




# Dynamax: A Python package for probabilistic state space modeling with JAX

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DOI: [10.xxxxxx/draft](https://doi.org/10.xxxxxx/draft)

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Submitted: 01 January 1970

Published: unpublished

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## Summary

State space models (SSMs) are fundamental tools for modeling sequential data. They are broadly used across engineering disciplines like signal processing and control theory, as well as scientific domains like neuroscience (Vyas et al., 2020), genetics (Durbin et al., 1998), ecology (Patterson et al., 2008), computational ethology (Weinreb et al., 2024), economics (Jacquier et al., 2002), and climate science (Ott et al., 2004). Fast and robust tools for state space modeling are crucial to researchers in all of these application areas.

State space models specify a probability distribution over a sequence of observations,  $y_1, \dots, y_T$ , where  $y_t$  denotes the observation at time  $t$ . The key assumption of an SSM is that the observations arise from a sequence of *latent states*,  $z_1, \dots, z_T$ , which evolve according to a *dynamics model* (aka transition model). An SSM may also use inputs (aka controls or covariates),  $u_1, \dots, u_T$ , to steer the latent state dynamics and influence the observations. For example, in a neuroscience application from Vyas et al. (2020),  $y_t$  represents a vector of spike counts from  $\sim 1000$  measured neurons, and  $z_t$  is a lower dimensional latent state that changes slowly over time and captures correlations among the measured neurons. If sensory inputs to the neural circuit are known, they can be encoded in  $u_t$ . In the computational ethology application of Weinreb et al. (2024),  $y_t$  represents a vector of 3D locations for several key points on an animal's body, and  $z_t$  is a discrete behavioral state that specifies how the animal's posture changes over time. In both examples, there are two main objectives: First, we aim to infer the latent states  $z_t$  that best explain the observed data; formally, this is called *state inference*. Second, we need to estimate the dynamics that govern how latent states evolve; formally, this is part of the *parameter estimation* process. Dynamax provides algorithms for state inference and parameter estimation in a variety of SSMs.

There are a few key design choices to make when constructing an SSM:

- What is the type of latent state? E.g., is  $z_t$  a continuous or discrete random variable?
- How do the latent states evolve over time? E.g., are the dynamics linear or nonlinear?
- How are the observations distributed? E.g., are they Gaussian, Poisson, etc.?

Some design choices are so common they have their own names. Hidden Markov models (HMM) are SSMs with discrete latent states, and linear dynamical systems (LDS) are SSMs with continuous latent states, linear dynamics, and additive Gaussian noise. Dynamax supports canonical SSMs and allows the user to construct bespoke models as needed, simply by inheriting from a base class and specifying a few model-specific functions. For example, see the *Creating Custom HMMs* tutorial in the Dynamax documentation.

43 Finally, even for canonical models, there are several algorithms for state inference and parameter  
44 estimation. Dynamax provides robust implementations of several low-level inference algorithms  
45 to suit a variety of applications, allowing users to choose among a host of models and algorithms  
46 for their application. More information about state space models and algorithms for state  
47 inference and parameter estimation can be found in the textbooks by Murphy (2023) and  
48 Särkkä & Svensson (2023).

## 49 Statement of need

50 Dynamax is an open-source Python package for state space modeling. Since it is built with JAX  
51 (Bradbury et al., 2018), it supports just-in-time (JIT) compilation for hardware acceleration on  
52 CPU, GPU, and TPU machines. It also supports automatic differentiation for gradient-based  
53 model learning. While other libraries exist for state space modeling in Python (Corenflos  
54 & Särkkä, 2021; Johnson, 2020; Linderman et al., 2020; Seabold & Perktold, 2010; Weiss  
55 et al., 2024) and Julia (Dalle, 2024), Dynamax provides a diverse combination of low-level  
56 inference algorithms and high-level modeling objects that can support a wide range of research  
57 applications in JAX. Additionally, Dynamax implements parallel message passing algorithms  
58 that leverage the associative scan (a.k.a., parallel scan) primitive in JAX to take full advantage  
59 of modern hardware accelerators. Currently, these primitives are not natively supported in  
60 other frameworks like PyTorch. While various subsets of these models and algorithms may be  
61 found in other libraries, Dynamax is a “one stop shop” for state space modeling in JAX.

62 The API for Dynamax is divided into two parts: a set of core, functionally pure, low-level  
63 inference algorithms, and a high-level, object oriented module for constructing and fitting  
64 probabilistic SSMs. The low-level inference API provides message passing algorithms for several  
65 common types of SSMs. For example, Dynamax provides JAX implementations for:

- 66 ■ Forward-Backward algorithms for discrete-state hidden Markov models (HMMs),
- 67 ■ Kalman filtering and smoothing algorithms for linear Gaussian SSMs,
- 68 ■ Extended and unscented generalized Kalman filtering and smoothing for nonlinear and/or
- 69 non-Gaussian SSMs, and
- 70 ■ Parallel message passing routines that leverage GPU or TPU acceleration to perform
- 71 message passing in  $O(\log T)$  time on a parallel machine (Hassan et al., 2021; Särkkä
- 72 & García-Fernández, 2020; Stone, 1975). Note that these routines are not simply
- 73 parallelizing over batches of time series, but rather using a parallel algorithm with
- 74 sublinear depth or span.

75 The high-level model API makes it easy to construct, fit, and inspect HMMs and linear Gaussian  
76 SSMs. Finally, the online Dynamax documentation and tutorials provide a wealth of resources  
77 for state space modeling experts and newcomers alike.

78 Dynamax has supported several publications. The low-level API has been used in machine  
79 learning research (Chang et al., 2023; Lee et al., 2023; Zhao & Linderman, 2023). Special  
80 purpose libraries have been built on top of Dynamax, like the Keypoint-MoSeq library for  
81 modeling animal behavior (Weinreb et al., 2024) and the Structural Time Series in JAX library,  
82 sts-jax (Li & Murphy, 2022). Finally, the Dynamax tutorials are used as reference examples in  
83 a major machine learning textbook (Murphy, 2023).

## 84 Acknowledgements

85 A significant portion of this library was developed while S.W.L. was a Visiting Faculty Researcher  
86 at Google and P.C., G.H.D., A.K., and X.L. were Google Summer of Code participants.

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