Digital Image Processing
Assignment 3

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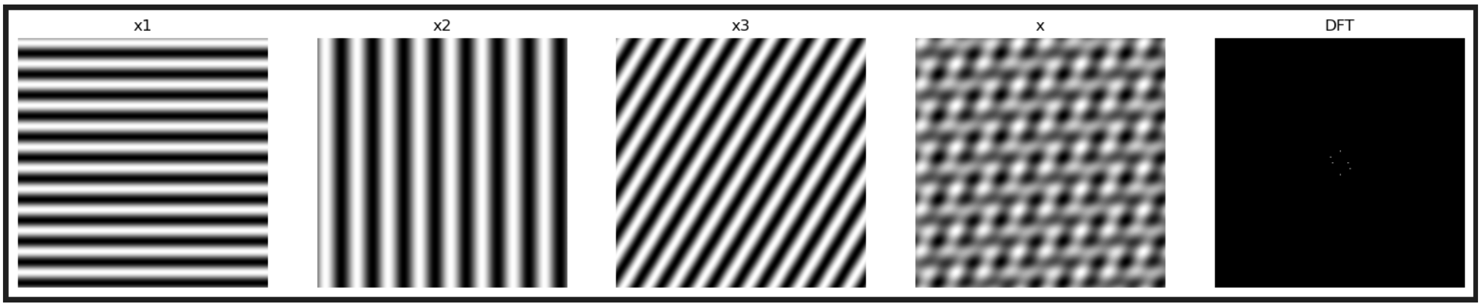
M.Tech SP

Packages used: NumPy, Pillow, Matplotlib

1. Directional Filtering

Three images were created using the sine function with different frequencies, and the fourth image is the average of the three images.

All the operations will be done in the 4th image.
Its DFT has been computed using NumPy (np.fft.fft2), and then it was centred with the help of the NumPy function (np.fft.fftshift). Since the difference between the pixel values was too huge, for visualisation purposes, I used the log function.



Observation:

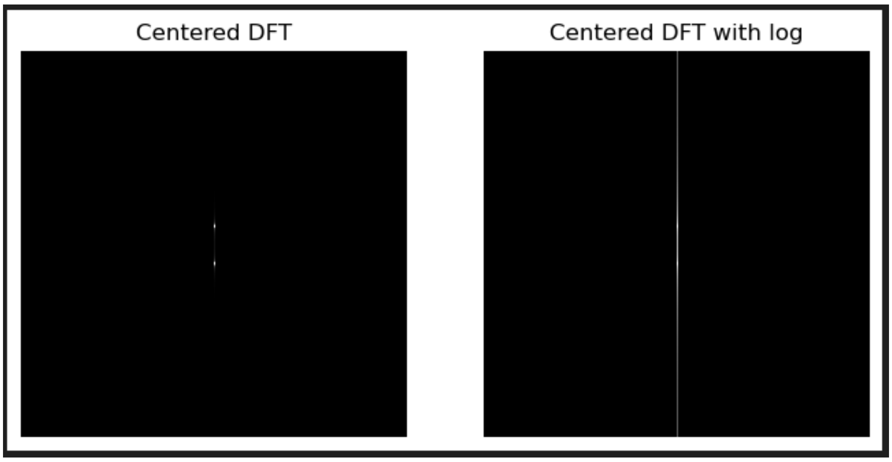
In the image x1, along the rows, the frequency is changing, but it is constant along the columns. Therefore, in the DFT, there are 2 bright spots in the index (12,0) and (-12,0). This index is with respect to the centred DFT. In the uncentred DFT, the index will be (12,0) and (244, 0).

Similarly, frequency associated with the image x2 is in the index (0,8) and (0, -8). And Frequency associated with the image x3, is in the index (6, 10) and (-6, -10).

In the final Image x, all three images' frequencies are present in equal strength as the image x has equal weightage of all three ingredient images. If, say, the weight of x1 is more, then the frequency associated with the image x1 will have more strength and thus, the resultant image will have more intensity for the corresponding frequency.

Note:

What if the frequency was, say 12.5, instead of 12? Then the frequency will get spread to the neighbouring frequencies, with decreasing strength. This is called Spectral Leakage.



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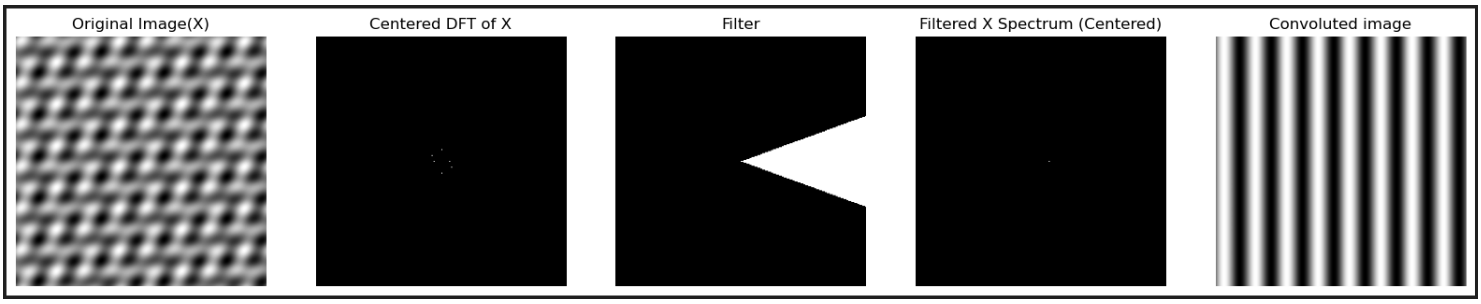
Four Directional Filters are created. The filter will take the (min, max) angle from the centre as thresholds, and for all the frequencies which has an angle outside the threshold range of angles will be filtered out.

a) (-20, 20)
b) (70, 110)
c) (25, 65)

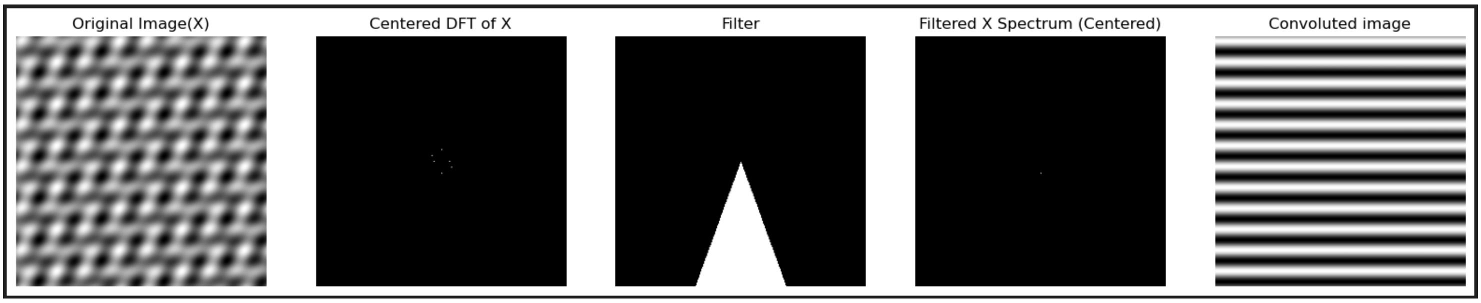
Three filters will have the above threshold angles. Fourth Filter will have the element-wise maximum of these 3 Filters. In other words, the angles that it will not filter out will be the union of the above angles.

a) (-20, 20)



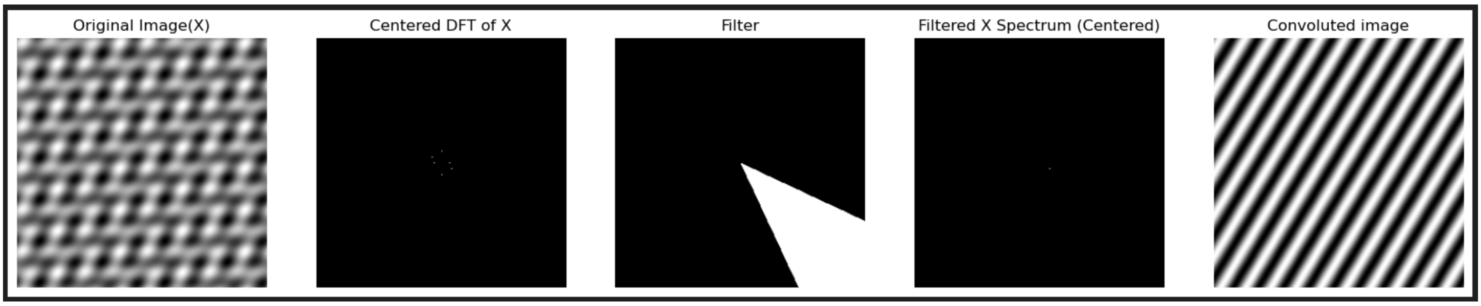
Mean square error between the original image x and convoluted image is: 0.12499999999999994

b) (70, 110)



Mean square error between the image x and the convoluted image is: 0.125

c) (25, 65)



Mean square error between the image x and the convoluted image is: 0.12499999999999993

d) Union(a, b, c)

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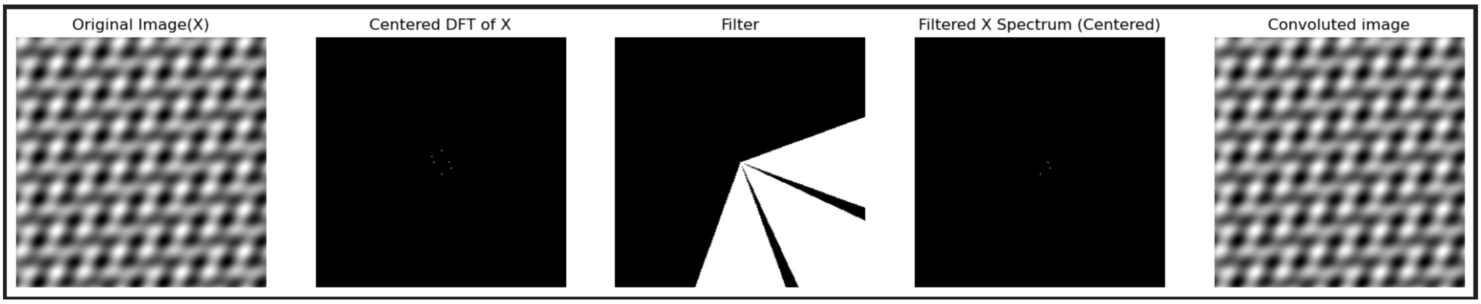


Mean square error between the image x and the convoluted image is: 0.04166666666666665

Observation:

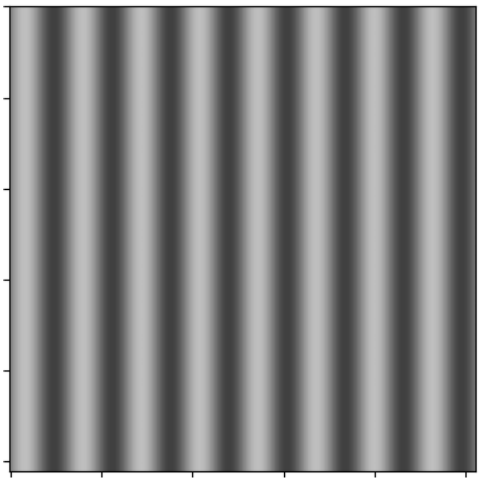
The white area in the Filter H shows that if any frequency is present in that area, Filter H will pass only those frequencies. Mean Square Error shows the element-wise difference between the original Image x and the reconstructed Image after passing it through the filters. It can be observed that the MSE for the first 3 images are high as it only allowing the frequencies on one orientation, whereas in the fourth image, MSE is comparatively less. This is because of the reason written below:

Note:

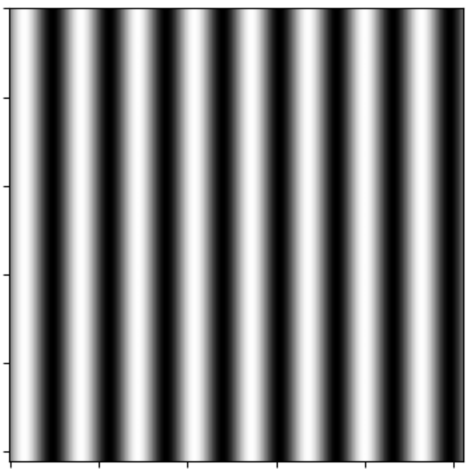
i) To show the reconstructed image, which required the usage of inverse fft. The image will have complex values here, as only one frequency of the 2 frequencies is present. For example, in (b), only the frequency present in index (12, 0) is present, whereas in image x1, there are 2 frequencies with equal strength present in index (12, 0) and (-12, 0). When both of these frequencies are used to reconstruct the image, the complex part gets cancelled, and only the real part is present.

   Now, since one of these frequencies is removed after the filtration, the complex part is present in the reconstructed image. Therefore, usage of np.real(inverse_image) was required to plot only the real values.

ii) Further, the strength of the real values (that is, the brightness) will be half of what is present in the original image. This will not be observed using plt.imshow because of its auto-scaling feature, but with the help of vmin and vmax, it can be shown.



x2 with single freq



x2 with both freq

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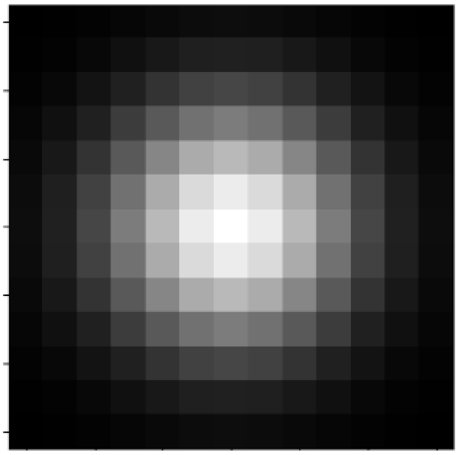
2. Gaussian Blurring and Inverse Filtering

a. Read the image. Design the Gaussian Kernel with SD = 2.5

Original Image (1024x1024)

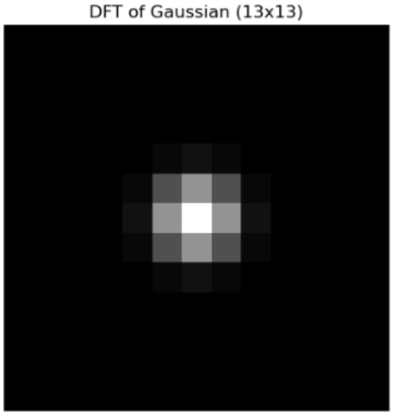


Gaussian Filter (13x13)

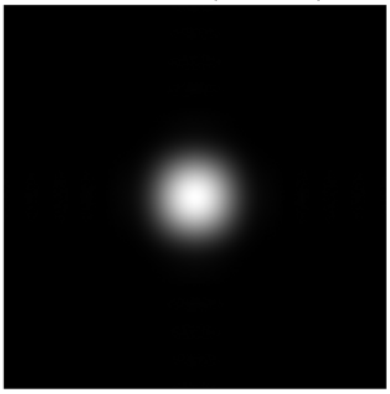
The image will be blurred using the Gaussian filter in the frequency domain. From the Multiplication property, we know that Convolution in the Spatial Domain is Element-wise multiplication in the Frequency domain. For the Element-wise multiplication, the sizes must be the same.

Zero Padding has been done to make them the size of (Image_Size + Kernel_Size – 1), which will be (1036x1036). Padding has been equally on all sides.

DFT of Gaussian (13x13)



DFT of Gaussian (1036x1036)



Note:
Since Gaussian is centred, and DFT assumes the DC component to be at (0,0). Hence, After Padding, it needs to be reverse shifted to make the DC component at (0,0) before taking the DFT of it.

This padded DFT is then multiplied by the DFT of the image to do the convolution in the spatial dimension.

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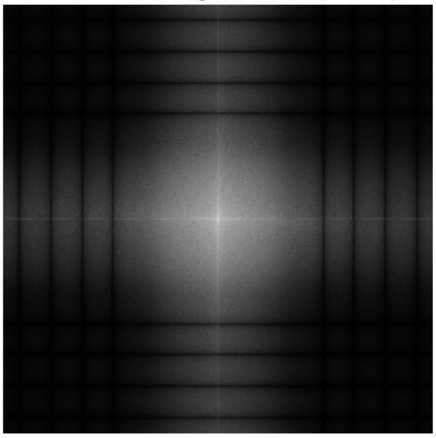
Original image

Convolution using FFT

FFT of Convoluted image Blue Channel (Centered)

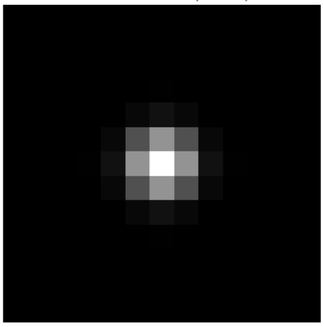


FFT of the convoluted image is for one channel only*.

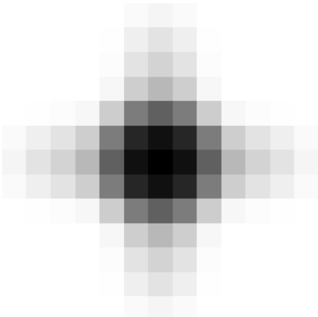
It was for the observation that the lines in the high frequency region shows that those frequencies are absent and the bright + sign in the middle explains the presence of lower frequencies in the image. Thus, the result will be a blurred image.

b. Design Gaussian Frequency Response fit

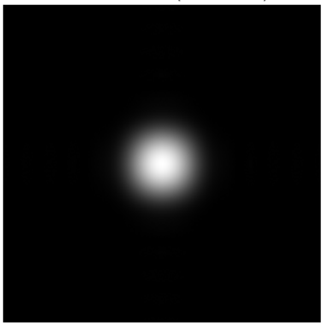
DFT of Gaussian (13x13)



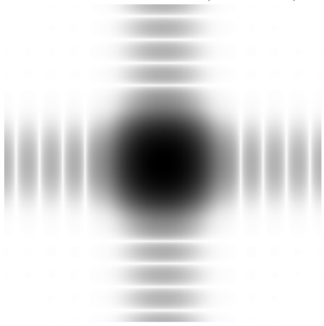
DFT of Inverse Gaussian (13x13)



DFT of Gaussian (1036x1036)



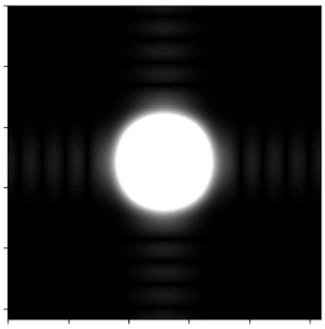
DFT of Inverse Gaussian (1036x1036)



Inverse of DFT is the element wise reciprocal of the Filter.

Note:

In the DFT of inverse Gaussian, there is a ripple like effect, which is not visible in the DFT of Gaussian but it is present there too.



But they are so light that with proper vmin, vmax, it can be seen that similar ripple like effect is there in the DFT of Gaussian as well.

The reason for this ripple effect is convolution with the sinc function. Gaussian is infinitely continuous but to make a filter, we are sliding a window over it as per our requirement. By the Convolution Property, multiplication in spatial

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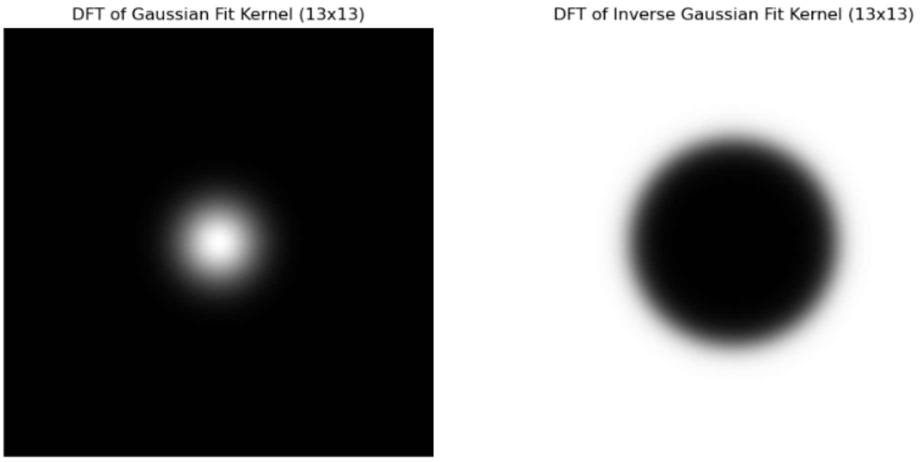
domain is convolution in frequency domain. Therefore, It is a convolution between gaussian and Sinc function and thus, ripple like effect is present.

c. Restore the blurred image

Restoring the blurred image is done by the inverse Filter. The small constant value chosen for mathematical stability of the inverse Filter is 0.001.

Two types of inverse filters are used here :

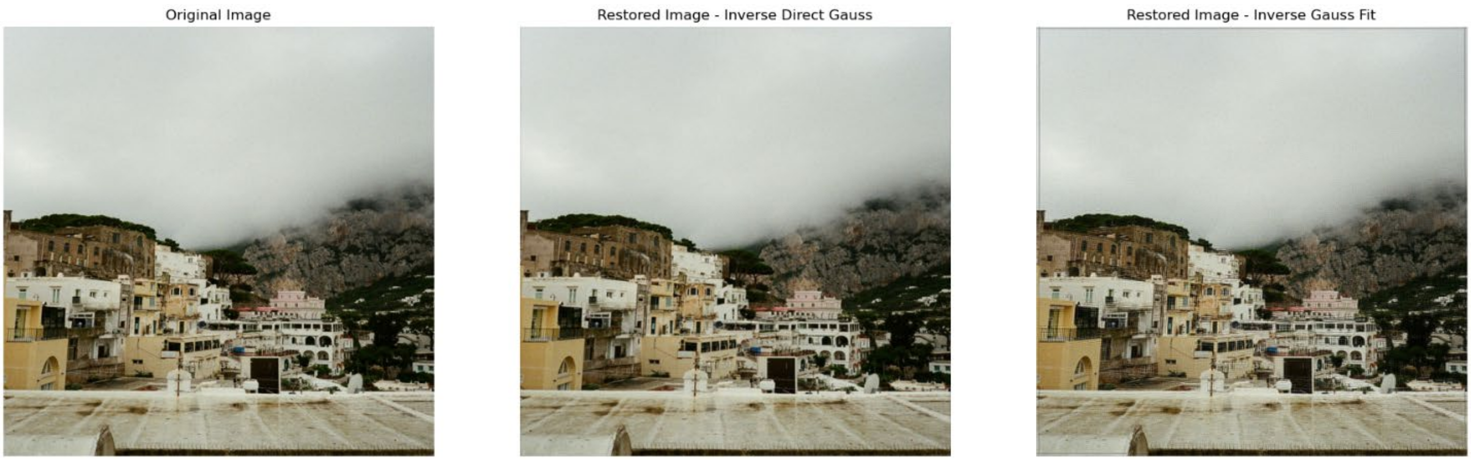
i) Direct Inverse Filter
ii) Gaussian Fit Inverse Filter : A Gaussian Filter that resembles the Direct Inverse Filter as closely as possible without the ripple artefact.



Note that, the Gaussian Fit Inverse Filter here does not have any Ripple Artifact.

Output: K_optimal recorded is: 0.000113
Min_error between the Gaussian Fit and Direct Fit Inverse Filter: 7.6893

Result:



Observation:

In the restored image using the Inverse Gauss Fit Filter, it can be observed that there are some strong border around the edge of the image, which is hardly present in the restored image using Direct Inverse Gauss Filter.

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The reason for this artefact is that the Inverse Direct Gauss Filter has ripple-like artefacts as well, which will cancel the high-frequency artefacts which is mildly present in the blurred image due to ripple-like artefacts present in the Direct Gauss Filter.

On the other hand, since the Inverse Gauss Fit Filter is smooth, it's not able to cancel those high-frequency artefacts. In fact, it amplifies those frequencies as the Inverse filter amplifies the high-frequency components.

Result:

MSE between the original Image and Restored Image using the inverse Direct Gaussian Filter is: 176.90

MSE between the original Image and Restored Image using the Inverse Gaussian Fit Filter is: 243.47