Data Analysis with Python Book

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Preface

This is the first edition of *Data Analysis with Python Book* by Chege G.B as **STATS_SOLUTIONS**@ryannthegeek.

Chapter 1

A case study on automobile dataset

1.1 Importing Datasets

1.1.1 Statement of the problem

Tom wants to sell his car, how much should he sell it? Help him determine the best for his car.

Here are the questions you should ask yourself as a data scientist.

- Is there data on the prices of other cars and their characteristics?
- What features of cars affect their prices?
- Color? Brand? Horsepower? Something else?
- Asking the right questions in terms of data

1.1.2 Python packages for data science

- **Scientifics Computing Libraries**
  - `Numpy`: arrays and matrices
  - `Pandas`: Data structures and tools
- **Visualization Libraries**
  - `Matplotlib`: Plots, graphs, most popular
  - `Seaborn`: advanced plotting, time series, violin plots
- **Algorithmic libraries**
  - `Scikit-learn`: machine learning: regression, classification,...
  - `Statsmodels`: explore data, estimation of statistical models and perform statistical tests

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
%matplotlib inline
import seaborn as sns
from scipy.stats import pearsonr # correlation
from scipy.stats import chi2_contingency # chi square test
from scipy.stats import zscore
import statsmodels.api as sm
from statsmodels.tools.eval_measures import rmse
```

```
from sklearn.preprocessing import PolynomialFeatures
from sklearn.model_selection import train_test_split, GridSearchCV
from sklearn.linear_model import Ridge
from sklearn.metrics import r2_score, mean_squared_error
#from latexifier import latexify
#from IPython.display import Latex
```

- **Global settings**

```
# Global figure size
sns.set(rc={
    'figure.figsize':(6, 3),
    'font.size':12,
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    'text.usetex': False
    })
# Set style
sns.set_style('whitegrid')
```

```
plt.rcParams
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```

```
automobile_df = pd.read_excel("/home/chege/Documents/Data-Science/Python/
    Data-Analysis-with-Python-Book/automobile.xlsx",
                              header=0, engine='openpyxl', index_col=0) #
    header=none because the first row have no headers
automobile_df.shape
```

```
(205, 26)
```

```
# headers = ["symboling", "normalized-losses", "make", "fuel-type", "
    aspiration", "num-of-doors",
#            "body-style", "drive-wheels","engine-location", "wheel-base
    ", "length", "width","height",
#            "curb-weight", "engine-type", "num-of-cylinders", "engine-
    size", "fuel-system", "bore",
#            "stroke", "compression-ratio", "horsepower", "peak-rpm", "
    city-mpg", "highway-mpg", "price"]
#automobile_df.columns = headers
```

```
automobile_df.head()
```

	symboling	normalized-losses	make	fuel-type	aspiration	num-of-doors	body-style	driv
0	3	?	alfa-romero	gas	std	two	convertible	rwd
1	3	?	alfa-romero	gas	std	two	convertible	rwd
2	1	?	alfa-romero	gas	std	two	hatchback	rwd
3	2	164	audi	gas	std	four	sedan	fwd
4	2	164	audi	gas	std	four	sedan	4wd

```
automobile_df.to_excel("/home/chege/Documents/Data-Science/Python/Data-
    Analysis-with-Python-Book/automobile.xlsx")
```

1.1.3 Data Description

No.	Attribute name	attribute range	No.	Attribute name	attribute range
1	symboling	-3, -2, -1, 0, 1, 2, 3.	14	curb-weight	continuous from 1488 to 4066.
2	normalized-losses	continuous from 65 to 256.	15	engine-type	dohc, dohcv, l, ohc, ohcf, ohcv, rotor.
3	make	audi, bmw, etc.	16	num-of-cylinders	eight, five, four, six, three, twelve, two.
4	fuel-type	diesel, gas.	17	engine-size	continuous from 61 to 326.
5	aspiration	std, turbo.	18	fuel-system	1bbl, 2bbl, 4bbl, idi, mfi, mpfi, spdi, spfi.
6	num-of-doors	four, two.	19	bore	continuous from 2.54 to 3.94.
7	body-style	hardtop, wagon, etc.	20	stroke	continuous from 2.07 to 4.17.
8	drive-wheels	4wd, fwd, rwd.	21	compression-ratio	continuous from 7 to 23.
9	engine-location	front, rear.	22	horsepower	continuous from 48 to 288.
10	wheel-base	continuous from 86.6 120.9.	23	peak-rpm	continuous from 4150 to 6600.
11	length	continuous from 141.1 to 208.1.	24	city-mpg	continuous from 13 to 49.
12	width	continuous from 60.3 to 72.3.	25	highway-mpg	continuous from 16 to 54.
13	height	continuous from 47.8 to 59.8.	26	price	continuous from 5118 to 45400.

Figure 1.1: Automobile data description

```
automobile_df.dtypes
```

```
symboling              int64
normalized-losses     object
make                  object
fuel-type             object
aspiration            object
num-of-doors          object
body-style            object
drive-wheels          object
engine-location       object
wheel-base           float64
length               float64
width                float64
height               float64
curb-weight            int64
engine-type           object
num-of-cylinders      object
engine-size            int64
fuel-system           object
bore                  object
stroke                object
```

```
compression-ratio    float64
horsepower            object
peak-rpm              object
city-mpg               int64
highway-mpg            int64
price                 object
dtype: object
```

```
automobile_df.describe(include='all')
```

	symboling	normalized-losses	make	fuel-type	aspiration	num-of-doors	body-style	dri
count	205.000000	205	205	205	205	205	205	205
unique	NaN	52	22	2	2	3	5	3
top	NaN	?	toyota	gas	std	four	sedan	fwd
freq	NaN	41	32	185	168	114	96	120
mean	0.834146	NaN	NaN	NaN	NaN	NaN	NaN	Nal
std	1.245307	NaN	NaN	NaN	NaN	NaN	NaN	Nal
min	-2.000000	NaN	NaN	NaN	NaN	NaN	NaN	Nal
25%	0.000000	NaN	NaN	NaN	NaN	NaN	NaN	Nal
50%	1.000000	NaN	NaN	NaN	NaN	NaN	NaN	Nal
75%	2.000000	NaN	NaN	NaN	NaN	NaN	NaN	Nal
max	3.000000	NaN	NaN	NaN	NaN	NaN	NaN	Nal

1.2 Data Wrangling/munging/cleaning/preprocessing

This is the process of cleaning and transforming raw data into a format suitable for analysis.

It involves various tasks, including: - removing duplicates - handling missing values - converting data types - merging datasets

1.2.1 Dealing with Missing Data

- Missing values occur when no data value is stored for a variable (feature) in an observation.
- Could be represented as "?" "N/A", 0 or just a blank cell.

How to deal with missing data

- Check with the data collection source
- Drop the missing values
  - drop the variable
  - drop the data entry
- Replace the missing values
  - replace it with an average (of similar datapoints)
  - replace it by frequency
  - interpolate missing values

  - replace it based on other functions
- Leave it as missing data

```
# Replace the question marks with NaN values
automobile_df.replace('?', pd.NaT, inplace=True)
# Handle missing values by replacing them with median
median = automobile_df.median()
automobile_df = automobile_df.fillna(median)
```

```
automobile_df.head()
```

	symboling	normalized-losses	make	fuel-type	aspiration	num-of-doors	body-style	driv
0	3	115.0	alfa-romero	gas	std	two	convertible	rwd
1	3	115.0	alfa-romero	gas	std	two	convertible	rwd
2	1	115.0	alfa-romero	gas	std	two	hatchback	rwd
3	2	164	audi	gas	std	four	sedan	fwd
4	2	164	audi	gas	std	four	sedan	4wd

1.2.2 Data formatting

- Putting data into a correct format for further processing and analysis

```
# changing miles per gallon to a litre per 100km
automobile_df['city-mpg'] = 235/automobile_df['city-mpg']
automobile_df.rename(columns={'city-mpg': 'city-L/100km'}, inplace=True)
```

```
automobile_df.dtypes
```

```
symboling              int64
normalized-losses     object
make                  object
fuel-type             object
aspiration            object
num-of-doors          object
body-style            object
drive-wheels          object
engine-location       object
wheel-base           float64
length               float64
width                float64
height               float64
curb-weight            int64
engine-type           object
num-of-cylinders      object
engine-size            int64
fuel-system           object
bore                  object
stroke                object
compression-ratio    float64
horsepower            object
peak-rpm              object
city-L/100km         float64
highway-mpg            int64
price                 object
dtype: object
```

```
# change datatypes of variables that need to be
automobile_df['normalized-losses'] = automobile_df['normalized-losses'].
    astype(float)
automobile_df['bore'] = automobile_df['bore'].astype(float)
automobile_df['stroke'] = automobile_df['stroke'].astype(float)
automobile_df['horsepower'] = automobile_df['horsepower'].astype(float)
automobile_df['peak-rpm'] = automobile_df['peak-rpm'].astype(float)
automobile_df['price'] = automobile_df['price'].astype(float)
```

```
automobile_df.dtypes
```

```
symboling              int64
normalized-losses    float64
make                  object
fuel-type             object
aspiration            object
num-of-doors          object
body-style            object
drive-wheels          object
engine-location       object
wheel-base           float64
length               float64
width                float64
height               float64
curb-weight            int64
engine-type           object
num-of-cylinders      object
engine-size            int64
fuel-system           object
bore                 float64
stroke               float64
compression-ratio    float64
horsepower           float64
peak-rpm             float64
city-L/100km         float64
highway-mpg            int64
price                float64
dtype: object
```

```
# create a dict that maps words to integers
word_to_int = {'four': 4, 'six': 6, 'five': 5, 'three': 3, 'twelve': 12,
    'two': 2, 'eight': 8}
# Define a list of words
words = automobile_df['num-of-cylinders']
# convert the list of words to integers
iin = [word_to_int[word] for word in words]
iinn = list(map(word_to_int.get, words))
```

1.2.3 Data normalization

```
norm = zscore(automobile_df[['length', 'horsepower']])
norm.head()
```

	length	horsepower
0	-0.426521	0.173309
1	-0.426521	0.173309
2	-0.231513	1.263761
3	0.207256	-0.054925
4	0.207256	0.274747

1.2.4 Binning in python

- **Binning:** Grouping of values into "bins"
- Converts numeric into categorical variables
- Group a set of numerical values into a set of "bins"

Let's bin prices column by categorizing them into low, medium, high

```
print(automobile_df['price'].min())
print(automobile_df['price'].max())
```

```
5118.0
45400.0
```

```
# `np.linspace()` function returns evenly spaced numbers over a specified
    interval, num evenly spaced samples, calculated over the interval [
   start, stop].
bins = np.linspace(min(automobile_df['price']), max(automobile_df['price'
   ]), 4)
grp_names = ['Low', 'Medium', 'High']
automobile_df['price-binned'] = pd.cut(automobile_df['price'], bins=bins,
    labels=grp_names, include_lowest=True)
```

```
# Remove index column and save the cleaned dataset
automobile_df.to_excel("/home/chege/Documents/Data-Science/Python/Data-
   Analysis-with-Python-Book/automobile_cleaned.xlsx")
```

```
automobile_df.head()
```

	symboling	normalized-losses	make	fuel-type	aspiration	num-of-doors	body-style	driv
0	3	115.0	alfa-romero	gas	std	two	convertible	rwd
1	3	115.0	alfa-romero	gas	std	two	convertible	rwd
2	1	115.0	alfa-romero	gas	std	two	hatchback	rwd
3	2	164.0	audi	gas	std	four	sedan	fwd
4	2	164.0	audi	gas	std	four	sedan	4wd

```
automobile_df['fuel-type'].unique()
```

```
array(['gas', 'diesel'], dtype=object)
```

```
pd.get_dummies(automobile_df['fuel-type']).head()
```

	diesel	gas
0	0	1
1	0	1
2	0	1
3	0	1
4	0	1

1.3 Exploratory Data Analysis

- Preliminary step in data analysis to:
  - Summarize main characteristics of the data
  - Gain better understanding of the data set
  - Uncover relationships between variables
  - Extract important variables

> **Note**
>
> **Question:** "What are the characteristics which have the most impact on the car price?"

1.3.1 Descriptive statistics

- Describe basic features of data
- Giving short summaries about the sample and measures of the data

```
automobile_df.describe()
```

	symboling	normalized-losses	wheel-base	length	width	height	curb-weight
count	205.000000	205.000000	205.000000	205.000000	205.000000	205.000000	205.000000
mean	0.834146	120.600000	98.756585	174.049268	65.907805	53.724878	2555.565854
std	1.245307	31.805105	6.021776	12.337289	2.145204	2.443522	520.680204
min	-2.000000	65.000000	86.600000	141.100000	60.300000	47.800000	1488.000000
25%	0.000000	101.000000	94.500000	166.300000	64.100000	52.000000	2145.000000
50%	1.000000	115.000000	97.000000	173.200000	65.500000	54.100000	2414.000000
75%	2.000000	137.000000	102.400000	183.100000	66.900000	55.500000	2935.000000
max	3.000000	256.000000	120.900000	208.100000	72.300000	59.800000	4066.000000

- Summarize the categorical variable using the `value_counts` function

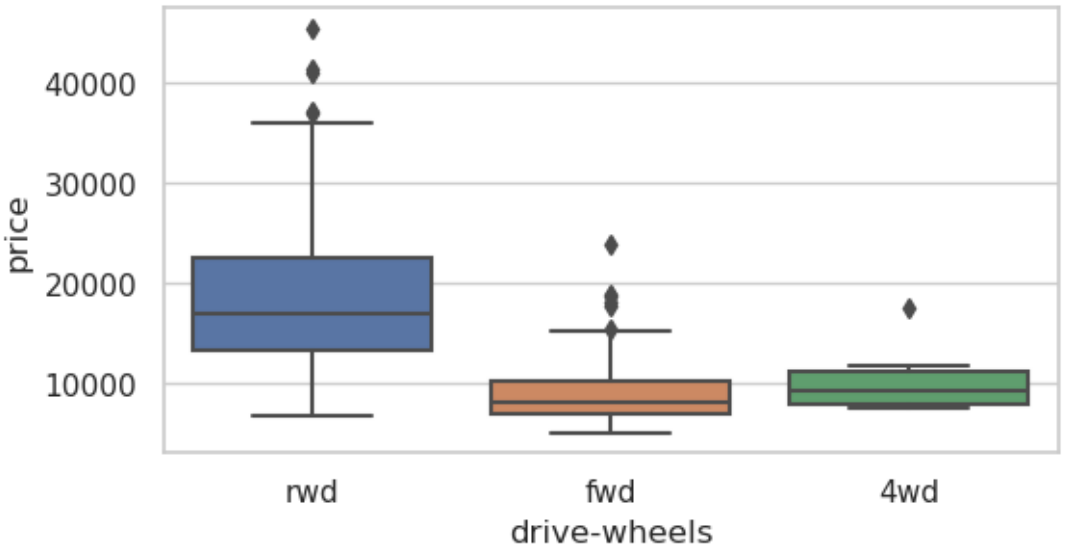
```
drive_wheels_counts = automobile_df['drive-wheels'].value_counts().
    to_frame()
drive_wheels_counts.rename(columns={'drive-wheels': 'value_counts'},
    inplace=True)
drive_wheels_counts
```

	value_counts
fwd	120
rwd	76
4wd	9

Box Plot Example

```
sns.boxplot(x='drive-wheels', y='price', data=automobile_df)
```

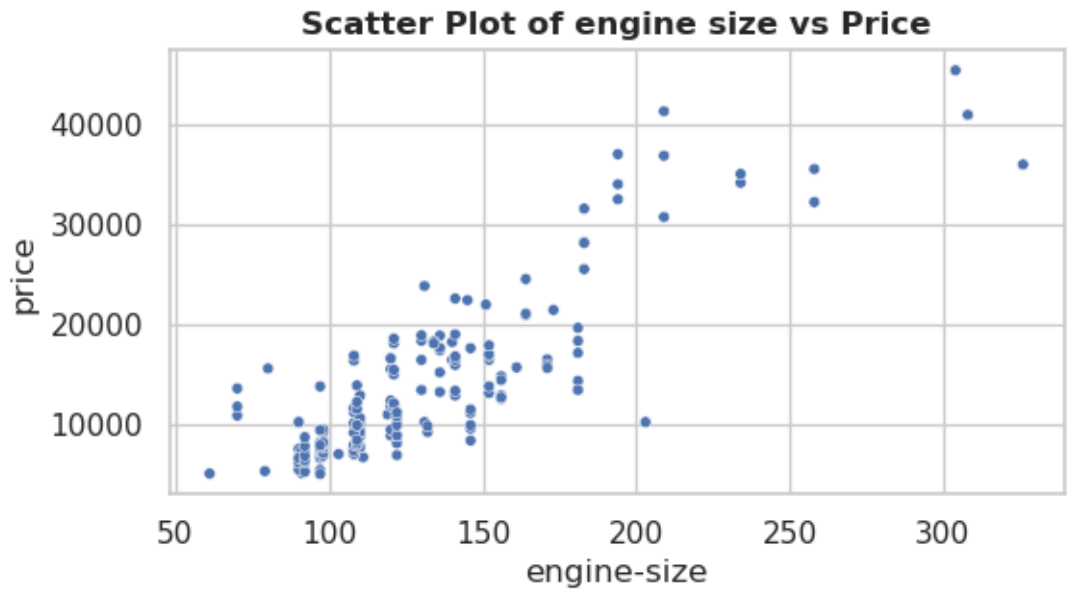
```
<Axes: xlabel='drive-wheels', ylabel='price'>
```



Scatter Plot Example

```
sns.scatterplot(x='engine-size', y='price', data=automobile_df, size=5,
    legend=False)
plt.title('Scatter Plot of engine size vs Price', fontweight='bold')
```

```
Text(0.5, 1.0, 'Scatter Plot of engine size vs Price')
```



- There is a positive correlation between price and engine size

1.3.2 groupby() Function

```
df_4grp = automobile_df[['drive-wheels', 'body-style', 'price']]
df_grp = df_4grp.groupby(['drive-wheels', 'body-style'], as_index=False).
    mean()
df_grp
```

	drive-wheels	body-style	price
0	4wd	hatchback	8949.000000
1	4wd	sedan	12647.333333
2	4wd	wagon	9095.750000
3	fwd	convertible	11595.000000
4	fwd	hardtop	8249.000000
5	fwd	hatchback	8396.387755
6	fwd	sedan	9828.754386
7	fwd	wagon	9997.333333
8	rwd	convertible	23949.600000
9	rwd	hardtop	24202.714286
10	rwd	hatchback	14125.000000
11	rwd	sedan	21711.833333
12	rwd	wagon	16994.222222

- The `pivot_table()` method

```
df_pv_table = pd.pivot_table(
    df_4grp,
    index='drive-wheels',
    columns='body-style'
)
df_pv_table
```

body-style drive-wheels	price convertible	hardtop	hatchback	sedan	wagon
4wd	NaN	NaN	8949.000000	12647.333333	9095.750000
fwd	11595.0	8249.000000	8396.387755	9828.754386	9997.333333
rwd	23949.6	24202.714286	14125.000000	21711.833333	16994.222222

1.3.3 Correlation

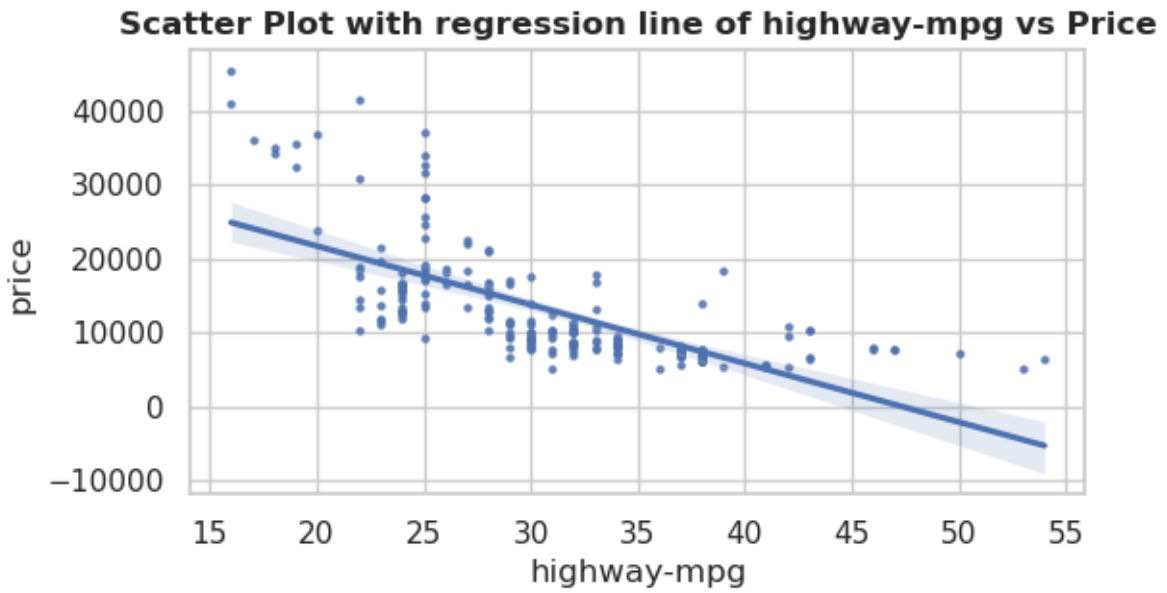
```
sns.regplot(x='engine-size', y='price', data=automobile_df, scatter_kws={
    's':5})
plt.title('Scatter Plot with regression line of engine size vs Price',
    weight='bold')
```

```
Text(0.5, 1.0, 'Scatter Plot with regression line of engine size vs Price
    ')
```

- When the highway miles per gallon value goes high, the price goes high.
- There is a positive linear correlation between price of the vehicle and its engine size.
- This means that engine size is a good predictor for price.
- The slope of correlation is positive

```
sns.regplot(x='highway-mpg', y='price', data=automobile_df, scatter_kws={
    's':5})
plt.title('Scatter Plot with regression line of highway-mpg vs Price',
    weight='bold')
```

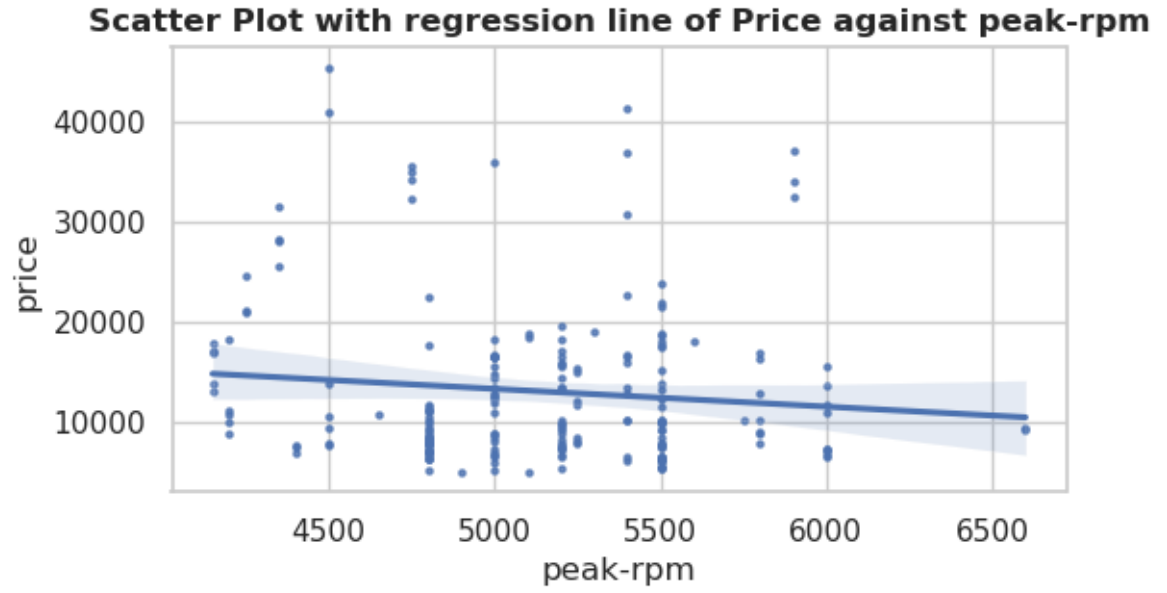
```
Text(0.5, 1.0, 'Scatter Plot with regression line of highway-mpg vs Price
    ')
```



- When the highway miles per gallon value goes high, the price goes down.
- There is a negative linear relationship between price of the vehicle and its highway miles per gallon (mpg).
- The slope of correlation is negative.
- This means that engine size is a good predictor for price indicated by the steepness of the slope.
- These two variables are said to have negative correlation

```
sns.regplot(x='peak-rpm', y='price', data=automobile_df, scatter_kws={'s'
    :5})
plt.title('Scatter Plot with regression line of Price against peak-rpm',
    weight='bold')
```

```
Text(0.5, 1.0, 'Scatter Plot with regression line of Price against peak-
    rpm')
```



- There is a weak correlation between the two variables
- The slope is almost zero
- peak-rpm is therefore a poor predictor for price

1.3.4 Correlation statistics

- Measure of the strength of the correlation among variables of interest
  - Correlation coefficient: $-1 < \rho < 1$
  - p value

```
pearson_coef, p_value = pearsonr(automobile_df['horsepower'],
   automobile_df['price'])
results = {'Pearson Correlation': [pearson_coef], 'P Value': [p_value]}
pd.DataFrame(results)
automobile_df.horsepower
```

```
0       111.0
1       111.0
2       154.0
3       102.0
4       115.0
        ...
200     114.0
201     160.0
202     134.0
203     106.0
204     114.0
Name: horsepower, Length: 205, dtype: float64
```

- Correlation for numeric variables only in a more general way

```
automobile_df.corr(method='pearson', numeric_only=True).round(3)
```

	symboling	normalized-losses	wheel-base	length	width	height	curb-weight	e
symboling	1.000	0.457	-0.532	-0.358	-0.233	-0.541	-0.228	-
normalized-losses	0.457	1.000	-0.074	-0.007	0.058	-0.366	0.064	0
wheel-base	-0.532	-0.074	1.000	0.875	0.795	0.589	0.776	0
length	-0.358	-0.007	0.875	1.000	0.841	0.491	0.878	0
width	-0.233	0.058	0.795	0.841	1.000	0.279	0.867	0
height	-0.541	-0.366	0.589	0.491	0.279	1.000	0.296	0
curb-weight	-0.228	0.064	0.776	0.878	0.867	0.296	1.000	0
engine-size	-0.106	0.073	0.569	0.683	0.735	0.067	0.851	1
bore	-0.133	-0.050	0.490	0.607	0.559	0.174	0.649	0
stroke	-0.005	0.047	0.160	0.129	0.183	-0.059	0.168	0
compression-ratio	-0.179	-0.115	0.250	0.158	0.181	0.261	0.151	0
horsepower	0.071	0.167	0.353	0.553	0.641	-0.109	0.751	0
peak-rpm	0.274	0.241	-0.361	-0.286	-0.219	-0.321	-0.266	-
city-L/100km	0.063	0.197	0.474	0.659	0.683	-0.002	0.792	0
highway-mpg	0.035	-0.149	-0.544	-0.705	-0.677	-0.107	-0.797	-
price	-0.080	0.095	0.585	0.687	0.725	0.140	0.820	0

- Correlation and p value for numeric only variables

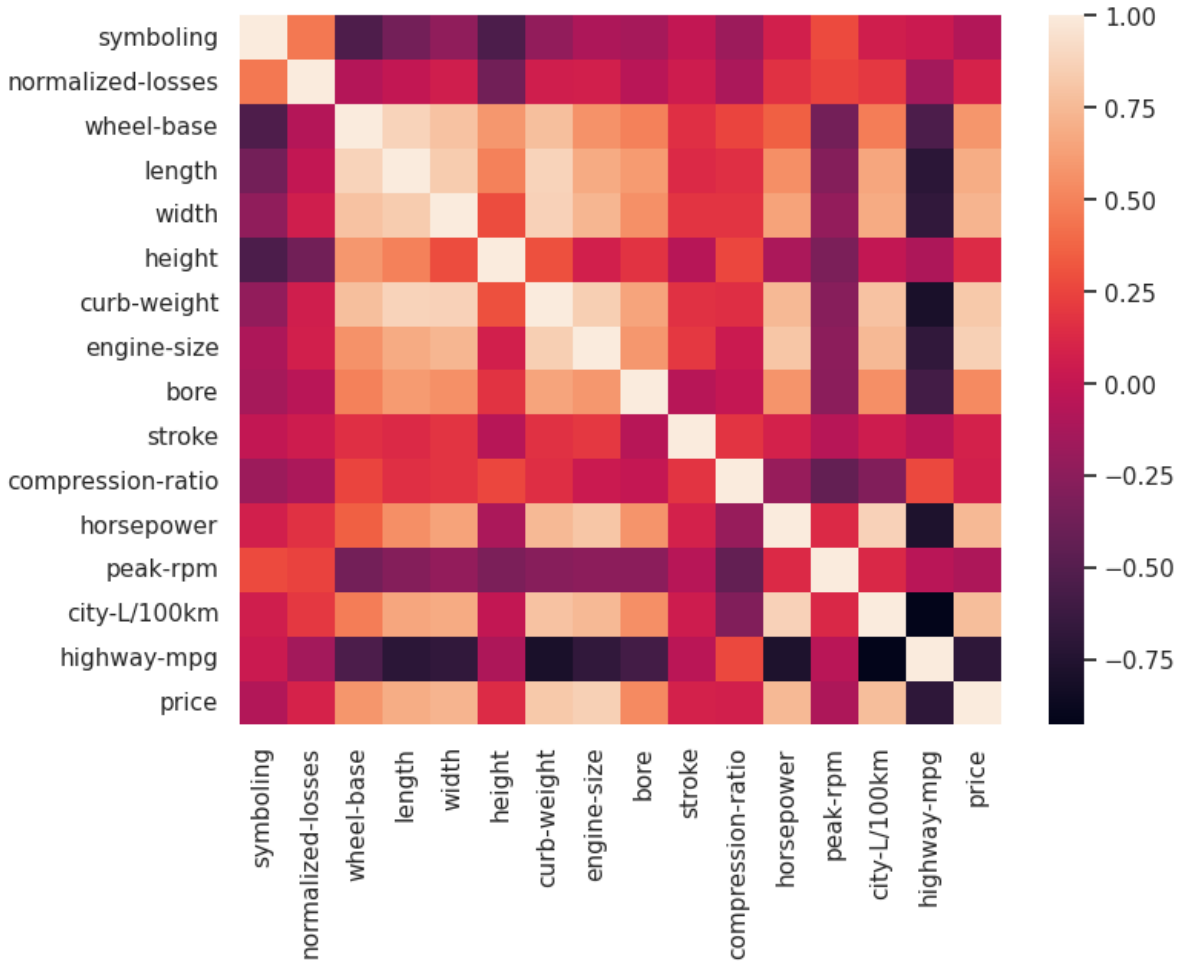
```
corr_df = pd.DataFrame(columns=['Pearson Correlation', 'p value'])
for col in automobile_df:
    if pd.api.types.is_numeric_dtype(automobile_df[col]) and col != '
   price':
        Pearson_rho, p_value = pearsonr(automobile_df.price,
   automobile_df[col])
        corr_df.loc[col] = [round(Pearson_rho, 3), round(p_value, 3)]

corr_df
```

	Pearson Correlation	p value
symboling	-0.080	0.253
normalized-losses	0.095	0.173
wheel-base	0.585	0.000
length	0.687	0.000
width	0.725	0.000
height	0.140	0.045
curb-weight	0.820	0.000
engine-size	0.860	0.000
bore	0.533	0.000
stroke	0.084	0.233
compression-ratio	0.073	0.299
horsepower	0.750	0.000
peak-rpm	-0.107	0.126
city-L/100km	0.769	0.000
highway-mpg	-0.693	0.000

- A heatmap that shows the correlations between one variable and another

```
# Select only integers and floats since only numerals are supported to
   create heatmap
automobile_df_num = automobile_df.select_dtypes(include=['int', 'float'])
# compute correlation matrix
corr = automobile_df_num.corr()
# create heatmap
fig = plt.figure(figsize=(8, 6))
sns.heatmap(corr)
```

```
<Axes: >
```



1.3.5 Association between two categorical variables: χ^2 test

- Chi-square tests for null hypothesis that variables are independent
- The Chi-square does not tell you the type of relationship that exists between both variables; but only that a relationship exists.

```
obs_data = automobile_df[['fuel-type', 'aspiration']]
obs_crosstab = pd.crosstab(obs_data['fuel-type'], obs_data['aspiration'],
    margins=True, margins_name='Total')
obs_crosstab
```

aspiration fuel-type	std	turbo	Total
diesel	7	13	20
gas	161	24	185
Total	168	37	205

The table above contains observed values from the dataset.

```
# Peform chi-square test and store the results in variables to be used
   later for visualization
```

```
chi2_stat, p_val, dof, ex = chi2_contingency(obs_crosstab, correction=
    True)

# visualize the expected outcome
exp_df = pd.DataFrame(ex, columns=obs_crosstab.columns, index=
    obs_crosstab.index)
exp_df
```

aspiration fuel-type	std	turbo	Total
diesel	16.390244	3.609756	20.0
gas	151.609756	33.390244	185.0
Total	168.000000	37.000000	205.0

- The table above shows the expected frequencies of the two variables after applying chi square test.

```
# get statistical test ouput in a data frame
output_df = pd.DataFrame({'Test Statistic': [chi2_stat], 'p-value': [
    p_val], 'Degrees of freedom': [dof]})
output_df
```

	Test Statistic	p-value	Degrees of freedom
0	33.029546	0.000001	4

- Testing at 5% level of significance
- P-value < 0.05, we reject the null hypothesis that the two variables are independent and conclude that there is evidence of association between fuel-type and aspiration.
- You can also use the test statistic in comparison to the critical value to assess whether the two variables are independent or not.

1.4 Model Development

In this module you will learn about:
1. Simple and Multiple Linear Regression 2. Model Evaluation using Visualization 3. Polynomial Regression and Pipelines 4. R-squared and MSE for In-Sample Evaluation 5. Prediction and Decision Making

::: callout-note **Question:**
How can you determine a fair value for a used car?

1.4.1 Linear Regression and Multiple Linear regression

Simple linear regresssion

```
# Define explanatory and response variables
x, y = automobile_df['highway-mpg'], automobile_df['price']
# Add a constant term to the x data for the intercept term
```

```
x = sm.add_constant(x)
# Build the Simple Linear Regression
slr = sm.OLS(y, x).fit()
```

```
slr.summary()
```

Table 1.16: OLS Regression Results

Dep. Variable:	price	R-squared:	0.480
Model:	OLS	Adj. R-squared:	0.478
Method:	Least Squares	F-statistic:	187.6
Date:	Tue, 20 Jun 2023	Prob (F-statistic):	1.13e-30
Time:	15:44:40	Log-Likelihood:	-2062.5
No. Observations:	205	AIC:	4129.
Df Residuals:	203	BIC:	4136.
Df Model:	1		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	3.753e+04	1824.098	20.577	0.000	3.39e+04	4.11e+04
highway-mpg	-792.9384	57.891	-13.697	0.000	-907.083	-678.794

Omnibus:	59.586	Durbin-Watson:	0.894
Prob(Omnibus):	0.000	Jarque-Bera (JB):	111.450
Skew:	1.465	Prob(JB):	6.29e-25
Kurtosis:	5.113	Cond. No.	145.

PLEASE FIND TIME TO INTEPRETE THESE ALL OUPUTS.

Multiple Linear Regression

```
# Define explanatory and response variables
x_mlr = automobile_df[['horsepower', 'curb-weight', 'engine-size', '
    highway-mpg']]
y_mlr = automobile_df[['price']]
# Add a constant term to the x data for the intercept term
x_mlr = sm.add_constant(x_mlr)
# Build the Multiple Linear Regression
mlr = sm.OLS(y_mlr, x_mlr).fit()
```

```
mlr.summary()
```

Table 1.19: OLS Regression Results

Dep. Variable:	price	R-squared:	0.773
Model:	OLS	Adj. R-squared:	0.769
Method:	Least Squares	F-statistic:	170.6

Date:	Tue, 20 Jun 2023	Prob (F-statistic):	2.65e-63
Time:	15:44:40	Log-Likelihood:	-1977.5
No. Observations:	205	AIC:	3965.
Df Residuals:	200	BIC:	3982.
Df Model:	4		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-7753.3835	4402.634	-1.761	0.080	-1.64e+04	928.153
horsepower	11.5370	13.472	0.856	0.393	-15.028	38.102
curb-weight	3.6310	1.192	3.047	0.003	1.281	5.981
engine-size	104.8075	14.488	7.234	0.000	76.238	133.377
highway-mpg	-93.5949	73.598	-1.272	0.205	-238.722	51.533

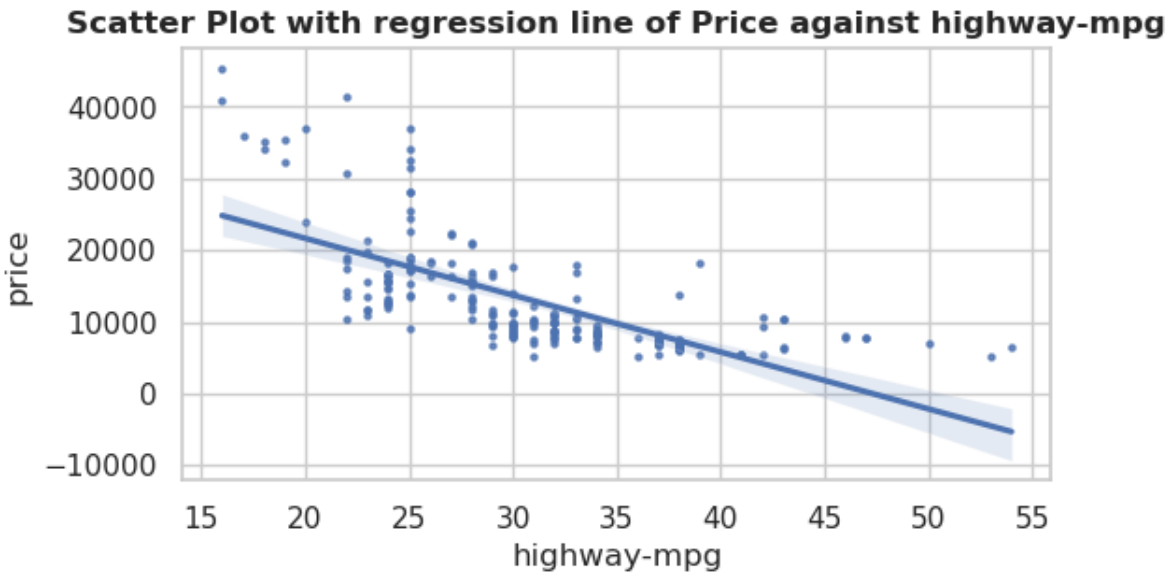
Omnibus:	25.979	Durbin-Watson:	1.040
Prob(Omnibus):	0.000	Jarque-Bera (JB):	94.516
Skew:	0.380	Prob(JB):	2.99e-21
Kurtosis:	6.238	Cond. No.	4.35e+04

PLEASE FIND TIME TO INTEPRETE THESE ALL OUPUTS.

1.4.2 Model Evaluation Using Visualization

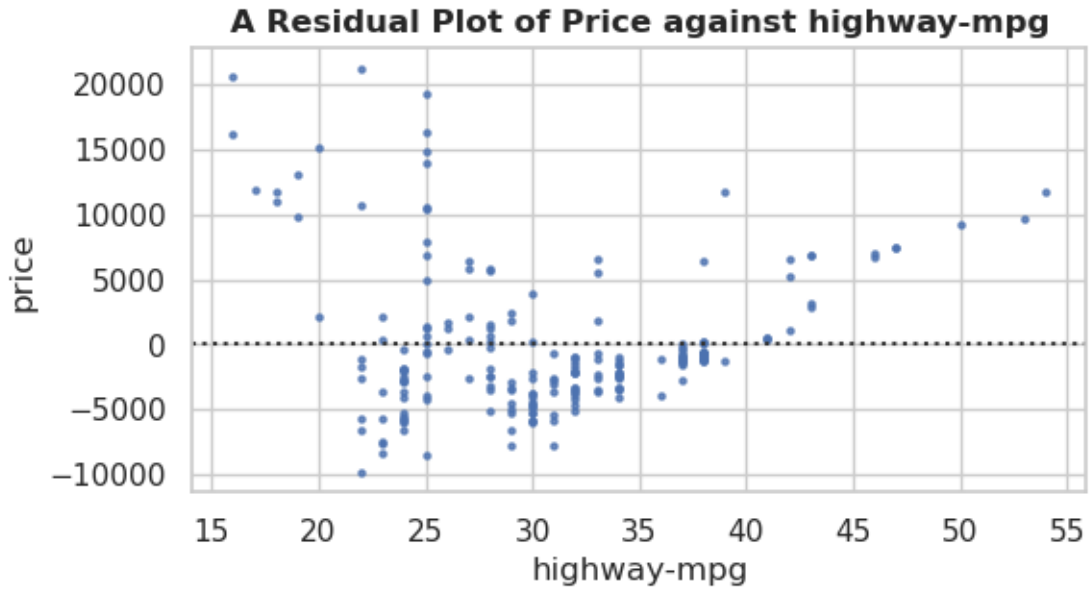
```
sns.regplot(x='highway-mpg', y='price', data=automobile_df, scatter_kws={
    's':5})
plt.title('Scatter Plot with regression line of Price against highway-mpg
    ', weight='bold')
```

```
Text(0.5, 1.0, 'Scatter Plot with regression line of Price against
    highway-mpg')
```



```
sns.residplot(x=automobile_df['highway-mpg'], y=automobile_df['price'],
    scatter_kws={'s':5})
plt.title('A Residual Plot of Price against highway-mpg', weight='bold')
```

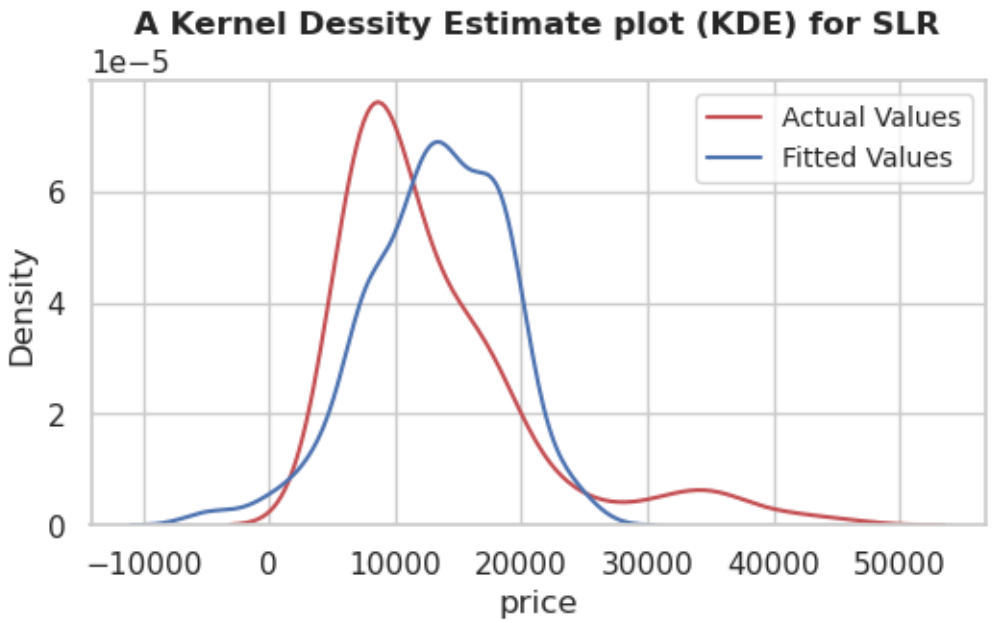
```
Text(0.5, 1.0, 'A Residual Plot of Price against highway-mpg')
```



A distribution plot - A distribution plot displays the predicted values vs the actual values.

```
# Get the fitted values
y_hat = slr.predict(x)
# First axis of actual values
ax1 = sns.kdeplot(automobile_df['price'], color='r', label='Actual Values
    ')
# Second axis of the fitted values and superimpose to the first axis
sns.kdeplot(y_hat, color='b', label='Fitted Values', ax=ax1)
plt.legend()
plt.title('A Kernel Dessity Estimate plot (KDE) for SLR', fontweight='
    bold')
```

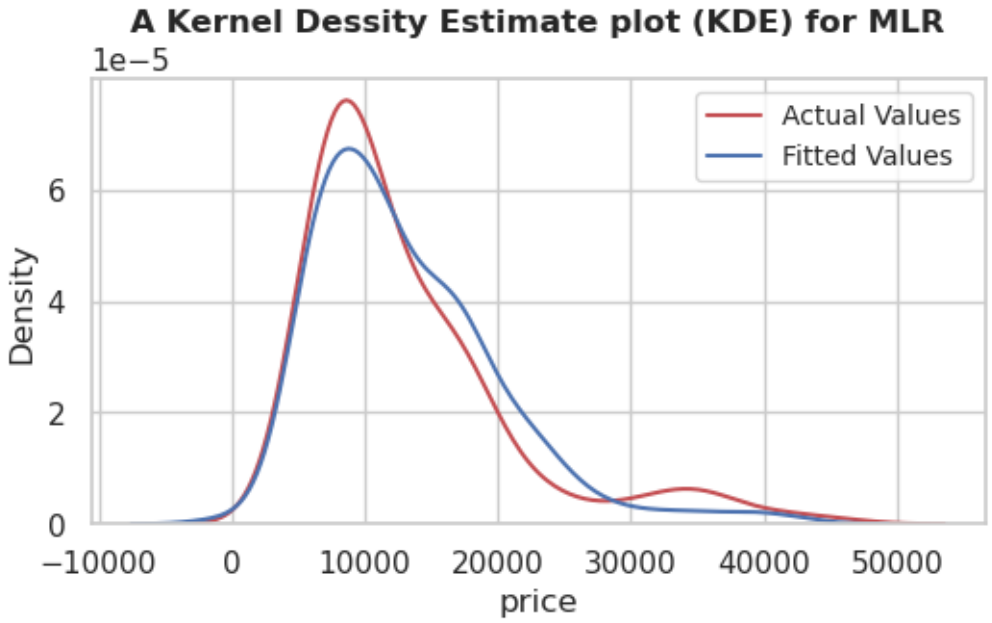
```
Text(0.5, 1.0, 'A Kernel Dessity Estimate plot (KDE) for SLR')
```



- The actual values are in red. We see the predicted values for prices in the range from 40,000 to 50,000 are inaccurate.
- The prices in the region from 10,000 to 20,000 are much closer to the target value.

```
y_hat_mlr = mlr.predict(x_mlr)
ax1_mlr = sns.kdeplot(automobile_df['price'], color='r', label='Actual
    Values')
sns.kdeplot(y_hat_mlr, color='b', label='Fitted Values', ax=ax1_mlr)
plt.legend()
plt.title('A Kernel Dessity Estimate plot (KDE) for MLR', fontweight='
    bold')
```

```
Text(0.5, 1.0, 'A Kernel Dessity Estimate plot (KDE) for MLR')
```



- In this example, we use multiple features or independent variables.
- Comparing it to the plot on the last slide, we see predicted values are much closer to the target values.

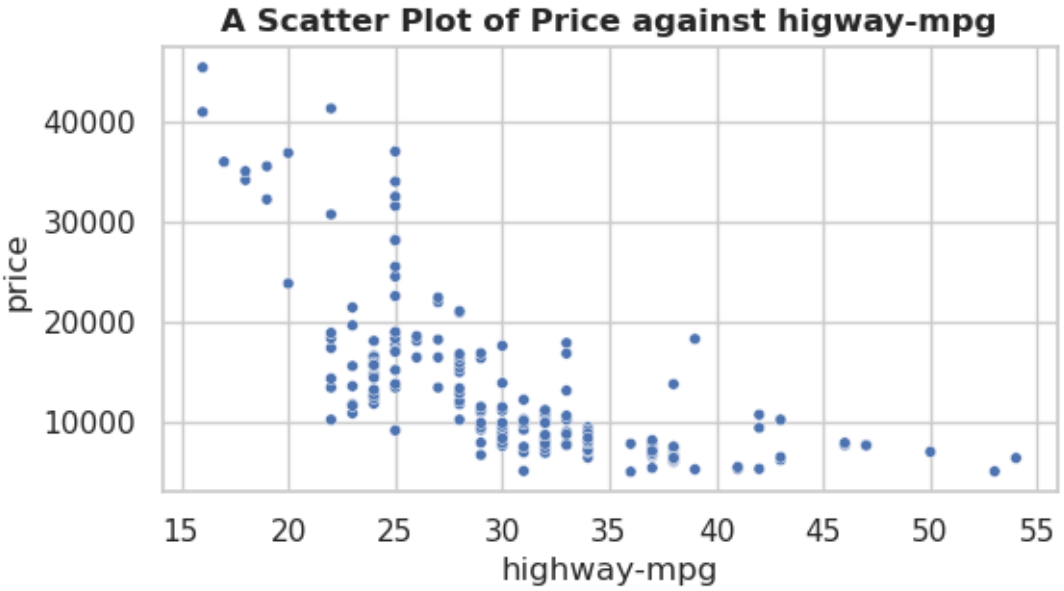
1.4.3 Polynomial regression

- A special case of the general linear regression
- Useful for describing curvilinear relationships
- **Curvilinear relationships:** Squaring or setting higher-order terms of the predictor variables
- Exists in three forms:
  - Quadratic - second order
  - Cubic - third order
  - Higher order

1.4.3.1 One-dimensional Polynomial Regression

- This incorporates only one predictor variable
- Easy to implement
- No scaling needed since it has one predictor variable

```
sns.scatterplot(x=automobile_df['highway-mpg'], y=automobile_df['price'],
    size=5, legend=False)
plt.title('A Scatter Plot of Price against higway-mpg', fontweight='bold'
    )
plt.show()
```



- The Relationship between price and highway-mpg looks like a poynomial of degree 5-10, lets try:

```
x = automobile_df[['highway-mpg']] # [[]] for convert 2D array suitable
    for model input
y = automobile_df['price']
```

```
poly_f = PolynomialFeatures(degree=5)
xp = poly_f.fit_transform(x)
model = sm.OLS(y, xp).fit()
ypred = model.predict(xp)
model.summary()
```

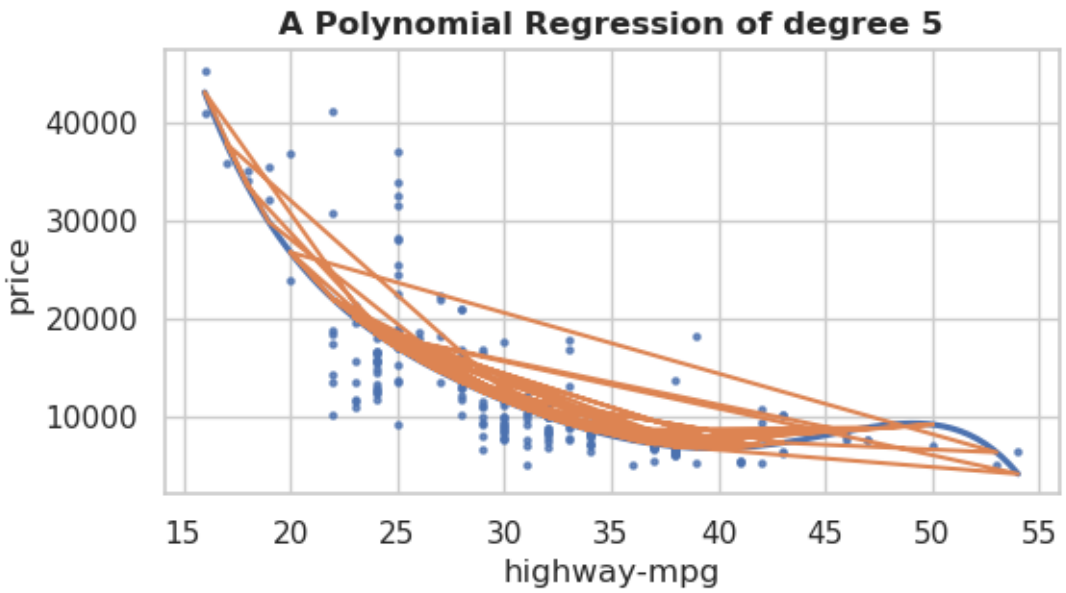
Table 1.22: OLS Regression Results

Dep. Variable:	price	R-squared:	0.666
Model:	OLS	Adj. R-squared:	0.658
Method:	Least Squares	F-statistic:	79.39
Date:	Tue, 20 Jun 2023	Prob (F-statistic):	1.66e-45
Time:	15:44:43	Log-Likelihood:	-2017.2
No. Observations:	205	AIC:	4046.
Df Residuals:	199	BIC:	4066.
Df Model:	5		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	4.317e+05	1.56e+05	2.768	0.006	1.24e+05	7.39e+05
x1	-5.768e+04	2.59e+04	-2.230	0.027	-1.09e+05	-6668.055
x2	3307.2116	1659.575	1.993	0.048	34.603	6579.820
x3	-96.7435	51.518	-1.878	0.062	-198.334	4.847
x4	1.4111	0.775	1.821	0.070	-0.117	2.939
x5	-0.0081	0.005	-1.791	0.075	-0.017	0.001

Omnibus:	75.738	Durbin-Watson:	0.938
Prob(Omnibus):	0.000	Jarque-Bera (JB):	243.964
Skew:	1.526	Prob(JB):	1.06e-53
Kurtosis:	7.387	Cond. No.	3.65e+10

```
sns.regplot(x=x, y=y, order=5, ci=None, scatter_kws={'s':5})
plt.plot(x, ypred)
plt.title('A Polynomial Regression of degree 5', fontweight='bold')
plt.show()
```



- The results show that the third and higher orders are insignificant at 5% significance level, lets reduce to second order

```
poly_f = PolynomialFeatures(degree=2)
xp = poly_f.fit_transform(x)
model = sm.OLS(y, xp).fit()
ypred = model.predict(xp)
model.summary()
```

Table 1.25: OLS Regression Results

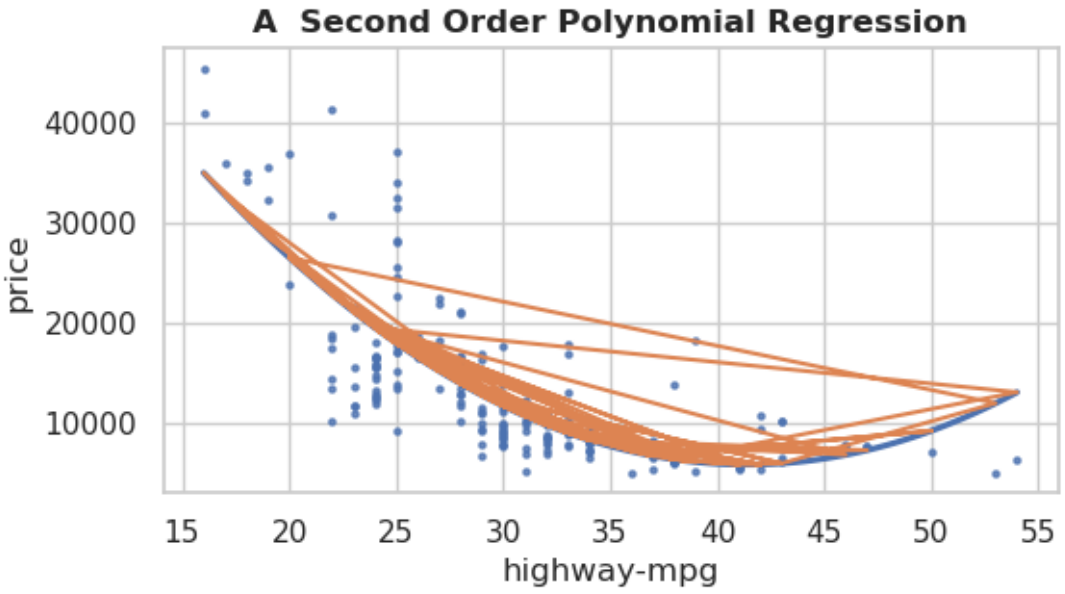
Dep. Variable:	price	R-squared:	0.636
Model:	OLS	Adj. R-squared:	0.632
Method:	Least Squares	F-statistic:	176.3
Date:	Tue, 20 Jun 2023	Prob (F-statistic):	4.92e-45
Time:	15:44:44	Log-Likelihood:	-2026.1
No. Observations:	205	AIC:	4058.
Df Residuals:	202	BIC:	4068.
Df Model:	2		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	8.325e+04	5154.375	16.151	0.000	7.31e+04	9.34e+04
x1	-3738.4212	320.823	-11.653	0.000	-4371.013	-3105.829
x2	45.1866	4.865	9.288	0.000	35.594	54.779

Omnibus:	45.733	Durbin-Watson:	1.051
Prob(Omnibus):	0.000	Jarque-Bera (JB):	101.386
Skew:	1.026	Prob(JB):	9.65e-23

Kurtosis:	5.768	Cond. No.	1.69e+04

```
sns.regplot(x=x, y=y, order=2, ci=None, scatter_kws={'s':5})
plt.plot(x, ypred)
plt.title('A  Second Order Polynomial Regression', fontweight='bold')
plt.show()
```



- The process of trial and error is abit tidious, lets use cross-validation to evaluate perfomance of models and pick the best to determine the optimal degree order

```
# Define explanatory and response variables
x = automobile_df[['highway-mpg']]
y = automobile_df['price']
# Define range of polynomial degrees to test, you can play with those
    values to get optimal one
degrees_1d = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11]
# Evaluate each polynomial degree
mse_scores = []
for degree in degrees_1d:
    # Generate polynomial features
    poly_f = PolynomialFeatures(degree=degree)
    xp = poly_f.fit_transform(x)
    # Fit linear regression model
    model = sm.OLS(y, xp).fit()
    # Calculate mean squared error
    ypred = model.predict(xp)
    mse_scores.append(model.mse_resid)

# Find the best degree based on mean squared error
best_degree_1d = degrees_1d[np.argmin(mse_scores)]
print('Best degree:', best_degree_1d)
```

```
Best degree: 7
```

```
# Fit the model with the best degree
poly_f_1d = PolynomialFeatures(degree=best_degree_1d)
xp_1d = poly_f_1d.fit_transform(x)
model_1d = sm.OLS(y, xp_1d).fit()
ypred_1d = model_1d.predict(xp_1d)
model_1d.summary()
```

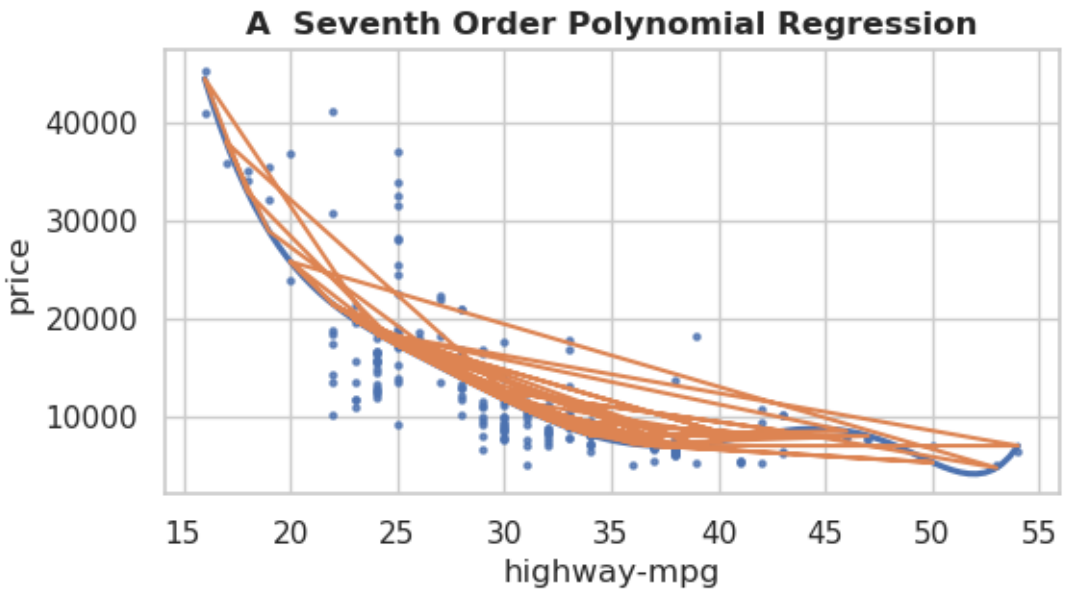
Table 1.28: OLS Regression Results

Dep. Variable:	price	R-squared:	0.671
Model:	OLS	Adj. R-squared:	0.661
Method:	Least Squares	F-statistic:	67.36
Date:	Tue, 20 Jun 2023	Prob (F-statistic):	3.46e-45
Time:	15:44:44	Log-Likelihood:	-2015.6
No. Observations:	205	AIC:	4045.
Df Residuals:	198	BIC:	4069.
Df Model:	6		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	4.206e+05	1.77e+06	0.238	0.812	-3.07e+06	3.91e+06
x1	-2.306e+04	4.14e+05	-0.056	0.956	-8.4e+05	7.93e+05
x2	-3351.0214	4.05e+04	-0.083	0.934	-8.32e+04	7.65e+04
x3	432.7990	2145.865	0.202	0.840	-3798.885	4664.483
x4	-20.5023	66.717	-0.307	0.759	-152.069	111.065
x5	0.4885	1.217	0.401	0.689	-1.912	2.889
x6	-0.0058	0.012	-0.484	0.629	-0.030	0.018
x7	2.796e-05	5.04e-05	0.555	0.580	-7.14e-05	0.000

Omnibus:	77.493	Durbin-Watson:	0.931
Prob(Omnibus):	0.000	Jarque-Bera (JB):	254.188
Skew:	1.558	Prob(JB):	6.37e-56
Kurtosis:	7.478	Cond. No.	9.49e+14

```
sns.regplot(x=x, y=y, order=best_degree_1d, ci=None, scatter_kws={'s':5})
plt.plot(x, ypred_1d)
plt.title('A  Seventh Order Polynomial Regression', fontweight='bold')
plt.show()
```



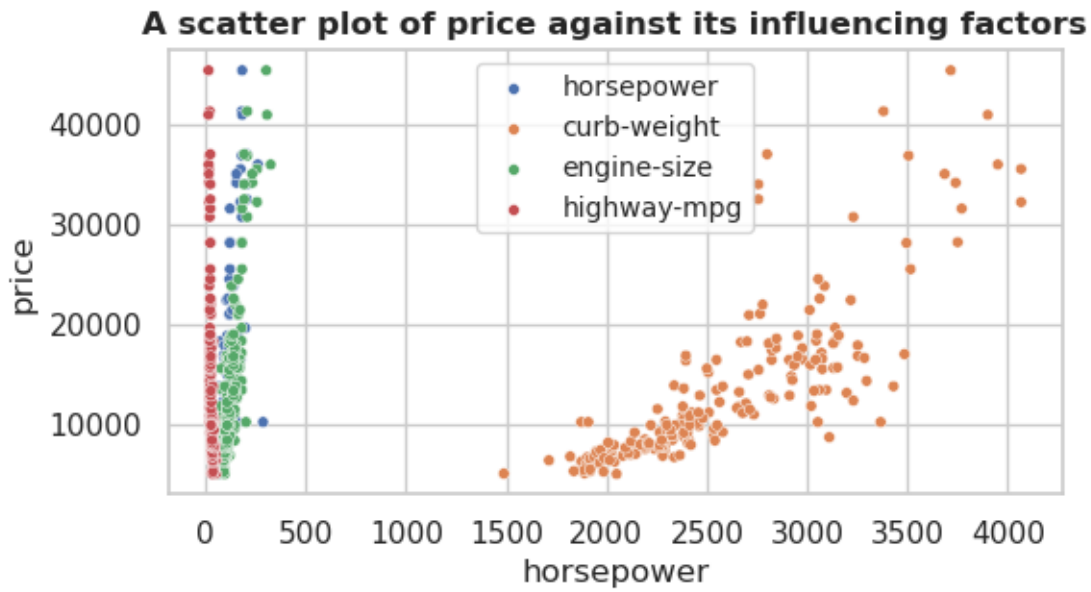
1.4.3.2 Multi-dimensional Polynomial Regression

- This incorporates more than one predictor variable
- Scaling(Normalizing) is needed to ensure standardizing of the data

```
x = automobile_df[['horsepower', 'curb-weight', 'engine-size', 'highway-
    mpg']]
y = automobile_df['price']
```

```
sns.scatterplot(x=automobile_df['horsepower'], y=y, label='horsepower',
    size=5, legend=False)
sns.scatterplot(x=automobile_df['curb-weight'], y=y, label='curb-weight',
     size=5, legend=False)
sns.scatterplot(x=automobile_df['engine-size'], y=y, label='engine-size',
     size=5, legend=False)
sns.scatterplot(x=automobile_df['highway-mpg'], y=y, label='highway-mpg',
     size=5, legend=False)
plt.title('A scatter plot of price against its influencing factors',
    fontweight='bold')
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7fa5533bd120>
```

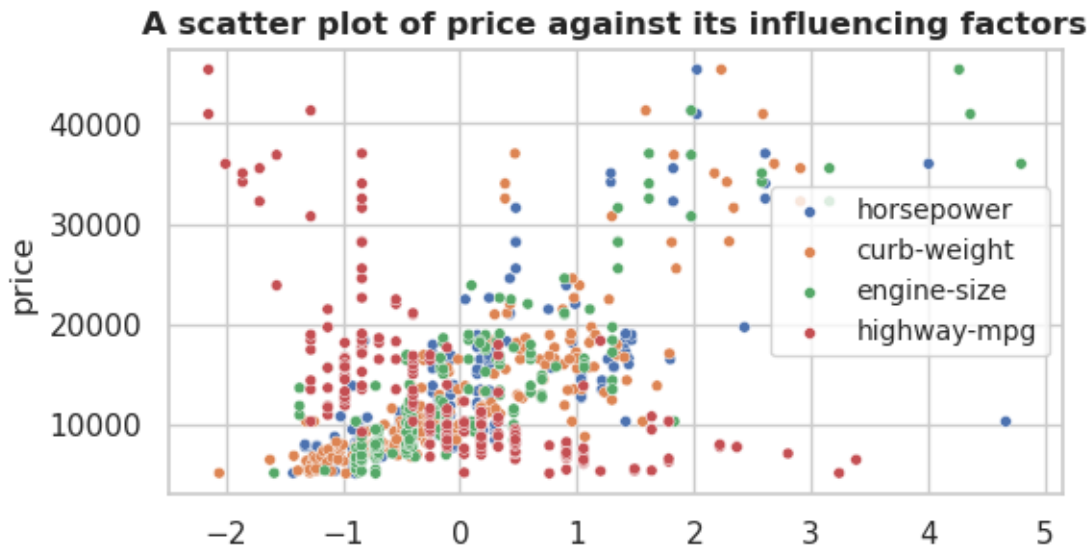


Looking at the scatter plot above, scaling is needed

```
# Standardize the predictors
x = zscore(x)
```

```
# Reproduce the scatter plot
sns.scatterplot(x=x['horsepower'], y=y, label='horsepower', size=5,
    legend=False)
sns.scatterplot(x=x['curb-weight'], y=y, label='curb-weight', size=5,
    legend=False)
sns.scatterplot(x=x['engine-size'], y=y, label='engine-size', size=5,
    legend=False)
sns.scatterplot(x=x['highway-mpg'], y=y, label='highway-mpg', size=5,
    legend=False)
plt.title('A scatter plot of price against its influencing factors',
    fontweight='bold')
plt.xlabel(None)
plt.legend()
```

```
<matplotlib.legend.Legend at 0x7fa5533e0130>
```



The plot shows that highway-mpg has negative relationship with price. Lets omit it and work with the rest

```
# Unstandardize the data so as to be accurate on predictions, to avoid
    inaccurate predictions and overfitting
x = automobile_df[['horsepower', 'curb-weight', 'engine-size', 'highway-
    mpg']]
x3 = x.drop('highway-mpg', axis=1)
y3 = automobile_df['price']
```

```
# Define range of polynomial degrees to test, play with it till you find
    the optimal degree
degrees_3d = [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17,
    18, 19, 20]
# Evaluate each polynomial degree
mse_scores = []
for degree in degrees_3d:
    # Add polynomial features to the predictors
    poly_f = PolynomialFeatures(degree=degree)
    xp = poly_f.fit_transform(x3)
    # Fit the model and compute MSE
    model = sm.OLS(y3, xp).fit()
    mse_scores.append(model.mse_resid)

# Select the best degree
best_degree_3d = degrees_3d[np.argmin(mse_scores)]
print('Best 3D polynomial regression degree:', best_degree_3d)
```

```
Best 3D polynomial regression degree: 8
```

```
# Fit the model with the best degree
poly_f_3d = PolynomialFeatures(degree=best_degree_3d)
xp_3d = poly_f_3d.fit_transform(x3)
model_3d = sm.OLS(y3, xp_3d).fit()
ypred_3d = model_3d.predict(xp_3d)
model_3d.summary()
```

Table 1.31: OLS Regression Results

Dep. Variable:	price	R-squared:	0.954
Model:	OLS	Adj. R-squared:	0.943
Method:	Least Squares	F-statistic:	86.76
Date:	Tue, 20 Jun 2023	Prob (F-statistic):	3.81e-91
Time:	15:44:47	Log-Likelihood:	-1815.1
No. Observations:	205	AIC:	3710.
Df Residuals:	165	BIC:	3843.
Df Model:	39		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
const	-1.946e-14	7.38e-15	-2.636	0.009	-3.4e-14	-4.89e-15
x1	3.156e-12	1.93e-12	1.633	0.104	-6.6e-13	6.97e-12
x2	2.651e-13	1.51e-13	1.754	0.081	-3.33e-14	5.64e-13
x3	-1.63e-13	8.99e-14	-1.813	0.072	-3.41e-13	1.45e-14
x4	1.925e-14	1.28e-14	1.498	0.136	-6.12e-15	4.46e-14
x5	-3.271e-14	1.95e-14	-1.677	0.095	-7.12e-14	5.8e-15
x6	-4.74e-15	4.17e-15	-1.138	0.257	-1.3e-14	3.49e-15
x7	1.895e-15	1.08e-15	1.749	0.082	-2.44e-16	4.03e-15
x8	-3.785e-16	1.7e-16	-2.230	0.027	-7.14e-16	-4.34e-17
x9	-6.286e-17	4.25e-17	-1.478	0.141	-1.47e-16	2.11e-17
x10	2.771e-16	1.38e-16	2.002	0.047	3.88e-18	5.5e-16
x11	-1.091e-15	6.88e-16	-1.586	0.115	-2.45e-15	2.67e-16
x12	-7.434e-17	7.85e-17	-0.947	0.345	-2.29e-16	8.06e-17
x13	-3.805e-17	3.73e-17	-1.021	0.309	-1.12e-16	3.55e-17
x14	-7.183e-18	6.23e-18	-1.153	0.250	-1.95e-17	5.11e-18
x15	-1.346e-17	1.17e-17	-1.148	0.253	-3.66e-17	9.69e-18
x16	1.054e-16	7.11e-17	1.482	0.140	-3.51e-17	2.46e-16
x17	3.062e-18	2.28e-18	1.343	0.181	-1.44e-18	7.56e-18
x18	5.865e-20	6.3e-20	0.931	0.353	-6.57e-20	1.83e-19
x19	-2.085e-21	1.72e-21	-1.215	0.226	-5.47e-21	1.3e-21
x20	2.916e-20	1.37e-19	0.213	0.831	-2.41e-19	2.99e-19
x21	2.377e-18	3.46e-18	0.687	0.493	-4.46e-18	9.21e-18
x22	5.668e-21	1.35e-19	0.042	0.966	-2.6e-19	2.71e-19
x23	7.511e-17	7.86e-17	0.956	0.341	-8.01e-17	2.3e-16
x24	8.81e-19	2.78e-18	0.316	0.752	-4.62e-18	6.38e-18
x25	-6.62e-20	1.25e-19	-0.531	0.596	-3.12e-19	1.8e-19
x26	2.066e-15	1.71e-15	1.206	0.230	-1.32e-15	5.45e-15
x27	2.558e-17	4.94e-17	0.518	0.605	-7.19e-17	1.23e-16
x28	-1.989e-18	1.85e-18	-1.076	0.284	-5.64e-18	1.66e-18
x29	-2.106e-19	1.18e-19	-1.785	0.076	-4.44e-19	2.24e-20
x30	5.294e-14	3.46e-14	1.532	0.127	-1.53e-14	1.21e-13
x31	6.031e-16	6.2e-16	0.973	0.332	-6.21e-16	1.83e-15
x32	-6.47e-17	3.12e-17	-2.072	0.040	-1.26e-16	-3.03e-18

x33	-7.091e-18	3.5e-18	-2.027	0.044	-1.4e-17	-1.83e-19
x34	-4.63e-19	2.26e-19	-2.045	0.042	-9.1e-19	-1.59e-20
x35	-1.306e-17	4.66e-18	-2.799	0.006	-2.23e-17	-3.85e-18
x36	-1.004e-16	5.78e-17	-1.736	0.084	-2.14e-16	1.38e-17
x37	-9.275e-18	3.92e-18	-2.365	0.019	-1.7e-17	-1.53e-18
x38	-2.477e-16	1.49e-15	-0.166	0.868	-3.19e-15	2.69e-15
x39	-8.482e-17	6.83e-17	-1.241	0.216	-2.2e-16	5.01e-17
x40	-6.633e-18	4.5e-18	-1.473	0.143	-1.55e-17	2.26e-18
x41	7.872e-15	2.82e-14	0.279	0.781	-4.79e-14	6.36e-14
x42	-1.373e-15	1.23e-15	-1.120	0.264	-3.79e-15	1.05e-15
x43	-1.092e-16	8.39e-17	-1.301	0.195	-2.75e-16	5.65e-17
x44	-4.491e-18	5.83e-18	-0.770	0.442	-1.6e-17	7.02e-18
x45	2.272e-13	4.01e-13	0.566	0.572	-5.65e-13	1.02e-12
x46	-3.791e-14	2.01e-14	-1.888	0.061	-7.76e-14	1.73e-15
x47	-3.595e-15	1.75e-15	-2.051	0.042	-7.06e-15	-1.33e-16
x48	-1.824e-16	9.79e-17	-1.864	0.064	-3.76e-16	1.08e-17
x49	-3.281e-18	5.89e-18	-0.557	0.578	-1.49e-17	8.34e-18
x50	2.27e-12	4.42e-13	5.140	0.000	1.4e-12	3.14e-12
x51	-1.139e-12	6.78e-13	-1.679	0.095	-2.48e-12	2e-13
x52	-1.155e-13	6.56e-14	-1.761	0.080	-2.45e-13	1.4e-14
x53	-7.483e-15	4.14e-15	-1.809	0.072	-1.57e-14	6.85e-16
x54	-3.262e-16	1.75e-16	-1.860	0.065	-6.73e-16	2.02e-17
x55	-4.246e-18	3.45e-18	-1.232	0.220	-1.11e-17	2.56e-18
x56	-1.167e-15	4.86e-16	-2.404	0.017	-2.13e-15	-2.08e-16
x57	-1.233e-14	5.67e-15	-2.174	0.031	-2.35e-14	-1.13e-15
x58	-9.336e-16	4.79e-16	-1.948	0.053	-1.88e-15	1.28e-17
x59	-1.125e-13	5.19e-14	-2.168	0.032	-2.15e-13	-1e-14
x60	-6.363e-15	3.59e-15	-1.774	0.078	-1.34e-14	7.17e-16
x61	-6.895e-16	4.1e-16	-1.681	0.095	-1.5e-15	1.2e-16
x62	-6.444e-13	2.41e-13	-2.673	0.008	-1.12e-12	-1.68e-13
x63	-2.156e-14	1.69e-14	-1.279	0.203	-5.48e-14	1.17e-14
x64	-2.859e-17	1.29e-15	-0.022	0.982	-2.58e-15	2.52e-15
x65	-2.16e-16	1.81e-16	-1.195	0.234	-5.73e-16	1.41e-16
x66	-2.546e-12	1.62e-12	-1.569	0.118	-5.75e-12	6.57e-13
x67	8.417e-14	2.21e-13	0.381	0.703	-3.52e-13	5.2e-13
x68	6.834e-14	4.71e-14	1.452	0.148	-2.46e-14	1.61e-13
x69	8.95e-15	5.62e-15	1.592	0.113	-2.15e-15	2.01e-14
x70	5.948e-16	3.62e-16	1.643	0.102	-1.2e-16	1.31e-15
x71	1.858e-13	1.22e-13	1.524	0.130	-5.5e-14	4.26e-13
x72	8.024e-14	3.15e-12	0.025	0.980	-6.15e-12	6.31e-12
x73	8.186e-13	5.36e-13	1.528	0.128	-2.39e-13	1.88e-12
x74	1.756e-13	1.04e-13	1.696	0.092	-2.88e-14	3.8e-13
x75	2.119e-14	1.22e-14	1.742	0.083	-2.83e-15	4.52e-14
x76	1.719e-15	9.67e-16	1.779	0.077	-1.89e-16	3.63e-15
x77	3.648e-15	3.84e-15	0.949	0.344	-3.94e-15	1.12e-14
x78	-3.483e-13	1.91e-13	-1.822	0.070	-7.26e-13	2.92e-14
x79	3.217e-12	2.71e-12	1.186	0.237	-2.14e-12	8.57e-12

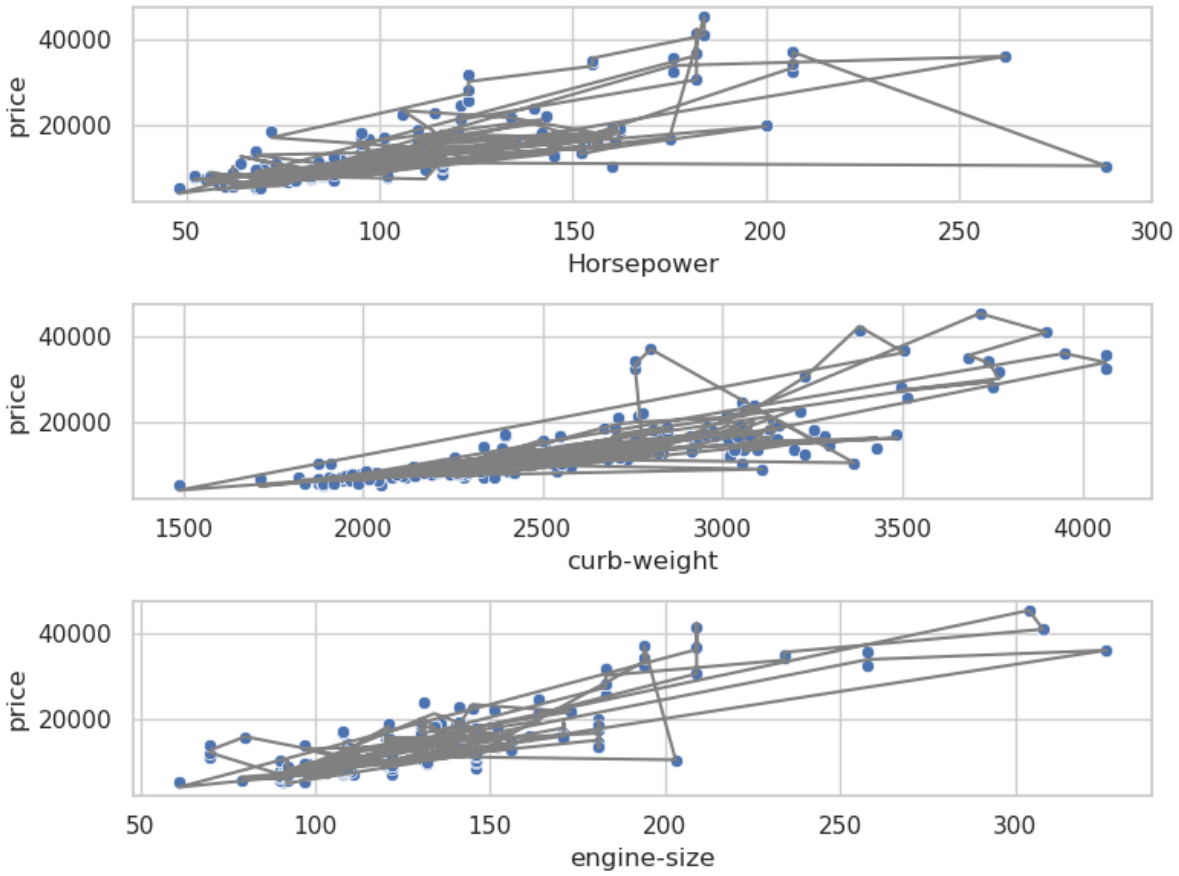
x80	1.694e-12	8.33e-13	2.033	0.044	4.85e-14	3.34e-12
x81	3.149e-13	1.62e-13	1.947	0.053	-4.48e-15	6.34e-13
x82	3.651e-14	1.92e-14	1.899	0.059	-1.46e-15	7.45e-14
x83	3.028e-15	1.63e-15	1.854	0.065	-1.96e-16	6.25e-15
x84	1.018e-14	7.71e-14	0.132	0.895	-1.42e-13	1.62e-13
x85	3.465e-13	6.36e-13	0.544	0.587	-9.1e-13	1.6e-12
x86	-4.749e-14	2.68e-14	-1.774	0.078	-1e-13	5.37e-15
x87	2.324e-12	2.96e-12	0.786	0.433	-3.51e-12	8.16e-12
x88	-1.64e-13	1.41e-13	-1.162	0.247	-4.43e-13	1.15e-13
x89	-1.143e-13	5.68e-14	-2.010	0.046	-2.26e-13	-2.04e-15
x90	-8.551e-13	1.06e-12	-0.808	0.420	-2.95e-12	1.23e-12
x91	-4.033e-13	3.59e-13	-1.125	0.262	-1.11e-12	3.05e-13
x92	-7.353e-13	4.02e-13	-1.828	0.069	-1.53e-12	5.91e-14
x93	-1.708e-13	9.88e-14	-1.728	0.086	-3.66e-13	2.43e-14
x94	1.232e-15	1.33e-13	0.009	0.993	-2.61e-13	2.63e-13
x95	1.809e-12	2.12e-12	0.851	0.396	-2.39e-12	6e-12
x96	-3.32e-12	2.02e-12	-1.646	0.102	-7.3e-12	6.63e-13
x97	-1.12e-12	6.72e-13	-1.666	0.098	-2.45e-12	2.07e-13
x98	-2.083e-13	1.25e-13	-1.666	0.098	-4.55e-13	3.86e-14
x99	3.839e-15	9.09e-15	0.422	0.673	-1.41e-14	2.18e-14
x100	-5.532e-14	2.65e-13	-0.209	0.835	-5.78e-13	4.68e-13
x101	-6.839e-13	2.76e-12	-0.248	0.805	-6.14e-12	4.77e-12
x102	-4.213e-12	2.53e-12	-1.663	0.098	-9.21e-12	7.89e-13
x103	-1.271e-12	7.63e-13	-1.666	0.098	-2.78e-12	2.35e-13
x104	-2.329e-13	1.38e-13	-1.684	0.094	-5.06e-13	4.01e-14
x105	-3.497e-16	2.73e-16	-1.282	0.202	-8.88e-16	1.89e-16
x106	7.854e-15	1.51e-14	0.520	0.603	-2.19e-14	3.76e-14
x107	-1.218e-13	4.28e-13	-0.285	0.776	-9.66e-13	7.23e-13
x108	1.522e-12	3.69e-12	0.412	0.681	-5.77e-12	8.82e-12
x109	-4.052e-12	2.39e-12	-1.693	0.092	-8.78e-12	6.74e-13
x110	-1.411e-12	7.99e-13	-1.766	0.079	-2.99e-12	1.67e-13
x111	-2.726e-13	1.51e-13	-1.809	0.072	-5.7e-13	2.5e-14
x112	5.945e-18	5.92e-18	1.004	0.317	-5.75e-18	1.76e-17
x113	-4.327e-16	6.2e-16	-0.698	0.486	-1.66e-15	7.91e-16
x114	2.03e-14	2.36e-14	0.861	0.390	-2.62e-14	6.69e-14
x115	-3.854e-13	3.91e-13	-0.986	0.326	-1.16e-12	3.87e-13
x116	2.888e-12	2.48e-12	1.166	0.245	-2e-12	7.78e-12
x117	-6.029e-12	2.99e-12	-2.014	0.046	-1.19e-11	-1.2e-13
x118	-2.016e-12	9.88e-13	-2.040	0.043	-3.97e-12	-6.52e-14
x119	-3.838e-13	1.87e-13	-2.054	0.042	-7.53e-13	-1.48e-14
x120	1.118e-12	1.36e-12	0.822	0.412	-1.57e-12	3.8e-12
x121	5.953e-12	4.03e-12	1.476	0.142	-2.01e-12	1.39e-11
x122	-2.243e-12	1.12e-12	-2.001	0.047	-4.46e-12	-2.93e-14
x123	-1.372e-12	1.41e-12	-0.974	0.332	-4.15e-12	1.41e-12
x124	-4.899e-12	2.29e-12	-2.141	0.034	-9.42e-12	-3.82e-13
x125	-2.911e-12	1.61e-12	-1.808	0.072	-6.09e-12	2.68e-13
x126	2.059e-13	1.83e-13	1.124	0.263	-1.56e-13	5.67e-13

x127	-2.474e-13	1.44e-12	-0.172	0.864	-3.09e-12	2.59e-12
x128	-5.607e-12	3.57e-12	-1.572	0.118	-1.26e-11	1.43e-12
x129	-2.062e-12	1.37e-12	-1.500	0.136	-4.78e-12	6.53e-13
x130	-1.166e-14	1.07e-14	-1.086	0.279	-3.28e-14	9.53e-15
x131	-2.755e-13	2.6e-13	-1.060	0.291	-7.89e-13	2.38e-13
x132	6.328e-12	3.36e-12	1.884	0.061	-3.04e-13	1.3e-11
x133	-2.65e-12	2.54e-12	-1.045	0.298	-7.66e-12	2.36e-12
x134	-4.393e-13	8.86e-13	-0.496	0.621	-2.19e-12	1.31e-12
x135	4.364e-17	4.01e-16	0.109	0.913	-7.48e-16	8.35e-16
x136	3.764e-14	2.34e-14	1.607	0.110	-8.61e-15	8.39e-14
x137	-5.323e-13	2.52e-13	-2.115	0.036	-1.03e-12	-3.53e-14
x138	-2.6e-12	3.42e-12	-0.760	0.448	-9.36e-12	4.16e-12
x139	1.66e-12	2.02e-12	0.820	0.413	-2.34e-12	5.66e-12
x140	1.527e-12	9.06e-13	1.686	0.094	-2.61e-13	3.31e-12
x141	1.681e-17	1.73e-17	0.972	0.333	-1.73e-17	5.1e-17
x142	-1.67e-15	1.02e-15	-1.640	0.103	-3.68e-15	3.4e-16
x143	9.506e-15	1.6e-14	0.593	0.554	-2.21e-14	4.11e-14
x144	3.534e-13	1.83e-13	1.935	0.055	-7.12e-15	7.14e-13
x145	-1.119e-12	2.09e-12	-0.536	0.593	-5.24e-12	3e-12
x146	6.887e-12	3.15e-12	2.189	0.030	6.76e-13	1.31e-11
x147	3.154e-12	1.54e-12	2.046	0.042	1.1e-13	6.2e-12
x148	-1.103e-20	2.99e-19	-0.037	0.971	-6.01e-19	5.79e-19
x149	-2.279e-17	2.1e-17	-1.084	0.280	-6.43e-17	1.87e-17
x150	2.445e-15	1.11e-15	2.202	0.029	2.52e-16	4.64e-15
x151	-6.363e-14	3.12e-14	-2.043	0.043	-1.25e-13	-2.12e-15
x152	6.637e-13	4.07e-13	1.632	0.105	-1.39e-13	1.47e-12
x153	-3.998e-12	2.27e-12	-1.762	0.080	-8.48e-12	4.83e-13
x154	6.849e-12	5.03e-12	1.361	0.175	-3.09e-12	1.68e-11
x155	2.494e-12	2.22e-12	1.125	0.262	-1.88e-12	6.87e-12
x156	-1.169e-21	9.44e-21	-0.124	0.902	-1.98e-20	1.75e-20
x157	4.943e-20	1.4e-18	0.035	0.972	-2.72e-18	2.82e-18
x158	1.196e-17	7.7e-17	0.155	0.877	-1.4e-16	1.64e-16
x159	-1.224e-15	2.25e-15	-0.543	0.588	-5.67e-15	3.22e-15
x160	3.826e-14	3.92e-14	0.976	0.330	-3.91e-14	1.16e-13
x161	-5.364e-13	4.13e-13	-1.298	0.196	-1.35e-12	2.79e-13
x162	3.663e-12	2.46e-12	1.491	0.138	-1.19e-12	8.51e-12
x163	-8.865e-12	5.89e-12	-1.504	0.134	-2.05e-11	2.77e-12
x164	-4.246e-12	2.79e-12	-1.520	0.130	-9.76e-12	1.27e-12

Omnibus:	28.617	Durbin-Watson:	1.420
Prob(Omnibus):	0.000	Jarque-Bera (JB):	54.826
Skew:	0.698	Prob(JB):	1.24e-12
Kurtosis:	5.114	Cond. No.	2.85e+16

```
fig, axs = plt.subplots(nrows=3, figsize=(8, 6))
```

```
# Plot each scatter plot with fitted polynomial regression
sns.scatterplot(x=x['horsepower'], y=y3, ax=axs[0])
axs[0].plot(x.values[:, 0], ypred_3d, color='grey',)

sns.scatterplot(x=x['curb-weight'], y=y3, ax=axs[1])
axs[1].plot(x.values[:, 1], ypred_3d, color='grey')

sns.scatterplot(x=x['engine-size'], y=y3, ax=axs[2])
axs[2].plot(x.values[:, 2], ypred_3d, color='grey')

# Set Plot labels
axs[0].set_xlabel('Horsepower')
axs[1].set_xlabel('curb-weight')
axs[2].set_xlabel('engine-size')

plt.tight_layout()
```



1.4.4 Measures for in-sample evaluation

- Mean squared error
- Coefficient of determination R^2

1.4.5 Prediction and Decision Making

Decision making:

1. Numerical measures for evaluation: MSE, RMSE among others
2. Visualization: residual plot, regression plot among others
3. Do the predicted values make sense: make a point prediction
4. Comparing models: MLR, SLR and Polynomial

> **NOTE**
>
> Lets compute RMSE and see if predicted values makes sense

- RMSE

```
# RMSE for 1D polynomial regression
rmse_1d = np.sqrt(mean_squared_error(y, ypred_1d))
print('The RMSE for 1D polynomial regression of price against high-mpg is
    :', rmse_1d)
```

```
The RMSE for 1D polynomial regression of price against high-mpg is:
    4507.054519308675
```

```
# RMSE for 3D polynomial regression
rmse_3d = rmse(y3, ypred_3d)
print('The RMSE for 3D polynomial regression of price ~ horsepower +
    engine-size + high-mpg is:', rmse_3d)
```

```
The RMSE for 3D polynomial regression of price ~ horsepower + engine-size
     + high-mpg is: 1694.8550199605947
```

- Make Point predictions of new data

```
# 1D polynomial regression model
# Predict price of a car that has a highway-mpg of 15

# Transform 15 suitable for input and predict price
y_new = model_1d.predict(poly_f_1d.fit_transform([[15]]))
print('The price of a car whose highway-mpg is 15 based on 7th order
    degree poly regression is', y_new)
```

```
The price of a car whose highway-mpg is 15 based on 7th order degree poly
     regression is [52635.58667572]
```

```
# 3D polynomial regression model
# Predict price of a car that has: horsepower:120, curb-weight:2500 and
    engine-size:130

# get data into a data frame
x3_new = {'horsepower': [120], 'curb-weight': [2500], 'engine-size':
    [130]}
x3_new = pd.DataFrame(x3_new)

# Transform and predict the price
y3_new = model_3d.predict(poly_f_3d.fit_transform(x3_new))
print('''The price of a car that has horsepower:120, curb-weight:2500 and
     engine-size:130
 based on 16th order degree 3D polynomial regression is''', y3_new)
```

```
The price of a car that has horsepower:120, curb-weight:2500 and engine-
    size:130
 based on 16th order degree 3D polynomial regression is [12033.40947437]
```

1.5 Model Evaluation

- Tells how the model performs in real world

1.5.1 Model Evaluation and Refinement

Let's fit polynomial regression that we have learnt to predict price of a vehicle

- Split the data into training and testing sets

Let's fit polynomial regression that we have learnt to predict price of a vehicle

- Split the data into training and testing sets

```
# Define respose and predictor variables data
x_pr_final = automobile_df_num[['wheel-base', 'horsepower', 'curb-weight'
    , 'engine-size', 'peak-rpm', 'highway-mpg']]
y_pr_final = automobile_df['price']

# Split data into training and testing sets
x_train, x_test, y_train, y_test = train_test_split(x_pr_final,
    y_pr_final, test_size=0.2, random_state=1234)
```

```
order = [1, 2, 3, 4, 5]
mse_scores = []
rsq = []
for n in order:
    pr = PolynomialFeatures(degree=n)
    x_train_pr = pr.fit_transform(x_train)
    x_test_pr = pr.fit_transform(x_test)
    model = sm.OLS(y_train, x_train_pr).fit()
    y_hat = model.predict(x_test_pr)
    mse_scores = mean_squared_error(y_test, y_hat)
    rsq = r2_score(y_test, y_hat)

best_degree_multi_d = order[np.argmin(mse_scores)]
print('Optimal polynomial regression degree based on MSE is:',
    best_degree_multi_d)
hi_rsq = order[np.argmax(rsq)]
print('Optimal polynomial regression degree based on R squared is:',
    hi_rsq)
```

```
Optimal polynomial regression degree based on MSE is: 1
Optimal polynomial regression degree based on R squared is: 1
```

```
# Fit the optimal model
pr_final = PolynomialFeatures(degree=best_degree_multi_d)
x_train_pr  = pr_final.fit_transform(x_train)
x_test_pr = pr_final.fit_transform(x_test)
model_final = sm.OLS(y_train, x_train_pr).fit()
y_hat_final = model_final.predict(x_test_pr)
```

```
print('R squared on test data is:', round(r2_score(y_test, y_hat_final),
    4))
```

```
R squared on test data is: 0.4958
```

```
y_hat_final[:6]
```

```
array([15571.2785236 , 12685.5674251 , 17361.16290682,  9647.28228682,
        5492.35823794, 15451.63416707])
```

Let's fit multiple linear regression model instead since there is are no aspects of polynomial regression on the dataset

- Split the data into training and testing sets

```
# Define respose and predictor variables data
x_mlr_final = automobile_df_num.drop('price', axis=1)
y_mlr_final = automobile_df['price']

# Split data into training and testing sets
x_train, x_test, y_train, y_test = train_test_split(x_mlr_final,
    y_mlr_final, test_size=0.2, random_state=1234)
```

```
# Add a constant term to the x data for the intercept term
x_mlr = sm.add_constant(x_mlr_final)
# Build the Multiple Linear Regression
mlr_final = sm.OLS(y_mlr_final, x_mlr_final).fit()
```

```
mlr_final.summary()
```

Table 1.34: OLS Regression Results

Dep. Variable:	price	R-squared (uncentered):	0.953
Model:	OLS	Adj. R-squared (uncentered):	0.949
Method:	Least Squares	F-statistic:	257.0
Date:	Tue, 20 Jun 2023	Prob (F-statistic):	2.43e-117
Time:	15:44:49	Log-Likelihood:	-1953.0
No. Observations:	205	AIC:	3936.
Df Residuals:	190	BIC:	3986.
Df Model:	15		
Covariance Type:	nonrobust		

	coef	std err	t	P>|t|	[0.025	0.975]
symboling	191.2443	295.302	0.648	0.518	-391.248	773.736
normalized-losses	-6.0363	9.456	-0.638	0.524	-24.688	12.615
wheel-base	183.4993	116.647	1.573	0.117	-46.591	413.589
length	-24.4833	57.492	-0.426	0.671	-137.889	88.922
width	-520.8529	200.533	-2.597	0.010	-916.410	-125.296
height	6.1387	142.369	0.043	0.966	-274.687	286.965
curb-weight	1.5049	1.872	0.804	0.422	-2.188	5.198
engine-size	130.4149	14.642	8.907	0.000	101.533	159.297
bore	-1358.0313	1242.939	-1.093	0.276	-3809.763	1093.701
stroke	-2898.9405	828.554	-3.499	0.001	-4533.288	-1264.593
compression-ratio	405.7156	90.232	4.496	0.000	227.730	583.701
horsepower	-23.9579	17.933	-1.336	0.183	-59.331	11.415
peak-rpm	1.9055	0.720	2.646	0.009	0.485	3.326
city-L/100km	1420.0755	366.842	3.871	0.000	696.468	2143.683

highway-mpg	62.2226	102.562	0.607	0.545	-140.084	264.529

Omnibus:	20.686	Durbin-Watson:	1.253
Prob(Omnibus):	0.000	Jarque-Bera (JB):	84.405
Skew:	0.087	Prob(JB):	4.70e-19
Kurtosis:	6.139	Cond. No.	3.00e+04

```
y_hat_mlr_final = mlr_final.predict(x_test)
print(y_hat_mlr_final.head(10))
print('The R squared on the test data is:', round(r2_score(y_test,
    y_hat_mlr_final), 4))
```

```
5      17187.948637
66     14900.247201
6      15327.869633
174    12665.093940
54      5569.766691
111    17925.339846
145     9855.497215
169    13196.559712
117    17164.492318
9      17439.493742
dtype: float64
The R squared on the test data is: 0.5864
```

1.5.2 Ridge regression

- Ridge regression prevents overfitting
- We will use the refined predictor variable prescribed by the multiple linear regression as significant

```
# Define respose and predictor variables data
x_ridge = automobile_df_num[['width', 'engine-size', 'stroke', '
    compression-ratio', 'peak-rpm', 'city-L/100km']]
y_ridge = automobile_df['price']

# Split data into training and testing sets
x_train, x_test, y_train, y_test = train_test_split(x_ridge, y_ridge,
    test_size=0.2, random_state=1234)
```

- Perform ridge regression

```
ridge_m = Ridge(alpha=0.1)
ridge_m.fit(x_train, y_train)
yhat = ridge_m.predict(x_test)
```

```
print('The R squared based on ridge regression on alpha=0.1 is:', round(
    r2_score(y_test, yhat), 4))
```

```
The R squared based on ridge regression on alpha=0.1 is: 0.6002
```

1.5.3 GridSearch

- GridSearch allows us to search into multiple free parameters
- Parameters like alpha, as discussed on ridge regression are not part of the fitting or training process, this values are called hyperparameters
- Scikit-Learn has a means of iterating over the hyperparameters using cross-validation called gridsearch

```
# Define respose and predictor variables data
x_grid = automobile_df_num[['width', 'engine-size', 'stroke', '
    compression-ratio', 'peak-rpm', 'city-L/100km']]
y_grid = automobile_df['price']

# Split data into training and testing sets
x_train, x_test, y_train, y_test = train_test_split(x_grid, y_grid,
    test_size=0.2, random_state=1234)
```

```
parameters1 = [{'alpha':[0.001, 0.1, 1, 10, 100, 1000, 10000, 100000], '
    random_state': [1234]}]
RR = Ridge()
Grid1 = GridSearchCV(RR, parameters1, cv=4)

Grid1.fit(x_grid, y_grid)
Grid1.best_estimator_
```

```
Ridge(alpha=100, random_state=1234)
```

```
scores = Grid1.cv_results_
scores['mean_test_score']
```

```
array([0.69125347, 0.69160829, 0.69428076, 0.70266428, 0.71208232,
       0.70087927, 0.67566231, 0.60792794])
```

- Print the scores of different free parameters

```
for param, mean_val in zip(scores['params'], scores['mean_test_score']):
    print(param, 'R^2 on test data:', mean_val)
```

```
{'alpha': 0.001, 'random_state': 1234} R^2 on test data:
    0.6912534679021181
{'alpha': 0.1, 'random_state': 1234} R^2 on test data: 0.6916082909114248
{'alpha': 1, 'random_state': 1234} R^2 on test data: 0.6942807618280009
{'alpha': 10, 'random_state': 1234} R^2 on test data: 0.7026642832286997
{'alpha': 100, 'random_state': 1234} R^2 on test data: 0.7120823229372935
{'alpha': 1000, 'random_state': 1234} R^2 on test data:
    0.7008792705314631
{'alpha': 10000, 'random_state': 1234} R^2 on test data:
    0.6756623124423294
{'alpha': 100000, 'random_state': 1234} R^2 on test data:
    0.60792794083413
```

Chapter 2

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