

FUNDAMENTALS OF FRICTION MODELING

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INTRODUCTION

Friction modeling has been steadily gaining in interest over the last few decades. However, owing to the complexity of the friction phenomenon, no comprehensive, practicable friction model that shows all of the experimentally observed aspects of friction force dynamics in one formulation is available. Most available friction models are, in essence, empirical, that is, based on limited observations and interpretations. In this sense, the resulting models are valid only for the specific scope of test conditions, such as the level and type of excitation, used to obtain the data. On the other hand, development of simulation models and, where possible, predictive theories, at scales from atomic, through continuum, to useful engineering models, can fill empty gaps in the toolboxes available to designers and analysts.

Besides the field of tribology, where the origin of friction is one of the main topics, modeling and compensation of friction dynamics are treated in several other domains. In the machining and assembly industry, demand for high-accuracy positioning systems and tracking systems is increasing. Research on controlled mechanical systems with friction is motivated by the increasing demand for these systems. Friction can severely deteriorate control system performance in the form of higher tracking errors, larger settling times, hunting, and stick-slip phenomena. In short, friction is one of the main players in a wide variety of mechanical systems.

This communication presents an overview of friction model-building, which starts from the generic mechanisms behind friction to construct models that simulate observed macroscopic friction behavior. First, basic friction properties are presented. Then, the generic friction model is outlined. Hereafter, simple heuristic/empirical models are discussed, which are suitable for quick simulation and control purposes. An example of these is the Generalized Maxwell-Slip model.

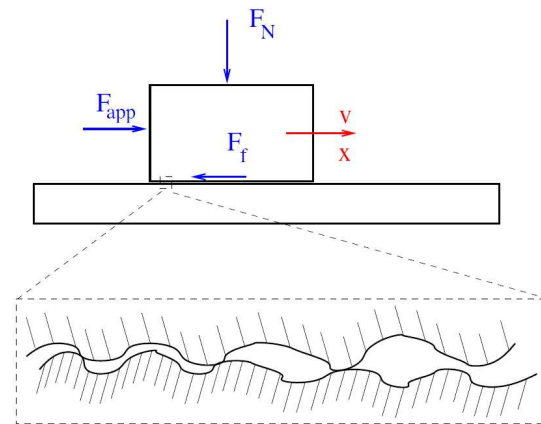


FIGURE 1. Basic friction configuration.

BASIC FRICTION BEHAVIOR

Considering friction as a mechanical system, (see Fig.1), a close examination of the sliding process reveals two friction regimes, namely, the pre-sliding regime and the gross sliding regime, (see Fig.2). In the pre-sliding regime the adhesive forces, owing to asperity contacts, are dominant, and thus the friction force is primarily a function of displacement rather than velocity. The reason for this behavior is that the asperity junctions deform elasto-plastically, thus behaving as nonlinear hysteretic springs.

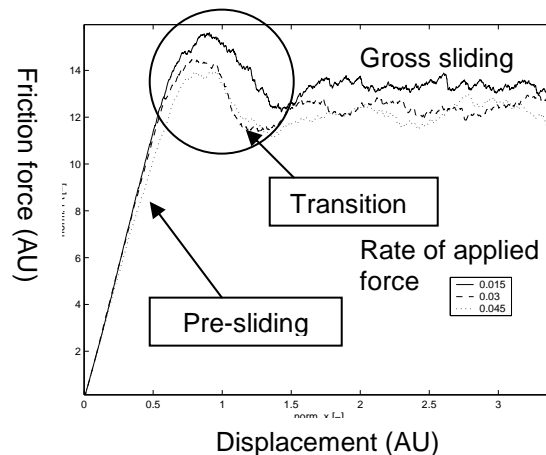


FIGURE 2. The two friction regimes and the transition between them.

As the displacement increases, more and more junctions break and have less time to reform, resulting eventually in gross sliding.

The sliding regime is, thus, characterized by a continuous process of asperity junction formation and breaking such that the friction force becomes predominantly a function of the velocity [1]. The transition from pre-sliding to gross sliding is a *criticality* that depends on many factors such as the relative velocity (to be envisaged as the displacement rate) and acceleration of the sliding objects, see [2].

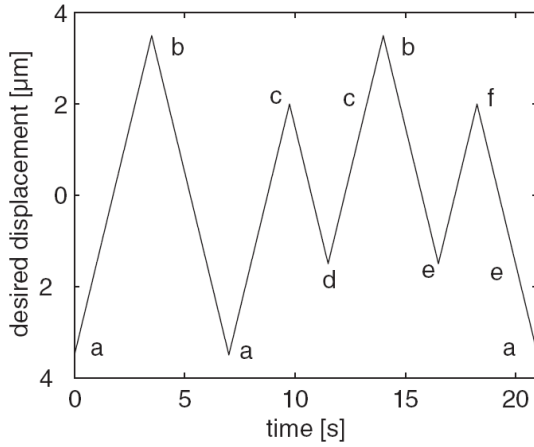


FIGURE 3. Example of desired motion in the pre-sliding regime.

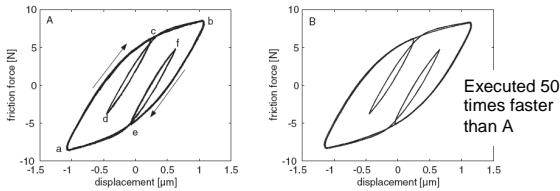


FIGURE 4. Hysteresis behavior as a result of the trajectory of Fig. 3.

Pre-sliding behavior

At very small displacements, that is, in the pre-sliding regime, experiments reveal a hysteretic displacement-dependent friction force [3,4]. When a pre-sliding displacement command, such as that shown in Fig. 3, is applied to the block, the force-displacement behavior of Fig.4 results. The position signal is chosen such that there is an inner loop within the outer hysteresis loop. The resulting friction-position curve is rate independent (compare the right and left panels of Fig.4). In other words, the friction-position curve is independent of the speed of the applied position signal. When an inner loop is closed, (c-

d-c), the curve of the outer loop (a-c-b) is followed again, proving the nonlocal memory characteristic of the hysteresis. The shape of the hysteresis function is determined by the distribution of the asperity heights, the tangential stiffness, and the normal stiffness of the contact.

This hysteresis behavior arises primarily from micro-slip, that is, the breaking of adhesive contacts, just as in the Maxwell-Slip model discussed further below. The contribution of deformation losses, which are hysteresis losses in the bulk materials, depends on the relative value of this part as compared to the adhesive part, as well as on the tangential stiffness of the asperities, which governs the extent of deformation before slip.

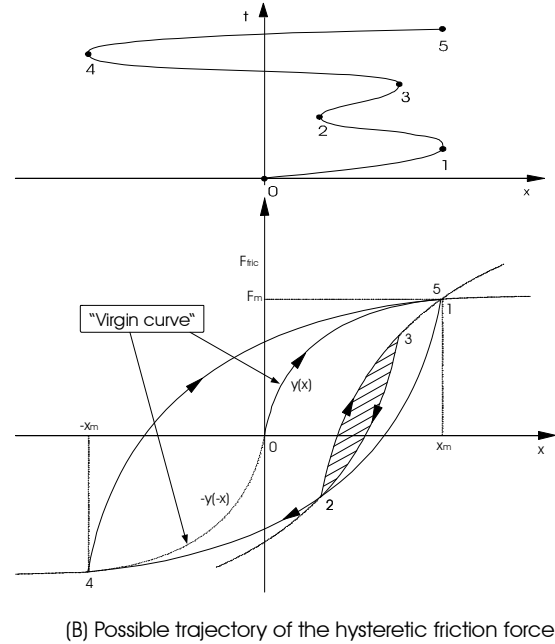


FIGURE 5. Characteristics of hysteresis with nonlocal memory.

Figure 5 explains the constitution of hysteresis according to Masing's rules. The force-displacement curve initially follows:

$$F_{\text{virgin}} = f(x)$$

$$\text{with } f(x) = \begin{cases} y(x) & ; x \geq 0 \\ -y(-x) & ; x \leq 0 \end{cases}$$

Upon reversal at any point x_m , a doubly dilated version of the virgin curve is followed:

$$F_{\text{fric}} = F_m + 2f\left(\frac{x - x_m}{2}\right)$$

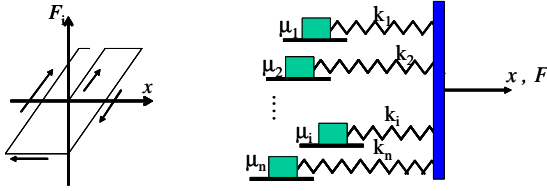


FIGURE 6. Modeling of hysteresis using Maxwell-Slip elements.

This behavior can also be modeled discretely by a parallel connection of Maxwell-Slip elements [5], as shown in Fig. 6.

Gross sliding

When the asperity junctions are continually being created and broken, the frictional interface is in the gross sliding regime. Two main characteristics are of interest here. The first is the steady-state friction force behavior with increasing steady-state sliding velocities, generally known as the Stribeck curve, Fig. 7. The second is the change of the friction force with the velocity variation, known as the *friction lag* or *friction memory* phenomenon, Fig. 8.

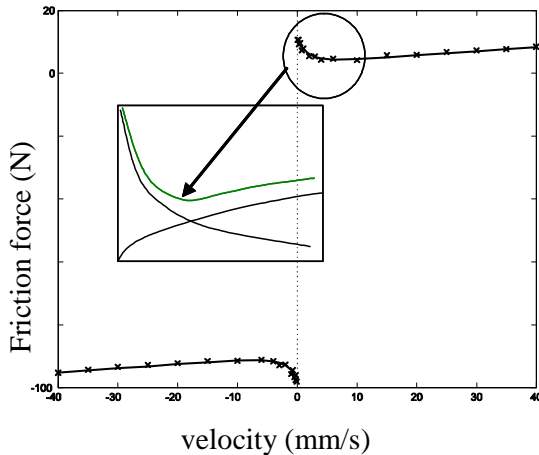


FIGURE 7. The Stribeck curve consists of velocity weakening and velocity strengthening.

The Stribeck curve

When the friction force is measured at constant velocity values (Fig. 7), the resulting functional relationship has a characteristic form. For increasing velocities, the friction force initially decreases to a minimum (*velocity weakening*) and then increases again (*velocity strengthening*). In lubricated sliding, this characteristic is known as the Stribeck curve, where the velocity weakening arises from the

initial buildup of hydrodynamic pressure, while the velocity strengthening is attributed to the viscous shear of the lubricating film.

The same behavior seems to hold true for dry friction, which justifies using the same name, that is, the Stribeck curve, to describe it. The actual form of the friction-velocity curve is determined by various process parameters, namely, the normal creep or, equivalently, the time evolution of adhesion, the surface topography, and the asperity parameters, primarily the tangential stiffness and inertia [2].

Friction lag

Friction lag, also called hysteresis in the velocity, or frictional memory, is manifested by a lag in the friction force relative to the sliding velocity. The origin of friction lag in lubricated friction relates to the time required to modify the lubricant film thickness, which is known as the *squeeze effect*. Friction lag is also observed in dry friction experiments (Fig. 8), where lubrication is not used. The mechanism causing friction lag in dry sliding is similar to that for lubricated friction, namely, that the local adhesion coefficient increases with the time of contact of two opposing asperities, owing to normal creep. In other words, time is required before the friction force changes with changing sliding velocity. Since the normal creep is caused by the sinking of the surfaces into each other, this mechanism is akin to the squeeze effect in lubricated friction. Thus, the friction force is higher for acceleration than for deceleration, so that the dynamic friction force curve circles around the steady-state curve. The Stribeck curve $s(v)$ acts as the attractor.

$$\text{Model: } \frac{dF}{dt} = f \left(1 - F / s(v) \right), \quad f(0) = 0.$$

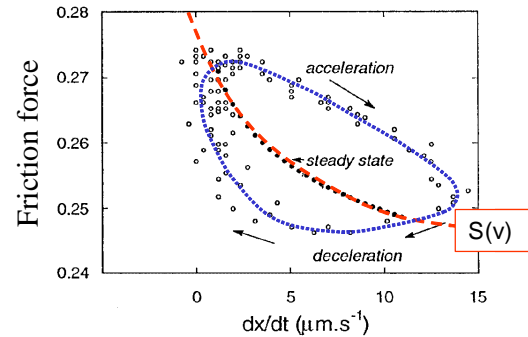


FIGURE 8. Friction lag and its constitutive equation. $s(v)$ is the velocity weakening curve also called the "Stribeck effect".

A GENERIC FRICTION MODEL

In order reconstruct the friction behavior outlined above, in the framework of a mechanical theory, a generic model was developed [6].

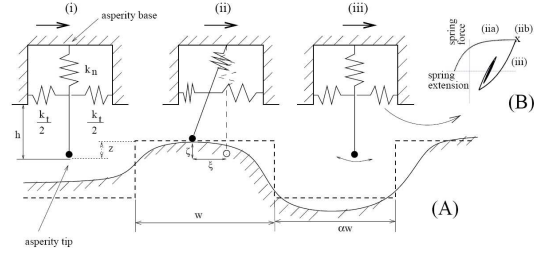


FIGURE 9. A generic representation of the sliding contact of rough bodies.

The model comprises an upper body containing point-mass asperities supported on hysteresis springs, which slides against a rigid, profiled lower surface, subject to normal creep, adhesion and deformation. The life cycle of an average asperity contact is depicted in Fig. 9. (A) An asperity is initially moving freely (i) until it touches the lower rigid surface (ii). After sticking and slipping, it breaks completely loose from the lower profile (iii). (B) depicts the hysteretic force-deformation diagram during a contact cycle of the asperity, where upon breaking loose, the asperity is assumed to dissipate all of its elastic energy through internal hysteresis losses.

Typical results of the generic model

The generic model yields the self-explanatory results reviewed in Fig. 10 through 12. The parameters used in the model have been chosen ad hoc to illustrate typical behavior, although they can also be identified from experimental results.

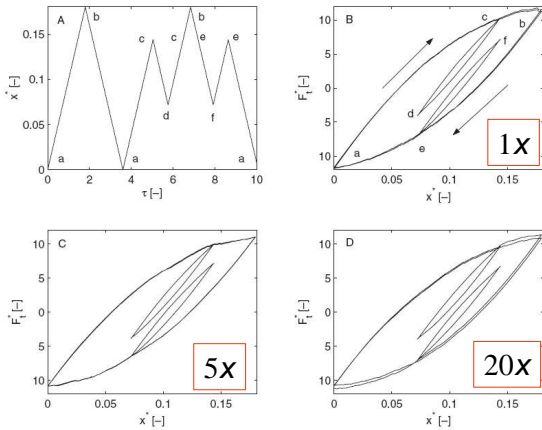


FIGURE 10. Simulation results of pre-sliding hysteresis using the generic model. Cf. Fig. 3.

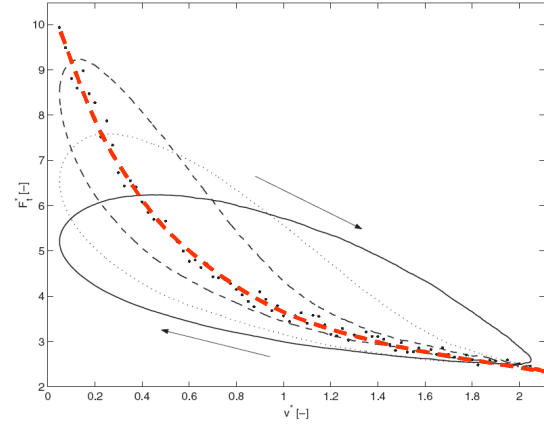


FIGURE 11. Friction lag using the generic model. The higher the acceleration, the more the loops depart from the Stribeck curve.

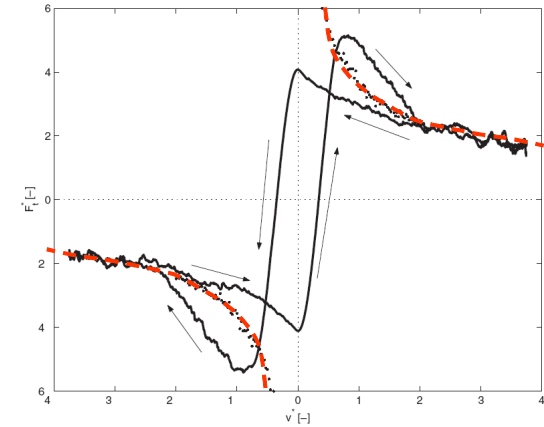


FIGURE 12. Periodic friction-velocity curves using the generic model.

Finally, simulation experiments with this model have contributed to the derivation of Generalized Maxwell-Slip (GMS) model belonging to the category of empirical models subject of the next section.

EMPIRICAL MODELS

Generalized Empirical Friction Model

Structure

Analysis of this class of model reveals that most of the existing empirical friction models correspond to a generalized friction model structure, which consists in a friction force equation and a state equation.

The friction force F_f is a generalized function of an internal state vector \mathbf{z} (often representing

asperity deflections), the velocity v , and the position x of the moving object, that is,

$$F_f = f(z, v, x)$$

The state equation that describes the dynamics of the internal state vector \mathbf{z} is a first-order differential equation of the form

$$\frac{dz}{dt} = g(z, v, x)$$

The functions f and g are generally nonlinear functions. In particular, it is shown that

$$f(z, v, x) = f_1(z, v, x) + f_2(v)$$

where f_1 is responsible for the transient response in the velocity, while f_2 represents the instantaneous response to velocity changes. However, this formulation corresponds to sliding and thus does not allow for pre-sliding or for passing through zero velocity.

Empirical friction modeling consists then in finding suitable expressions for the generalized functions f and g , such that the resulting model faithfully simulates all observed types of friction behavior.

Two generic conditions apply, which provide limiting conditions on the functions f and g . First, for constant velocities, the steady state ($dz/dt = 0$) friction force is a function of the velocity v only. Thus, if $v = \text{constant}$, then

$$g(z, v, x) = 0, \text{ and}$$

$$f(z, v, x) = F(v) = s(v) + f_2(v)$$

where $s(v)$ is the velocity-weakening curve, while $f_2(v)$ is the “viscous” or velocity-strengthening curve.

The second condition on the general functions is determined by the frictional behavior in the pre-sliding regime at small displacements. The friction force is then a hysteresis function of the position, with nonlocal memory characteristics [3] as given by

$$F_f = f(z, v, x) = F_h(x)$$

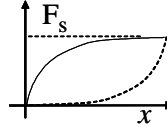
From the last two conditions, we can see that g is generally not continuous.

Evolution of the Empirical Friction Models

A possible way to evolutionarily classify existing empirical/heuristic friction models is according to the following main line.

Dahl Model

The friction force is a hysteresis function (without memory) of the displacement.

$$\frac{dF_f}{dx} = \sigma_0 \left(1 - \frac{F_f}{F_s} \text{sgn}(v) \right)$$


LuGre Model

Transforms the Dahl equation into the state equation:

$$\frac{dz}{dt} = v \left(1 - \frac{z}{s(v)} \text{sgn}(v) \right)$$

The friction force is given by:

$$F_f = \sigma_0 z + \sigma_1 \dot{z} + \sigma_2 v$$

Leuven Model

This adds a hysteresis function with nonlocal memory F_h to the formulation

$$\frac{dz}{dt} = v \left(1 - \frac{F_h(z)}{s(v)} \text{sgn}(v) \right), \text{ state}$$

$$F_f = F_h(z) + \sigma_1 \dot{z} + \sigma_2 v, \text{ Friction force.}$$

Generalized Maxwell-Slip Model

This was conceived as a match between the generic, LuGre and Leuven models [7]. It is constructed by imposing a rate-state behavior to the friction blocks of the Maxwell-Slip model. Referring to Fig. 13, the state equations are:

$$\text{If } \textit{stick} : \frac{dz_i}{dt} = v, \quad z_i \leq s_i(v)$$

$$\text{If } \textit{slip} : \frac{dz_i}{dt} = \text{sgn}(v) C_i \left(1 - \frac{z_i}{s_i(v)} \right)$$

The friction force is given by:

$$F_f = \sum_{i=1}^{i=N} (k_i z_i + \sigma_i \dot{z}_i) + f(v)$$

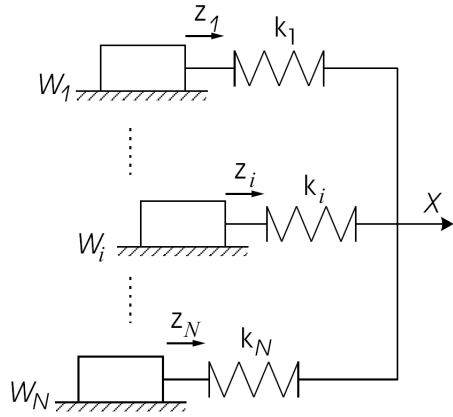


FIGURE 13. Maxwell-Slip to Generalized Maxwell-Slip model.

COMPARISON WITH EXPERIMENT

In how far the generic and GMS models agree with experimental observations depends on parameters chosen for the models. In other words, those models are effective in regard to structure. This is supported by the results of Fig. 14, which is at the same time an illustration of the stick-slip phenomenon and its intricate behavior.

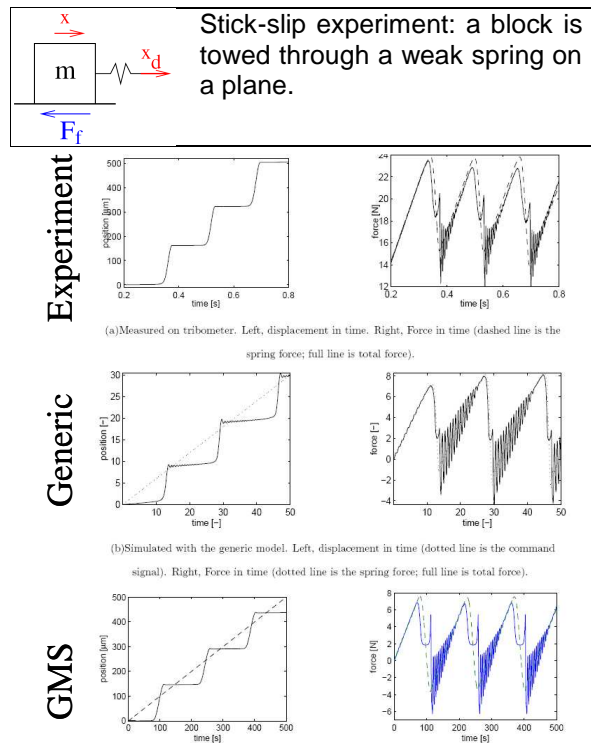


FIGURE 14. Stick-slip motion: comparison between models and experiment.

CONCLUSIONS

Friction is a complex, nonlinear phenomenon. Modeling of friction is important in many fields of science and engineering. We have reviewed a generic physical model and a number of empirical models; closing the circle by deriving the Generalized Maxwell-Slip (GMS) model, which is suitable for quick simulation and control purposes. The generic and GMS models agree well with experimental observations

Acknowledgements

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