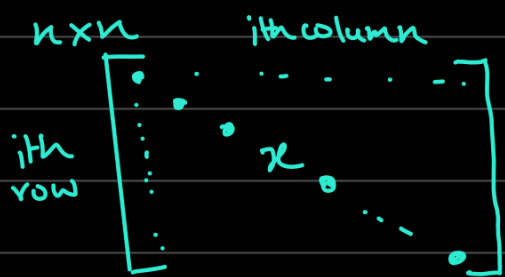


Operation Count for gaussian elimination

Consider an $n \times n$ matrix and an element 'x' in the i th row and column:



The total number of operations required for gaussian elimination from element 'x' is:

- ① $(n-i)$ multiplications of row i by $(n-i)$ elements under x .
- ② $(n-i)$ additions of multiplied row i members to rows underneath it.

$$\text{Total} = (n-i)^2 \text{ operations}$$

\therefore Total number of operations for every element along diagonal is:

$$T = \sum_{k=1}^{n-1} (n-i)^2$$

recall summation identity $\sum_{k=1}^n k^2 = \frac{1}{6} n(n+1)(n+1)$ — same

\therefore The order of the gaussian elimination algo is $O(n^3)$.

Operation count for forward and backward substitutions

Now we see why solving $LUX = B$ is faster than solving $AX = B$ using gaussian elimination.
consider —

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_{1n} \\ 0 & a_{22} & a_{23} & \dots & a_{2n} \\ \vdots & & & \ddots & \\ 0 & \dots & \dots & \dots & a_{nn} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}$$

$$x_i = \frac{1}{a_{ii}} \left(b_i - \sum_{j=i+1}^n (a_{ij} x_j) \right) \quad \text{--- ①}$$

\therefore # of operations for backward operation per row

= $n-i+1$ multiplications + additions for eq ①

of rows $\rightarrow n$, \therefore we do all these multiplications and additions for each of the n rows. The operation counts for each row keep summing up?

$$\therefore \text{Operations count} = \sum_{i=1}^n (n-i+1)$$

$$\text{same form as } \sum_{i=1}^n i = \frac{n(n+1)}{2}$$

\therefore Order: $O(n^2)$

\therefore LU decomposition then solution of $LUX = B$ is faster than gaussian elimination for solving $AX = B$