QuantA&M Book Club
Meeting 7

QuantA&M of Texas A&M University

Brief Recap

Quantum Fourier Transform

Large Summations

- Examples of sigma notation:

$$\sum_{n=1}^{3} n$$

$$= 1 + 2 + 3$$

$$\sum_{n=1}^{3} x^n$$

$$= x^1 + x^2 + x^3$$

Large Products

- Examples of pi notation:

$$\prod_{n=1}^{3} n$$

$$= 1 * 2 * 3$$

$$\prod_{n=1}^{3} x^n$$

$$= x^1 * x^2 * x^3$$

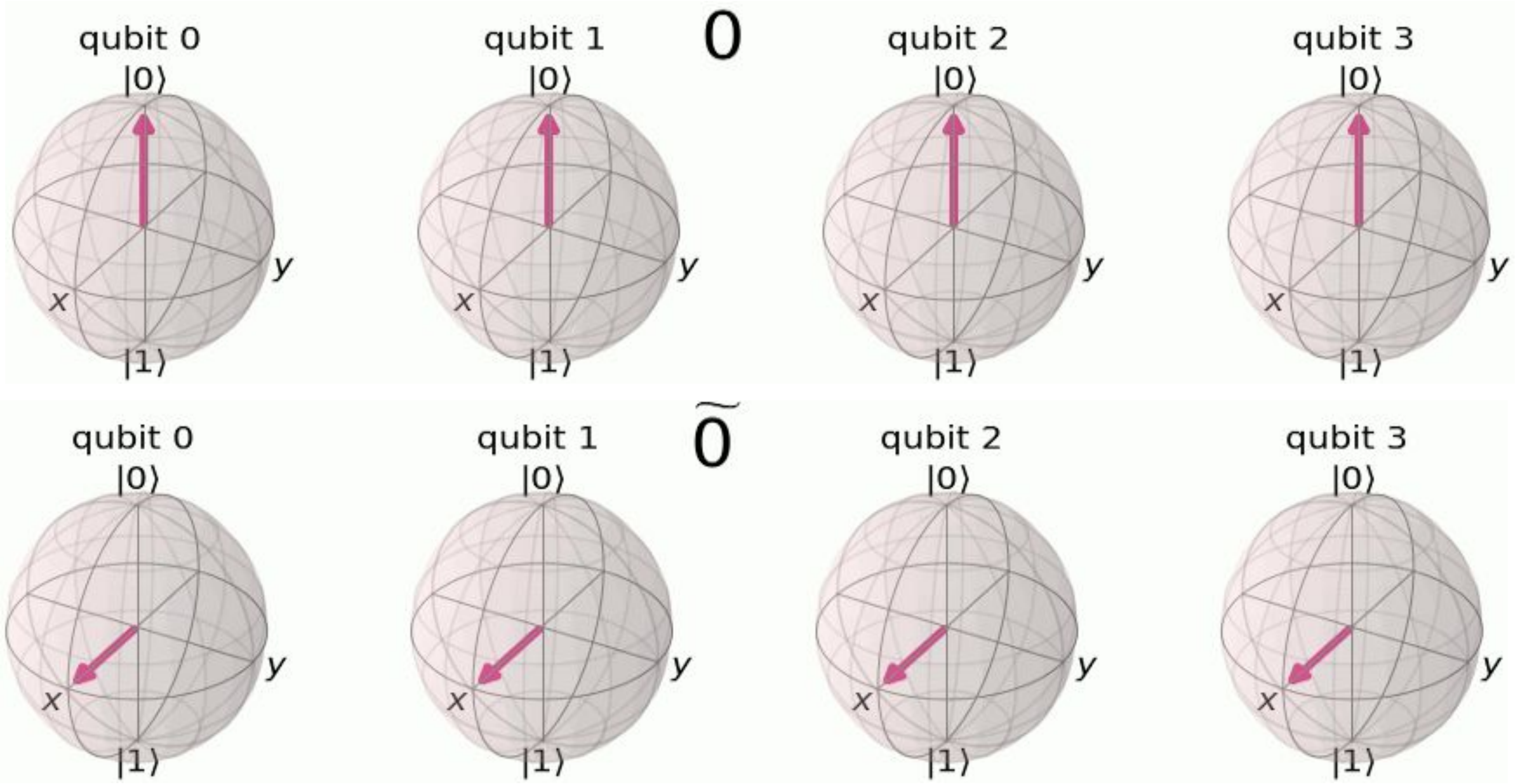
Large Tensor Products

- New tensor notation:

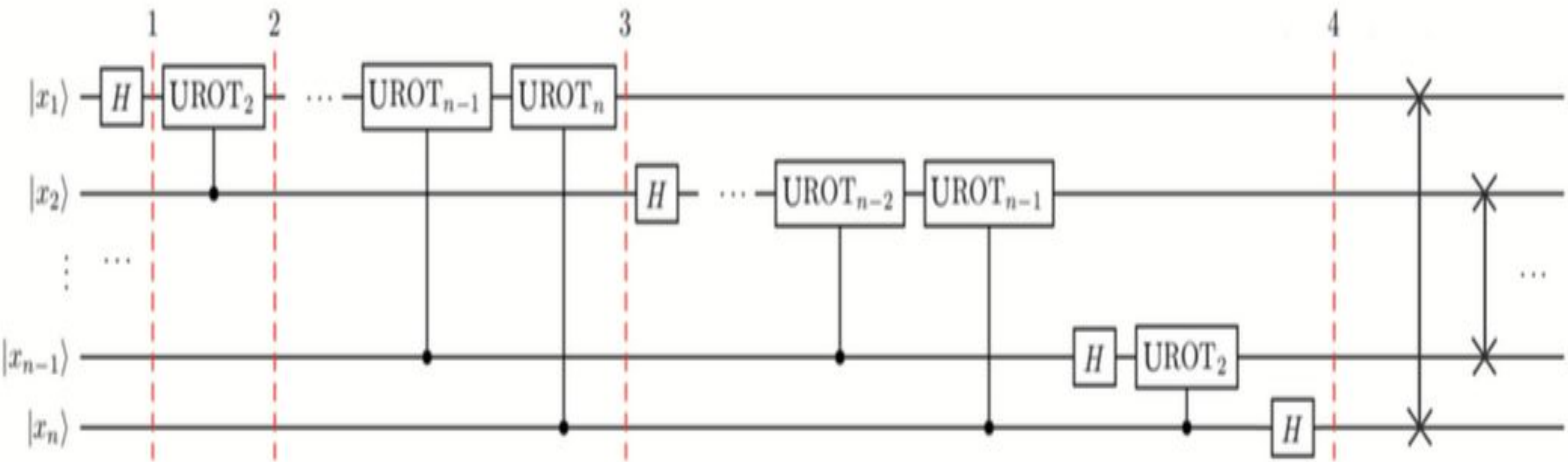
$$\bigotimes_{n=1}^{3} e^{i\pi * k}|1\rangle$$

$$= e^{i\pi*1}|1\rangle \otimes e^{i\pi*2}|1\rangle \otimes e^{i\pi*3}|1\rangle$$

What does a QFT do?



The QFT Circuit



The 'Unitary Rotation' Gate

The URot-gate rotates an amount depending on the value of k.

'k' is the power of the control bit.

$$UROT_k = \begin{bmatrix} 1 & 0 \\ 0 & \exp\left(\frac{2\pi i}{2^k}\right) \end{bmatrix}$$

The QFT Formula

The circuit we built gives us the formula for the QFT:

$$\frac{1}{\sqrt{2}}\left[|0\rangle + \exp\left(\frac{2\pi i}{2^n}x\right)|1\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0\rangle + \exp\left(\frac{2\pi i}{2^{n-1}}x\right)|1\rangle\right] \otimes \ldots \otimes \frac{1}{\sqrt{2}}\left[|0\rangle + \exp\left(\frac{2\pi i}{2^2}x\right)|1\rangle\right] \otimes \frac{1}{\sqrt{2}}\left[|0\rangle + \exp\left(\frac{2\pi i}{2^1}x\right)|1\rangle\right]$$

$$= \frac{1}{\sqrt{N}} \bigotimes_{k=1}^{n} \left(|0\rangle + e^{\frac{2\pi i x_{b10}}{2^k}} |1\rangle\right)$$

The QFT Formula: An Example

Plugging in x=6 provides the following answer:

$$\frac{1}{\sqrt{8}}\bigotimes_{k=1}^{n}\left(|0\rangle + e^{\frac{2\pi i6}{2^k}}|1\rangle\right)$$

$$= \frac{1}{\sqrt{8}}\left[\left(|0\rangle + e^{\frac{2\pi i6}{2^1}}|1\rangle\right)\otimes\left(|0\rangle + e^{\frac{2\pi i6}{2^2}}|1\rangle\right)\otimes\left(|0\rangle + e^{\frac{2\pi i6}{2^3}}|1\rangle\right)\right]$$

Binary and Decimal

- The notation for a string of binary and its decimal form can be shown as follows (b is short for base):

$$y_{b2} = y_1 y_2 \cdots y_n$$

$$y_{b10} = \sum_{k=1}^{n} 2^{n-k} * y_k$$

Quantum Phase Estimation (QPE)

The Question

The Question:

- Imagine we run a qubit through a circuit. This circuit is defined as the following:

$$U|\psi\rangle = e^{i\theta}|\psi\rangle$$

The Question:

- In other words, we rotate the qubit by $e^{i\theta}$.

$$U|\psi\rangle = e^{i\theta}|\psi\rangle$$

The Question:

- Of course, I did not define θ. That is because in this example, we do not know the value of it.

$$U|\psi\rangle = e^{i\theta}|\psi\rangle$$

$$\theta = ?$$

The Question:

- Is there a way to make a circuit that gets us this mystery θ-value?

$$U|\psi\rangle = e^{i\theta}|\psi\rangle$$

$$\theta = ?$$

Implementing a 1-bit QPE

1-Qubit QPE

- What we want is the exponent of our Unitary gate to be ‘saved’ in our circuit somehow.

$$U|\psi\rangle = e^{i\theta}|\psi\rangle$$

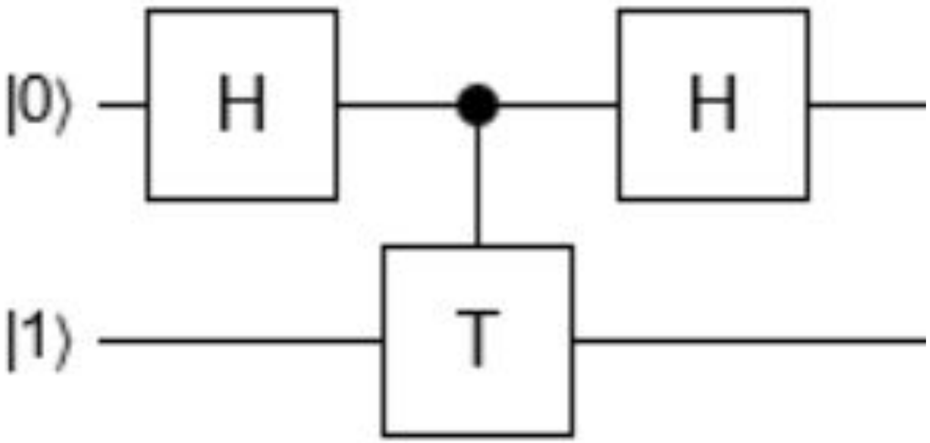
$$\theta = ?$$

1-Qubit QPE

- Going back to our previous lectures, does anyone remember how we were able to manipulate qubits outside of our gate, without modifying the qubits inside the main gate?

1-Qubit QPE

- We used phase kickback. Remembering this, we apply this concept to our naïve 1-qubit gate:



1-Qubit QPE

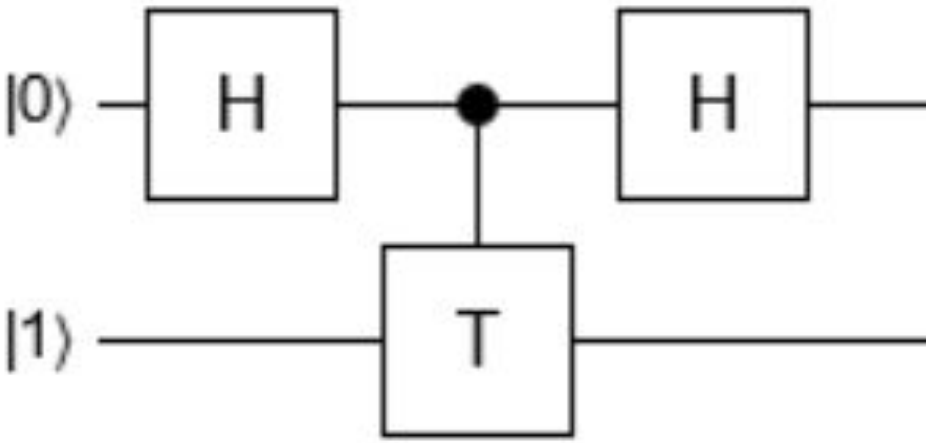
- Reminder, our “mystery” T-gate is equal to:

$$T = \begin{pmatrix} 1 & 0 \\ 0 & \exp\left(\frac{i\pi}{4}\right) \end{pmatrix}$$

- It rotates our qubit about 45 degrees on the Bloch sphere.

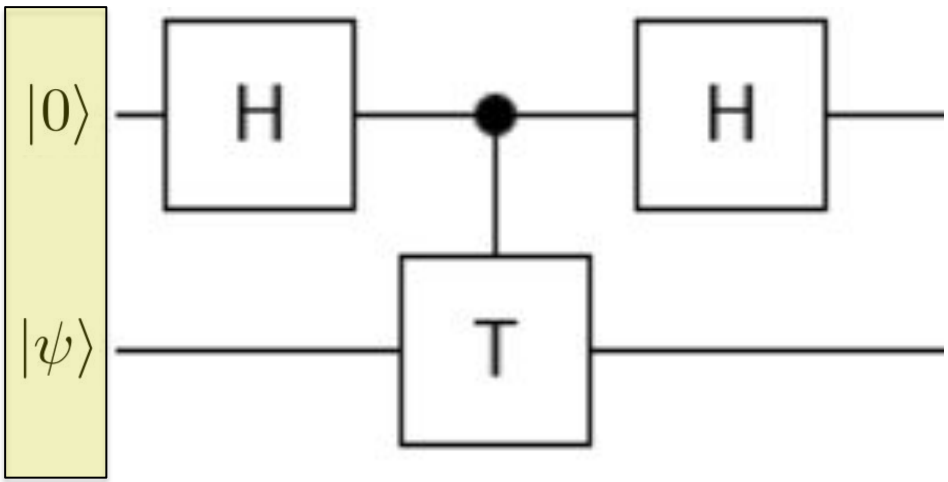
1-Qubit QPE

- Back to our circuit:



1-Qubit QPE

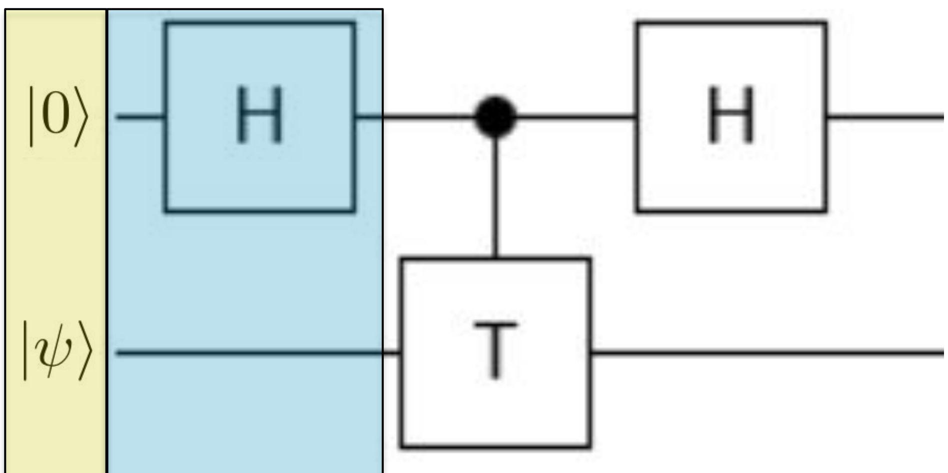
- Step 0:



$$= |0\rangle \otimes |\psi\rangle$$

1-Qubit QPE

- Step 1:

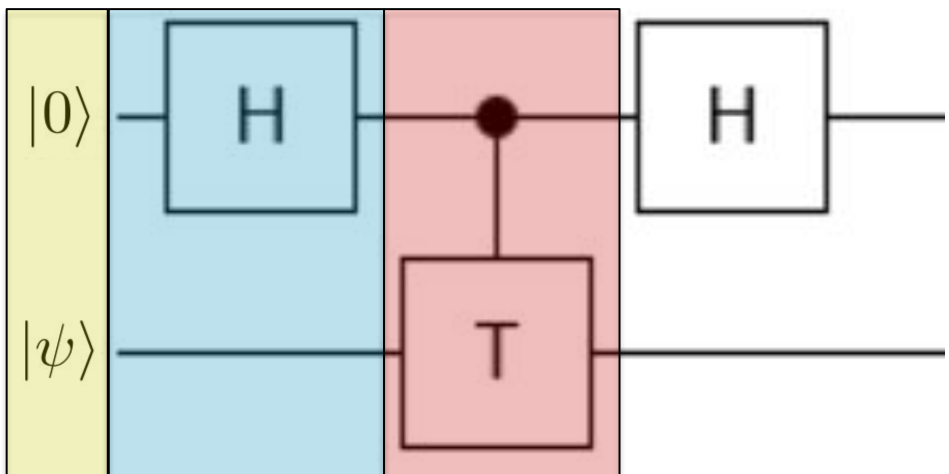


$$= |0\rangle \otimes |\psi\rangle$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |\psi\rangle$$
$$= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)$$

1-Qubit QPE

- Step 2:



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$$= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)$$

1-Qubit QPE

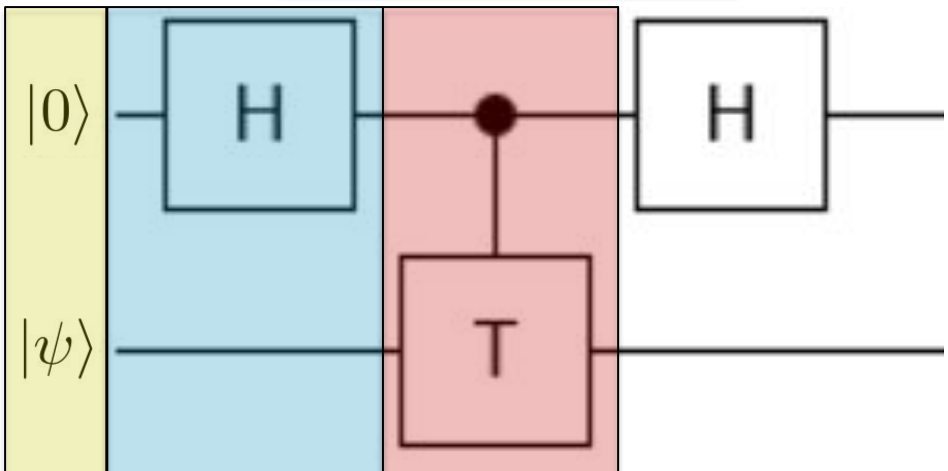
- Applying phase kickback in step 2 gives us:

$$
\begin{aligned}
&= T[c,\psi]\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle) \\
&= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + T|1\rangle|\psi\rangle) \\
&= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + e^{\frac{i\pi}{4}}|1\rangle|\psi\rangle)
\end{aligned}
$$

1-Qubit QPE

- Step 2:

$$= T[c,\psi]\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)$$
$$= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + e^{\frac{i\pi}{4}}|1\rangle|\psi\rangle)$$



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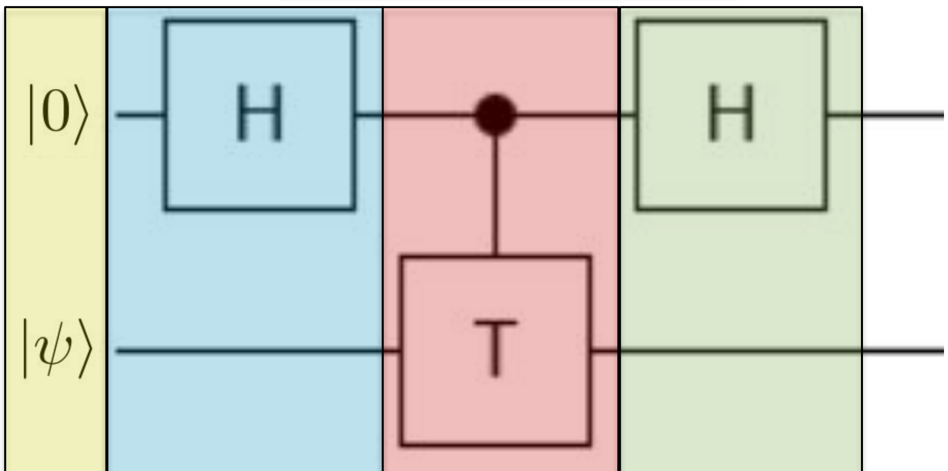
1-Qubit QPE

- Step 3:

$$= T[c,\psi]\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)$$
$$= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + e^{\frac{i\pi}{4}}|1\rangle|\psi\rangle)$$



$$= |0\rangle \otimes |\psi\rangle$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |\psi\rangle$$
$$= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)$$

1-Qubit QPE

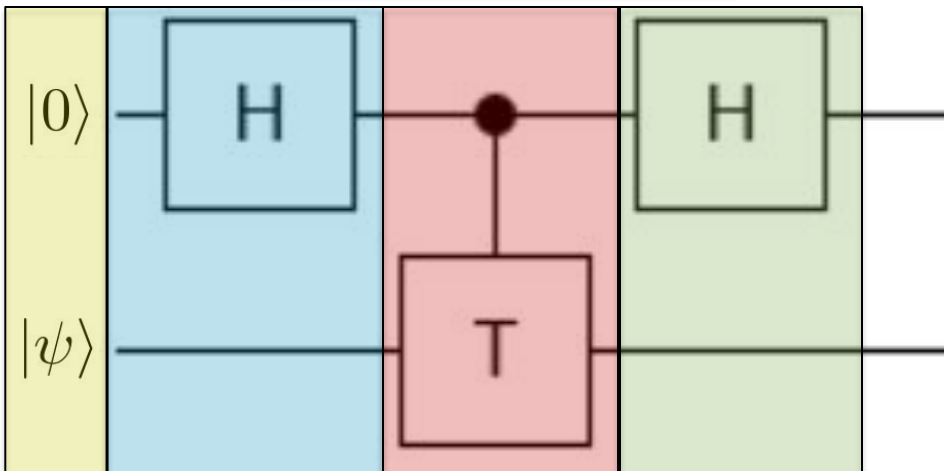
- Finishing the Hadamard gives us the following:

$$
\begin{aligned}
&H(\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + e^{\frac{i\pi}{4}}|1\rangle|\psi\rangle)) \\
&= \frac{1}{\sqrt{2}}(H|0\rangle|\psi\rangle + e^{\frac{i\pi}{4}}H|1\rangle|\psi\rangle) \\
&= \frac{1}{\sqrt{2}}(\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle) + e^{\frac{i\pi}{4}}\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle - |1\rangle|\psi\rangle)) \\
&= \frac{1}{2}(((|0\rangle|\psi\rangle + |1\rangle|\psi\rangle) + e^{\frac{i\pi}{4}}(|0\rangle|\psi\rangle - |1\rangle|\psi\rangle)) \\
&= \frac{1}{2}(((|0\rangle + |1\rangle) + e^{\frac{i\pi}{4}}(|0\rangle - |1\rangle))|\psi\rangle \\
&= \frac{1}{2}(|0\rangle(1 + e^{\frac{i\pi}{4}}) + |1\rangle(1 - e^{\frac{i\pi}{4}}))|\psi\rangle
\end{aligned}
$$

1-Qubit QPE

- Step 3:

$$= T[c,\psi]\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)$$
$$= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + e^{\frac{i\pi}{4}}|1\rangle|\psi\rangle)$$



$$= |0\rangle \otimes |\psi\rangle$$

$$= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \otimes |\psi\rangle$$
$$= \frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + |1\rangle|\psi\rangle)$$

$$H(\frac{1}{\sqrt{2}}(|0\rangle|\psi\rangle + e^{\frac{i\pi}{4}}|1\rangle|\psi\rangle))$$
$$= \frac{1}{2}(|0\rangle(1 + e^{\frac{i\pi}{4}}) + |1\rangle(1 - e^{\frac{i\pi}{4}}))|\psi\rangle$$

Interpreting Our Results

Interpreting Our Results

- Let's look a little closer at the output. Our mystery function (The T-gate) rotates our qubits by $e^{\frac{\pi i}{4}}$.

$$= \frac{1}{2}(|0\rangle(1 + e^{\frac{i\pi}{4}}) + |1\rangle(1 - e^{\frac{i\pi}{4}}))|\psi\rangle$$

Interpreting Our Results

- We can generalize this with any gate rotation using a variable θ. In the T-gate's case, θ=1/8.

$$= \frac{1}{2}(|0\rangle(1 + e^{\frac{i\pi}{4}}) + |1\rangle(1 - e^{\frac{i\pi}{4}}))|\psi\rangle$$

$$= \frac{1}{2}(|0\rangle(1 + e^{2\pi i\theta}) + |1\rangle(1 - e^{2\pi i\theta}))|\psi\rangle$$

Interpreting Our Results

- As you can see here, we have skewed the probability of measuring 1 or 0 a certain way depending on our angle.

$$Prob.|0\rangle = ||\frac{1}{2}(|0\rangle(1 + e^{2\pi i\theta})||$$

$$Prob.|1\rangle = ||\frac{1}{2}(|1\rangle(1 - e^{2\pi i\theta})||$$

Interpreting Our Results

- Obviously, we are looking for θ, not for 1 or 0. Whatever we are doing, 1 qubit of calculations is not precise enough.

$$Prob.|0\rangle = ||\frac{1}{2}(|0\rangle(1 + e^{2\pi i\theta})||$$

$$Prob.|1\rangle = ||\frac{1}{2}(|1\rangle(1 - e^{2\pi i\theta})||$$

Interpreting Our Results

- We can further skew our output by repeating this phase kickback. Will this give us θ?

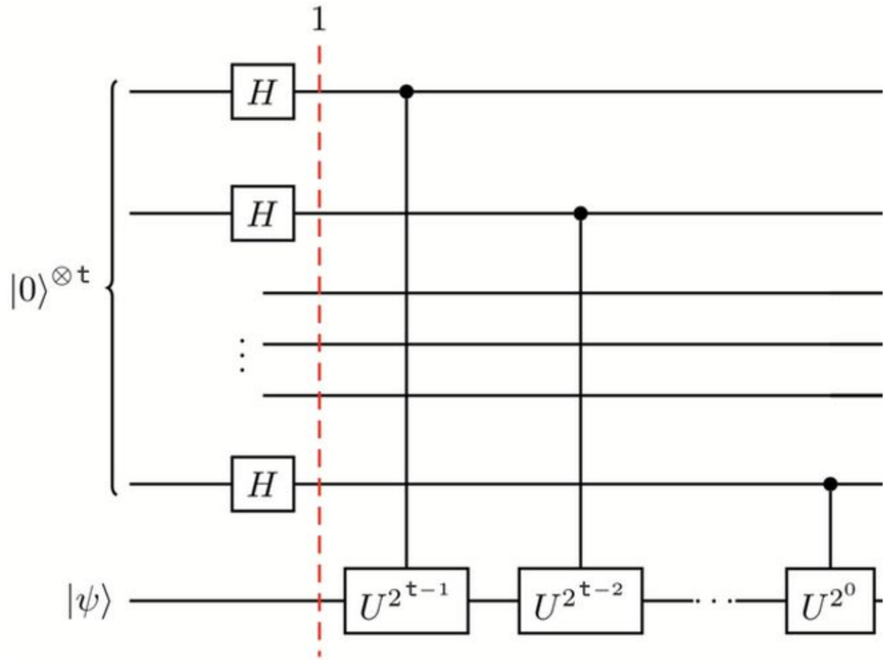
$$Prob.|0\rangle = ||\frac{1}{2}(|0\rangle(1+e^{2\pi i\theta})||$$

$$Prob.|1\rangle = ||\frac{1}{2}(|0\rangle(1-e^{2\pi i\theta})||$$

Implementing a N-bit QPE

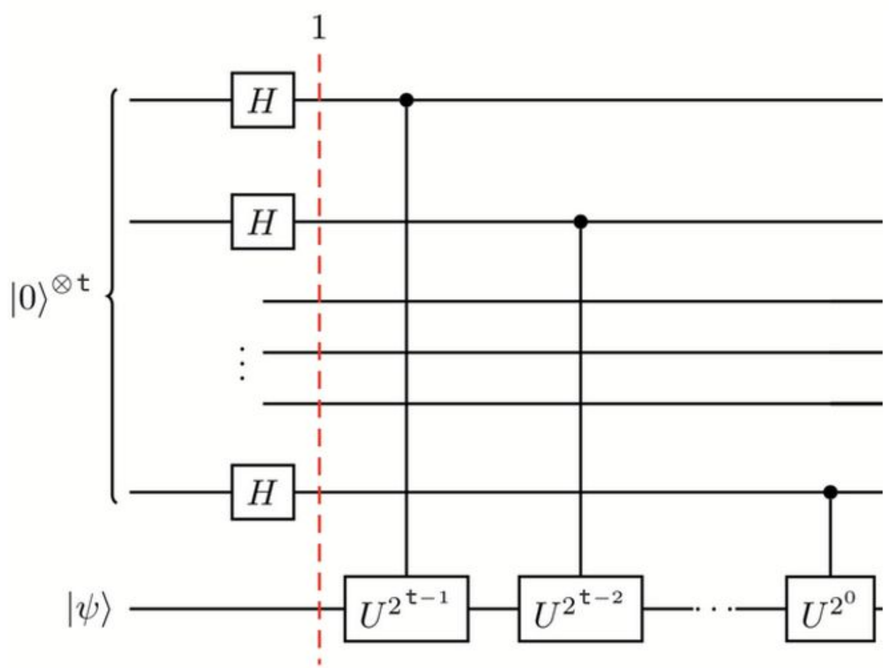
n-Qubit QPE

- Let's generalize our previous circuit to repeat over n-qubits instead of 1-qubit.



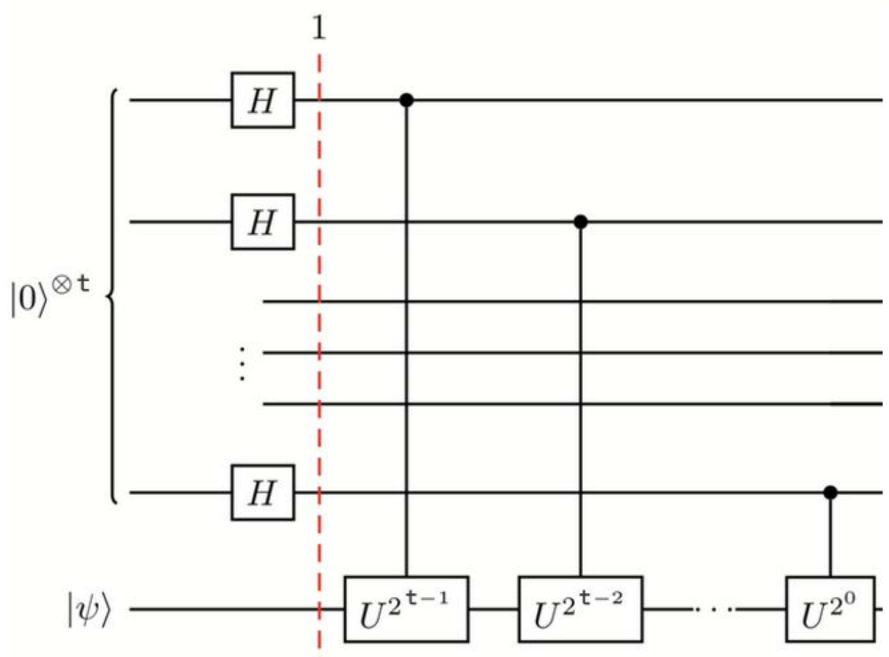
n-Qubit QPE

- The pattern seems simple enough, we simply repeat the U-gate for each control bit.



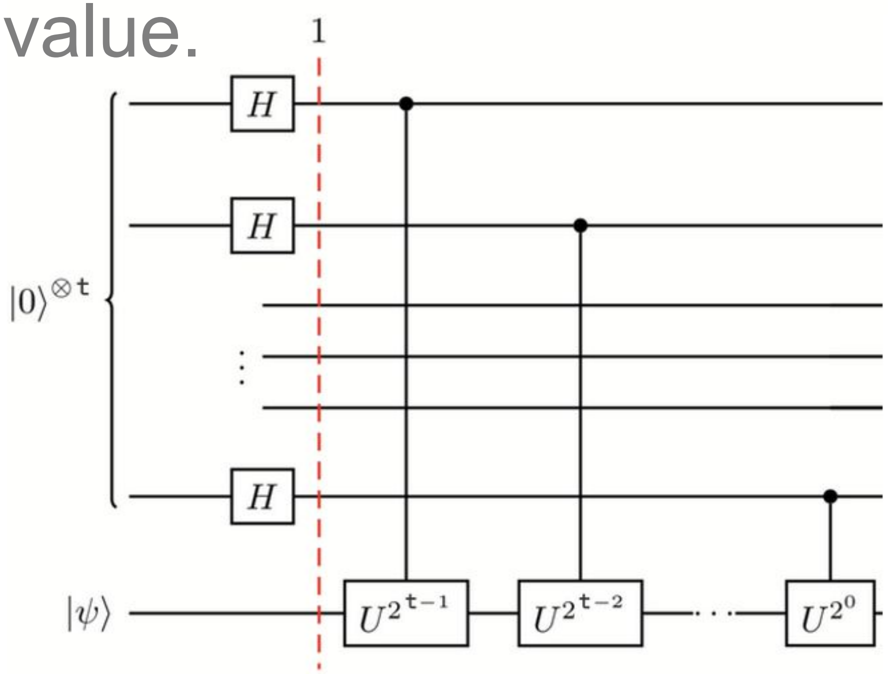
n-Qubit QPE

- However, what are these 2^{t-1}, 2^{t-2}, etc. values and how do they effect our U-gate?



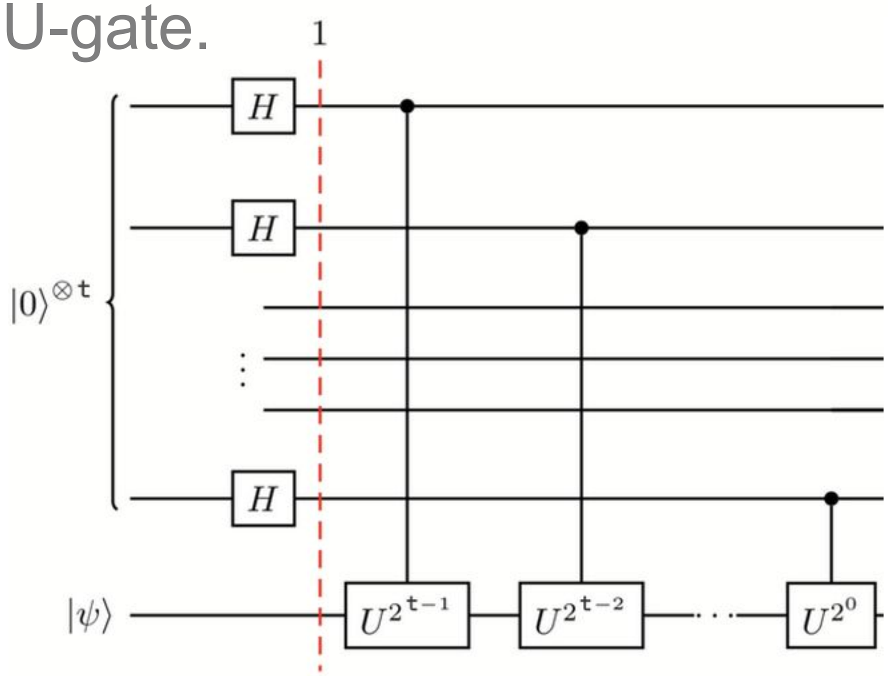
n-Qubit QPE

- For this notation, t is the length of our Hadamard-string. The larger the t, the more accurate our value.



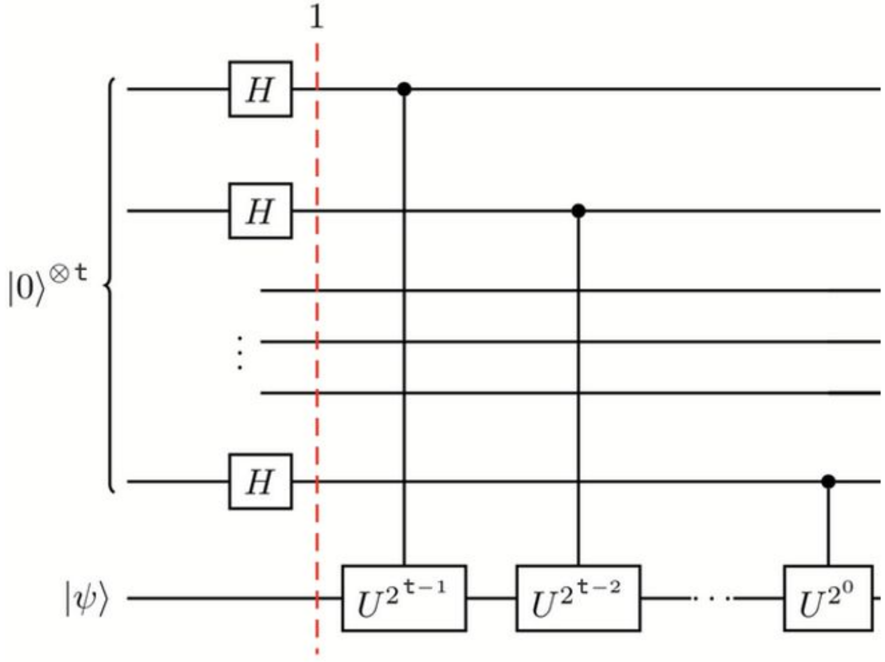
n-Qubit QPE

- $U^{2\wedge(t-1)}$ means U*U*U…U repeated $2^{(t-1)}$-times.
- $U^{2\wedge(t-2)}$ is 1 less repetitions of U.
- $U^{2\wedge 0}$ is a single U-gate.



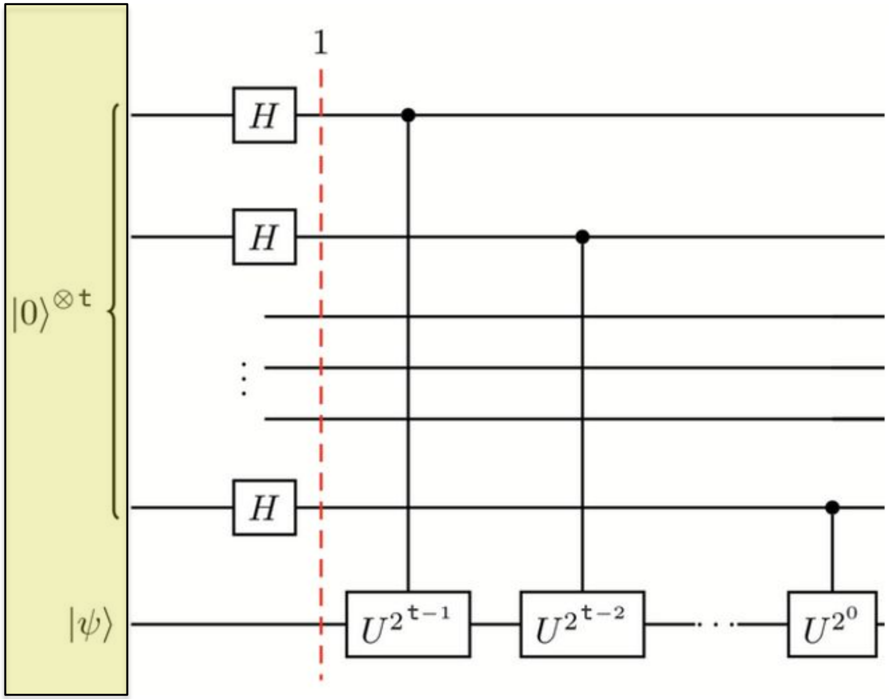
n-Qubit QPE

- So, what does this do?



n-Qubit QPE

- Step 0:

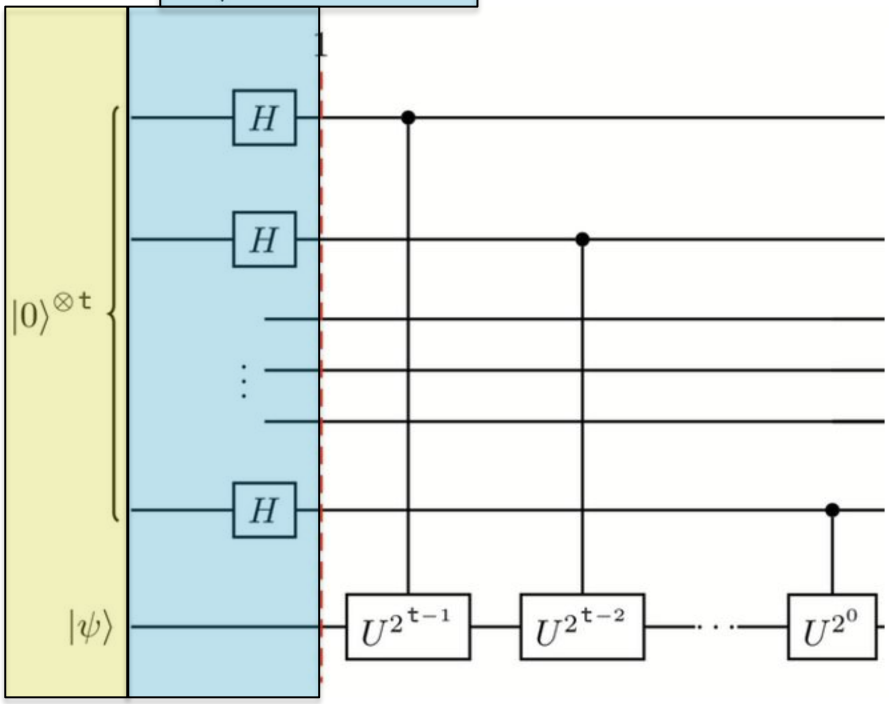


$|0\rangle^{\otimes t} \otimes |\psi\rangle$

n-Qubit QPE

- Step 1:

$$(H|0\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= (\frac{1}{\sqrt{2}})^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$

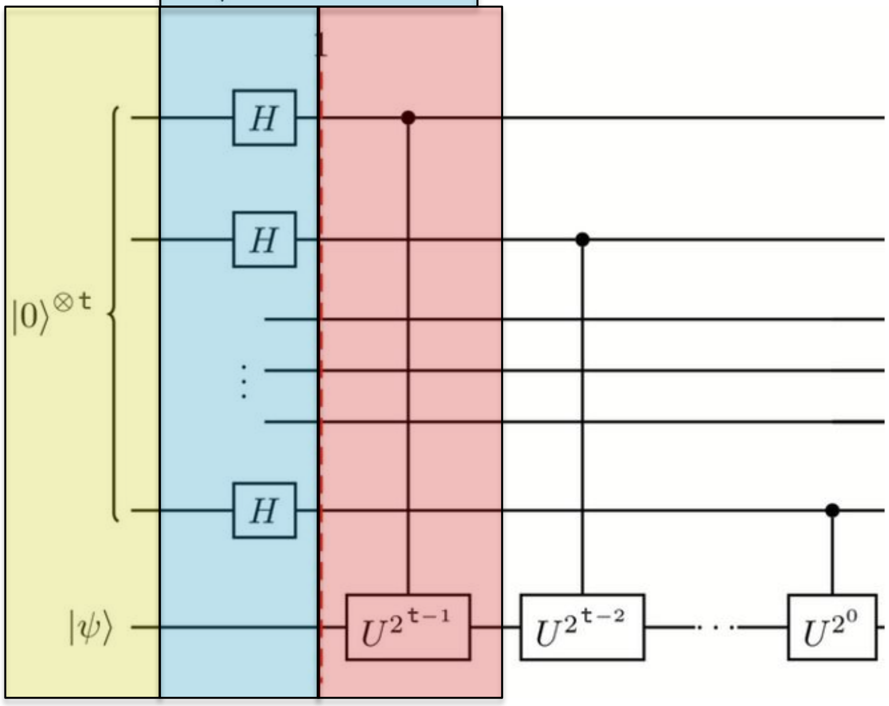


$$|0\rangle^{\otimes t} \otimes |\psi\rangle$$

n-Qubit QPE

- Step 2:

$$(H|0\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= (\frac{1}{\sqrt{2}})^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$



$$|0\rangle^{\otimes t} \otimes |\psi\rangle$$

n-Qubit QPE

- Applying phase kickback in step 2 gives us:

$$U^{2^{t-1}}[c,\psi](\frac{1}{\sqrt{2}})^t(|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$

$$= (\frac{1}{\sqrt{2}})^t * (|0\rangle + U^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$

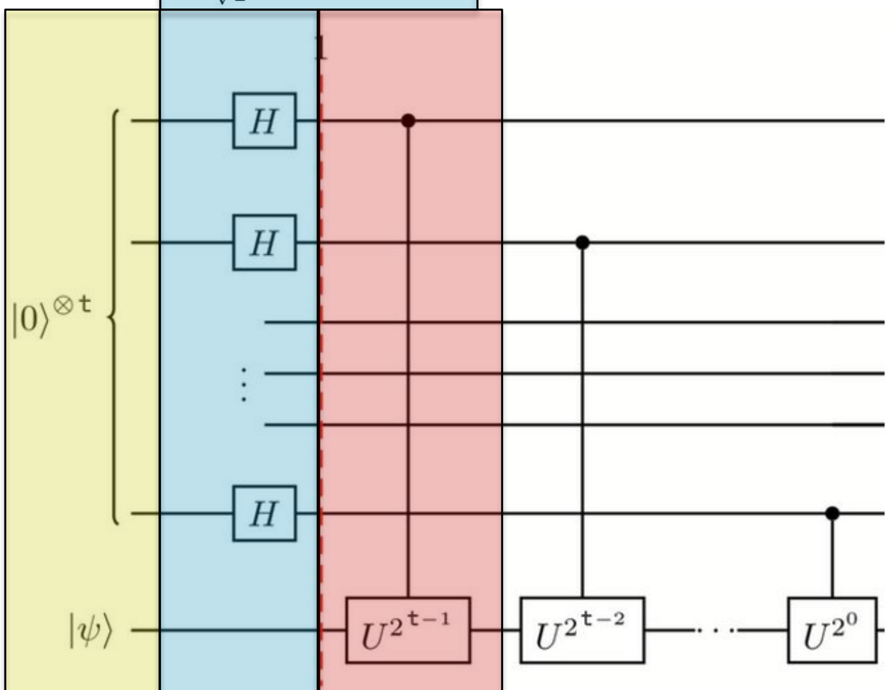
- Which if we substitute U with $e^{2\pi i\theta}$ gives us:

$$= (\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$

n-Qubit QPE

- Step 2:

$$(H|0\rangle)^{\otimes t} \otimes |\psi\rangle = (\frac{1}{\sqrt{2}})^t(|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$



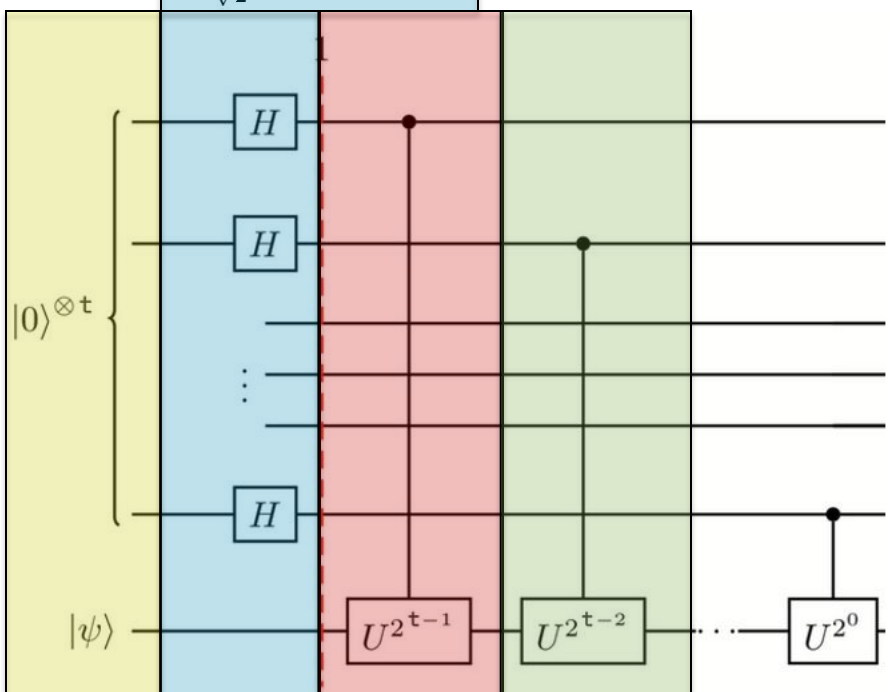
$$|0\rangle^{\otimes t} \otimes |\psi\rangle$$

$$U^{2^{t-1}}[c, \psi](\frac{1}{\sqrt{2}})^t(|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= (\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$

n-Qubit QPE

- Step 3:

$$(H|0\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= (\frac{1}{\sqrt{2}})^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$



$$|0\rangle^{\otimes t} \otimes |\psi\rangle$$

$$U^{2^{t-1}}[c, \psi](\frac{1}{\sqrt{2}})^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= (\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$

n-Qubit QPE

- Because we are just repeating step 2 but with the second qubit and one less U gate, the logic done before can be applied again:

$$U^{2^{t-2}}[c,\psi](\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$

$$= (\frac{1}{\sqrt{2}})^t * (|0\rangle + U^{2^{t-1}}|1\rangle) \otimes (|0\rangle + U^{2^{t-2}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-2} \otimes |\psi\rangle$$

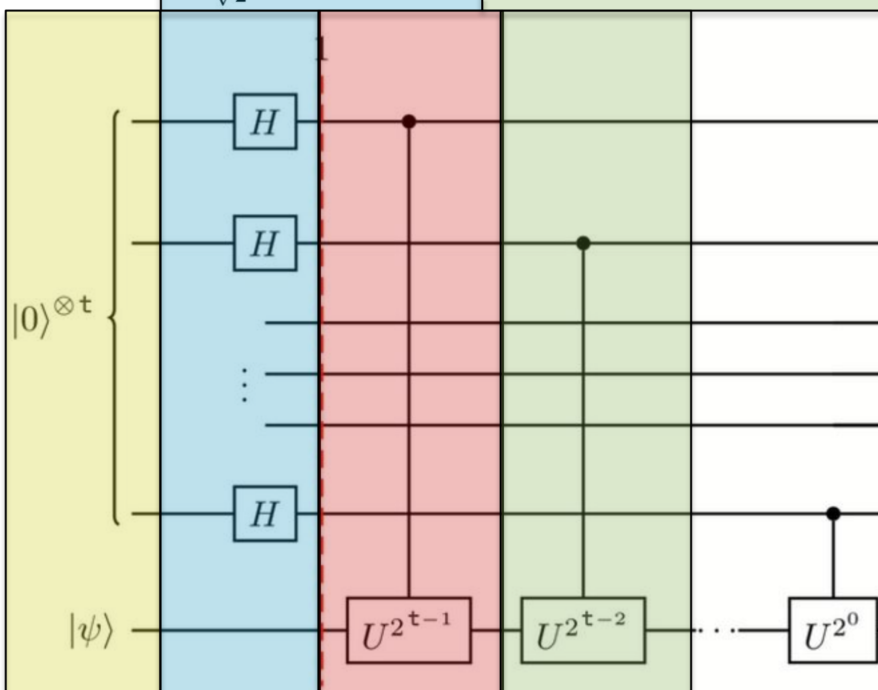
$$= (\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + (e^{2\pi i\theta})^{2^{t-2}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-2} \otimes |\psi\rangle$$

n-Qubit QPE

- Step 3:

$$(H|0\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= \left(\frac{1}{\sqrt{2}}\right)^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$

$$U^{2^{t-2}}[c,\psi]\left(\frac{1}{\sqrt{2}}\right)^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$
$$= \left(\frac{1}{\sqrt{2}}\right)^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + (e^{2\pi i\theta})^{2^{t-2}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-2} \otimes |\psi\rangle$$



$$|0\rangle^{\otimes t} \otimes |\psi\rangle$$

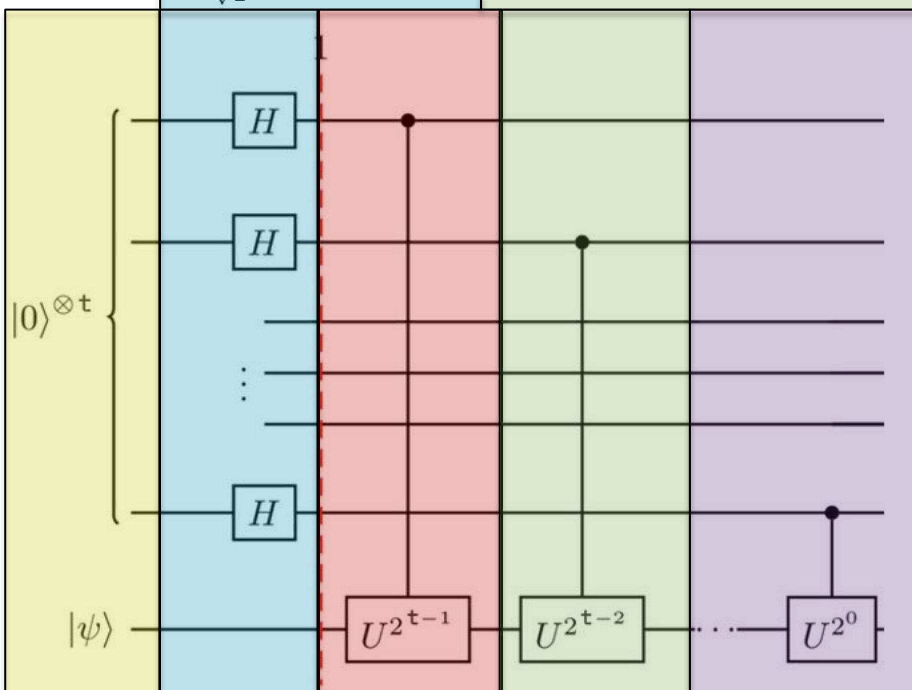
$$U^{2^{t-1}}[c,\psi]\left(\frac{1}{\sqrt{2}}\right)^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= \left(\frac{1}{\sqrt{2}}\right)^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$

n-Qubit QPE

- Step n:

$$(H|0\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= \left(\frac{1}{\sqrt{2}}\right)^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$

$$U^{2^{t-2}}[c,\psi]\left(\frac{1}{\sqrt{2}}\right)^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$
$$= \left(\frac{1}{\sqrt{2}}\right)^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + (e^{2\pi i\theta})^{2^{t-2}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-2} \otimes |\psi\rangle$$



$$|0\rangle^{\otimes t} \otimes |\psi\rangle$$

$$U^{2^{t-1}}[c,\psi]\left(\frac{1}{\sqrt{2}}\right)^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$$
$$= \left(\frac{1}{\sqrt{2}}\right)^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$$

n-Qubit QPE

- Repeating our steps over and over gives us the following formula:

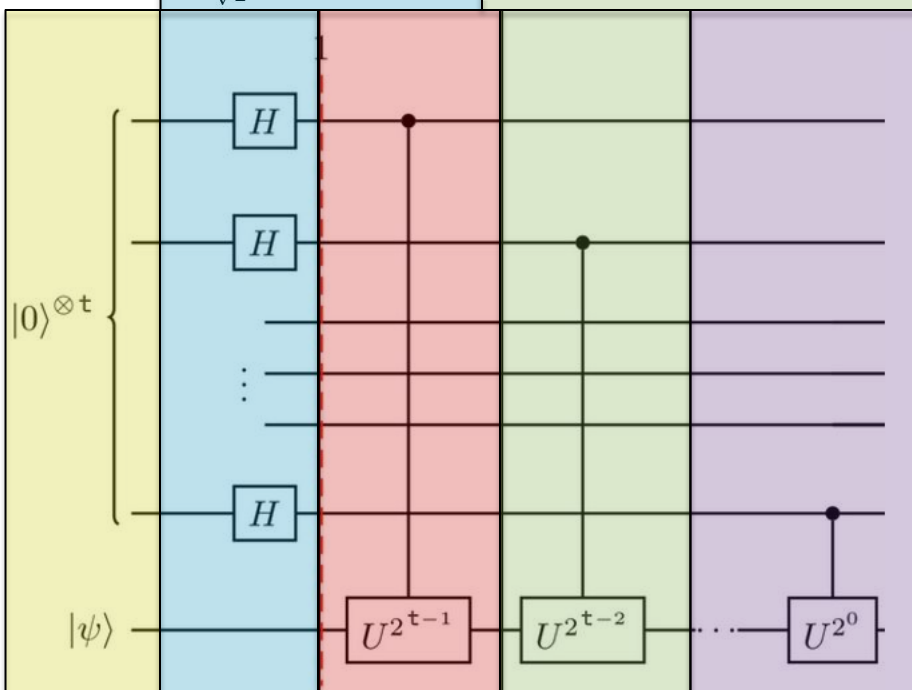
$$\implies (\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + (e^{2\pi i\theta})^{2^{t-2}}|1\rangle) \otimes ... \otimes (|0\rangle + (e^{2\pi i\theta})^{2^0}|1\rangle)$$

$$= \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} (|0\rangle + e^{2\pi i\theta * 2^k}|1\rangle)$$

- The first control qubit is modified by the (t-1) gate, the second qubit is modified by the (t-2) gate, etc.

n-Qubit QPE

- Step n:



$|0\rangle^{\otimes t} \otimes |\psi\rangle$

$(H|0\rangle)^{\otimes t} \otimes |\psi\rangle$

$= (\frac{1}{\sqrt{2}})^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$

$U^{2^{t-1}}[c,\psi](\frac{1}{\sqrt{2}})^t (|0\rangle + |1\rangle)^{\otimes t} \otimes |\psi\rangle$

$= (\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$

$U^{2^{t-2}}[c,\psi](\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-1} \otimes |\psi\rangle$

$= (\frac{1}{\sqrt{2}})^t * (|0\rangle + (e^{2\pi i\theta})^{2^{t-1}}|1\rangle) \otimes (|0\rangle + (e^{2\pi i\theta})^{2^{t-2}}|1\rangle) \otimes (|0\rangle + |1\rangle)^{\otimes t-2} \otimes |\psi\rangle$

$= \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} (|0\rangle + e^{2\pi i\theta * 2^k}|1\rangle)$

Interpreting Our Results

Interpreting Our Results

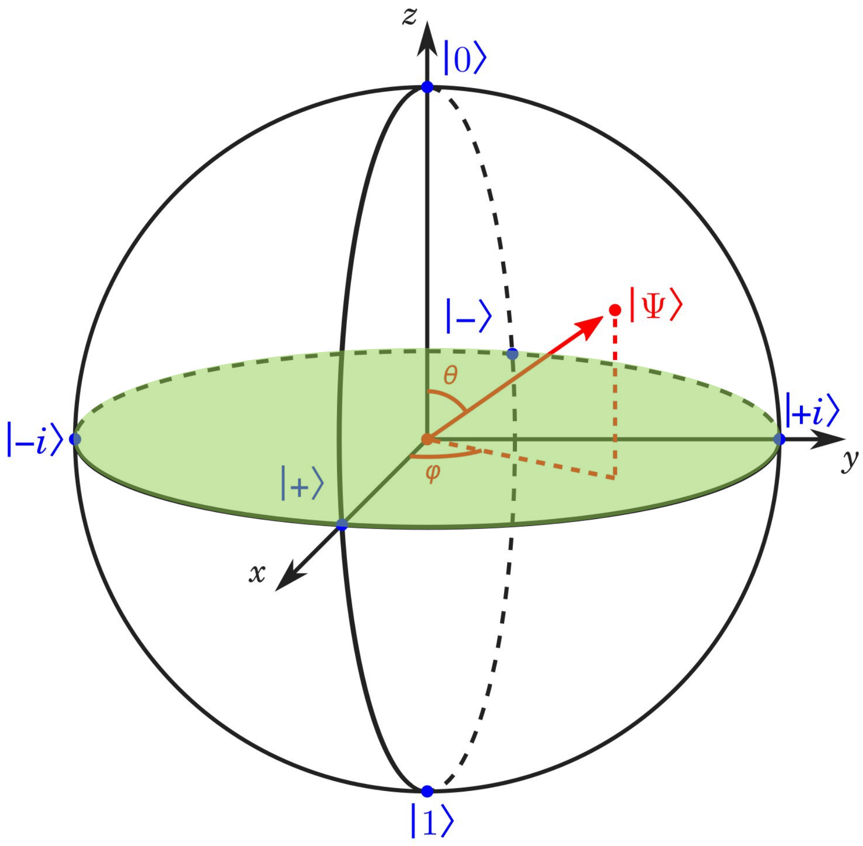
- We simply repeated our naïve 1-qubit algorithm iteratively until we reached our nth qubit.

Interpreting Our Results

- As such, we shifted our probabilities of measuring 1 and 0 depending on our angle θ.
- This shifted is more and more precise the more qubits we use.

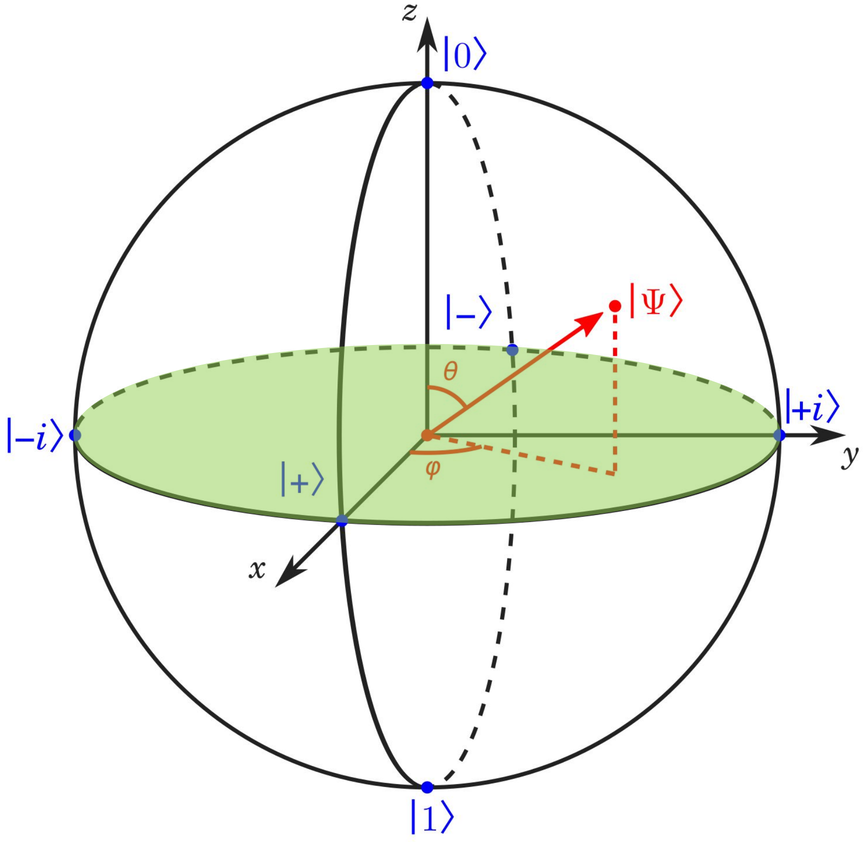
Interpreting Our Results

- Because of the precision, each value of θ should have a unique probability.
- To reword this, each value of θ should have a unique coordinate on the Bloch Sphere's x/y axis.



Interpreting Our Results

Sound Familiar?



Interpreting Our Results

Turns out, the formula is almost the exact same as the QFT, with the difference being $1/2^n$.

$$\frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} (|0\rangle + e^{2\pi i\theta * 2^k} |1\rangle)$$

$$\frac{1}{\sqrt{N}} \bigotimes_{k=1}^{n} (|0\rangle + e^{\frac{2\pi i x_{b10}}{2^k}} |1\rangle)$$

Interpreting Our Results

$$= \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} \left(|0\rangle + e^{2\pi i\theta * 2^k} |1\rangle \right)$$

$$\implies \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} \left(|0\rangle + e^{\frac{1}{2^n}} * e^{2\pi i\theta * 2^k} |1\rangle \right)$$

$$= \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} \left(|0\rangle + e^{\frac{1}{2^n}} * e^{2\pi i\theta * 2^k} |1\rangle \right)$$

$$= \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} \left(|0\rangle + e^{\frac{2\pi i\theta * 2^k}{2^n}} |1\rangle \right)$$

$$= \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} \left(|0\rangle + e^{\frac{2\pi i\theta}{2^k}} |1\rangle \right)$$

Interpreting Our Results

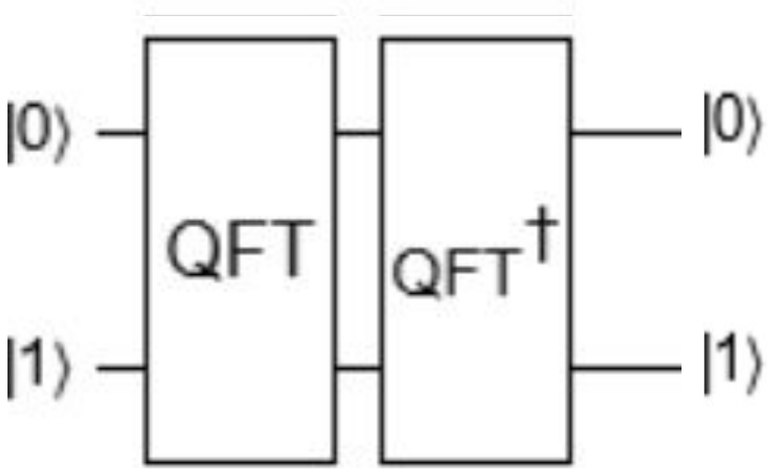
When you multiply out the constant, it is the same formula, with some variables renamed.

$$= \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} (|0\rangle + e^{2\pi i\theta * 2^k} |1\rangle)$$

$$= \frac{1}{\sqrt{N}} \bigotimes_{k=1}^{n} (|0\rangle + e^{\frac{2\pi i x_{b10}}{2^k}} |1\rangle)$$

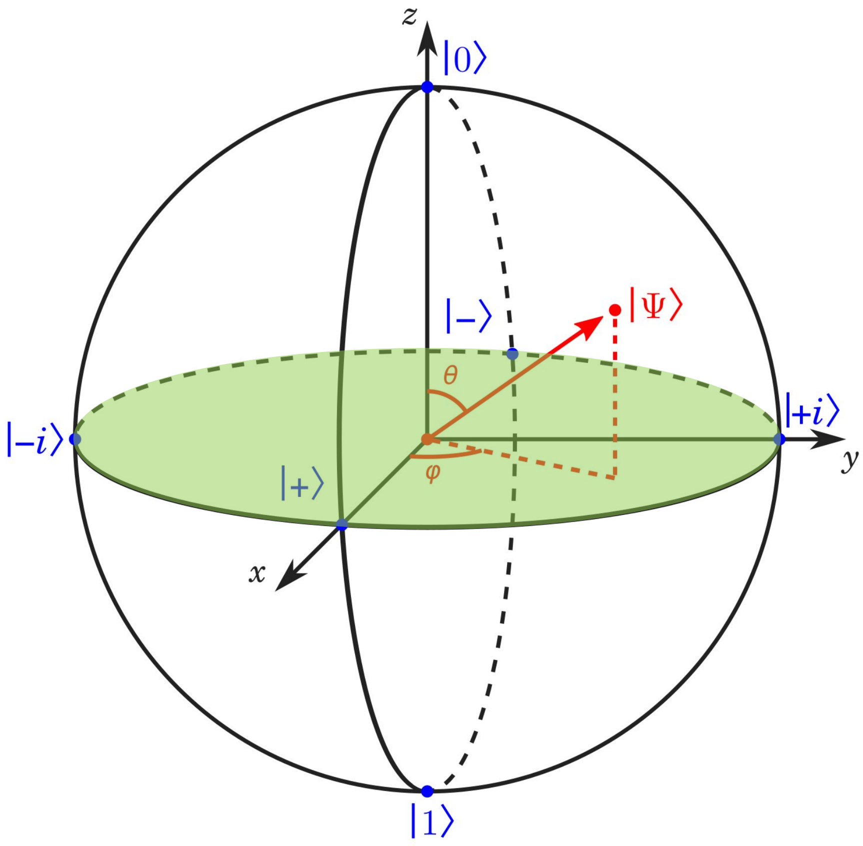
Interpreting Our Results

- Finally, we can remember that every function is unitary in Quantum Computing.
- This means that for every function, there is an inverse function.



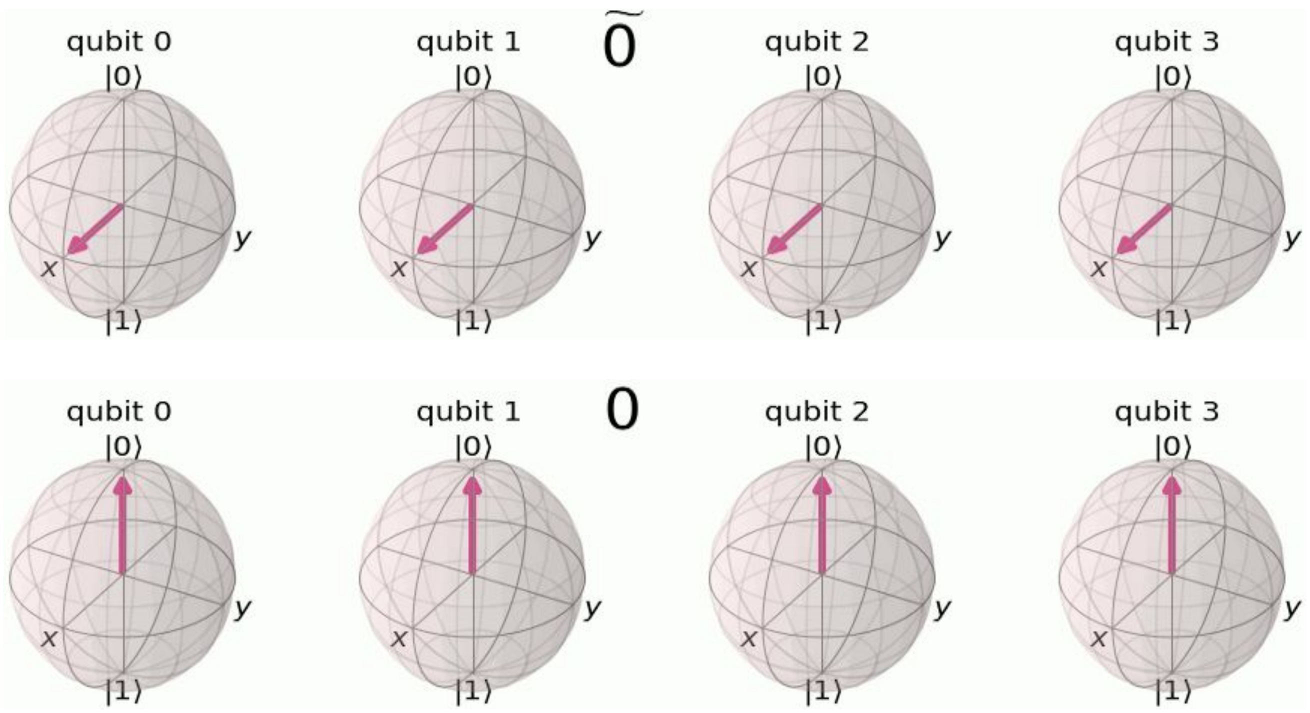
Interpreting Our Results

- If a QFT takes a number and turns it into a rotation angle along the Bloch Sphere…
- Then the Inverse QFT takes that rotation and gets us our number.



Interpreting Our Results

- The input of a QFT is our unknown θ!



Interpreting Our Results

- …Except our QPE is equal to the QFT function, multiplied by a constant 2^n. which means we must divide by 2^n to get our true value for θ.

$$QPE = \frac{1}{\sqrt{T}} \bigotimes_{k=1}^{t} (|0\rangle + e^{2\pi i\theta * 2^k} |1\rangle)$$

$$QFT = \frac{1}{\sqrt{N}} \bigotimes_{k=1}^{n} (|0\rangle + e^{\frac{2\pi i x_{b10}}{2^k}} |1\rangle)$$

Interpreting Our Results

- In the base case of 1-qubit, we only skewed our algorithm slightly. This resulted in an output with low precision and only gave us 0 or ½ as our estimation for θ. With value closest to the true answer having the highest probability.

$$\theta \in \begin{cases} 0 & a = 0 \\ \frac{1}{2} & a = 1. \end{cases}$$

Interpreting Our Results

- In the case of a 2-qubit QPE, we skewed our algorithm slightly more. This resulted in an output with higher precision and gave us more potential answers. With value closest to the true answer having the highest probability.

$$\theta \in \{0, 1/4, 1/2, 3/4\}$$

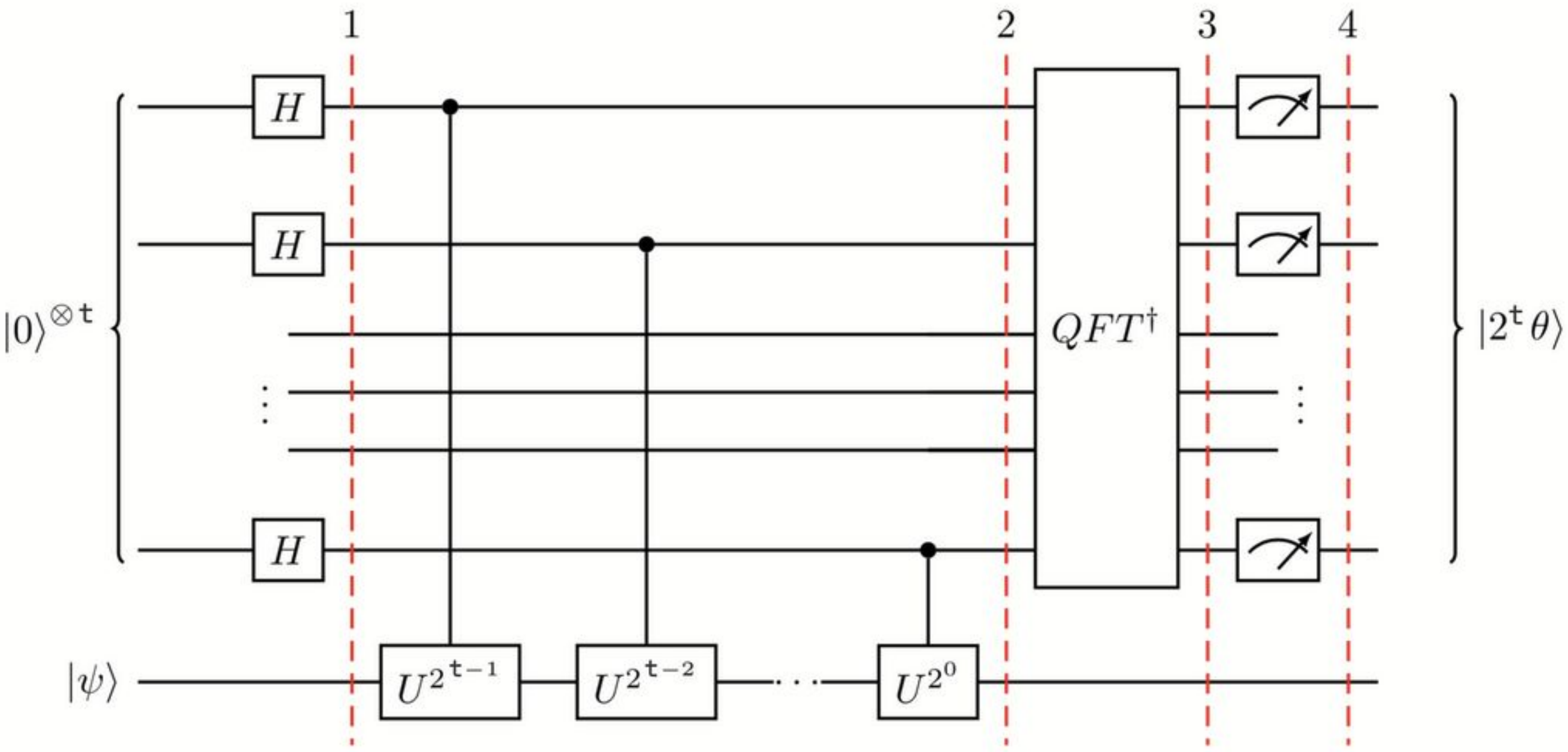
Interpreting Our Results

- Naturally, as n increases, so does our decimal range. With enough precision, we can get our desired angle with any number of decimal places.

$$\theta \in \{\mathbb{R}\}$$

The Final Circuit

The Final Circuit



Lets Get Started