Exercise 9. Answer Sheet

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Problem 1. (40 points) Find an optimal parenthesization of a matrix-chain product whose sequence of dimensions is <5, 10, 3, 12, 5, 50, 6>. Show your work (costs matrix **m,** number of multiplications).

	Rows	Columns
A	5	10
B	10	3
C	3	12
D	12	5
E	5	50
F	50	6

The cost table for the parenthesization of the matrix chain multiplication:

	B	C	D	E	F
A	150 [A][B]	330 [AB][C]	405 [AB][CD]	1655 [ABCD][E]	2010 [AB][CDEF]
B		360 [B][C]	330 [B][CD]	2430 [B][CDE]	1950 [B][CDEF]
C			180 [C][D]	930 [CD][E]	1770 [CD][EF]
D				3000 [D][E]	1860 [D][EF]
E					1500 [E][F]

Therefore, the optimal parenthesization of the matrix chain multiplication is (AB)((CD)(EF)) with the optimal cost 2010

Problem 2. (60 points) Write a program implementing the algorithms `Matrix-Chain-Order` and `Print-Optimal-Parens` given in the lecture. Upload your code. Using your program, find the optimal parenthesization for the following matrix-chain products and show your **m** and **s** matrices.

To compile and run the program, run the folloing command lines:
        g++ -std=c++11 -o matrixChainMultiplication matrixChainMultiplication.cpp
        ./matrixChainMultiplication
Change the value of the initial array to change the input of the program
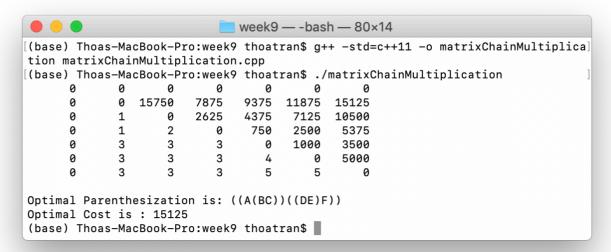In the program,   m[i, j] = m[i,j] in the pseudo code
                m[j,i] = s[i, j] in the pseudo code

The cost table (m and s matrices) for the parenthisization of the matrix chain multiplication:

	B	C	D	E	F
A	15750 [A][B]	7875 [A][BC]	9375 [ABC][D]	11875 [ABC][DE]	15125 [ABC][DEF]
B		2625 [B][C]	4375 [BC][D]	7125 [BC][DE]	10500 [BC][DEF]
C			750 [C][D]	2500 [C][DE]	5375 [C][DEF]
D				1000 [D][E]	3500 [DE][F]
E					5000 [E][F]

When running the program, the output is:

```
                                    week9 — -bash — 80×14
[(base) Thoas-MacBook-Pro:week9 thoatran$ g++ -std=c++11 -o matrixChainMultiplica]
tion matrixChainMultiplication.cpp
[(base) Thoas-MacBook-Pro:week9 thoatran$ ./matrixChainMultiplication          ]
        0       0       0       0       0       0       0
        0       0   15750    7875    9375   11875   15125
        0       1       0    2625    4375    7125   10500
        0       1       2       0     750    2500    5375
        0       3       3       3       0    1000    3500
        0       3       3       3       4       0    5000
        0       3       3       3       5       5       0

Optimal Parenthesization is: ((A(BC))((DE)F))
Optimal Cost is : 15125
(base) Thoas-MacBook-Pro:week9 thoatran$ 
```

Therefore, the optimal parenthesization of the matrix chain multiplication is (A(BC))((DE)F) with the optimal cost is 15125

b) (20 points) p = [10,20,10,15,20,10]

The cost table (m and s matrices) for the parenthisization of the matrix chain multiplication:

	B	C	D	E
A	2000 [A][B]	3500 [AB][C]	6500 [ABC][D]	7500 [AB][CDE]
B		3000 [B][C]	7000 [B][CD]	6500 [B][CDE]
C			3000 [C][D]	4500 [C][DE]
D				3000 [D][E]

When running the program, the output is:

```
● ● ●                    📁 week9 — -bash — 80×14
tion matrixChainMultiplication.cpp
[(base) Thoas-MacBook-Pro:week9 thoatran$ g++ -std=c++11 -o matrixChainMultiplica]
tion matrixChainMultiplication.cpp
[(base) Thoas-MacBook-Pro:week9 thoatran$ ./matrixChainMultiplication            ]
        0       0       0       0       0       0
        0       0    2000    3500    6500    7500
        0       1       0    3000    7000    6500
        0       2       2       0    3000    4500
        0       3       2       3       0    3000
        0       2       2       3       4       0

Optimal Parenthesization is: ((AB)(C(DE)))
Optimal Cost is : 7500
(base) Thoas-MacBook-Pro:week9 thoatran$ █
```

Therefore, the optimal parenthesization of the matrix chain multiplication is: (AB)(C(DE)) with the optimal cost is 7500

c) (20 points) p = [100,10,100,1,1000,100]

The cost table (m and s matrices) for the parenthisization of the matrix chain multiplication:

	B	C	D	E
A	100000 [A][B]	2000 [A][BC]	102000 [ABC][D]	112000 [ABC][DE]
B		1000 [B][C]	11000 [BC][D]	102000 [BC][DE]
C			100000 [C][D]	110000 [C][DE]
D				100000 [D][E]

When running the program, the output is:

```
● ● ●                    📁 week9 — -bash — 80×14
Optimal Cost is : 7500
[(base) Thoas-MacBook-Pro:week9 thoatran$ g++ -std=c++11 -o matrixChainMultiplica]
tion matrixChainMultiplication.cpp
[(base) Thoas-MacBook-Pro:week9 thoatran$ ./matrixChainMultiplication            ]
        0       0       0       0       0       0
        0       0  100000    2000  102000  112000
        0       1       0    1000   11000  102000
        0       1       2       0  100000  110000
        0       3       3       3       0  100000
        0       3       3       3       4       0

Optimal Parenthesization is: ((A(BC))(DE))
Optimal Cost is : 112000
(base) Thoas-MacBook-Pro:week9 thoatran$ █
```

Therefore, the optimal parenthesization of the matrix chain multiplication is (A(BC))(DE) with the optimal cost is 112000