$\boxed{\text{Fractal}}$ Geometrical object that is similar on all scales

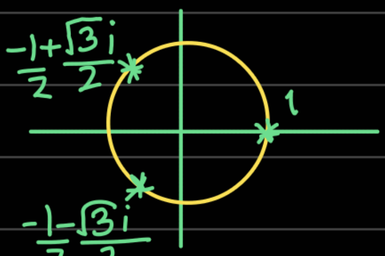
We will be generating a fractal for the equation used to find cube root of 1:

$z^3 = 1$

or, $f(z) = z^3 - 1 \longrightarrow f'(z) = 3z^2$

Now we use know the roots of the equation:

$z = 1, -\frac{1}{2} \pm \frac{\sqrt{3}\,i}{2}$



The roots are spread equally over a unit circle.

Since it simulates a circle in the complex plane, we can use euler's formula: $\cos\theta + i\sin\theta = e^{i\theta}$

Writing $e^{i\theta}$ as z^3: $e^{i\theta} = z^3$

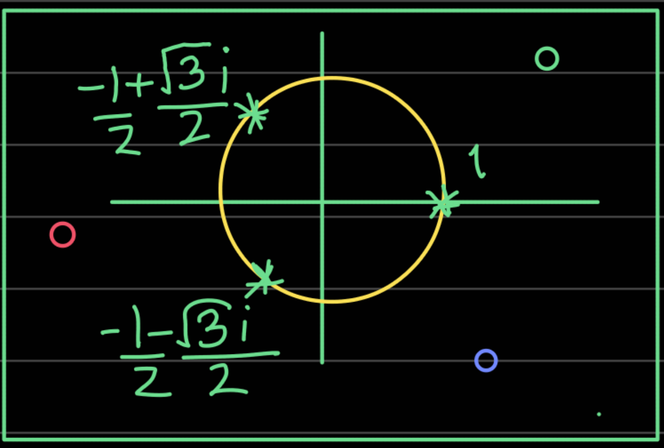
or, $z = e^{i2\pi n/3}$

$\therefore$ The solution to z is:

$$z = \begin{cases} 1 \\ e^{i2\pi/3} \\ e^{i-2\pi/3} \end{cases}$$ — polar forms

These are roots r_1, r_2, r_3

In Newton's method we start with a point in the complex plane and consider the convergence to the 3 roots each: $z_{n+1} = z_n - \frac{f(z_n)}{f'(z_n)}$



o $\longrightarrow$ 3 proposed points in the plane that will converge to each of the 3 roots.

The proposed points will have rgb colours and will create a fractal as they converge to the roots.

Exercise fourth roots of unity:

$z^4 = 1$

$f(z) = z^4 - 1$

$z-1$ is a root:

$$\begin{array}{r|l} & z^3+z^2+z+1 \\ \hline z-1 & z^4-1 \\ & \underline{z^4-z^3} \\ & -1+z^3 \\ & \underline{-z^2+z^3} \\ & -1+z^2 \\ & \underline{-z+z^2} \\ & -1+z \\ & \underline{-1+z} \\ & \times \end{array}$$

$\therefore f(z) = (z-1)(z^3+z^2+z+1)$

let $c(z) = (z^3+z^2+z+1)$

$c(-1) = -1+1-1+1 = 0$

$\therefore$ $z+1$ is a root of $c(z)$:

$$\begin{array}{r|l} & z^2+1 \\ \hline z+1 & z^3+z^2+z+1 \\ & z^3+z^2 \\ \hline & z+1 \end{array}$$

$\therefore f(z) = (z-1)(z+1)(z^2+1)$

or $f(z) = (z-1)(z+1)(z+i)(z-i)$

$\boxed{\text{Matlab}}$

How meshgrid works:

$nx = 10$, $ny = 10$, $xmin/ymin = -2$
$xmax/ymax = 2$

```
X = linspace(xmin, xmax, nx);
Y = linspace(ymin, ymax, ny);
[X, Y] = meshgrid(X, Y);  %meshgrid(X, Y) creates a 2D grid of same
                           number of points as X and Y
Z = X + 1i*Y;  %our complex gridpoints used for creating the plane
```

Z × 2D array of Z created

10x10 complex double

	1	2	3	4	5	6	7	8	9	10
1	-2.0000 - 2.0000i	-1.5556 - 2.0000i	-1.1111 - 2.0000i	-0.6667 - 2.0000i	-0.2222 - 2.0000i	0.2222 - 2.0000i	0.6667 - 2.0000i	1.1111 - 2.0000i	1.5556 - 2.0000i	2.0000 - 2.0000i
2	-2.0000 - 1.5556i	-1.5556 - 1.5556i	-1.1111 - 1.5556i	-0.6667 - 1.5556i	-0.2222 - 1.5556i	0.2222 - 1.5556i	0.6667 - 1.5556i	1.1111 - 1.5556i	1.5556 - 1.5556i	2.0000 - 1.5556i
3	-2.0000 - 1.1111i	-1.5556 - 1.1111i	-1.1111 - 1.1111i	-0.6667 - 1.1111i	-0.2222 - 1.1111i	0.2222 - 1.1111i	0.6667 - 1.1111i	1.1111 - 1.1111i	1.5556 - 1.1111i	2.0000 - 1.1111i
4	-2.0000 - 0.6667i	-1.5556 - 0.6667i	-1.1111 - 0.6667i	-0.6667 - 0.6667i	-0.2222 - 0.6667i	0.2222 - 0.6667i	0.6667 - 0.6667i	1.1111 - 0.6667i	1.5556 - 0.6667i	2.0000 - 0.6667i
5	-2.0000 - 0.2222i	-1.5556 - 0.2222i	-1.1111 - 0.2222i	-0.6667 - 0.2222i	-0.2222 - 0.2222i	0.2222 - 0.2222i	0.6667 - 0.2222i	1.1111 - 0.2222i	1.5556 - 0.2222i	2.0000 - 0.2222i
6	-2.0000 + 0.2222i	-1.5556 + 0.2222i	-1.1111 + 0.2222i	-0.6667 + 0.2222i	-0.2222 + 0.2222i	0.2222 + 0.2222i	0.6667 + 0.2222i	1.1111 + 0.2222i	1.5556 + 0.2222i	2.0000 + 0.2222i
7	-2.0000 + 0.6667i	-1.5556 + 0.6667i	-1.1111 + 0.6667i	-0.6667 + 0.6667i	-0.2222 + 0.6667i	0.2222 + 0.6667i	0.6667 + 0.6667i	1.1111 + 0.6667i	1.5556 + 0.6667i	2.0000 + 0.6667i
8	-2.0000 + 1.1111i	-1.5556 + 1.1111i	-1.1111 + 1.1111i	-0.6667 + 1.1111i	-0.2222 + 1.1111i	0.2222 + 1.1111i	0.6667 + 1.1111i	1.1111 + 1.1111i	1.5556 + 1.1111i	2.0000 + 1.1111i
9	-2.0000 + 1.5556i	-1.5556 + 1.5556i	-1.1111 + 1.5556i	-0.6667 + 1.5556i	-0.2222 + 1.5556i	0.2222 + 1.5556i	0.6667 + 1.5556i	1.1111 + 1.5556i	1.5556 + 1.5556i	2.0000 + 1.5556i
10	-2.0000 + 2.0000i	-1.5556 + 2.0000i	-1.1111 + 2.0000i	-0.6667 + 2.0000i	-0.2222 + 2.0000i	0.2222 + 2.0000i	0.6667 + 2.0000i	1.1111 + 2.0000i	1.5556 + 2.0000i	2.0000 + 2.0000i
11										

After z has been computed in newton method loop:

Z ×

100x100 complex double

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	
1	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
2	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
3	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
4	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
5	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
6	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
7	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
8	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
9	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
10	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
11	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
12	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
13	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
14	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
15	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0.8660i	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.5000 - 0....	-0.50
16	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0.8660	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.5000 - 0	-0.50

Command Window

We can see a few convergences here