Homework 3

Sandy Auttelet

Due: 01/30/2024 at 11:59PM

Question:
A producer of trail mix has year-end left-over stock of peanuts, chocolate, raisins, pretzels and dried papayas. They would like to make and sell a holiday mix from this stock. The goal is to create a mix of these ingredients that has close to 235 calories per bag as possible. Processing machinery dispenses each ingredient in integer unit amounts. The following table describes the stock.

Table 1: Itemization for Trail Mix

Item	**Weight** (grams per unit)	**Calories** (calories per unit)
peanuts	6	35
chocolate	5	26
raisins	7	21
pretzels	4	15
papaya	8	4

Any optimal recipe for trail mix must contain at least one unit of each of the five items. The total bag weight cannot exceed 57 grams (about 2 oz.). Construct a linear optimization model whose solution provides the optimal trail mix recipe. Solve using software. Discuss how the solution to this problem may not be possible to implement, and consider reasonable modifications to address your concerns.

Answer:
First, we should attempt to define the variables and their constrains. Logically, we have 5 dimensions in our x-vector as there are five different food items to add to each bag. Since each bag must contain at least one unit of each of the items, this implies that $x \geq 1$. Because the machinery dispenses each ingredient in integer units, then all x's are integers in our linear program. We are also constrained by weight, meaning

$$6x_1 + 5x_2 + 7x_3 + 4x_4 + 8x_5 \leq 57.$$

We also have the constraint on calories such that

$$35x_1 + 26x_2 + 21x_3 + 15x_4 + 4x_5 \leq 235 + \delta,$$

$$35x_1 + 26x_2 + 21x_3 + 15x_4 + 4x_5 \geq 235 - \delta.$$

The δ appears in this constraint because we want to get as close to 235 calories as possible, so we must account for some variance in that number.

Therefore, the problem becomes

$$\max_{z} 35x_1 + 26x_2 + 21x_3 + 15x_4 + 4x_5$$

$$\text{subject to } 35x_1 + 26x_2 + 21x_3 + 15x_4 + 4x_5 \leq 235 + \delta$$

$$35x_1 + 26x_2 + 21x_3 + 15x_4 + 4x_5 \geq 235 - \delta$$

$$6x_1 + 5x_2 + 7x_3 + 4x_4 + 8x_5 \leq 57$$

$$x \geq 1$$

$$x \in \mathbb{Z}^5$$

To find a solution, I implemented the following code:

```
[IN]
import numpy as np
import scipy.optimize as opt

deltas = np.linspace(0,30,7)
for delta in deltas:
  c=np.array([35,26,21,15,4])

  A = np.array([[6,5,7,4,8],\
  [35,26,21,15,4],\
    [-35,-26,-21,-15,-4]] )

  b = np.array([57,235+delta,-235+delta])

  bounds=((1,np.inf),(1,np.inf),(1,np.inf),(1,np.inf),(1,np.inf))

  isint=[1,1,1,1,1]

  res=opt.linprog(c,A,b,A_eq=None,b_eq=None,bounds=bounds,integrality=isint)

  print("Delta: ", delta)
  print("zstar: ",round(res['fun'],1))
  print("xstar: ",res['x'],"\n")

[OUT]
Delta:  0.0
zstar:  None
xstar:  None

Delta:  5.0
zstar:  231.0
xstar:  [1. 6. 1. 1. 1.]

Delta:  10.0
zstar:  226.0
```

```
xstar:  [1. 5. 2. 1. 1.]

Delta:  15.0
zstar:  220.0
xstar:  [1. 5. 1. 2. 1.]

Delta:  20.0
zstar:  215.0
xstar:  [1. 4. 2. 2. 1.]

Delta:  25.0
zstar:  210.0
xstar:  [4. 1. 1. 1. 2.]

Delta:  30.0
zstar:  205.0
xstar:  [1. 5. 1. 1. 1.]

```

As you can see, the first iteration of the solution resulted in no solution because it is not attainable to make a bag of trail mix at exactly 235 calories under the additional weight constraint. If the store is comfortable with selling bags that are ± 5 calories from their target of 235 calories, then solutions will be attainable.

If they choose to increase the tolerance further, bags with more variety can be implemented, although I wouldn't complain about trail mix with mostly chocolate. The bag with the most variety of items from my sample of tolerance is when the bags are sitting at 235 ± 20 calories. In this bag, a customer will receive one serving of peanuts and papaya, two servings of raisins and pretzels and four servings of chocolate.