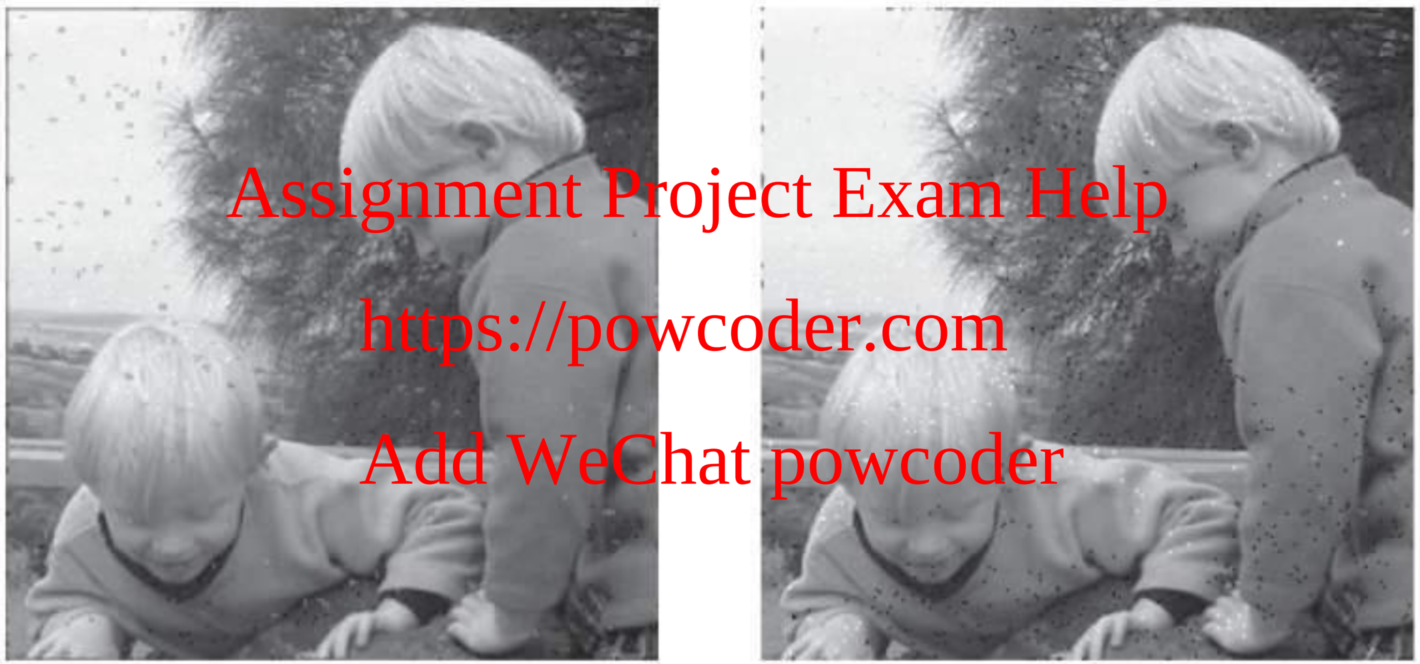
Salt and Pepper Noise Removal: Outlier Rejection Method

- **The brute force implementation of median filtering is very slow.**
  - Different method was proposed to remove the salt-and-pepper noise efficiently.

- **Key idea: Outlier detection → rejection**
  - Outliers usually tend to be different from neighboring pixels' intensities.

- **Outlier rejection method**
  1. Choose a threshold value D
  2. For a given pixel, compare its value p with the mean m of the values of its eight neighbors
  3. If $|p - m| > D$, then classify the pixel as noisy, otherwise not.
  4. If the pixel is noisy, replace its value with m.

Salt and Pepper Noise Removal: Outlier Rejection Method



(a) (b)

FIGURE 8.9 *Applying the outlier method to 10% salt and pepper noise. (a)* $D = 0.2$. *(b)* $D = 0.4$.

Gaussian Noise Removal

- **Why does an image average filter work well for Gaussian Noise removal?**

- **Additive White Gaussian Noise (AWGN) $N(x, y)$ is added as below.**
  - $I_G(x, y) = I(x, y) + N(x, y)$

1. **Suppose we have 100 noisy images I_G.**
   - $I_G{}^i(x, y) = I(x, y) + N^i(x, y) \quad i = 1, \ldots, 100$

2. **Average 100 noisy images**

10x10 kernel

$$\frac{1}{100}\sum_{i=1}^{100} I_G{}^i(x, y) = I(x, y) + \frac{1}{100}\sum_{i=1}^{100} N^i(x, y)$$

This will be close to 0, as AWGN N is a zero-mean Gaussian PDF.

Gaussian Noise Removal

- **Why does an image average filter work well for Gaussian Noise removal?**

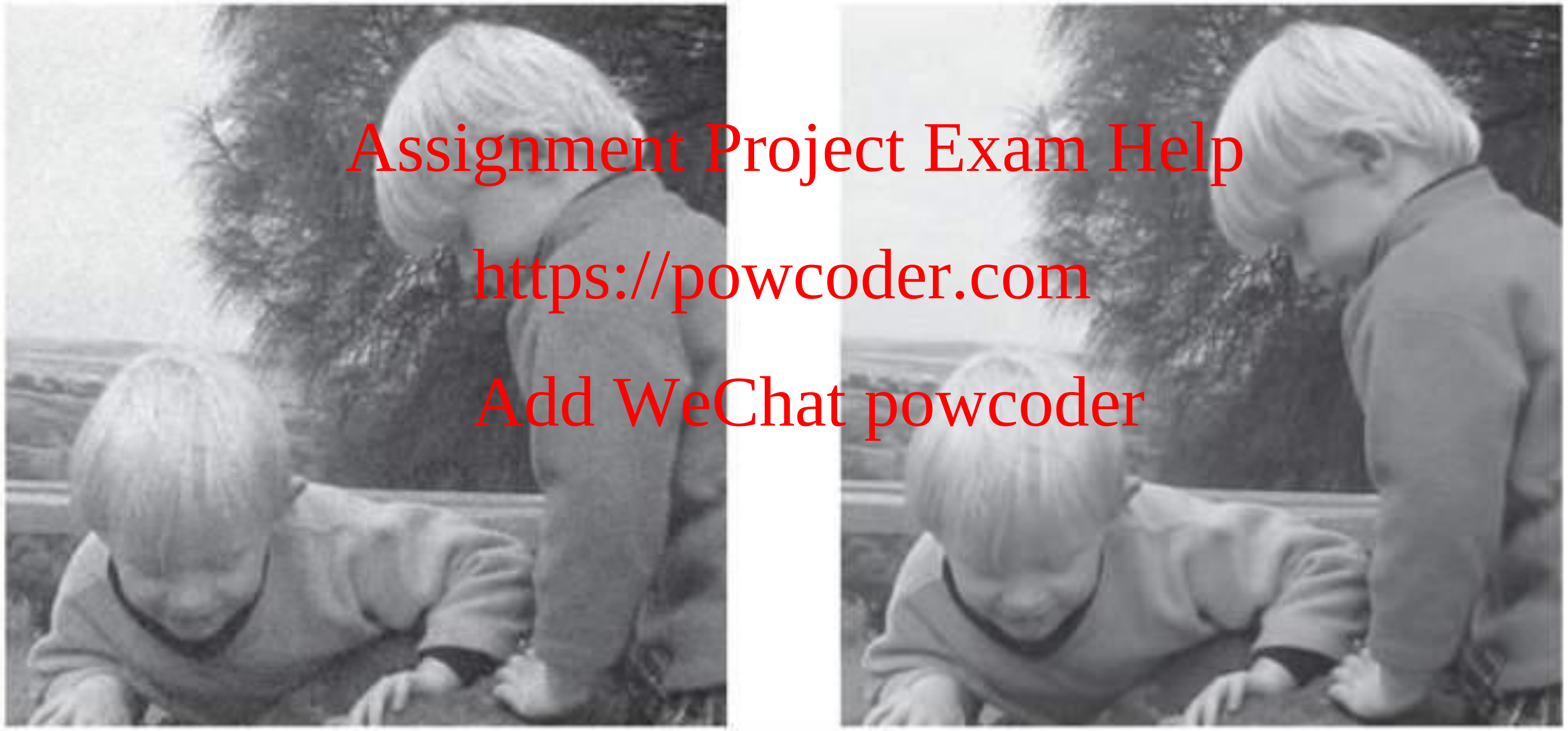
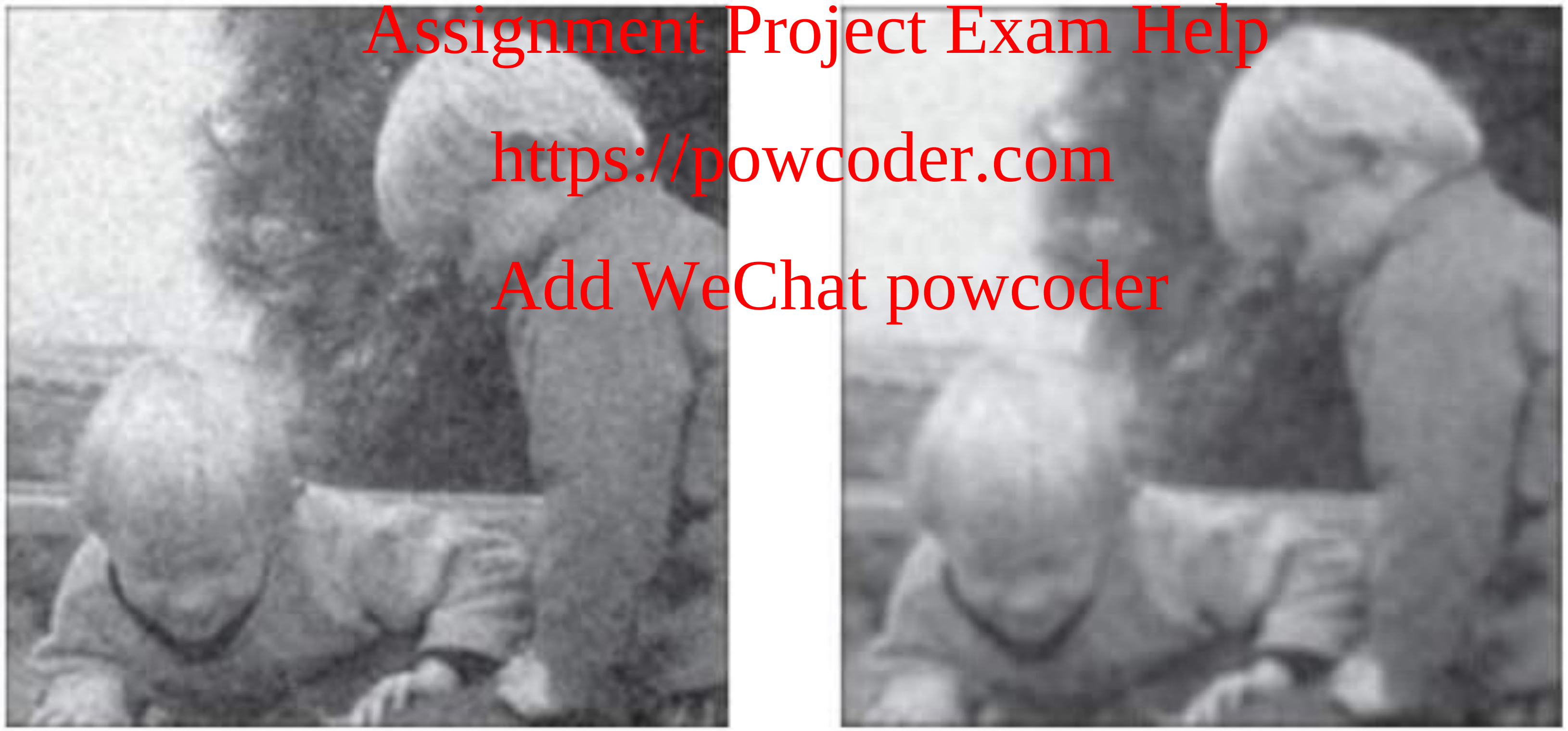


Image averaging to remove Gaussian noise. (a) 10 images (b) 100 images

Gaussian Noise Removal: Simple Average Filtering

- **Simple Average Filtering: Uniform mean filter or Gaussian filter**
  - Using a small window: not so effective in noise removal.
  - Using a large window: effective in noise removal, but the output is over-smoothed.

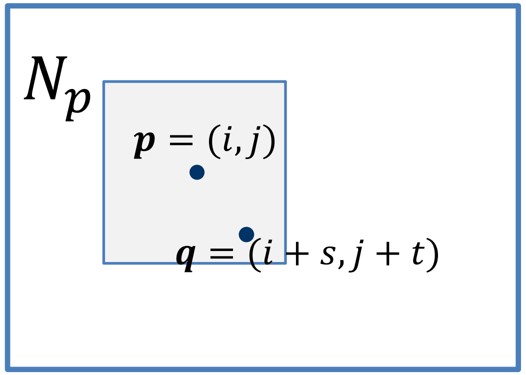


Uniform mean filter to remove Gaussian noise. (a) 3x3 averaging (b) 5x5 averaging

Gaussian Noise Removal: Bilateral Filtering

- **Bilateral filter for grayscale image**
  - One of the most popular filters with various applications
  - Considers both spatial and intensity distances

$$O(i,j) = \sum_{s=-a}^{a} \sum_{t=-b}^{b} w(s,t) I(i+s, j+t)$$



$$w(s,t) = \frac{1}{W(i,j)} \exp\left(-\frac{s^2}{2\sigma_s{}^2} - \frac{t^2}{2\sigma_t{}^2}\right) \exp\left(-\frac{(I(i,j) - I(i+s,j+t))^2}{2\sigma_r{}^2}\right)$$

$$W(i,j) = \sum_{m=-a}^{a} \sum_{n=-b}^{b} \exp\left(-\frac{m^2}{2\sigma_s{}^2} - \frac{n^2}{2\sigma_t{}^2}\right) \exp\left(-\frac{(I(i,j) - I(i+m,j+n))^2}{2\sigma_r{}^2}\right)$$

  - This can be rewritten as:

$$O_{\boldsymbol{p}} = \frac{1}{W_{\boldsymbol{p}}} \sum_{\boldsymbol{q} \in N_p} G_{\sigma_s}(|\boldsymbol{p} - \boldsymbol{q}|) G_{\sigma_r}\left(\left|I_{\boldsymbol{p}} - I_{\boldsymbol{q}}\right|\right) I_{\boldsymbol{q}}$$

$$W_{\boldsymbol{p}} = \sum_{\boldsymbol{q} \in N_p} G_{\sigma_s}(|\boldsymbol{p} - \boldsymbol{q}|) G_{\sigma_r}\left(\left|I_{\boldsymbol{p}} - I_{\boldsymbol{q}}\right|\right)$$

Gaussian Noise Removal: Bilateral Filtering

- **Bilateral filter for color image**
  - Applying filter to each channel

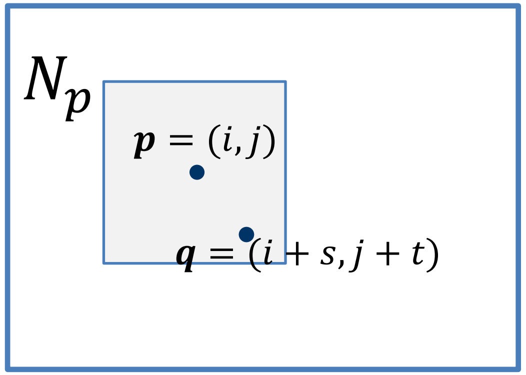
$$C_{\boldsymbol{p}} = (R_{\boldsymbol{p}}, G_{\boldsymbol{p}}, B_{\boldsymbol{p}})^{\mathrm{T}}$$

$$R_{\boldsymbol{p}} = \frac{1}{W_{\boldsymbol{p}}} \sum_{\boldsymbol{q} \in N_p} G_{\sigma_s}(|\boldsymbol{p} - \boldsymbol{q}|) G_{\sigma_r}(|C_{\boldsymbol{p}} - C_{\boldsymbol{q}}|) R_{\boldsymbol{q}}$$

$$G_{\boldsymbol{p}} = \frac{1}{W_{\boldsymbol{p}}} \sum_{\boldsymbol{q} \in N_p} G_{\sigma_s}(|\boldsymbol{p} - \boldsymbol{q}|) G_{\sigma_r}(|C_{\boldsymbol{p}} - C_{\boldsymbol{q}}|) G_{\boldsymbol{q}}$$

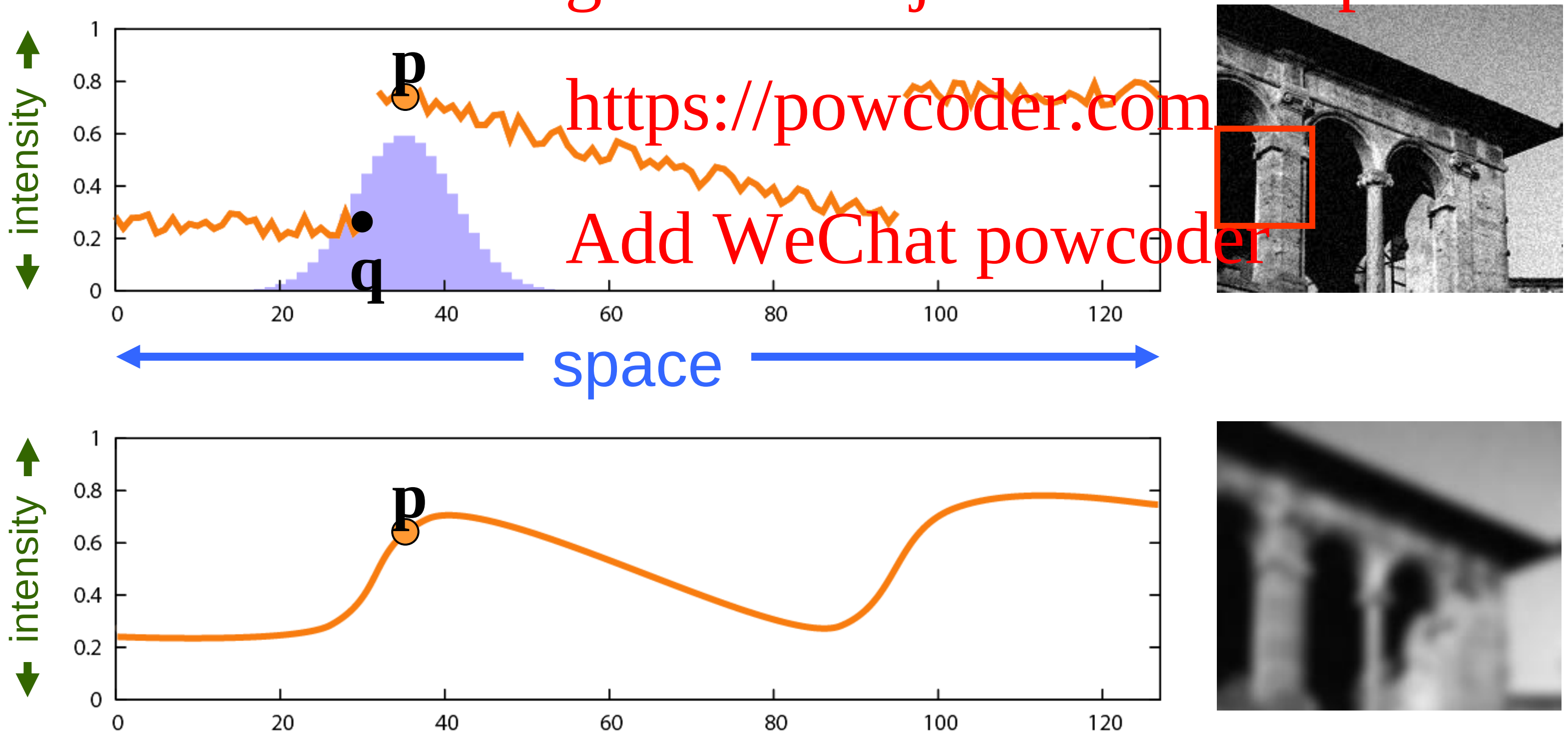
$$B_{\boldsymbol{p}} = \frac{1}{W_{\boldsymbol{p}}} \sum_{\boldsymbol{q} \in N_p} G_{\sigma_s}(|\boldsymbol{p} - \boldsymbol{q}|) G_{\sigma_r}(|C_{\boldsymbol{p}} - C_{\boldsymbol{q}}|) B_{\boldsymbol{q}}$$

$$W_{\boldsymbol{p}} = \sum_{\boldsymbol{q} \in N_p} G_{\sigma_s}(|\boldsymbol{p} - \boldsymbol{q}|) G_{\sigma_r}(|C_{\boldsymbol{p}} - C_{\boldsymbol{q}}|)$$



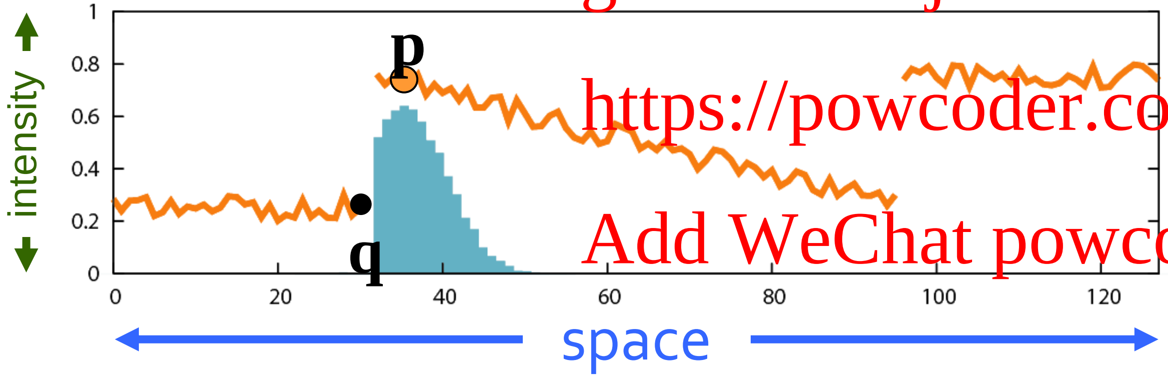
Gaussian filter vs. Bilateral filter

- **Gaussian filter**
  - Weighted average of neighbors
  - Depends only on spatial distance
  - No edge term

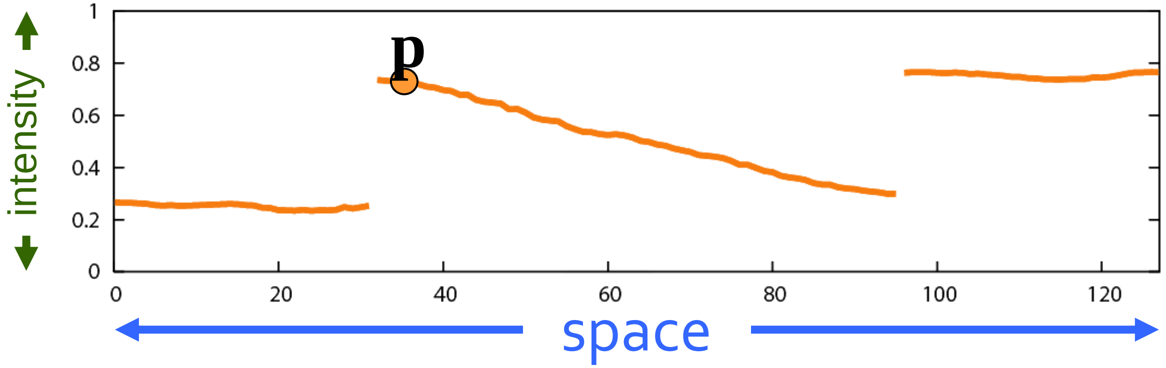


Gaussian filter vs. Bilateral filter

- **Bilateral filter**
  - Weighted average of neighbors
  - Depends on spatial and range difference



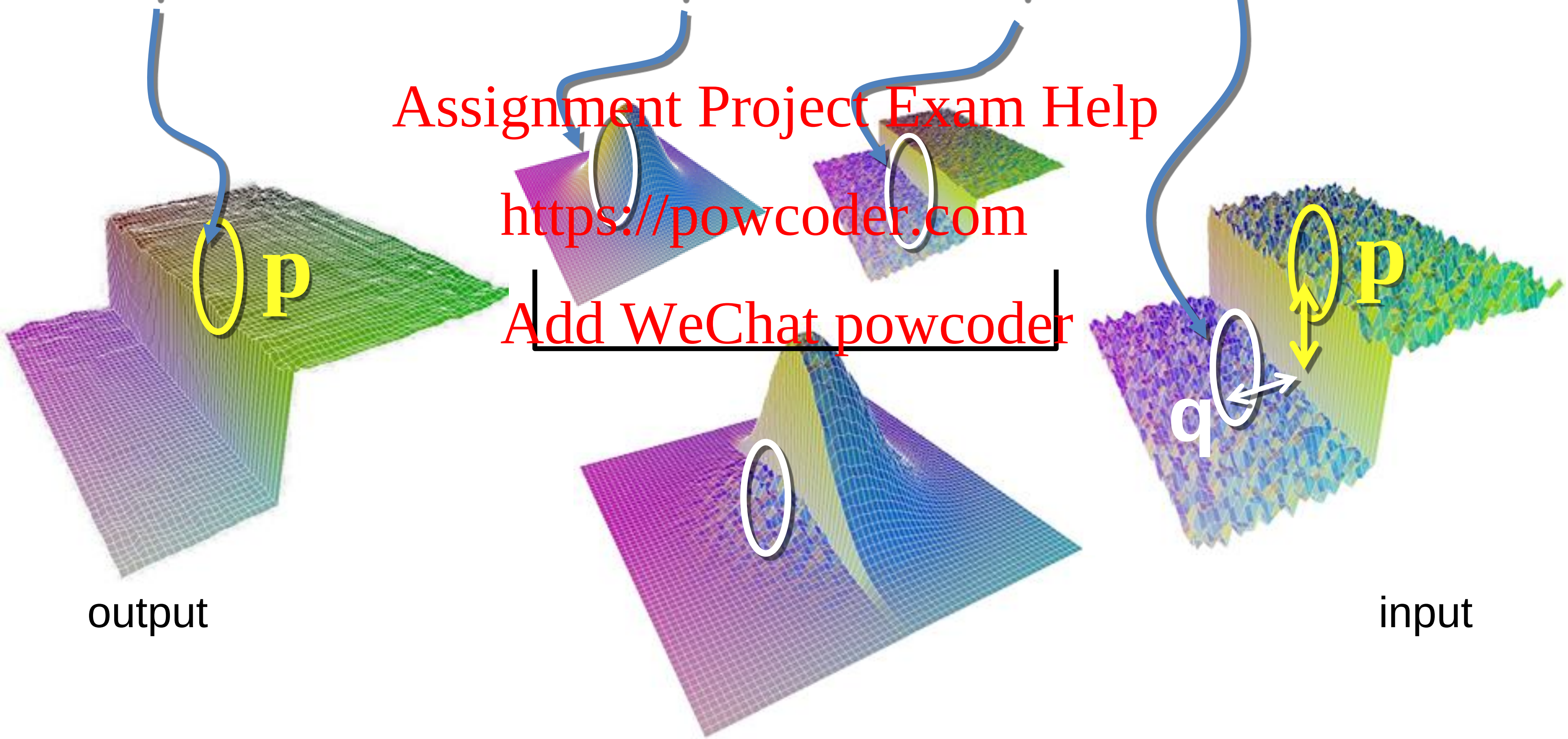
Input



Output

Bilateral filter on a height field

$$BF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q}\in S} G_{\sigma_s}(||\mathbf{p}-\mathbf{q}||)\; G_{\sigma_r}(|I_{\mathbf{p}}-I_{\mathbf{q}}|)\; I_{\mathbf{q}}$$



reproduced from [Durand 02]

Gaussian Noise Removal: Non-local Means Filtering

- **Same goals:**
  - Smooth within Similar Regions
- **KEY INSIGHT:**
  - Generalize, extend 'Similarity'

- **Bilateral:**
  - Averages neighbors with similar intensities;
- **NL-Means:**
  - Averages neighbors with similar neighborhoods!

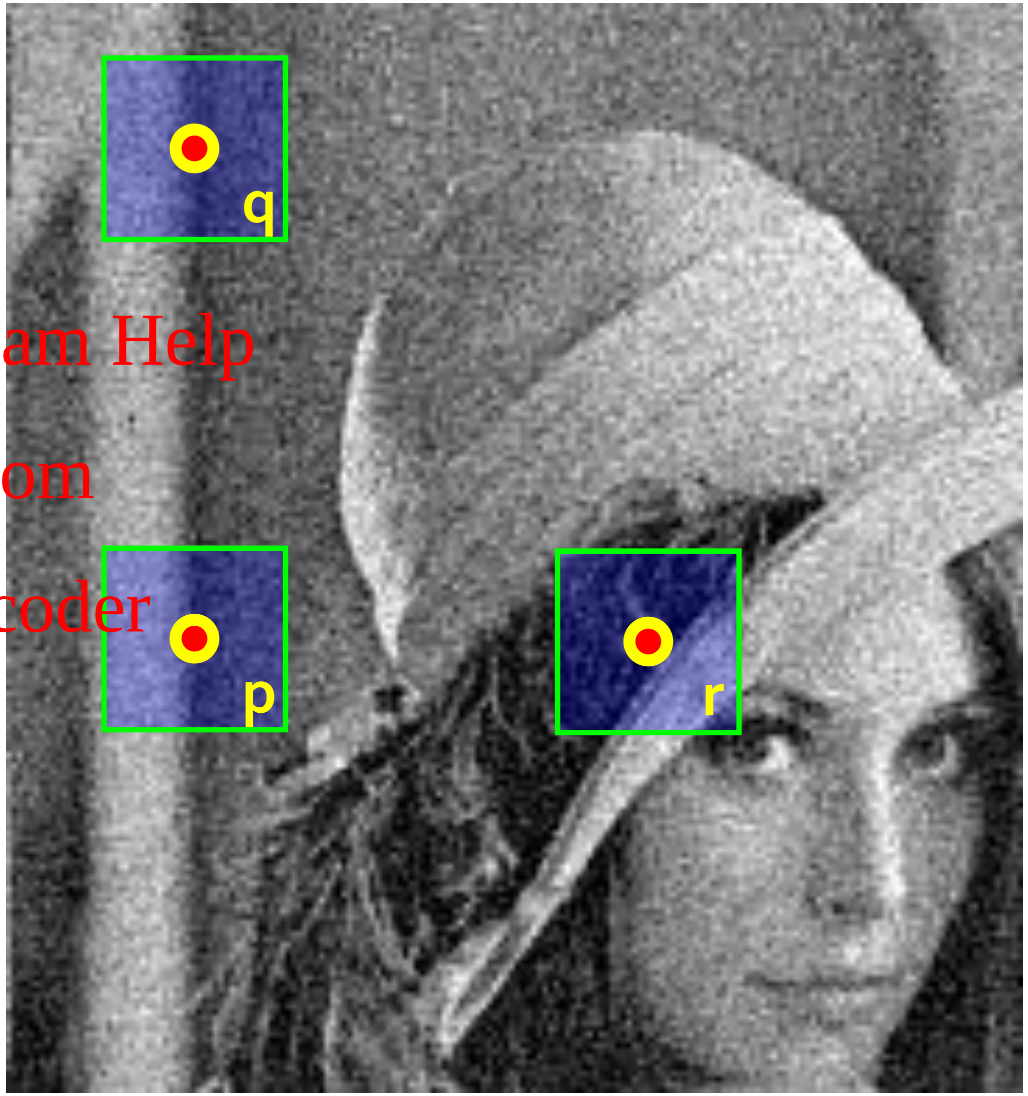
Gaussian Noise Removal: Non-local Means Filtering

- **For each pixel p:**
  - Define a small, simple fixed size neighborhood;
  - Define vector $\mathbf{V}_p$: a list of neighboring pixel values.

$$\mathbf{V}_p = \begin{bmatrix} 0.74 \\ 0.32 \\ 0.41 \\ 0.55 \\ \ldots \\ \ldots \\ \ldots \end{bmatrix}$$

Gaussian Noise Removal: Non-local Means Filtering

- **For each pixel p:**
  - Define a small, simple fixed size neighborhood;
  - Define vector V_p: a list of neighboring pixel values.
  - 'Similar' pixels **p, q** → SMALL distance $\|V_p - V_q\|^2$
  - 'Dissimilar' pixels **p, r** → LARGE distance $\|V_p - V_r\|^2$

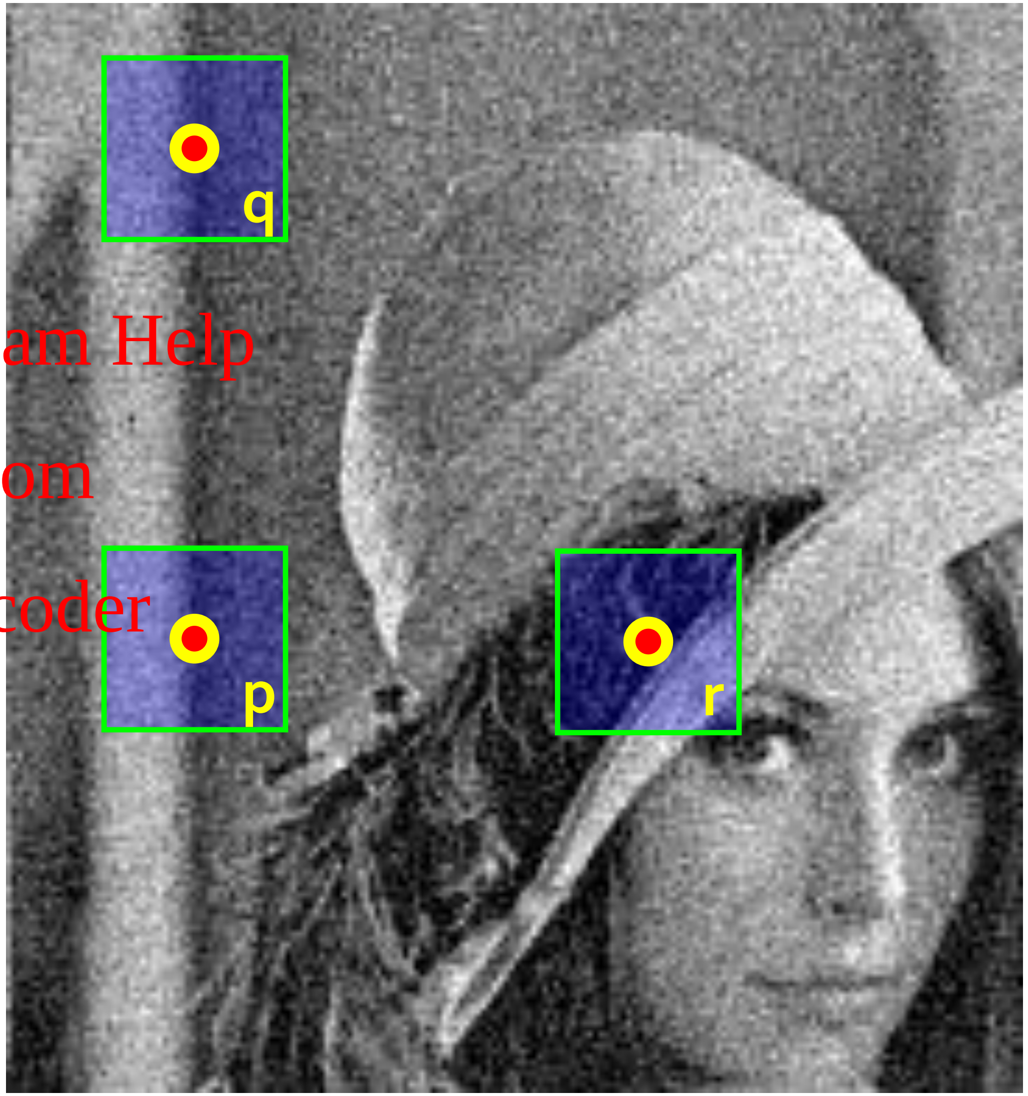


Gaussian Noise Removal: Non-local Means Filtering

- **For each pixel p:**
  - Define a small, simple fixed size neighborhood;
  - Define vector $\mathbf{V}_\mathbf{p}$: a list of neighboring pixel values.
  - 'Similar' pixels **p, q** → SMALL distance $\|\mathbf{V}_\mathbf{p} - \mathbf{V}_\mathbf{q}\|^2$
  - 'Dissimilar' pixels **p, r** → LARGE distance $\|\mathbf{V}_\mathbf{p} - \mathbf{V}_\mathbf{r}\|^2$

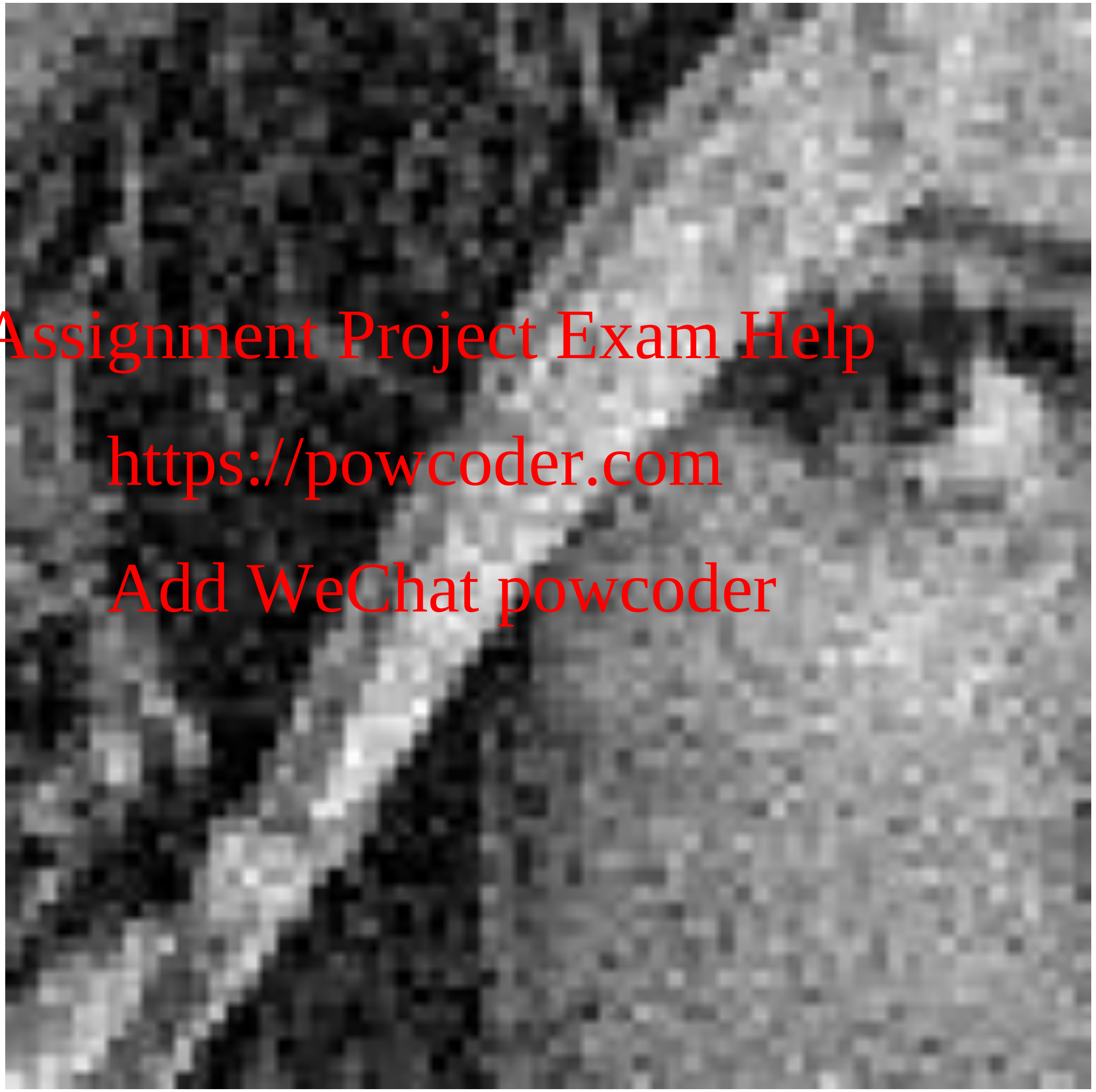
- **Filtering with this neighboring pixels!**
  - No spatial terms,
  - Measures the distance between patches (neighbor pixels)

$$NLMF[I]_\mathbf{p} = \frac{1}{W_\mathbf{p}} \sum_{\mathbf{q}\in S} \cancel{G_{\sigma_s}(||\mathbf{p}-\mathbf{q}||)} \boxed{G_{\sigma_r}\left(||\vec{V}_\mathbf{p} - \vec{V}_\mathbf{q}||^2\right)} I_\mathbf{q}$$



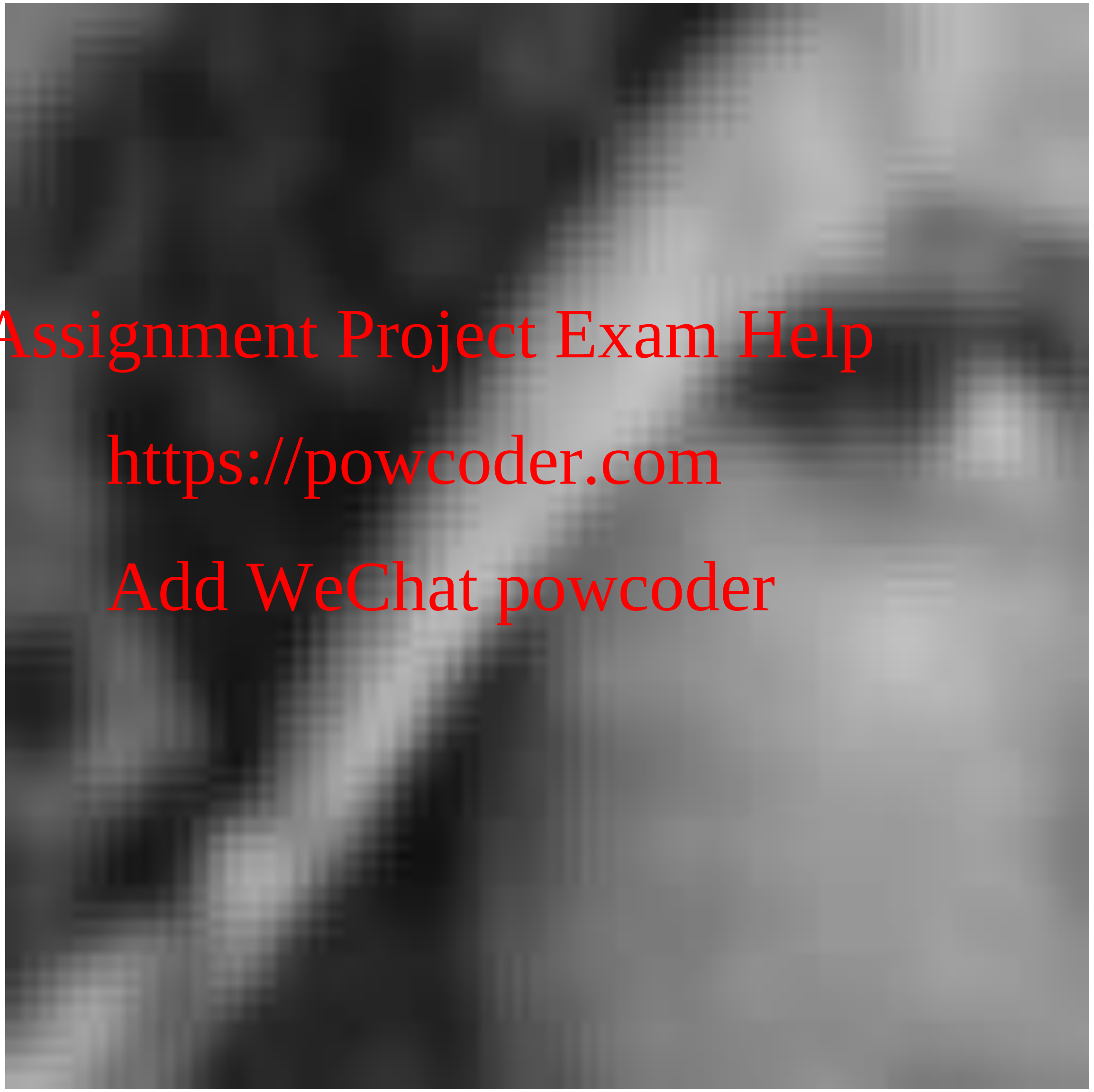
Gaussian Noise Removal: Non-local Means Filtering

- Noisy source image:



Gaussian Noise Removal: Non-local Means Filtering

- **Gaussian Filter**
  - Low noise
  - Low detail

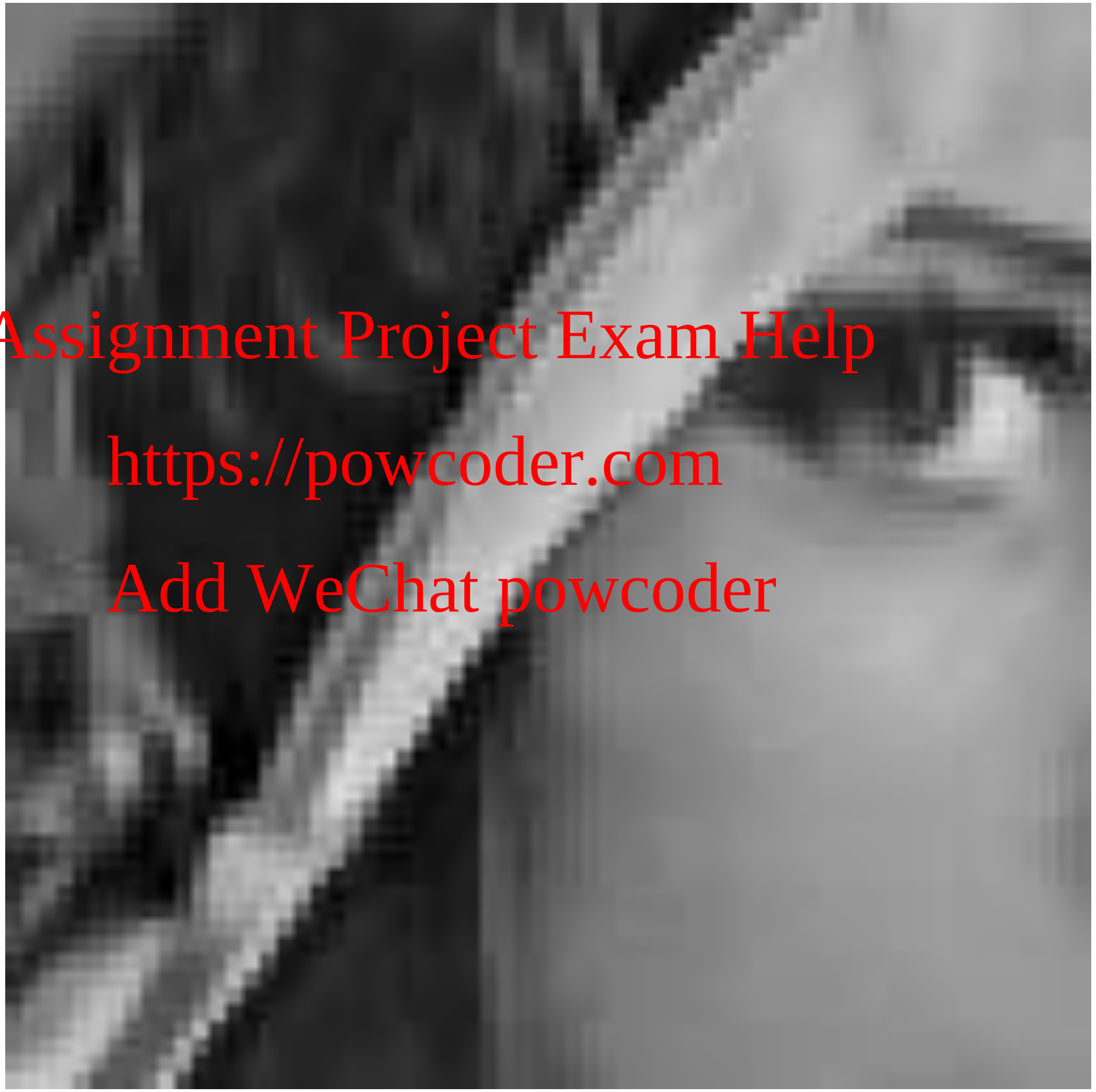


Gaussian Noise Removal: Non-local Means Filtering

- **Bilateral Filter**
  - Better at noise removal
  - but 'stairsteps'

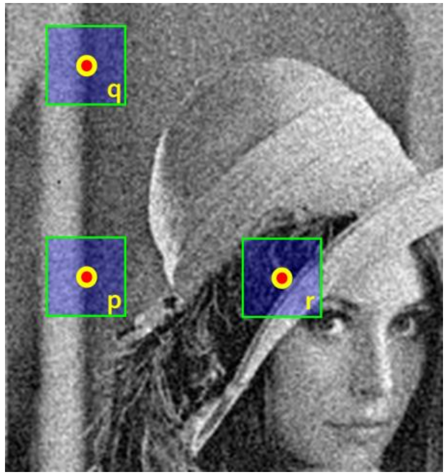
Gaussian Noise Removal: Non-local Means Filtering

- **NL-Means**
  - Sharp
  - Low noise
  - Few artifacts

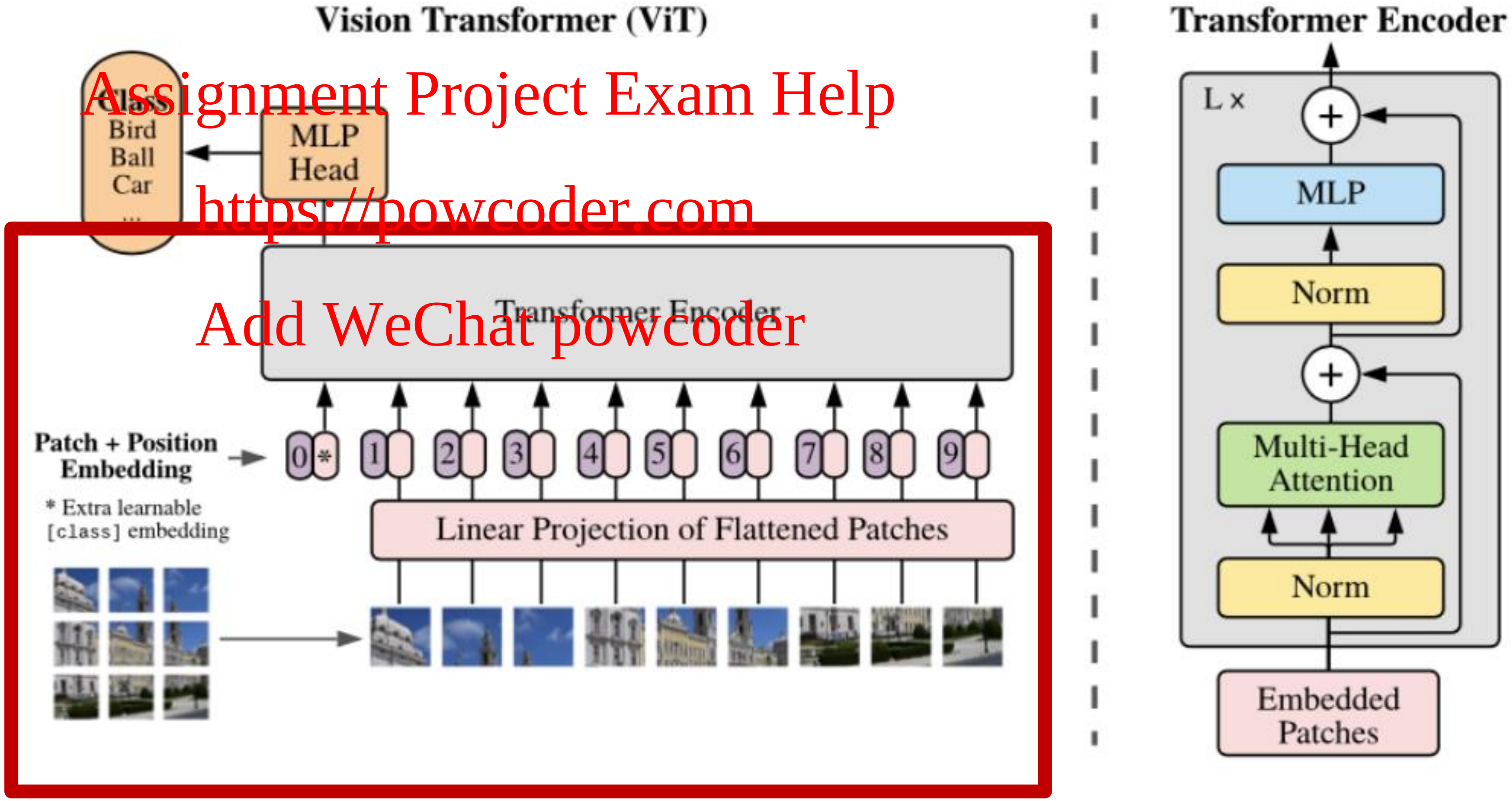


Non-local Means Filtering: Old-fashioned?

- **Vision Transformer (in deep learning)**
  - A new architecture that shows state-of-the-art performance in computer vision tasks

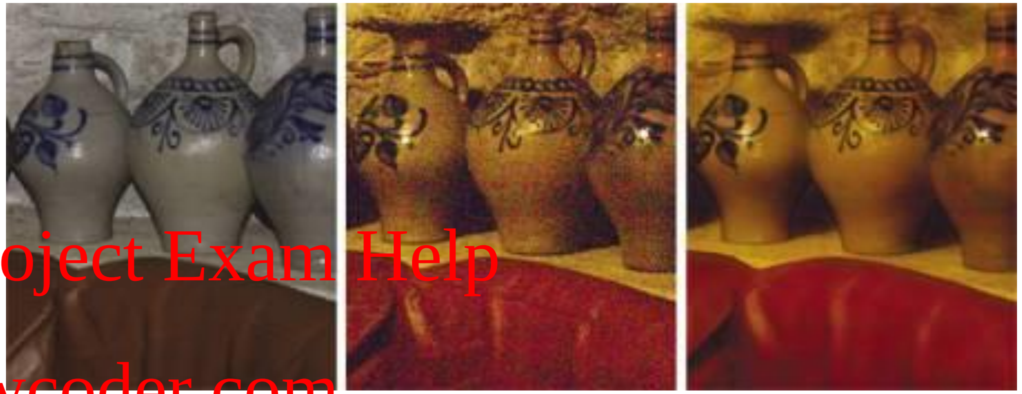


$$NLMF[I]_{\mathbf{p}} = \frac{1}{W_{\mathbf{p}}} \sum_{\mathbf{q} \in S} G_{\sigma_r}(||\vec{V}_{\mathbf{p}} - \vec{V}_{\mathbf{q}}||^2)\, I_{\mathbf{q}}$$

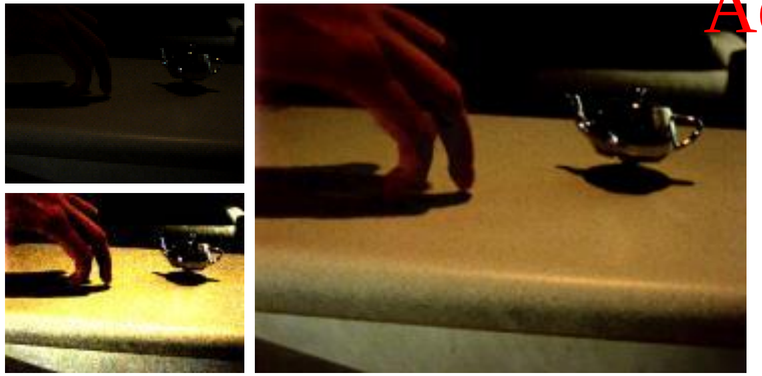


Many applications, not limited to denoising

Tone Mapping [Durand 02]



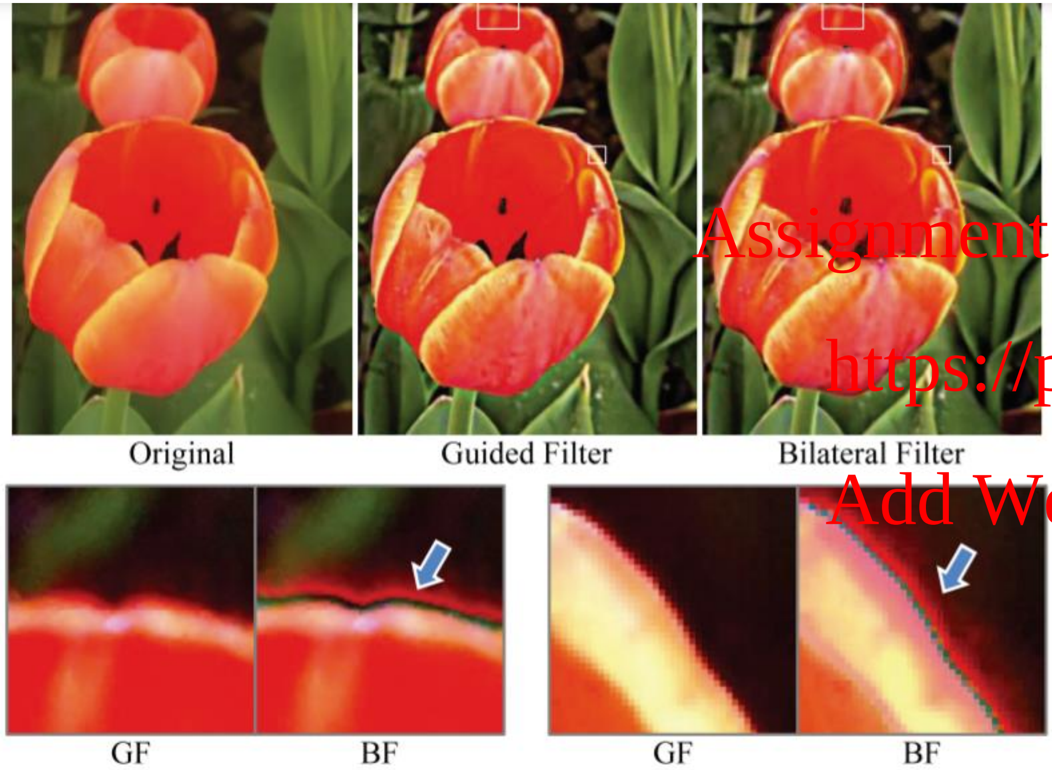
Flash / No-Flash [Eisemann 04, Petschnigg 04]



Virtual Video Exposure [Bennett 05]

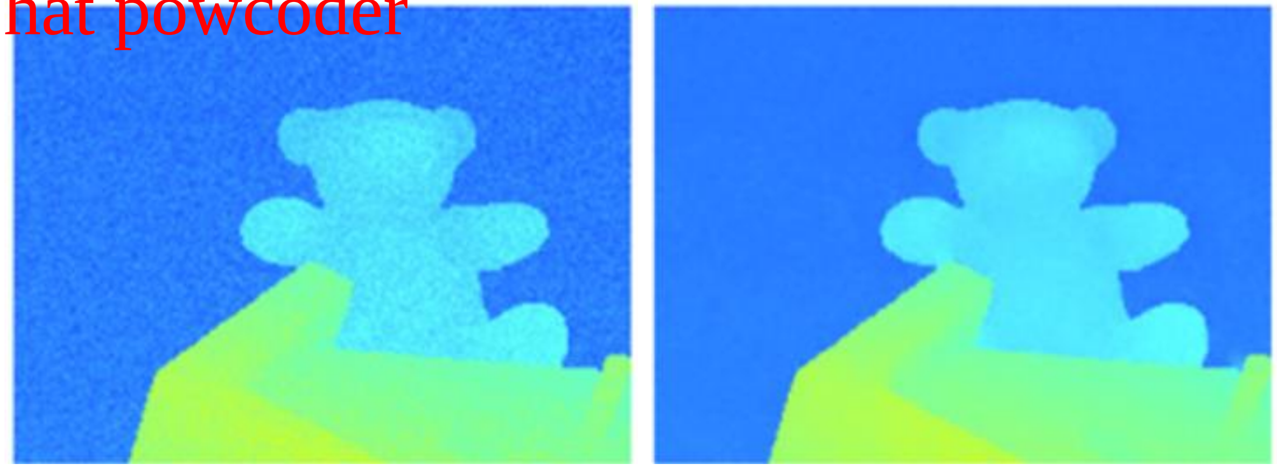
Tone Management [Bae 06]

Edge-aware smoothing filters



[He 2010]

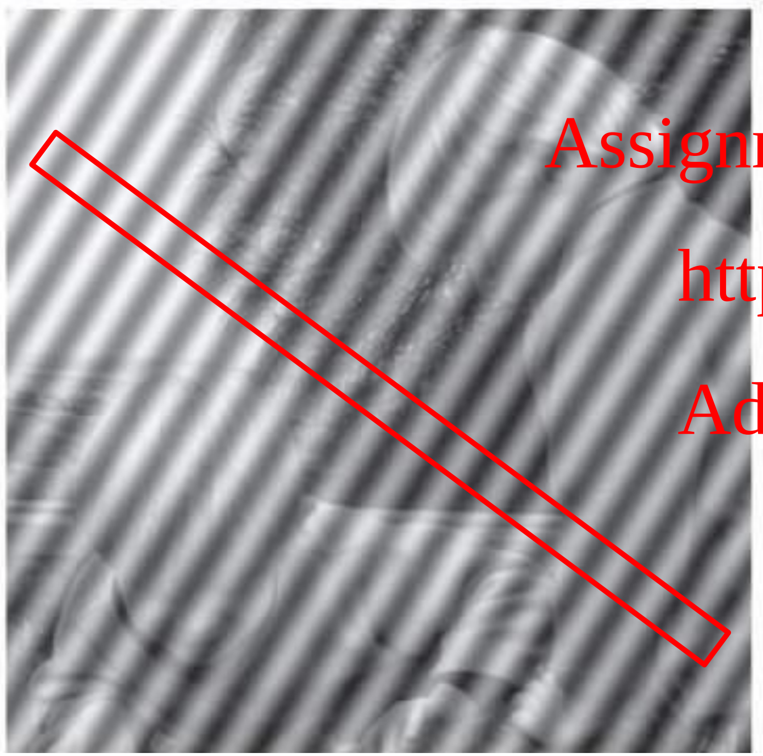
[Li 2011]

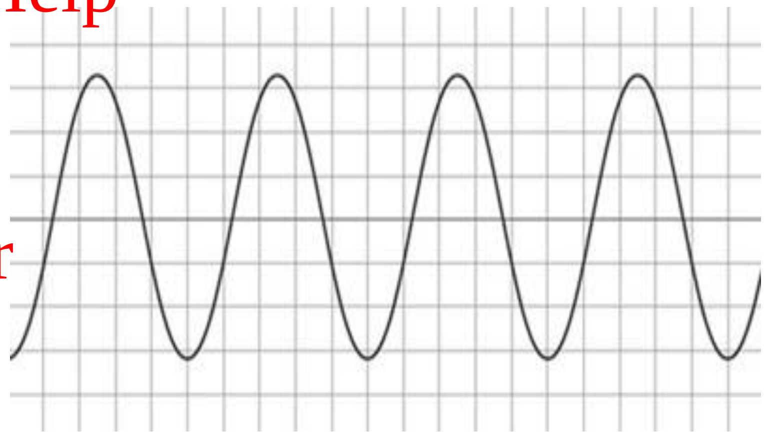


[Shen 2015]

Periodic Noise Removal: Frequency Domain Filtering

- **Periodic fluctuation can be modeled as sinusoid waves**





Periodic Noise Removal: Frequency Domain Filtering

- **Periodic fluctuation can be modeled as sinusoid waves**

- Spatial domain

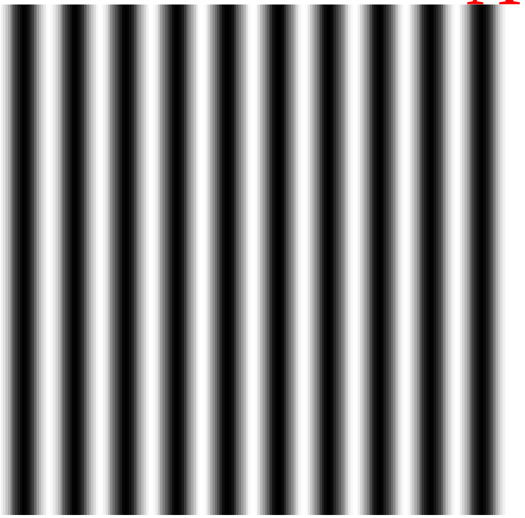
$$r(x,y) = A\sin\left[\frac{2\pi u_0(x+B_x)}{M} + \frac{2\pi v_0(y+B_y)}{N}\right]$$

A : amplitude
u_0, v_0 : sinusoidal frequencies
B_x, B_y : phase displacement w.r.t. the origin

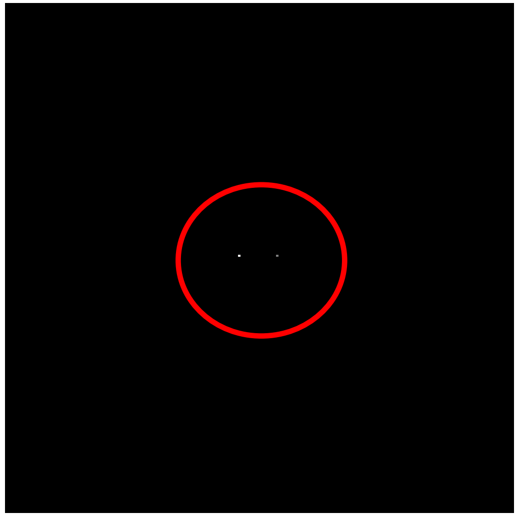
- Frequency domain

$$R(u,v) = j\frac{AMN}{2}\left[e^{-j2\pi\left(\frac{u_0B_x}{M}+\frac{v_0B_y}{N}\right)}\delta(u+u_0,v+v_0) - e^{j2\pi\left(\frac{u_0B_x}{M}+\frac{v_0B_y}{N}\right)}\delta(u-u_0,v-v_0)\right]$$

Sinusoidal wave in the spatial domain



Two symmetric spikes in the Fourier domain



Periodic Noise Removal: Frequency Domain Filtering

- **Periodic noise reduction by selective filters**
  - Notch filters can efficiently remove the periodic noise
  - Step 1: analyze the Fourier spectrum F of the image

https://docs.opencv.org/3.4/d2/d0b/tutorial_periodic_noise_removing_filter.html

Periodic Noise Removal: Frequency Domain Filtering

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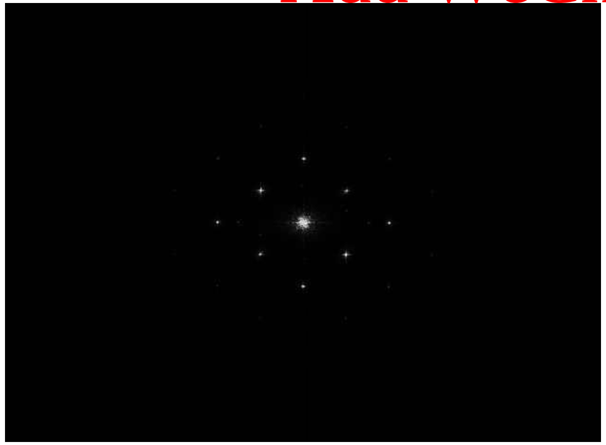

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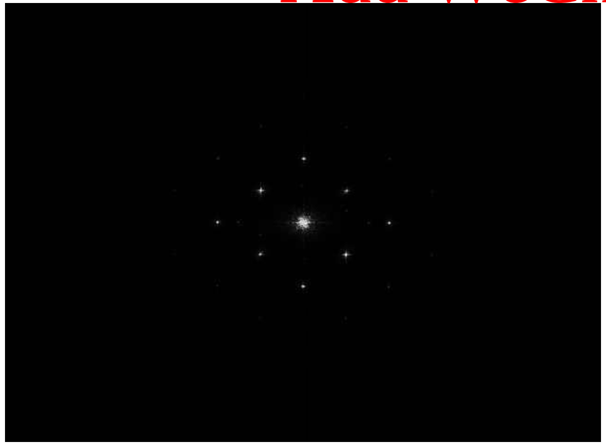
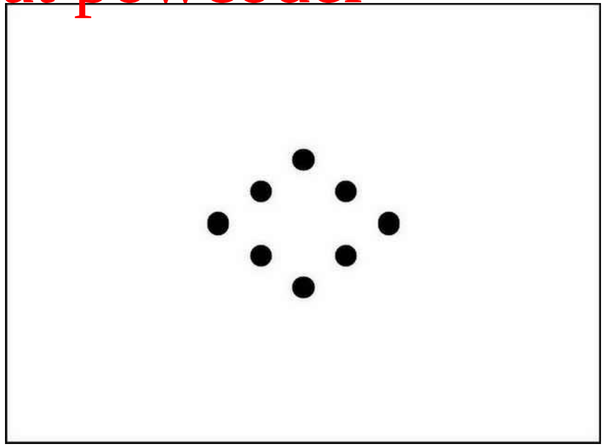
  - Step 3: construct a notch reject filter H in Fourier domain, whose centers are at peaks

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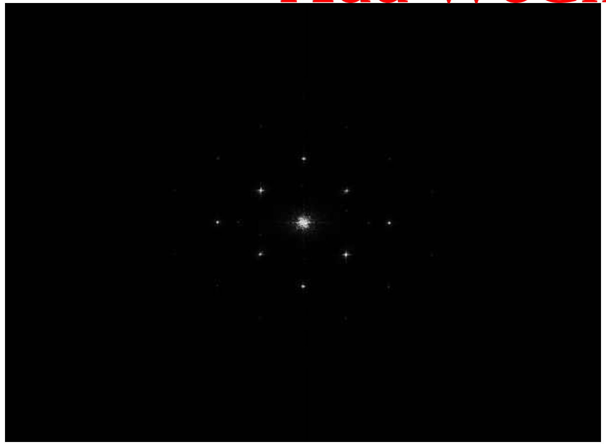
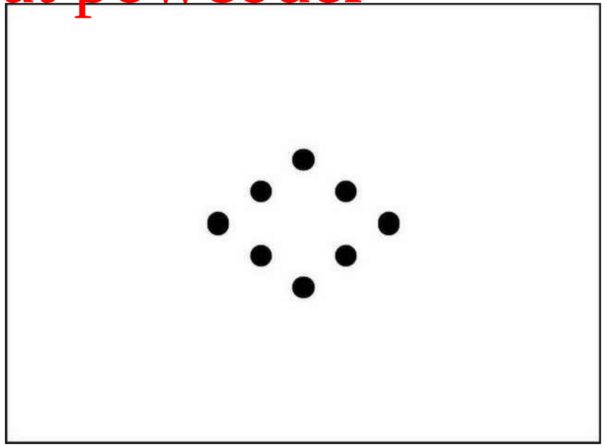
  - Step 4: use H to filter F to get the result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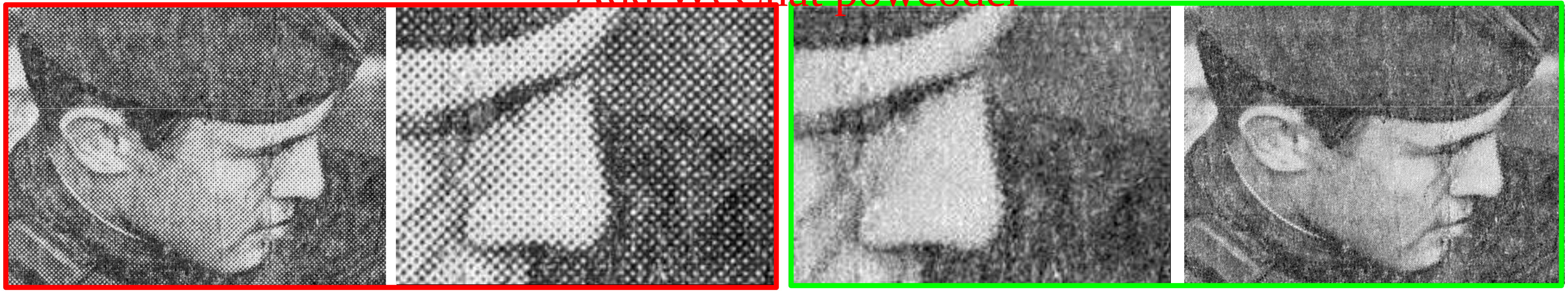

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Summary

- **4 different type of noises**
  - Salt and Pepper noise, Gaussian noise, Speckle noise, Periodic noise

- **Salt-and-Pepper Noise Removal**
  - Median filtering produces the best results.
- **Gaussian Noise Removal**
  - Additive White Gaussian Noise (AWGN): the most widely used noise model
  - Adaptive filtering produces satisfactory results.
- **Periodic Noise Removal**
  - Filtering in Frequency domain

Next topic

- **How can we represent images more effectively?**
  - Prerequisite
    - Review EBU6230 Image/Video Processing – Week3: Edges
    - Review EBU6230 Image/Video Processing – Week3: Interest Points