Orthogonal Codes for Acoustic UnderwaterLocalization

by

Sergej Keller

April 2022

Supervised by

Christoph Weyer
Institute of Telematics, Hamburg University of Technology

First Examiner	Prof. Dr.-Ing. Bernd-Christian Renner
	Institute of Autonomous Cyber-Physical Systems
	Hamburg University of Technology
Second Examiner	Prof. Dr. Volker Turau
	Institute of Telematics
	Hamburg University of Technology

Institute of Autonomous Cyber-Physical Systems

Hamburg University of Technology

TUHH

Acknowledgment

This is the place to thank all the people involved with your thesis / project. Examples would be your family, friends, and of course your supervisor. The acknowledgement will not have any influence on your grade; however, we think it is good style to have an acknowledgement in your thesis.

Abstract

The abstract of your thesis goes here. There may be formal requirements on it that can be found in the corresponding examination guidelines (Prüfungsordnung). If there are none, ask your supervisor. As a rule of thumb, the abstract should be concise and focused. It is not a shortened introduction to your work. We also suggest that—if an abstract is not required—only write one if it is really well done.

Table of Contents

Table of Contents

List of Symbols

$\mathbb{N}, \mathbb{Z}, \mathbb{Q}, \mathbb{R}, \mathbb{C}$	Regular sets of numbers
$\mathcal{NP}, \mathcal{P}$	Complexity classes
$\mathcal{V} = \{v_0, \ldots, v_{N-1}\}$	Set of N nodes v_i belonging to a network with sink v_0
ϱ	Node density, i.e., the average number of nodes within another node's communication range
$(v_i, v_j) \in \mathcal{E}$	Set of bidirectional communication links in the network
$G = (\mathcal{V}, \mathcal{E})$	Graph representation of a wireless sensor network
$\mathcal{N}_i = \{v_j \in \mathcal{V} \mid i \neq j \wedge (v_i, v_j) \in \mathcal{E}\}$	The set of bidirectional communication partners of node v_i
$\mathcal{T} \subseteq \mathcal{E}, \quad |\mathcal{T}| = N-1$	Routing tree rooted in the sink
$\mathcal{T}_i$	Subtree rooted in (and including) node v_i
$\mathcal{C}_i, \quad C_i = |\mathcal{C}_i|$	The set and number of children of node v_i in $\mathcal{T}$
$\mathcal{F} = \{v_i \in \mathcal{V} \mid C_i = \varnothing\}$	Set of leafs in $\mathcal{T}$ and the number of leafs
$\mathcal{F}_i$	Set of leafs in the subtree $\mathcal{T}_i$ of $\mathcal{T}$

List of Symbols

Introduction

1.1 Motivation

1.2 Setup

1.3 Principle

1 Introduction

PN and orthogonal sequences

There are two main goals need to be pursued for receiving higher localization accuracy.

First the code which is used for the under water localization needs its auto-correlation approaching a Dirac impulse. Thus one get advantageous detection capabilities.

The next factor are cross-correlation properties, which should meet certain criteria for improving the separation from other sequences. These will come in handy if noise, reflections and other artifacts emerge.

2.1 Pseudo-random codes

There are a couple of principles to generate PN sequences. Most of these methods use linear feedback shift registers to generate the codes by an initial condition. In this project I will concertize my research on gold codes, Kasami-Codes and the classical m-sequences which are also used for generating gold codes.

M-sequences are binary PN codes, which are generated by linear shift registers with feedback. The sequences are periodic and contain the same number of zeros and ones [PS08]. M-sequences need to fulfill certain criteria. First its length is defined by $N = 2^n - 1$ where n is the maximum degree of the generator polynomial $f(X)$ [SP80].

$$I. \quad |u| = 2^n - 1 = N, \quad \text{from polynomial } h(x) \text{of degree } n \tag{2.1}$$

$$II. \quad \frac{N}{gcd(N,q) = N}, \quad \text{from decimation polynomials } \widetilde{h(x)} \tag{2.2}$$

Second the cross-correlation between m-sequences needs to take three values only, which are $-1, -t(n), t(n) - 2$. With it $t(n)$ is defined by $1 + 2^{\lfloor 0.5(n+2) \rfloor}$ [SP80]. If every pair of m-sequences is a preferred pair, they form a maximal connected set and these sets have a

2 PN and orthogonal sequences
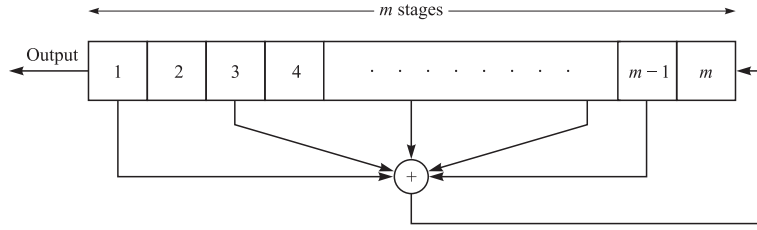
Figure 2.1: Basic structure of an LFSR (Linear Feedback Register). [PS08]
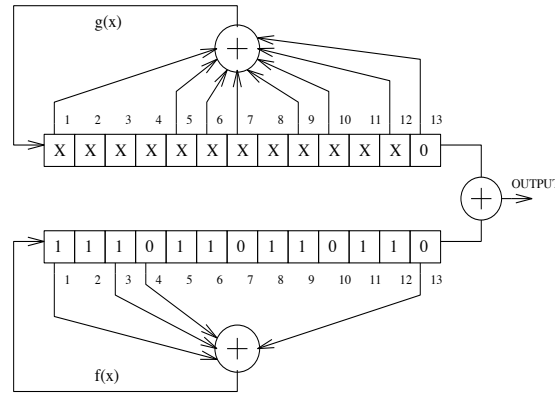
Figure 2.2: LFSR structure of preferred generator polynom of degree 13). [MD]

limited cardinality. Experiments from Gold and Koptizke showed that the number of such connected pairs is limited. Between degrees

[GK65]. To get an m-sequence we need a primitive which needs to fulfill certain citeria.

2.1.1 Gold Codes

Because of not optimal cross-correlation properties m-sequences alone are not applicable for the project. But if these type of codes are combined their correlation qualities can change. Gold Codes are m-sequences where two of them with same length are modulo-2 summed. [PS08]

The gold code which has the highest similarity to Gaussian random variable has a degree of 13. Thus, a generator polynomial pair of $x^13 + x^4 + x^3 + x + 1$ and $x^13 + x^12 + x^10 + x^9 + x^7 + x^6 + x^5 + x + 1$ is chosen [MD] .

$$Gold(u,v) = \{u, v, u \oplus v, u \oplus (v \ll 1), \ldots, u \oplus (v \ll N-1)\} \tag{2.3}$$

2.1.2 Kasami Codes

Kasami sequences are constructed in the same fashion by using m-sequences with the exception that a second sequence, which is used in the modulo sum, is formed by decimating the default m-sequence by $2^{m/2}$ [PS08] [SP80] [PPWW72].

$$w = u[2^{N/2} + 1] = \{u_1, \ldots, u_i, \ldots, u_N | \text{take every } i\text{-th bit of u}\} \tag{2.4}$$

$$Kasami(u) = \{u, u \oplus w, u \oplus (w \ll 1), \ldots, u \oplus (w \ll 2^{N/2} - 2)\} \tag{2.5}$$

2.2 Comparison

For the localization process by orthogonal codes certain criteria needs to be met, which were named in the first chapter. To compare the before explained code types three measures are introduced.

The first one is the peak to side-lobe ratio (PSR) 2.6. This measure is defined by subtracting the mean from the peak of the auto-correlation. Then this value get divided by the standard deviation of the same auto-correlation. A higher PSR value signifies a smaller error between the auto correlation and the perfect Dirac rsulting in better detection capability. The second one is the ratio between the auto-correlation peak and the maximum of the cross-correlation (ACR) 2.7. Here a higher value indicates good code separation qualities. The last one is the correlation coefficient showing if there are similar anchros 2.8, which would be a bad indicator.

$$PSR = \frac{max\{x_{ac}\} - \overline{x_{ac}}}{\sigma_{ac}} \tag{2.6}$$

$$ACR = \frac{max\{x_{ac}\}}{max\{x_{cc}\}} \tag{2.7}$$

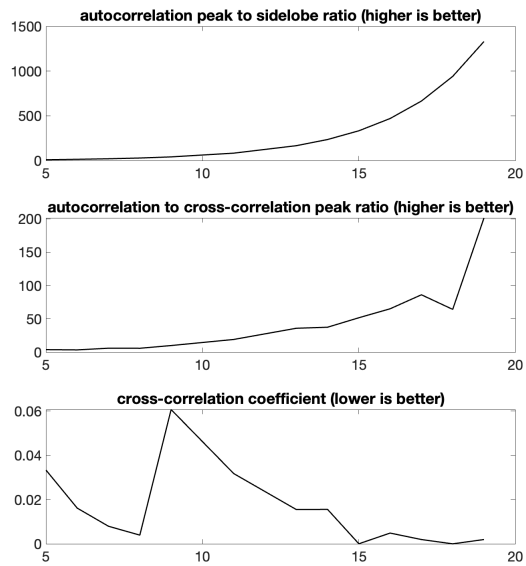$$\rho(a1, a2) = \frac{cov\{a1, a2\}}{\sigma_{a1} \cdot \sigma_{a2}} \tag{2.8}$$
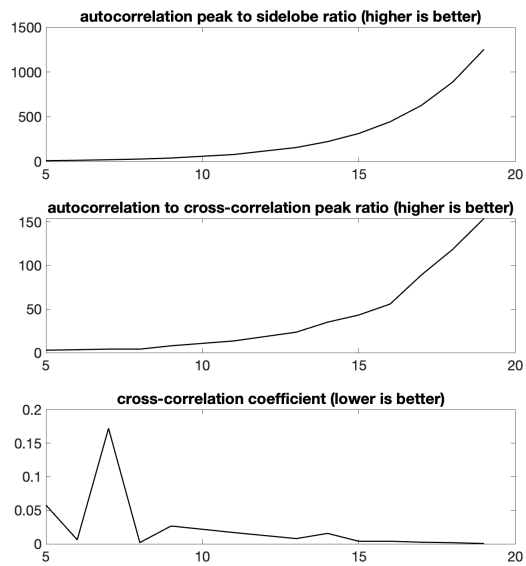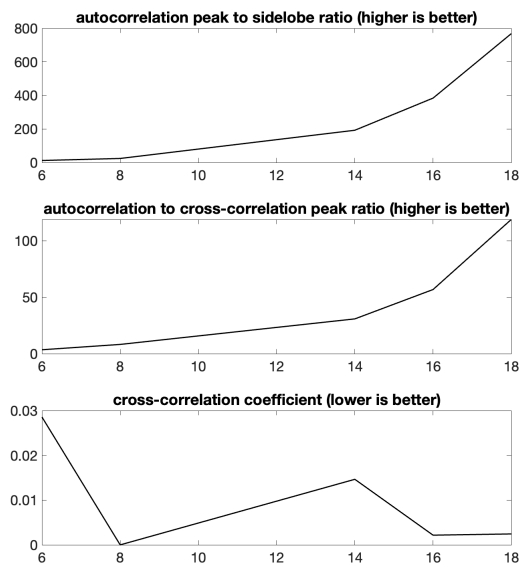
Figure 2.3: Maximum Length Sequence evaluation

Figure 2.4: Gold sequence evaluation

Figure 2.5: Kasami sequence evaluation

2 PN and orthogonal sequences

Bibliography

[GK65]   R. Gold and E. Kopitzke. Study of correlation properties of binary sequences. *Interim Tech. Rep.*, 1–4(1), 1965.

[MD]   Steven J. Merrifield and John C. Devlin. The implementation of a multiplexing gold codegenerator using a xilinx logic cell array. *School of Electronic Engineering, La Trobe University*.

[PPWW72]   William Wesley Peterson, Wesley Peterson, Edward J Weldon, and Edward J Weldon. *Error-correcting codes*. MIT press, 1972.

[PS08]   John G. Proakis and Masoud Salehi. *Digital Communications*. McGraw-Hill Higher Education, New York, 2008.

[SP80]   Dilip V Sarwate and Michael B Pursley. Crosscorrelation properties of pseudorandom and related sequences. *Proceedings of the IEEE*, 68(5):593–619, 1980.

Bibliography

10

Content of the DVD

In this chapter, you should explain the content of your DVD.