1.) SSE

$$e_i = y_i - b_0 + b_1 z_{i1} + b_2 z_{i2}$$

$$SSE = \sum_{i=1}^{N} (y_i - b_0 \mp b_1 z_{i1} \mp b_2 z_{i2})^2$$

2.) $$\frac{\partial SSE}{\partial b_0} = -2 \sum_{i=1}^{N} (y_i - b_0 \mp b_1 z_{i1} \mp b_2 z_{i2})$$

$$\frac{\partial SSE}{\partial b_1} = -2 \sum_{i=1}^{N} (y_i - b_0 \mp b_1 z_{i1} \mp b_2 z_{i2})(z_{i1})$$

$$\frac{\partial SSE}{\partial b_2} = -2 \sum_{i=1}^{N} (y_i - b_0 \mp b_1 z_{i1} \mp b_2 z_{i2})(z_{i2})$$

3.) $$\frac{1}{N} \sum_{i=1}^{N} z_{ij} = 0 \implies \frac{1}{N} \sum_{i=1}^{N} e_i = 0$$

$$\sum_{i=1}^{N} (y_i - b_0 - b_1 x_i) x_i = e \cdot x$$

$$-2 \sum_{i=1}^{2} (y_i - b_0 \mp b_1 z_{i1} \mp b_2 z_{i2}) = e_i = 0$$

$$-2 \sum_{i=1}^{N} (y_i - b_0 \mp b_1 z_{j1} \mp b_2 z_{i2})(z_{i1}) = e_i z_{i1} = 0$$

$$-2 \sum_{i=1}^{N} (y_i - b_0 \mp b_1 z_{j1} \mp b_2 z_{i2})(z_{i2}) = e_i z_{i2} = 0$$

4.) $$\frac{-2 \sum_{i=1}^{N} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2})}{-2} = \frac{0}{-2}$$

$$= \sum_{i=0}^{N} (y_i - b_0 - b_1 z_{i1} - b_2 z_{i2}) = 0$$

$$= \sum_{i=0}^{N} y_i - \sum_{i=0}^{N} b_0 - b_1 \sum_{i=1}^{N} z_{i1} - b_2 \sum z_{i2}$$

$$= \sum_{i=0}^{N} y_i - N b_0 - b_1 \sum_{i=1}^{N} z_{i1} - b_2 \sum_{i=1}^{N} z_{i2}$$

$$= \bar{y} - b_0 - \cancel{\frac{b_1 \sum_{i=1}^{N} z_{i1}}{N}} - \cancel{\frac{b_2 \sum_{i=1}^{N} z_{i2}}{N}} \quad \text{since } \sum_{i=1}^{N} z_{ij} = 0$$

$$b_0 = \bar{y} \implies b_0^* = \bar{y}$$

eliminating b_0^*

$$0 = \sum_{i=1}^{N} (y_i - \bar{y} - b_1 z_{i1} - b_2 z_{i2})(z_{i1})$$

$$= \sum_{i=1}^{N} (z_{i1} y_i - z_{i1}\bar{y} - b_1 z_{i1}^2 - b_2 z_{i2} z_{i1})$$

$$\cancel{= \left(\sum_{i=1}^{N} z_{i1} y_i - \bar{y}\sum_{i=1}^{N} z_{i1} - b_1 \sum_{i=1}^{N} z_{i1}^2 + b_2 \sum_{i=1}^{N} z_{i2} z_{i1}\right) / N}$$

$$= \sum_{i=1}^{N} z_{i1}(y_i - \bar{y}) - b_1 \sum_{i=1}^{N} z_{i1}^2 - b_2 \sum_{i=1}^{N} z_{i2} z_{i1}$$

$$\sum_{i=1}^{N} z_{i1}(y_i - \bar{y}) = b_1 \sum z_{i1}^2 + b_2 \sum_{i}^{N} z_{i2} z_{i1}$$

$$\sum_{i=1}^{N} z_{i1} y_i = b_1 \sum_{i=1}^{N} z_{i1}^2 + b_2 \sum_{i=1}^{N} z_{i2} z_{i1}$$

$$0 = \sum_{i=1}^{N} (y_i - \bar{y} - b_1 z_{i1} - b_2 z_{i2})(z_{i2})$$

$$= \sum_{i=1}^{N} y_i z_{i2} - \bar{y} z_{i2} - b_1 z_{i1} z_{i2} - b_2 z_{i2}^2$$

$$= \sum_{i=1}^{N} z_{i2}(y_i - \bar{y}) - b_1 \sum z_{i1} z_{i2} - b_2 \sum z_{i2}^2$$

$$\sum_{i=1}^{N} z_{i2}(y_i - \bar{y}) = b_1 \sum_{i=1}^{N} z_{i1} z_{i2} + b_2 \sum_{i=1}^{N} z_{i2}^2$$

$$\sum_{i=1}^{N} z_{i2} y_i = b_1 \sum_{i=1}^{N} z_{i1} z_{i2} + b_2 \sum_{i=1}^{N} z_{i2}^2$$

5.) $Ab = C$

$$\begin{bmatrix} \sum z_{i1}^2 & \sum z_{i2} z_{i1} \\ \sum z_{i1} z_{i2} & \sum z_{i2}^2 \end{bmatrix} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} \sum z_{i1} y_i \\ \sum z_{i2} y_i \end{bmatrix}$$

6.) $AB/N = C/N$ where $z_{ij} = x_{ij} - m_j$, $m_j = \bar{x}_j$

$$\overset{A}{\begin{bmatrix} \sum \frac{1}{N}(x_{i1} - m_1)(x_{i1} - m_1) & \frac{1}{N}\sum(x_{i2} - m_2)(x_{i1} - m_1) \\ \sum \frac{1}{N}(x_{i1} - m_1)(x_{i2} - m_2) & \frac{1}{N}\sum(x_{i2} - m_2)(x_{i2} - m_2) \end{bmatrix}} = \begin{bmatrix} var(x_1) & cov(x_1, x_2) \\ cov(x_1, x_2) & var(x_2) \end{bmatrix}$$

$$\overset{C}{\begin{bmatrix} \frac{1}{N}\sum(x_{i1} - m_1)(y_i) \\ \frac{1}{N}\sum(x_{i2} - m_2)(y_i) \end{bmatrix}} = \begin{bmatrix} cov(x_1, y) \\ cov(x_2, y) \end{bmatrix}$$

Dividing by N changes the sums into averages and substituting $z_{ij} = x_{ij} - m_j$ shows that coefficients are determined by covariances & structures.

Matrix A tells us about the relation between predictors while vector c contains the relations between predictors & y.