Midterm Test of Complex Analysis

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一、(20%) For the next statements, mark the correct ones with $\surd$, and the wrong ones with $\times$.

(a) $\lim\limits_{z\to 0}\overline{z}=0$ (____)

(b) $w=\overline{z}$ is not differential everywhere in $\mathbb{C}$ (____)

(c) For any simple closed contour C in $\mathbb{C}$, $\int_C (z^2+2\sin z-3e^z)dz=0$ (____)

(d) For $w=f(z)$ continuous in a domain $\Omega\subset\mathbb{C}$, then $f(z)$ is analytic in Ω if and only if $\int_C f(z)dz=0$ with C any closed contour interior to Ω (____)

(e) If f is analytic in a domain Ω, and $f\equiv 0$ on the curve $S\subset\Omega$, then $f\equiv 0$ in Ω (____)

二、(20%) Putting your answers in the paces

(a) For $z=1-i$, its principal argument (_____) and argument (_____)

(b) Write $w=\dfrac{1-2i}{2+i}+\dfrac{1}{i}$ in form $a+ib$ (_____)

(c) The derivative of the power function $(1+i)^z$ is (_____)

(d) The set of points at which $w=z^n\overline{z}$, $n\in\mathbb{N}$, differentiable is (_____)

(e) The set of points at which $\mathrm{Log}z$ analytic is (_____)

三、(10%) Write $(1-i)^5$ is rectangular form, and point out its principal argument and argument.

四、(10%) Present all three 3th roots $z^{1/3}$ of $z = -\frac{1}{\sqrt{2}} + i\frac{1}{\sqrt{2}}$, and compute the logarithm of the second one

五、(10%) Compute the limits

$$\lim_{z\to i}\frac{3\overline{z}-iz^3}{2-z^2}, \qquad \lim_{z\to\infty}\frac{5z^3-iz+1}{2z^3-z^2-1}$$

六、(10%) (*a*) Verify that $\overline{z}^2$ is not differentiable at any $z_0 \neq 0$, and is differential at $z_0 = 0$.

(*b*) By (*a*), explain that why $\overline{z}^2$ is nowhere analytic in $\mathbb{C}$

七、(10%) Evaluate the integral

$$\int_C (2z - 3z^2)dz$$

with C the contour $z = z(\theta) = e^{i\theta}$, $\theta \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

八、(10%) Let $z_0 = -2 - 2i$, and C be a positively oriented regular octagon centered at z_0. Compute the integral

$$\int_C (z - z_0)^{n+\frac{1}{2}} dz, \quad n \in \mathbb{Z}$$

[Needs the detail derivation on your conclusion]