Performance Comparison of TOA and TDOA Based Location Estimation Algorithms in LOS Environment

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Abstract—**In this paper, various positioning algorithms for range-based TOA and TDOA localization have been analyzed, which include the Analytical method, Least Square method, Approximate Maximum Likelihood method, Taylor Series method, Two-Stage Maximum Likelihood method and Genetic Algorithm. The assumed scenario is an overdetermined system in a 3D space under Line of Sight (LOS) situation and a number of sensor nodes placed arbitrarily across this area. The performance of the algorithms has been compared in the assumed scenario. Both the average error and the failure rate have been investigated in terms of the number of reference nodes and the root mean squared error (RMSE) of the range estimation.**

Index Terms—**Localization, Location Estimation, TDOA, TOA, UWB**

I. Introduction

LOCALIZATION in distributed UWB sensor networks is an important area that is attracted significant research interest. It is required for many sensor network applications, such as the environment imaging, moving objects detection and tracking and so on. For the localization in distributed sensor networks, firstly, measurement is needed for obtaining the channel impulse response (CIR) by using the radio signals. Then, proper parameters of the signal, such as angle, time delay, or amplitude, are acquired by the parameter extraction. The parameters are used for data fusion to estimate the location of the target node by signal processing [1] [2].

Ultra-Wideband (UWB) sensor networks promise interesting perspectives for position location in the short-range environment [17]. There are various existing location estimation approaches by making use of UWB technology [4] - [11], such as received signal strength intensity (RSSI), angle of arrival (AOA), time of arrival (TOA) and time difference of arrival (TDOA). Among these techniques, the ranged-based schemes, TOA and TDOA, are proved to have a very good accuracy [12] due to the high time resolution (large bandwidth) of the UWB signals.

Manuscript received October 2, 2007. This work was supported in part by the European integrated project PULSERS-II, and project "UKoLoS" which is supported by DFG (German Research Foundation).

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For the range-based schemes, the parameter extraction step is to gain ranges between the sensor nodes, so it can be called range estimation. The data fusion is the location estimation step.

Many range-based location estimation methods with different complexity and restrictions have been proposed, such as Analytical method (AM), Least Squares method (LS), Taylor Series method (TS), Approximate Maximum Likelihood method (AML), Two-Stage Maximum Likelihood method (TSML) and Genetic Algorithm (GA).

The assumed scenario for localization in this paper is an overdetermined system where the number of the measured ranges is greater than the number of the unknowns. In other words, it consists of one target node, more than three known reference nodes in TOA based scenario, and more than four in TDOA based scenario. The sensor nodes are placed arbitrarily inside a cube (3D space) under Line of Sight (LOS) situation. The position of the target node is localized by the noisy range estimation.

The remainder of the paper is organized as follows: Section II analyzes various location estimation algorithms based on TOA. Section III describes the methods based on TDOA. In section IV, performance is evaluated by computer simulation in the assumed scenario. Finally, section V gives the conclusion.

II. TOA Based Location Estimation Algorithms

In this section, we analyze some typical TOA based location estimation algorithms.

The position of the target node is determined as the intersection of all the spheres, of which centers are the coordinates of the reference nodes and radiuses are the ranges between the reference nodes and the target node. The spheres can be described as below,

$$(x-x_i)^2+(y-y_i)^2+(z-z_i)^2=m_i^2,(i=1,2,...,n) \quad (1)$$

where (x_i, y_i, z_i) and m_i $(i=1,2,\ldots,n)$ are the known coordinates of the reference nodes and the range estimations respectively. n is the number of reference nodes. The coordinates of the target node to be estimated are referred to as (x, y, z).

The accuracy of range estimation is affected by noise and the multipath components, thus the spheres will not always intersect at one single point. The goal of the location estimation is to find out the closest coordinates to the actual position.

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A. Analytical Method

A straightforward method for determining the target node position is to solve the nonlinear equations directly [3] [13].

If there are only three reference nodes, it is a set of three equations with three unknowns. Substituting

$$x' = x - x_1, y' = y - y_1, z' = z - z_1 \tag{2}$$

and

$$x_i' = x_i - x_1 \qquad (i = 2,3) \tag{3}$$

into (1) and subtracting the first one (i=1) successively from it for i=2,3 results in an equation set in the matrix form as

$$\begin{bmatrix} x_2' & y_2' & z_2' \\ x_3' & y_3' & z_3' \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \frac{1}{2} \begin{bmatrix} m_1^2 - m_2^2 + x_2'^2 + y_2'^2 + z_2'^2 \\ m_1^2 - m_2^2 + x_2'^2 + y_2'^2 + z_2'^2 \end{bmatrix} \tag{4}$$

There are two equations with three unknowns, thus we make one of the unknowns, such as x', a parameter, and the other two as functions of this parameter,

$$y' = f_1(x'), z' = f_2(x') \tag{5}$$

Then we put (5) into (1) with i=1 obtaining a quadratic equation

$$x'^2 + f_1^2(x') + f_2^2(x') = m_1^2 \tag{6}$$

After we find the solution x', the coordinates of the target node can be derived from (5) and (2).

There are three possibilities of the solution to equation (6). If there is only one solution, we can get the coordinates of the target node directly. If there are two solutions, we need one more reference node to compare the distance between this node and the two solutions, and select the one that is close to the measured distance. If there is no solution, it is caused by the range estimation error.

The assumed scenario is an overdetermined system of equations with more than three equations in (1). To find location estimate, we can use the method of combination of analytical solution. Its main idea is to solve all combinations equation including three equations. The final location estimation is obtained by averaging of all the results.

B. Least Squares Method

From (1), subtracting the first one (i=1) from other equations results in an equation set in the matrix as

$$2\begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ & \cdots & \\ x_n - x_1 & y_n - y_1 & z_n - z_1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} m_1^2 - m_2^2 + k_2 - k_1 \\ m_1^2 - m_3^2 + k_3 - k_1 \\ \cdots \\ m_1^2 - m_n^2 + k_n - k_1 \end{bmatrix} \tag{7}$$

where

$$k_i = x_i^2 + y_i^2 + z_i^2, (i = 1,2,...,n) \tag{8}$$

It can be denoted as

$$2\mathbf{At} = \mathbf{b} \tag{9}$$

where

$$\mathbf{t} = [x \quad y \quad z]^T,$$

$$\mathbf{A} = \begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ & \cdots & \\ x_n - x_1 & y_n - y_1 & z_n - z_1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} m_1^2 - m_2^2 + k_2 - k_1 \\ m_1^2 - m_3^2 + k_3 - k_1 \\ \cdots \\ m_1^2 - m_n^2 + k_n - k_1 \end{bmatrix} \tag{10}$$

The solution can be obtained by using the least square method [13],

$$\mathbf{t} = \tfrac{1}{2}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T\mathbf{b}. \tag{11}$$

C. Taylor Series Method

We define a function, describing the actual ranges as shown below [12] [14]

$$\begin{aligned} f_i(x,y,z) &= \sqrt{(x-x_i)^2 + (y-y_i)^2 + (z-z_i)^2}, \\ &= m_i + \varepsilon_i \qquad (i = 1,2,...,n) \end{aligned} \tag{12}$$

where m_i is the TOA estimation at reference node i, and ε_i is the corresponding range estimation error and the elements of it are independent and zero-mean Gaussian random variables with covariance matrix

$$\mathbf{Q} = E\{\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T\} = diag[\sigma^2 \quad \ldots \quad \sigma^2]. \tag{13}$$

in which σ is the range estimation error.

Given an initial estimation of (x, y, z) is (x_v, y_v, z_v), then,

$$x = x_v + \delta_x, y = y_v + \delta_y, z = z_v + \delta_z,$$

where δ_x, δ_y and δ_z are the location estimation errors to be determined.

Expanding (12) into Taylor series and retaining the first two terms produce

$$f_{i,v} + a_{i,1}\delta_x + a_{i,2}\delta_y + a_{i,3}\delta_z \approx m_i + \varepsilon_i, \tag{14}$$

where

$$f_{i,v} = f_i(x_v, y_v, z_v),\ a_{i,1} = \left.\frac{\partial f_i}{\partial x}\right|_{x_v, y_v, z_v} = \frac{x_v - x_i}{r_i},$$

$$a_{i,2} = \left.\frac{\partial f_i}{\partial y}\right|_{x_v, y_v, z_v} = \frac{y_v - y_i}{r_i},\ a_{i,3} = \left.\frac{\partial f_i}{\partial z}\right|_{x_v, y_v, z_v} = \frac{z_v - z_i}{r_i},$$

in which

$$r_i = \sqrt{(x_v - x_i)^2 + (y_v - y_i)^2 + (z_v - z_i)^2}.$$

We can rewrite (14) as

$$\mathbf{A\delta} = \mathbf{D} + \mathbf{e} \tag{15}$$

where

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ & \cdots & \\ a_{n,1} & a_{n,2} & a_{n,3} \end{bmatrix},$$

$$\boldsymbol{\delta} = \begin{bmatrix}\delta_x & \delta_y & \delta_z\end{bmatrix}^T, \mathbf{e} = [\varepsilon_1, \varepsilon_2, \cdots, \varepsilon_n]^T,$$

$$\mathbf{D} = [m_1 - f_{1,v}, m_2 - f_{2,v}, \cdots, m_n - f_{n,v}]^T.$$

The weight least square estimation of (15) is

$$\boldsymbol{\delta} = \left(\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{A}\right)^{-1}\mathbf{A}^{\mathrm{T}}\mathbf{Q}^{-1}\mathbf{D} \tag{16}$$

From the initial position guess (x_v, y_v, z_v) and $\boldsymbol{\delta}$ computed from (16), the location estimation can be updated according to

$$x_v = x_v + \delta_x, y_v = y_v + \delta_y, z_v = z_v + \delta_z \tag{17}$$

The location estimation can be continually refined by iterating the above procedure.

D. Approximate Maximum Likelihood Method

Let the measured TOA ranges be

$$\mathbf{m} = [m_1 \quad m_2 \quad \dots \quad m_n]^T = \mathbf{r} + \boldsymbol{\varepsilon} \tag{18}$$

where $\boldsymbol{\varepsilon}$ is the range estimation errors with covariance matrix $\mathbf{Q}$ which is denoted in (13),

$$\mathbf{r} = [r_1 \quad r_2 \quad \dots \quad r_n]^T = \mathbf{r}(\boldsymbol{\theta})$$

is the vector of true range and $\boldsymbol{\theta}$ are the coordinates of the target node, $\boldsymbol{\theta}=[x\ y\ z]^T$.

The probability density function [18] of $\mathbf{m}$ given $\boldsymbol{\theta}$ is

$$f(\mathbf{m}/\boldsymbol{\theta}) = (2\pi)^{-n/2}(\det \mathbf{Q})^{-1/2}\exp\{-J/2\} \tag{19}$$

where

$$J = [\mathbf{m} - \mathbf{r}(\boldsymbol{\theta})]^T \mathbf{Q}^{-1}[\mathbf{m} - \mathbf{r}(\boldsymbol{\theta})] \tag{20}$$

The ML estimate is the $\boldsymbol{\theta}$ that minimizes J [15].

From $\frac{\partial J(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = 0$, we get

$$2\begin{bmatrix} \sum g_i x_i & \sum g_i y_i & \sum g_i z_i \\ \sum h_i x_i & \sum h_i y_i & \sum h_i z_i \\ \sum f_i x_i & \sum f_i y_i & \sum f_i z_i \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \sum g_i(s + k_i - m_i) \\ \sum h_i(s + k_i - m_i) \\ \sum f_i(s + k_i - m_i) \end{bmatrix} \tag{21}$$

where

$$g_i = \frac{x - x_i}{r_i(r_i + m_i)}, h_i = \frac{y - y_i}{r_i(r_i + m_i)}, f_i = \frac{z - z_i}{r_i(r_i + m_i)},$$

$$s = x^2 + y^2 + z^2 \tag{22}$$

Given an initial guess of (x, y, z), firstly we compute g_i, h_i and f_i, and also the least square estimation of (x, y, z) from (21) in terms of s. Putting them into (22) produces a quadratic equation in s, allowing for a new estimation.

By repeating the above procedure with the new (x, y, z) estimates for several times, we can get an accurate result by selecting the one which gives the smallest J.

E. Two-Stage Maximum Likelihood Method

We can rewrite (1) as

$$(m_i^2 - k_i) - [-2(x_i x + y_i y + z_i z) + s] = 0 \tag{23}$$

where k_i and s are defined in (8) and (22).

With range estimation error, the left of (23) is not always equal to zero. Let $\mathbf{z}=[x\ y\ z\ s]^T$ be an unknown vector [16], a cost function derived from (23) is

$$\varphi = \mathbf{h} - \mathbf{G}\mathbf{z} \tag{24}$$

Where

$$\mathbf{h} = \begin{bmatrix} m_1^2 - k_1 \\ m_2^2 - k_2 \\ \dots \\ m_n^2 - k_n \end{bmatrix}, \mathbf{G} = -2\begin{bmatrix} x_1 & y_1 & z_1 & -0.5 \\ x_2 & y_2 & z_2 & -0.5 \\ & \dots & & \\ x_n & y_n & z_n & -0.5 \end{bmatrix}.$$

From (18), φ is found to be [16],

$$\varphi = \mathbf{2B}\boldsymbol{\varepsilon} + \boldsymbol{\varepsilon} \cdot \boldsymbol{\varepsilon} \tag{25}$$

where

$$\mathbf{B} = diag\{r_1 \quad r_2 \quad \dots \quad r_n\}$$

When ignoring the second term on the right of (25), φ becomes a Gaussian random vector with covariance matrix given by

$$\Phi = E[\varphi\varphi^T] = 4\mathbf{BQB} \tag{26}$$

Then the ML estimation of $\mathbf{z}$ from (24) is

$$\begin{aligned} \mathbf{z} &= \arg\min\{(\mathbf{h} - \mathbf{Gz})^T \Phi^{-1}(\mathbf{h} - \mathbf{Gz})\} \\ &= (\mathbf{G}^T\Phi^{-1}\mathbf{G})^{-1}\mathbf{G}^T\Phi^{-1}\mathbf{h} \end{aligned} \tag{27}$$

However, Φ is not known in practice as $\mathbf{B}$ contains the true distance, we can use m_i as the true to compute $\mathbf{B}$ and then get an initial $\mathbf{z}$. The covariance matrix of $\mathbf{z}$ is

$$\operatorname{cov}(\mathbf{z}) = E[\Delta\mathbf{z}\Delta\mathbf{z}^T] = (\mathbf{G}^T\Phi^{-1}\mathbf{G})^{-1} \tag{28}$$

where

$$\Delta\mathbf{z} = (\mathbf{G}^T\Phi^{-1}\mathbf{G})^{-1}\mathbf{G}^T\Phi^{-1}\mathbf{B}\boldsymbol{\varepsilon}$$

During the derivation above, we regard x, y, z and s as independent but they are related by (22). The elements of $\mathbf{z}$ can be expressed as

$$z_1 = x + e_1,\ z_2 = y + e_2,\ z_3 = z + e_3,\ z_4 = s + e_4, \tag{29}$$

where e_1, e_2, e_3 and e_4 are estimation errors of $\mathbf{z}$. Squaring the first three elements gives another cost function

$$\varphi' = \mathbf{h}' - \mathbf{G}'\mathbf{z}' \tag{30}$$

where φ' is a vector denoting the errors in $\mathbf{z}$ and

$$\mathbf{h}' = \begin{bmatrix} z_1^2 \\ z_2^2 \\ z_3^2 \\ z_4 \end{bmatrix}, \mathbf{G}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \mathbf{z}' = \begin{bmatrix} x^2 \\ y^2 \\ z^2 \end{bmatrix},$$

Substituting (29) into (30) gives

$$\begin{aligned} &\varphi'_1 = 2xe_1 + e_1^2 \approx 2xe_1; \quad \varphi'_2 = 2ye_2 + e_2^2 \approx 2ye_2; \\ &\varphi'_3 = 2ze_3 + e_3^2 \approx 2ze_3; \quad \varphi'_4 = e_4. \end{aligned} \tag{31}$$

in which the approximation is valid as e_1, e_2 and e_3 are small.

The covariance matrix of φ' is given by

$$\Phi' = E[\varphi'\varphi'^T] = 4\mathbf{B}'\operatorname{cov}(\mathbf{z})\mathbf{B}' \tag{32}$$

where

$$\mathbf{B}' = diag\{x \quad y \quad z \quad 0.5\}$$

Then the ML estimate of $\mathbf{z}'$ is

$$\mathbf{z}' = (\mathbf{G}'^T\Phi'^{-1}\mathbf{G}')^{-1}\mathbf{G}'^T\Phi'^{-1}\mathbf{h}' \tag{33}$$

The final estimation is then obtained by

$$[x \quad y \quad z]^T = \pm\sqrt{\mathbf{z}'} \tag{34}$$

The proper solution is selected to be the one that lies in the monitored area. If there are more than one solutions lying in the area, we select the one that gives the smallest function value that is shown below,

$$f = \sum_{i=1}^{n}\left[\sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} - m_i\right]^2 \tag{35}$$

F. Genetic Algorithm

The Genetic Algorithm (GA) is a searching process based on the law of natural selection and genetics [19] [20]. We can apply it into the positioning problem.

The unknown variables of this problem are the coordinates of the target node (x, y, z). If we obtain an estimate of the target node, there are some distance differences between the measurement ranges and the ranges by using the estimated coordinates. The best estimation should offer the smallest distance difference. Thus, a simple fitness function can be given by using (35), which describes the quadratic sum of the distance differences. The initial population can be derived by an initial estimation of the coordinates of the target node.

Usually, GA consists of three operations: reproduction, crossover, and mutation. By repeating several times, we can get the generation, with a smaller value of f than the initial estimation.

III. TDOA Based Location Estimation Algorithms

In this section, we analyze the location estimation algorithms that were motioned in section II however based on TDOA.

Without loss of generality, let all the TDOA be measured with respect to the first reference node, therefore

$$m_{i,1} = r_i - r_1, (i = 2,3,...n) \tag{36}$$

where $m_{i,1}$ (i=2,3,…,n) are the TDOA range estimations. r_i ($i=1,2,...,n$) are the unknown parameters of true distances between the reference nodes and the target node. n is the number of the reference nodes.

We denote (x_i, y_i, z_i) as the known coordinates of the reference nodes. The coordinates of the target node to be determined are referred to as (x, y, z). Thus, (1) can be rewritten as

$$\begin{cases} (x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2 = r_1^2 \\ (x-x_2)^2 + (y-y_2)^2 + (z-z_2)^2 = (r_1 + m_{2,1})^2 \\ \qquad \cdots \\ (x-x_n)^2 + (y-y_n)^2 + (z-z_n)^2 = (r_1 + m_{n,1})^2 \end{cases} \tag{37}$$

where there are four unknowns, x, y, z and r_1.

Since the methods in this section contain almost the same idea with them that based on TOA, we only give a rough introduction of them here. The detailed description of them can be referred to the corresponding references that motioned blow.

A. Analytical Method

The minimum number of reference nodes is four because there are four unknowns, (x, y, z) and r_1, in (37).

Substituting (2) and (3) (but here, i=2,3,4) into (37) and subtracting the first one (i=1) from it for i=2,3,4 results in an equation set in the matrix form as

$$\begin{bmatrix} x_2' & y_2' & z_2' \\ x_3' & y_3' & z_3' \\ x_4' & y_4' & z_4' \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \\ r_1 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} x_2'^2 + y_2'^2 + z_2'^2 - m_{2,1}^2 \\ x_2'^2 + y_2'^2 + z_2'^2 - m_{3,1}^2 \\ x_2'^2 + y_2'^2 + z_2'^2 - m_{4,1}^2 \end{bmatrix} \tag{38}$$

Where

$$r_1 = x'^2 + y'^2 + z'^2 \tag{39}$$

We make one of the unknowns, such as r_1, a parameter, and the other three as functions of this parameter. Putting them into (39), we get a quadratic equation in terms of r_1. The solution of the quadratic equation will lead to the solution of the coordinates of the target node. We can use the combination of this method for overdetermined system of equations with combinations of four equations.

B. Least Squares Method

Subtracting the first one (i=1) from other equations of (37) results in

$$2\begin{bmatrix} x_2' & y_2' & z_2' \\ x_3' & y_3' & z_3' \\ & \cdots & \\ x_n' & y_n' & z_n' \end{bmatrix} \begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} k_2' - m_{2,1}^2 \\ k_3' - m_{3,1}^2 \\ \cdots \\ k_n' - m_{n,1}^2 \end{bmatrix} + r_1 \begin{bmatrix} -m_{21} \\ -m_{31} \\ \cdots \\ -m_{n1} \end{bmatrix} \tag{40}$$

where (x', y', z'), (x_i', y_i', z_i') are defined in (2) and (3), and

$$k_i' = x_i'^2 + y_i'^2 + z_i'^2, (i = 2,...,n) \tag{41}$$

It can be described by

$$2\mathbf{At} = \mathbf{c} + r_1\mathbf{d} \tag{42}$$

where A is defined in (10) and

$$\mathbf{t} = [x' \quad y' \quad z']^T,$$

$$\mathbf{c} = \begin{bmatrix} k_2' - m_{2,1}^2 \\ k_3' - m_{3,1}^2 \\ \cdots \\ k_n' - m_{n,1}^2 \end{bmatrix}, \mathbf{d} = \begin{bmatrix} -m_{21} \\ -m_{31} \\ \cdots \\ -m_{n1} \end{bmatrix}$$

The least square solution [13] is

$$\mathbf{t} = \tfrac{1}{2}\left(\mathbf{A}^T\mathbf{A}\right)^{-1}\mathbf{A}^T(\mathbf{c} + r_1\mathbf{d}) \tag{43}$$

Then the solution, which contains parameter r_1, forms a quadratic equation. The solution of r_1 delivers the finial result.

C. Taylor Series Method

The idea is the same as TOA, but the function is defined as [12] [14]

$$\begin{aligned} f_i(x,y,z) &= \sqrt{(x-x_{i+1})^2 + (y-y_{i+1})^2 + (z-z_{i+1})^2} \\ &\quad - \sqrt{(x-x_1)^2 + (y-y_1)^2 + (z-z_1)^2} \\ &= m_{i+1,1} + \varepsilon_{i+1,1} (i = 1,2,...,n-1) \end{aligned} \tag{44}$$

where $m_{i+1,1}$ is the TDOA estimate of the $(i+1)$th reference node with the first one and $\varepsilon_{i+1,1}$ is the corresponding range difference estimation error with covariance matrix $\mathbf{Q}$.

Expanding (44) into Taylor series by using an initial estimation (x_v, y_v, z_v) and retaining the first two terms produce

$$f_{i,v} + a_{i,1}\delta_x + a_{i,2}\delta_y + a_{i,3}\delta_z \approx m_{i+1,1} + \varepsilon_{i+1,1} \tag{45}$$

where δ_x, δ_y and δ_z are the location estimation errors to be determined and

$$f_{i,v} = f_i(x_v, y_v, z_v),$$

$$a_{i,1} = \left.\frac{\partial f_i}{\partial x}\right|_{x_v, y_v, z_v} = \frac{x_1 - x_v}{r_1} - \frac{x_{i+1} - x_v}{r_{i+1}},$$

$$a_{i,2} = \left.\frac{\partial f_i}{\partial y}\right|_{x_v, y_v, z_v} = \frac{x_1 - x_v}{r_1} - \frac{x_{i+1} - x_v}{r_{i+1}},$$

$$a_{i,3} = \frac{\partial f_i}{\partial z}\bigg|_{x_v, y_v, z_v} = \frac{x_1 - x_v}{r_1} - \frac{x_{i+1} - x_v}{r_{i+1}},$$

In which

$$r_i = \sqrt{(x_v - x_i)^2 + (y_v - y_i)^2 + (z_v - z_i)^2},$$

It can be rewritten as (12) but where

$$\mathbf{A} = \begin{bmatrix} a_{1,1} & a_{1,2} & a_{1,3} \\ a_{2,1} & a_{2,2} & a_{2,3} \\ & \cdots & \\ a_{n-1,1} & a_{n-1,2} & a_{n-1,3} \end{bmatrix},$$

$$\boldsymbol{\delta} = [\delta_x \quad \delta_y \quad \delta_z]^T, \mathbf{e} = [\varepsilon_{2,1}, \varepsilon_{3,1}, \cdots, \varepsilon_{n,1}]^T,$$

$$\mathbf{D} = [m_{2,1} - f_{1,v}, m_{3,1} - f_{2,v}, \cdots, m_{n,1} - f_{n,v}]^T.$$

The weight least square estimator of it is denoted as (16).The same as this method for TOA, compute δ with (16) by some initial position guess, and then update the location estimation according to (17).

D. Approximate Maximum Likelihood Method

Let the measured TDOA ranges be

$$\mathbf{m} = [m_{2,1} \quad m_{3,1} \quad \ldots \quad m_{n,1}]^T = \mathbf{d} + \boldsymbol{\varepsilon} \tag{46}$$

where

$$\mathbf{d}(\boldsymbol{\theta}) = [r_{2,1} \quad r_{3,1} \quad \ldots \quad r_{n,1}] \tag{47}$$

The probability density function of **m** given **θ** is (19). The ML estimate is the **θ** that minimizes J [15]. Thus, we obtained that

$$2\mathbf{HD}\boldsymbol{\theta} = \mathbf{H}(\mathbf{v} + 2r_1\mathbf{m}) \tag{48}$$

where

$$\mathbf{H} = \left[\frac{\partial \mathbf{d}(\theta)}{\partial \boldsymbol{\theta}}\right]^{\mathrm{T}} \mathbf{Q}^{-1} diag\left[\frac{1}{r_2 + r_1 + m_{2,1}} \quad \cdots \quad \frac{1}{r_n + r_1 + m_{n,1}}\right],$$

$$\mathbf{D} = -\begin{bmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ & \cdots & \\ x_n - x_1 & y_n - y_1 & z_n - z_1 \end{bmatrix},$$

$$\mathbf{v} = [m_{2,1}{}^2 + k_1 - k_2, \cdots, m_{n,1}{}^2 + k_1 - k_n]^T.$$

Firstly, letting **H** be an identity matrix. From (48), we can get **θ** in terms of r_1. Substituting it into the first equation of (37) deliver the solution of r_1. Then, we obtain the result of **H** by using this (x, y, z). From (48), another **θ** in terms of r_1 is acquired. By repeating of the above procedure with the new estimation several times results several solution. Select the one that gives the smallest J.

The detail of this method can be found in [15] [18].

E. Two-Stage Maximum Likelihood Method

The set (34) can be rewritten as

$$m_{i,1}{}^2 + 2m_{i,1}r_1 = -2(x_i x + y_i y + z_i z) + k_i - k_1 \tag{49}$$

Similarly as in TOA case, let $\mathbf{z}=[x\ y\ z\ s]^{\mathrm{T}}$ be an unknown vector. With TDOA range estimation error, the first cost function can be derived as (24) but where

$$\mathbf{h} = \begin{bmatrix} m_{2,1}{}^2 - k_2 + k_1 \\ m_{3,1}{}^2 - k_3 + k_1 \\ \cdots \\ m_{n,1}{}^2 - k_n + k_1 \end{bmatrix}, \mathbf{G} = -2\begin{bmatrix} x_{21} & y_{21} & z_{21} & m_{2,1} \\ x_{31} & y_{31} & z_{31} & m_{3,1} \\ & \cdots & & \\ x_{n1} & y_{n1} & z_{n1} & m_{n,1} \end{bmatrix}$$

The same as it in case of TOA, φ is found to be (25) but where

$$\mathbf{B} = diag\{r_2 \quad r_3 \quad \ldots \quad r_n\} \tag{50}$$

The covariance matrix of φ is given by (26). Then the first ML estimate of **z** from (24) is described as (27).

The elements of **z** can be denoted as

$$\mathrm{z}_1 = x + e_1,\ \mathrm{z}_2 = y + e_2,\ \mathrm{z}_3 = z + e_3,\ \mathrm{z}_4 = r_1 + e_4, \tag{51}$$

Subtracting the first three elements by x_1, y_1 and z_1, and then squaring the elements, another cost function is derived as (30), but where

$$\mathbf{h}' = \begin{bmatrix} (\mathrm{z}_1 - x_1)^2 \\ (\mathrm{z}_2 - y_1)^2 \\ (\mathrm{z}_3 - z_1)^2 \\ \mathrm{z}_4 \end{bmatrix}, \mathbf{G}' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \mathbf{z}' = \begin{bmatrix} (x - x_1)^2 \\ (y - y_1)^2 \\ (z - z_1)^2 \end{bmatrix}$$

Substituting (51) into (30) gives

$$\varphi_1' = 2(x - x_1)e_1 + e_1{}^2 \approx 2(x - x_1)e_1$$

$$\varphi_2' = 2(y - y_1)e_2 + e_2{}^2 \approx 2(y - y_1)e_2$$

$$\varphi_3' = 2(z - z_1)e_3 + e_3{}^2 \approx 2(z - z_1)e_3$$

$$\varphi_4' = 2r_1 e_4 + e_4{}^2 \approx 2r_1 e_4$$

where the approximation is valid as the errors e_1, e_2, e_3 and e_4 are small.

The covariance matrix of φ' is given by (32), where

$$\mathbf{B}' = diag\{x - x_1 \quad y - y_1 \quad z - z_1 \quad r_1\}$$

Then the ML estimate of **z'** is (30), thus, the final estimation is then obtained by

$$[x \quad y \quad z]^T = [x_1 \quad y_1 \quad z_1]^T \pm \sqrt{\mathbf{z}'} \tag{52}$$

The detail description of this method can be found in [16].

F. Genetic Algorithm Method

Similarly as in TOA case, the fitness function can be given by the function below.

$$f = \sum_{i=2}^{n} \left[\begin{array}{l} \sqrt{(x_i - x)^2 + (y_i - y)^2 + (z_i - z)^2} \\ -\sqrt{(x_1 - x)^2 + (y_1 - y)^2 + (z_1 - z)^2} - m_{i,1} \end{array} \right]^2 \tag{53}$$

The algorithm is to find the generation with a smaller value of f than the initial estimation.

IV. Performance Comparison of the Methods

The location estimation algorithms offer different accuracies and complexities. In this section, we examine the performance of location estimation methods described in this paper.

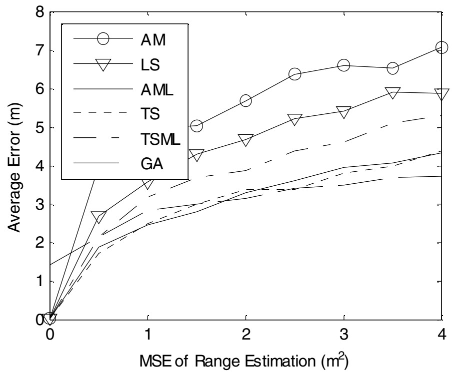

Fig. 1. Average error of TOA based location estimation methods with 5 reference nodes and 500 times' average in a 10m×10m×10m 3D space.

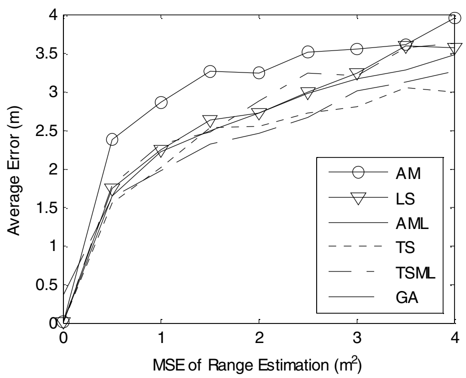

Fig. 2. Average error of the TDOA based location estimation methods with 6 reference nodes and 500 times' average in a 10m*10m*10m 3D space.

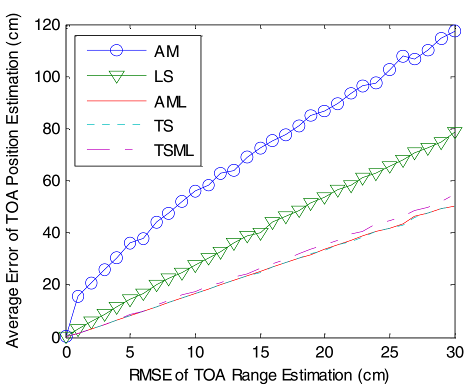

Fig. 3. Average error of TOA based location estimation methods with 5 reference nodes and 10,000 times' average in a 10m×10m×10m 3D space.

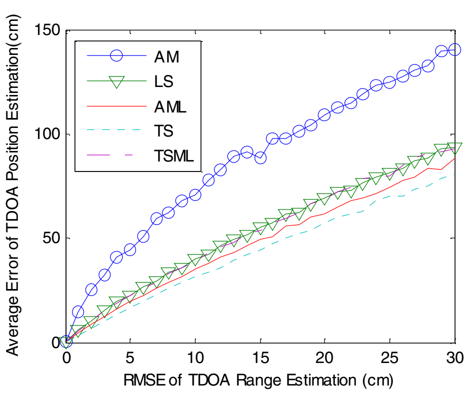

Fig. 4. Average error of TDOA based location estimation methods with 6 reference nodes and 10,000 times' average in a 10m×10m×10m 3D space.

Within the assumed scenario, n (e.g. 5 for TOA and 6 for TDOA) reference nodes and one target node were randomly placed in a 10m×10m×10m 3D space. We know the exact positions of all the sensor nodes including the target node.

At each placement, given the possible root mean squared error (RMSE) of range estimation of the measurement, the possible "measured" TOA and TDOA ranges were produced by using the true ranges.

We did the localization through the positions of the reference nodes and the "measured" ranges by using location estimation methods described in this paper. Then, we examined the localization performance of these methods by comparing the solution of each method with the actual position.

The performance evaluation was performed in terms of two important characteristics, including the average error and the failure rate of the location estimation methods.

The failure solution includes when there is no solution, the solution is unreasonable, the solution is beyond the monitored area, or the method is not converging to a solution (the iterative algorithms). The failure rate was defined as the percentage of the failed times in the total simulation times.

The average error was obtained by averaging all the solutions by the times that we got the solutions.

With 5 reference nodes for TOA and 6 for TDOA, the average errors with 500 times average of each method in this paper are displayed in Fig.1 and Fig.2, of which the mean squared error (MSE) of the range estimation is up to 4 m^2.

From Fig.1 and Fig.2, the average error of GA is the lowest among all the methods when the RMSE of range estimation is bigger than 1m in the assumed scenario. However, in real measurement by using UWB technology, the RMSE of the range estimation is generally not so big. Thus, the GA method is not suitable for the localization in UWB sensor networks.

The following figures in this paper give the performance comparison of the methods in this paper except the GA. The RMSE of range estimation was limit to 30 cm.

The simulation results of the average error are displayed in Fig.3 and Fig.4 with 5 reference nodes for TOA and 6 for TDOA and 10,000 times' average, and the failure rate are displayed in Fig.5 and Fig.6.

We also compared the performance with different number of the reference nodes. Fig. 7 and Fig. 8 present the average error and the failure rate of the TOA based AML method with different numbers of arbitrarily placed reference nodes.

Inside a specific area, it would be interested to know the best method for the location estimation. Therefore, we examine the distribution of the location estimation errors of each method in a specific area with a certain fixed arrangement of the reference nodes.

We produced an L-shaped 2D area in which a number of reference nodes were placed and fixed. The location estimation errors of each point inside the area were estimated by using all the LOS reference nodes to this point. An example contour map of the error distribution is displayed in Fig.9. In this simulation, the Error values and axis description have the same unit. A mean square error of the range estimation was assumed 0.1.

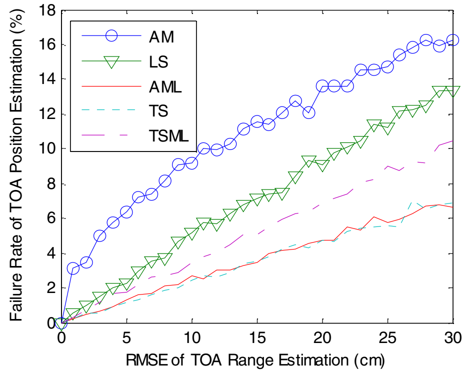

Fig. 5. Failure rate of TOA based location estimation methods with 5 reference nodes of 10,000 times' average in a 10m×10m×10m 3D space.

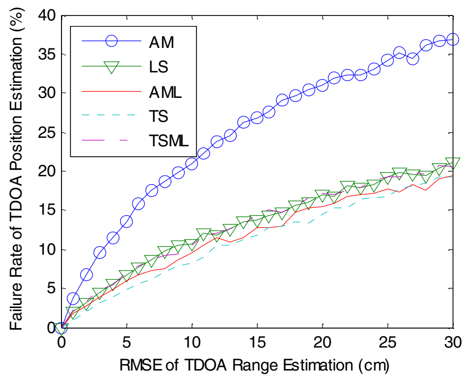

Fig. 6. Failure rate of TDOA based location estimation methods with 6 reference nodes of 10,000 times' average in a 10m×10m×10m 3D space.

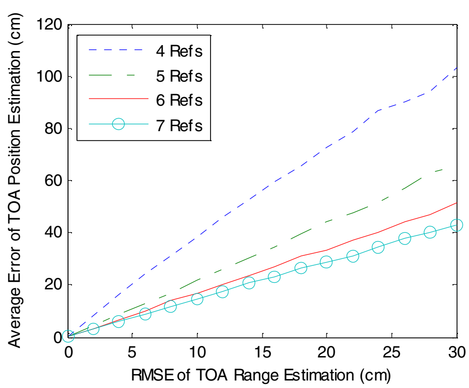

Fig. 7. Average error by using TOA based AML method with different number of reference nodes and 500 times' average in a 10m×10m×10m 3D space.

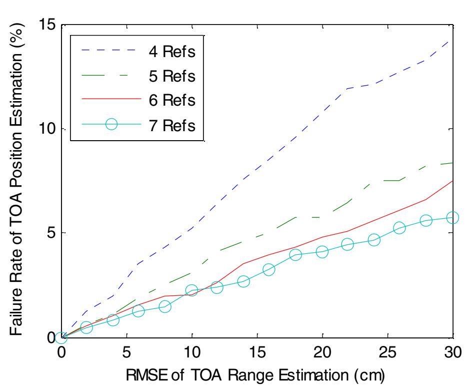

Fig. 8. Failure rate by using TOA based AML method with different number of reference nodes and 500 times' average in a 10m×10m×10m 3D space.

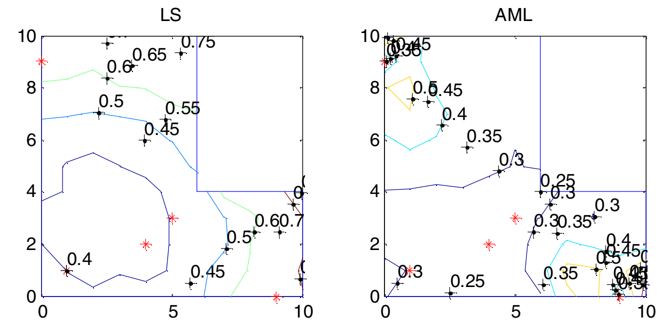

Fig. 9. Distribution of positioning errors across a 2D area of LS and AML method. (The five stars represent the five reference nodes)

From all the simulation results above, we can get some general conclusions as below.

a) For an arbitrary topology of sensor nodes, the AML and the TS method offer accurate location estimation at reasonable noise levels and provide the best tradeoff between average error and failure rate among all the methods in this paper. However, they require a close initial guess to guarantee convergence and they are computational intensive.
b) The simple GA method described in this paper provides an accurate solution when the noise level is high (RMSE of the range estimation is bigger than 1m in a 10m×10m×10m space), so it is not usable for the location estimation by using UWB technology.
c) The TSML method offers a good estimation precision and it need not iterations as AML and TS, but it requires a *prior* information of the approximate location to eliminate the ambiguities.
d) The location estimation performance in terms of the average error and the failure rate improved by decreasing the range estimation error, for all the location estimation algorithms in this paper of both the TOA and TDOA approach. Thus, we should try to get higher precision range estimation to improve the location estimation.
e) The location estimation performance can be improved by increasing the number of reference nodes.
f) The location estimation performance is different with different positions of the target node, although with the same placement of reference nodes within the same area.
g) The location estimation performance is significantly different with a different distribution of sensor nodes although with same number of reference nodes and same range estimation error. Therefore, we should find out some algorithms for an optimum fixed node arrangement and some criterion like GDOP for the quality estimation.

V. Conclusion

In this paper, we studied the location estimation algorithms in distributed UWB sensor networks. We analyzed some typical location estimation algorithms in 3D space, and gave the comparison of them by computer simulation.

Our research showed that there were significant differences in the obtained precession and availability of the location estimates comparing various algorithms. The performance of the algorithms depends on the network topology and the position of the target node. Therefore, in our future work, we will focus on the adaptive selection of the location estimation algorithms and look for suitable quality criteria that will allow optimum selection of the location estimation algorithm for the given network topology and position of the mobile node.

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