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MST - 2

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Sol 1.

a) $(((a \vee b) \rightarrow c), (c \rightarrow (d \wedge e))) \Rightarrow (a \rightarrow d)$

$$\left.\begin{array}{l}(a \vee b) \rightarrow c \\ c \rightarrow (d \wedge e)\end{array}\right\} - \text{Prove}$$

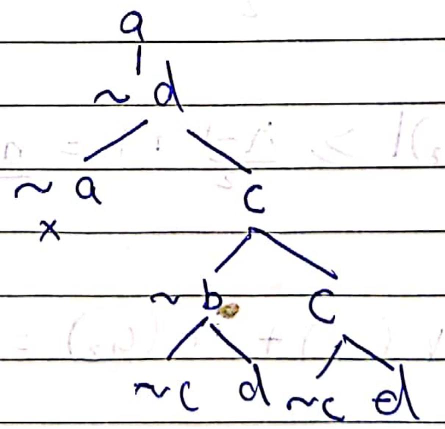
$a \rightarrow d \;\} \rightarrow$ result

take the negotion of result

$$a \rightarrow d$$
$$= \sim (a \rightarrow d)$$
$$= \sim (\sim a \vee d)$$
$$= a \wedge \sim d$$

Solving it by tree methods

$$(a \vee b) \rightarrow c = \sim (a \vee b) \vee c$$
$$= (\sim a \wedge \sim b) \vee c$$
$$= (\sim a \vee c) \wedge (\sim b \vee c)$$
$$c \rightarrow (d \wedge e) = (\sim c) \vee (d \wedge e)$$
$$= (\sim c \vee d) \wedge (\sim c \vee e)$$



negotiation of result is false
So,
Ans :- Valid.

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MST - 2

Sol 1.

a) $(((a \vee b) \to c), (c \to (d \wedge e)))) \Rightarrow (a \to d)$

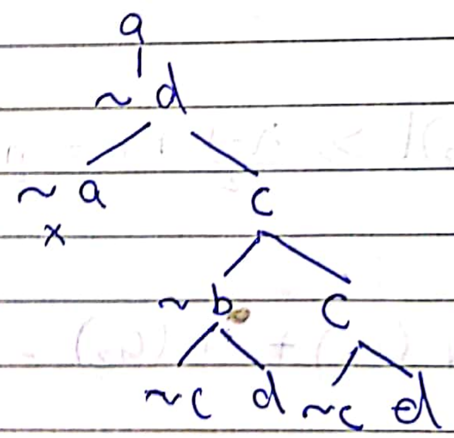
$(a \vee) b) \to c$ } – Prove
$c \to (d \wedge e)$ }
$a \to d$ } result

take the negation of result
$a \to d$
$= \sim (a \to d)$
$= \sim (\sim a \vee d)$
$= a \wedge \sim d$

Solving it by tree methods

$(a \vee b) \to c = \sim (a \vee b) \vee c$
$= (\sim a \wedge \sim b) \vee c$
$= (\sim a \vee c) \wedge (\sim b \vee c)$

$c \to (d \wedge e) = (\sim c) \vee (d \wedge e)$
$= (\sim c \vee d) \wedge (\sim c \vee e)$



negotiation of result is false
So,
Ans :- Valid.

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Sol 3 a) If a simple graph is G is Connected, then its Compliment is not connected.

~~False~~ True

let G" denote the compliment of G. Consider any two vertices U, V in G. If U and V are in Some connected components in G. then a edge in G connects them, the vertex be w; so By our argument the Edges (u, w) & (v, w) does not Exist in G".
Hence G" is not connected.

3 b) True

A simple graph with n vertices is Connected if

$$\delta(G) = \frac{n-1}{2}$$

$$|V(G_1)| \geq \frac{n-1}{2} + 2 = \frac{n+1}{2}$$

$$|V(G_2)| \geq \frac{n-1}{2} + 1 = \frac{n+1}{2}$$

$$V(G_1) + V(G_2) = n+1$$

But in orignal graph, n vertices
So,
Assumption was wrong & hence its connected when $\delta(G) = \frac{n-1}{2}$

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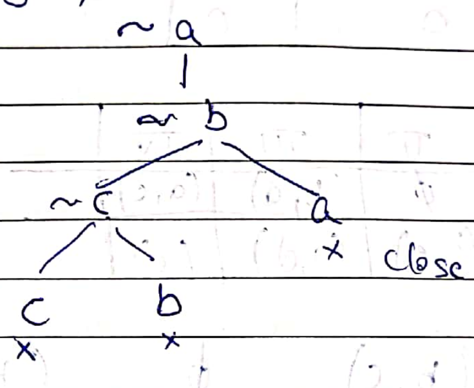
1 b) Prove $\left\{ \begin{array}{ll} c \to a & = \sim c \vee a \\ \sim c \to b & = (c \vee b) \end{array} \right.$

result $a \vee b$

take negation of result $= \sim (a \vee b)$
$= \sim a \wedge \sim b$

Solving by graph or tree methods



So, negation of result is false so of result is valid.

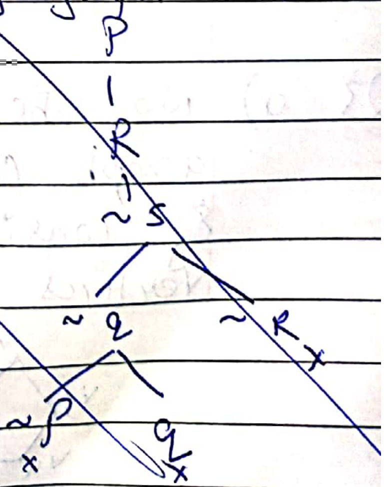
Ans:- valid

c) $\{P \to q, P \to R, Q \to \sim R, P\} \Rightarrow (R \to S)$

By graph methods

$P \to q$ $\sim P \vee q$
$P \to R$ $(\sim P \vee R)$
$Q \to \sim R$ $(\sim q \vee \sim R)$
P P

$R \to S$ $\sim R \vee S$



Take negation of result $= R \wedge \sim S$

Ans:- valid as ~~not~~ assumption was false.

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Ans 5 → A = [a, b, c, d]
R = [(a,d), (b,a), (b,c), (c,a), (c,d), (d,c)]

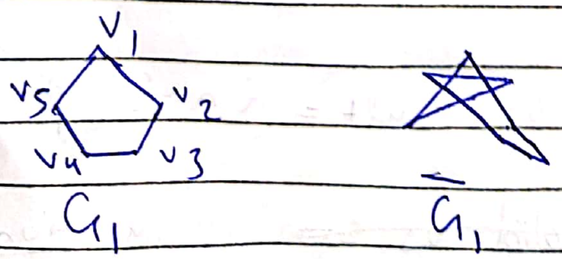
$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 0 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 1 & 0 & 0 & 1 \\ d & 0 & 0 & 1 & 0 \end{array}$$

	I	II	III	IV
Col	(b,c)	ϕ	(b,d)	(a,c)
Row	(d)	(a,c)	(a,d)	(c)

~~(b,c)~~ ✓(b,c) ϕ (b,d) (a,c)
(b,d) (b,a) (c,c)
(d,a)
(d,d)

Ans → (b,d) (d,a) (d,d) (a,c) (c,c)

Q3 a) No, the compliment of a simple connected graph can be connected or disconnected. For Eg consider the case of cyclic graph with 5 Vertices



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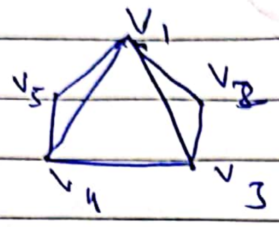
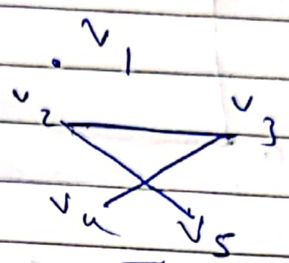
But for a modified cycle graph G_2,



G_2



$\overline{G_2}$

~~Only~~ clearly, $\overline{G_1}$ is connected while $\overline{G_2}$ is not when both G_1 & G_2 were simple connected graph.

Q1 c) $(P \to Q,\ P \to R,\ Q \to \sim R,\ P) \Rightarrow (R \to S)$

$P \to Q$ $\quad$ $P \to$ true

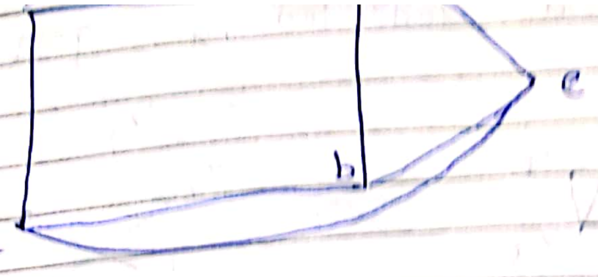
$P \to R$ $\quad$ $P \to Q$ is true $\Rightarrow Q$ is true

$Q \to \sim R$ $\quad$ (I) $P \to R$ is true $\Rightarrow R$ is true

P $\quad$ (II) $Q \to \sim R$ is true if $\sim R$ is true $\Rightarrow R$ is false

(I) and (II) contradict each other hence not valid.

Q4)



$$
\begin{array}{c|ccccc}
 & A & B & C & D & E \\
\hline
A & 3 & -1 & 0 & -1 & -1 \\
B & -1 & 3 & -1 & 0 & -1 \\
C & 0 & -1 & 2 & -1 & 0 \\
D & -1 & 0 & -1 & 3 & -1 \\
E & -1 & -1 & 0 & -1 & 3
\end{array}
$$

drop ① row and ① column

~~find det~~

$$
\begin{matrix}
3 & -1 & 0 & -1 \\
-1 & 2 & -1 & 0 \\
0 & -1 & 3 & -1 \\
-1 & 0 & -1 & 0
\end{matrix}
$$

Find determinant of above matrix

$$
3 \times \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{vmatrix} + 1 \times \begin{vmatrix} -1 & -1 & 0 \\ 0 & 3 & -1 \\ -1 & -1 & 0 \end{vmatrix}
$$

$$
+ 1 \times \begin{vmatrix} -1 & 2 & -1 \\ 0 & -1 & 3 \\ -1 & 0 & -1 \end{vmatrix}
$$

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$= 3\,|2\,(9-1) + 1(-3)| + 1\,(-1(9-1) + 1(1)) + 1\,(-1-6+1)$

$= 3\,(16-3) + 1\,(-8-1) + 1\,(-6)$

$= 29$

no. of spanning tree $= 29$

Q2 b) $\forall_x (P(x) \rightarrow Q(x)),\ \exists_y P(y) \Rightarrow \exists_z Q(z)$

let us assume (a, b) be in universal set Primes

$$\forall_x (P(x) \rightarrow Q(x))$$
$$\underline{\exists_y\ P_y}$$

result $\exists_z Q(z)$

Take negation of result

$\exists_z Q(z) = \sim \exists_z Q(z)$
$= \forall_x (\sim Q(z))$

$\Rightarrow \sim Q(a) \wedge \sim Q(b)$

1 $= \forall_x (\sim P(x) \vee Q(x))$
$= \forall_x (\sim P(x) \vee q(x))$
$= (\sim P(a) \vee q(a)) \wedge (\sim P(b) \vee q(b))$

2 $= \exists_y P(y)$
$= P(a) \vee P(b)$

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By graph method

$\sim q(a)$
- $\sim q(b)$
  - $\sim P(a)$
    - $\sim P(a)$
      - $\sim P(a)$ ×
      - $P(b)$ ×
    - $P(b)$ ×
  - $q(a)$ × (close)

All branch are close

Since negation of result is false so result is true

So statement is valid.

Q2 a) $\exists_x (P(x) \wedge Q(x)) \Rightarrow (\exists_x P(x) \wedge \exists_x Q(x))$

take negation of above

let us asume we have (a, b) in universe set

$\sim (\sim [\exists_x (P(x) \wedge q(x)) \vee (\exists_x P(x) \wedge \exists_x (Q(x))]$

$\exists_x (P(x) \wedge q(x)) \wedge \sim (\exists_x (P(x) \wedge \exists_x q(x))$

$\sim [\; \because \sim (\exists_x) = \forall_x]$

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So, above statement is Equal to $= a \wedge a'$

$(\because a \wedge a' = F)$

So,

ans = Valid.