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Homework 2

1.) Proof that addition is commutative (continued from lecture 4):

$(q + n)' = q + n'$	but we need $q' + n$
$q + n' = q + (n + 1)$	by def of +
$q + (n + 1) = (q + n) + 1$	by associative law proved before
$(q + n) + 1 = 1 + (q + n)$	because $(q + n) + 1 = (q + n)'$ and $1 + (q + n) = (q + n)'$
$1 + (q + n) = (1 + q) + n$	by associative law
$(1 + q) + n = q' + n$	by def of +

Therefore since $q' + n = (q + n)' = (n + q)' = n + q'$ -> $q' + n = n + q'$ and q' is in Q.

5.)

b.) F(n) = [a, b, c, d, e] where a is negative and absval(a) >= absval(b + c + d + e)
F(n+1) = [-a, b + a, c, d, e + a]

Since a was added twice (to b and e) and also was subtracted twice (from itself) the sum of [a, b, c, d, e] does not change from one iteration to the next. Therefore it will remain negative, so at least one number in the list will always be negative and OPR will never halt.

7.)

$$F(n) = F(n-1) + n^2$$
$$= \frac{(n - 1)((n - 1) + 1)(2(n - 1) + 1)}{6} + n^2$$
$$= \frac{(n - 1)n(2n - 1) + 6n^2}{6}$$
$$= \frac{n((n - 1)(2n - 1) + 6n)}{6}$$
$$= \frac{n(2n^2 + 3n + 1)}{6}$$
$$= \frac{n(n + 1)(2n + 1)}{6}$$

8.)

$$F(n - 1) = 11^{(n - 1) + 2} + 12^{2(n - 1) + 1}$$
$$= 11^{n + 1} + 12^{-2} + 12^{2n + 1}$$
$$= 11^{n + 2} * 11^{-1} + 12^{-2} * 12^{2n + 1}$$
$$= (1 / 11) * 11^{n + 2} + (1 / 144) * 12^{2n + 1}$$
$$= (1 / 11) * 11^{n + 2} + ((1 / 11) + (1 / 133)) * 12^{2n + 1}$$
$$= (1 / 11)(11^{n + 2} + 12^{2n + 1}) + (1 / 133) * 12^{2n + 1}$$

With this equation we can deduce that the next number (n) is just created by the previous number's combination multiplied by 2 constants. Since that number was divisible by 133, any number multiplied by a constant will also be divisible by 133.

9.) If tree T has an infinite number of nodes but a finite number of children, then it follows that one of these children must also be an infinite tree. If we then move to this child that is an infinite tree, but also has a finite number of children, then it must again be the case that one of its children is an infinite tree. This must be the case at every step of an infinite tree. Therefore, you can always move to the next child that is itself an infinite tree. This leads you to an infinite path on tree T.

10.)

b.) The relationship between L and I in a full binary tree is that L is always one greater than I.

c.) I = Internal Node
L = Leaf Node
We start with I = 1 with a root node that has 2 children (L = 2)
One of these children can then become an internal node (I + 1 = 2) by adding two leaf nodes to it (L - 1 + 2 = L + 1 = 3). For each leaf node we convert to an internal node, we must take away the converted node from the leaves and add it to the internals, and then add 2 leaves for the new internal.

I -> L + 1 = L'
I + 1 -> (L' - 1) + 2 = L + 2

Therefore, L always stays one ahead of I.