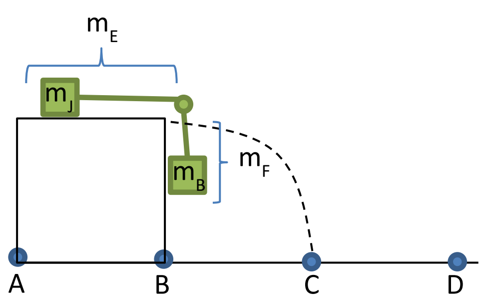
Julia Rasmussen — Uber Pulley Problem — October 29, 2019 — Section K

Description:
"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the 3-chain.

Diagram



Given Values:

m_J=62 kg	x_i=0 m	a_E=-a_F
m_B=149 kg	h=15 m	v_i=0 m/s
m_C=42 kg	x_{BD}=87 m	
	L_C=87 m	μ_P=0.21

Strategy:

1. **Determine m_E and m_F:**
   a. The masses are dependent on the chain as well as the jumper and pile of rocks. As such, the changes in the chain must be taken into account in calculating the masses. The mass of the chain for m_E starts with the full mass of the chain, and the chain gets taken away as time goes on. The converse happens for m_F.
   b. $m_E=(m_J+m_c)-\left(\frac{\Delta x}{L_c}\cdot m_C\right)$
   c. $m_F=m_B+(\frac{\Delta x}{L_c}\cdot m_C)$
   d. $\underline{m_E=(104)-\left(\frac{\Delta x}{8}\cdot 42\right)}$
   e. $\underline{m_F=149+(\frac{\Delta x}{8}\cdot 42)}$
2. **Find a[x]:**
   a. $\sum F_{yE}=0$
   b. $F_{NE}-F_{GE}=0$
   c. $\underline{F_{NE}=m_J\cdot g}$
   d. $\sum F_{xE}=ma$
   e. $F_{TE}-F_{FE}=m_E\cdot a_E$
   f. $F_{TE}=m_E\cdot a_E+F_{FE}$
   g. $F_{TE}=m_E\cdot a_E+\mu_P\cdot F_{NE}$
   h. $\underline{F_{TE}=m_E\cdot a_E+\mu_P\cdot m_J\cdot g}$
   i. $\sum F_{xF}=ma$
   j. $F_{TF}-F_{GF}=m_F\cdot a_F$
   k. $F_{TF}=-(m_F\cdot a_E)+F_{GF}$
   l. $\underline{F_{TF}=(m_F\cdot g)-(m_F\cdot a_E)}$
   m. Set F_{TE} equal to F_{TF}
   n. $F_{TE}=F_{TF}$
   o. $m_E\cdot a_E+\mu_P\cdot m_J\cdot g=(m_F\cdot g)-(m_F\cdot a_E)$
   p. $m_E\cdot a_E+(m_F\cdot a_E)=(m_F\cdot g)-\mu_P\cdot m_J\cdot g$
   q. $a_E=\frac{(m_F\cdot g)-\mu_P\cdot m_J\cdot g}{m_E+m_F}$
   r. $a_E=\frac{g\cdot((m_B+(\frac{\Delta x}{L_c}\cdot m_C))-\mu_P\cdot m_J)}{((m_J+m_c)-(\frac{\Delta x}{L_c}\cdot m_C))+(m_B+(\frac{\Delta x}{L_c}\cdot m_C))}$
   s. $a_E=\frac{9.8\cdot((149+(\frac{\Delta x}{8}\cdot 42))-(0.21\cdot 62)}{((104)-(\frac{\Delta x}{8}\cdot 42))+(149+(\frac{\Delta x}{8}\cdot 42))}$
   t. $a_E=\frac{9.8\cdot((149+5.25\cdot\Delta x)-(13.02)}{(104-5.25\cdot\Delta x)+(149+5.25\cdot\Delta x)}$
   u. $a_E=\frac{9.8\cdot(135.98+5.25\cdot\Delta x)}{(253)}$
   v. $\underline{a_E=0.20336\cdot\Delta x+5.26721}$
3. **Find v[8]:**
   a. $a[x]=\frac{dv}{dt}$
   b. $a[x]=\frac{dv}{dt}\cdot\frac{dx}{dx}$
   c. $a[x]=\frac{dv}{dx}v$
   d. $a[x]\cdot dx=v\cdot dv$
   e. $\int_0^8 a[x]\cdot dx=\int_0^v v\cdot dv$
   f. $\int_0^8(0.20336\cdot\Delta x+5.26721)\cdot dx=\int_0^v v\cdot dv$
   g. $\left(\frac{1}{2}\cdot 0.20336x^2+5.26721x+C\right)|_0^8=\left(\frac{1}{2}v^2+C\right)|_0^v$
   h. $(0.20336\cdot 32+5.26721\cdot 8+C)-C=\left(\frac{1}{2}v^2+C\right)-C$
   i. $\frac{1}{2}v^2=6.50751+42.1377$
   j. $v^2=97.2904$
   k. $\underline{v_B=9.8636\ m/s}$
4. **Find t_{BC} (y-dir):**
   a. $x_f=\frac{1}{2}a\cdot\Delta t^2+v_i\cdot\Delta t+x_i$
   b. $0=\frac{1}{2}(a_g)\cdot {t_{BC}}^2+h$
   c. ${t_{BC}}^2=\frac{15}{\frac{1}{2}(9.8)}$
   d. $\underline{t_{BC}=1.7496\ s}$
5. **Find x_{BC}:**
   a. $x_f=\frac{1}{2}a\cdot\Delta t^2+v_i\cdot\Delta t+x_i$
   b. $x_f=0+v_B\cdot t_{BC}+0$
   c. $x_f=9.8636\cdot 1.7496$
   d. $\underline{x_f=17.2577\ m}$
6. **Find v_C:**
   a. $\underline{v_{cx}=9.8636\ m/s}$
   b. $v_{cy}=a\Delta t+v_i$
   c. $v_{cy}=-9.8\cdot 1.7496+0$
   d. $\underline{v_{cy}=-17.1464}$
   e. $a^2+b^2=c^2$
   f. ${v_{cx}}^2+{v_{cy}}^2={v_c}^2$
   g. $9.8636^2+-17.1464^2={v_c}^2$
   h. ${v_c}^2=391.29$
   i. $v_c=19.7811\ m/s$
   j. But actually, because 25% of the speed is lost:
   k. $v_c=14.8358\ m/s$

7. **Find the acceleration on the jumper from C to D:**
    a. $v_f{}^2 = v_i{}^2 + 2a\Delta x$
    b. $0 = v_c{}^2 + 2a(x_{BD} - x_{BC})$
    c. $2a(87 - 17.2577) = -220.101$
    d. $a = \frac{-220.101}{2 \cdot 69.7423}$
    e. $a = 1.57796\ m/s^2$
8. **Find μ_G:**
    a. $\mu_G = \frac{F_F}{F_N}$
    b. $\mu_G = \frac{m \cdot a}{m \cdot g}$
    c. $\mu_G = \frac{a}{g}$
    d. $\mu_G = \frac{1.57796}{9.8}$

$$\boxed{\mu_G = 0.16102}$$

Answer:

The coefficient of kinetic friction between the jumper and the ground, or μ_G, is 0.16102.