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1.)

a)

$$\left. \begin{array}{l} (a \vee b) \rightarrow c \\ c \rightarrow (d \wedge e) \end{array} \right\} \text{Premises}$$

$$a \rightarrow d \quad \} \rightarrow \text{result}$$

Take the negation of result.

$$\begin{aligned} & (a \rightarrow d) \\ &= \sim(a \rightarrow d) \\ &= \sim(\sim a \vee d) \\ &= a \wedge \sim d \end{aligned}$$

Solving it by tree method.

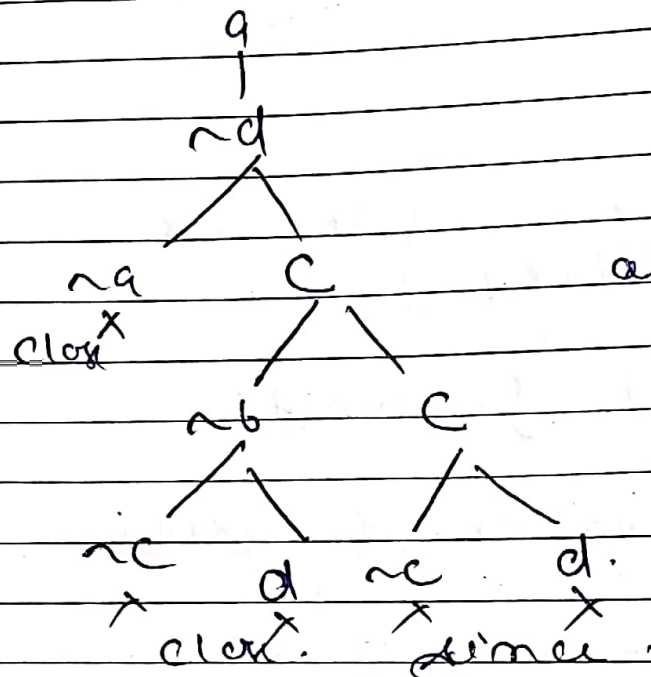
Premises.

$$\begin{aligned} (a \vee b) \rightarrow c &= \sim(a \vee b) \vee c \\ &= (\sim a \wedge \sim b) \vee c \\ &= (\sim a \vee c) \wedge (\sim b \vee c) \\ c \rightarrow d \wedge e &= (\sim c) \vee (d \wedge e) \\ &= (\sim c \vee d) \wedge (\sim c \vee e) \end{aligned}$$

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Solving it by graph method.



all branches are close

negation of result is False.

of result is valid.

a.) Ans: valid.

b.) premises.

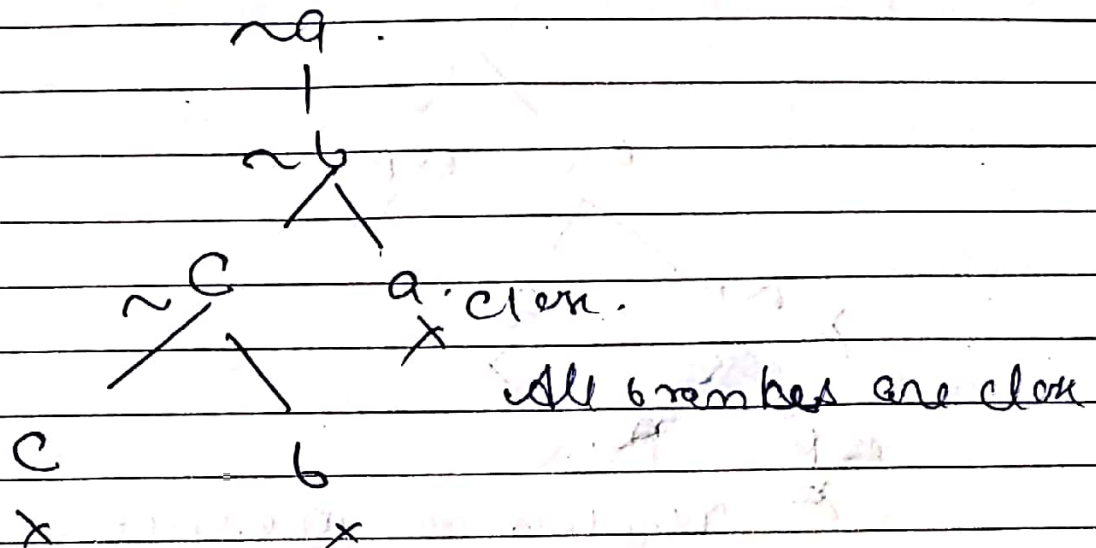
$$\begin{array}{lcl}
 C \rightarrow A & = & (C \vee A) \\
 \neg C \rightarrow B & = & (C \vee B) \\
 \hline
 \text{result } A \vee B & = & (C \vee A) \wedge (C \vee B)
 \end{array}$$

take negation of result = $\neg(A \vee B)$

$$= \neg A \wedge \neg B.$$

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Solving by graph or tree method.


Since negation of result is false.
of result is valid.

b) Ans: valid.

c) $(P \rightarrow Q, P \rightarrow R, Q \rightarrow \sim R, P) \Rightarrow (R \rightarrow S)$

$$\begin{array}{lcl}
 P \rightarrow Q & (\sim P \vee Q) & \\
 P \rightarrow R & (\sim P \vee R) & \\
 Q \rightarrow \sim R & (\sim Q \vee \sim R) & \\
 P & P & \\
 \hline
 R \rightarrow S & \sim R \vee S &
 \end{array}$$

Take negation of result $\Rightarrow R \wedge \sim S$

Q3.)

3.1, 3.2

Sol:

3.2. A simple graph is connected if $\delta(G) = \frac{n-1}{2}$

$$|V(G_1)| \geq \frac{n-1}{2} + 1 = \frac{n+1}{2}$$

$$|V(G_2)| \geq \frac{n-1}{2} + 1 = \frac{n+1}{2}$$

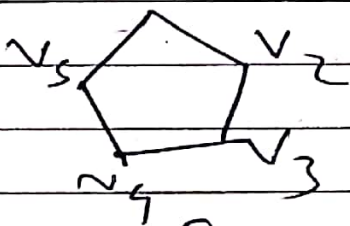
$$V(G_1) + V(G_2) \geq n+1$$

Assumption \rightarrow False.

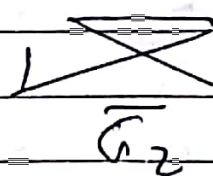
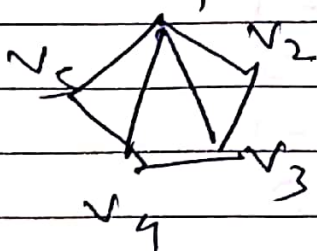
Statement \rightarrow True.

Q4) 3.1

Ans: NO, the complement of a simple connected graph can be connected or disconnected.

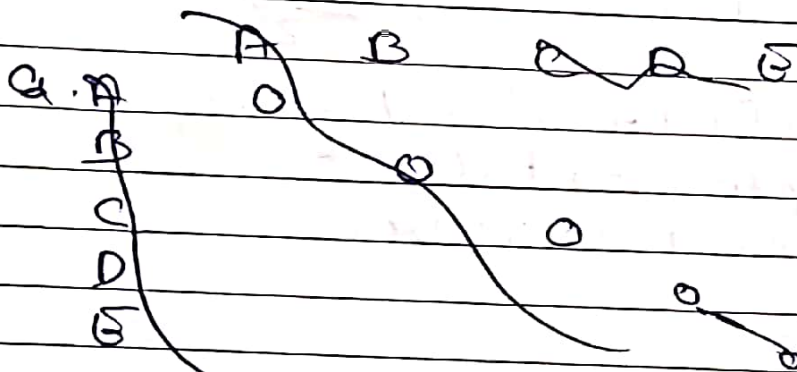
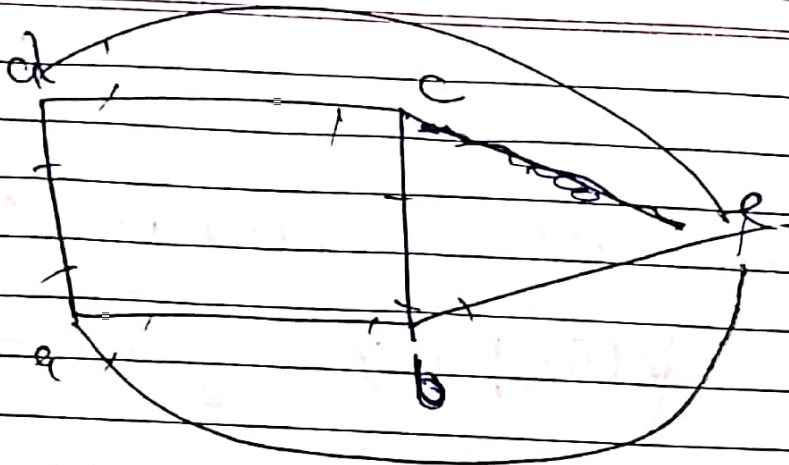


But for a modified cycle graph.



G_1 is connected while G_2 is not when both G_1 & G_2 were simple connected graphs.

Q 9.3



	A	B	C	D	E
A	3	-1	0	-1	-1
B	-1	3	-1	0	-1
C	0	-1	2	-1	0
D	-1	0	-1	3	-1
E	-1	-1	0	-1	3

Drop 1 row and 1 column.

3	-1	0	-1
-1	2	-1	0
0	-1	3	-1
-1	0	-1	3

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Find determinant of above matrix.

$$3 \begin{vmatrix} 2 & -1 & 0 \\ -1 & 3 & -1 \\ 0 & -1 & 3 \end{vmatrix} + 1 \begin{vmatrix} -1 & -1 & 0 \\ 0 & 3 & -1 \\ -1 & -1 & 3 \end{vmatrix}$$

$$+ 1 \begin{vmatrix} -1 & 2 & -1 \\ 0 & -1 & 3 \\ -1 & 0 & -1 \end{vmatrix}$$

$$3 \left[2(9-1) + 1(-3) \right] + 1 \left[-1(9-1) + 1(-2) \right]$$

$$+ 1 \left[-1(1) - 2(+3) - 1(-2) \right]$$

$$= 3 \times (16-3) + 1(-8-1) + 1(-1-6+2)$$

$$= 3 \times 13 - 9 - 6$$

$$= 39 - 9 - 6 = 30 - 6 = 24.$$

no of spanning tree = 24.

Q5.)

Sol:

$$A = \{a, b, c, d\}$$

$$R = \{ (a, d), (b, a), (b, c), (c, d), (c, d), (d, c) \}$$

	a	b	c	d
a	0	0	0	1
b	1	0	1	0
c	1	0	0	1
d	0	0	1	0

	I	II	III	IV
Col	$\{b, c\}$	$\{a\}$	$\{b, d\}$	$\{a, c\}$
Row	$\{d\}$	$\{a, c\}$	$\{a, d\}$	$\{c\}$

$$\begin{aligned} & \neg (b, c) \quad \& \quad \neg (b, a) \quad \neg (a, c) \\ & (b, d) \quad \quad (b, d) \quad \neg (c, c) \\ & \quad \quad (d, a) \\ & \quad \quad (d, d) \end{aligned}$$

$$\text{Ans: } (b, d) (d, a) (d, d) (a, c) (c, d)$$

Q 2.)

$$a) \exists x (P(x) \wedge Q(x)) \Rightarrow (\exists x (P(x) \wedge \exists x (Q(x)))$$

Take negation of above.

Let us assume we have $\{a, b\}$ it's universe

$$\sim (\sim (\exists x (P(x) \wedge Q(x))) \vee (\exists x (P(x) \wedge \exists x (Q(x)))$$

$$\exists x (P(x) \wedge Q(x)) \wedge \sim (\exists x (P(x) \wedge \exists x (Q(x)))$$

$$\therefore \sim (\exists x) = \forall x$$

$$\exists x (P(x) \wedge Q(x)) \wedge (\forall x (\sim P(x) \vee \sim Q(x)))$$

$$(\because \exists x (P(x)) \Rightarrow P(a) \vee P(b))$$

$$\Rightarrow (P(a) \wedge Q(a) \vee (P(b) \wedge Q(b))) \wedge (\sim P(a) \wedge \sim P(b) \vee (\sim Q(a) \wedge \sim Q(b)))$$

$$\Rightarrow \text{Let's let } a = (P(a) \wedge Q(a)) \vee (P(b) \wedge Q(b))$$

$$a' \text{ or } \sim a = (\sim P(a) \wedge \sim P(b) \vee (\sim Q(a) \wedge \sim Q(b)))$$

\therefore Above statement is Equ. to $a \wedge a'$

$$(\because a \wedge a' = F)$$

Since negation of statement is false so.

Result or statement is true or valid.

So ans is valid.

b.) $\forall x (P(x) \rightarrow Q(x))$, $\exists x P(y) \Rightarrow \exists x Q(z)$
let us assume a, b are universal set.
Primes.

$$\frac{\forall x (P(x) \rightarrow Q(x))}{\exists x P(y)}$$

Result $\exists x Q(z)$.

Take negation of result.

$$\exists x Q(z) = \sim \exists x (Q(z))$$

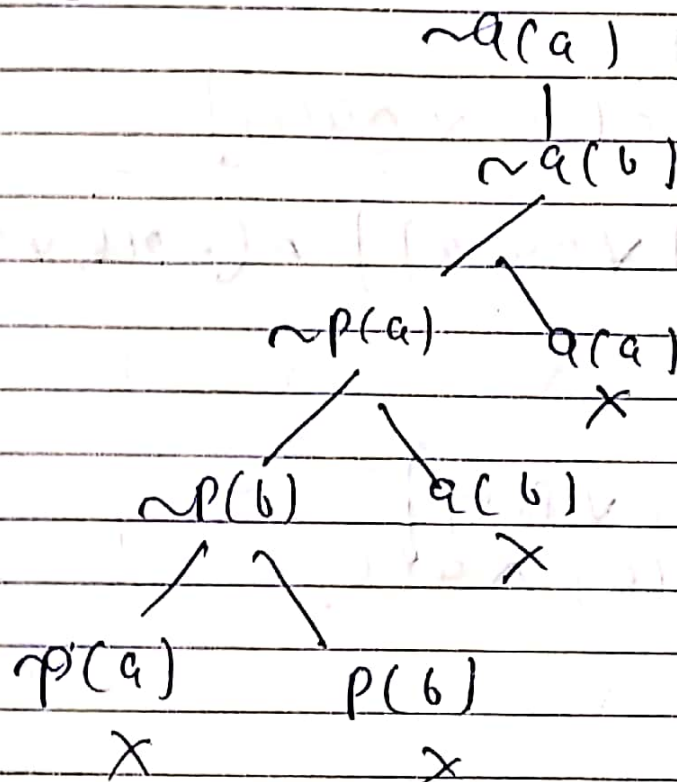
$$= \forall x (\sim Q(z))$$

$$\Rightarrow \sim Q(a) \wedge \sim Q(b)$$

$$\begin{aligned} \text{Primes. 2} &= \forall x (\sim P(x) \vee Q(x)) \\ &= \forall x (\sim P(x) \vee Q(x)) \\ &= (\sim P(a) \vee Q(a)) \wedge (\sim P(b) \vee Q(b)) \end{aligned}$$

$$\begin{aligned} \text{Primes. 2} &= \exists x P(x) \\ &= P(a) \vee P(b) \end{aligned}$$

By graph method.



all branch are close.

Since ~~result~~ is negation of result
is false so result is true.

So statement is valid.

c) Premises.

$$\exists x P(x), \forall x Q(x)$$

result.

$$\exists x \{P(x) \wedge Q(x)\}.$$

let us assume we have $\{a, b\}$ u'n universal set.

take negation of result.

$$\sim(\exists x (P(x) \wedge Q(x)))$$

$$\equiv \forall x (\sim P(x) \vee \sim Q(x))$$

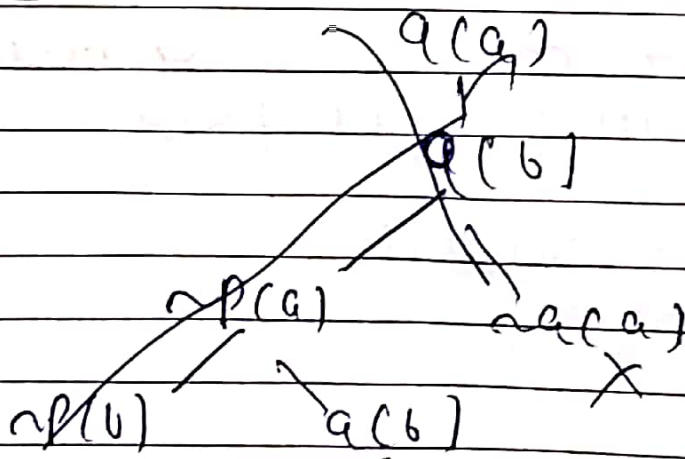
$$\equiv (\sim P(a) \vee \sim Q(a)) \wedge (\sim P(b) \vee \sim Q(b))$$

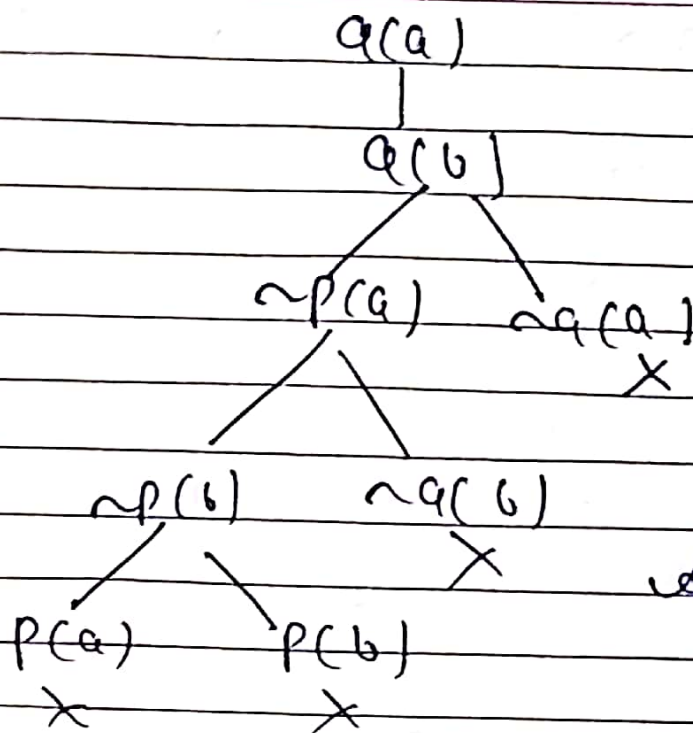
Premises.

$$\exists x P(x) \equiv (P(a) \vee P(b))$$

$$\forall x Q(x) \equiv Q(a) \wedge Q(b)$$

By tree method.





all branches are
close.

Since negation of statement is false so
above statement is valid or true.