Homework 4

Sandy Auttelet

Due: 02/06/2024 at 11:59PM

Question:
A farmer is planning an orchard of mixed apple, pear and cherry trees that can hold a maximum of 345 trees. The seasonal cost of labor per tree is \$150, \$200 and \$240, respectively. The seasonal cost of materials per tree is \$275, \$180 and \$125, respectively. Planting logistics require that **IF** any apple trees are planted then at least 150 must be planted. Similar limits on pear and cherry trees are 50 and 80, respectively. Write and solve an optimization model that maximizes the number of trees which the farmer can plant when labor costs are limited to a maximum of \$50,000 and materials costs are limited to a maximum of \$60,000.

Answer:
x-vector has three dimensions:

$$x_1 := \text{number of apple trees},$$
$$x_2 := \text{number of pear trees},$$
$$x_3 := \text{number of cherry trees}.$$

Our $x \in \mathbb{Z}^3$ for $x \geq 0$ since you cannot plant part of a tree, nor can you plant negative trees, but you can plant no trees.

We want to maximize number of trees under financial constraints.
Constriant 1, c_1: At most \$50,000 in labor cost.
Constraint 2, c_2: At most \$60,000 in material cost.
Tree limit, x_{max}: Only 345 trees will fit in the orchard.
Each x_i for $i = 1, 2, 3$ has a constraint such that there is a minimal number of trees can be planted if that type of tree is planted. Then we have to introduce a binary variable for each x_i, call this x_{i+3} such that $x_{i+3} \in \mathbb{Z}$ and $x_{i+3} \in [0, 1]$. Then,

$$345x_4 \leq x_1 \leq 150x_4$$
$$345x_5 \leq x_2 \leq 50x_5$$
$$345x_6 \leq x_3 \leq 80x_6.$$

These terms can be split such that each of the above conditions become two separate conditions for our linear program. This approach causes the linear program solution for the x-vector to be now be in $\mathbb{Z}^6$. Then,

$$x_4 := \begin{cases} 0 \text{ if we don't plant apple trees} \\ 1 \text{ if we do plant apple trees} \end{cases}.$$

$$x_5 := \begin{cases} 0 \text{ if we don't plant pear trees} \\ 1 \text{ if we do plant pear trees} \end{cases}.$$

$$x_6 := \begin{cases} 0 \text{ if we don't plant cherry trees} \\ 1 \text{ if we do plant cherry trees} \end{cases}.$$

Then the problem becomes,

$$\begin{gathered}
\max_{z} x_1 + x_2 + x_3 \\
\text{subject to } 150x_1 + 200x_2 + 240x_3 \leq 50000 \\
275x_1 + 180x_2 + 125x_3 \leq 60000 \\
x_1 + x_2 + x_3 \leq 345 \\
150x_4 - x_1 \leq 0 \\
x_1 - 345x_4 \leq 0 \\
50x_5 - x_2 \leq 0 \\
x_2 - 345x_5 \leq 0 \\
80x_6 - x_3 \leq 0 \\
x_3 - 345x_6 \leq 0 \\
x \geq 0 \\
x \in \mathbb{Z}^6
\end{gathered}$$

Finally, the solution to this linear program is:

$$z^* = 270 \text{ and } x = [172, 0, 100, 1, 0, 1].$$

This result implies we should plant 172 apple trees, 0 pear trees and 100 cherry trees. Therefore, $x_4 = 1$ since we are planting apple trees, $x_5 = 0$ since we are not planting pear trees and $x_6 = 1$ since we are planting cherry trees. This modification to the program allows us to model this system such that we can choose to either plant 0 trees or the minimum number of trees by the above parameters.