Ordered Statistic CFAR Technique – an Overview

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Abstract: *Adaptive radar target detection in a noise or clutter environment is a very important device in each radar receiver. In almost all detection procedures the received echo signal amplitude is simply compared with a certain threshold. The main objective in target detection is to maximize the target detection probability under the constraints of very low and Constant False Alarm Rate (CFAR). The noise and clutter background will be described by a statistical model with e.g. independent and identically Rayleigh or exponentially distributed random variables of known average noise power. But in practical applications this average noise or clutter power is absolutely unknown and can additionally vary over range, time and azimuth angle.*
Therefore this paper describes some of the so-called range CFAR techniques for several different background signal situations in which the average noise power and some other additional statistical parameter are assumed to be unknown. All range CFAR techniques combine therefore an estimation procedure (to get precise or estimated knowledge about the noise power) and a decision step by applying an amplitude threshold to the echo signal amplitude inside the test cell. This general detection scheme has been analysed in many research activities and a large effort has been spent on this topic. The objective of this paper is to give a short characterisation and technical comparison of some of these important range CFAR detection schemes.

1. Introduction

The general task of radar systems used in air or vessel traffic control is to detect all targets inside the observation area and to estimate their range, azimuth angle and radial velocity parameters respectively, even in a multiple target situation. The target detection scheme would be an easy task if the echo signal is observed before an empty or statistically completely known and homogeneous noise or clutter signal background. In this case all received echo signal amplitudes would be compared with a fixed threshold, which is based on the noise and clutter statistic only, and targets are detected in all cases when this threshold is exceeded by the echo signal inside the test cell.

But in real radar applications many different noise and clutter background signal situations can occur. The target echo signal practically always appears before a background signal, which is filled with point, area or even extended clutter and additional superimposed noise signals. Furthermore, the location of this background clutter varies in time, position, and intensity. Clutter is a complicated time and space variant stochastic process in real applications.

All these different conditions call for an adaptive thresholding in the detection procedure. In a first step, the unknown stochastic parameter of a certain background signal will always be estimated by analysing the signal inside a fixed window size, which is oriented in the range direction surrounding the radar test cell. The general detection procedure is shown in a block diagram in Figure 1 where the sliding range window is split into two parts, the leading and lagging part surrounding the test cell. Additionally, some guard cells are introduced to reduce self-interferences in a real target echo situation. All data inside the reference window will be used to estimate the unknown clutter parameter and to calculate the adaptive threshold for target detection.

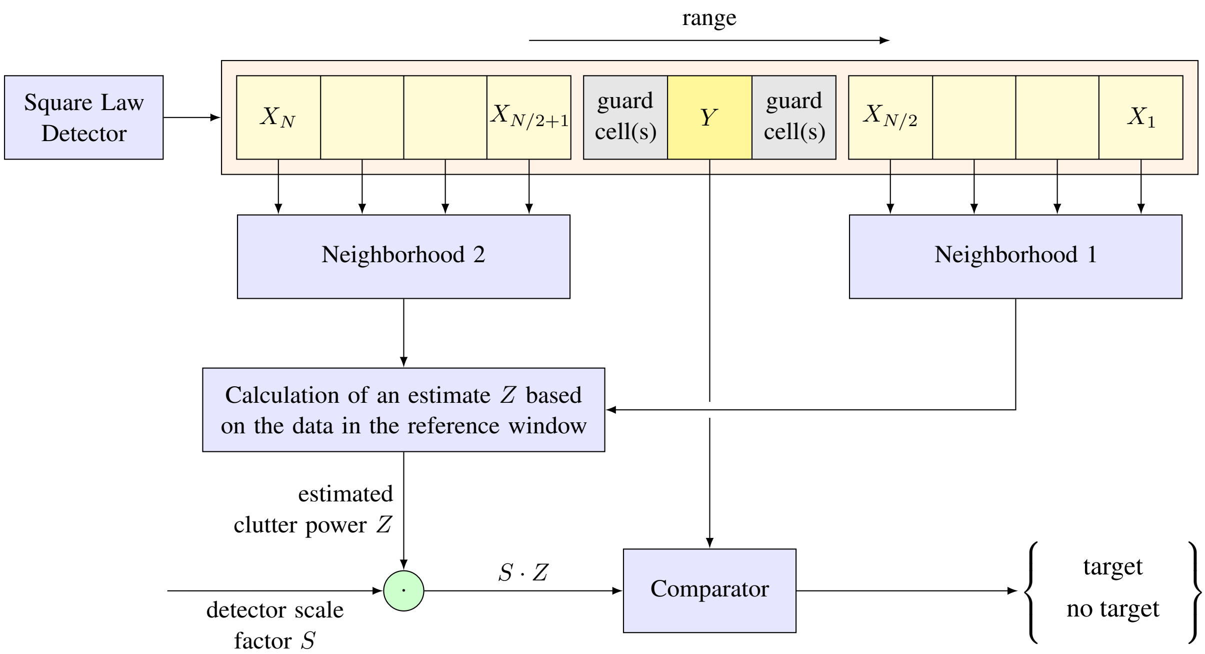


Figure 1: General architecture of range CFAR procedures

All the background signals, undesired as they are from the standpoint of target detection and tracking, are just denoted as "clutter". The detection procedure has to distinguish between useful target echoes and all possible clutter situations. Clutter is not just a uniformly distributed sequence of random variables, but can be caused in practical applications by a number of different physical sources. Therefore, the length N of the sliding window will be chosen already as a compromise based on rough knowledge about the typical clutter extension. To get good estimation performance (low variance) in a homogeneous clutter environment, the window size N should be as large as possible. However, the window length N must be adapted simultaneously to the typical extension of homogeneous clutter. In typical air traffic control radars the number of range cells in the reference windows is e.g. between $N = 16$ and $N = 32$.

2. Radar target detection in noise

In a first model for target detection it is assumed that the background clutter in a square law detector can be described by a statistical model, in which the different range cells inside the sliding window contain statistically independent and identically distributed (iid) exponential random variables. The probability density function (pdf) of exponentially distributed clutter variables are fully described by the following equation:

$$p_0(x) = \begin{cases} \frac{1}{\mu} e^{-\frac{x}{\mu}} & x \geq 0 \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

In this ideal case it is assumed that μ is a known parameter, which describes the expectation value of the exponential distributed random variables. In this case, the false alarm probability P_{fa} depends on the noise and clutter statistic only and a certain threshold $T = S \cdot \mu$ will be calculated analytically as follows:

$$P_{\text{fa}} = \int_{T=S\cdot\mu}^{\infty} p_0(x)\,\mathrm{d}x \tag{2}$$

The scaling factor S of the threshold T can be described for a given false alarm probability P_{fa} in this case analytically by the following equation:

$$S = \ln \frac{1}{P_{\text{fa}}} \tag{3}$$

The non-fluctuating target amplitude statistic can be described by the following pdf:

$$p_1(x) = \begin{cases} \frac{1}{2\sigma_0^2} e^{-\frac{x+c^2}{2\sigma_0^2}} \cdot I_0\left(\frac{\sqrt{x}c}{\sigma_0^2}\right) & x \geq 0 \\ 0 & \text{otherwise} \end{cases}, \qquad c \text{ is the RICE parameter} \tag{4}$$

Therefore the detection probability (P_{d}) for a non-fluctuating target in homogeneous noise is:

$$P_{\text{d}} = \int_{S\cdot\mu}^{\infty} p_1(x)\,\mathrm{d}x \tag{5}$$

The general objectives of all radar detection procedures is to get a constant false alarm rate (CFAR property). However, in real radar applications the average noise and clutter power level μ is absolutely unknown and has to be estimated in the detection procedure first. This is done by the several published CFAR procedures, which will be discussed in this paper.

3. CFAR procedures with sliding window techniques

Each developed and published CFAR technique implicitly refers to a certain background clutter or even target model. Therefore, in the following, these assumptions will be described explicitly and the different considered CFAR procedures will be compared in some typical clutter and target situations inside the reference window. The amplitude in each individual range cell is tested. The amplitudes in the leading and lagging part of the reference cells are used in a signal processing procedure to estimate the unknown statistical parameters of the clutter background signal. Based on these estimated parameters, the detection threshold T will be calculated.

To demonstrate the behaviour of some different and specifically designed range CFAR procedures, in total four different, however characteristic clutter, noise and target situations are considered. A pure homogeneous noise background signal with iid random variables, a local clutter area over 10 adjacent range cells superimposed with noise, a single target and a two target situation with $30\,\mathrm{dB}$ and $35\,\mathrm{dB}$ SNR, respectively. Figure 2 shows these four different signal situations (blue line) in 100 adjacent range gates. It is the task for any CFAR procedure to detect all targets and suppress all local clutter in a self-organized way. All considered range CFAR procedures are optimised in accordance with the pure noise situation. The estimation procedure Z and the related scaling factor S define the detection threshold T:

$$T = S \cdot Z \tag{6}$$

To analyse the performance for different CFAR procedures, the average detection threshold (ADT) has been designed in [3], which is a useful single value parameter for system comparison in pure noise situations. The ADT is the expectation of the detection threshold T normalised by the average noise power μ.

$$\mathrm{ADT} = \frac{1}{\mu} \cdot \mathrm{E}\left[T\right] = \frac{1}{\mu} \cdot \mathrm{E}\left[S \cdot Z\right] = \frac{S}{\mu} \cdot \mathrm{E}\left[Z\right] \tag{7}$$

3.1. Cell averaging CA-CFAR

In this first signal model it is assumed that the clutter and noise background can be described by statistically iid exponential random variables with a single exception: the average clutter plus noise power level is unknown [1, 2]. The optimised signal processing technique in this situation and from a statistical point of view is to calculate an estimation of the clutter power level simply by applying the arithmetic mean to the received amplitudes inside the considered reference window with N range gates.

$$Z_{\mathrm{CA}} = \frac{1}{N} \sum_{i=1}^{N} X_i \tag{8}$$

The arithmetic mean has excellent estimation performance. The estimation result is unbiased, which means

$$\mathrm{E}\left[Z_{\mathrm{CA}}\right] = \mathrm{E}\left[\frac{1}{N} \sum_{i=1}^{N} X_i\right] = \frac{1}{N} \sum_{i=1}^{N} \mathrm{E}\left[X_i\right] = \mu \tag{9}$$

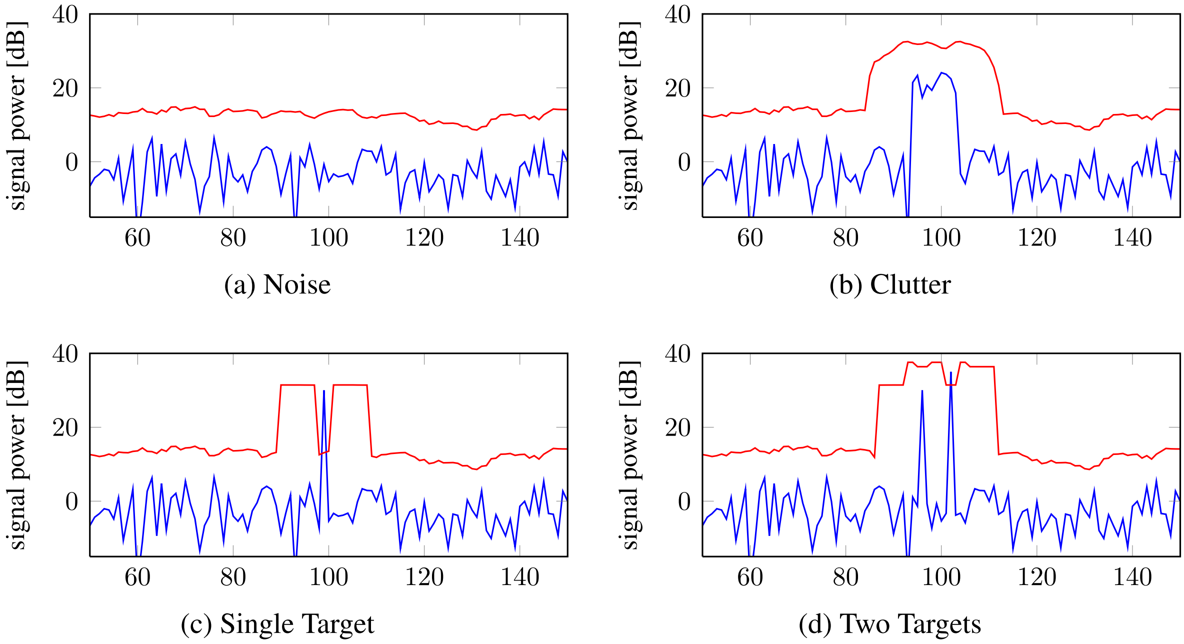
and additionally it shows a minimum estimation variance

$$\mathrm{Var}\left[Z_{\mathrm{CA}}\right] = \mathrm{Var}\left[\frac{1}{N} \sum_{i=1}^{N} X_i\right] = \frac{1}{N} \sum_{i=1}^{N} \mathrm{Var}\left[X_i\right] = \frac{1}{N}\mu^2 \tag{10}$$

If this estimation procedure is applied to the random variables inside the range window, this CFAR technique is called "cell averaging" CA-CFAR. The statistical performance is excellent, if the assumptions of homogeneous clutter inside the reference window are fulfilled in the statistical model and in the real world application. The estimation performance is increased with increasing window length N. The first analysed range CFAR procedure is the CA-CFAR with a reference window length of $N = 16$, the noise level estimation result Z_{CA}, the scaling factor $S_{\text{CA}} = 21.94$ [3], the detection threshold T_{CA}:

$$T_{\text{CA}} = S_{\text{CA}} \cdot Z_{\text{CA}}, \qquad \text{ADT}_{\text{CA}} = \frac{1}{\mu} \cdot \mathrm{E}\left[T_{\text{CA}}\right] = \frac{1}{\mu} \cdot \mathrm{E}\left[S_{\text{CA}} \cdot Z_{\text{CA}}\right] \tag{11}$$

and a false alarm rate of P_{fa} of 10^{-6}. The resulting detection threshold T_{CA} (red line) is shown in Figure 2 for the four considered signal situations.



(a) Noise

(b) Clutter

(c) Single Target

(d) Two Targets

Figure 2: CA-CFAR ($N = 16$), $P_{\text{fa}} = 10^{-6}$

The choice of the reference window length N is already a critical design parameter for the CA-CFAR. From a pure homogeneous clutter point of view, the window length N should be as large as possible. However, the window length N influences the CFAR behaviour in the other signal and noise situations significantly.

This homogeneous noise model with unknown noise power was the motivation to develop the CA-CFAR. In this case a very good detection threshold based on CA-CFAR noise power estimation can be observed in Figure 2(a). Furthermore, the CA-CFAR detection procedure adapts quite well in the local clutter situations, however, with some losses (increased P_{fa}) at the clutter edges, see Figure 2(b). The most important behaviour of CA-CFAR procedure is observed in single and in multiple target situations. A single target will be detected, see Figure 2(c), however, strong masking effects are observed in multiple target situations, see Figure 2(d). This CA-CFAR behaviour is not acceptable in practical applications.

3.2. OS-CFAR

The CA-CFAR detection procedure behaves very sensitively in multiple target situations and shows poor target detection performance in such situations. This observation has been described in [3]. From an estimation point of view the estimation results become poor if the clutter is not homogeneous inside the reference window. This is a real challenge for the adaptive detection procedure. When the drawbacks and limitations of CA-CFAR had been analysed, a totally different noise power estimation procedures was designed which is based on an ordered statistic (OS). While in the CA-CFAR and the arithmetic mean procedure all signal amplitudes inside the reference window contribute to the detection threshold, in the OS-CFAR case only a single amplitude will be selected.

It is well known from general signal processing topics that estimation procedures are much more robust if they are based on ordered statistics. This means all N amplitudes inside the reference window are sorted according to increasing magnitude, resulting in the ordered sequence

$$X_{(1)} \leq \ldots \leq X_{(k)} \leq \ldots \leq X_{(N)} \tag{12}$$

The general idea of an ordered statistic CFAR procedure is therefore technically simple. To estimate the average noise and clutter power in a homogeneous signal situation, see Figure 3(a), the arithmetic mean Z_{CA} is replaced by a single rank k of the ordered statistic (12).

$$Z_{\mathrm{OS}} = X_{(k)} \tag{13}$$

The OS-CFAR decision threshold T_{OS} is given by the product of the estimation Z_{OS} and the corresponding scaling factor S_{OS}

$$T_{\mathrm{OS}} = S_{\mathrm{OS}} \cdot Z_{\mathrm{OS}}, \qquad \mathrm{ADT}_{\mathrm{OS}} = \frac{1}{\mu} \cdot \mathrm{E}\left[T_{\mathrm{OS}}\right] = \frac{1}{\mu} \cdot \mathrm{E}\left[S_{\mathrm{OS}} \cdot Z_{\mathrm{OS}}\right] \tag{14}$$

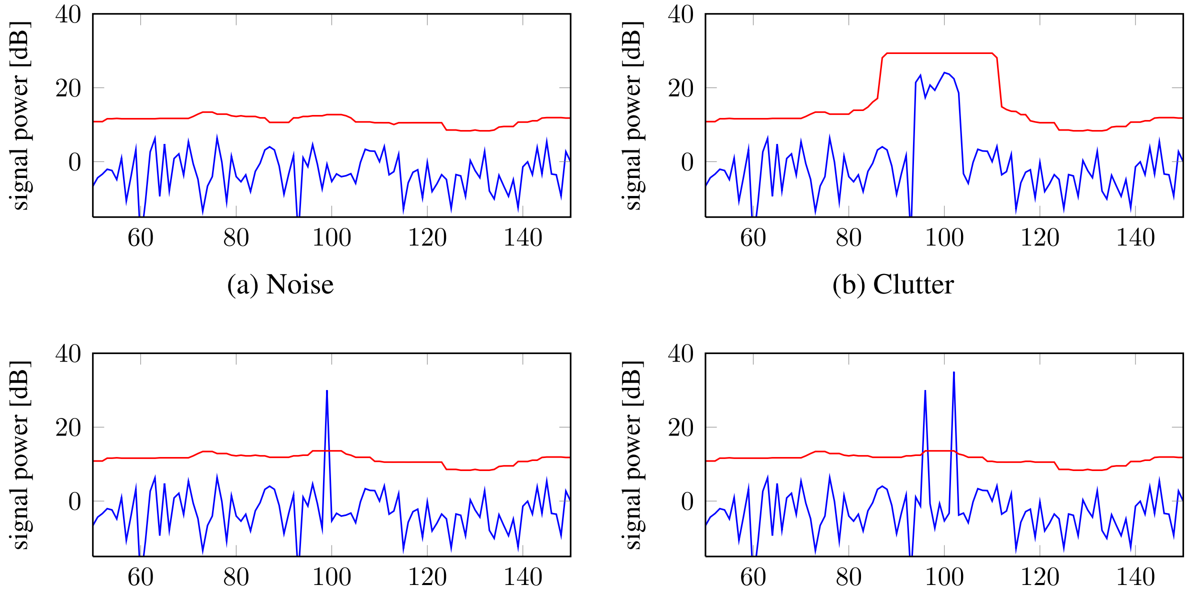
With this OS-CFAR procedure it was shown that very good performance can be observed in all four considered signal situations, see Figure 3. Especially the masking effects of the CA-CFAR in two target situations could be avoided completely. Even weak targets in the neighbourhood of strong targets are not any longer masked in almost all cases.

The OS-CFAR is not any longer based on the assumption of homogeneous clutter inside the reference window. The choice of the reference window length N is in the OS-CFAR case less important compared to the CA-CFAR case. Therefore the window length N can be extended in the OS-CFAR case to get better clutter estimation results in homogeneous signal situations. For comparison reasons, the OS-CFAR is based on a reference window length of $N = 32$ while the CA-CFAR uses a window length of $N = 16$. With these features the OS-CFAR shows already better detection performance in the pure homogeneous noise and clutter situation. The CA- and

OS-CFAR procedures are quantitatively compared in a pure noise situation based on the ratio of single value ADTs (see [3]) measured in dB

$$\Delta_{\text{dB}} = 10 \cdot \log \frac{\text{ADT}_{\text{OS}}}{\text{ADT}_{\text{CA}}} = 10 \cdot \log \frac{19.3}{21.94} = -0.56\,\text{dB} \tag{15}$$

Furthermore the "ordered statistic" OS-CFAR shows clearly a much better adaptation capability and detection performance compared to CA-CFAR in all other clutter and considered target situations. Additionally, the detection threshold T_{OS} follows the clutter contour with a certain safety distance. In both target situations the detection threshold is more or less unchanged compared to a pure noise situation.



(a) Noise (b) Clutter

(c) Single Target (d) Two Targets

Figure 3: OS-CFAR ($N = 32, k = 24$), $P_{\text{fa}} = 10^{-6}$

The robustness of OS-CFAR in all the different signal situations and applications is important to notice. The OS-CFAR can even be applied to linear and to square law detectors simultaneously without any further adaptation steps while the CA-CFAR is fully based on the square law detector. In the OS-CFAR case it is even not necessary to exclude the test and guard cells from the reference window.

3.3. Some other CFAR procedures

Several papers have been published which propose some extensions of CA-CFAR and OS-CFAR procedures or applications in many different background situations. In [4], a CFAR procedure has been proposed which is based on the CA-CFAR, however with an additionally censoring procedure to reduce some masking effects in multiple target situations. In [5], three generalised modified OS-CFAR techniques and an automatic censoring are discussed. In [6, 7], the classical OS-CFAR has been analysed in detail and applied to WEIBULL clutter background signals. In [8], the design of a modified CA-CFAR is described and analysed in multiple target situations. In [9], two alternative OS-CFAR systems are considered and compared. In [10], CA-CFAR is applied in spatially correlated K-distributed clutter which can be used as a model for sea clutter. In [11], the OS-CFAR is applied and analysed in non-uniform clutter. This is just a short selection of a large number of other contributions to the range CFAR topic.

In [12], a totally new metric for space-time covariance matrices has been developed and applied to the information geometry topic. This is a very promising new research area for radar signal processing and for space time adaptive processing (STAP). It has been found that the FRÉCHET median procedure is much more robust compared to the centre of mass technology applied to the metric of covariance matrices. That means the general idea of OS-CFAR technique is in this case shifted to the information geometry subject which is a robust version to new metric considerations in the field of information geometry. The median value can even be replaced by a single value of the ordered sequence.

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