1. Considering the following two-dimensional measurements:

	<i>y</i> ₁	y ₂
x_1	-2	2
x_2	-1	3
x_3	0	1
x_4	-2	1

- (a) What are the maximum likelihood parameters of a multivariate Gaussian distribution for this data set?
- (b) What is the Gaussian's shape? Draw its contour plot.
- (a)
- (b)

2. Consider the following data-set, paired with a query vector $x_{new} = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix}^T$:

	<i>y</i> ₁	<i>y</i> ₂	<i>y</i> ₃	y ₄	<i>y</i> ₅	z
$\overline{x_1}$	1	1	0	1	0	1
x_2	1	1	1	0	0	0
x_3	0	1	1	1	0	0
x_4	0	0	0	1	1	0
<i>x</i> ₅	1	0	1	1	1	1
x_6	0	0	1	0	0	1
<i>x</i> ₇	0	0	0	0	1	1

- (a) Using Bayes' rule, without making any assumptions, compute the posterior probabilities for the query vector. How is it classified?
- (b) What is the problem of working without assumptions?
- (c) Compute the class for the same query vector under the naive Bayes assumption?
- (d) Consider the presence of missings. Under the same naive Bayes assumption, how would you classify the query vector $x_{new} = \begin{bmatrix} 1 & ? & 1 & ? & 1 \end{bmatrix}^T$?
- (a)
- (b)
- (c)
- (d)

3. Considering the following data set, paired with the query vector $x_{new} = \begin{bmatrix} 100 & 225 \end{bmatrix}^T$:

	<i>y</i> ₁	y ₂	z
x_1	170	160	0
x_2	80	220	1
x_3	90	200	1
x_4	60	160	0
<i>x</i> ₅	50	150	0
x_6	70	190	1

- (a) Compute the most probable class for the query vector assuming that the likelihoods are 2-dimensional Gaussians.
- (b) Compute the most probable class for the query vector, under the Naive Bayes assumption, using 1-dimensional Gaussians to model the likelihoods.

(a)

(b)

- 4. Assuming training examples with m features and a binary class:
- (a) How many parameters are needed to estimate, considering boolean features and:
 - i. No assumptions regarding the data's distribution.
 - ii. Naive Bayes assumption.
- (b) How many parameters are needed to estimate, considering numeric-valued features and:
 - i. Multivariate Gaussian assumption.
 - ii. Naive Bayes assumption.
- (a) i.

ii.

(b) i.

ii.