# Book Club Meeting 4

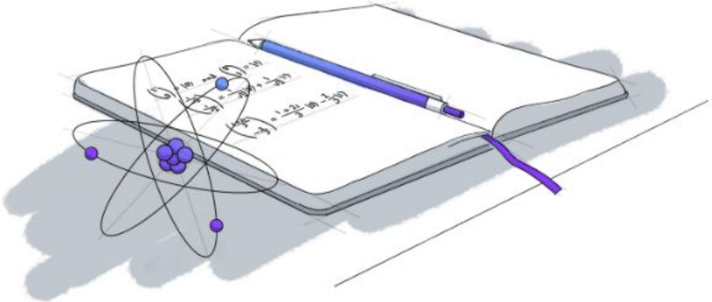
*Sam Bieberich*

# Pages

Page preview



# Single systems



This lesson introduces the basic framework of quantum information, including the description of quantum states as vectors with complex number entries, measurements that allow classical information to be extracted from quantum states, and operations on quantum states that are described by unitary matrices. We will restrict our attention in this lesson to the comparatively simple setting in which a single system is considered in isolation. In the next lesson, we will expand our view to multiple systems, which can interact with one another and be correlated, for instance.

There are, in fact, two common mathematical descriptions of quantum information. The one introduced in this lesson is the simpler of the two. This description is sufficient for understanding many (or perhaps most) quantum algorithms, and is a natural place to start from a pedagogical viewpoint.

A more general, and ultimately more powerful description of quantum information, in which quantum states are represented by *density matrices*, will be introduced in a later lesson. The density matrix descr
essential for the study of quantum information for reasons that will be explained as they arise. As
the density matrix description is used to model the effects of noise on quantum computations, an
generally it serves as a mathematical basis for quantum information theory and quantum cryptog

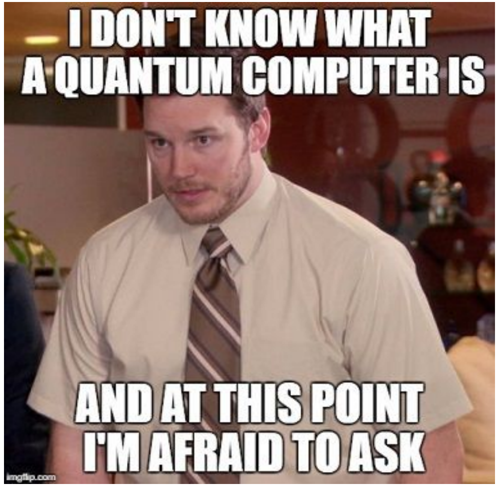
Go to page

We've only made it this far…

**Single systems**

1. Classical information
   - 1.1 Classical states and probability vectors
   - 1.2 Measuring probabilistic states
   - 1.3 Classical operations
2. Quantum information
   - 2.1 Quantum state vectors
   - 2.2 Measuring quantum states
   - **2.3 Unitary operations**
3. Code examples
   - 3.1 Vectors and matrices in Python
   - 3.2 States, measurements, and operations in Qiskit

Multiple systems



# Unitaries

# Operations

Quantum computers can only do some operations, and in many (most) cases these are fundamentally different from classical operations.

Classical operations include:

- AND
- OR
- NAND
- XOR

# Operations

Quantum physics is *unitary*, meaning that operations on a quantum state must be unitary.

Unitary -

In linear algebra, an invertible complex square matrix $U$ is **unitary** if its conjugate transpose $U^*$ is also its inverse, that is, if

$$U^*U = UU^* = UU^{-1} = I,$$

where $I$ is the identity matrix.

# Conjugate Transpose

Since this math stuff is not super necessary to understand, here is a super fast and easy example. Note that the operation is depicted as a dagger superscript

**Example** Define the matrix

$$A = \begin{bmatrix} 3i & -i \\ 1 & 2-i \end{bmatrix}$$

Its conjugate is

$$\overline{A} = \begin{bmatrix} -3i & i \\ 1 & 2+i \end{bmatrix}$$

and its conjugate transpose is

$$A^* = \overline{A}^\top = \begin{bmatrix} -3i & 1 \\ i & 2+i \end{bmatrix}$$

# Unitary Operations

There are actually quite a few you already would recognize (remember QC is limited to these). They expand to encompass multiple qubits, and may be composed by multiplying the matrices together.

1. *Pauli operations.* The four Pauli matrices are as follows:

$$\mathbb{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

A common notation, which we will often use, is $X = \sigma_x$, $Y = \sigma_y$, and $Z = \sigma_z$ — but be aware that the letters $X, Y$, and $Z$ are also commonly used for other purposes. The $\sigma_x$ (or $X$) operation is also called a *bit flip* or a *NOT operation* because it induces this action on bits:

$$\sigma_x|0\rangle = |1\rangle \quad \text{and} \quad \sigma_x|1\rangle = |0\rangle.$$

The $\sigma_z$ (or $Z$) operation is also called a *phase flip* because it has this action:

$$\sigma_z|0\rangle = |0\rangle \quad \text{and} \quad \sigma_z|1\rangle = -|1\rangle.$$

2. *Hadamard operation.* The Hadamard operation is described by this matrix:

$$H = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

3. *Phase operations.* A phase operation is one described by the matrix

$$P_\theta = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$$

for any choice of a real number $\theta$. The operations

$$S = P_{\pi/2} = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix} \quad \text{and} \quad T = P_{\pi/4} = \begin{pmatrix} 1 & 0 \\ 0 & \frac{1+i}{\sqrt{2}} \end{pmatrix}$$

are particularly important examples. Other examples include $\mathbb{1} = P_0$ and $\sigma_z = P_\pi$.

All of the matrices just defined are unitary, and therefore represent quantum operations on a single qubit.

# Code Break

*https://learn.qiskit.org/course/basics/single-systems#single-14-0*


Quantum Particles: *Vibing*
Human: *observes them*
Quantum Particles:
well now I am not doing it

# Questions?

# Multiple Systems

# Quantum Systems

Classical computing and quantum computing are combined when optimizing several qubits working together and controlling one another. This is best exhibited with multiple systems.

Since we kinda already have been covering this, I am just going to add a few things.

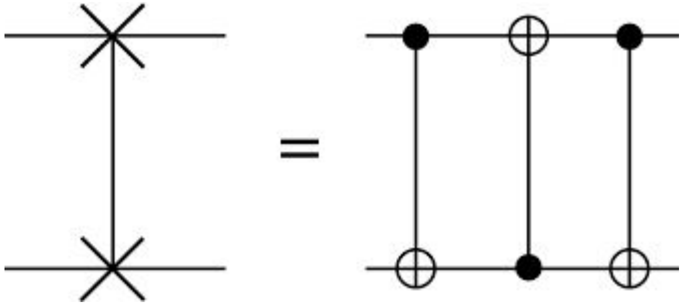
# Any Unitary can be a gate or be made of quantum gates

$$U = \begin{pmatrix} \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & 0 & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{i}{2} & -\frac{1}{2} & 0 & 0 & -\frac{i}{2} \\ \frac{1}{2} & -\frac{1}{2} & \frac{1}{2} & 0 & 0 & -\frac{1}{2} \\ 0 & 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ \frac{1}{2} & -\frac{i}{2} & -\frac{1}{2} & 0 & 0 & \frac{i}{2} \\ 0 & 0 & 0 & -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \end{pmatrix}.$$

# Swapping

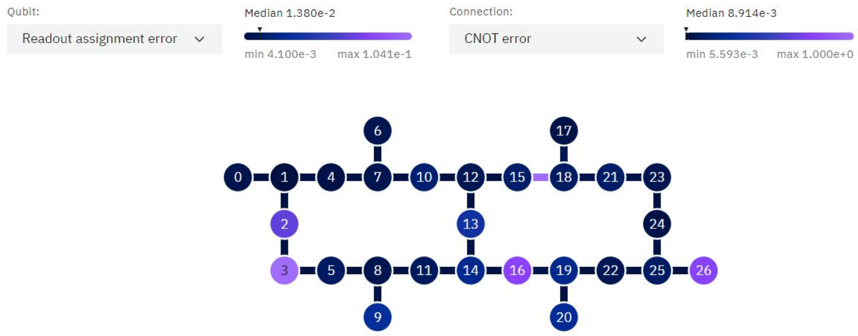
$$\mathrm{SWAP}\,|a\rangle|b\rangle = |b\rangle|a\rangle.$$

$$\mathrm{SWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$



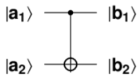
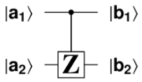
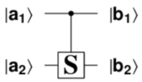
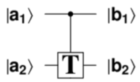
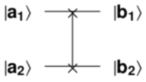
# Lots and lots of swapping…

Since superconducting supercomputers can’t have connections between all qubits due to hardware limitations, a lot of qubits need to be swapped to utilize large algorithms, making the swap and thus the CNOT gate the most common in quantum programming.



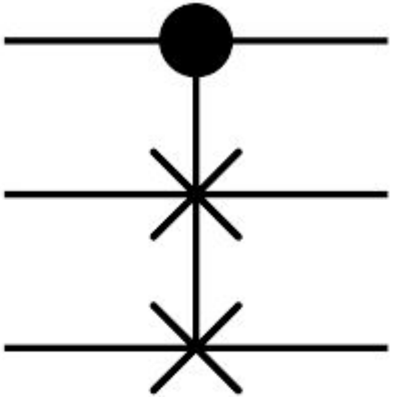

# Control a state

If you want to control another qubit, just add the top left matrix and zero everything else.

| | | |
|---|---|---|
| CNOT gate |  | $\begin{pmatrix} \boxed{\begin{matrix}1 & 0 \\ 0 & 1\end{matrix}} & \begin{matrix}0 & 0 \\ 0 & 0\end{matrix} \\ \begin{matrix}0 & 0 \\ 0 & 0\end{matrix} & \begin{matrix}0 & 1 \\ 1 & 0\end{matrix} \end{pmatrix}$ |
| Controlled - Z |  | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$ |
| Controlled - S |  | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & i \end{pmatrix}$ |
| Controlled - T |  | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\frac{\pi}{4}} \end{pmatrix}$ |
| SWAP gate |  | $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ |

# It extrapolates too lol

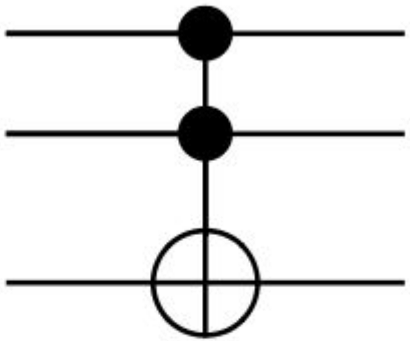
$$\text{CSWAP} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$



# It can't possibly extrapolate any mo…

CCX or Toffoli Gate

- Universal (along with H gate)



$$CCX = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

# Code Break

*https://learn.qiskit.org/course/basics/multiple-systems#multiple-29-0*

Questions?

emma
Follow
name a more iconic duo.. I'll wait.
Quantum Computation and Quantum Information
MICHAEL A. NIELSEN and ISAAC L. CHUANG

# Limitations on Quantum Computation

*Aw man*

math

# Limitations

There are three main quantum limitations. These are never going to be fixed unless we break physics, as they are defined by quantum mechanics.

# Global Phases are Irrelevant

Due to math shenanigans, this can be easily explained.

From Medium:

“Making an analogy, let’s say I want to leave my house and walk to the bakery. What matters is the relative position between my position and the destination.

However, strictly speaking, we can say that the planet Earth is rotating, and at every moment, my position is modified in relation to the center of the Earth. The same happens with the bakery, which has its position modified by the rotation of the Earth at every moment.

Therefore, the global phase affects both terms of the equation equally and can be eliminated. I don’t need to map the absolute position in relation to the universe (and, according to Einstein, there’s no absolute position).“

# No Cloning

The no-cloning theorem shows it is impossible to create a perfect copy of an unknown quantum state.

> **Theorem (No-cloning theorem)**:
>
> Let X and Y be systems sharing the same classical state set $\Sigma$ having at least two elements. There does not exist a quantum state $|\phi\rangle$ of Y and a unitary operation $U$ on the pair $(\mathsf{X}, \mathsf{Y})$ such that
>
> $$U\big(|\psi\rangle \otimes |\phi\rangle\big) = |\psi\rangle \otimes |\psi\rangle \tag{6}$$
>
> for every state $|\psi\rangle$ of X.

# Why no cloning?

The necessary cloning operation would never be able to be both linear and unitary, making it not possible to manipulate.

This is kinda a problem, considering much of what computers do is read bits and “clone” them

# Non-orthogonal states cannot be perfectly discriminated

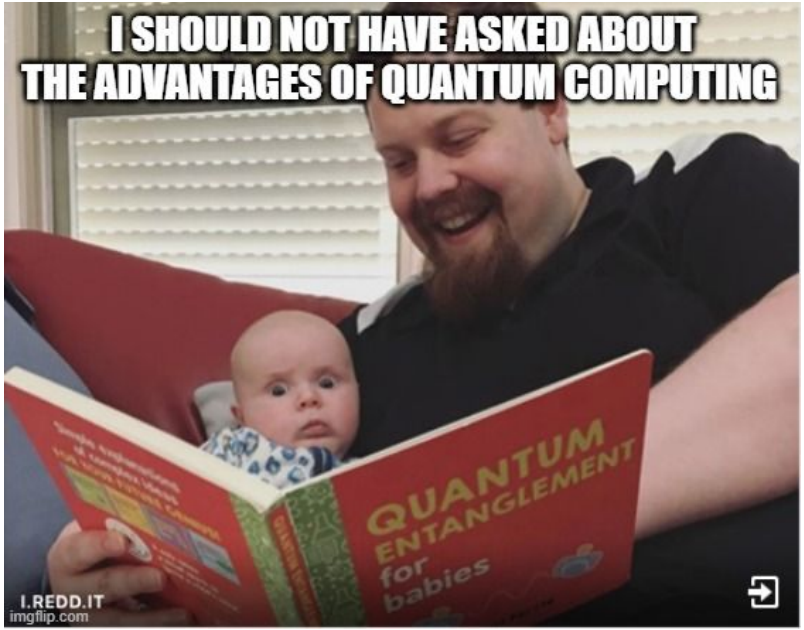
If we now have a state $|\psi\rangle$ which is either $|\psi_a\rangle$ or $|\psi_b\rangle$ which are not orthogonal, then there is no measurement that can perfectly tell us which of these states $|\psi\rangle$ is.

... but we can perform a measurement that tells us something about the likelihood of whether $|\psi\rangle = |\psi_a\rangle$ or $|\psi\rangle = |\psi_b\rangle$.

Intuitively:

- If we just guess, we will be correct with probability equal to one half, so we expect to be able to do better than this.
- The "closer together" $|\psi_a\rangle$ and $|\psi_b\rangle$ are, the harder they will be to distinguish (i.e., the lower the probability of correctly inferring $|\psi\rangle$)

# Questions?


I SHOULD NOT HAVE ASKED ABOUT
THE ADVANTAGES OF QUANTUM COMPUTING
QUANTUM
ENTANGLEMENT
for
babies
I.REDD.IT
imgflip.com

See you two weeks from now for:

# Entanglement in Action