Book Club Meeting 6

Zach Lewis

Announcements with Sam

- Goodbye Zach!
- Positions opening up
  - Secretary
  - Vice President
  - Treasurer
- SQUID information
- Project Update

Phase Estimation

Phase Estimation

Link:
https://learning.quantum.ibm.com/course/fundamentals-of-quantum-algorithms/phase-estimation-and-factoring

Problem statement

In the *phase estimation problem*, we're given a quantum state $|\psi\rangle$ of n qubits, along with a unitary quantum circuit that acts on n qubits. We're *promised* that $|\psi\rangle$ is an eigenvector of the unitary matrix U that describes the action of the circuit, and our goal is to either identify or approximate the eigenvalue λ to which $|\psi\rangle$ corresponds.

More precisely, because λ lies on the complex unit circle we can write

$$\lambda = e^{2\pi i\theta}$$

for a unique real number θ satisfying $0 \leq \theta < 1$. The goal of the problem is to compute or approximate this real number θ.

> **Phase estimation problem**
> Input: An n qubit quantum state $|\psi\rangle$ and a unitary quantum circuit for an n-qubit operation U
> Promise: $|\psi\rangle$ is an eigenvector of U
> Output: an approximation to the number $\theta \in [0, 1)$ satisfying $U|\psi\rangle = e^{2\pi i\theta}|\psi\rangle$

So what does that mean?

- The Phase Estimation Algorithm **estimates** the phase corresponding to an eigenvalue of a given **unitary** operator.
- The eigenvalues of a unitary operator are characterized by their phase, and therefore the algorithm can be equivalently described as retrieving either the **phase** or the **eigenvalue** itself.
- Phase estimation is often considered a **subroutine**, particularly and most famously in **Shor's** Factoring Algorithm

Additional Note

The phase estimation problem is different from other problems we've seen so far in the series in that the input includes a quantum state.

This means that the math is different than that which was studied last week, albeit it is not much harder (especially if you make the computer do the math for you.