DPCM for Quantizated Block-Based Compressed Sensing of Images

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Problem

CS measurement with random projections produces ***dimensionality reduction***—some form of quantization is necessary to yield a ***compressed bitstream***. We propose combining differential pulse-code modulation (DPCM) and uniform scalar quantization (SQ) to block-based CS (BCS) of images.

Basic Concepts

In BCS, image is divided into $B \times B$ blocks; measurement matrix applied to blocks independently:

$$\mathbf{y}^{(j)} = \mathbf{\Phi}_B \mathbf{x}^{(j)}$$

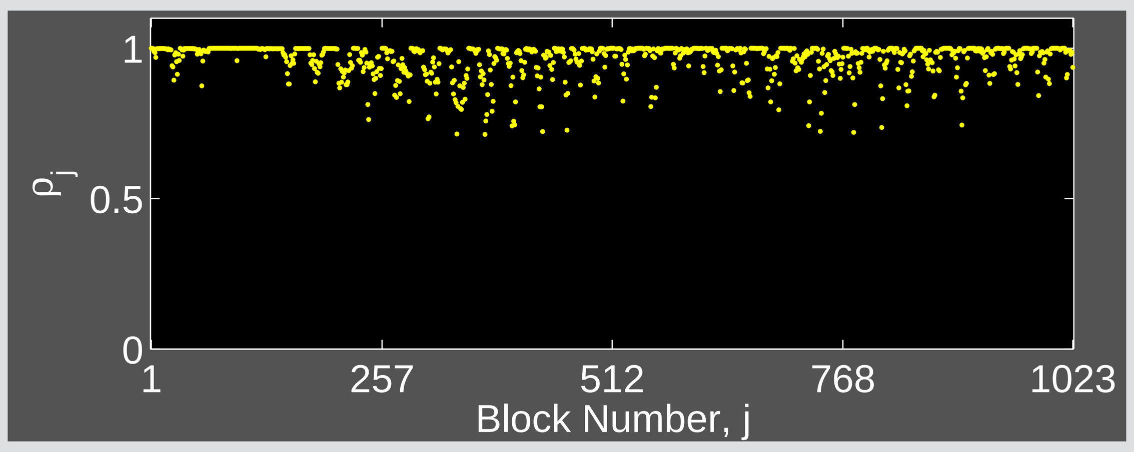
$\mathbf{\Phi}_B$: $M_B \times B^2$ measurement matrix
(subrate: $S = M_B/B^2$)
$\mathbf{x}^{(j)}$: block j of input image $\mathbf{x}$
$\mathbf{y}^{(j)}$: corresponding block j measurements

- **BCS-SPL**: BCS combined with a smoothed PL reconstruction [1]
- **MS-BCS-SPL**: extension to incorporate block-based measurement in the domain of a wavelet transform [2]
- **MH-BCS-SPL**: extension whose reconstruction was driven by the measurement-domain residual resulting from multiple predictions culled from the image to be reconstructed. [3]

Correlation Coefficients of BCS

DPCM works when signals possess high correlation from one time to the next. Such correlation typically exists in image blocks, and random projection in the form of $\mathbf{y}^{(j)} = \mathbf{\Phi}_B \mathbf{x}^{(j)}$ preserves this correlation.

$$\rho_j = \frac{{\mathbf{y}^{(j)}}^T \mathbf{y}^{(j-1)}}{\left\|\mathbf{y}^{(j)}\right\| \left\|\mathbf{y}^{(j-1)}\right\|}.$$



16×16 blocks of Lenna, subrate $= 0.5$
$\bar{\rho} = 0.971$

References

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[2] J. E. Fowler, S. Mun, and E. W. Tramel, “Multiscale block compressed sensing with smoother projected Landweber reconstruction,” in *Proceedings of the European Signal Processing Conference*, Barcelona, Spain, August 2011, pp. 564–568.

[3] C. Chen, E. W. Tramel, and J. E. Fowler, “Compressed-sensing recovery of images and video using multihypothesis predictions,” in *Proceedings of the 45^{th} Asilomar Conference on Signals, Systems, and Computers*, Pacific Grove, CA, November 2011, pp. 1193–1198.

[4] X. Wu, X. Zhang, and J. Wang, “Model-guided adaptive recovery of compressive sensing,” in *Proceedings of the Data Compression Conference*, J. A. Storer and M. W. Marcellin, Eds., Snowbird, UT, March 2009, pp. 123–132.

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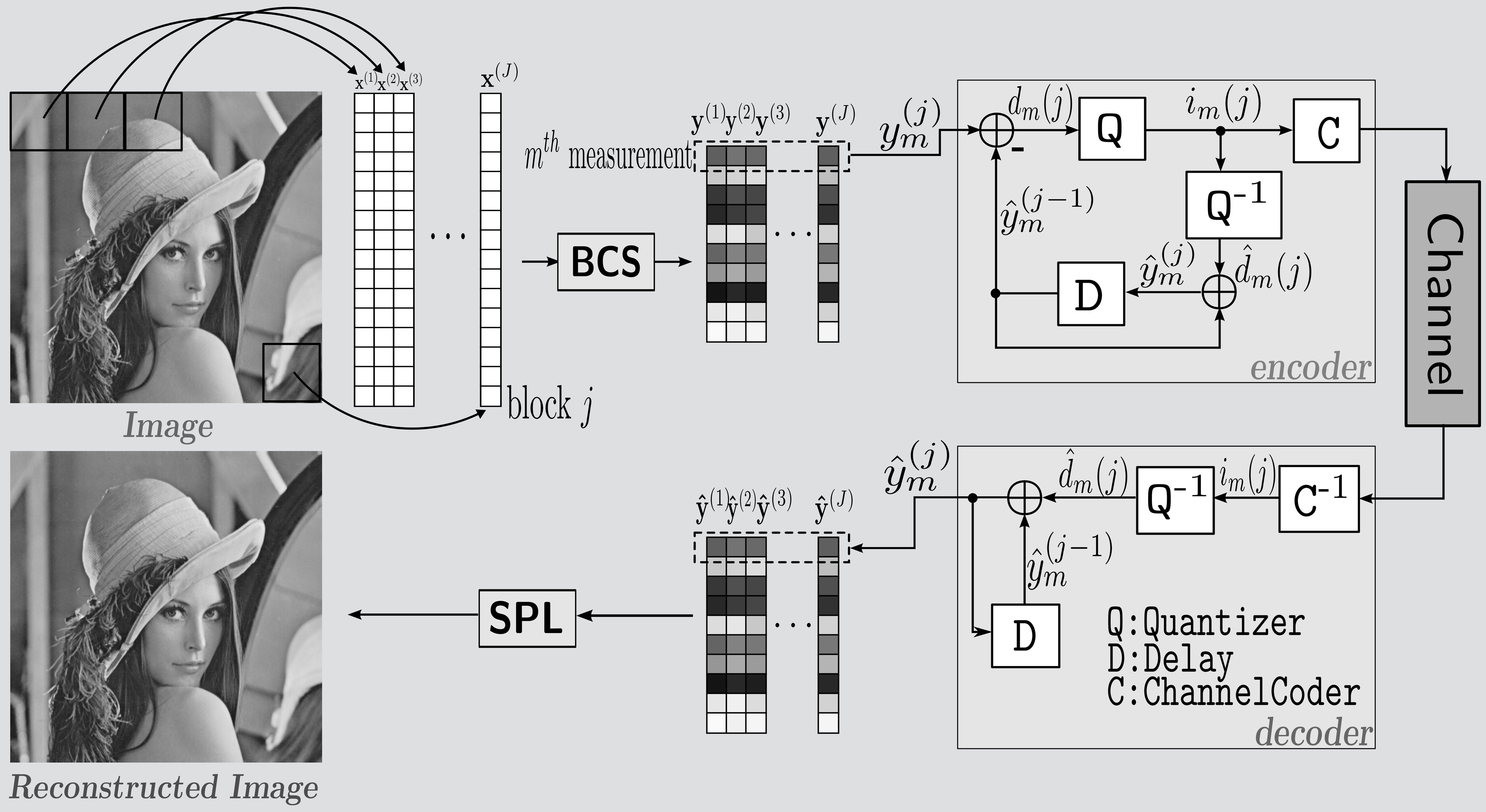
[6] L. Jacques, D. K. Hammond, and J. M. Fadili, “Dequantizing compressed sensing: When oversampling and non-Gaussian constraints combine,” *IEEE Transactions on Information Theory*, vol. 57, no. 1, pp. 559–571, January 2011.

Application of DPCM and SQ to BCS-SPL

Our proposed approach applies DPCM and SQ onto the CS measurements within the BCS-SPL architecture of [1]. On the sensor side of the system, BCS measurements are acquired as usual using $B \times B$ blocks from the original image, producing M-dimensional measurement vector

$$\mathbf{y}^{(j)} = \begin{bmatrix} y_1^{(j)} & \cdots & y_m^{(j)} & \cdots & y_{M_B}^{(j)} \end{bmatrix}^T = \mathbf{\Phi}_B \mathbf{x}^{(j)}$$

for block j of the image, $\mathbf{x}^{(j)}$. For component m in measurement vector $\mathbf{y}^{(j)}$, a prediction is subtracted and the residual is scalar-quantized. Specifically, to predict $y_m^{(j)}$, we use the corresponding vector component of the previously processed block $\hat{\mathbf{y}}^{(j-1)}$. That is, the residual $d_m^{(j)} = y_m^{(j)} - \hat{y}_m^{(j-1)}$ is scalar-quantized to produce quantization index $i_m^{(j)} = Q\left[d_m^{(j)}\right]$ which is then entropy coded. The DPCM feedback loop consists of dequantization of $i_m^{(j)}$, producing the quantized residual, $\hat{d}_m^{(j)} = Q^{-1}\left[i_m^{(j)}\right]$ such that $\hat{y}_m^{(j)} = \hat{d}_m^{(j)} + \hat{y}_m^{(j-1)}$. Finally, the prediction is implemented with a one-block delay buffer. We note that the set of measurements in the first block is processed in the same manner by initializing $\hat{\mathbf{y}}^{(0)}$ to be the zero vector.

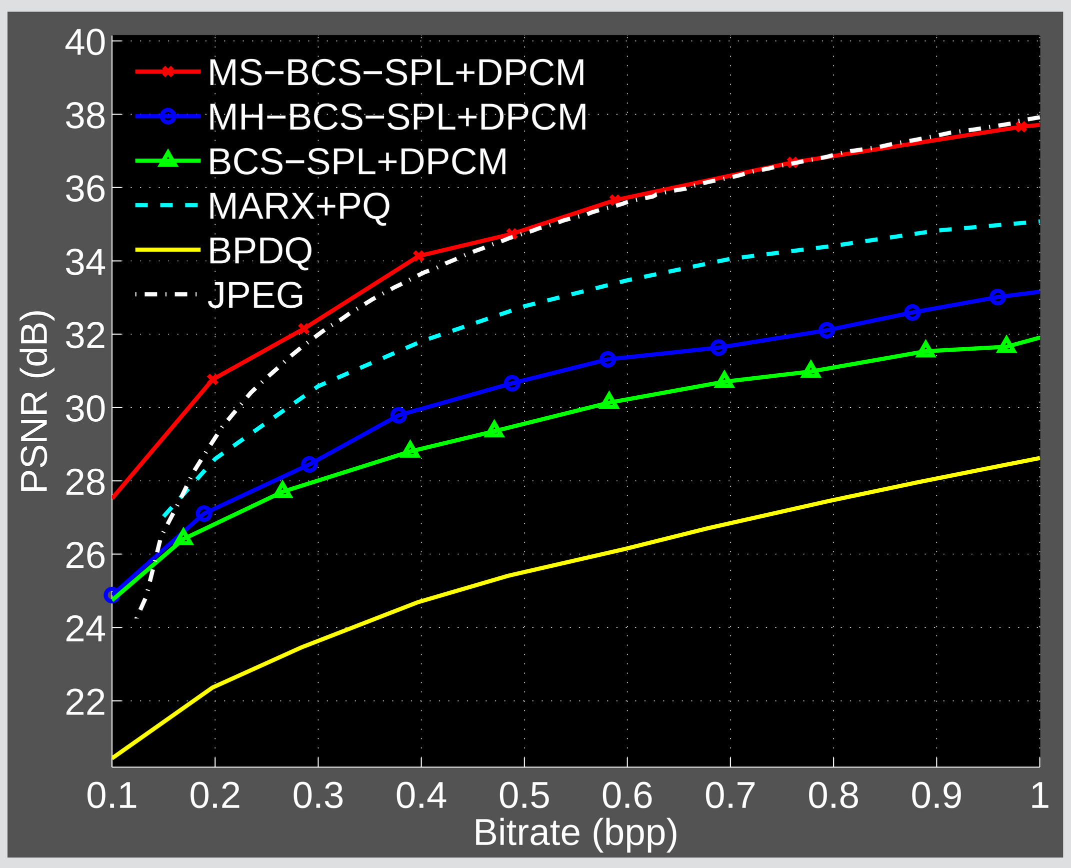


Results

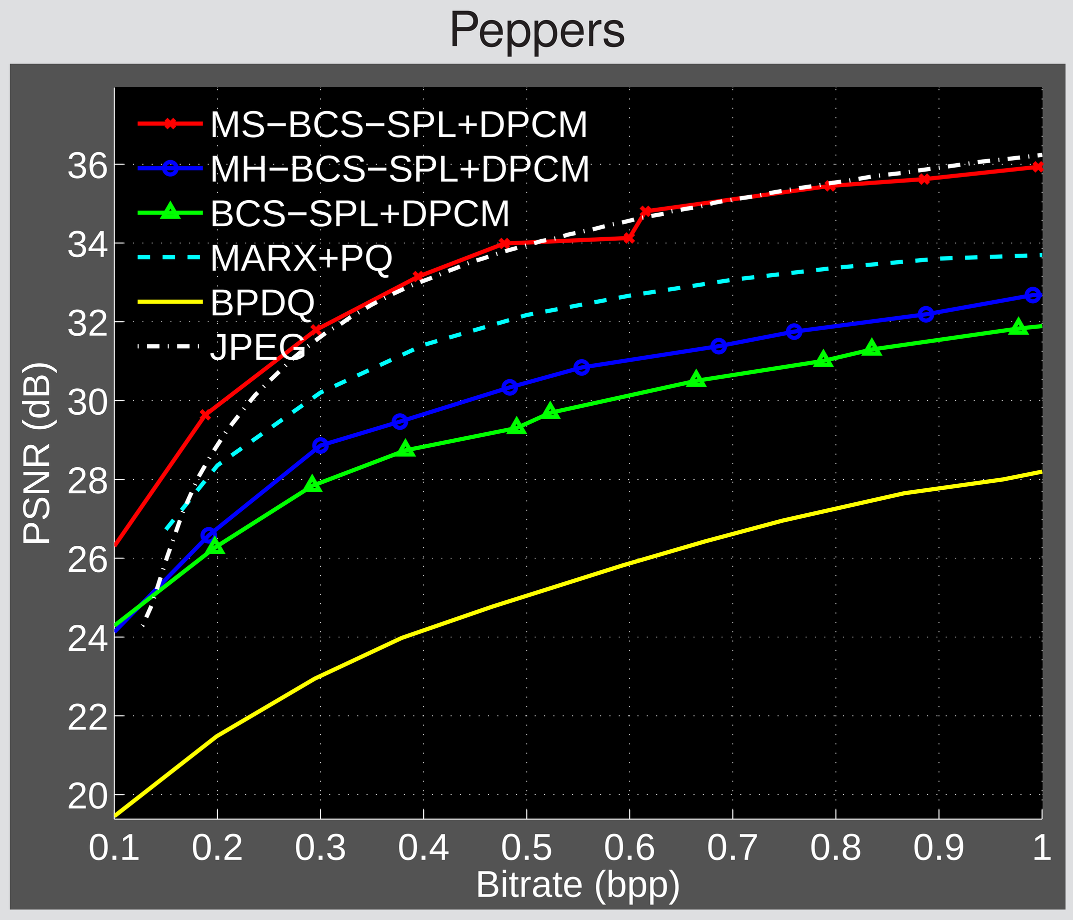
PSNR (dB) for a bitrate of 0.5 bpp

	BCS-SPL [1]			MS-BCS-SPL [2]			MH-BCS-SPL [3]		
Image	SQ	DPCM	*Gain*	SQ	DPCM	*Gain*	SQ	DPCM	*Gain*
Lenna	27.7	29.4	+1.7	33.9	34.7	+0.9	29.2	30.7	+1.5
Barbara	22.9	23.6	+0.7	26.6	27.4	+0.8	27.3	28.2	+0.9
Peppers	28.6	29.5	+0.9	33.8	34	+0.2	29.6	30.3	+0.7
Goldhill	26.7	27.4	+0.7	30.6	31	+0.5	27.0	28.2	+1.2
Man	26.2	26.9	+0.7	30.5	30.7	+0.2	26.5	27.3	+0.8
Clown	26.7	27.6	+0.9	32.7	33.2	+0.5	28.8	29.8	+1.0
Average	26.5	27.4	+0.9	31.3	31.8	+0.5	28.1	29.1	+1.0

Lenna



Peppers



Source Code

http://www.ece.msstate.edu/~fowler/BCSSPL/